INTRODUCTION TO LINEAR SYSTEMS : ASSIGNMENT 2

AAKASH JOG
ID : 989323563

Exercise 1.

(1) The transfer function of a system is

$$G(s) = \frac{1}{(s+2)(s+5)}$$

Find the output, i.e. the particular solution for the input

$$u(t) = \cos(3t)\delta_{-1}(t)$$

(2) Consider the signal

$$p(t) = \left(3 + 2e^{-t} - e^{-2t}\right)\delta_{-1}(t)$$

(a) Suppose it represents the impulse response of a system. Is the system stable?
(b) Suppose it does not represent the response of a system to a step input, with zero initial conditions, is the system stable?

Solution 1.

(1)

$$u(t) = \cos(3t)\delta_{-1}(t)$$
$$\therefore U(s) = \frac{s}{s^2+9}$$

Therefore,

$$Y_p(s) = G(s)U(s)$$
$$= \frac{1}{(s+1)(s+5)}\frac{s}{s^2+9}$$

Let

$$Y_p(s) = \frac{A}{s+2} + \frac{B}{s+5} + \frac{Cs+D}{s^2+9}$$

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Therefore,

$$\begin{aligned} A &= \lim_{s\to -2} \frac{1}{s+5}\frac{s}{s^2+9} \\ &= -\frac{2}{39} \\ B &= \lim_{s\to -5} \frac{1}{s+2}\frac{s}{s^2+9} \\ &= \frac{5}{102} \end{aligned}$$

Therefore,

$$\begin{aligned} Y_p(s) &= \frac{-\frac{2}{39}(s^2+9)(s+5) + \frac{5}{102}(s^2+9)(s+2) + (Cs+D)(s+2)(s+5)}{(s+2)(s+5)(s^2+9)} \\ \therefore s = &\quad -\frac{2}{39}\left(s^3+5s^2+9s+45\right) \\ &+ \frac{5}{102}\left(s^3+2s^2+9s+18\right) \\ &+ (Cs+D)\left(s^2+7s+10\right) \end{aligned}$$

Therefore, solving,

$$\begin{aligned} C &= \frac{1}{442} \\ D &= \frac{63}{442} \end{aligned}$$

Therefore,

$$Y_p(s) = -\frac{2}{39}\frac{1}{s+2} + \frac{5}{102}\frac{1}{s+5} + \frac{21}{442}\frac{3}{s^2+9} + \frac{1}{442}\frac{s}{s^2+9}$$

Therefore,

$$y_p(t) = \left(-\frac{2}{39}e^{-2t} + \frac{5}{102}e^{-5t} + \frac{21}{442}\sin(3t) + \frac{1}{442}\cos(3t)\right)\delta_{-1}(t)$$

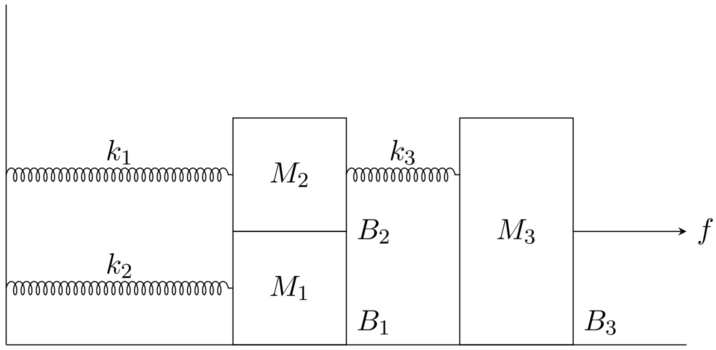
(2) (a) As $p(t)$ is the impulse response of a system, the system is stable if and only is all roots of $P(s) = G(s) = 0$ are in the left half-plane of the complex plane.

$$\begin{aligned} p(t) &= \left(3 + 2e^{-t} - e^{-2t}\right)\delta_{-1}(t) \\ \therefore P(s) &= \frac{3}{s} + \frac{2}{s+1} - \frac{1}{s+2} \\ &= \frac{4s^2+12s+6}{s(s+1)(s+2)} \end{aligned}$$

Therefore, as $P(s) = 0$ has a root at $s = 0$, which is not in the left half-plane, the system is not stable.

(b) If $p(t)$ does not represent the step response of a system, nothing can be said about the system, as the information is not sufficient.

Exercise 2.
Consider the system in the following illustration.

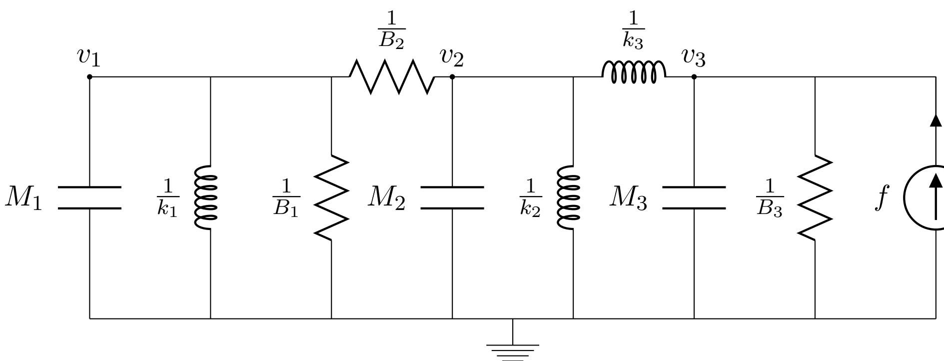


k_i represent springs, B_i represent viscous friction, and M_i represent masses. The positive direction is defined to the right.

(1) Draw an equivalent electrical diagram.
(2) Write the Laplace set of equations in matrix form whose unknowns are the velocities of the three masses, v_1, v_2, and v_3.
(3) For this part only, assume that mass M_2 is being held, i.e. it is constrained to $v_2 = 0$. Draw the electrical diagrams for this case and find the Laplace and the differential equations for v_1 and v_3.
(4) Assume a constant force $f(t) = f_0\delta_{-1}(t)$. Find the velocities of the masses M_1, M_2, M_3, in the steady state.

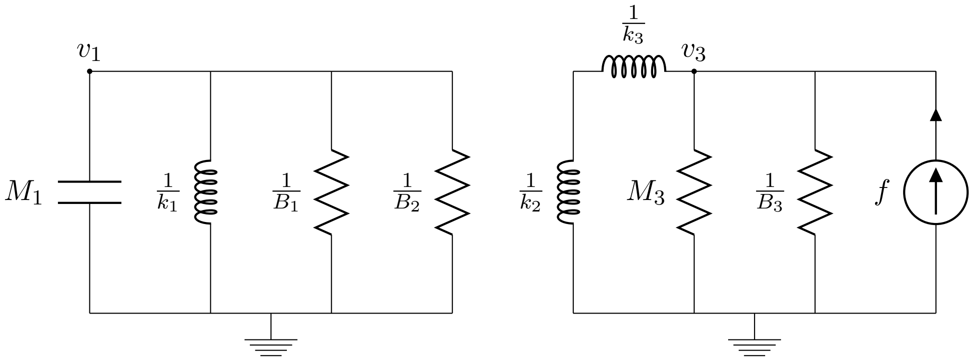
Exercise 3.

(1) For an equivalent electrical system, springs are equivalent to inductors, friction is equivalent to resistors, and masses are equivalent to capacitors.
Let the velocities of each M_i be v_i.
Therefore,



(2) Using the method of voltage nodes,

$$\begin{pmatrix} M_1 s + \frac{k_1}{s} + B_1 + B_2 & -B_2 & -\frac{k_1}{s} \\ -B_2 & B_2 + M_2 s + \frac{k_2}{s} + \frac{k_3}{s} & -\frac{k_3}{s} \\ -\frac{k_3}{s} & -\frac{k_3}{s} & \frac{k_3}{s} + M_3 s + B_3 \end{pmatrix} \begin{pmatrix} V_1(s) \\ V_2(s) \\ V_3(s) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ F(s) \end{pmatrix}$$



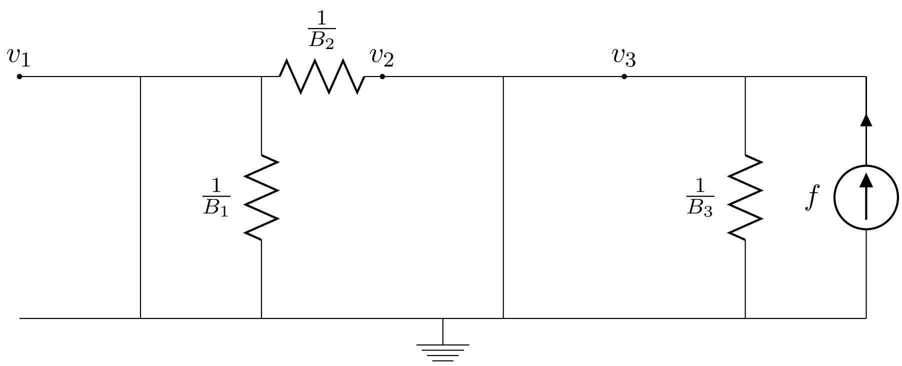
(3) Therefore,

$$\left(M_1 s + \frac{k_1}{s} + B_1 + B_2\right) V_1(s) = 0$$
$$\therefore M_1 v_1'' + (B_1 + B_2) v_1' + k_1 v_1 = 0$$

and

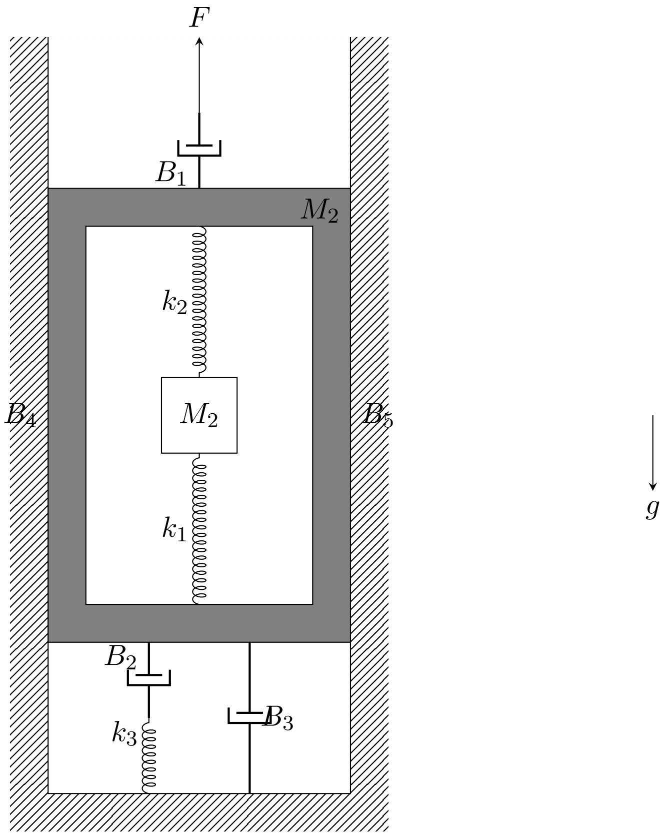
$$\left(\frac{k_3}{s} + M_3 s + B_3\right) = 0$$
$$\therefore M_3 v_3'' + k_3 v_3' + B_3 v_3 = 0$$

(4) In the steady state, all capacitors are equivalent to breaks and all inductors are equivalent to shorts.
Therefore, the equivalent circuit is



Therefore, as v_1, v_2, and v_3 are zero, all three masses, M_1, M_2, and M_3 are stationary.

Exercise 4.
Consider the system in the following illustration.

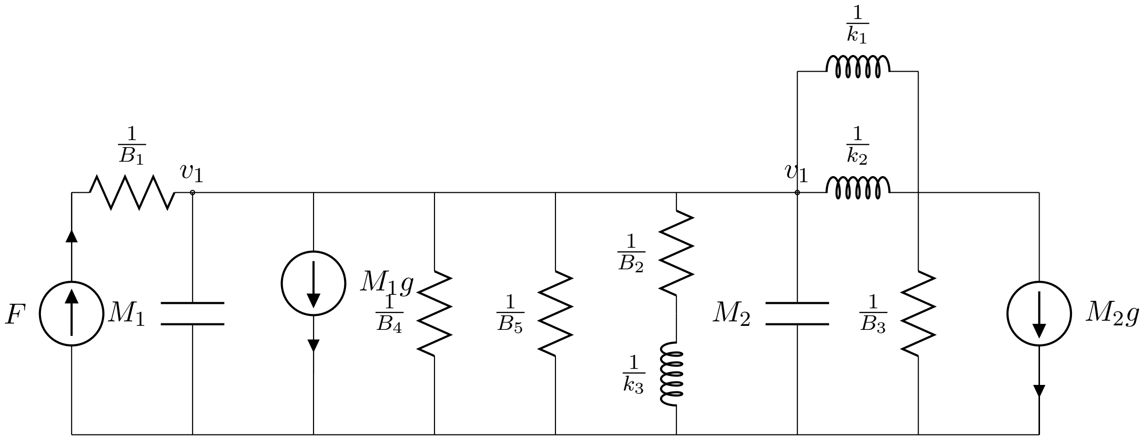


The illustration describes a system of an elevator with mass M_1, and a mass M_2 connected to the elevator. The positive direction is defined upwards. It is given that $\forall i$,

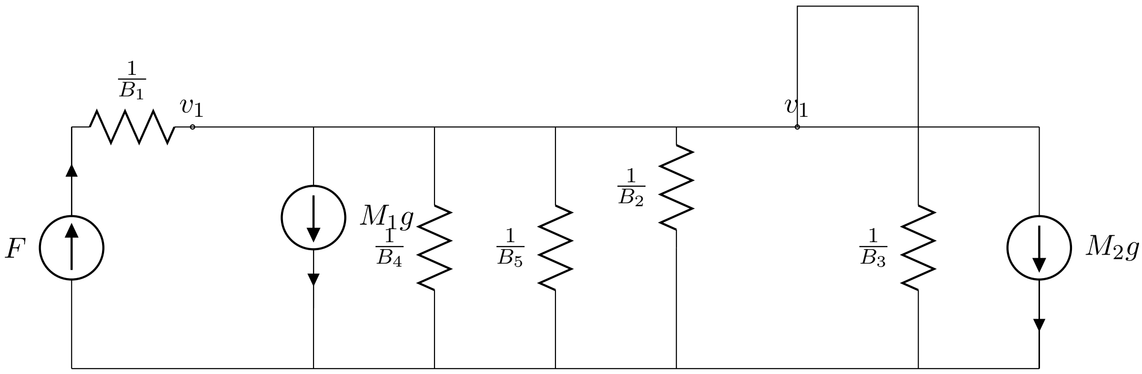
$$B_i = B$$
$$k_i = k$$

(1) Draw an equivalent electrical diagram.
(2) Find the velocities of the masses M_1 and M_2 in the steady state. Assume a constant force $f(t) = f_0\delta_{-1}(t)$.

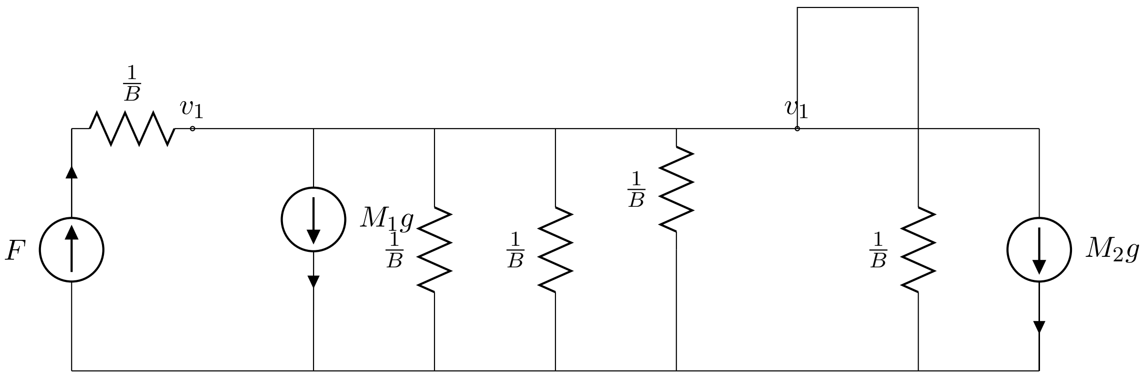
Solution 4.
(1) The equivalent electrical system is



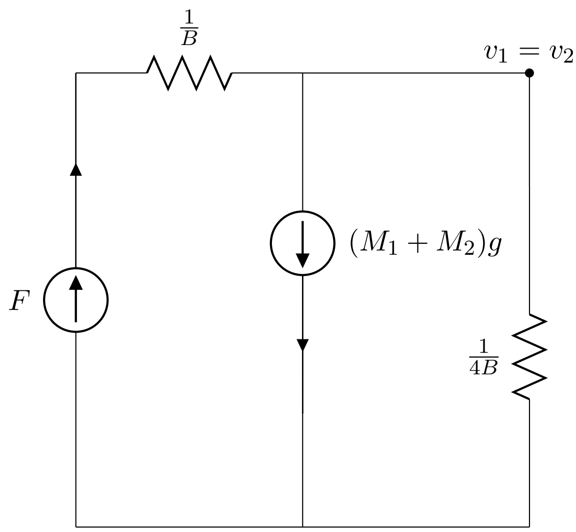
(2) In the steady state, all capacitors are equivalent to breaks and all inductors are equivalent to shorts.
Therefore, the equivalent circuit is



Therefore, substituting the given values for all B_i and k_i, the equivalent circuit is



Therefore, simplifying, the circuit is equivalent to



Therefore, the velocities of the masses are equal, and are
$v_1 = v_2 = \big(F - (M_1 + M_2)g\big)\,\dfrac{1}{4B}$