Review Session 2

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Sunday 25$^{\text{th}}$ January, 2015

Example 1. Consider $V = M_{2\times 2}(\mathbb{R})$ with inner product

$$\langle A, B\rangle = \operatorname{trace}\left(B^T \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} A\right)$$

Find an orthonormal basis of V with respect to this inner product.

Solution.

$$\begin{aligned}
v_1 &= e_1 \\
v_2 &= e_2 - \frac{\langle e_2, v_1\rangle}{\|v_1\|^2} v_1 \\
&= e_2 - \frac{\operatorname{trace}\left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right)}{\operatorname{trace}\left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right)} e_1 \\
&= e_2 \\
v_3 &= \begin{pmatrix} -2 & 0 \\ 1 & 0 \end{pmatrix} \\
v_4 &= \begin{pmatrix} 0 & -2 \\ 0 & 1 \end{pmatrix}
\end{aligned}$$

Example 2. Let $A \in M_{6\times 6}(\mathbb{C})$. Given

$$p_A(\lambda) = \lambda^2(\lambda - 2)(\lambda + 1)^3$$

find $\det(A^2 + A)$.

Solution.

$$\begin{aligned}\det(A + A^2) &= \det(A(A + I)) \\ &= \det(A)\det(A + I)\end{aligned}$$

As 0 is an eigenvalue of A, the product of all eigenvalues is 0. Therefore, $\det(A) = 0$.
Therefore, $\det(A^2 + A) = 0$.