Numerical Analysis : Recitations

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Contents

1 Instructor Information

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2 Errors

Definition 1 (Error)**.** The absolute error in representation is defined as

$$e_x = x - \widetilde{x}$$

The relative error in representation is defined as

$$\delta = \frac{x - \widetilde{x}}{x}$$

Recitation 1 – Exercise 1.
The dimensions of a field are measured. The length is measured to be $\widetilde{x} = 800\text{m}$, with an absolute error bounded by 16. The width is measured to be $\widetilde{y} = 30\text{m}$, with an absolute error e_y, such that $|e_y| \leq 6$.

1. Find the approximate bounds for $|\delta_x|$ and $|\delta_y|$.

2. Find the bounds on the absolute error in the calculated area of the field.

Recitation 1 – Solution 1.
1.

$$\begin{aligned}
|\delta_x| &= \frac{|e_x|}{|x|} \\
&\leq \frac{16}{|x|} \\
&\approx \frac{16}{800} \\
&= 0.02 \\
\therefore |\delta_x| &\leq 0.02
\end{aligned}$$

$$
\begin{aligned}
|\delta_y| &= \frac{|e_y|}{|y|} \\
&\leq \frac{6}{|y|} \\
&\approx \frac{6}{300} \\
&= 0.02 \\
\therefore |\delta_y| &\leq 0.02
\end{aligned}
$$

2. The measured area of the field is

$$
\begin{aligned}
\widetilde{A} &= \widetilde{x}\widetilde{y} \\
&= 800 \cdot 300 \\
&= 240000
\end{aligned}
$$

The maximum area of the field is

$$
\begin{aligned}
A_{\max} &= (\widetilde{x} + e_{x\max})(\widetilde{y} + e_{y_{\max}}) \\
&= (800 + 16)(300 + 6) \\
&= 249696
\end{aligned}
$$

The maximum area of the field is

$$
\begin{aligned}
A_{\min} &= (\widetilde{x} + e_{x\min})(\widetilde{y} + e_{y_{\min}}) \\
&= (800 - 16)(300 - 6) \\
&= 230496
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
|e_{xy}| &\leq (A_{\max} - A_{\min}) \\
&\leq 9696
\end{aligned}
$$

3.

$$
\begin{aligned}
|\delta_{xy}| &= \frac{|e_{xy}}{|xy|} \\
&\leq \frac{9696}{|xy|} \\
&\leq \frac{9696}{230496} \\
&\approx 0.042
\end{aligned}
$$

2.1 Propagation of Error

Recitation 1 – Exercise 2.
Let $\widetilde{x}$, $\widetilde{y}$ be approximations of x, y.

1. Find a formula for the absolute error in $x + y$ in terms of e_x and e_y.

2. Find a formula for δ_{x+y}, δ_{x-y} in terms of δ_x, δ_y, x, y.

3. Let $\delta = \max\{\delta_x, \delta_y\}$. Assuming $x, y > 0$, show

$$|\delta_{x-y}| \leq \frac{x+y}{|x-y|}\delta$$

Recitation 1 – Solution 2.
1.

$$\begin{aligned} e_{x+y} &= (x+y) - (\widetilde{x} + \widetilde{y}) \\ &= (x - \widetilde{x}) + (y - \widetilde{y}) \\ &= e_x + e_y \end{aligned}$$

2.

$$\begin{aligned} \delta_{x+y} &= \frac{e_{x+y}}{x+y} \\ &= \frac{e_x + e_y}{x+y} \\ &= \frac{x\delta_x + y\delta_y}{x+y} \end{aligned}$$

Similarly,

$$\begin{aligned} \delta_{x-y} &= \frac{e_{x-y}}{x-y} \\ &= \frac{e_x - e_y}{x-y} \\ &= \frac{x\delta_x - y\delta_y}{x-y} \end{aligned}$$

3.

$$
\begin{aligned}
|\delta_{x-y}| &= \left|\frac{x\delta_x - y\delta_y}{x-y}\right| \\
&\leq \frac{|x||\delta_x| + |y||\delta_y|}{|x-y|} \\
&\leq \frac{x\delta + y\delta}{|x-y|} \\
&= \frac{x+y}{|x-y|}\delta
\end{aligned}
$$

Recitation 1 – Exercise 3.
Find a formula for δ_{xy}, in terms of x, y, δ_x, δ_y.

Recitation 1 – Solution 3.

$$
\begin{aligned}
\delta_a &= \frac{a - \widetilde{a}}{a} \\
\therefore \widetilde{a} &= a(1-\delta_a)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\widetilde{x}\widetilde{y} &= (x(1-\delta_x))\Big(y(1-\delta_y)\Big) \\
&= xy(1-\delta_x-\delta_y+\delta_x\delta_y)
\end{aligned}
$$

Also,

$$\widetilde{x}\widetilde{y} = xy(1-\delta_{xy})$$

Therefore,

$$\delta_{xy} = \delta_x + \delta_y - \delta_x\delta_y$$