

INTRODUCTION TO LINEAR SYSTEMS : ASSIGNMENT 2

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Exercise 1.

- (1) The transfer function of a system is

$$G(s) = \frac{1}{(s+2)(s+5)}$$

Find the output, i.e. the particular solution for the input

$$u(t) = \cos(3t)\delta_{-1}(t)$$

- (2) Consider the signal

$$p(t) = \left(3 + 2e^{-t} - e^{-2t}\right) \delta_{-1}(t)$$

- (a) Suppose it represents the impulse response of a system. Is the system stable?
(b) Suppose it does not represent the response of a system to a step input, with zero initial conditions, is the system stable?

Solution 1.

- (1)

$$u(t) = \cos(3t)\delta_{-1}(t)$$

$$\therefore U(s) = \frac{s}{s^2 + 9}$$

Therefore,

$$\begin{aligned} Y_p(s) &= G(s)U(s) \\ &= \frac{1}{(s+2)(s+5)} \frac{s}{s^2 + 9} \end{aligned}$$

Let

$$Y_p(s) = \frac{A}{s+2} + \frac{B}{s+5} + \frac{Cs+D}{s^2+9}$$

Therefore,

$$A = \lim_{s \rightarrow -2} \frac{1}{s+5} \frac{s}{s^2+9}$$

$$= -\frac{2}{39}$$

$$B = \lim_{s \rightarrow -5} \frac{1}{s+2} \frac{s}{s^2+9}$$

$$= \frac{5}{102}$$

Therefore,

$$Y_p(s) = \frac{-\frac{2}{39}(s^2+9)(s+5) + \frac{5}{102}(s^2+9)(s+2) + (Cs+D)(s+2)(s+5)}{(s+2)(s+5)(s^2+9)}$$

$$\begin{aligned} \therefore s = & -\frac{2}{39} (s^3 + 5s^2 + 9s + 45) \\ & + \frac{5}{102} (s^3 + 2s^2 + 9s + 18) \\ & + (Cs + D) (s^2 + 7s + 10) \end{aligned}$$

Therefore, solving,

$$C = \frac{1}{442}$$

$$D = \frac{63}{442}$$

Therefore,

$$Y_p(s) = -\frac{2}{39} \frac{1}{s+2} + \frac{5}{102} \frac{1}{s+5} + \frac{21}{442} \frac{3}{s^2+9} + \frac{1}{442} \frac{s}{s^2+9}$$

Therefore,

$$y_p(t) = \left(-\frac{2}{39} e^{-2t} + \frac{5}{102} e^{-5t} + \frac{21}{442} \sin(3t) + \frac{1}{442} \cos(3t) \right) \delta_{-1}(t)$$

- (2) (a) As $p(t)$ is the impulse response of a system, the system is stable if and only if all roots of $P(s) = G(s) = 0$ are in the left half-plane of the complex plane.

$$p(t) = \left(3 + 2e^{-t} - e^{-2t} \right) \delta_{-1}(t)$$

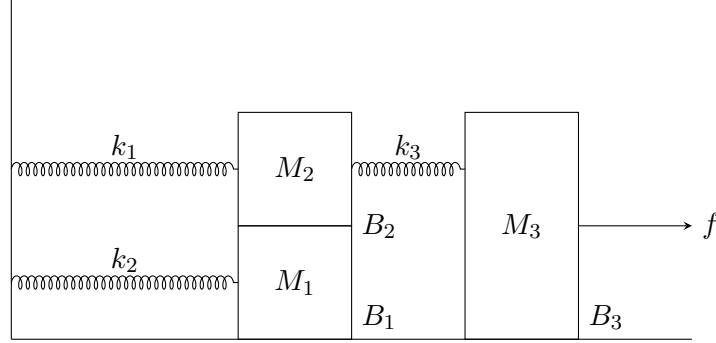
$$\begin{aligned} \therefore P(s) &= \frac{3}{s} + \frac{2}{s+1} - \frac{1}{s+2} \\ &= \frac{4s^2 + 12s + 6}{s(s+1)(s+2)} \end{aligned}$$

Therefore, as $P(s) = 0$ has a root at $s = 0$, which is not in the left half-plane, the system is not stable.

- (b) If $p(t)$ does not represent the step response of a system, nothing can be said about the system, as the information is not sufficient.

Exercise 2.

Consider the system in the following illustration.

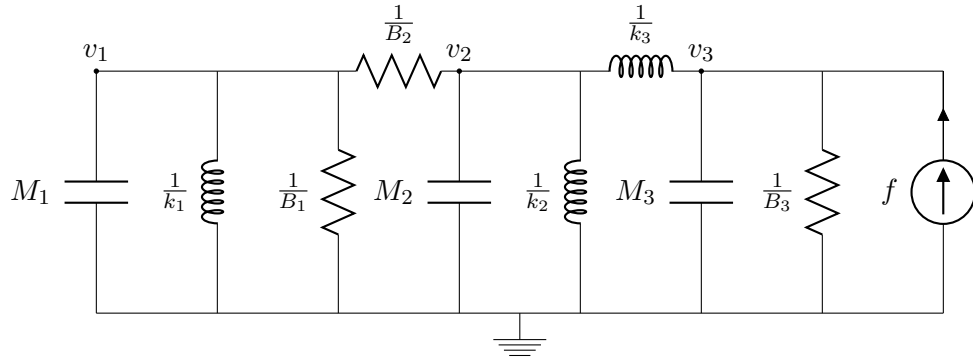


k_i represent springs, B_i represent viscous friction, and M_i represent masses. The positive direction is defined to the right.

- (1) Draw an equivalent electrical diagram.
- (2) Write the Laplace set of equations in matrix form whose unknowns are the velocities of the three masses, v_1 , v_2 , and v_3 .
- (3) For this part only, assume that mass M_2 is being held, i.e. it is constrained to $v_2 = 0$. Draw the electrical diagrams for this case and find the Laplace and the differential equations for v_1 and v_3 .
- (4) Assume a constant force $f(t) = f_0\delta_{-1}(t)$. Find the velocities of the masses M_1 , M_2 , M_3 , in the steady state.

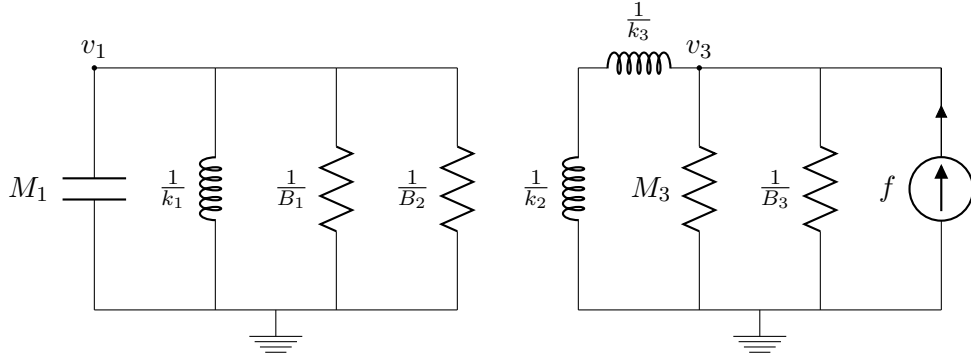
Solution 2.

- (1) For an equivalent electrical system, springs are equivalent to inductors, friction is equivalent to resistors, and masses are equivalent to capacitors. Let the velocities of each M_i be v_i . Therefore,



- (2) Using the method of voltage nodes,

$$\begin{pmatrix} M_1s + \frac{k_1}{s} + B_1 + B_2 & -B_2 & -\frac{k_1}{s} \\ -B_2 & B_2 + M_2s + \frac{k_2}{s} + \frac{k_3}{s} & -\frac{k_3}{s} \\ -\frac{k_3}{s} & -\frac{k_3}{s} & \frac{k_3}{s} + M_3s + B_3 \end{pmatrix} \begin{pmatrix} V_1(s) \\ V_2(s) \\ V_3(s) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ F(s) \end{pmatrix}$$



(3) Therefore,

$$\left(M_1 s + \frac{k_1}{s} + B_1 + B_2 \right) V_1(s) = 0$$

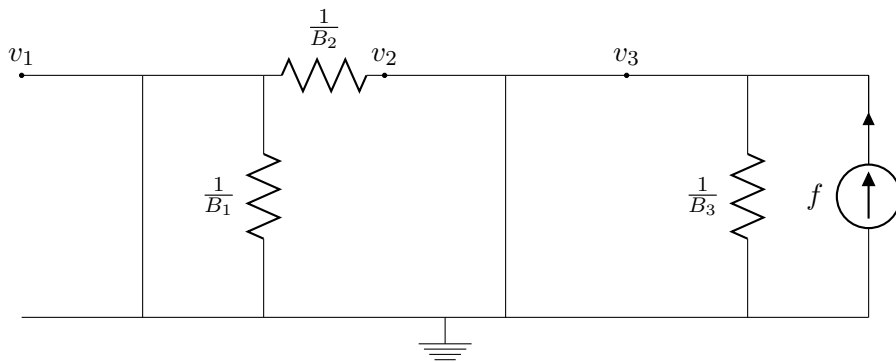
$$\therefore M_1 v_1'' + (B_1 + B_2) v_1' + k_1 v_1 = 0$$

and

$$\left(\frac{k_3}{s} + M_3 s + B_3 \right) = 0$$

$$\therefore M_3 v_3'' + k_3 v_3' + B_3 v_3 = 0$$

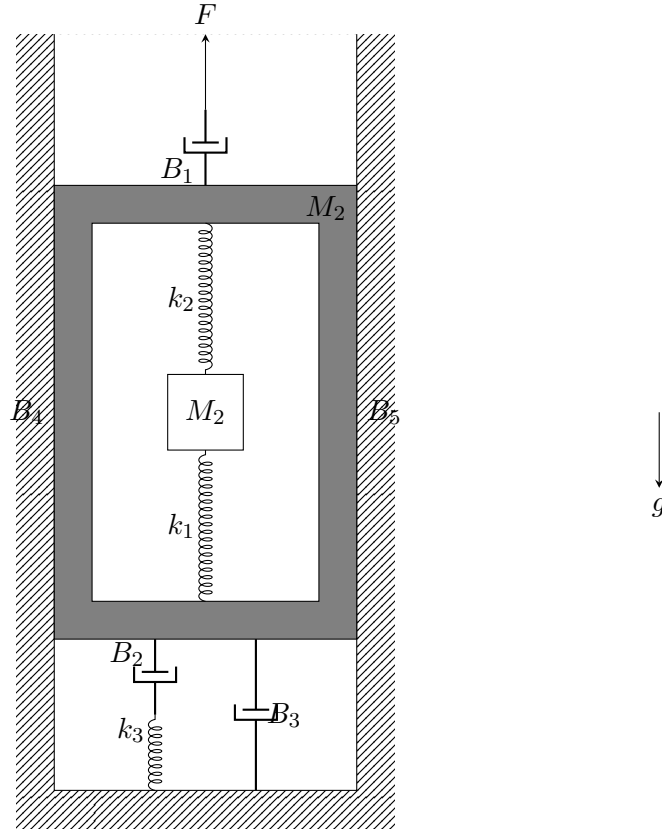
(4) In the steady state, all capacitors are equivalent to breaks and all inductors are equivalent to shorts. Therefore, the equivalent circuit is



Therefore, as v_1 , v_2 , and v_3 are zero, all three masses, M_1 , M_2 , and M_3 are stationary.

Exercise 3.

Consider the system in the following illustration.



The illustration describes a system of an elevator with mass M_1 , and a mass M_2 connected to the elevator. The positive direction is defined upwards. It is given that $\forall i$,

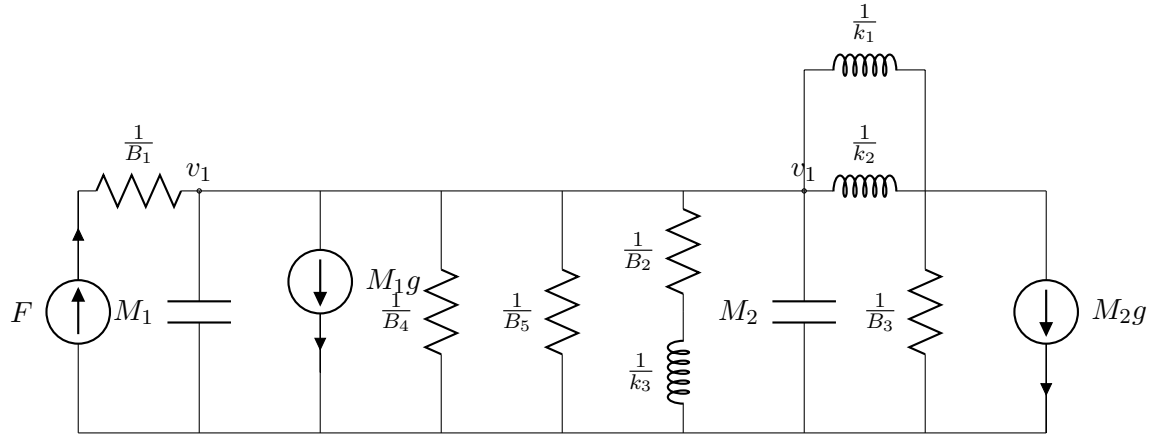
$$B_i = B$$

$$k_i = k$$

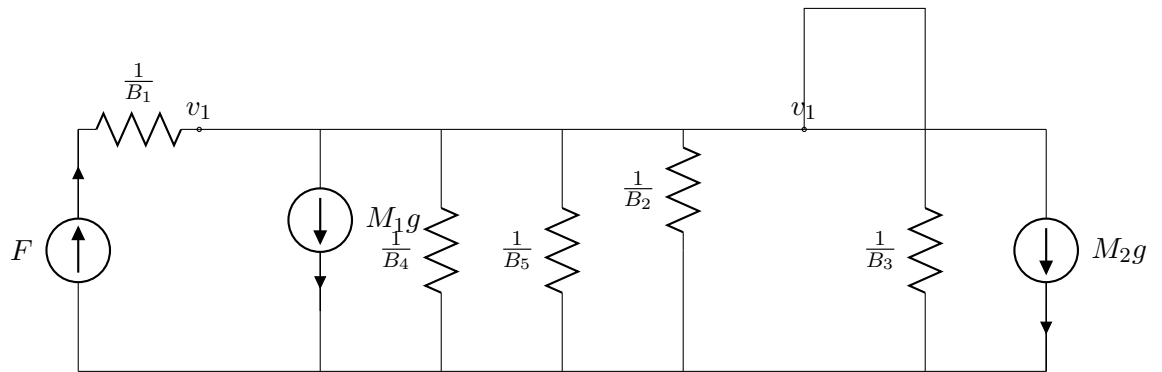
- (1) Draw an equivalent electrical diagram.
- (2) Find the velocities of the masses M_1 and M_2 in the steady state. Assume a constant force $f(t) = f_0 \delta_{-1}(t)$.

Solution 3.

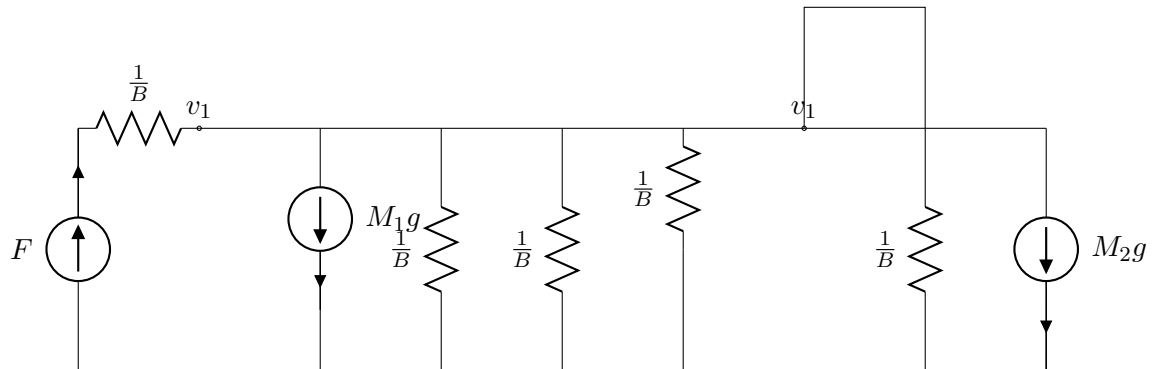
- (1) The equivalent electrical system is



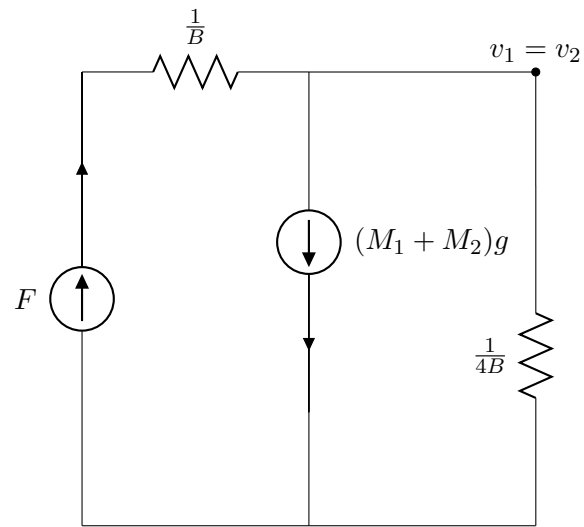
- (2) In the steady state, all capacitors are equivalent to breaks and all inductors are equivalent to shorts. Therefore, the equivalent circuit is



Therefore, substituting the given values for all B_i and k_i , the equivalent circuit is



Therefore, simplifying, the circuit is equivalent to



Therefore, the velocities of the masses are equal, and are

$$v_1 = v_2 = (F - (M_1 + M_2)g) \frac{1}{4B}$$