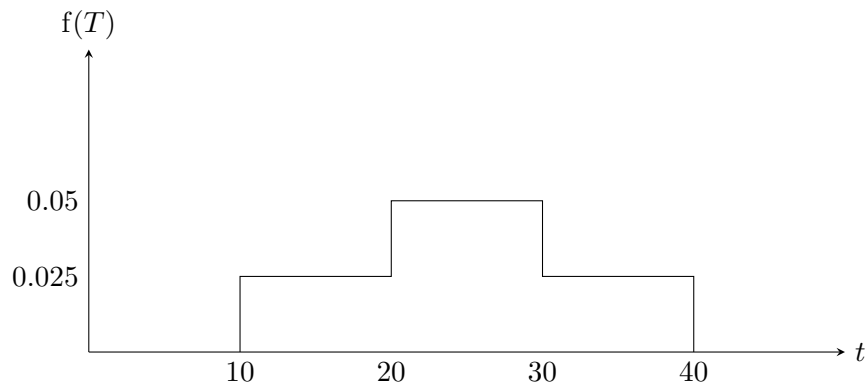


**INTRODUCTION TO PROBABILITY AND STATISTICS
ASSIGNMENT 6**

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Exercise 1.

The number of minutes of playing time of a certain high school basketball player in a randomly chosen game is a random variable whose probability density function is given.



Find the probability that the player plays

- (1) over 15 minutes.
- (2) between 20 and 35 minutes.
- (3) less than 30 minutes.
- (4) more than 36 minutes.

Solution 1.

(1)

$$\begin{aligned} P(T > 15) &= \int_{15}^{\infty} f(T) dt \\ &= 0.875 \end{aligned}$$

(2)

$$\begin{aligned} P(20 < T < 35) &= \int_{20}^{35} f(T) dt \\ &= 0.625 \end{aligned}$$

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(3)

$$\begin{aligned} P(T < 30) &= \int_{30}^{\infty} f(T) dt \\ &= 0.75 \end{aligned}$$

(4)

$$\begin{aligned} P(T > 36) &= \int_{36}^{\infty} f(T) dt \\ &= 0.1 \end{aligned}$$

Exercise 2.

For some constant c , the random variable X has probability density function

$$f(x) = \begin{cases} cx^4 & ; \quad 0 < x < 2 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Find $E[X]$ and $V(X)$.

Solution 2.

As $f(x)$ is a probability density function,

$$\begin{aligned} 1 &= \int_{-\infty}^{\infty} f(x) dx \\ &= \int_0^2 cx^4 dx \\ &= c \left(\frac{32}{5} \right) \end{aligned}$$

Therefore,

$$c = \frac{5}{32}$$

Therefore,

$$\begin{aligned} E[X] &= \int_{-\infty}^{\infty} x f(x) dx \\ &= \frac{5}{32} \int_0^2 x^5 dx \\ &= \frac{5}{32} \frac{64}{6} \\ &= \frac{5}{3} \end{aligned}$$

Therefore,

$$\begin{aligned} E[g(X)] &= \int_{-\infty}^{\infty} g(x) f(x) dx \\ \therefore E[X^2] &= \frac{5}{32} \int_0^2 x^2 x^4 dx \\ &= \frac{5}{32} \int_0^2 x^6 dx \\ &= \frac{5}{32} \frac{128}{7} \\ &= \frac{20}{7} \end{aligned}$$

Therefore,

$$\begin{aligned} V(X) &= E[X^2] - E[X]^2 \\ &= \frac{20}{7} - \frac{25}{9} \\ &= \frac{5}{63} \end{aligned}$$

Exercise 3.

Your company must make a scaled bid for a construction project. If you succeed in winning the contract by having the lowest bid, then you plan to pay another firm \$100,000 to do the work. If you believe that the minimum bid of the other participating companies can be modelled as the value of a random variable that is uniformly distributed on $(70, 140)$, how much should you bid to maximize your profit?

Exercise 4.

At a certain bank, the amount of time that a customer spends being served by a teller is an exponential random variable with mean of 5 minutes. If there is a customer in service when you enter the bank, what is the probability that he will still be with the teller after an additional 4 minutes?

Exercise 5.

Alice is leaving work in a uniformly distributed time between 7 and 9, i.e.

$$X \sim U(7, 9)$$

The time it takes her to commute home is

$$Y = 1 + \frac{1}{X}$$

Find the density and distribution of Y .

Exercise 6.

A randomly chosen IQ test taker obtains a score that is approximately a normal random variable with mean 100 and standard deviation 15. Find the probability that the score of such a person is

- (1) above 125.
- (2) between 90 and 110.

Solution 6.

- (1) Let X be the obtained score.

Let

$$\begin{aligned} Z &= \frac{X - \mu}{\sigma} \\ &= \frac{X - 100}{15} \end{aligned}$$

Therefore, Z is a standard normal random variable. Therefore,

$$\begin{aligned} P(X > 125) &= P\left(Z > \frac{125 - 100}{15}\right) \\ &= P\left(\frac{5}{3}\right) \\ &= 1 - \Phi\left(\frac{5}{3}\right) \end{aligned}$$

- (2) Let X be the obtained score.

Let

$$\begin{aligned} Z &= \frac{X - \mu}{\sigma} \\ &= \frac{X - 100}{15} \end{aligned}$$

Therefore, Z is a standard normal random variable. Therefore,

$$\begin{aligned} P(90 < X < 100) &= P\left(\frac{90 - 100}{15} < Z < \frac{110 - 100}{15}\right) \\ &= P\left(\frac{-2}{3} < Z < \frac{2}{3}\right) \\ &= \Phi\left(\frac{2}{3}\right) - \Phi\left(-\frac{2}{3}\right) \\ &= \Phi\left(\frac{2}{3}\right) - \left(1 - \Phi\left(\frac{2}{3}\right)\right) \\ &= 2\Phi\left(\frac{2}{3}\right) - 1 \end{aligned}$$

Exercise 7.

The life of a certain type of automobile tyre is normally distributed with mean 34,000 miles and standard deviation 4,000 miles.

- (1) What is the probability that such a tyre lasts over 40,000 miles?

- (2) What is the probability that such a tyre lasts between 30,000 and 35,000 miles?
- (3) Given that such a tyre has survived 30,000 miles, what is the conditional probability that the tyre survives another 10,000 miles?

Solution 7.

- (1) Let X be the life of the tyre.

Let

$$\begin{aligned} Z &= \frac{X - \mu}{\sigma} \\ &= \frac{X - 34000}{4000} \end{aligned}$$

Therefore, Z is a standard normal random variable. Therefore,

$$\begin{aligned} P(X > 40000) &= P\left(Z > \frac{40000 - 34000}{4000}\right) \\ &= P\left(Z > \frac{3}{2}\right) \\ &= 1 - \Phi\left(\frac{3}{2}\right) \end{aligned}$$

- (2) Let X be the life of the tyre.

Let

$$\begin{aligned} Z &= \frac{X - \mu}{\sigma} \\ &= \frac{X - 34000}{4000} \end{aligned}$$

Therefore, Z is a standard normal random variable. Therefore,

$$\begin{aligned} P(30000 < X < 35000) &= P\left(\frac{30000 - 34000}{4000} < Z < \frac{35000 - 34000}{4000}\right) \\ &= P\left(-1 < Z < \frac{1}{4}\right) \\ &= \Phi\left(\frac{1}{4}\right) - \Phi(-1) \\ &= \Phi\left(\frac{1}{4}\right) - (1 - \Phi(1)) \\ &= \Phi\left(\frac{1}{4}\right) + \Phi(1) - 1 \end{aligned}$$

- (3) Let X be the life of the tyre.

Let

$$\begin{aligned} Z &= \frac{X - \mu}{\sigma} \\ &= \frac{X - 34000}{4000} \end{aligned}$$

Therefore, Z is a standard normal random variable. Therefore,

$$\begin{aligned}
 P(X > 30000) &= P\left(Z > \frac{30000 - 34000}{4000}\right) \\
 &= P(Z > -1) \\
 &= 1 - \Phi(1) \\
 P(X > 40000) &= P\left(Z > \frac{40000 - 34000}{4000}\right) \\
 &= P\left(Z > \frac{3}{2}\right) \\
 &= 1 - \Phi\left(\frac{3}{2}\right)
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 P(X > 40000 | X > 30000) &= \frac{P(X > 40000 \cap X > 30000)}{P(X > 30000)} \\
 &= \frac{P(X > 40000)}{P(X > 30000)} \\
 &= \frac{1 - \Phi\left(\frac{3}{2}\right)}{1 - \Phi(1)}
 \end{aligned}$$

Exercise 8.

The annual rainfall in Cleveland, Ohio, is approximately a normal random variable with mean 40.2 inches and standard deviation 8.4 inches. Let A_i be the event that the rainfall in the next i th year exceeds 44 inches, and fume that all A_i are independent. Find the probability that

- (1) next year's rainfall will exceed 44 inches?
- (2) the yearly rainfall in exactly 3 of the next 7 years will exceed 44 inches?

Solution 8.

- (1) Let X be the rainfall in a year.

Let

$$\begin{aligned}
 Z &= \frac{X - \mu}{\sigma} \\
 &= \frac{X - 40.2}{8.4}
 \end{aligned}$$

Therefore, Z is a standard normal random variable. Therefore,

$$\begin{aligned} P(X > 44) &= P\left(Z > \frac{44 - 30.2}{8.4}\right) \\ &= P\left(Z > \frac{13.8}{8.4}\right) \\ &= P\left(Z > \frac{23}{14}\right) \\ &= 1 - \Phi\left(\frac{23}{14}\right) \end{aligned}$$

- (2) For any year, as the rainfall in every year is independent of the previous years, the probability that the rainfall will exceed 44 inches is

$$P(X > 44) = 1 - \Phi\left(\frac{23}{14}\right)$$

Therefore, the probability that the rainfall will exceed 44 inches in exactly 3 of the next 7 years is $\frac{7}{3} \left(1 - \Phi\left(\frac{23}{14}\right)\right)^3 \left(\Phi\left(\frac{23}{14}\right)\right)^4$.