Recitation 7

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1 Transformation matrices

Example 1. B and E are bases of V, which is the set of all 2×2 matrices.

$$B = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}$$
$$E = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

Find $[E]_B$ which holds $[m]_B = [E]_B[m]_E$, $\forall m \in V$.

Solution. For ease of calculation, $[E]_B$ can be written as $[B]_E^{-1}$.

$$\begin{aligned}
[E]_B &= [B]_E^{-1} \\
&= \left(\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]_E \; \left[\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right]_E \; \left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right]_E \; \left[\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right]_E \right)^{-1} \\
&= \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{pmatrix}^{-1} \\
&= \begin{pmatrix} 1/2 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 0 \\ 1/2 & 0 & 0 & -1/2 \\ 0 & 0 & 1 & 0 \end{pmatrix}
\end{aligned}$$

2 Completing to a Basis

Definition 1. Let V be a n dimensional vector space. Let $W \subset V$ be a subset of V and let its basis B have $m < n$ vectors.
We can find a set D having $n - m$ vectors, s.t. $B \cup D$ will be a basis of V.

The vectors in D will be linearly independent, and $\text{span}(B) \cap \text{span}(D) = \{0\}$.

2.1 Method

In order to find the set D, we will write a matrix that has the vectors of B as its rows, and will add row vectors having all zeroes except in the positions corresponding to the free variables, which have 1.