Linear Algebra: Homework 11
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Due: 14.01.15

EIGENVALUES AND EIGENVECTORS CONTINUED, DIAGONALIZATION

1) For each the following matrices:

- Find the eigenvalues and a basis for the eigenspace of each eigenvalue.
- What are the geometric and algebraic multiplicities?
- Is the matrix diagonizable? If so, find invertible matrix P, and diagonal matrix D so that $A = P^{-1}DP$.

a. $\begin{pmatrix} 7 & -1 & -4 \\ 14 & 1 & -12 \\ 8 & -1 & -5 \end{pmatrix}$ $over\ \mathbb{R}$ b. same matrix as in c, but $over\ \mathbb{C}$

2) For the following linear transformation find the eigenvalues and a basis for the eigenspace of each eigenvalue. Determine whether the transformation is diagonizable.

$$T: \mathbb{R}^3 \to \mathbb{R}^3 \qquad\qquad T(x, y, z) - (2x + y, y - z, 2y + 4z)$$

3) Let A be a 3×3 matrix that fulfills $\det(A - I) = 0, rank(A + I) = 2$ and let there exist a nonzero vector x so that $Ax = 3x$.

a) What is the characteristic polynomial of A?

*<u>hint:</u> If $(A - I)$ is not invertible then for some nonzero vector $(A - I)v = 0$. If you write this in another way, what do you get? What does it mean about the invertibility of (A+I) if the rank is 2?

b) What is the trace of A?

c) What is the determinant of A?

c) Is $(A + 3I)$ an invertible matrix?

ORTHOGONALITY

1) Which of the following vector spaces is an inner product space?

$a.$ $\boldsymbol{V} = \boldsymbol{R}^2,\ \boldsymbol{v_1} = \binom{x_1}{x_2}, \boldsymbol{v_2} = \binom{y_1}{y_2}, \langle \boldsymbol{v_1}, \boldsymbol{v_2} \rangle = x_1x_2 + 7y_1y_2$

$b.$ $\boldsymbol{V} = \boldsymbol{R}^2,\ \boldsymbol{v_1} = \binom{x_1}{x_2}, \boldsymbol{v_2} = \binom{y_1}{y_2}, \langle \boldsymbol{v_1}, \boldsymbol{v_2} \rangle = x_1y_1 + 7x_2y_2$

$c.$ $\boldsymbol{A}, \boldsymbol{B} \in \boldsymbol{V} = \boldsymbol{M_2}(\boldsymbol{R}), \qquad (\boldsymbol{A}, \boldsymbol{B}) = tr(\boldsymbol{AB})$

Good luck!