Lecture 21

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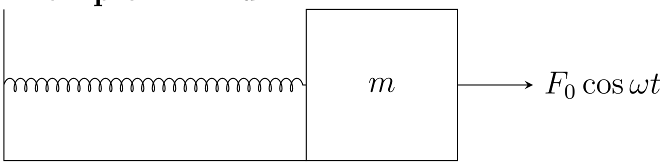
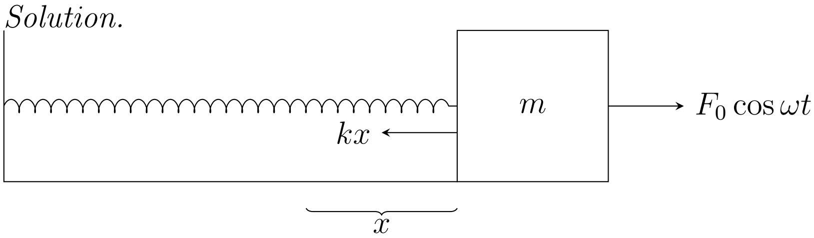
Contents

1 Forced Oscillations

Example 1. Find



Solution.



$$
\begin{aligned}
m\ddot{x} + \frac{k}{m}x &= \frac{F_0}{m}\cos \omega t \\
\therefore \ddot{x} + \frac{k}{m}x &= \frac{F_0}{m}\cos \omega t
\end{aligned}
$$

Therefore, solving

$$
\begin{aligned}
x &= A\cos \omega_0 t + B \sin \omega_0 t + \frac{F_0}{k - m\omega^2}\cos \omega t \\
\therefore \dot{x} &= \omega_0(-A \sin \omega_0 t + B \cos \omega_0 t) - \frac{F_0}{k - m\omega^2}\omega \sin \omega t
\end{aligned}
$$

Substituting initial conditions,

$$
x = \frac{\frac{F_0}{m}}{\frac{k}{m} - \omega^2}(-\cos \omega_0 t + \cos \omega t)
$$

Let $\dfrac{F_0}{m} = f_0$

$$
\therefore x = -\frac{2f_0}{{\omega_0}^2 - \omega^2}\sin\left(\frac{\omega t - \omega_0 t}{2}\right)\sin\left(\frac{\omega t + \omega_0 t}{2}\right)
$$

$\omega - \omega_0 = \Delta\omega$ and $\omega + \omega_0 \approx 2\omega_0$

$$
\therefore x \approx \frac{2f_0}{\Delta\omega \cdot 2\omega_0}\sin\left(\frac{\Delta\omega}{2}t\right) \cdot \sin(\omega_0 t)
$$