Recitation 1

Wednesday 29th October, 2014

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1 General Information

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Reception Hour: Sunday, 11:00 - 11:00

2 Complex Numbers

Let us set $i = \sqrt{-1}$.
A complex number is a number of the form $z = a + ib$, where $a, b \in \mathbb{R}$.
$\Re(z) = Re(z) = a$
$\Im(z) = Im(z) = b$

2.1 Addition

$(a + ib) + (c + id) = (a + c) + i(b + d)$

2.2 Multiplication

$(a + ib)(c + id) = ac + ibc + iad - bd = (ac - bd) + i(bc + ad)$

2.3 Complex Conjugate

For $z = a + ib$, its conjugate is $\overline{z} = a - ib$
$z + \overline{z} = (a + ib) + (a - ib) = 2a = 2\Re(z)$
$z - \overline{z} = (a + ib) - (a - ib) = 2ib = 2\Im(z)$
$z \cdot \overline{z} = (a + ib)(a - ib) = a^2 + b^2$

2.4 Absolute Value

$|z| = \sqrt{z\overline{z}} = \sqrt{\Re^2(z) + \Im^2(z)}$

2.5 Complex Division

$$\frac{z}{w} = \frac{z}{w} \cdot \frac{\overline{z}}{\overline{w}} = \frac{z \cdot \overline{w}}{|w|^2}$$

2.6 Examples

2.6.1 Example 1

2.6.1.1 Express $\frac{(2+i)(4+i)}{(1+i)}$ as a complex number.

$(2 + i)(4 + i) = 8 + 6i - 1 = 7 + 6i$

$$\therefore \frac{(2+i)(4+i)}{(1+i)} = \frac{7+6i}{1+i} = \frac{7+6i}{1+i} \cdot \frac{1-i}{1+i} = \frac{7+6-i}{2} = \frac{13}{2} - \frac{i}{2}$$

2.6.1.2 Show that $i^{77} = i$

$i^1 = i, i^2 = -1, i^3 = -i, i^4 = 1$
$n = 4m + k$
$\therefore i^n = i^{4m+k} = (i^{4m})i^k = (i^4)^m i^k = i^k$
$\therefore i^{77} = i^{4 \cdot 19+1} = i$

2.7 Complex Plane

For every complex number, there is a corresponding point in the complex plane.
This plane is called Complex plane or Gauss plane.
The x-axis represents the real part and the y-axis represents the imaginary part.

$$|z| = \sqrt{a^2 + b^2} = \text{the distance from} O(0,0)$$
$$z = r\cos\theta + r\sin\theta = r(\cos\theta + i\sin\theta)$$

$$r = |z| = \sqrt{a^2 + b^2}$$
$$\theta = \arctan\frac{b}{a}$$
$$z = r(\cos\theta + i\sin\theta) = r\text{cis}\theta = re^{i\theta}$$
$$z_1 z_2 = (r_1 e^{i\theta_1})(r_2 e^{i\theta_2}) = r_1 r_2 e^{i(\theta_1 + \theta_2)}$$

2.8 De Moivre's Formula

$(re^{i\theta})^n = r^n e^{in\theta}$

3 Matrices

A matrix of order $m \times n$, over a field $\mathbb{F}$ is a table with n rows and m columns.

$$A = \begin{matrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{matrix} = (a_{ij})$$

3.1 Row Vector

Row vector is a matrix of order $1 \times m$

3.2 Column Vector

Column vector is a matrix of order $n \times 1$

3.3 Multiplying a Matrix by a Scalar

$\alpha A = (\alpha a_{ij})_{ij}$

3.4 Addition of Matrices

Addition of matrices is defined only if the matrices are of same order.

$A + B = (a_{ij} + b_{ij})$

3.5 Multiplication of Matrices

Multiplication of matrices is defined for only matrices of the type $A_{n\times m}$ and $B_{m\times p}$.

$$\begin{aligned} C_{n\times p} &= A_{n\times m} \cdot B_{m\times p} \\ c_{ik} &= \sum_{j=1}^{n} a_{ij} b_{jk} \end{aligned}$$