DIGITAL LOGIC SYSTEMS : ASSIGNMENT 1

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1.4.

1.

$$c(b_1, b_2, b_3) = 1 \iff b_1 + b_2 + b_3 \geq 2$$

Let

$$d = (b_1 \wedge b_2) \vee (b_2 \wedge b_3) \vee (b_1 \wedge b_3)$$

b_1	b_2	b_3	$(b_1 \wedge b_2)$	$(b_2 \wedge b_3)$	$(b_1 \wedge b_3)$	d
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	1	0	1
1	0	0	0	0	0	0
1	0	1	0	0	1	1
1	1	0	1	0	0	1
1	1	1	1	1	1	1

b_1	b_2	b_3	$c(b_1, b_2, b_3)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

The columns of truth values of $(b_1 \wedge b_2) \vee (b_2 \wedge b_3) \vee (b_1 \wedge b_3)$ and $c(b_1, b_2, b_3)$ are identical. Hence,

$$c(b_1, b_2, b_3) = (b_1 \wedge b_2) \vee (b_2 \wedge b_3) \vee (b_1 \wedge b_3)$$

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2.

Let $b_1 = 1$. Therefore,

$$\begin{aligned}(b_1 \wedge b_2) \vee (b_2 \wedge b_3) \vee (b_1 \wedge b_3) &= (1 \wedge b_2) \vee (b_2 \wedge b_3) \vee (1 \wedge b_3) \\ &= b_2 \vee (b_2 \wedge b_3) \vee b_3 \\ &= \big((b_2 \vee b_2) \wedge (b_2 \vee b_3)\big) \vee b_3 \\ &= (b_2 \vee b_2 \vee b_3) \wedge (b_2 \vee b_3 \vee b_3) \\ &= (b_2 \vee b_3) \wedge (b_2 \vee b_3) \\ &= b_2 \vee b_3\end{aligned}$$

3.

Let $b_1 = 0$. Therefore,

$$\begin{aligned}(b_1 \wedge b_2) \vee (b_2 \wedge b_3) \vee (b_1 \wedge b_3) &= (0 \wedge b_2) \vee (b_2 \wedge b_3) \vee (0 \wedge b_3) \\ &= 0 \vee (b_2 \wedge b_3) \vee 0 \\ &= (b_2 \wedge b_3)\end{aligned}$$

1.6.

The addition operator is commutative, and the subtraction operator is not commutative.

+	0	1	2
0	0	1	2
1	1	2	3
2	2	3	4

-	0	1	2
0	0	-1	-2
1	1	0	-1
2	2	1	0

The multiplication table of a commutative operator is symmetric, while the one of a non-commutative operator is asymmetric. This property is sufficient to determine whether an operator is commutative.

1.8.

a	b	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
0	0	0	1	0	0	0	1	1	1
0	1	0	0	1	0	0	1	0	0
1	0	0	0	0	1	0	0	1	0
1	1	0	0	0	0	1	0	0	1

a	b	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}
0	0	0	0	0	1	1	1	0	1
0	1	1	1	0	1	1	0	1	1
1	0	1	0	1	1	0	1	1	1
1	1	0	1	1	0	1	1	1	1

The following functions correspond to the following known functions.

AND	f_5
OR	f_{15}
XOR	f_9
implication	f_{13}
equivalence	f_8
NAND	f_{12}
NOR	f_2

1.12.

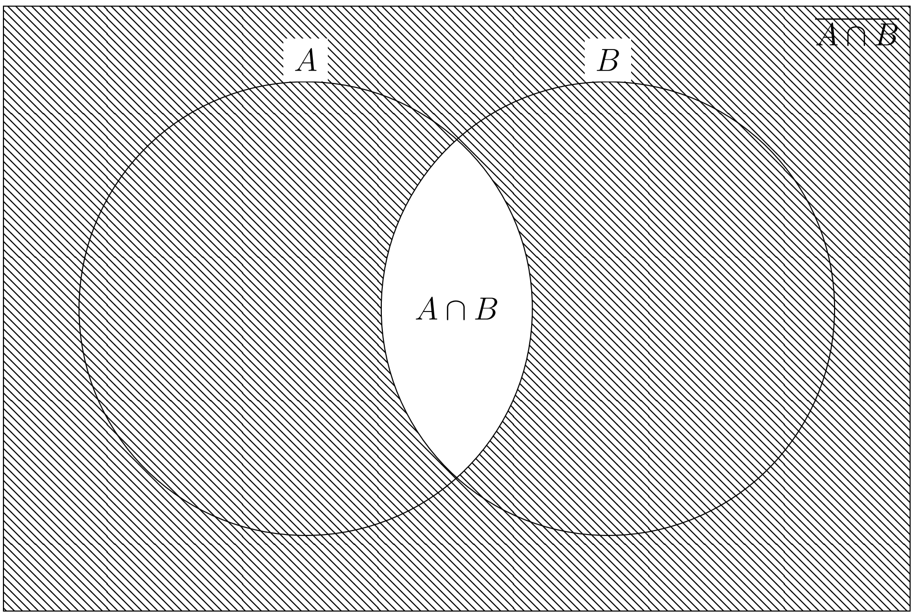

FIGURE 1. $\overline{A \cap B}$

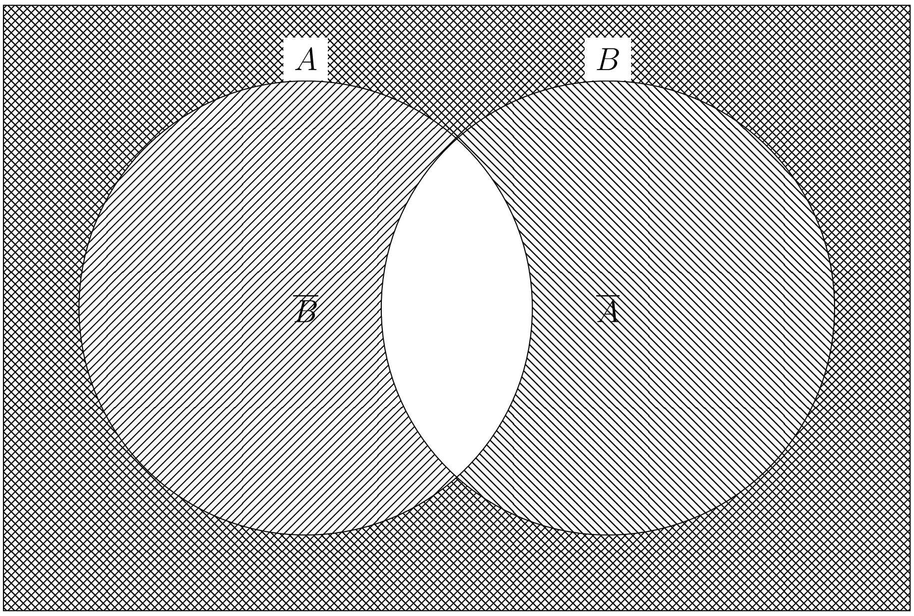

FIGURE 2. $\overline{A} \cup \overline{B}$

1.19.

2.

Let

$$f(x) = x^2$$
$$g(x) = 2x$$

Therefore,

$$\begin{aligned} f\left(g(x)\right) &= (2x)^2 \\ &= 4x^2 \\ g\left(f(x)\right) &= 2x^2 \\ \therefore f(g(x)) &\neq g(f(x)) \end{aligned}$$

Hence, composition is not commutative.

2.5.

$$|\{0,1\}^k| = |\{0,1\} \times \{0,1\}^{k-1}|$$

For any finite sets A, B, $|A \times B| = |A| \cdot |B|$. Therefore,

$$\begin{aligned}
|\{0,1\}^k| &= |\{0,1\}| \cdot |\{0,1\}^{k-1}| \\
&= |\{0,1\}| \cdot |\{0,1\}| \cdot |\{0,1\}^{k-2}| \\
&\ \ \vdots \\
&= \overbrace{|\{0,1\}| \cdot \cdots \cdot |\{0,1\}|}^{k \text{ times}} \\
&= 2^k
\end{aligned}$$

2.6.

Let

$$|A| = n$$

The number of subsets of A is

$$\begin{aligned}
&\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n-1} + \binom{n}{n} \\
&= \sum_{r=0}^{n} \binom{n}{r} \\
&= \sum_{r=0}^{n} \binom{n}{r} 1^r \cdot 1^{n-r} \\
&= (1+1)^n && \text{(by Binomial theorem)} \\
&= 2^n \\
&= 2^{|A|}
\end{aligned}$$

2.7.

Let

$$\begin{aligned}
A &= \{a_1, \ldots, a_m\} \\
B &= \{b_1, \ldots, b_n\}
\end{aligned}$$

Therefore, $f(a_1)$ can be exactly one out of $b_1, \ldots, b_n$. Similarly for all $a_2, \ldots, a_m$.
Therefore, there are n possible combinations for every $a_i \in A$.
Therefore, the total number of possible combinations between A and B are $\overbrace{n \cdot \cdots \cdot n}^{m \text{ times}}$.
Therefore,

$$\begin{aligned}
|F| &= n^m \\
\therefore |F| &= |B|^{|A|}
\end{aligned}$$

$\square$