DIGITAL LOGIC SYSTEMS : ASSIGNMENT 3

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Exercise 1.
Suggest an algorithm for computing the binary representation of a number $x \in [0, 2^k - 1]$ using k-bit strings. Your algorithm should compute the representation from the LSB to the MSB.

Solution 1.

Algorithm 1 An algorithm for computing the binary representation of a number using k-bit strings.

1: Let $i = 0$
2: Let $B[(k-1):0] = 0$
3: **while** $i < k$ **do**
4: $\quad B[i] = (x\%2)$
5: $\quad x = \left\lfloor \frac{x}{2} \right\rfloor$
6: $\quad i++$
7: **return** $B[(k-1):0]$

Exercise 2.
This question deals with the conversion of a hexadecimal string to a binary string such that both strings represent the same natural number. Let $H[k-1:0]$ denote a k-digit hexadecimal string. Let $X_H[n-1:0]$ denote an n-bit binary string. Answer the following questions.

1. Define the conversion, i.e., define the binary string X_H as a function of the hexadecimal string H.
2. Let h denote the number represented by the hexadecimal string H. Prove that
$$\langle X_H \rangle = h$$

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Solution 2.

1. Let $\text{hex} - \text{to} - \text{bin}(h)$ be a function defined as

$$\text{hex} - \text{to} - \text{bin}(h) = \begin{cases} 0000 & ; \quad h = 0 \\ 0001 & ; \quad h = 1 \\ 0010 & ; \quad h = 2 \\ 0011 & ; \quad h = 3 \\ 0100 & ; \quad h = 4 \\ 0101 & ; \quad h = 5 \\ 0110 & ; \quad h = 6 \\ 0111 & ; \quad h = 7 \\ 1000 & ; \quad h = 8 \\ 1001 & ; \quad h = 9 \\ 1010 & ; \quad h = A \\ 1011 & ; \quad h = B \\ 1100 & ; \quad h = C \\ 1101 & ; \quad h = D \\ 1110 & ; \quad h = E \\ 1111 & ; \quad h = F \end{cases}$$

   Assuming that X_H and H represent the same number,

$$X_H \left[(n-1) : (n-4)\right] = \text{hex} - \text{to} - \text{bin}\left(H[k-1]\right)$$

2. $\langle X_H \rangle$ is the number represented by X_H. By definition, h is the number represented by H.
   Therefore, by the initial assumptions, X_H and H represent the same number.