Recitation 10

Wednesday 31st December, 2014

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Translational motion	Rotational Motion
x	θ
v	ω
a	α
$\overrightarrow{p} = m\overrightarrow{v}$	$\overrightarrow{L} = I\overrightarrow{\omega}$
$\overrightarrow{F} = m\overrightarrow{a}$	$\overrightarrow{\tau} = I\overrightarrow{\alpha}$
$\overrightarrow{F} = \dfrac{\mathrm{d}\overrightarrow{p}}{\mathrm{d}t}$	$\overrightarrow{\tau} = \dfrac{\mathrm{d}\overrightarrow{L}}{\mathrm{d}t}$

Example 1. A thin cylindrical chimney of length L is falling down, rotating around its base, point B, untill it breaks at some point P. Find the breaking point of the chimney.

Solution. Let the angle between the chimney and the vetical be α and let the distance between B and P be x.
The torque acting on the upper part of the chimney, about point P is

$$\tau_p = \left(\frac{m}{L}(L-x)\right) g \left(\frac{L-x}{2}\right) \sin\alpha$$

Let

$$m_1 = \frac{m}{L}(L-x)$$

Therefore, the moment of inertia of the upper part of the chimney is

$$\begin{aligned} I_1 &= \frac{m_1(L-x)^2}{3} \\ &= \frac{m}{L} \cdot \frac{(L-x)^3}{3} \end{aligned}$$

$$
\begin{aligned}
\tau &= \tau_P + \tau(x) \\
&= I_1 \ddot{\alpha} \\
\therefore \tau(x) &= I_1 \ddot{\alpha} \\
&= \left(\frac{m}{L}\right) \frac{(L-x)^3}{3} \ddot{\alpha} - \frac{Mg}{2L} (L-x)^2 \sin\alpha \\
&= \left(\frac{m}{L}\right) \frac{(L-x)^3}{3} \left(\frac{3g\sin\alpha}{2L}\right) - \frac{Mg}{2L} (L-x)^2 \sin\alpha \\
&= \left(\frac{Mg\sin\alpha}{2L}\right) \left(\frac{(L-x)^3}{L} - (L-x)^2\right)
\end{aligned}
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Differentiating, $\tau(x)$ is maximum at $x = \dfrac{L}{3}$.