INTRODUCTION TO PROBABILITY AND STATISTICS
ASSIGNMENT 3

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Exercise 1.
You ask your neighbour to water a sickly plant while you are on vacation.
Without water, it will die with probability 0.8.
With water, it will die with probability 0.15.
You are 90% certain that your neighbour will remember to water the plant.

(1) What is the probability that the plant will be alive when you return?
(2) If the plant is dead upon your return, what is the probability that your neighbour forgot to water it?

Solution 1.
(1) The probability of the plant being alive is $(0.9)(0.85) + (0.1)(0.2)$.
Therefore, the probability of the plant being alive is 0.785.
(2) The total probability that the plant will be dead is 0.215.
The probability that the plant will be dead and the neighbour watered it is $(0.9)(0.15)$.
The probability that the plant will be dead and the neighbour did not water it is $(0.1)(0.8)$. Therefore, if the plant is dead, the probability that the neighbour did not water it is $\frac{0.08}{0.215}$.

Exercise 2.
Six balls are to be randomly chosen from an urn containing 8 red, 10 green, and 12 blue balls.
(1) What is the probability that at least one red ball is chosen?
(2) Given that no red balls are chosen, what is the conditional probability that there are exactly 2 green balls among the 6 chosen balls?

Solution 2.
(1) The total number of ways to choose 6 balls is ${}^{30}\mathrm{C}_6$.
The number of ways to choose 0 red balls is ${}^{22}\mathrm{C}_6$.
Therefore, the probability of selecting 0 red balls is $\frac{{}^{22}\mathrm{C}_6}{{}^{30}\mathrm{C}_6}$. Therefore, the probability of selecting at least 1 red ball is $1 - \frac{{}^{22}\mathrm{C}_6}{{}^{30}\mathrm{C}_6}$.
(2) If 0 red balls are chosen and exactly 2 green balls are chosen, the remaining 4 balls must be blue.
Therefore, the number of ways of choosing 2 green balls and 4 blue balls is ${}^{10}\mathrm{C}_2 {}^{12}\mathrm{C}_4$.

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The total number of ways to choose 6 non-red balls is ${}^{22}\mathrm{C}_6$.
Therefore, the probability is $\frac{{}^{10}\mathrm{C}_2 {}^{12}\mathrm{C}_4}{{}^{22}\mathrm{C}_6}$.

Exercise 3.
A coin having probability 0.8 of landing on 'Heads' is flipped. Alice observes the result, and rushes off to tell Bob. However, with probability 0.4, Alice will have forgotten the result by the time she reaches Bob. If Alice has forgotten, then rather than admitting this to Bob, she is equally likely to tell Bob that the coin landed on 'Heads' or that it landed on 'Tails'. If she does remember, she tells Bob the correct result.

(1) What is the probability that Bob is told that the coin landed on heads?
(2) What is the probability that Bob is told the correct result?
(3) Given that Bob is told that the coin landed on heads, what is the probability that it did in fact land on 'Heads'?

Solution 3.
(1) If Alice forgets the result, the probability that Bob is told that the result is 'Heads' is 0.5.
If Alice remembers the result, the probability that Bob is told that the result is 'Heads' is 0.8
Therefore, the total probability that Bob is told that the result is 'Heads' is $(0.4)(0.5)+(0.6)(0.8)$. Therefore, the probability is 0.68.
(2) The probability that Bob is told the correct result if Alice remembers the result is 1.
The probability that Bob is told the correct result if Alice forgets the result is 0.5.
Therefore, the probability that Bob is told the correct result is $(0.6)(1)+(0.4)(0.5)$ Therefore, the probability is 0.8.

Exercise 4.
In the urn, there are 5 red and 4 white balls. You sequentially draw 3 balls randomly without replacement. What is the probability that all balls are white?

Solution 4.
The total number of ways to select 3 balls from the urn is ${}^{9}\mathrm{C}_3$.
The number of ways to select 3 white balls from 4 white balls is ${}^{4}\mathrm{C}_3$.
Therefore, the probability is $\frac{{}^{4}\mathrm{C}_3}{{}^{9}\mathrm{C}_3}$.

Exercise 5.
Let A and B be events having positive probability. State whether each of the following statements is necessarily true, necessarily false, or possibly true.

(1) If A and B are mutually exclusive, then they are independent.
(2) If A and B are independent, then they are mutually exclusive.
(3) $\mathrm{P}(A)=\mathrm{P}(B)=0.6$, and A and B are mutually exclusive.
(4) $\mathrm{P}(A)=\mathrm{P}(B)=0.6$, and A and B are independent.

Solution 5.

(1) If A and B are mutually exclusive,

$$\mathrm{P}(A \cap B) = 0$$

A and B can be independent if and only if

$$\begin{aligned} & \mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B) \\ \iff\ & 0 = \mathrm{P}(A)\,\mathrm{P}(B) \end{aligned}$$

However, as A and B have positive probability, this statement is necessarily false.

(2) If A and B are independent,

$$\begin{aligned} & \mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B) \\ \therefore\ & \mathrm{P}(A \cap B) > 0 \end{aligned}$$

A and B can be mutually exclusive if and only if

$$\begin{aligned} & \mathrm{P}(A)\,\mathrm{P}(B) = 0 \\ \iff\ & \mathrm{P}(A \cap B) = 0 \end{aligned}$$

However, as A and B have positive probability, this statement is necessarily false.

(3) A and B are mutually exclusive if and only if

$$\begin{aligned} & \mathrm{P}(A) + \mathrm{P}(B) \leq 1 \\ \iff\ & 0.6 + 0.6 \leq 1 \end{aligned}$$

Therefore, this statement is necessarily false.

(4) A and B are independent if and only if

$$\begin{aligned} & \mathrm{P}(A)\,\mathrm{P}(B) = \mathrm{P}(A \cap B) \\ \iff\ & (0.6)(0.6) = \mathrm{P}(A \cap B) \\ \iff\ & \mathrm{P}(A \cap B) = 0.36 \end{aligned}$$

Therefore, this statement is possibly true.

Exercise 6.

Arrange the following from most likely to occur to least likely to occur.

(1) A fair coin lands on ‘Heads’.
(2) Three independent trials, each of which is a success with probability 0.8, all result in successes.
(3) Seven independent trials, each of which is a success with probability 0.9, all result in successes.

Solution 6.

The probability that a fair coin lands on ‘Heads’ is 0.5.
The probability that three independent trials with probability of success 0.8 all result in successes is $(0.8)^3$. Therefore, the probability is 0.512.
The probability that seven independent trials with probability of success 0.9 all result in successes is $(0.9)^7$. Therefore, the probability is 0.4782969.
Therefore, the descending order of probabilities is

(1) Three independent trials, each of which is a success with probability 0.8, all result in successes.
(2) A fair coin lands on ‘Heads’.
(3) Seven independent trials, each of which is a success with probability 0.9, all result in successes.