Recitation 5

Wednesday 26^{th} November, 2014

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Example 1. Let $B = \{v_1, v_2, v_3, v_4\}$ be a base of a vector space V. If $C = \{v_1 + v_2, v_2 + v_3, v_3 + v_4, v_4 + v_1\}$ a base of V?

Solution. Let

$$\begin{aligned}
&\alpha(v_1 + v_2) + \beta(v_2 + v_3) + \gamma(v_3 + v_4) + \delta(v_4 + v_1) = 0 \\
&\therefore v_1(\delta + \alpha) + v_2(\alpha + \beta) + v_3(\beta + \gamma) + v_4(\gamma + \delta) = 0
\end{aligned}$$

As $B = \{v_1, v_2, v_3, v_4\}$ is a basis of V,

$$\begin{aligned}
\delta + \alpha &= 0 \\
\alpha + \beta &= 0 \\
\beta + \gamma &= 0 \\
\gamma + \delta &= 0
\end{aligned}$$

If

$$\alpha = -1 \qquad \beta = 1 \qquad \gamma = -1 \qquad \delta = 1$$

the above system of equations hold.
Therefore, C is linearly dependent. Therefore, it is not a base of V.

Definition 1 (Row space)**.** The row space of $A \in M_{n\times m}(\mathbb{F})$ is a subspace of $\mathbb{F}^m$ spanned by the rows of A. The row space of A is denoted by $R(A)$.

Definition 2 (Column space)**.** The column space of $A \in M_{n\times m}(\mathbb{F})$ is a subspace of $\mathbb{F}^n$ spanned by the columns of A. The column space of A is denoted by $C(A)$.

Definition 3. The rank of $A \in M_{n\times m}(\mathbb{F})$ is defined as

1. the number of dependent variables in the REF of A
2. the number of non-zero rows on the REF of A

   $$\operatorname{rk}(A) = \dim(C(A)) = \dim(R(A))$$