INTRODUCTION TO PROBABILITY AND STATISTICS
ASSIGNMENT 1

AAKASH JOG
ID : 989323563

Exercise 1.
In a certain class, the following groups were defined.

(1) A: Hebrew speaking students
(2) B: English speaking students
(3) C: Yiddish speaking students

Use the union, intersection, and complement actions to describe the following groups.

(1) Students who speak all three languages.
(2) Students who speak exactly one language.
(3) Students who speak at least one of the languages.
(4) Students who do not speak Hebrew.
(5) Students who speak exactly two languages.
(6) Students who speak at least two languages.

Which of the following groups are mutually exclusive?

Solution 1.

(1) $A \cap B \cap C$
(2) $(A \cap B^{\mathrm{C}} \cap C^{\mathrm{C}}) \cup (A^{\mathrm{C}} \cap B \cap C^{\mathrm{C}}) \cup (A^{\mathrm{C}} \cap B^{\mathrm{C}} \cap C)$
(3) $A \cup B \cup C$
(4) A^{C}
(5) $(A^{\mathrm{C}} \cap B \cap C) \cup (A \cap B^{\mathrm{C}} \cap C) \cup (A \cap B \cap C^{\mathrm{C}})$
(6) $(A \cap B) \cup (B \cap C) \cup (C \cap A)$

The following pairs of groups are mutually exclusive.

(1) 1, 2
(2) 1, 5
(3) 1, 4
(4) 2, 5
(5) 2, 6

Exercise 2.
A series of ten coin flips is performed. The possible results of each flip are 'Head' or 'Tail'. Describe with words the complement of each of the following events.

(1) 'Head' appeared at least 6 times.
(2) 'Head' appeared no more than 4 times.
(3) 'Head' never appeared.

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(4) In the first two flips, 'Head' appeared.
(5) Number of 'Head's is greater than the number of 'Tail's.
(6) Less than 8 'Tails' appeared.

Solution 2.
(1) 'Head' appeared at most 5 times.
(2) 'Head' appeared more than 4 times.
(3) 'Head' appeared at least once.
(4) In the first two flips, 'Tail' appeared at least once.
(5) Number of 'Head's is less than or equal to the number of 'Tail's.
(6) 8 or more 'Tails' appeared.

Exercise 3.
Let A and B be events such that

$$\begin{aligned} \mathrm{P}(A) &= \frac{3}{5} \\ \mathrm{P}(B) &= \frac{1}{3} \\ \mathrm{P}(A \cap B) &= \frac{1}{10} \end{aligned}$$

Calculate

(1) $\mathrm{P}(A \cup B)$
(2) $\mathrm{P}\left(A^{\mathrm{C}} \cap B^{\mathrm{C}}\right)$
(3) $\mathrm{P}\left(A^{\mathrm{C}} \cup B^{\mathrm{C}}\right)$
(4) $\mathrm{P}\left(A \cap B^{\mathrm{C}}\right)$

Solution 3.
(1)

$$\begin{aligned} \mathrm{P}(A \cup B) &= \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B) \\ &= \frac{3}{5} + \frac{1}{3} - \frac{1}{10} \\ &= \frac{5}{6} \end{aligned}$$

(2)

$$\begin{aligned} \mathrm{P}\left(A^{\mathrm{C}} \cap B^{\mathrm{C}}\right) &= \mathrm{P}\left((A \cup B)^{\mathrm{C}}\right) \\ &= 1 - \mathrm{P}(A \cup B) \\ &= 1 - \frac{5}{6} \\ &= \frac{1}{6} \end{aligned}$$

Course	Choices
Entree	Chicken or roast beef
Starch	Pasta or rice or potatoes
Dessert	Ice cream or jello or apple pie or a peach

(3)

$$\begin{aligned} \mathrm{P}\left(A^{\mathrm{C}} \cup B^{\mathrm{C}}\right) &= \mathrm{P}\left((A \cap B)^{\mathrm{C}}\right) \\ &= 1 - \mathrm{P}(A \cap B) \\ &= 1 - \frac{1}{10} \\ &= \frac{9}{10} \end{aligned}$$

(4)

$$\begin{aligned} \mathrm{P}\left(A \cap B^{\mathrm{C}}\right) &= \mathrm{P}(A) - \mathrm{P}(A \cap B) \\ &= \frac{3}{5} - \frac{1}{10} \\ &= \frac{1}{2} \end{aligned}$$

Exercise 4.
A cafeteria offers a three-course meal consisting of an entree, a starch, and a dessert. The possible choices are given in the following table. A person is to choose one course from each category.

(1) How many outcomes are in the sample space?
(2) Let A be the event that ice cream is chosen. How many outcomes are in A?
(3) Let B be the event that chicken is chosen. How many outcomes are in B?
(4) List all the outcomes in the event AB.
(5) Let C be the event that rice is chosen. How many outcomes are in C?
(6) List all the outcomes in the event ABC.

Solution 4.
(1)

$$\begin{aligned} |S| &= {}^{2}\mathrm{C}_1 \cdot {}^{3}\mathrm{C}_1 \cdot {}^{4}\mathrm{C}_1 \\ &= 2 \cdot 3 \cdot 4 \\ &= 24 \end{aligned}$$

(2)

$$\begin{aligned} |A| &= {}^{2}\mathrm{C}_1 \cdot {}^{3}\mathrm{C}_1 \\ &= 2 \cdot 3 \\ &= 6 \end{aligned}$$

Entree	Starch	Dessert
Chicken	Pasta	Ice cream
Chicken	Rice	Ice cream
Chicken	Potatoes	Ice cream

Entree	Starch	Dessert
Chicken	Rice	Ice cream

(3)

$$\begin{aligned} |B| &= {}^3\mathrm{C}_1 \cdot {}^4\mathrm{C}_1 \\ &= 3 \cdot 4 \\ &= 12 \end{aligned}$$

(4) $A \cap B$ is the event in which chicken and ice cream are chosen. Therefore, the outcomes are

(5)

$$\begin{aligned} |C| &= {}^2\mathrm{C}_1 \cdot {}^4\mathrm{C}_1 \\ &= 2 \cdot 4 \\ &= 8 \end{aligned}$$

(6) $A \cap B \cap C$ is the event in which chicken, rice, and ice cream are chosen. Therefore, the only outcome is

Exercise 5.
A customer visiting the suit department of a certian store will purchase a suit with probability 0.22, a shirt with probability 0.30, and a tie with probability 0.28. The customer wil purchase both a suit and a shirt with probability 0.11, both a suit an a tie with probability 0.14, and both a shirt and a tie with probability 0.10. A customer will purchase all three items with probability 0.06.
Find the probability that a customer purchases

(1) None of these items.
(2) Exactly one of these items?

Answer the questions both with the inclusion-exclusion identity, and a Venn diagram.

Solution 5.
(1) Let A be the event that a customer buys a suit.
Let B be the event that a customer buys a shirt
Let C be the event that a customer buys a tie.

Therefore,

$$\begin{aligned}\mathrm{P}\left((A \cup B \cup C)^{\mathrm{C}}\right) = \quad & 1 \\ & - \big(\mathrm{P}(A) + \mathrm{P}(B) + \mathrm{P}(C)\big) \\ & - \big(-\mathrm{P}(A \cap B) - \mathrm{P}(B \cap C) - \mathrm{P}(C \cap A)\big) \\ & - \big(\mathrm{P}(A \cap B \cap C)\big) \\ = \ & 1 - (0.22 + 0.30 + 0.28 - 0.11 - 0.14 - 0.10 + 0.06) \\ = \ & 1 - (0.51) \\ = \ & 0.49\end{aligned}$$

(2) Let the required outcome be D. Therefore,

$$\begin{aligned}\mathrm{P}(D) &= \mathrm{P}\Big(\big(A \setminus (B \cup C)\big) \cup \big(B \setminus (C \cup A)\big) \cup \big(C \setminus (A \cup B)\big)\Big) \\ &= \mathrm{P}(A \cup B \cup C) - \mathrm{P}(A \cap B) - \mathrm{P}(B \cap C) - \mathrm{P}(C \cap A) + 2\,\mathrm{P}(A \cap B \cap C) \\ &= 0.51 - 0.11 - 0.14 - 0.10 + 2(0.06) \\ &= 0.28\end{aligned}$$