Recitation 7

Wednesday 10$^{\text{th}}$ December, 2014

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Example 1. On one side of a boat of mass M, a box of mass m is placed and the boat's engine is pulling the box via a massless rope. At $t = 0$ the entire system is at rest. Then the engine is turned on and it starts pulling the box with force $\overrightarrow{F}(t) = \alpha t$ where α is a positive constant. The friction coefficients between the box and the boat are μ_s and μ_k. The engine is working for time $\tau = \dfrac{mg}{\alpha}$ and then stops. Assuming that the box does not reach the end of the boat nor collide with any other object, what will be the boat's velocity after a very long time $t >> \tau$? Find the box's velocity w.r.t. the boar from the moment the engine is turned on.

Solution.

$$
\begin{aligned}
F &= \alpha t \qquad\qquad ; \quad 0 \le t \le \tau = \frac{mg}{\alpha} \\
N_s &= \mu_s mg \\
N_k &= \mu_k mg
\end{aligned}
$$

Let t_0 be the time when the box starts moving.

$$
\begin{aligned}
F(t) &= \mu_s mg \\
&= \alpha t \\
\therefore t_0 &= \frac{\mu_s mg}{\alpha}
\end{aligned}
$$

For $t_0 < t < \tau$,

$$
\begin{aligned}
\alpha t - \mu_k mg &= ma \\
\therefore a &= \frac{\alpha t}{m} - \mu_k g \\
\therefore v &= \int_{t_0}^{\tau} a(t)\,\mathrm{d}t \\
&= \frac{\alpha}{2m}(\tau^2 - t_0^2) - \mu_k g(\tau - t_0)
\end{aligned}
$$

For $t > \tau$,

$$
\begin{aligned}
a &= \mu_k g \\
\therefore v &= v_\tau + \int_{\tau}^{t} g\,\mathrm{d}t \\
&= v_\tau - \mu_k g(t - \tau)
\end{aligned}
$$