DIGITAL LOGIC SYSTEMS : ASSIGNMENT 1

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Exercise 1.
Prove that if the vertices of a directed graph G admit a topological ordering (i.e. $(u, v) \in E$ implies that $\pi(u) < \pi(v)$), then G is acyclic.

Solution 1.
If possible, let $G = (V, E)$ be cyclic.
Therefore, after a finite number of runs of the algorithm $TS(V, E)$, there will be a case where there are no sinks. In such a case, the algorithm fails. Therefore, the vertices of G do not assume a topological sorting.
This contradicts the given condition.
Hence, G must be acyclic. □

Exercise 2.
Suggest an algorithm that is input a directed graph and outputs whether the graph is acyclic.

Algorithm 1 An algorithm that is input a directed graph and outputs whether the graph is acyclic.

if $|V| = 0$ **then**
  let $v \in V$ and **return** $\pi(v) = 0$ and **return** acyclic
else if $\exists v \in V$, such that $\deg_{\text{out}} = 0$ **then**
  return $\left(\text{TS}(V, E)\left(V \setminus v, E \setminus E_v\right)\right)$ extended by $\pi(v) = |V| - 1$.
else
  return cyclic

Exercise 3.
Suggest an algorithm that is input a DAG $G = (V, E)$, and outputs a function $s : V \to N$, where $s(v)$ denote the length of a shortest path from a sink in G to v.

Solution 3.

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Algorithm 2 shortest-path-lengths(V, E) - An algorithm for computing the lengths of shortest paths from every node to a sink, in a DAG.

topological sort: $(v_0, \ldots, v_{n-1}) \leftarrow \text{TS}(V, E)$.
for $j = 0$ to $(n-1)$ **do**
  if v_j is a sink **then**
    $d(v_j) \leftarrow 0$
  else
    $d(v_j) \leftarrow 1 + \min\{d(v_i) | i > j \text{ and } (v_i, v_j) \in E\}$.