INTRODUCTION TO SIGNAL ANALYSIS
ASSIGNMENT 1

AAKASH JOG
ID : 989323563

Exercise 1.
Sort the following systems based on linearity, time invariance, memory, causality, invertibility, and stability.

(1) $y(t) = 1 + x(t)\cos(\omega t)$
(2) $y(t) = \sum\limits_{n=-\infty}^{\infty} x(t)\delta(t - nT)$
(3) $y(t) = \int\limits_{-\infty}^{\infty} \sin(t + \tau)x(\tau)\,\mathrm{d}\tau$

Solution 1.
(1) (a)

$$\begin{aligned} H\left\{x(t)\right\} &= y(t) \\ &= 1 + x(t)\cos(\omega t) \end{aligned}$$

Therefore,

$$\begin{aligned} H\left\{ax_1(t) + bx_2(t)\right\} &= 1 + \big(ax_1(t) + bx_2(t)\big)\cos(\omega t) \\ &= 1 + ax_1(t)\cos(\omega t) + bx_2(t)\cos(\omega t) \\ &\neq aH\left\{x_1(t)\right\} + bH\left\{x_2(t)\right\} \end{aligned}$$

Therefore, the system is non-linear.

(b)

$$\begin{aligned} H\left\{x(t)\right\} &= y(t) \\ &= 1 + x(t)\cos(\omega t) \end{aligned}$$

Therefore,

$$\begin{aligned} H\left\{x(t - t_0)\right\} &= 1 + x(t - t_0)\cos(\omega t) \\ &= 1 + x(t - t_0)\cos(\omega t) \\ &\neq y(t - t_0) \end{aligned}$$

Therefore, the system is time-variant.

(c)

$$y(t) = 1 + x(t)\cos(\omega t)$$

Therefore, as $y(t_0)$ is dependent only on $x(t_0)$, the system is memoryless.

Date: Wednesday 23^{rd} March, 2016.

(d)

$$y(t) = 1 + x(t)\cos(\omega t)$$

Therefore, as $y(t_0)$ is independent of $x(t > t_0)$, the system is causal.

(e)

$$\begin{aligned} y(t) &= 1 + x(t)\cos(\omega t) \\ \therefore x(t) &= \frac{y(t) - 1}{\cos(\omega t)} \end{aligned}$$

Therefore, the system is invertible.

(f) Let $x(t)$ be bounded. Therefore, as $\cos(\omega t)$ is also bounded, $x(t)\cos(\omega t)$ is bounded. Therefore, as $y(t)$ is the sum of a finite number and a bounded function, it is also bounded. Therefore, the system is BIBO stable.

(2) (a)

$$\begin{aligned} H\left\{x(t)\right\} &= y(t) \\ &= \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT) \end{aligned}$$

Therefore,

$$\begin{aligned} H\left\{ax_1(t) + bx_2(t)\right\} &= \sum_{n=-\infty}^{\infty} \big(ax_1(t) + bx_2(t)\big)\,\delta(t - nT) \\ &= a\sum_{n=-\infty}^{\infty} x_1\delta(t - nT) + b\sum_{n=-\infty}^{\infty} x_2(t)\delta(t - nT) \\ &= aH\left\{x_1(t)\right\} + bH\left\{x_2(t)\right\} \end{aligned}$$

Therefore, the system is linear.

(b)

$$\begin{aligned} H\left\{x(t)\right\} &= y(t) \\ &= \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT) \end{aligned}$$

Therefore,

$$\begin{aligned} H\left\{x(t - t_0)\right\} &= \sum_{n=-\infty}^{\infty} x(t - t_0)\delta(t - nT) \\ &\neq y(t - t_0) \end{aligned}$$

Therefore, the system is time-variant.

(c)

$$y(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT)$$

Therefore, as $y(t_0)$ is dependent only on $x(t_0)$, the system is memoryless.

(d)

$$y(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT)$$

Therefore, as $y(t_0)$ is independent of $x(t > t_0)$, the system is causal.

(e)

$$y(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT)$$

Therefore, as $x(t)$ cannot be written in tersm of $y(t)$, the system is not invertible.

(f) Let $x(t)$ be bounded. However, as the Dirac delta function is not bounded, $y(t)$ is also unbounded. Therefore, the system is BIBO unstable.

(3) (a)

$$\begin{aligned} H\left\{x(t)\right\} &= y(t) \\ &= \int\limits_{-\infty}^{\infty} \sin(t + \tau)x(\tau)\,\mathrm{d}\tau \end{aligned}$$

Therefore,

$$\begin{aligned} H\left\{ax_1(t) + bx_2(t)\right\} &= \int\limits_{-\infty}^{\infty} \sin(t + \tau)\left(ax_1(\tau) + bx_2(\tau)\right)\mathrm{d}\tau \\ &= a\int\limits_{-\infty}^{\infty} \sin(t + \tau)x_1(\tau)\,\mathrm{d}\tau + b\int\limits_{-\infty}^{\infty} \sin(t + \tau)x_2(\tau)\,\mathrm{d}\tau \\ &= aH\left\{x_1(t)\right\} + bH\left\{x_2(t)\right\} \end{aligned}$$

Therefore, the system is linear.

(b)

$$\begin{aligned} H\left\{x(t)\right\} &= y(t) \\ &= \int\limits_{-\infty}^{\infty} \sin(t + \tau)x(\tau)\,\mathrm{d}\tau \end{aligned}$$

Therefore,

$$\begin{aligned} H\left\{x(t - t_0)\right\} &= \int\limits_{-\infty}^{\infty} \sin(t + \tau)x(\tau - t_0)\,\mathrm{d}\tau \\ &\neq y(t - t_0) \end{aligned}$$

Therefore, the system is time-variant.

(c)

$$y(t) = \int\limits_{-\infty}^{\infty} \sin(t + \tau)x(\tau)\,\mathrm{d}\tau$$

Therefore, as $y(t_0)$ is dependent on $x(t < t_0)$, the system is not memoryless.

(d)

$$y(t) = \int\limits_{-\infty}^{\infty} \sin(t+\tau)x(\tau)\,\mathrm{d}\tau$$

Therefore, as $y(t_0)$ is dependent on $x(t > t_0)$, the system is causal.

(e)

$$\begin{aligned} y(t) &= \int\limits_{-\infty}^{\infty} \sin(t+\tau)x(\tau)\,\mathrm{d}\tau \\ \therefore \frac{\mathrm{d}y(t)}{\mathrm{d}\tau} &= \sin(t+\tau)x(\tau) \\ \therefore x(\tau) &= \frac{\frac{\mathrm{d}y(t)}{\mathrm{d}\tau}}{\sin(t+\tau)} \end{aligned}$$

Therefore, the system is invertible.

(f) Let $x(t)$ be bounded. Therefore, $\sin(t+\tau)x(\tau)$ is also bounded. However, as the integral is from $-\infty$ to ∞, $y(t)$ is not bounded. Therefore, the system is BIBO unstable.

Exercise 2.

Is the following system linear time invariant?

$$y(t) = \begin{cases} 0 & ; \quad t < T_1 \\ 2x(t) & ; \quad t \geq T_1 \end{cases}$$

Solution 2.

$$y(t) = \begin{cases} 0 & ; \quad t < t_1 \\ 2x(t) & ; \quad t \geq T_1 \end{cases}$$

Therefore,

$$\begin{aligned} H\left\{x(t)\right\} &= y(t) \\ &= \begin{cases} 0 & ; \quad t < T_1 \\ 2x(t) & ; \quad t \geq T_1 \end{cases} \end{aligned}$$

Therefore,

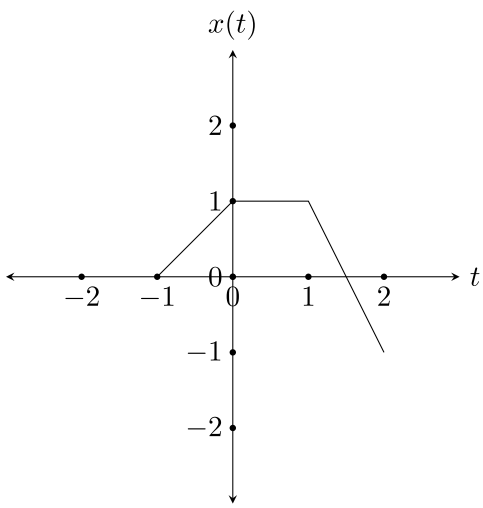
$$\begin{aligned} H\left\{ax_1(t) + bx_2(t)\right\} &= \begin{cases} 0 & ; \quad t < T_1 \\ 2\left(ax_1(t) + bx_2(t)\right) & ; \quad t \geq T_1 \end{cases} \\ &= \begin{cases} 0 & ; \quad t < T_1 \\ 2ax_1(t) + 2bx_2(t) & ; \quad t \geq T_1 \end{cases} \\ &= aH\left\{x_1(t)\right\} + bH\left\{x_2(t)\right\} \end{aligned}$$

Therefore, the system is linear.

$$
\begin{aligned}
H\left\{x(t)\right\} &= \begin{cases} 0 & ; \quad t < T_1 \\ 2x(t) & ; \quad t \geq T_1 \end{cases} \\
&= 2x(t)\delta_{-1}(t - T_1)
\end{aligned}
$$

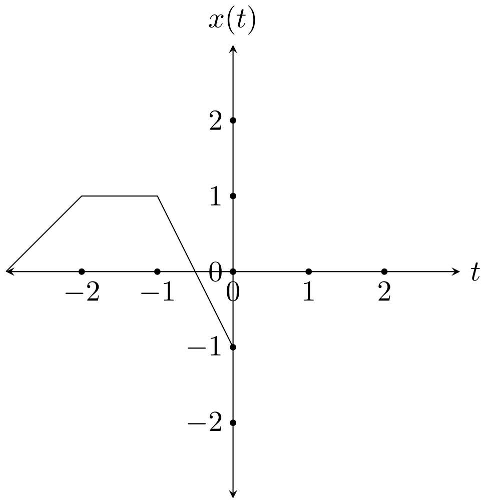
Therefore, the system in time-variant. Therefore, the system is not a LTI system.

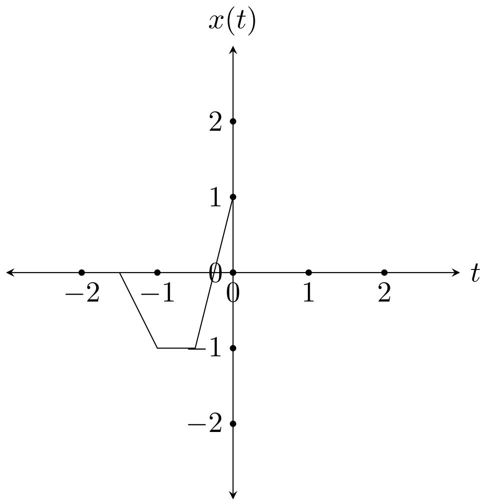
Exercise 3.
Consider the following signal.



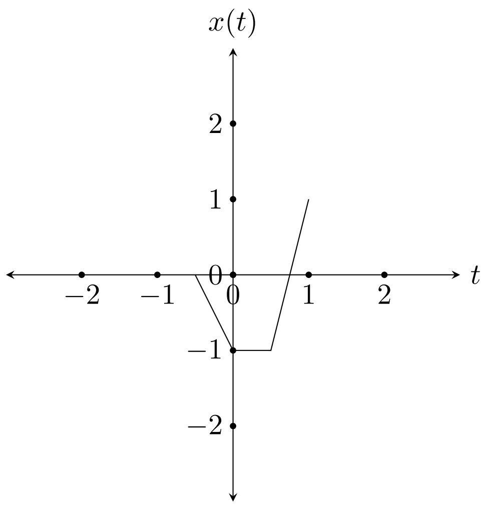
(1) Draw $x(-2t + 2)$ by shifting and then downscaling the time axis. Point out the shifting and downscaling parameters.
(2) Draw $x(-2t + 2)$ by downscaling and then shifting the time axis. Point out the downscaling and shifting parameters.

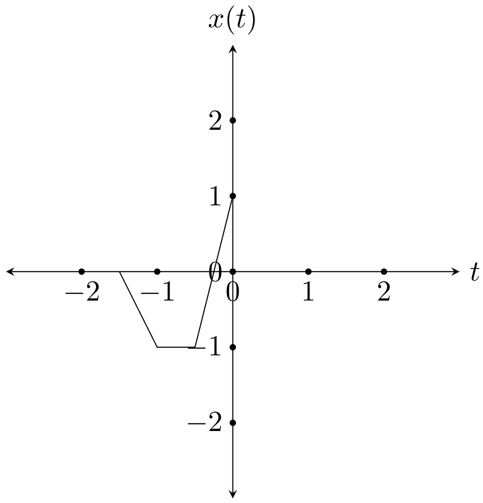
Solution 3.
(1)



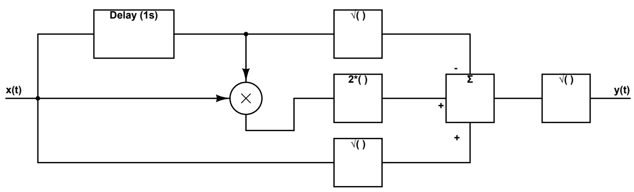


(2)





Exercise 4.
Consider a system T as shown.



(1) Find the expression that represents $y(t)$ as a function of $x(t)$.
(2) Is the system linear?
(3) Is the system time invariant?
(4) Is the system stable?

Solution 4.
(1)

$$y(t) = \sqrt{-\sqrt{x(t-1)} + 2\left(x(t-1)x(t)\right) + \sqrt{x(t)}}$$

(2) As $y(t)$ is dependent on $x(t-1)x(t)$.
(3) As the system is not dependent on the absolute time, it is time invariant.

(4) For a bounded $x(t)$, $y(t)$ is also bounded. Therefore, the system is stable.

Exercise 5.
Determine the stability for the following LTI systems using their impulse response.

(1) $h[n] = (-0.9)^n u[n] + (1.01)^n u[n-1]$
(2) $h[n] = \left(\frac{1}{a}\right)^n u[-n]$
(3) $h(t) = e^{-5t} u(1-t)$

Solution 5.
(1) Comparing to the standard form, the roots of the characteristic equation are -0.9 and 1.01. Therefore, as the magnitude of one of them is greater than 1, the system is unstable.
(2) Comparing to the standard for, the root of the characteristic equation is $\frac{1}{a}$. Therefore, the system is stable if and only if

$$a > 1$$

(3) Comparing to the standard form, the root of the characteristic equation is -5. Therefore, as the root is negative, the system is stable.

Exercise 6.
Consider

$$x(t) = u(t+0.25) - u(t-0.25)$$
$$h(t) = e^{j\omega t}$$
$$y(t) = x(t) * h(t)$$

(1) Find ω such that $y(0) = 0$.
(2) Is the solution above unique?

Solution 6.

$$
\begin{aligned}
y(t) &= x(t) * h(t) \\
&= \int_{-\infty}^{\infty} \big(u(\tau + 0.25) - u(\tau - 0.25)\big)\, e^{j\omega(t-\tau)} \,\mathrm{d}\tau \\
&= e^{j\omega t} \int_{-\infty}^{\infty} u(\tau + 0.25) e^{j\omega\tau} \,\mathrm{d}\tau + e^{j\omega t} \int_{-\infty}^{\infty} u(\tau - 0.25) e^{j\omega\tau} \,\mathrm{d}\tau \\
&= e^{j\omega t} \int_{-0.25}^{t} e^{j\omega\tau} \,\mathrm{d}\tau + e^{j\omega t} \int_{0.25}^{t} e^{j\omega\tau} \,\mathrm{d}\tau \\
&= e^{j\omega t} \left. \frac{e^{j\omega\tau}}{j\omega} \right|_{-0.25}^{t} + e^{j\omega t} \left. \frac{e^{j\omega\tau}}{j\omega} \right|_{0.25}^{t} \\
&= e^{j\omega t} \left(\frac{e^{j\omega t} - e^{-0.25 j\omega}}{j\omega} \right) + e^{j\omega t} \left(\frac{e^{j\omega t} - e^{0.25 j\omega}}{j\omega} \right) \\
&= \frac{1}{j\omega} \left(e^{2j\omega t} - e^{j\omega(t-0.25)} + e^{2j\omega t} - e^{j\omega(t+0.25)} \right)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
y(0) &= \frac{1}{j\omega} \left(e^{0} - e^{-0.25 j\omega} + e^{0} - e^{0.25 j\omega} \right) \\
&= \frac{1}{j\omega} \left(2 - e^{0.25 j\omega} - e^{0.25 j\omega} \right)
\end{aligned}
$$

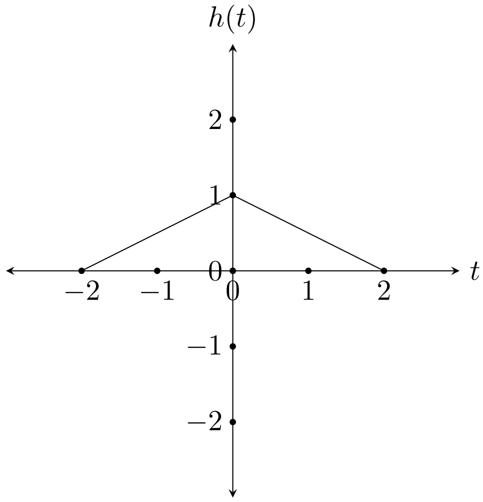
Therefore,

$$
\begin{aligned}
& y(0) = 0 \\
\iff & \frac{1}{j\omega} \left(2 - e^{-0.25 j\omega} - e^{0.25 j\omega} \right) = 0 \\
\iff & 2 - e^{-0.25 j\omega} - e^{0.25 j\omega} = 0 \\
\iff & e^{-0.25 j\omega} + e^{0.25 j\omega} = 2 \\
\iff & e^{j\omega} \left(e^{-0.25} + e^{0.25} \right) = 2 \\
\iff & e^{j\omega} = \frac{2}{e^{-0.25} + e^{0.25}} \\
\iff & j\omega = \ln \left(\frac{2}{e^{-0.25} + e^{0.25}} \right) \\
\iff & \omega = -j \ln \left(\frac{2}{e^{-0.25} + e^{0.25}} \right)
\end{aligned}
$$

This solution is unique.

Exercise 7.
Consider the following system and input.

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$



Draw $y(t) = x(t) * h(t)$, the output of the system, for the following values of T.

(1) $T = 4$
(2) $T = 3$
(3) $T = 1$

Solution 7.
(1)

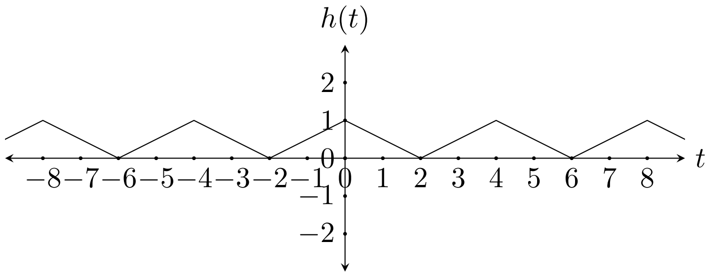
$$T = 4$$

Therefore,

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - 4k)$$

Therefore,

$$\begin{aligned} y(t) &= x(t) * h(t) \\ &= h(t) * \sum_{k=-\infty}^{\infty} \delta(t - 4k) \\ &= \sum_{k=-\infty}^{\infty} h(t - 4k) \end{aligned}$$



(2)

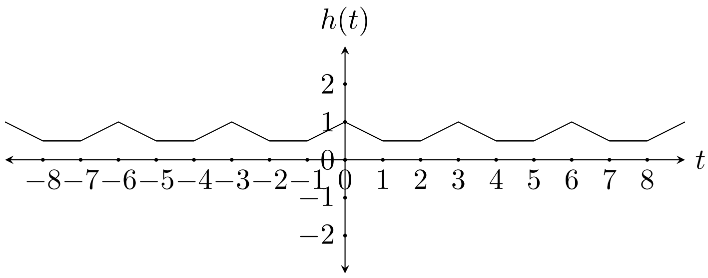
$$T = 3$$

Therefore,

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - 3k)$$

Therefore,

$$\begin{aligned} y(t) &= x(t) * h(t) \\ &= h(t) * \sum_{k=-\infty}^{\infty} \delta(t - 3k) \\ &= \sum_{k=-\infty}^{\infty} h(t - 3k) \end{aligned}$$



(3)

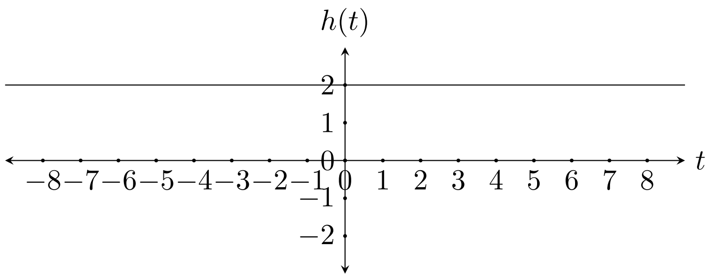
$$T = 1$$

Therefore,

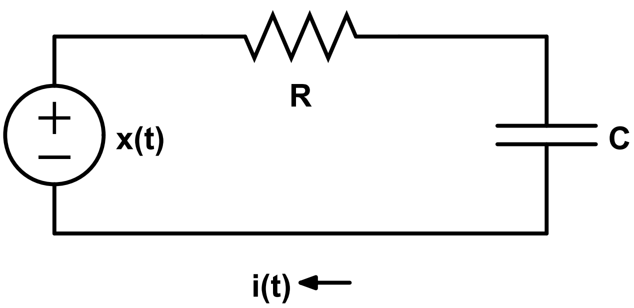
$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - k)$$

Therefore,

$$
\begin{aligned}
y(t) &= x(t) * h(t) \\
&= h(t) * \sum_{k=-\infty}^{\infty} \delta(t-k) \\
&= \sum_{k=-\infty}^{\infty} h(t-k)
\end{aligned}
$$



Exercise 8.
Consider the following RC circuit.



(1) Using Kirchoff's Laws, find a differential equation of the voltage of the capacitor $y(t)$, as a function of the original voltage $x(t)$.
(2) Under the assumption of initial time $-\infty$, solve the differential equation. The solution should be represented as an integral that depends on the input $x(t)$.
(3) Find the impulse response.
(4) Find the unit step response.

(5) Find the frequency response and draw the phase and amplitude of the system. Is it HP/BP/LP?
(6) Derive the frequency response from voltage division considerations using impedances.

Solution 8.
(1)

$$\begin{aligned} y(t) &= x(t) - i(t)R \\ \therefore y(t) &= x(t) - RC\frac{\mathrm{d}y(t)}{\mathrm{d}t} \\ \therefore y'(t) + \frac{1}{RC}y(t) &= \frac{1}{RC}x(t) \end{aligned}$$

(2)

$$y'(t) + \frac{1}{RC}y(t) = \frac{1}{RC}x(t)$$

Therefore, the integrating factor is

$$\begin{aligned} \mu(t) &= e^{\int \frac{1}{RC}\,\mathrm{d}t} \\ &= e^{\frac{t}{RC}} \end{aligned}$$

Therefore,

$$\begin{aligned} y(t) &= \frac{1}{\mu(t)}\int\limits_{-\infty}^{t} \mu(t)\frac{1}{RC}x(t)\,\mathrm{d}t \\ &= \frac{e^{-\frac{t}{RC}}}{RC}\int\limits_{-\infty}^{t} e^{\frac{t}{RC}}x(t)\,\mathrm{d}t \end{aligned}$$

(3)

$$\begin{aligned} h(t) &= \frac{e^{-\frac{t}{RC}}}{RC}\int\limits_{-\infty}^{t} e^{\frac{t}{RC}}x(t)\,\mathrm{d}t \\ &= \frac{e^{-\frac{t}{RC}}}{RC}\int\limits_{-\infty}^{t} e^{\frac{t}{RC}}\delta(t)\,\mathrm{d}t \\ &= \frac{e^{-\frac{t}{RC}}}{RC}e^{0} \\ &= \frac{e^{-\frac{t}{RC}}}{RC} \end{aligned}$$

(4) Let

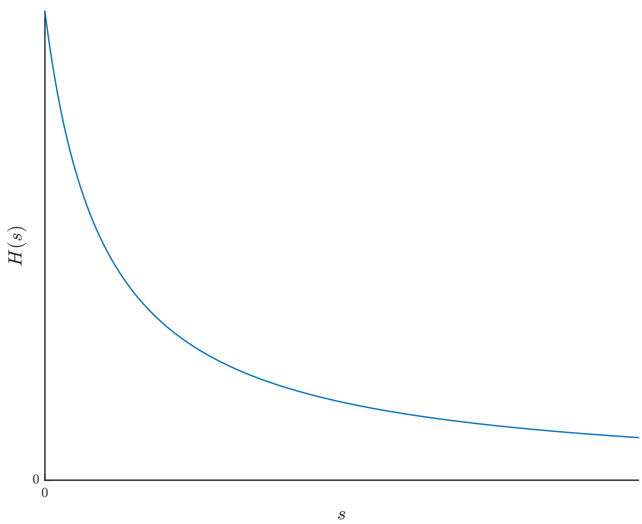
$$x(t) = u(t)$$

Therefore,

$$\begin{aligned} y(t) &= \int_0^t h(t)\,\mathrm{d}t \\ &= \int_0^t \frac{e^{-\frac{t}{RC}}}{RC} \\ &= -\frac{e^{-\frac{t}{RC}}}{R^2C^2}\bigg|_0^t \\ &= \frac{1+e^{-\frac{t}{RC}}}{R^2C^2} \end{aligned}$$

(5)

$$\begin{aligned} H(s) &= \mathcal{L}\left\{h(t)\right\} \\ &= \mathcal{L}\left\{\frac{e^{-\frac{t}{RC}}}{RC}\right\} \\ &= \frac{1}{1+RCs} \end{aligned}$$



Therefore, it is a low pass filter.

(6)

$$\begin{aligned}
X(s) &= I(s)R + Y(s) \\
&= sY(s)CR + Y(s) \\
\therefore Y(s) &= \frac{X(s)}{1 + sCR} \\
&= \frac{\mathcal{L}\left\{\delta(t)\right\}}{1 + sCR} \\
&= \frac{1}{1 + sCR}
\end{aligned}$$

Exercise 9.
Given

$$\begin{aligned}
x[n] &= a^n u[n] \\
h[n] &= u[n]
\end{aligned}$$

Find $y[n]$ using convolution.

Solution 9.

$$\begin{aligned}
y[n] &= x[n] * h[n] \\
&= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\
&= \sum_{k=-\infty}^{\infty} a^k u[k]u[n-k] \\
&= \sum_{k=-\infty}^{n} a^k u[k] \\
&= \sum_{k=0}^{n} a^k
\end{aligned}$$

Exercise 10.
For each of the following signals, determine whether it is an energy signal or a power signal. In case the signal is an energy signal, calculate the energy. In case the signal is a power signal, calculate the average power.

(1) $x(t) = \begin{cases} t & ; \quad 0 \le t \le 1 \\ 2-t & ; \quad 1 \le t \le 2 \\ 0 & ; \quad \text{otherwise} \end{cases}$

(2) $x[n] = \begin{cases} n & ; \quad 0 \le t \le 5 \\ 10-n & ; \quad 5 \le t \le 10 \\ 0 & ; \quad \text{otherwise} \end{cases}$

(3) $x(t) = 5\cos(\pi t) + \sin(5\pi t)$, $-\infty < t < \infty$

(4) $x(t) = \begin{cases} 5\cos(\pi t) & ; \quad -1 \le t \le 1 \\ 0 & ; \quad \text{otherwise} \end{cases}$

(5) $x(t) = \begin{cases} 5\cos(\pi t) & ; \quad -0.5 \le t \le 0.5 \\ 0 & ; \quad \text{otherwise} \end{cases}$

(6) $x[n] = \begin{cases} \sin(\pi n) & ; \quad -4 \le n \le 4 \\ 0 & ; \quad \text{otherwise} \end{cases}$

(7) $x[n] = \begin{cases} \cos(\pi n) & ; \quad -4 \le n \le 4 \\ 0 & ; \quad \text{otherwise} \end{cases}$

(8) $x[n] = \begin{cases} \cos(\pi n) & ; \quad 0 \le n \\ 0 & ; \quad \text{otherwise} \end{cases}$

Solution 10.
(1)

$$
\begin{aligned}
E_\infty &= \lim_{T\to\infty} \int_{-T}^{T} \left|x(t)\right|^2 \mathrm{d}t \\
&= \lim_{T\to\infty} \left(\int_0^1 t^2 \,\mathrm{d}t + \int_1^2 (2-t)^2 \,\mathrm{d}t \right) \\
&= \lim_{T\to\infty} \left(\left.\frac{t^3}{3}\right|_0^1 + \left. 4t - 2t^2 + \frac{t^3}{2} \right|_1^2 \right) \\
&= \lim_{T\to\infty} \left(\frac{1}{3} + 8 - 8 + \frac{8}{2} - 4 + 2 - \frac{1}{2} \right) \\
&= \lim_{T\to\infty} \left(\frac{11}{6} \right) \\
&= \frac{11}{6}
\end{aligned}
$$

Therefore, as E_∞ is finite, the signal is an energy signal.
(2)

$$
\begin{aligned}
E_\infty &= \lim_{N\to\infty} \sum_{n=-N}^{N} \left|x[n]\right|^2 \\
&= \lim_{N\to\infty} \sum_{n=0}^{5} n^2 + \sum_{n=6}^{10} (10-n)^2 \\
&= \lim_{N\to\infty} 85 \\
&= 85
\end{aligned}
$$

Therefore, as E_∞ is finite, the signal is an energy signal.

(3)

$$
\begin{aligned}
E_\infty &= \lim_{T\to\infty} \int_{-T}^{T} \left|x(t)\right|^2 \mathrm{d}t \\
&= \lim_{T\to\infty} \int_{-T}^{T} \left|5\cos(5t) + \sin(5\pi t)\right|^2 \mathrm{d}t \\
&= \lim_{T\to\infty} \int_{-T}^{T} \Big(25\cos^2(5t) + 10\cos(5t)\sin(5\pi t) + \sin^2(5\pi t)\Big)\,\mathrm{d}t \\
&= \lim_{T\to\infty} 26T + \frac{5}{2}\sin(10T) - \frac{\sin(10\pi T)}{10\pi} \\
&\to \infty
\end{aligned}
$$

Therefore, as E_∞ is not finite, the signal is not an energy signal.

$$
\begin{aligned}
P_\infty &= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left|x(t)\right|^2 \mathrm{d}t \\
&= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left|5\cos(5t) + \sin(5\pi t)\right|^2 \mathrm{d}t \\
&= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \Big(25\cos^2(5t) + 10\cos(5t)\sin(5\pi t) + \sin^2(5\pi t)\Big)\,\mathrm{d}t \\
&= \lim_{T\to\infty} \frac{1}{2T} \left(26T + \frac{5\sin(10T)}{2} - \frac{\sin(10\pi T)}{10\pi}\right) \\
&= \lim_{T\to\infty} 26 + \frac{5\sin(10T)}{2T} - \frac{\sin(10\pi T)}{10\pi T} \\
&= 26
\end{aligned}
$$

Therefore, as P_∞ is finite, the signal is an power signal.

(4)

$$
\begin{aligned}
E_\infty &= \lim_{T\to\infty} \int_{-T}^{T} \left|x(t)\right|^2 \mathrm{d}t \\
&= \lim_{T\to\infty} \int_{-1}^{1} 25\cos^2(\pi t)\,\mathrm{d}t \\
&= 25
\end{aligned}
$$

Therefore, as E_∞ is finite, the signal is an energy signal.

(5)

$$
\begin{aligned}
E_\infty &= \lim_{T\to\infty} \int_{-T}^{T} \left|x(t)\right|^2 \mathrm{d}t \\
&= \lim_{T\to\infty} \int_{-0.5}^{0.5} 25\cos^2(\pi t)\,\mathrm{d}t \\
&= \frac{25}{2}
\end{aligned}
$$

Therefore, as E_∞ is finite, the signal is an energy signal.

(6)

$$
\begin{aligned}
E_\infty &= \lim_{N\to\infty} \sum_{n=-N}^{N} \left|x[n]\right|^2 \\
&= \lim_{N\to\infty} \sum_{n=-4}^{4} \sin^2(n\pi) \\
&= \sum_{n=-4}^{4} \sin^2(n\pi) \\
&= 0
\end{aligned}
$$

Therefore, as E_∞ is finite, the signal is an energy signal.

(7)

$$
\begin{aligned}
E_\infty &= \lim_{N\to\infty} \sum_{n=-N}^{N} \left|x[n]\right|^2 \\
&= \lim_{N\to\infty} \sum_{n=-4}^{4} \cos^2(n\pi) \\
&= \sum_{n=-4}^{4} \cos^2(n\pi) \\
&= 9
\end{aligned}
$$

Therefore, as E_∞ is finite, the signal is an energy signal.

(8)

$$
\begin{aligned}
E_\infty &= \lim_{N\to\infty} \sum_{n=-N}^{N} \left|x[n]\right|^2 \\
&= \lim_{N\to\infty} \sum_{n=0}^{N} \cos^2(n\pi) \\
&= \lim_{N\to\infty} \sum_{n=0}^{N} \cos^2(n\pi) \\
&= \lim_{N\to\infty} \sum_{n=0}^{N} 1 \\
&= \lim_{N\to\infty} N+1 \\
&\to \infty
\end{aligned}
$$

Therefore, as E_∞ is not finite, the signal is not an energy signal.

$$
\begin{aligned}
P_\infty &= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|x[n]\right|^2 \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} \cos^2(n\pi) \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1 \\
&= \lim_{N\to\infty} \frac{N+1}{2N+1} \\
&= \lim_{N\to\infty} \frac{1+\frac{1}{N}}{2+\frac{1}{N}} \\
&= \frac{1}{2}
\end{aligned}
$$

Therefore, as P_∞ is finite, the signal is an power signal.