Digital Logic Systems

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1 Lecturer Information

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2 Required Reading

Guy Even and Moti Medina: *Digital Logic Design*

Part I
Introduction to Discrete Math

1 Sets and Functions

Definition 1 (Universal set)**.** The universal set is a set that contains all the possible objects.

Definition 2 (Set)**.** A set is a collection of objects from a universal set.

Definition 3 (Subset)**.** A is a subset of B if every element in A is also an element in B. It is denoted as $A \subseteq B$

Definition 4 (Equal sets)**.** Two sets A and B are said to be equal if $A \subseteq B$ and $B \subseteq A$.

Definition 5 (Strict containment)**.** $A \subsetneq B \iff A \subseteq B$ and $A \neq B$.

Definition 6 (Empty set)**.** The empty set is the set that does not contain any element. It is usually denoted by $\emptyset$.

Definition 7 (Power set)**.** The power set of a set A is the set of all the subsets of A. The power set of A is denoted by $P(A)$ or 2^A.

Definition 8 (Ordered pair)**.** Two objects (possibly equal) with an order (i.e., the first object and the second object) are called an ordered pair.

Definition 9 (Cartesian product)**.** The Cartesian product of the sets A and B is the set

$$A \times B \triangleq \{(a, b) | a \in A \text{ and } b \in B\}$$

Theorem 1 (De Morgan's Laws)**.**

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$
$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

Theorem 2 (Binary relation)**.** *A subset $R \subseteq A \times B$ is called a binary relation.*

Definition 10 (Function)**.** A binary relation $R \subseteq A \times B$ is a function if for every $a \in A$ there exists a unique element $b \in B$ such that $(a, b) \in R$.

Definition 11 (Extension)**.** Let f and g denote two functions. g is an extension of f if $f \subseteq g$, i.e., if every ordered pair in f is also an ordered pair in g.

Definition 12 (Boolean function)**.** A function B : 0, 1n 0, 1k is called a Boolean function.

1.1 Important Boolean Functions

Definition 13 (NOT)**.**

$$\text{NOT}(x) = 1 - x$$

Definition 14 (AND)**.**

$$\text{AND}(x) = x \cdot y$$

Definition 15 (OR)**.**

$$\text{OR}(x) = x + y - (x \cdot y)$$

Definition 16 (XOR)**.**

$$\text{XOR}(x) = (x + y) \bmod 2$$

2 Mathematical Induction

Theorem 3. *For every $n \geq 2$, and for sets $A_1, \ldots, A_n$,*

$$\overline{A_1 \cup \cdots \cup A_n} = \overline{A_1} \cap \cdots \cap \overline{A_n}$$

Proof. If $n = 2$, the statement is true, by De Morgan's Laws.
Let

$$B = A_1 \cup \cdots \cup A_n$$

If possible, let

$$\overline{B} = \overline{A_1} \cap \cdots \cap \overline{A_n}$$

Therefore,

$$\begin{aligned} \overline{A_1 \cup \cdots \cup A_n \cup A_{n+1}} &= \overline{B \cup A_{n+1}} \\ &= \overline{B} \cap \overline{A_{n+1}} \\ &= (\overline{A_1} \cap \cdots \cap \overline{A_n}) \cap \overline{A_{n+1}} \end{aligned}$$

□

Theorem 4 (Pigeon-hole Principle)**.** *If there are n holes and $m > n$ pigeons then there exists at least one hole with more than one pigeon in it. Let $f : A \to \{1, ..., n\}$, and $|A| > n$, then f is not one-to-one, i.e., there are distinct $a_1, a_2 \in A; a_1 \neq a_2$, such that $f(a_1) = f(a_2)$.*