PHYSICS 1 : ASSIGNMENT 12

AAKASH JOG

Week 4 : Class Exercises

Exercise 1

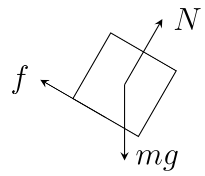

Figure 1. Forces acting on the block in a rest frame if the block tends to move down the incline.

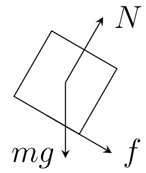

Figure 2. Forces acting on the block in a rest frame if the block tends to move up the incline.

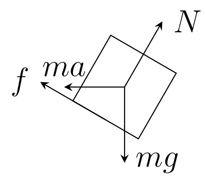

Figure 3. Forces acting on the block in the wedge frame if the block tends to move down the incline.

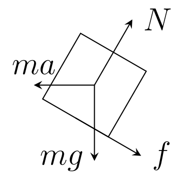

Figure 4. Forces acting on the block in the wedge frame if the block tends to move up the incline.

For the block to be stationary with respect to the wedge, the net forces on the block, in

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the frame of the reference of the wedge, are 0.
Therefore, if the block tends to move down the incline, at the threshold condition

$$\begin{aligned} f_{\max}\cos\theta + ma &= N\sin\theta \\ f_{\max}\sin\theta + N\cos\theta &= mg \end{aligned}$$

Therefore,

$$\begin{aligned} \mu N\cos\theta + ma &= N\sin\theta \\ \therefore N\sin\theta - \mu N\cos\theta &= ma \\ \mu N\sin\theta + N\cos\theta &= mg \\ \therefore \frac{\sin\theta - \mu\cos\theta}{\mu\sin\theta + \cos\theta} &= \frac{a}{g} \\ \therefore a &= g\left(\frac{\sin\theta - \mu\cos\theta}{\mu\sin\theta + \cos\theta}\right) \end{aligned}$$

Therefore, if the block tends to move up the incline, at the threshold condition

$$\begin{aligned} ma &= N\sin\theta + f_{\max}\cos\theta \\ N\cos\theta &= mg + f_{\max}\sin\theta \end{aligned}$$

Therefore,

$$\begin{aligned} ma &= N\sin\theta + \mu N\cos\theta \\ N\cos\theta &= mg + \mu N\sin\theta \\ \therefore mg &= N\cos\theta - \mu N\sin\theta \\ \therefore \frac{a}{g} &= \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} \\ \therefore a &= g\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right) \end{aligned}$$

If the block tends to move down the incline,
If $\theta = 0$,

$$a = -\mu g$$

If $\theta = \frac{\pi}{2}$,

$$a = \frac{g}{\mu}$$

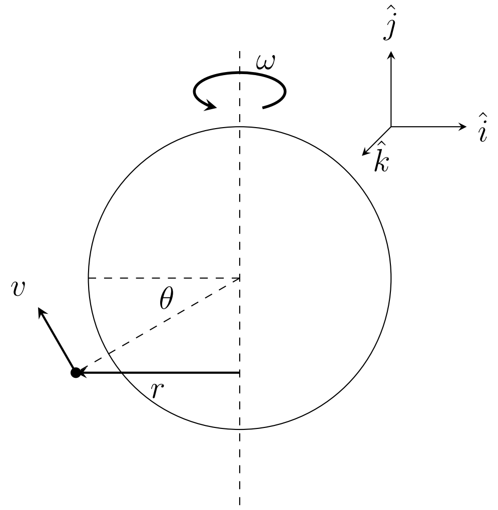
If the block tends to move up the incline,
If $\theta = 0$,

$$a = \mu g$$

If $\theta = \dfrac{\pi}{2}$,

$$a = -\frac{g}{\mu}$$

Exercise 2



$$\begin{aligned}\overrightarrow{F_{\text{centrifugal}}} &= -\overrightarrow{\omega} \times \left(\overrightarrow{\omega} \times \overrightarrow{r}\right) \\ &= -m\omega^2 R_e \cos\theta \hat{i}\end{aligned}$$

$$\begin{aligned}\overrightarrow{F_{\text{coriolis}}} &= -2m\overrightarrow{\omega} \times \overrightarrow{v} \\ &= -2m\omega v \sin\theta \hat{k}\end{aligned}$$

Week 4 : Home Assignment

Exercise 1

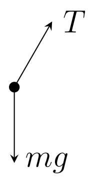


Figure 5. Forces on the ball as seen from the rest frame.

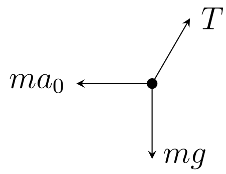


FIGURE 6. Forces on the ball as seen from the train frame of reference.

In the frame of reference of the train,

$$
\begin{aligned}
mg &= T\cos\theta \\
ma_0 &= T\sin\alpha \\
\therefore \frac{a_0}{g} &= \tan\theta \\
\therefore \theta &= \tan^{-1}\frac{a_0}{g}
\end{aligned}
$$

Let the position of the ball at $t = T$ be 0.
In the frame of reference of the train,

$$\overrightarrow{a'} = -a_0\hat{i} - g\hat{j}$$

As $v'(t = T) = 0$,

$$\therefore \overrightarrow{v'} = -a_0(t - T)\hat{i} - g(t - T)\hat{j}$$

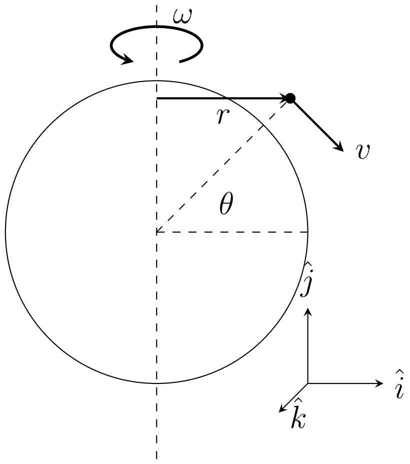
As $r'(t = T) = 0$,

$$\therefore \overrightarrow{r'} = -\frac{1}{2}a_0(t - T)^2\hat{i} - \frac{1}{2}g(t - T)^2\hat{j}$$

In the rest frame,

$$
\begin{aligned}
\overrightarrow{a} &= -g\hat{j} \\
\overrightarrow{v}(t = 0) &= a_0T\hat{i} \\
\therefore \overrightarrow{r} &= a_0T(t - T)\hat{i} - \frac{1}{2}g(t - T)^2
\end{aligned}
$$

Exercise 2



$$
\begin{aligned}
m &= 2\text{kg} \\
v &= 500\,\text{m s}^{-1} \\
\theta &= 45^\circ \\
R_e &= 6.37 \times 10^6\text{m}
\end{aligned}
$$

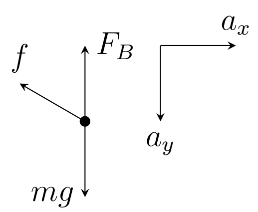
$$
\begin{aligned}
\overrightarrow{F_{\text{centrifugal}}} &= -m\overrightarrow{\omega} \times \left(\overrightarrow{\omega} \times \overrightarrow{r}\right) \\
&= m\omega^2 R_e \cos\theta \hat{i} \\
&= m\left(\frac{2\pi}{T}\right)^2 R_e \cos\theta \hat{i} \\
&= m\frac{4\pi^2}{T^2} R_e \cos\theta \hat{i} \\
&= (2)\frac{4\pi^2}{(86400)^2}(6.37 \times 10^6)\frac{1}{\sqrt{2}}\hat{i} \\
&= \frac{8\pi^2}{746496}(6.37 \times 10^2)\frac{1}{\sqrt{2}}\hat{i} \\
&= \frac{(8)(6.37)}{(746496)(\sqrt{2})} \times 10^2 \hat{i} \\
&= \frac{5096}{1055704.7675}\hat{i} \\
&= 0.00482710712\hat{i}\text{N} \\
&= 4.82710712 \times 10^{-3}\hat{i}\text{N} \\
\therefore F_{\text{centrifugal}} &= 4.82710712 \times 10^{-3}\text{N}
\end{aligned}
$$

$$
\begin{aligned}
\overrightarrow{F_{\text{coriolis}}} &= -2m\overrightarrow{\omega} \times \overrightarrow{v} \\
&= 2m\omega v \frac{1}{\sqrt{2}}\hat{k} \\
&= (2)(2)\frac{2\pi}{86400}(500)\frac{1}{\sqrt{2}}\hat{k} \\
&= \frac{40}{864} \cdot \frac{1}{\sqrt{2}}\hat{k} \\
&= \frac{5\sqrt{2}}{216}\pi\hat{k} \\
&= 0.102845\hat{k}\text{N} \\
\therefore F_{\text{coriolis}} &= 1.02845 \times 10^{-1}\text{N}
\end{aligned}
$$

$$
\begin{aligned}
F_{\text{gravity}} &= mg \\
&= (2)(9.81) \\
&= 19.62\text{N}
\end{aligned}
$$

Therefore, the Coriolis and centrifugal forces are very small compared to gravity.

Exercise 5



$$
\begin{aligned}
ma_y &= -F_B + mg - f\sin\theta \\
&= -\rho_W V g + mg - kv_x \\
&= -\frac{4}{3}\pi\rho_W R^3 g + mg - kv_y \\
ma_x &= -f\cos\theta \\
&= -kv_x
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
m\ddot{x} &= -k\dot{x} \\
m\ddot{y} &= -k\dot{y} - \frac{4}{3}\pi\rho_W R^3 g + mg
\end{aligned}
$$

Therefore,

$$\begin{aligned}
\frac{\mathrm{d}v_x}{\mathrm{d}t} &= -\frac{k}{m}v_x \\
\therefore \frac{\mathrm{d}v_x}{v_x} &= -\frac{k}{m}\,\mathrm{d}t \\
\therefore \ln v_x - \ln v_0\cos\theta &= -\frac{k}{m}t \\
\therefore \frac{v_x}{v_0\cos\theta} &= e^{-kt/m} \\
\therefore v_x &= v_0\cos\theta e^{-kt/m}
\end{aligned}$$

$$\begin{aligned}
\frac{\mathrm{d}v_y}{\mathrm{d}t} &= -\frac{k}{m}v_y - \frac{\rho_W V g}{\rho_B g} + g \\
&= g\left(1 - \frac{\rho_W}{\rho_B}\right) - \frac{k}{m}v_y
\end{aligned}$$

Let

$$\begin{aligned}
u &= g\left(1 - \frac{\rho_W}{\rho_B}\right) - \frac{k}{m}v_y \\
\therefore \mathrm{d}u &= -\frac{k}{m}\,\mathrm{d}v_y
\end{aligned}$$

Let

$$\begin{aligned}
A &= g\left(1 - \frac{\rho_W}{\rho_B}\right) \\
B &= \frac{k}{m}
\end{aligned}$$

Therefore,

$$\begin{aligned}
\therefore \frac{\mathrm{d}u}{u} &= -\frac{k}{m}v_y\,\mathrm{d}t \\
\therefore \ln u &= -\frac{k}{m}v_y t + c \\
\therefore v_y &= \frac{A}{B}t + \left(v_0\sin\theta - \frac{A}{B}\right)e^{-Bt}
\end{aligned}$$

At $t = t_0$, $v_y(t) = 0$.

$$\begin{aligned}
t_0 &= \frac{1}{B}\ln\left(\frac{v_y\sin\theta + {}^{A}\!/_{B}}{{}^{A}\!/_{B}}\right) \\
\therefore y_{\max} &= \frac{v_0\sin\theta}{B} - \frac{A}{B^2}\ln\left(\frac{Bv_0\sin\theta}{A} + 1\right)
\end{aligned}$$