Lecture 18

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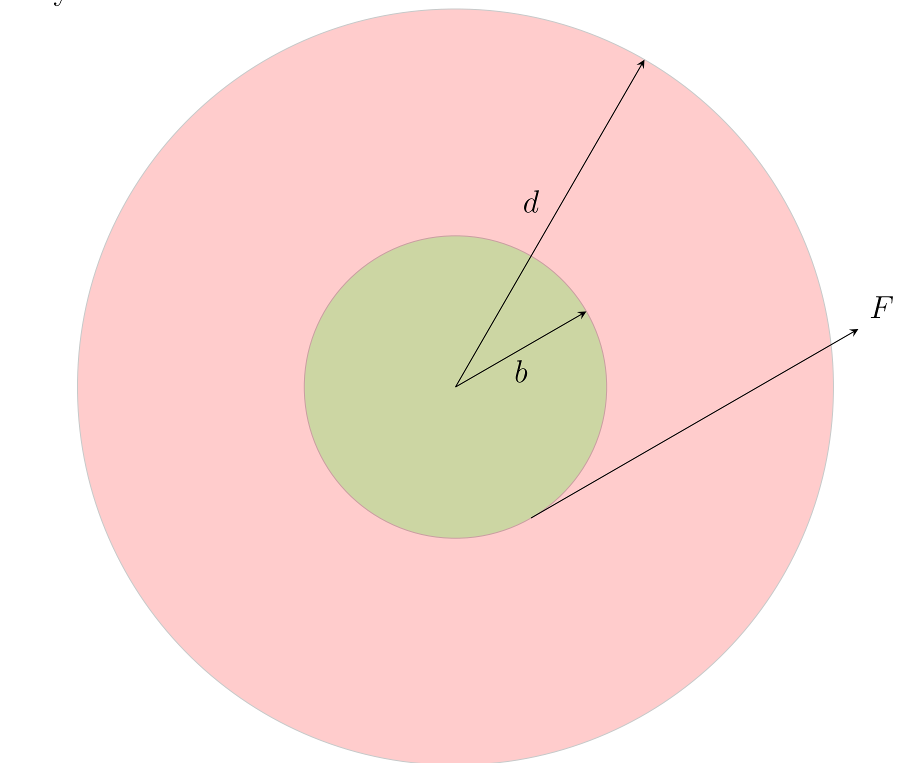
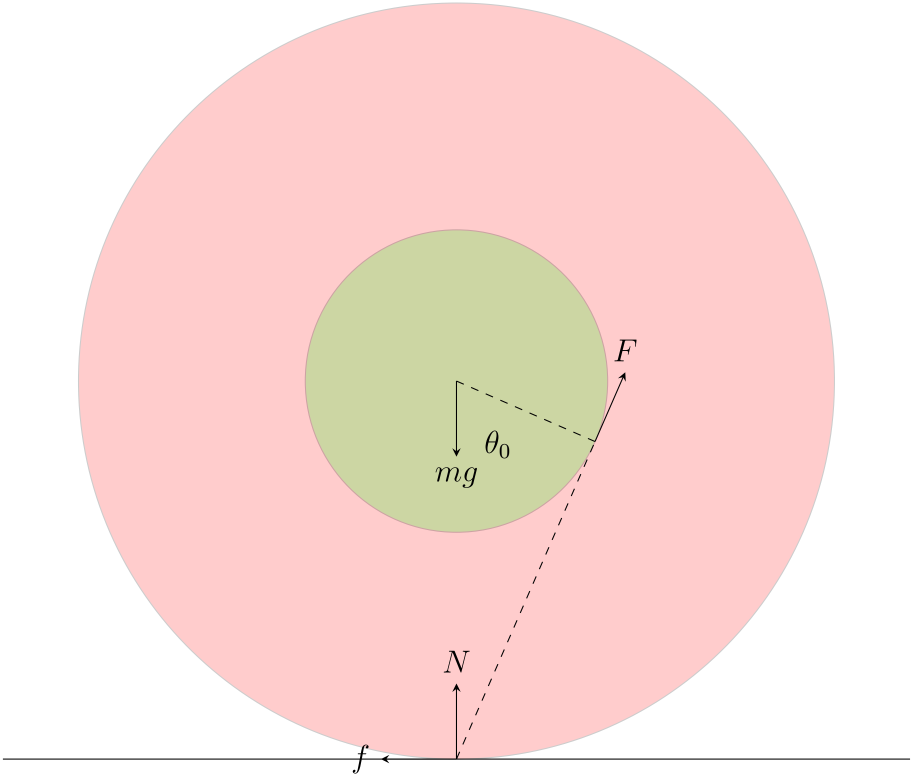
Contents

1 Rigid Body Mechanics

Example 1. A body is constructed using two concentric cylinders, of radii d and b as shown. A string is wound around the inner cylinder and is pulled with force F. The whole body has moment of inertia kmd^2. The ground has friction such that the body rolls without slipping. Find the condition for the body to not move.



Solution. For the torque about the IAOR to be zero, F must be in the direction of the tangent from the point of contact to the inner cylinder.



As the body is stationary,

$$F \sin\theta_0 + N = mg$$
$$F \cos\theta_0 = f$$

About the centre, as $\tau = 0$,

$$\cos\theta_0 = \frac{b}{d}$$

Therefore,

$$Fb = fd$$

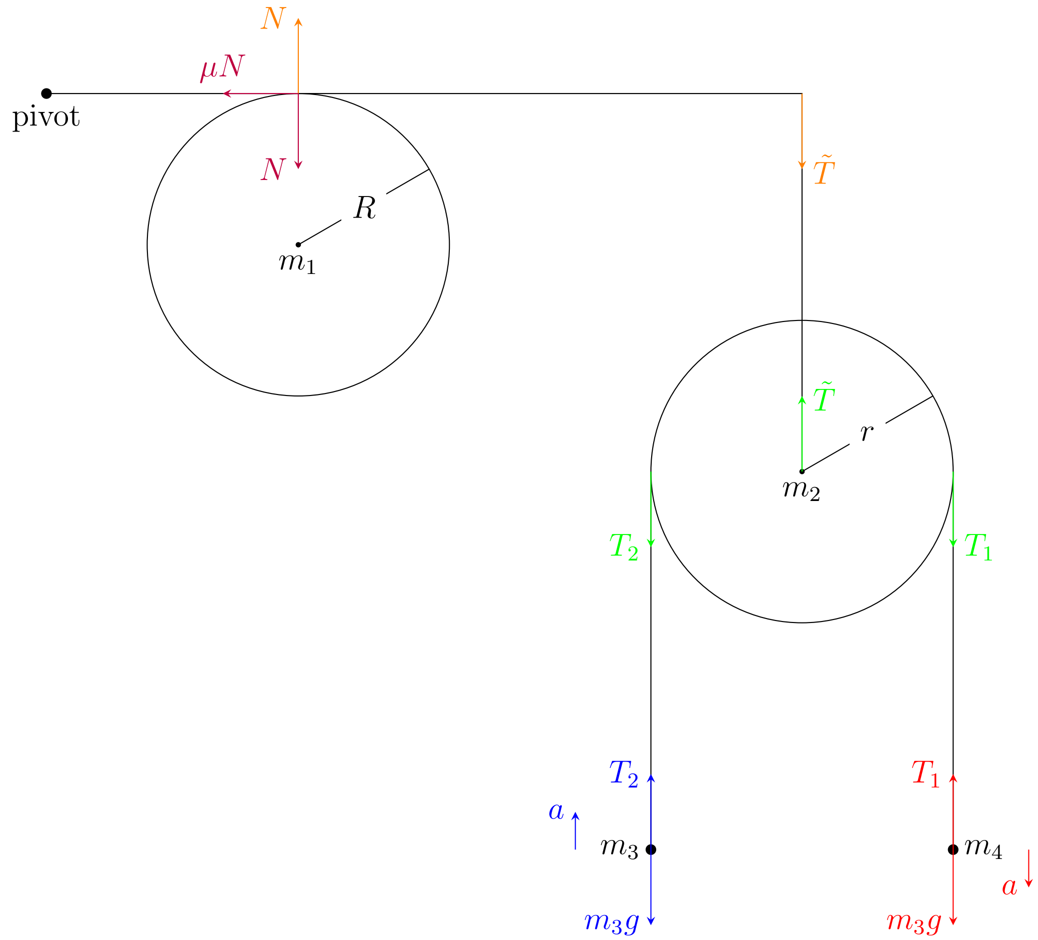
If $\theta < \theta_0$,

$$\begin{aligned} F\cos\theta - f &= ma_{\text{COM}} \\ F\sin\theta + N &= mg \\ fd - bF &= kmd^2\alpha \\ &= kmda_{\text{COM}} \end{aligned}$$

Solving,

$$a_{\text{COM}} = \frac{Fd(\cos\theta - {}^{b}/_{d})}{md(1+k)}$$

Example 2. A disk is of mass m_1 is fixed at its centre and is rotating with ω_0. A system with a small brake pad is arranged such that the brake pad touches the disk, as shown. Find the angular velocity of the disk as a function of time.



Solution. For the pulley and the masses,

$$\begin{aligned}
m_4 g - T_1 &= m_4 a \\
T_2 - m_3 g &= m_3 a \\
T_1 r - T_2 r &= \frac{1}{2} m_2 r^2 \alpha \\
\tilde{T} &= T_1 + T_2 + m_2 g
\end{aligned}$$

For the lever, as $\tau = 0$ about the pivot point,

$$\begin{aligned}
0 &= -\tilde{T} L + N \frac{L}{3} \\
\therefore N &= 3\tilde{T}
\end{aligned}$$

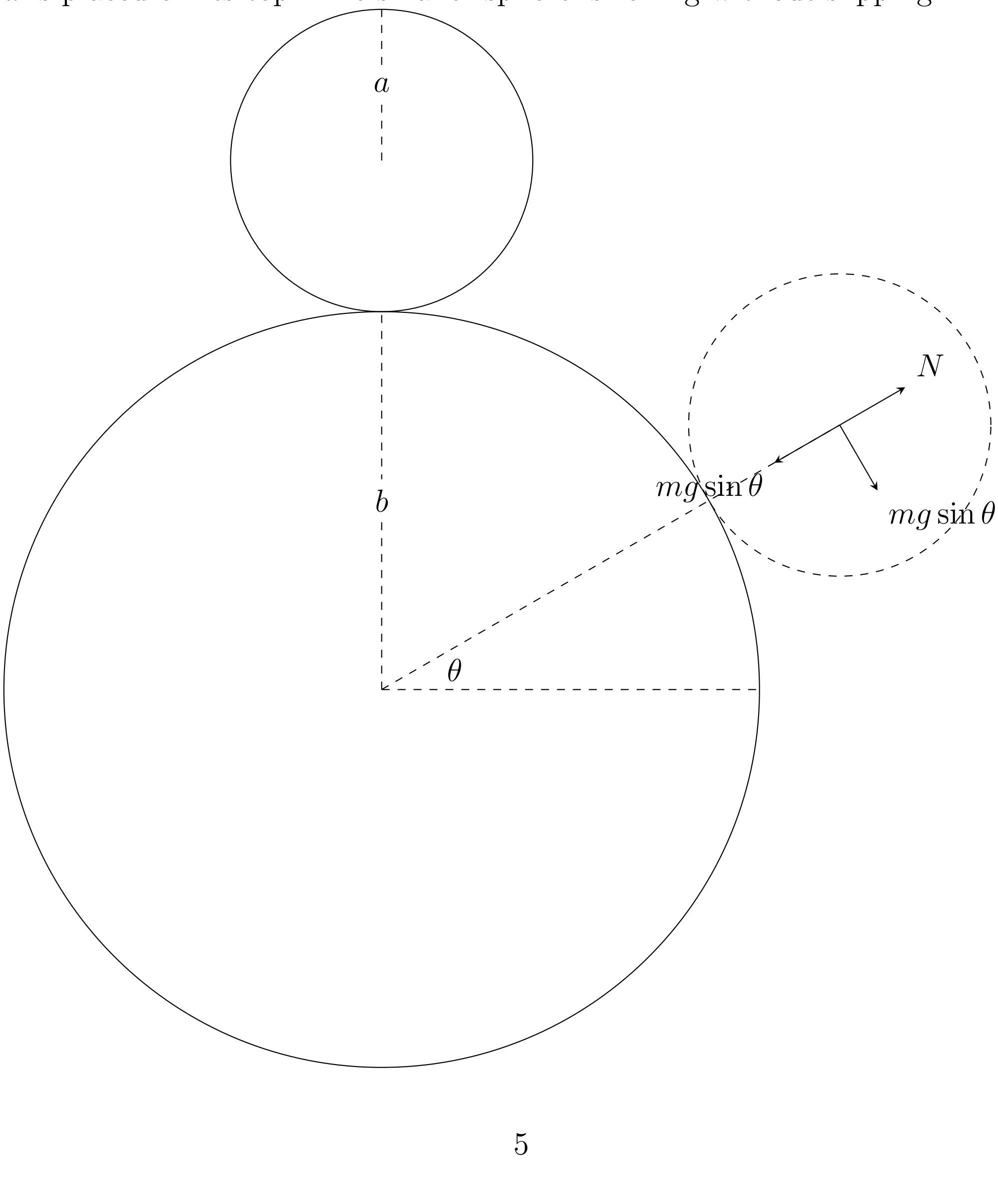
For the disk,

$$
\begin{aligned}
fR &= \frac{1}{2} m_1 R^2 \alpha \\
\therefore \alpha &= \frac{3\mu\tilde{T}}{{}^1\!/_2 \cdot m_1 R}
\end{aligned}
$$

Therefore,

$$
\omega = \omega_0 - \frac{6\mu\tilde{T}}{m_1 R} \cdot t
$$

Example 3. A sphere of radius b is fixed to the ground. A sphere of radius a is placed on its top. The smaller sphere is rolling without slipping.



Solution.

$$mg\sin\theta - N = m\frac{v_{\text{COM}}^2}{a+b}$$

When the ball loses contact, $N = 0$. Therefore,

$$mg\sin\theta = m\frac{v_{\text{COM}}^2}{a+b}$$

By COME,

$$0 = -mg(a+b)(1-\sin\theta) + \frac{1}{2}\left(\frac{2}{5}ma^2\right)\omega^2 + \frac{1}{2}mv_{\text{COM}}^2$$

As the ball is purely rolling,

$$v_{\text{COM}} = \omega a$$

Solving,

$$\theta = \sin^{-1}\left(\frac{10}{17}\right)$$

Example 4. A pool ball of radius R is at rest on the ground. A cue hits the ball at h from the ground. Find h such that the ball starts purely rolling.

Solution. By COLM,

$$p_0 = mv_{\text{COM}}$$

By COAM, with respect to the IAOR,

$$hp_0 = \frac{7}{5}mR^2\omega$$

As the ball is purely rolling,

$$v_{\text{COM}} = \omega R$$

Therefore,

$$\begin{aligned} hmv_{\text{COM}} &= \frac{7}{5}mR^2\omega \\ \therefore hm\omega R &= \frac{7}{5}mR^2\omega \\ \therefore h &= \frac{7}{5}R \end{aligned}$$

Example 5. A pool ball of mass m and radius R is at rest on a rough ground. The coefficient of friction between the ball and the ground is μ. A cue hits the ball at R from the ground. Find

Solution.

$$p_0 = mv_{\text{COM}}$$

$$\begin{aligned} ma_{\text{COM}} &= -\mu mg \\ \therefore v_{\text{COM}} &= -\mu gt \end{aligned}$$

$$\begin{aligned} \mu mgR &= \frac{2}{5}mR^2\alpha \\ \therefore \alpha &= \frac{5\mu g}{2R} \\ \therefore \omega &= \frac{5\mu g}{2R}t \end{aligned}$$

The ball will start at some t_1, such that

$$\begin{aligned} v_{\text{COM}}(t_1) &= \omega(t_1)R \\ \frac{p_0}{m} - \mu g t_1 &= \frac{5\mu g}{2R}t_1 R \\ \therefore t &= \frac{2p_0}{7m} \end{aligned}$$