## DIGITAL LOGIC SYSTEMS : ASSIGNMENT 1

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**Exercise 1.**

Prove that if the vertices of a directed graph G admit a topological ordering (i.e. $(u, v) \in E$ implies that $\pi(u) < \pi(v)$), then $G$ is acyclic.

**Solution 1.**

If possible, let $G = (V, E)$ be cyclic.
Therefore, after a finite number of runs of the algorithm $TS(V, E)$, there will be a case where there are no sinks. In such a case, the algorithm fails. Therefore, the vertices of $G$ do not assume a topological sorting.
This contradicts the given condition.
Hence, $G$ must be acyclic. □

**Exercise 2.**

Suggest an algorithm that is input a directed graph and outputs whether the graph is acyclic.

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**Algorithm 1** An algorithm that is input a directed graph and outputs whether the graph is acyclic.

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**if** $|V| = 0$ **then**
  let $v \in V$ **and**
  **return** $\pi(v) = 0$ **and**
  **return** acyclic
**else if** $\exists v \in V$, such that $\deg_{\text{out}} = 0$ **then**
  **return** $\left(\text{TS}(V, E)\left(V \setminus v, E \setminus E_v\right)\right)$ extended by $\pi(v) = |V| - 1$.
**else**
  **return** cyclic
**end if**

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*Date*: Tuesday $17^{\text{th}}$ March, 2015.