PHYSICS 1 : COMPENDIUM

AAKASH JOG

1. Non-Linear Motion

$$\begin{aligned}
\overrightarrow{r} &= r\hat{r} \\
\overrightarrow{v} = \dot{\overrightarrow{r}} &= \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} \\
\overrightarrow{a} = \ddot{\overrightarrow{r}} &= \left(\ddot{r} - r(\dot{\theta})^2\right)\hat{r} + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)\hat{\theta}
\end{aligned}$$

2. Conservative Forces

$$\begin{aligned}
\operatorname{curl}\overrightarrow{F} &\doteq \overrightarrow{\nabla} \times \overrightarrow{F} \\
&= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}
\end{aligned}$$

If $\overrightarrow{F}$ is conservative, $\operatorname{curl}\overrightarrow{F} = 0$.

3. Variable Mass Systems

If a rocket is releasing gasses with velocity u with respect to it,

$$\frac{\mathrm{d}p}{\mathrm{d}t} = m\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{\mathrm{d}m}{\mathrm{d}t}u$$

4. Centres of Mass

Solid hemisphere	$\left(3/8\right) R$
Hollow hemisphere	$\left(1/2\right) R$
Solid cone (from vertex)	$\left(3/4\right) h$
Hollow cone (from vertex)	$\left(2/3\right) h$

5. Moments of Inertia

$$I = \int r^2 \, \mathrm{d}m$$

Ring ($\perp$ to plane)	mR^2
Disk ($\perp$ to plane)	$\left({}^1/_2\right) mR^2$
Solid sphere	$\left({}^2/_5\right) mR^2$
Hollow sphere	$\left({}^2/_3\right) mR^2$
Rod (centre)	$\left({}^1/_{12}\right) ml^2$
Rod (end)	$\left({}^1/_3\right) ml^2$
Cone (axis of symmetry)	$\left({}^3/_{10}\right) mR^2$

6. Accelerating Systems

$$F_{\text{centrifugal}} = -m\vec{\omega} \times \left(\vec{\omega} \times \vec{r}\right)$$

$$F_{\text{coriolis}} = -2m\vec{\omega} \times \vec{v}$$

7. Oscillations

7.1. Simple Oscillations.

$$\ddot{x} = -\omega^2 x$$

$$\omega_{\text{physical pendulum}} = \sqrt{\frac{d_{\text{axis,COM}} mg}{I_{\text{axis}}}}$$

7.2. Damped Oscillations.

$$\ddot{x} + \frac{\beta}{m}\dot{x} + {\omega_0}^2 x = 0$$

Strong damping	$\frac{\beta}{2m} > \omega_0$
Critical damping	$\frac{\beta}{2m} = \omega_0$
Weak damping	$\frac{\beta}{2m} < \omega_0$

Oscillations occur in case of weak damping.

Let $\omega_1 = \sqrt{{\omega_0}^2 - \left(\frac{\beta}{2m}\right)^2}$.

For weak damping,

$$x = e^{-\beta/2m \cdot t}\left(\widetilde{A}\cos\omega_1 t + \widetilde{B}\sin\omega_1 t\right)$$

For critical damping,

$$x = e^{-\beta/2m \cdot t}\left(\widetilde{A} + \widetilde{B}t\right)$$

In case of strong damping,

$$x = \widetilde{A}e^{\left(-\beta/2m + \sqrt{-{\omega_1}^2}\right)} + \widetilde{B}e^{\left(-\beta/2m - \sqrt{-{\omega_1}^2}\right)t}$$

7.3. Forced Oscillations.

$$m\ddot{x} + \frac{k}{m}x = \frac{F_0}{m}\cos\omega t$$

$$\therefore \ddot{x} + \frac{k}{m}x = \frac{F_0}{m}\cos\omega t$$

Therefore, solving

$$x = A\cos\omega_0 t + B\sin\omega_0 t + \frac{F_0}{k - m\omega^2}\cos\omega t$$

$$\therefore \dot{x} = \omega_0(-A\sin\omega_0 t + B\cos\omega_0 t) - \frac{F_0}{k - m\omega^2}\omega\sin\omega t$$

Substituting initial conditions,

$$x = \frac{\frac{F_0}{m}}{\frac{k}{m} - \omega^2}(-\cos\omega_0 t + \cos\omega t)$$

Let $\frac{F_0}{m} = f_0$

$$\therefore x = -\frac{2f_0}{{\omega_0}^2 - \omega^2}\sin\left(\frac{\omega t - \omega_0 t}{2}\right)\sin\left(\frac{\omega t + \omega_0 t}{2}\right)$$

$\omega - \omega_0 = \Delta\omega$ and $\omega + \omega_0 \approx 2\omega_0$

$$\therefore x \approx \frac{2f_0}{\Delta\omega \cdot 2\omega_0}\sin\left(\frac{\Delta\omega}{2}t\right) \cdot \sin(\omega_0 t)$$