Lecture 22

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Contents

1 Simple Harmonic Oscillators

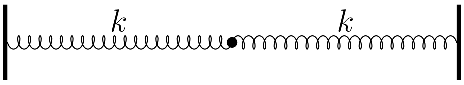
Example 1. A system is arranged as follows, with the walls moving with

$$x_1 = A \sin \omega t$$
$$x_2 = 2L + A \cos \omega t$$

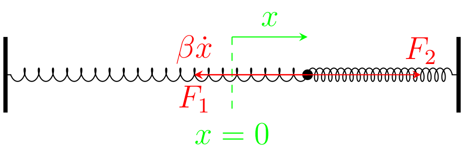
The entire system is in submerged in a fluid, s.t. the damping force is

$$\overrightarrow{f} = -\beta \overrightarrow{v}$$

The mass is displaced by a small x to the right. Write the equation of motion.



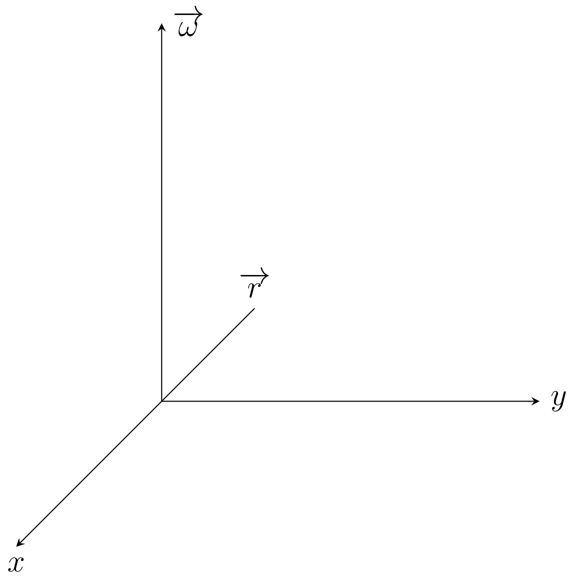
Solution.



$$\begin{aligned} m\ddot{x} &= -F_1 + F_2 - \beta \dot{x} \\ &= -k(L + x - A \sin \omega t - L) + k(L - x + A \cos \omega t - L) - \beta \dot{x} \end{aligned}$$

$$\therefore m\ddot{x} + \beta \dot{x} + 2kx = kA(\sin \omega t + \cos \omega t)$$
$$\therefore \ddot{x} + \frac{\beta}{m}\dot{x} + \frac{2k}{m}x = \frac{kA}{m}(\sin \omega t + \cos \omega t)$$

2 Coriolis Force



$$
\begin{aligned}
\overrightarrow{r} &= (r\sin\theta\cos\omega t, r\sin\theta\sin\omega t, r\cos\theta) \\
\overrightarrow{v} &= \frac{\mathrm{d}\overrightarrow{r}}{\mathrm{d}t} \\
&= (-\omega r\sin\theta\sin\omega t, \omega r\sin\theta, 0) \\
\overrightarrow{\omega}\times\overrightarrow{r} &= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \omega \\ r\sin\theta\cos\omega t & r\sin\theta\sin\omega t & r\cos\theta \end{vmatrix} \\
&= -\omega r\sin\theta\sin\omega t\hat{x} + \omega r\sin\theta\cos\omega t\hat{y} \\
\therefore \overrightarrow{v} &= \overrightarrow{\omega}\times\overrightarrow{r}
\end{aligned}
$$

In general, for any vector $\overrightarrow{A}$,

$$
\frac{\mathrm{d}\overrightarrow{A}}{\mathrm{d}t} = \overrightarrow{\omega}\times\overrightarrow{A}
$$

The system $S : xyz$ is inertial, and the system $S' : x'y'z'$ is rotating around $\overrightarrow{\omega}$ relative to S.
Therefore, the position vector $\overrightarrow{r}$ can be written as follows.

$$
\begin{aligned}
\overrightarrow{r} &= x\hat{x} + y\hat{y} + z\hat{z} \\
\overrightarrow{r'} &= x'\hat{x'} + y'\hat{y'} + z'\hat{z'}
\end{aligned}
$$

Therefore,

$$\begin{aligned}
\overrightarrow{v} &= \dot{x}\hat{x} + \dot{y}\hat{y} + \dot{z}\hat{z} \\
\overrightarrow{v'} &= \dot{x}'\hat{x'} + \dot{y}'\hat{y'} + \dot{z}'\hat{z'} \\
\overrightarrow{a} &= \ddot{x}\hat{x} + \ddot{y}\hat{y} + \ddot{z}\hat{z} \\
\overrightarrow{a'} &= \ddot{x}'\hat{x'} + \ddot{y}'\hat{y'} + \ddot{z}'\hat{z'}
\end{aligned}$$

Therefore, in S,

$$\begin{aligned}
\overrightarrow{v} &= \frac{\mathrm{d}\overrightarrow{r}}{\mathrm{d}t} \\
&= \frac{\mathrm{d}\overrightarrow{r'}}{\mathrm{d}t} \\
&= \dot{x'}\hat{x'} + \dot{y'}\hat{y'} + \dot{z'}\hat{z'} \\
&\quad + x'\frac{\mathrm{d}\hat{x'}}{\mathrm{d}t} + y'\frac{\mathrm{d}\hat{y'}}{\mathrm{d}t} + z'\frac{\mathrm{d}\hat{z'}}{\mathrm{d}t} \\
&= \overrightarrow{v'} + x'\left(\overrightarrow{\omega} \times \hat{x'}\right) + y'\left(\overrightarrow{\omega} \times \hat{y'}\right) + z'\left(\overrightarrow{\omega} \times \hat{z'}\right) \\
&= \overrightarrow{v'} + \overrightarrow{\omega} \times \left(x'\hat{x'} + y'\hat{y'} + z'\hat{z'}\right) \\
&= \overrightarrow{v'} + \overrightarrow{\omega} \times \overrightarrow{r}
\end{aligned}$$

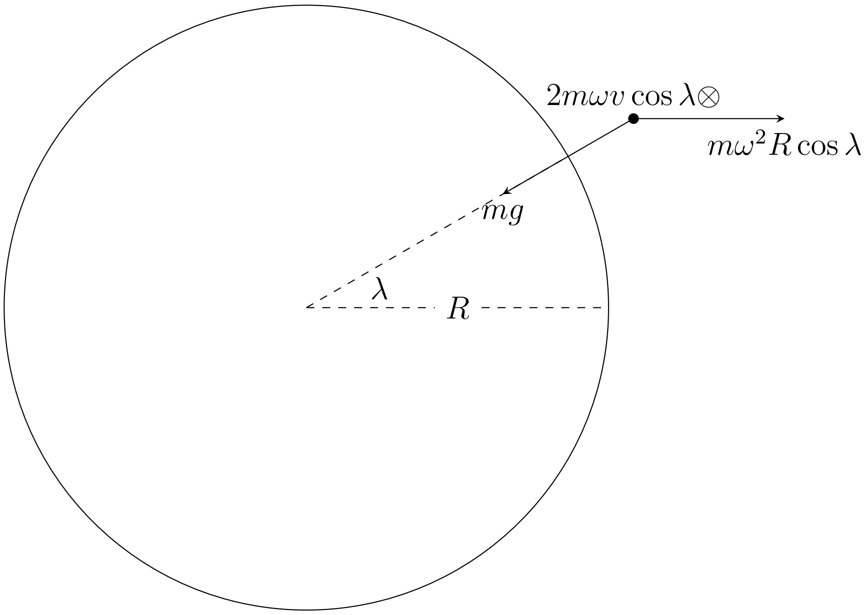
Similarly,

$$\begin{aligned}
\overrightarrow{a} &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\overrightarrow{v'} + \overrightarrow{\omega} \times \overrightarrow{r}\right) \\
&= \frac{\mathrm{d}\overrightarrow{v'}}{\mathrm{d}t} + \overrightarrow{\omega} \times \frac{\mathrm{d}\overrightarrow{r}}{\mathrm{d}t} \\
&= \frac{\mathrm{d}}{\mathrm{d}t}\left(\dot{x}'\hat{x'} + \dot{y}'\hat{y'} + \dot{z}'\hat{z'}\right) + \overrightarrow{\omega} \times \left(\overrightarrow{v'} + \overrightarrow{\omega} \times \overrightarrow{r}\right) \\
&= \ddot{x'}\hat{x'} + \dot{x'}\frac{\mathrm{d}\hat{x'}}{\mathrm{d}t} + \ddot{y'}\hat{y'} + \dot{y'}\frac{\mathrm{d}\hat{y'}}{\mathrm{d}t} + \ddot{z'}\hat{z'} + \dot{z'}\frac{\mathrm{d}\hat{z'}}{\mathrm{d}t} \\
&\quad + \overrightarrow{\omega} \times \overrightarrow{v'} + \overrightarrow{\omega} \times (\overrightarrow{\omega} \times \overrightarrow{r}) \\
&= \overrightarrow{a'} + 2\overrightarrow{\omega} \times \overrightarrow{v'} + \overrightarrow{\omega} \times (\overrightarrow{\omega} \times \overrightarrow{r})
\end{aligned}$$

Therefore,

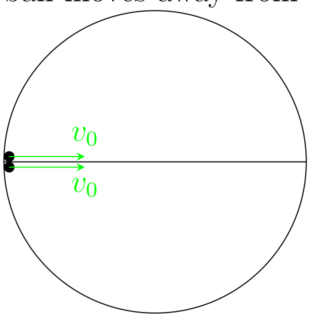
$$m\overrightarrow{a'} = \underbrace{m\overrightarrow{a}}_{\sum \overrightarrow{F}} - \underbrace{2m\overrightarrow{\omega} \times \overrightarrow{v}}_{\text{coriolis force}} - \underbrace{m\overrightarrow{\omega} \times (\overrightarrow{\omega} \times \overrightarrow{r})}_{\text{centrifugal force}}$$

Example 2. Find the forces acting on a body at a small distance above the surface of the earth, at angle λ from the equator, moving with ω.

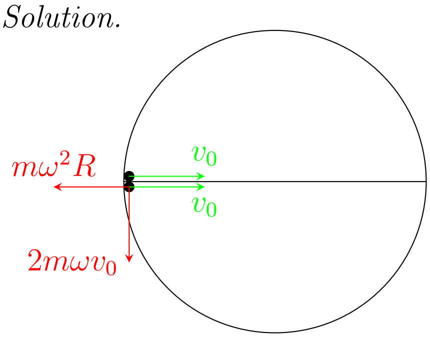


Solution.

Example 3. A disk with a diametrical partition is rotating anti-clockwise with ω, as shown. Two small masses start moving with v_0 as shown. Which ball moves away from the partition?



Solution.



Therefore, the ball which is below the partition moves away from the partition.