Lecture 23

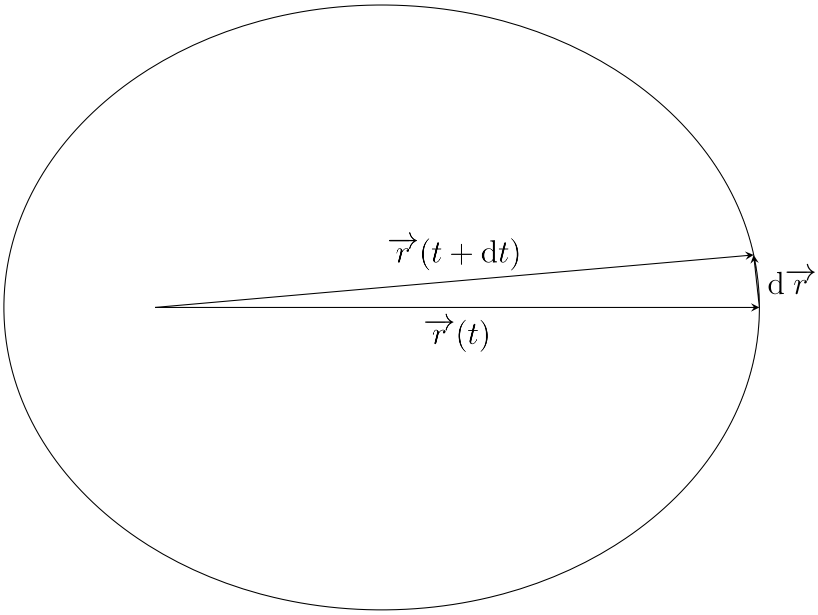
Aakash Jog

Thursday 15^{th} January, 2015

Contents

1 Gravitation

1.1 Kepler's Second Law



$$\begin{aligned}
\mathrm{d}\overrightarrow{A} &= \frac{1}{2}\overrightarrow{r} \times \mathrm{d}\overrightarrow{r} \\
\therefore \frac{\mathrm{d}\overrightarrow{A}}{\mathrm{d}t} &= \frac{1}{2}\overrightarrow{r} \times \frac{\mathrm{d}\overrightarrow{r}}{\mathrm{d}t} \\
&= \frac{1}{2}\overrightarrow{r} \times \overrightarrow{v} \\
&= \frac{\overrightarrow{L}}{2m}
\end{aligned}$$

1.2 Newton's Law of Gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

$$
\begin{aligned}
\overrightarrow{F} &= \frac{c}{r^2}\hat{r} \\
&= \frac{c}{x^2+y^2+z^2}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}\right) \\
&= \frac{c}{(x^2+y^2+z^2)^{3/2}}(x, y, z) \\
\therefore \operatorname{curl}\left(\overrightarrow{F}\right) &= 0
\end{aligned}
$$

Therefore, $\overrightarrow{F}$ is conservative.

$$
\begin{aligned}
\overrightarrow{F} &= -\overrightarrow{\nabla} U \\
\frac{\partial U}{\partial x} &= -\frac{cx}{(x^2+y^2+z^2)^{3/2}} \\
\frac{\partial U}{\partial y} &= -\frac{cy}{(x^2+y^2+z^2)^{3/2}} \\
\frac{\partial U}{\partial z} &= -\frac{cz}{(x^2+y^2+z^2)^{3/2}}
\end{aligned}
$$

Therefore,

$$
U = \frac{c}{r} + d
$$

Considering the potential to be zero at $r = \infty$,

$$
\begin{aligned}
U &= \frac{c}{r} \\
&= -G\frac{m_1 m_2}{r}
\end{aligned}
$$