Lecture 3

Tuesday 4^{th} November, 2014

Contents

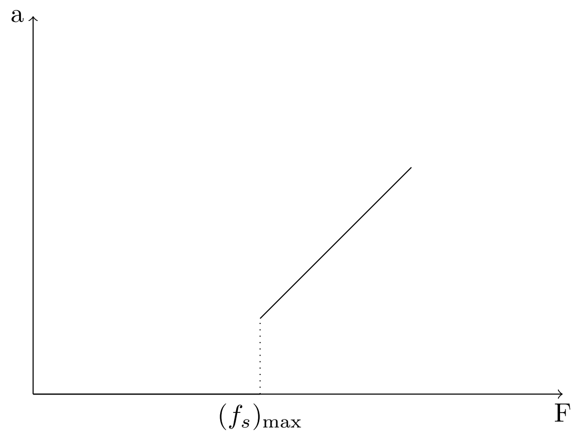
1 Friction

Friction is an adaptive force, which inhibits the
Experimentally, it is observed that $(f_s)_{\max} \propto N$

$$
\begin{aligned}
(f_s)_{\max} &\doteq \mu_s N \\
f_k &\doteq \mu_k N
\end{aligned}
$$

It is observed that for most pairs of materials, $f_k < f_s$
Hence, against an external force F,

$$
f = \begin{cases} F; F \leq (f_s)_{\max} \\ \mu_k N; F > (f_s)_{\max} \end{cases}
$$



1.1 Body on a rough inclined plane

For the body to be in static equilibrium,

$$
\begin{aligned}
f &\leq \mu_s N \\
&\leq \mu_s mg\cos\theta \\
f &= mg\sin\theta \\
\therefore mg\sin\theta &\leq \mu_s mg\cos\theta \\
\therefore \mu_s &\geq \tan\theta
\end{aligned}
$$

1.2 Two bodies across a pulley

$$
\begin{aligned}
N &= m_1 g \\
T &= m_1 a \\
m_2 g - T &= m_2 a
\end{aligned}
$$

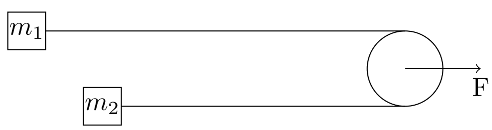
Solving the above equations,

$$m_2 g = (m_1 + m_2)a$$
$$\therefore a = \frac{m_2}{m_1 + m_2} g$$

1.3 Two bodies across 2 pullies

$$m_1 g = N$$
$$T_1 = m_1 a_1$$
$$m_2 g - T_2 = m_2 a_2$$
$$T_2 - 2T_1 = (0)(a_2) = 0$$
$$a_2 = \frac{a_1}{a_2}$$

1.4 2 bodies and a pulley in horizontal plane



If the pulley moves by a distance x_3, m_1 moves by x_1, and m_2 moves by x_2. The length of the rope is constant.

$$\therefore x_1 + x_2 = 2x_3$$
$$\therefore x_3 = \frac{x_1 + x_2}{2}$$
$$\therefore a_3 = \frac{a_1 + a_2}{2}$$

1.5 Body on another body with pulley

1.5.1 Case I: No relative motion

$$(m_1 + m_2)a = F$$
$$\therefore a = \frac{F}{m_1 + m_2}$$

1.5.2 Case II: Relative motion exists

$$m_1 a = \frac{F}{2} + f$$
$$m_2 a = \frac{F}{2} - f$$

No relative motion will persist as long as

$$f < (f_s)_{\max} = \mu_s N_{12} = \mu_s m_2$$

1.6 2 Bodies and 3 pulleys

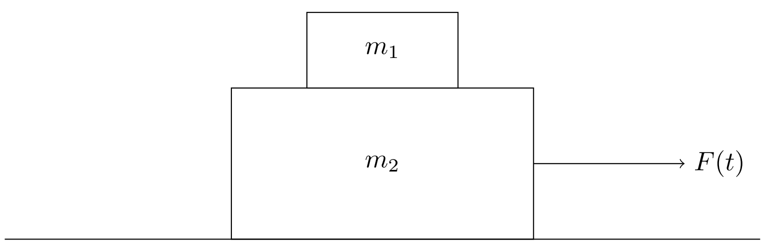
For m_1,

$$\hat{y} : N_{\text{floor}} = m_1 g + T$$
$$\hat{x} : 2T - N = m_1 a_1$$

For m_2,

$$\hat{y} : m_2 g - T = m_2 a_2 \tag{1}$$
$$\hat{x} : N = m_2 a_1 \tag{2}$$
$$a_2 = 2a_1 \tag{3}$$

1.7



Given : The force varies as $F = bt; b > 0$, and the coefficients of friction between all surfaces are μ_s, μ_k.

Stage I: No movement
No movement will occur till $f = \mu_s N_{\text{floor}} = \mu_s (m_1 + m_2) g$
If movement starts at say t_0,

$$f = F(t_0) = bt_0$$
$$\therefore \mu_s (m_1 + m_2) g = bt_0$$
$$\therefore t_0 = \frac{\mu_s (m_1 + m_2) g}{b}$$

Stage II: m_1 and m_2 move together

$$bt - \mu_k N_{\text{floor}} = (m_1 + m_2) a$$
$$\therefore bt - \mu_k (m_1 + m_2) g = (m_1 + m_2) a$$
$$\therefore a = \frac{b}{m_1 + m_2} t - \mu_k g$$

For m_1,

$$\begin{aligned}\hat{x} &: f_{12} = m_1 a \\ \hat{y} &: N_{12} = m_1 g\end{aligned}$$

For m_2,

$$\begin{aligned}\hat{x} &: F - f - f_{12} = m_2 a \\ \hat{y} &: N_{\text{floor}} = N_{12} + m_2 g\end{aligned}$$

At $t = t_1$,

$$f_{12} = \mu_s n_{12} = \mu_s m_1 g$$

Therefore,

$$t_1 = \frac{(\mu_s + \mu_k) g (m_1 + m_2)}{b}$$

Stage III: m_1 **and** m_2 **are in relative motion**
For m_1,

$$\begin{aligned}\hat{x} &: \mu_k N_{12} = m_1 a_1 \\ \hat{y} &: N_{12} = m_1 g\end{aligned}$$

For m_2,

$$\begin{aligned}\hat{x} &: bt - \mu_k N_{\text{floor}} - \mu_k n_{12} = m_2 a_2 \\ \hat{y} &: N_{\text{floor}} = N_{12} + m_2 g\end{aligned}$$

Therefore,

$$\begin{aligned}a_1 &= \frac{\mu_k N_{12}}{m_1} \\ &= \frac{\mu_k m_1 g}{m_1} \\ &= \mu_k g \\ a_2 &= \frac{b}{m_2} t - \frac{\mu_k}{m_2}(m_1 g + m_2 g) - \frac{\mu_k m_1 g}{m_2} \\ &= \frac{b}{m_2} t - \mu_k g \left(2\frac{m_1}{m_2} + 1\right)\end{aligned}$$

At $t = t_1$,

$$\begin{aligned}a_2 &= \frac{b}{m_2} t_1 - \mu_k g \left(2\frac{m_1}{m_2} + 1\right) \\ &= \mu_s g + \frac{m_1}{m_2}(\mu_s - \mu_k) g\end{aligned}$$

Therefore, for $t > t_1$

$$a_2 > a_1$$