Recitation 8

Wednesday 17th December, 2014

Contents

1 Centre of Mass

Example 1. Find the centre of mass of an eighth of a solid sphere.

Solution. Consider an elemental mass $\mathrm{d}m$ at (r, θ, φ).

$$\begin{aligned} x_{\text{COM}} &= \frac{\int_{r=0}^{R} \int_{\theta=0}^{\frac{\pi}{2}} \int_{\varphi=0}^{\frac{\pi}{2}} r \sin\theta \cos\varphi \, \mathrm{d}V}{\iiint \mathrm{d}V} \\ &= \frac{\int_{r=0}^{R} \int_{\theta=0}^{\frac{\pi}{2}} \int_{\varphi=0}^{\frac{\pi}{2}} r \sin\theta \cos\varphi (r^2 \sin\theta \, \mathrm{d}r \, \mathrm{d}\theta \, \mathrm{d}\varphi)}{\frac{1}{8} \cdot \frac{4}{3}\pi R^3} \end{aligned}$$

Therefore,

$$\therefore x_{\text{COM}} = y_{\text{COM}} = z_{\text{COM}} = \frac{3}{8}R$$