Introduction to Electrical Engineering

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Contents

1 Lecturer Information

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2 Required Reading

C.A. Desoer and E.S. Kuh: *Basic Circuit Theory*, Mc-Graw-Hill, International Edition.

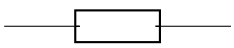
Part I
Basic Definitions and Laws

1 Basic Definitions

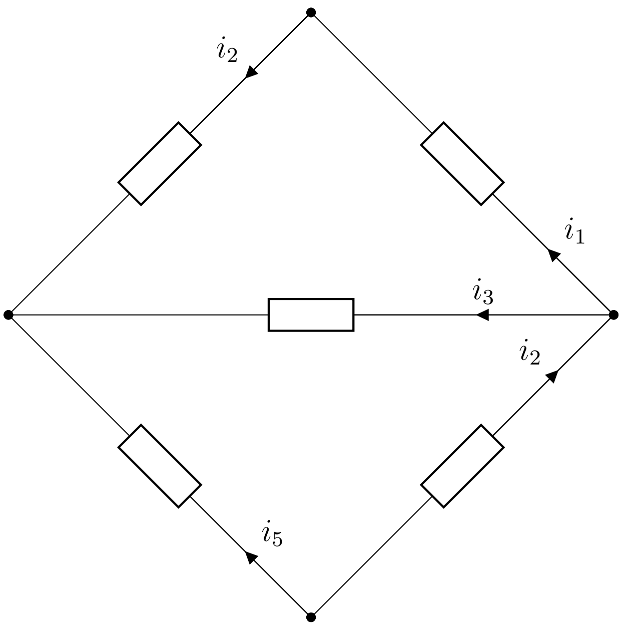
Definition 1 (Electrical circuit)**.** A collection of interconnected components.

Definition 2 (Lumped component)**.** An electrical component whose dimensions are very very small compared to the wavelength of the electromagnetic waves passing through it is called a lumped component.

Definition 3 (One port device)**.** An electrical component with two terminals is called a one port device.



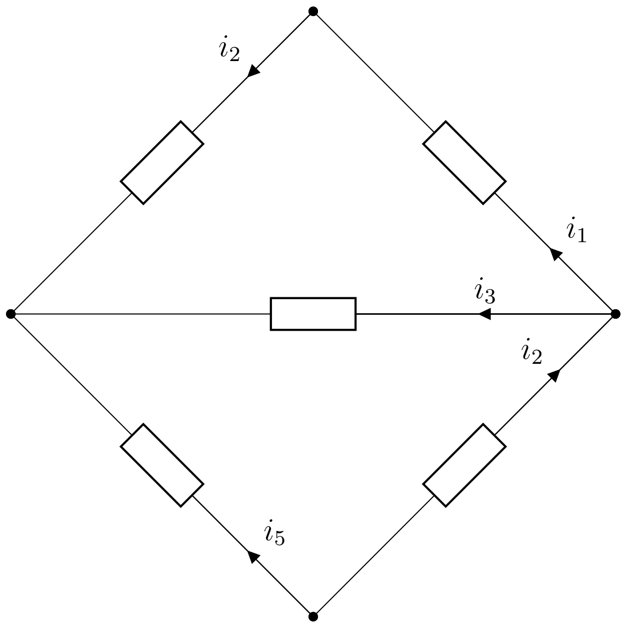
Definition 4 (Nodes and branches)**.** In the figure, all the black dots are called nodes. The parts of the circuit between two nodes are called branches.



2 Kirchoff's Laws

2.1 Kirchoff's Current Law

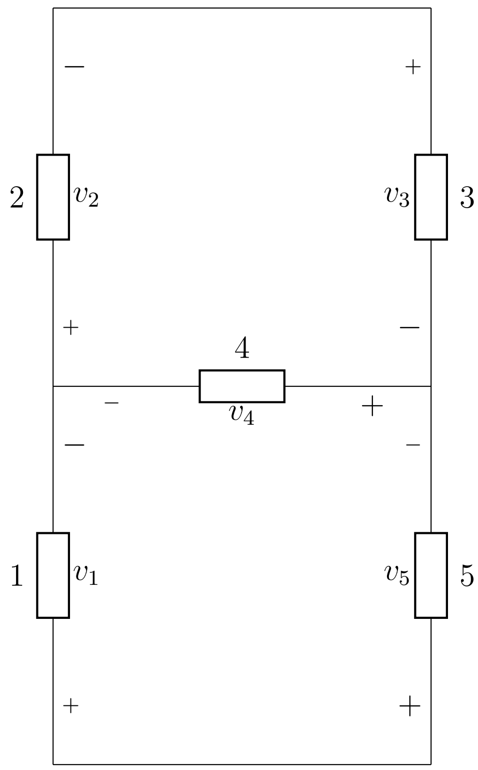
Law 1 (Kirchoff's Current Law)**.** *The sum of all currents entering or exiting a node is zero.*



$i_1 + i_3 - i_4 = 0$

2.2 Kirchoff's Voltage Law

Law 2 (Kirchoff's Voltage Law)**.** *The sum of all branch voltages along a closed loop is zero.*



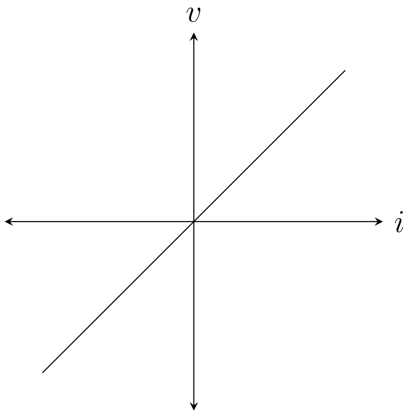
$$
\begin{aligned}
v_1 - v_4 - v_5 &= 0 \\
v_2 + v_3 + v_4 &= 0 \\
v_1 + v_2 + v_3 - v_5 &= 0
\end{aligned}
$$

3 Components

3.1 Resistors

Definition 5 (Resistor)**.** A two terminal component is called a resistor if the voltage across it at any given time t is a function of the current at the same time t.

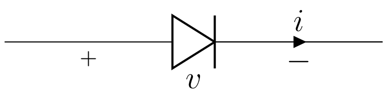
3.1.1 Linear Time Independent Resistor



$$
\begin{aligned}
v(t) &= R \cdot i(t) \\
i(t) &= G \cdot v(t)
\end{aligned}
$$

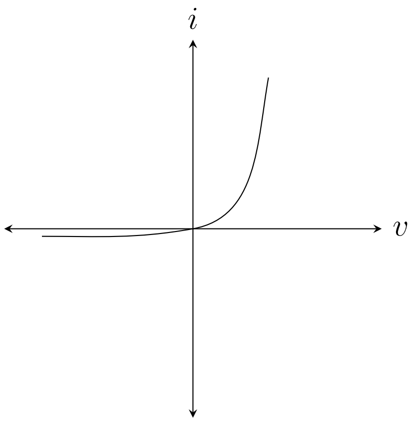
R is called the resistance and G is called the conductance.

3.1.2 Non-linear Resistors (Diodes)

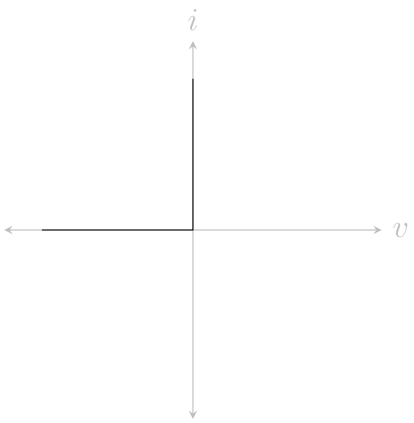


$$
i(t) = I_s \left(e^{\dfrac{q \cdot v(t)}{kT}} - 1 \right)
$$

$$
\begin{aligned}
I_s &= \text{reverse current} \\
k &= \text{Boltzman constant} \\
T &= a\text{bsolute temperature} \\
q &= \text{electronic change} \\
\frac{kT}{q} &= 0.026(\text{ at 300K})
\end{aligned}
$$



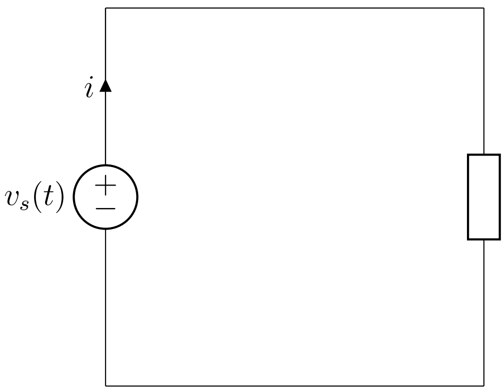
An ideal diode has a i-v graph like

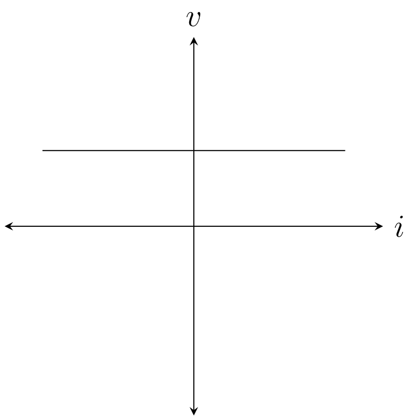


3.2 Independent Sources

3.2.1 Voltage Sources

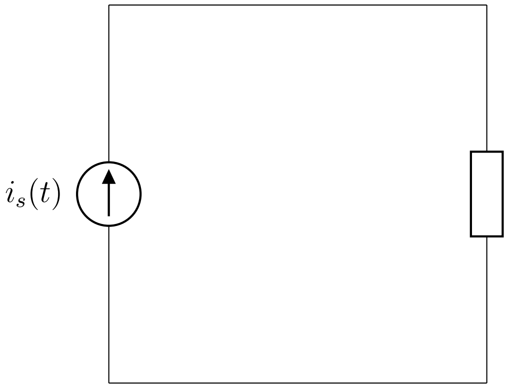
Definition 6 (Voltage source)**.** A two terminal component is called a voltage source if the voltage on its terminals is independent of the current through it.

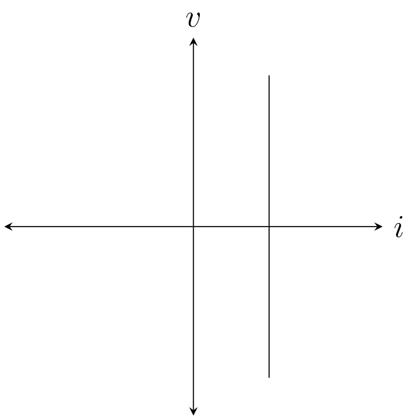




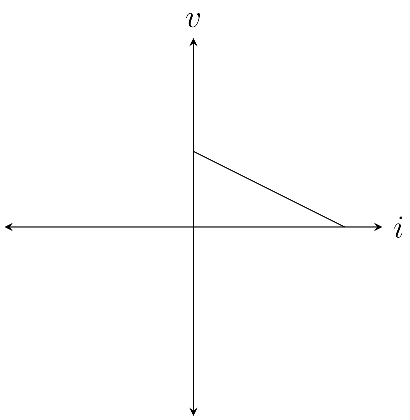
3.2.2 Current Sources

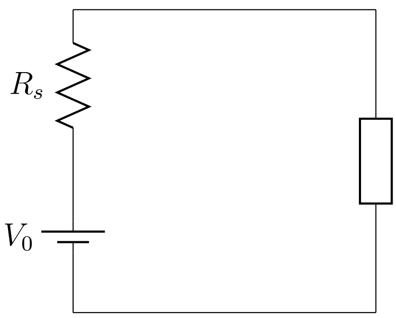
Definition 7 (Current source)**.** A two terminal component is called a current source if it can supply a current $i_s(t)$ independent of the voltage across its terminals.





3.2.3 Real Batteries

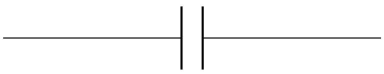




$$
\begin{aligned}
0 &= -V_0 + v_R + v \\
v &= V_0 - v_R \\
\therefore v &= V_0 - R_s i
\end{aligned}
$$

3.3 Capacitor

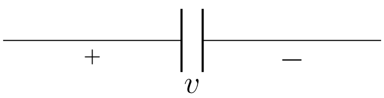
Definition 8 (Capacitor)**.** A capacitor is a two terminal device where V is a fucntion of q.



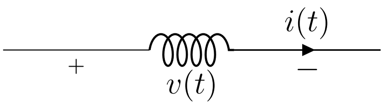
3.3.1 Linear Capacitors

If the charges on the terminals of a capacitor are $+q$ and $-q$, and the potential difference across it is v, the ratio between q and v is said to be the capacitance. The unit of capacitance is farad or F.

$$\begin{aligned} q &= Cv \\ \therefore i &= C\,\frac{\mathrm{d}v}{\mathrm{d}t} \\ \therefore v(t) &= v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(t)\,\mathrm{d}t \end{aligned}$$



3.4 Inductor

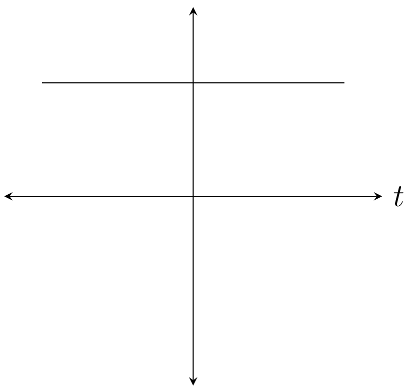


Definition 9 (Inductor)**.**

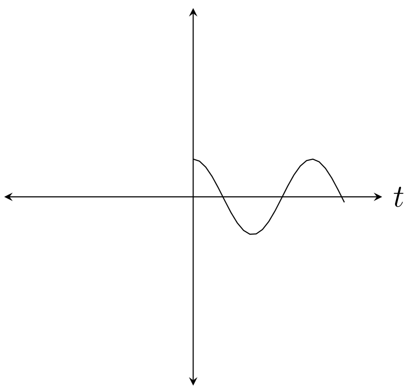
$$\begin{aligned} v(t) &= \frac{\mathrm{d}\varphi}{\mathrm{d}t} \\ \therefore v(t) &= L\,\frac{\mathrm{d}i}{\mathrm{d}t} \\ \therefore i(t) &= i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(t)\,\mathrm{d}t \end{aligned}$$

4 Waveforms

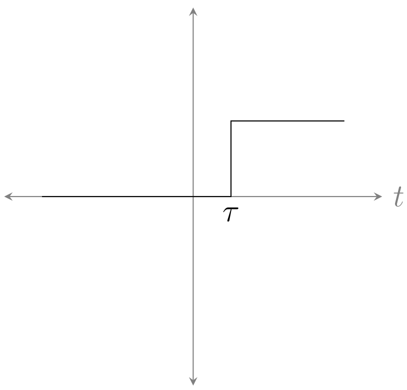
4.1 DC (Constant Function)



4.2 Sinusoidal Wave

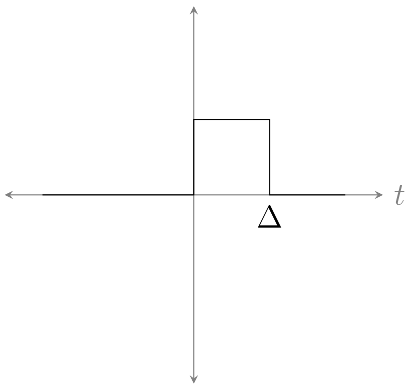


4.3 Step Function



$$u(t) = \begin{cases} 0 & ; \quad t < 0 \\ \frac{1}{2} & ; \quad t = \tau \\ 1 & ; \quad t > \tau \end{cases}$$

4.4 Rectangular Pulse



$$P_\Delta(t) = \frac{u(t) - u(t-\Delta)}{\Delta}$$

$$P_\Delta(t) = \begin{cases} 0 & ; \quad t < 0 \\ \frac{1}{\Delta} & ; \quad t = 0 \\ 1 & ; \quad t > 0 \end{cases}$$

4.5 Dirac δ function

$$\delta(t) = \lim_{\Delta \to 0} P_\Delta(t)$$

$$S(\Delta) = \int_{-\infty}^{\infty} P_\Delta(t) f(t) \, \mathrm{d}t$$

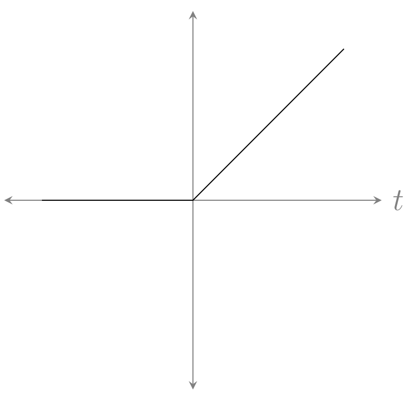
As $\Delta \to 0$,

$$
\begin{aligned}
S(\Delta) &= \int_{-\infty}^{\infty} P_\Delta(t) f(0)\,\mathrm{d}t \\
&= f(0) \int_{-\infty}^{\infty} P_0(t)\,\mathrm{d}t \\
&= f(0)
\end{aligned}
$$

$$
\delta(t) = \begin{cases} 0 & ; \quad t \neq 0 \\ \infty & ; \quad t = 0 \end{cases}
$$

$$
\begin{aligned}
\int_{-\infty}^{\infty} \delta(t) f(t)\,\mathrm{d}t &= f(0) \\
\int_{-\infty}^{\infty} \delta(t-\tau) f(t)\,\mathrm{d}t &= f(\tau) \\
\int_{-\infty}^{\infty} \delta(at) f(t)\,\mathrm{d}t &= \frac{1}{|a|}\delta(t)
\end{aligned}
$$

4.6 Ramp Function



$$r(t) = tu(t)$$

4.7 Doublet Function

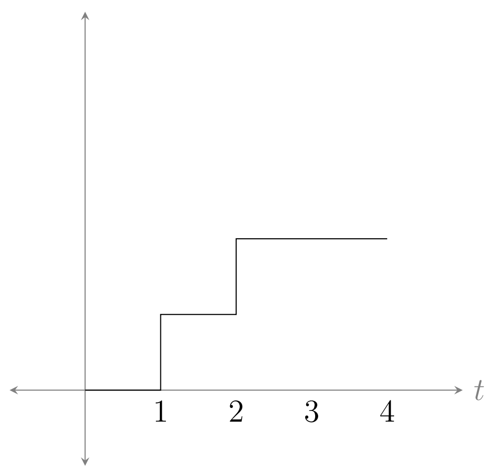
$$\delta'(t) = \frac{\mathrm{d}\delta(t)}{\mathrm{d}t}$$

4.8 Relation Between Standard Waveforms

$$r(t) \underset{\int_{-\infty}^{t}}{\overset{\frac{\mathrm{d}}{\mathrm{d}t}}{\rightleftharpoons}} u(t) \underset{\int_{-\infty}^{t}}{\overset{\frac{\mathrm{d}}{\mathrm{d}t}}{\rightleftharpoons}} \delta(t) \underset{\int_{-\infty}^{t}}{\overset{\frac{\mathrm{d}}{\mathrm{d}t}}{\rightleftharpoons}} \delta'(t)$$

Exercise 1.

Express the following wave as a sum of standard waveforms.

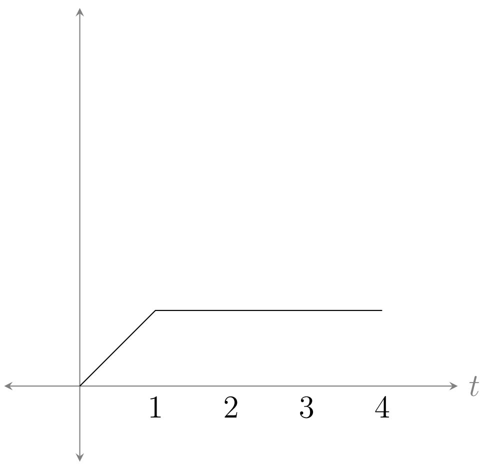


Solution 1.

$$f(t) = u(t-1) + u(t-2)$$

Exercise 2.

Express the following wave as a sum of standard waveforms.



Solution 2.

$$f(t) = r(t) + r(t-1)$$

5 Power and Energy

The instantaneous power supplied to a load is

$$P(t) = v(t) \cdot i(t)$$

where $v(t)$ and $i(t)$ are in matched directions.

The energy supplied to a load from time t_0 to time t is

$$\begin{aligned} W(t_0, t) &= \int_{t_0}^{t} P(t)\,\mathrm{d}t \\ &= \int_{t_0}^{t} v(t) \cdot i(t)\,\mathrm{d}t \end{aligned}$$

5.1 Energy Stored in a Capacitor

$$W(t_0, t) = \int_{t_0}^{t} v(t)i(t)\,\mathrm{d}t$$

As $i(t) = \dfrac{\mathrm{d}q}{\mathrm{d}t}$, $\mathrm{d}q = i(t)\,\mathrm{d}t$. Therefore,

$$\begin{aligned}
W(t_0, t) &= \int\limits_{q(t_0)}^{q(t)} v(q)\,\mathrm{d}q \\
&= \int\limits_{q(t_0)}^{q(t)} \frac{q}{C}\,\mathrm{d}q \\
&= \frac{q^2}{2c} \\
&= \frac{1}{2}cv^2
\end{aligned}$$

5.2 Energy Stored in an Inductor

$$W(t_0, t) = \int\limits_{t_0}^{t} v(t)i(t)\,\mathrm{d}t$$

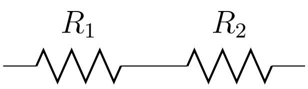
As $v(t) = \dfrac{\mathrm{d}\varphi}{\mathrm{d}t}$, $\mathrm{d}\varphi = v(t)\,\mathrm{d}t$. Therefore,

$$\begin{aligned}
W(t_0, t) &= \int\limits_{\varphi(t_0)}^{\varphi(t)} i(\varphi)\,\mathrm{d}\varphi \\
&= \frac{\varphi^2}{2L} \\
&= \frac{1}{2}Li^2
\end{aligned}$$

Part II
Simple Circuits

1 Equivalent Circuits

Definition 10. Two circuits are said to be equivalent if they have the same $v(t)$-$i(t)$ relationships.



Example 1. Let the voltage across R_1 be v_1 and across R_2 be v_2.
Let the current through R_1 be i_1 and through R_2 be i_2.
Therefore,

$$\begin{aligned} v_1 &= f_1(i_1) \\ v_2 &= f_2(i_2) \end{aligned}$$

Therefore,

$$\begin{aligned} v &= v_1 + v_2 \\ &= f_1(i_1) + f_2(i_2) \\ &= f_1(i) + f_2(i) \\ &= f_3(i) \end{aligned}$$

Therefore, the system of resistors is equivalent to a single resistor R_3.

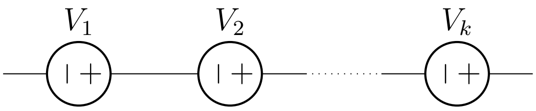
1.1 Series Connections

If circuit elements are connected in series, by Kirchoff's Current Law, the current passing through all of them is equal. By Kirchoff's Voltage Law, the net potential difference is the sum of the voltages across each of them.

1.1.1 Resistors in Series

$$
\begin{aligned}
v &= \sum_{k=1}^{m} v_k \\
&= \sum_{k=1}^{m} R_k i_k \\
&= \sum_{k=1}^{m} R_k i \\
&= i \sum_{k=1}^{m} R_k \\
\therefore v &= Ri \\
\therefore R &= \sum_{k=1}^{m} R_k
\end{aligned}
$$

1.1.2 Voltage Sources in Series



By Kirchoff's Voltage Law,

$$
v = \sum_{k=1}^{m} v_k
$$

1.1.3 Half Wave Rectifier

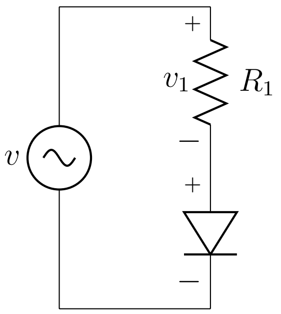
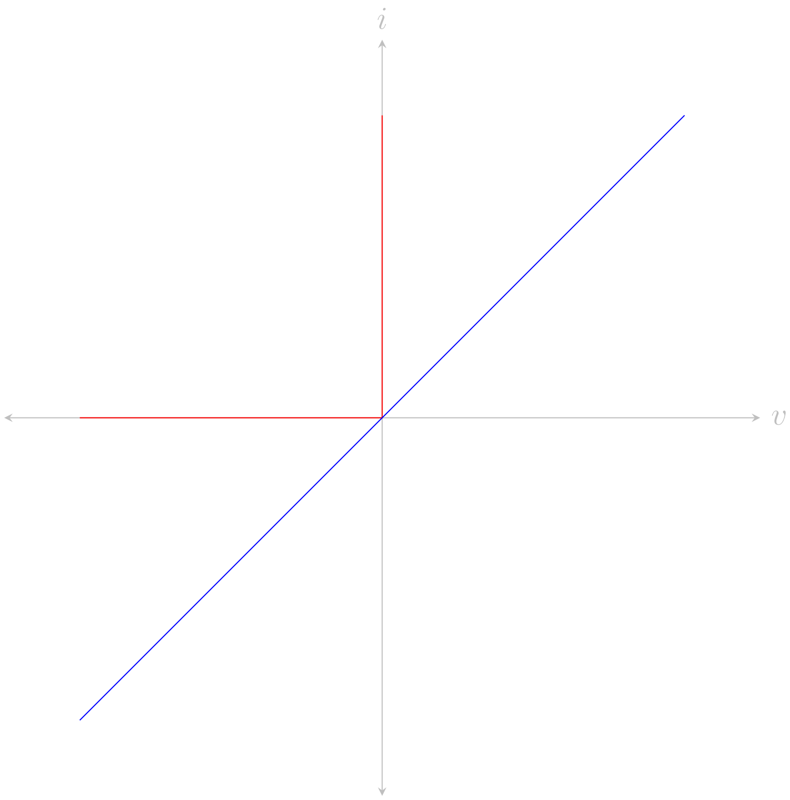
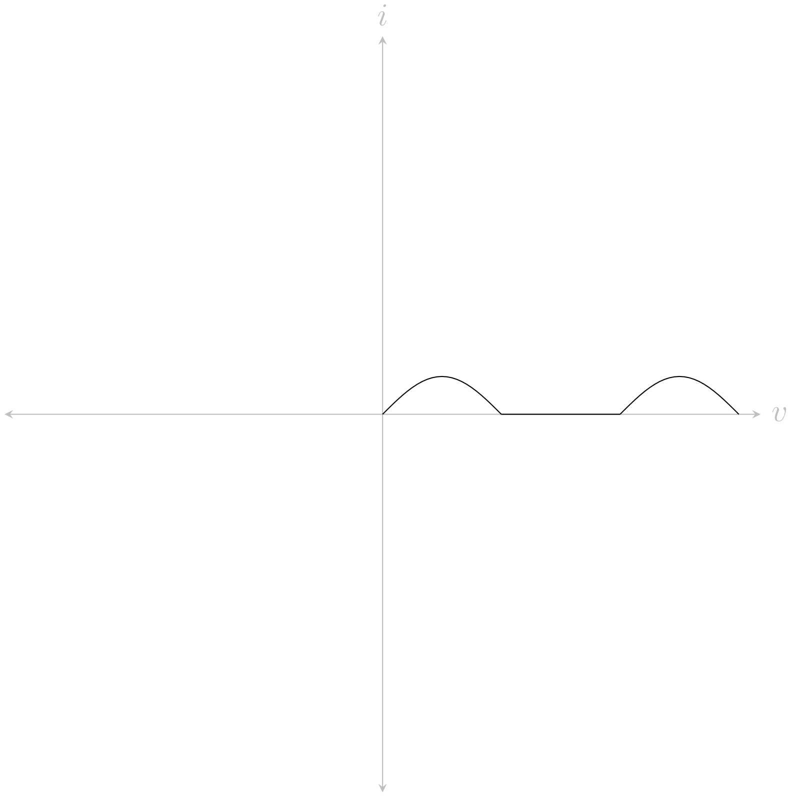


Figure 1: Half Wave Rectifier



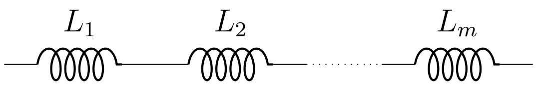
If the current flows in a direction such that the diode is in forward bias, it will allow the current to pass through. If the current is in the opposite direction, it will provide infinite resistance.
Therefore, the wave representing the current in the circuit will be made up of only upper halves of the sine wave.



1.1.4 Capacitors in Series

$$
\begin{aligned}
v_k(t) &= v_k(0) + \frac{1}{C_k} \int\limits_0^t i_k(t)\,\mathrm{d}t \\
v &= \sum_{k=1}^{m} v_k(t) \\
&= \sum_{k=1}^{m} v_k(0) + \left(\sum_{k=1}^{m} \frac{1}{C_k} \right) \int\limits_0^t i(t)\,\mathrm{d}t \\
\therefore v &= v(0) + \frac{1}{C} \int\limits_0^t i(t)\,\mathrm{d}t \\
\therefore \frac{1}{C} &= \sum_{k=1}^{m} \frac{1}{C_k}
\end{aligned}
$$

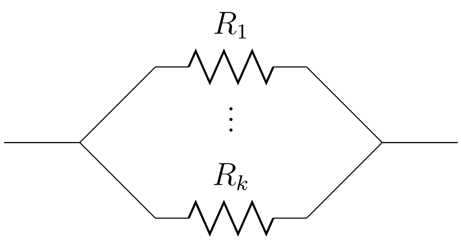
1.1.5 Resistors in Series



$$
\begin{aligned}
v &= \sum_{k=1}^{m} v_k \\
&= \sum_{k=1}^{m} L_k \frac{\mathrm{d}i_k}{\mathrm{d}t} \\
&= \sum_{k=1}^{m} L_k \frac{\mathrm{d}i}{\mathrm{d}t} \\
&= \frac{\mathrm{d}i}{\mathrm{d}t} \sum_{k=1}^{m} L_k \\
\therefore v &= L \frac{\mathrm{d}i}{\mathrm{d}t} \\
\therefore L &= \sum_{k=1}^{m} L_k
\end{aligned}
$$

1.2 Parallel Connections

1.2.1 Resistors in Parallel

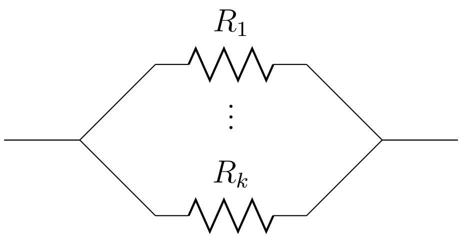


$$
\begin{aligned}
i &= \sum_{k=1}^{m} i_k \\
&= \sum_{k=1}^{m} G_k v_k \\
&= v \sum_{k=1}^{m} G_k \\
\therefore v &= Gi \\
\therefore G &= \sum_{k=1}^{m} G_k \\
\therefore \frac{1}{R} &= \sum_{k=1}^{m} \frac{1}{R_k}
\end{aligned}
$$

1.2.2 Capacitors in Parallel

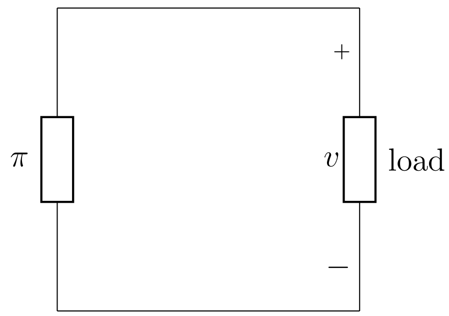
$$
\begin{aligned}
i &= \sum_{k=1}^{m} i_k \\
&= \sum_{k=1}^{m} C_k \frac{\mathrm{d}v_k}{\mathrm{d}t} \\
&= \sum_{k=1}^{m} C_k \frac{\mathrm{d}v}{\mathrm{d}t} \\
&= \frac{\mathrm{d}v}{\mathrm{d}t} \sum_{k=1}^{m} C_k \\
\therefore i &= C \frac{\mathrm{d}v}{\mathrm{d}t} \\
\therefore C &= \sum_{k=1}^{m} C_k
\end{aligned}
$$

1.2.3 Resistors in Parallel



$$
\begin{aligned}
i_k(t) &= i_k(0) + \frac{1}{L_k}\int\limits_0^{(} t)v_k(t)\,\mathrm{d}t \\
i &= \sum_{k=1}^{m} i_k(t) \\
&= \sum_{k=1}^{m} i_k(0) + \sum_{k=1}^{m}\left(\frac{1}{L_k}\int\limits_0^{t} v_k(t)\,\mathrm{d}t\right) \quad = \sum_{k=1}^{m} i_k(0) + \sum_{k=1}^{m}\left(\frac{1}{L_k}\int\limits_0^{t} v(t)\,\mathrm{d}t\right) \\
&= \sum_{k=1}^{m} i_k(0) + \left(\sum_{k=1}^{m}\frac{1}{L_k}\right)\int\limits_0^{t} v(t)\,\mathrm{d}t \\
\therefore i &= i(0) + \frac{1}{L}\int\limits_0^{t} v(t)\,\mathrm{d}t \\
\therefore \frac{1}{L} &= \sum_{k=1}^{m}\frac{1}{L_k}
\end{aligned}
$$

Theorem 1 (Thévenin - Norton Theorem)**.** *Let π be a linear network connected by two terminals to an arbitrary load. π comprises of independent sources, linear resistors, linear capacitors and linear inductors. These components may be time dependent. Let π_0 be the network derived from π by zeroing all independent sources(A short is inserted in place of a voltage source, and a break is inserted in place of a current source). Let e_{OC} be the voltage of the open circuit π, i.e. without the load, between 1 and $1'$. Let i_{SC} be the short circuit current flowing from 1 to $1'$. Under these conditions for an arbitrary load, the voltage v between 1 and $1'$ and the current i between 1 and $1'$ through the load will not change if the network π is replaced by the equivalent circuit of either Thévenin or Norton.*



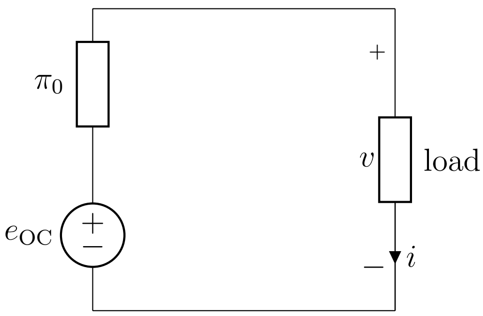


Figure 2: Thévenin Equivalent Circuit

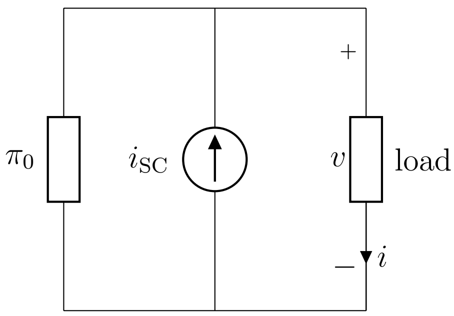
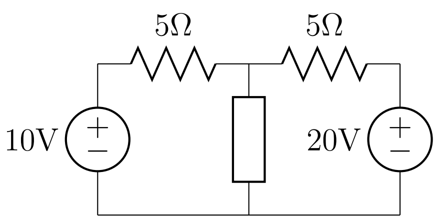
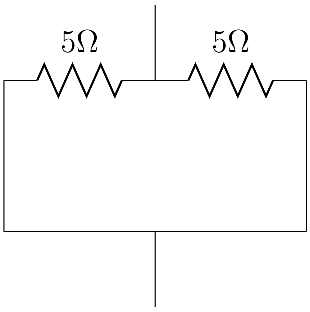


Figure 3: Norton Equivalent Circuit

Exercise 3.




5Ω
5Ω