Recitation 12

Wednesday 14^{th} January, 2015

Week 12 : Class Exercises

Exercise 3

1

$$mg = kx_0$$
$$\therefore x_0 = \frac{mg}{k}$$

2

Let the distance about the equilibrium position be

$$z = x - x_0$$
$$\therefore \ddot{z} = \ddot{x}$$

Therefore,

$$\ddot{z} + \frac{k}{m}z = 0$$
$$\therefore z(t) = A\cos\frac{k}{m}t + B\sin\frac{k}{m}t$$

According to the initial conditions,

$$z(t) = -\frac{mv_0}{k}\sin\frac{k}{m}t$$

Week 12 : Home Assignment

Exercise 5

$$-kx - f = m\ddot{x}$$

Considering torques about the centre,

$$Rf = \frac{mR^2}{2}\ddot{\theta}$$

As the cylinder is purely rolling,

$$\begin{aligned}\dot{x} &= \dot{\theta}R \\ \ddot{x} &= \ddot{\theta}R\end{aligned}$$

Therefore,

$$\begin{aligned}f &= \frac{m\ddot{x}}{2} \\ \therefore -kx + \frac{m\ddot{x}}{2} &= m\ddot{x} \\ \therefore \ddot{x} + \frac{2k}{3m}x &= 0 \\ \therefore \omega &= \sqrt{\frac{2k}{3m}}\end{aligned}$$

Also, as the cylinder is purely rolling, mechanical energy is conserved.
Therefore,

$$\begin{aligned}E &= \frac{1}{2}kx^2 + \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\theta}^2 \\ &= \frac{1}{2}kx^2 + \frac{1}{2}m\dot{x}^2 + \frac{1}{4}m\dot{x}^2 \\ \therefore 0 &= kx\dot{x} + m\dot{x}\ddot{x} + \frac{1}{2}m\dot{x}\ddot{x} \\ \therefore 0 &= \ddot{x} + \frac{2k}{3m}x \\ \therefore \omega &= \sqrt{\frac{2k}{3m}}\end{aligned}$$

Week 13 : Class Exercises

Exercise 1

$$\begin{aligned} v_{m,M} &= a\dot{\theta}\hat{\theta} \\ &= a\dot{\theta}(\cos\theta\hat{i} + \sin\theta\hat{j}) \\ &= a\omega(\cos\theta\hat{i} + \sin\theta\hat{j}) \end{aligned}$$

By COLM in the horizontal direction,

$$\begin{aligned} 0 &= m(v_m)_x + M(v_M)_x \\ \therefore (v_m)_x &= -\frac{M}{m}(v_M)_x \end{aligned}$$

$$\begin{aligned} (v_{m,M})_x &= (v_m)_x - (v_M)_x \\ &= -\left(\frac{M}{m} + 1\right)(v_M)_x \\ \therefore -(v_M)_x\left(\frac{M+m}{m}\right) &= a\dot{\theta}\cos\theta\dot{i} \\ \therefore (v_M)_x &= -a\dot{\theta}\cos\theta\dot{i}\left(\frac{m}{M+m}\right) \\ \therefore (v_m)_x &= -\frac{M}{m}\cdot -a\dot{\theta}\cos\theta\dot{i}\left(\frac{m}{M+m}\right) \\ &= \left(\frac{M}{M+m}\right)a\dot{\theta}\cos\theta\dot{i} \end{aligned}$$

$$(v_m)_y = a\dot{\theta}\sin\theta\hat{j}$$

Therefore,

$$\begin{aligned} v_m &= \left(\frac{M}{M+m}\right)a\dot{\theta}\cos\theta\dot{i} + a\dot{\theta}\sin\theta\hat{j} \\ v_M &= -\left(\frac{m}{M+m}\right)a\dot{\theta}\cos\theta\hat{i} \end{aligned}$$