Harmonic Analysis

Fall Semester

LECTURER

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INSTRUCTOR

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COURSE DESCRIPTION

The aim of this course is to introduce the fundamentals of Harmonic analysis. In particular, we focus on three main subjects: the theory of Fourier series, approximation in Hilbert spaces by a general orthogonal system, and the basics of the theory of Fourier transform.

Harmonic analysis is the study of objects (functions, measures, etc.), defined on different mathematical spaces. Specifically, we study the question of finding the "elementary components" of functions, and how to analyse a given function based on its elementary components. The trigonometric system of cosine and sine functions plays a major role in our presentation of the theory.

The course is intended for undergraduate students of engineering, mathematics and physics, although we deal almost exclusively with aspects of Fourier analysis that are usefull in physics and engeneering rather than those of pure mathematics. We presume knowledge in: linear algebra, calculus, and some acquaintance with the system of complex numbers.

COURSE TOPICS

Week 1: Fourier Series of piecewise continuous functions on a symmetrical segment. Complex and real representations of the Fourier Series.
Week 2: Bessel Inequality, the Riemann-Lebesgue Lemma, partial sums.
Week 3: Convergence theory, the Dirichlet kernel, the Dirichlet Theorem
Week 4: Fourier series on general segments, differentiability and integrability
Week 5: Smoothness and coefficients decay, the Gibbs phenomenon, the Riemann localization principle

Week 6: Inner product spaces, orthonormal bases
Week 7: Cauchy sequences and complete spaces, complete systems, the completeness of the trigonometric system
Week 8: Generalized complete systems, convergence in norm, back to Bessel and the Parseval's equation
Week 9: Hilbert spaces, Banach spaces, best approximation in Hilbert spaces, generalized Phythagoras Theorem
Week 10: Fourier Transform for functions L1, basis properties and convolution
Week 11: The inverse Fourier transform, Plancherel's Theorem
Week 12: The definition of the Fourier transfer in L2, smoothness theorems and the Riemann-Lebesgue for the Fourier transform
Week 13: (As time permits) An introduction to Nyquist-Shannon sampling theorem and ideal low pass filter.

ASSIGNMENTS

75% of all homework assignments must be handed in for evaluation

MIDTERM COURSE POLICY

A midterm exam will be scheduled in the beginning of the semester. During an examination, student shall not use books, papers, or other materials not authorized by the instructor. The midterm will count for 15% of the total course grade but it is magen.

FINAL COURSE POLICY

The final exam will cover the entire course material and will count for 85% of the total course grade (unless the midterm is not counted then it will count for 100%). The duration will be 3 hours. During an examination, student shall not use books, papers, or other materials not authorized by the instructor.
Students will have a first exam, Moed A. If the student does not pass, they can retake the exam, Moed B. The last exam taken will be the student's final grade for the exam.

REQUIRED READING

Folland, G.B.: *Fourier Analysis and its applications*, Wadsworth & Brooks/Cole mathematics series 1992 (available in the library of Exact Sciences & Engineering, location 515:3 FOL).

ADDITIONAL READING

Katznelson, Yitzhak. *An introduction to Harmonic analysis*. Cambridge University Press, 2004. Available online, for example in google books.