INTRODUCTION TO LINEAR SYSTEMS : ASSIGNMENT 5A

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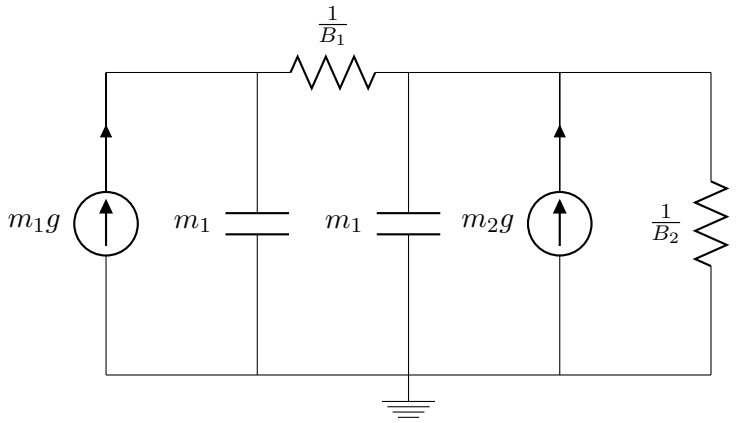
Exercise 1.
A mass M_1 is placed on a dashpot having a damping coefficient B_1. The dashpot at its other side is connected to a mass M_2. The latter is lying on a dashpot with damping coefficient B_2. The system is situated in a gravity field g.

(1) Draw an equivalent electrical diagram and write a set of differential equations of order 1, for the velocities v_1, v_2 of the two masses.
(2) Arrange your above solution in a state space presentation with the state vector
$$x(t) = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$
(3) Find the suitable output matrix C for the following cases.
   (a) The output of the system is the average of the two velocities.
   (b) The output of the system is the difference between the two velocities.

Solution 1.



(1)
$$v_1{}' = -\frac{B_1}{m_1} v_1 + \frac{B_1}{m_1} v_2 + g$$
$$v_2{}' = \frac{B_2}{m_2} v_1 - \frac{B_1 + B_2}{m_2} v_2 + g$$

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(2)

$$v_1{}' = -\frac{B_1}{m_1}v_1 + \frac{B_1}{m_1}v_2 + g$$
$$v_2{}' = \frac{B_2}{m_2}v_1 - \frac{B_1 + B_2}{m_2}v_2 + g$$

Therefore,

$$\begin{pmatrix} v_1{}' \\ v_2{}' \end{pmatrix} = \begin{pmatrix} -\frac{B_1}{m_1} & \frac{B_1}{m_2} \\ \frac{B_2}{m_2} & -\frac{B_1+B_2}{m_2} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} g$$

(3) (a)

$$C = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

(b) If $v_1 \geq v_2$,

$$C = \begin{pmatrix} 1 & -1 \end{pmatrix}$$

(c) If $v_2 \geq v_1$,

$$C = \begin{pmatrix} -1 & 1 \end{pmatrix}$$