

Lecture 14

Aakash Jog

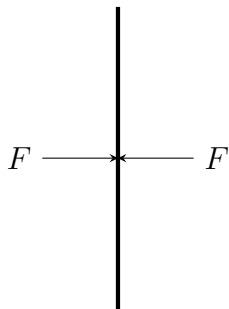
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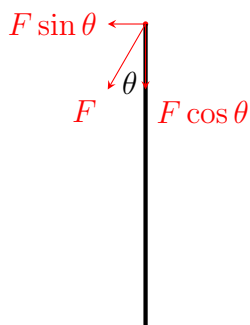
1 Rigid Body Mechanics

1.1 Definitions



$$M_{\text{total}} \overrightarrow{a_{\text{COM}}} = \sum \overrightarrow{F_{\text{ext}}}$$

$$\therefore v_{\text{COM}} = \text{constant}$$



Definition 1 (Torque).

$$\tau \doteq r F \sin \theta$$

$$\overrightarrow{\tau} \doteq \overrightarrow{r} \times \overrightarrow{F}$$

Definition 2 (Angular momentum).

$$\overrightarrow{L} \doteq \overrightarrow{r} \times \overrightarrow{p}$$

$$\begin{aligned}
\vec{\tau}_A &= 0 \\
\therefore \frac{d\vec{L}_A}{dt} &= 0 \\
\therefore \vec{L}_A &= \text{constant} \\
&= rmv\hat{z} \\
&= mr^2\omega\hat{z} \\
&= (mr^2)\vec{\omega}
\end{aligned}$$

Definition 3 (Moment of Inertia).

$$I \doteq mr^2$$

$$\begin{aligned}
E_k &= \frac{1}{2}mv^2 \\
&= \frac{1}{2}mr^2\omega^2 \\
&= \frac{1}{2}I\omega^2
\end{aligned}$$

Example 1. A particle attached to a string is on a horizontal table, moving in a circle of radius r_0 with v_0 . The other end on the string goes through the table, through a hole in the centre of the circle. It is pulled down by a force F . Find the velocity of the particle as a function of the radius.

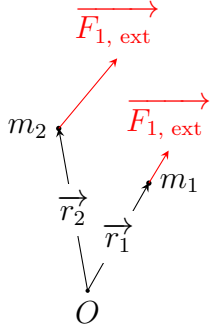
Solution.

$$\begin{aligned}
\vec{\tau} &= 0 \\
\therefore \vec{L}_A &= r_0mv_0\hat{z} \\
\therefore \vec{L} &= rmv\hat{z}
\end{aligned}$$

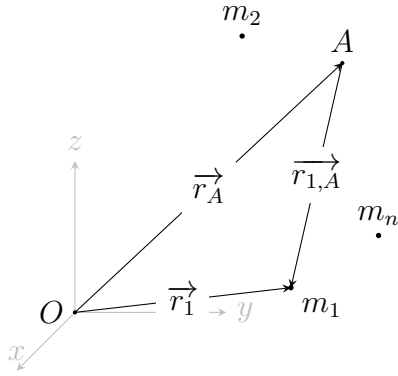
Example 2.

Solution.

$$\begin{aligned}
\vec{F}_{1,2} &= -\vec{F}_{2,1} \\
\vec{\tau}_{\text{total},0} &= \vec{\tau}_{1,0} + \vec{\tau}_{2,0} \\
&= \vec{r}_1 \times \vec{F}_{1,\text{ext}} + \vec{r}_1 \times \vec{F}_{1,2} + \vec{r}_2 \times \vec{F}_{2,\text{ext}} + \vec{r}_2 \times \vec{F}_{2,1} \\
&= \vec{r}_1 \times \vec{F}_{1,\text{ext}} + \vec{r}_2 \times \vec{F}_{2,\text{ext}} + \underbrace{(\vec{r}_1 - \vec{r}_2) \times \vec{F}_{1,2}}_{\vec{0}} \\
&= \vec{\tau}_{\text{total},O,\text{ext}}
\end{aligned}$$



1.2 Conditions for Equilibrium



$$\sum \vec{F}_{i, \text{ext}} = 0$$

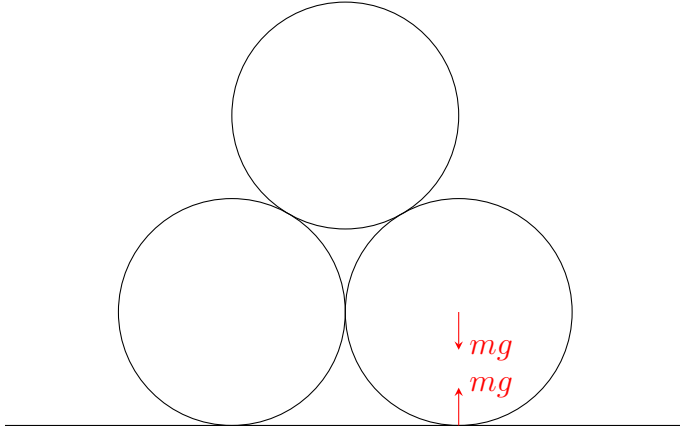
$$\sum \vec{\tau}_{i, O, \text{ext}} = 0$$

$$\begin{aligned} \sum \vec{\tau}_{i, A, \text{ext}} &= \sum \vec{r}_{i, A} \times \vec{F}_{i, \text{ext}} \\ &= \sum \vec{r}_{i, O} \times \vec{F}_{i, \text{ext}} - \sum \vec{r}_A \times \vec{F}_{i, \text{ext}} \\ &= 0 \end{aligned}$$

Therefore, equilibrium is independent of the point of reference.
Hence the conditions for equilibrium are

$$\sum \vec{F}_{i, \text{ext}} = 0$$

$$\sum \vec{\tau}_{i, \text{ext}} = 0$$



Example 3. Find the minimum μ for which the bodies remain at rest.

Solution.

$$\begin{aligned}\sum \overrightarrow{F_{i, \text{ext}}} &= 0 \\ \therefore 2N_f &= 3mg \\ \therefore N_f &= \frac{3}{2}mg\end{aligned}$$

With respect to an axis passing through A,

$$\begin{aligned}\sum \overrightarrow{\tau_{i, \text{ext}}} &= 0 \\ \therefore (-\hat{z})dmg + \hat{z}d \cdot \frac{3}{2}mg + (-\hat{z})Hf_f &= 0 \\ \therefore f_f &= \frac{^{1/2} \cdot mgd}{H} \\ &= \frac{^{1/2} \cdot mgR \cos 60}{R + R \sin 60} \\ &\leq \mu N_f \\ \therefore \frac{^{1/2} \cdot mg \cdot ^{1/2}}{1 + \sqrt{3}/2} &= \mu \cdot ^{3/2} \cdot mg \\ \therefore \mu &\geq \frac{^{1/6}}{1 + \sqrt{3}/2}\end{aligned}$$

2 Coordinate Systems

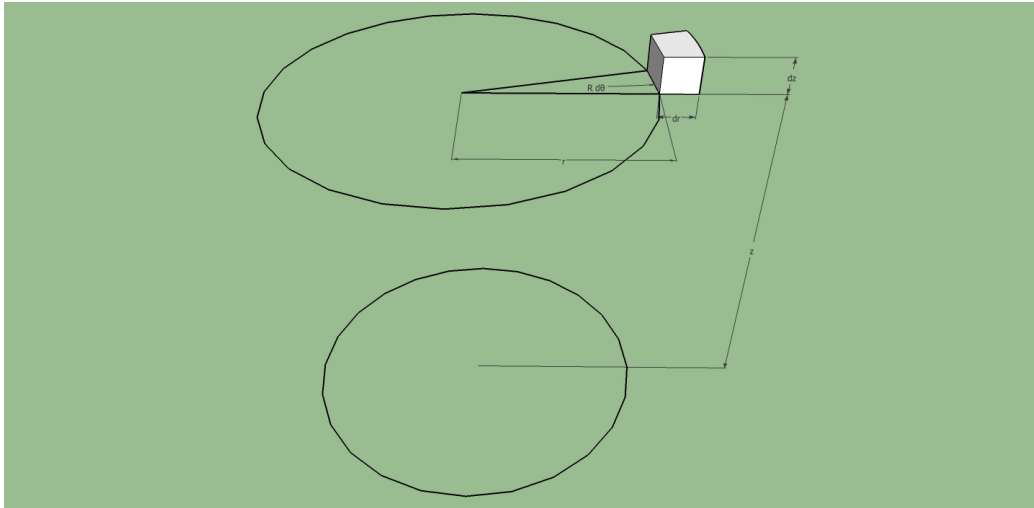
2.1 Cylindrical Coordinates

$$(x, y, z) \rightarrow (r, \theta, z)$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$z = z$$



$$dV = r d\theta dr dz$$

2.2 Spherical Coordinates

$$(x, y, z) \rightarrow (r, \theta, \varphi)$$

$$\begin{aligned} dV &= r d\theta \cdot r \sin \theta d\varphi dr \\ &= r^2 \sin \theta d\varphi dr d\theta \end{aligned}$$

