Introduction to Electrical Engineering

Aakash Jog

2014-15

Contents

1 Lecturer Information

Prof. Moshe Tur

Office: Wolfson 413
Telephone: 03-640-8125
E-mail: tur@post.tau.ac.il

2 Required Reading

C.A. Desoer and E.S. Kuh: *Basic Circuit Theory*, Mc-Graw-Hill, International Edition.

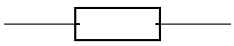
Part I
Basic Definitions and Laws

1 Basic Definitions

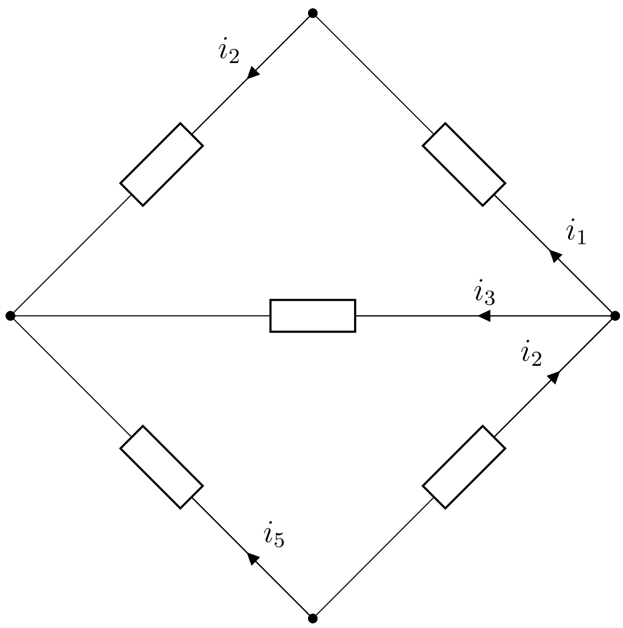
Definition 1 (Electrical circuit)**.** A collection of interconnected components.

Definition 2 (Lumped component)**.** An electrical component whose dimensions are very very small compared to the wavelength of the electromagnetic waves passing through it is called a lumped component.

Definition 3 (One port device)**.** An electrical component with two terminals is called a one port device.



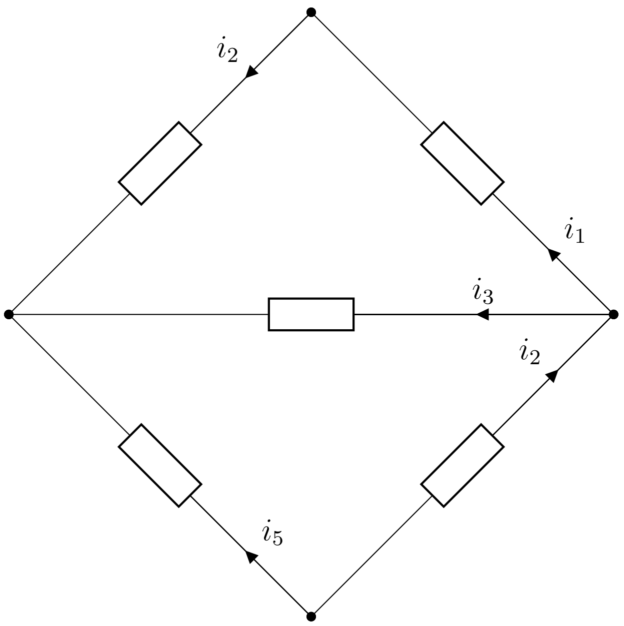
Definition 4 (Nodes and branches)**.** In the figure, all the black dots are called nodes. The parts of the circuit between two nodes are called branches.



2 Kirchoff's Laws

2.1 Kirchoff's Current Law

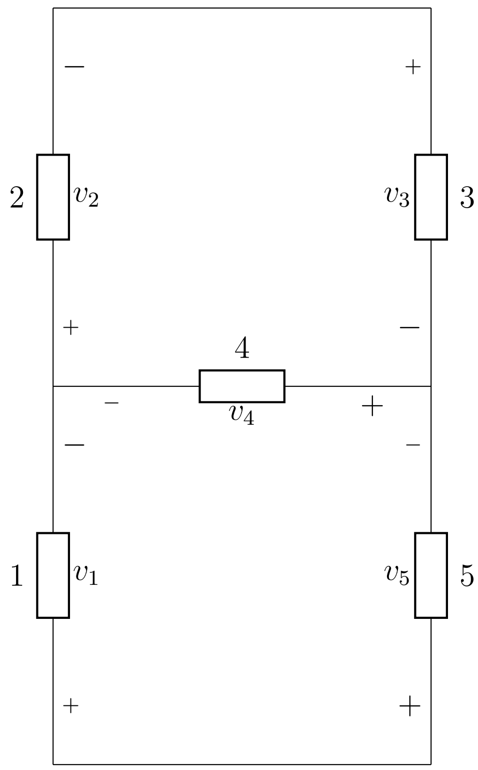
The sum of all currents entering or exiting a node is zero.



$$i_1 + i_3 - i_4 = 0$$

2.2 Kirchoff's Voltage Law

The sum of all branch voltages along a closed loop is zero.



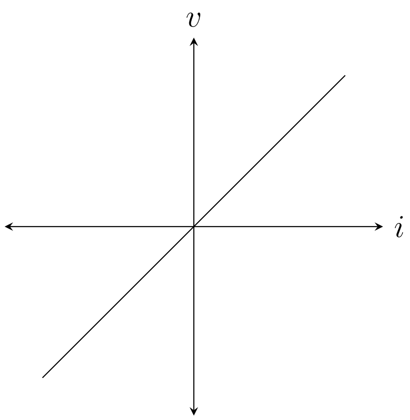
$$\begin{aligned} v_1 - v_4 - v_5 &= 0 \\ v_2 + v_3 + v_4 &= 0 \\ v_1 + v_2 + v_3 - v_5 &= 0 \end{aligned}$$

3 Components

3.1 Resistors

Definition 5 (Resistor)**.** A two terminal component is called a resistor if the voltage across it at any given time t is a function of the current at the same time t.

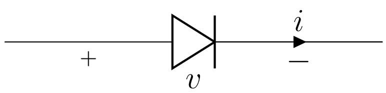
3.1.1 Linear Time Independent Resistor



$$v(t) = R \cdot i(t)$$
$$i(t) = G \cdot v(t)$$

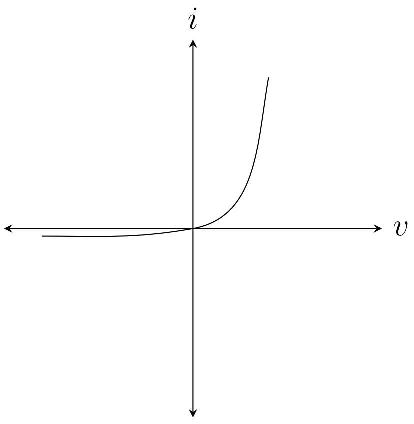
R is called the resistance and G is called the conductance.

3.1.2 Non-linear Resistors (Diodes)



$$i(t) = I_s \left(e^{\frac{q \cdot v(t)}{kT}} - 1 \right)$$

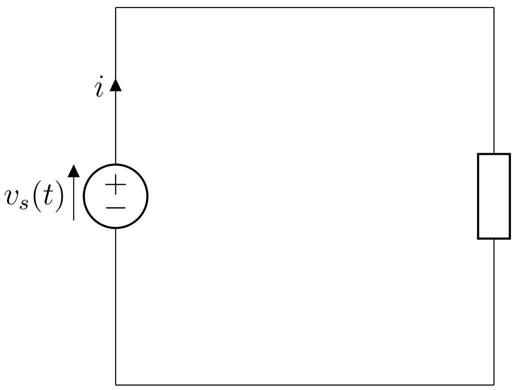
$$\begin{aligned} I_s &= \text{reverse current} \\ k &= \text{Boltzman constant} \\ T &= a\text{bsolute temperature} \\ q &= \text{electronic change} \\ \frac{kT}{q} &= 0.026(\text{ at } 300\text{K}) \end{aligned}$$

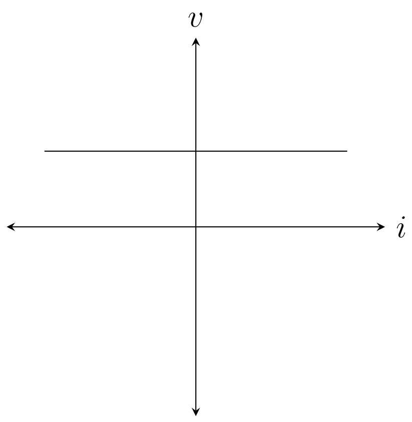


3.2 Independent Sources

3.2.1 Voltage Sources

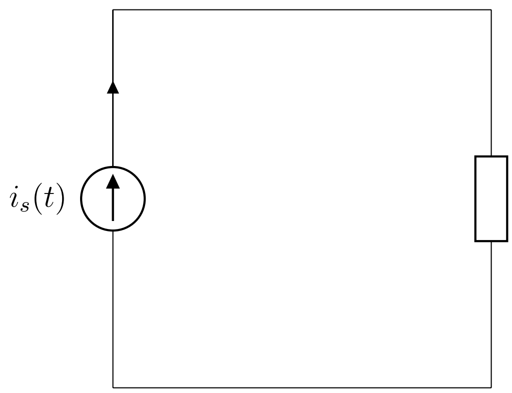
Definition 6 (Voltage source)**.** A two terminal component is called a voltage source if the voltage on its terminals is independent of the current through it.

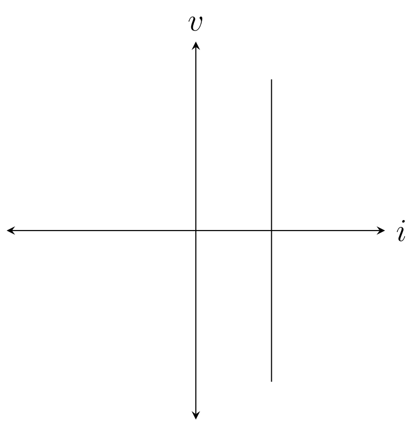




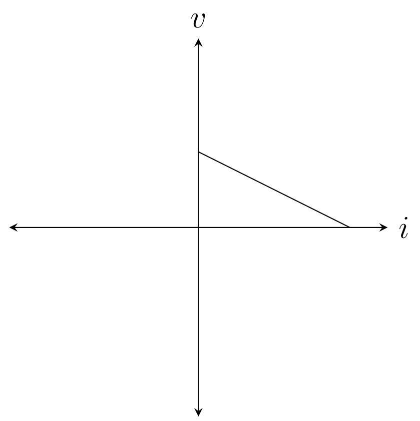
3.2.2 Current Sources

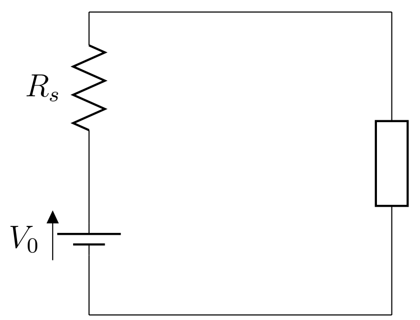
Definition 7 (Current source)**.** A two terminal component is called a current source if it can supply a current $i_s(t)$ independent of the voltage across its terminals.





3.2.3 Real Batteries

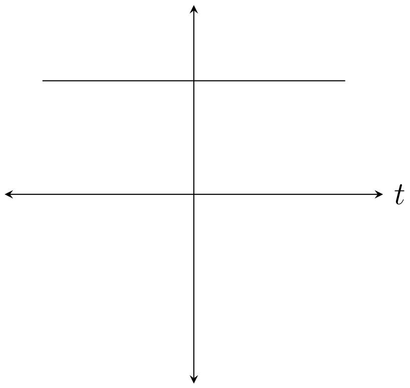




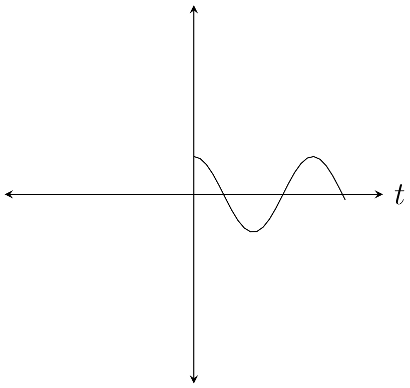
$$
\begin{aligned}
0 &= -V_0 + v_R + v \\
v &= V_0 - v_R \\
\therefore v &= V_0 - R_s i
\end{aligned}
$$

4 Waveforms

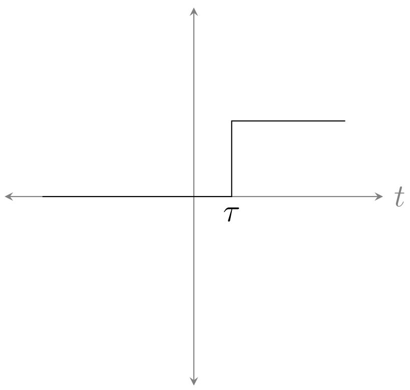
4.1 DC (Constant Function)



4.2 Sinusoidal Wave

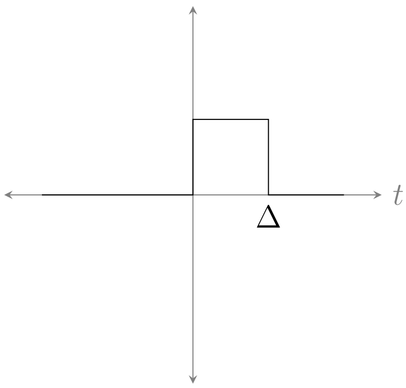


4.3 Step Function



$$
u(t) = \begin{cases} 0 & ; \quad t < 0 \\ \dfrac{1}{2} & ; \quad t = \tau \\ 1 & ; \quad t > \tau \end{cases}
$$

4.4 Rectangular Pulse



$$
P_\Delta(t) = \frac{u(t) - u(t - \Delta)}{\Delta}
$$

$$
P_\Delta(t) = \begin{cases} 0 & ; \quad t < 0 \\ \dfrac{1}{\Delta} & ; \quad t = 0 \\ 1 & ; \quad t > 0 \end{cases}
$$

4.5 Dirac δ function

$$
\delta(t) = \lim_{\Delta \to 0} P_\Delta(t)
$$

$$
S(\Delta) = \int\limits_{-\infty}^{\infty} P_\Delta(t) f(t) \, \mathrm{d}t
$$

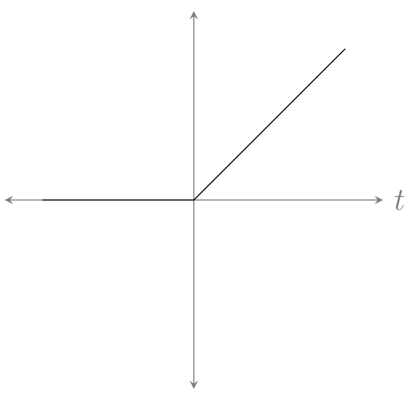
As $\Delta \to 0$,

$$
\begin{aligned}
S(\Delta) &= \int_{-\infty}^{\infty} P_\Delta(t) f(0)\,\mathrm{d}t \\
&= f(0) \int_{-\infty}^{\infty} P_0(t)\,\mathrm{d}t \\
&= f(0)
\end{aligned}
$$

$$
\delta(t) = \begin{cases} 0 & ; \quad t \neq 0 \\ \infty & ; \quad t = 0 \end{cases}
$$

$$
\begin{aligned}
\int_{-\infty}^{\infty} \delta(t) f(t)\,\mathrm{d}t &= f(0) \\
\int_{-\infty}^{\infty} \delta(t-\tau) f(t)\,\mathrm{d}t &= f(\tau) \\
\int_{-\infty}^{\infty} \delta(at) f(t)\,\mathrm{d}t &= \frac{1}{|a|}\delta(t)
\end{aligned}
$$

4.6 Ramp Function



$$
r(t) = tu(t)
$$

4.7 Doublet Function

$$\delta'(t) = \frac{\mathrm{d}\delta(t)}{\mathrm{d}t}$$

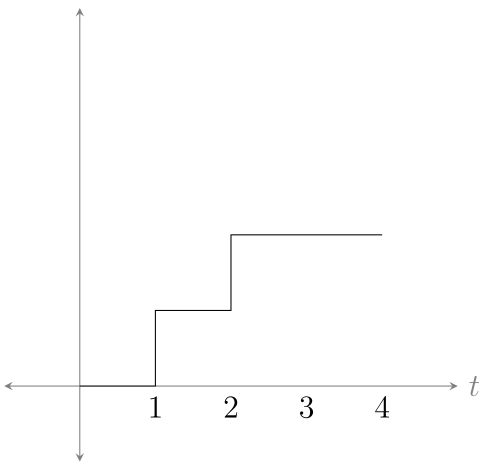
$$\int\limits_{-\infty}^{t} u(t')\,\mathrm{d}t' = r(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\, r(t) = u(t)$$

$$u(t) = \int\limits_{-\infty}^{t} \delta(t')\,\mathrm{d}t$$

$$r(t) \underset{\int_{-\infty}^{t}}{\overset{\frac{\mathrm{d}}{\mathrm{d}t}}{\rightleftharpoons}} u(t) \underset{\int_{-\infty}^{t}}{\overset{\frac{\mathrm{d}}{\mathrm{d}t}}{\rightleftharpoons}} \delta(t) \underset{\int_{-\infty}^{t}}{\overset{\frac{\mathrm{d}}{\mathrm{d}t}}{\rightleftharpoons}} \delta'(t)$$

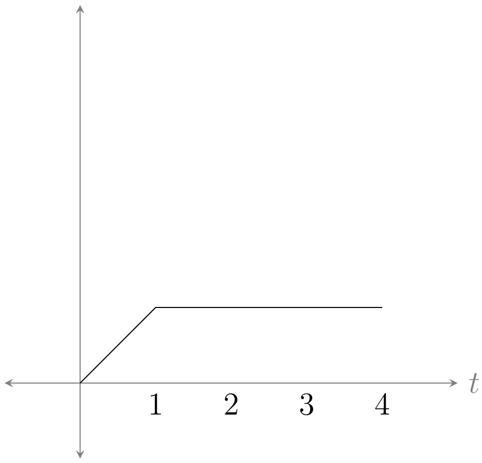
Exercise 1.
Express the following wave as a sum of standard waveforms.



Solution 1.

$$f(t) = u(t-1) + u(t-2)$$

Exercise 2.
Express the following wave as a sum of standard waveforms.



Solution 2.

$$f(t) = r(t) + r(t-1)$$