Introduction to Electrical Engineering

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Contents

1 Lecturer Information

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2 Required Reading

C.A. Desoer and E.S. Kuh: *Basic Circuit Theory*, Mc-Graw-Hill, International Edition.

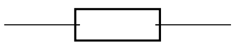
Part I
Basic Definitions and Laws

1 Basic Definitions

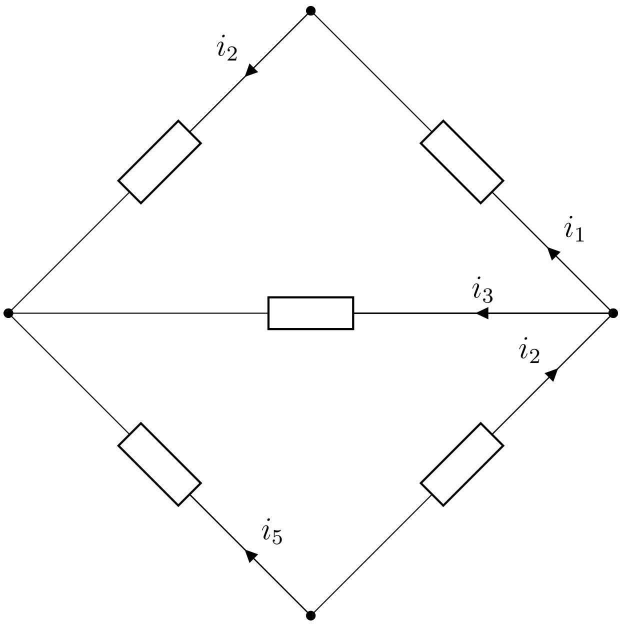
Definition 1 (Electrical circuit)**.** A collection of interconnected components.

Definition 2 (Lumped component)**.** An electrical component whose dimensions are very very small compared to the wavelength of the electromagnetic waves passing through it is called a lumped component.

Definition 3 (One port device)**.** An electrical component with two terminals is called a one port device.



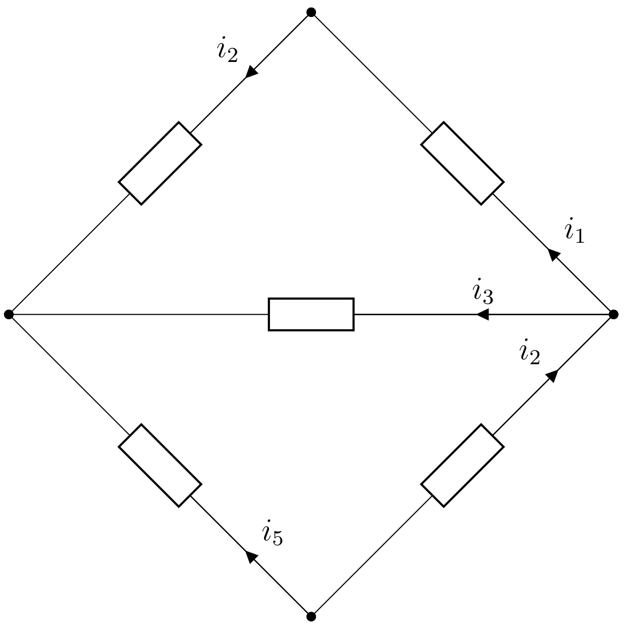
Definition 4 (Nodes and branches)**.** In the figure, all the black dots are called nodes. The parts of the circuit between two nodes are called branches.



2 Kirchoff's Laws

2.1 Kirchoff's Current Law

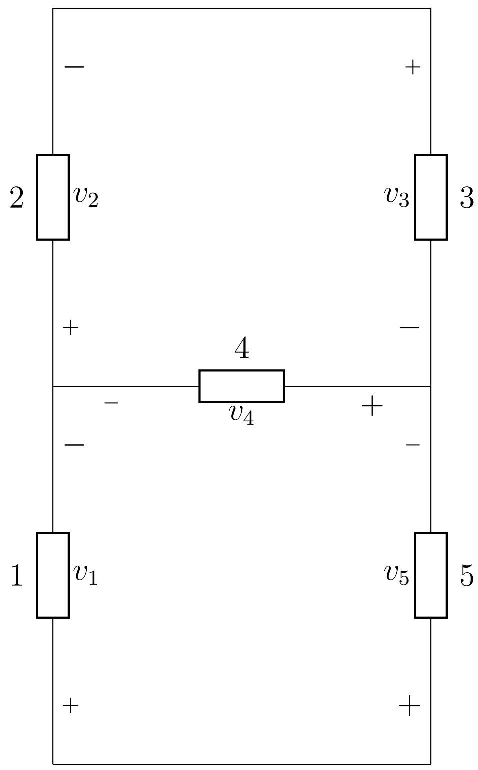
The sum of all currents entering or exiting a node is zero.



$$i_1 + i_3 - i_4 = 0$$

2.2 Kirchoff's Voltage Law

The sum of all branch voltages along a closed loop is zero.



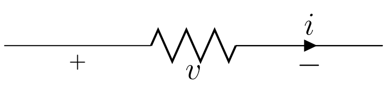
$$
\begin{aligned}
v_1 - v_4 - v_5 &= 0 \\
v_2 + v_3 + v_4 &= 0 \\
v_1 + v_2 + v_3 - v_5 &= 0
\end{aligned}
$$

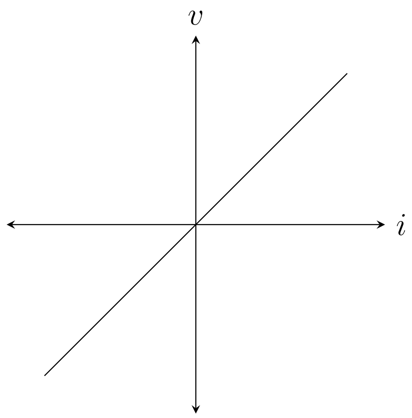
3 Components

3.1 Resistors

Definition 5 (Resistor)**.** A two terminal component is called a resistor if the voltage across it at any given time t is a function of the current at the same time t.

3.1.1 Linear Time Independent Resistor

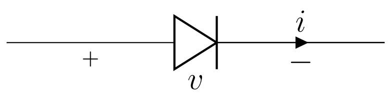




$$v(t) = R \cdot i(t)$$
$$i(t) = G \cdot v(t)$$

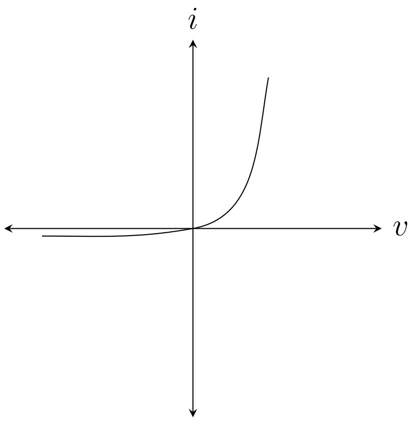
R is called the resistance and G is called the conductance.

3.1.2 Non-linear Resistors (Diodes)



$$i(t) = I_s \left(e^{\dfrac{q \cdot v(t)}{kT}} - 1 \right)$$

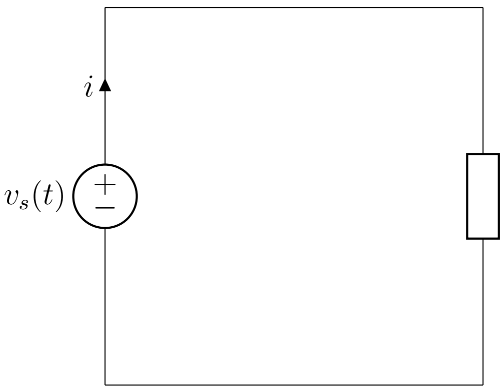
$$I_s = \text{reverse current}$$
$$k = \text{Boltzman constant}$$
$$T = a\text{bsolute temperature}$$
$$q = \text{electronic change}$$
$$\frac{kT}{q} = 0.026(\text{ at } 300\text{K})$$

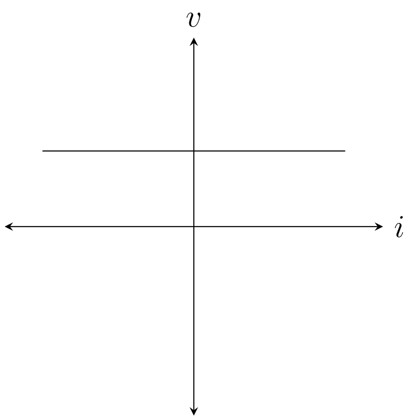


3.2 Independent Sources

3.2.1 Voltage Sources

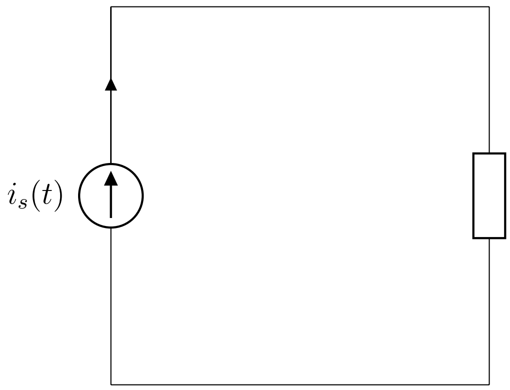
Definition 6 (Voltage source)**.** A two terminal component is called a voltage source if the voltage on its terminals is independent of the current through it.

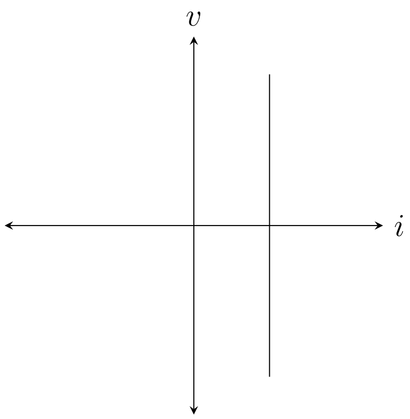




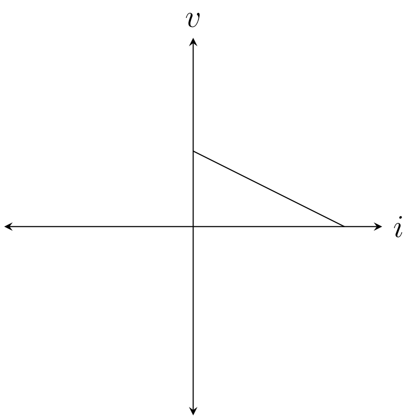
3.2.2 Current Sources

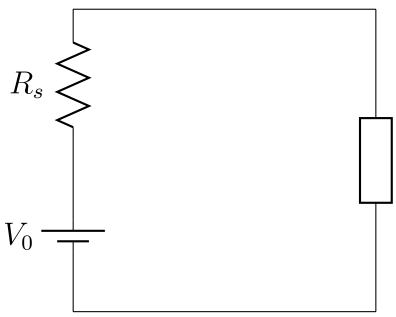
Definition 7 (Current source)**.** A two terminal component is called a current source if it can supply a current $i_s(t)$ independent of the voltage across its terminals.





3.2.3 Real Batteries

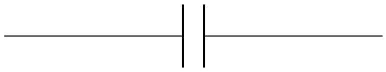




$$\begin{aligned} 0 &= -V_0 + v_R + v \\ v &= V_0 - v_R \\ \therefore v &= V_0 - R_s i \end{aligned}$$

3.3 Capacitor

Definition 8 (Capacitor)**.** A capacitor is a two terminal device where V is a fucntion of q.

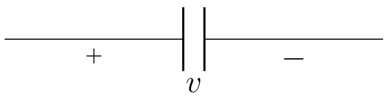


3.3.1 Linear Capacitors

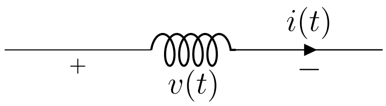
If the charges on the terminals of a capacitor are $+q$ and $-q$, and the potential difference across it is v, the ratio between q and v is said to be the capacitance.

The unit of capacitance is farad or F.

$$q = Cv$$
$$\therefore i = C \frac{\mathrm{d}v}{\mathrm{d}t}$$
$$\therefore v(t) = v(t_0) + \frac{1}{C} \int\limits_{t_0}^{t} i(t)\,\mathrm{d}t$$



3.4 Inductor

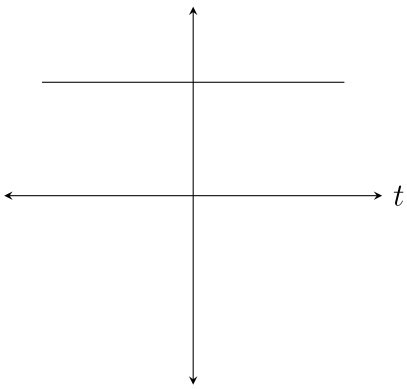


Definition 9 (Inductor)**.**

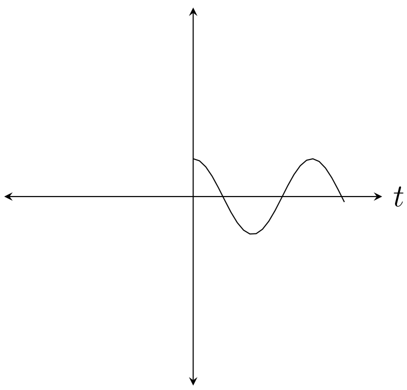
$$v(t) = \frac{\mathrm{d}\varphi}{\mathrm{d}t}$$
$$\therefore v(t) = L \frac{\mathrm{d}i}{\mathrm{d}t}$$
$$\therefore i(t) = i(t_0) + \frac{1}{L} \int\limits_{t_0}^{t} v(t)\,\mathrm{d}t$$

4 Waveforms

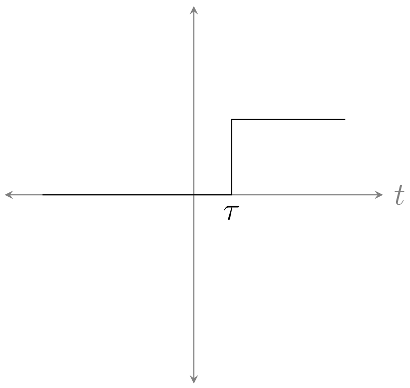
4.1 DC (Constant Function)



4.2 Sinusoidal Wave

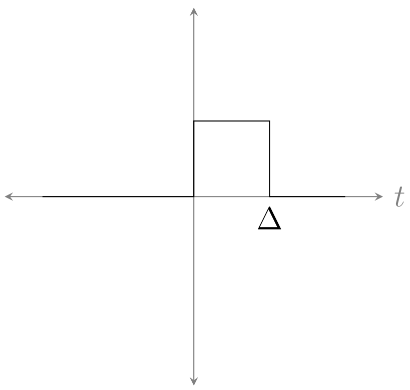


4.3 Step Function



$$u(t) = \begin{cases} 0 & ; \quad t < 0 \\ \dfrac{1}{2} & ; \quad t = \tau \\ 1 & ; \quad t > \tau \end{cases}$$

4.4 Rectangular Pulse



$$P_{\Delta}(t) = \frac{u(t) - u(t - \Delta)}{\Delta}$$

$$P_{\Delta}(t) = \begin{cases} 0 & ; \quad t < 0 \\ \dfrac{1}{\Delta} & ; \quad t = 0 \\ 1 & ; \quad t > 0 \end{cases}$$

4.5 Dirac δ function

$$\delta(t) = \lim_{\Delta \to 0} P_\Delta(t)$$

$$S(\Delta) = \int\limits_{-\infty}^{\infty} P_\Delta(t) f(t)\,\mathrm{d}t$$

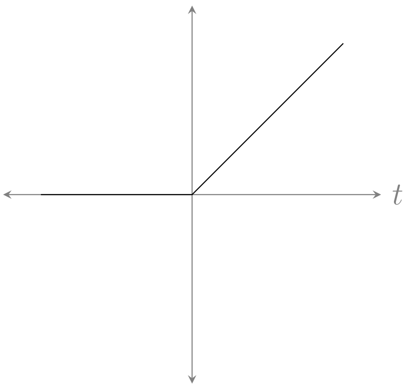
As $\Delta \to 0$,

$$\begin{aligned} S(\Delta) &= \int\limits_{-\infty}^{\infty} P_\Delta(t) f(0)\,\mathrm{d}t \\ &= f(0) \int\limits_{-\infty}^{\infty} P_0(t)\,\mathrm{d}t \\ &= f(0) \end{aligned}$$

$$\delta(t) = \begin{cases} 0 & ; \quad t \neq 0 \\ \infty & ; \quad t = 0 \end{cases}$$

$$\begin{aligned} \int\limits_{-\infty}^{\infty} \delta(t) f(t)\,\mathrm{d}t &= f(0) \\ \int\limits_{-\infty}^{\infty} \delta(t-\tau) f(t)\,\mathrm{d}t &= f(\tau) \\ \int\limits_{-\infty}^{\infty} \delta(at) f(t)\,\mathrm{d}t &= \frac{1}{|a|}\delta(t) \end{aligned}$$

4.6 Ramp Function



$$r(t) = tu(t)$$

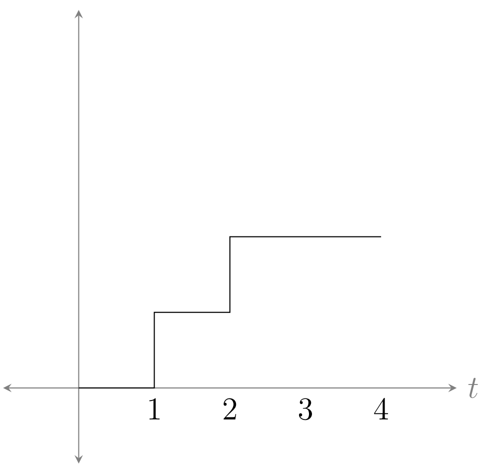
4.7 Doublet Function

$$\delta'(t) = \frac{\mathrm{d}\delta(t)}{\mathrm{d}t}$$

4.8 Relation Between Standard Waveforms

$$r(t) \underset{\int_{-\infty}^{t}}{\overset{\frac{\mathrm{d}}{\mathrm{d}t}}{\rightleftarrows}} u(t) \underset{\int_{-\infty}^{t}}{\overset{\frac{\mathrm{d}}{\mathrm{d}t}}{\rightleftarrows}} \delta(t) \underset{\int_{-\infty}^{t}}{\overset{\frac{\mathrm{d}}{\mathrm{d}t}}{\rightleftarrows}} \delta'(t)$$

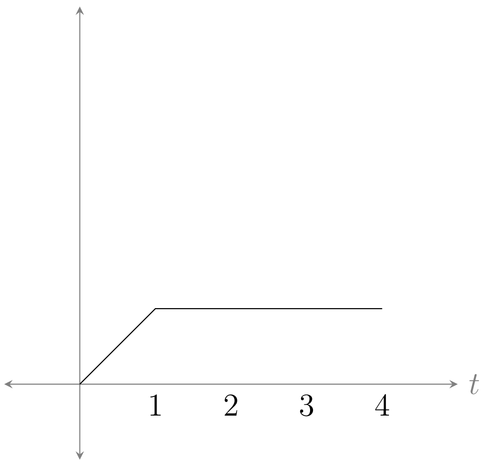
Exercise 1.
Express the following wave as a sum of standard waveforms.



Solution 1.

$$f(t) = u(t-1) + u(t-2)$$

Exercise 2.
Express the following wave as a sum of standard waveforms.



Solution 2.

$$f(t) = r(t) + r(t-1)$$

5 Power and Energy

The instantaneous power supplied to a load is

$$P(t) = v(t) \cdot i(t)$$

where $v(t)$ and $i(t)$ are in matched directions.

The energy supplied to a load from time t_0 to time t is

$$\begin{aligned} W(t_0, t) &= \int_{t_0}^{t} P(t)\,\mathrm{d}t \\ &= \int_{t_0}^{t} v(t) \cdot i(t)\,\mathrm{d}t \end{aligned}$$

5.1 Energy Stored in a Capacitor

$$W(t_0, t) = \int_{t_0}^{t} v(t)i(t)\,\mathrm{d}t$$

As $i(t) = \dfrac{\mathrm{d}q}{\mathrm{d}t}$, $\mathrm{d}q = i(t)\,\mathrm{d}t$. Therefore,

$$\begin{aligned}
W(t_0, t) &= \int_{q(t_0)}^{q(t)} v(q)\,\mathrm{d}q \\
&= \int_{q(t_0)}^{q(t)} \frac{q}{C}\,\mathrm{d}q \\
&= \frac{q^2}{2c} \\
&= \frac{1}{2}cv^2
\end{aligned}$$

5.2 Energy Stored in an Inductor

$$W(t_0, t) = \int_{t_0}^{t} v(t)i(t)\,\mathrm{d}t$$

As $v(t) = \dfrac{\mathrm{d}\varphi}{\mathrm{d}t}$, $\mathrm{d}\varphi = v(t)\,\mathrm{d}t$. Therefore,

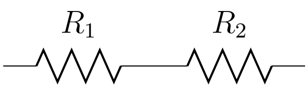
$$\begin{aligned}
W(t_0, t) &= \int_{\varphi(t_0)}^{\varphi(t)} i(\varphi)\,\mathrm{d}\varphi \\
&= \frac{\varphi^2}{2L} \\
&= \frac{1}{2}Li^2
\end{aligned}$$

Part II

Simple Circuits

1 Equivalent Circuits

Definition 10. Two circuits are said to be equivalent if they have the same $v(t)$-$i(t)$ relationships.



Let the voltage across R_1 be v_1 and across R_2 be v_2.
Let the current through R_1 be i_1 and through R_2 be i_2.
Therefore,

$$\begin{aligned} v_1 &= f_1(i_1) \\ v_2 &= f_2(i_2) \end{aligned}$$

Therefore,

$$\begin{aligned} v &= v_1 + v_2 \\ &= f_1(i_1) + f_2(i_2) \\ &= f_1(i) + f_2(i) \\ &= f_3(i) \end{aligned}$$

Therefore, the system of resistors is equivalent to a single resistor R_3.