Lecture 4

Thursday 6th November, 2014

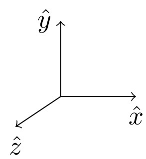
Contents

1 Vectors

1.1 Notation

$$A = \left|\overrightarrow{A}\right| = \left\|\overrightarrow{A}\right\|$$

$$\hat{A} \doteq \frac{\overrightarrow{A}}{A}$$

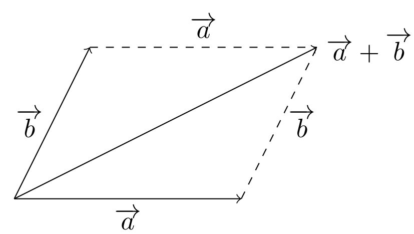


$$\hat{z} \doteq \hat{x} \times \hat{y}$$

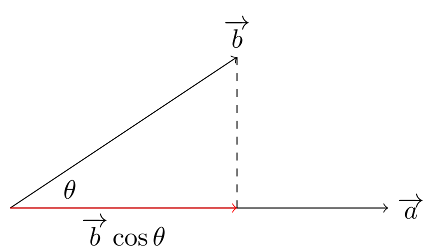
$$\overrightarrow{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

1.2 Vector Addition



1.3 Scalar Product/ Dot Product



$$\overrightarrow{A} \cdot \overrightarrow{B} = AB\cos\theta$$

1.3.1 Properties

$$\overrightarrow{A} \cdot \overrightarrow{B} = \overrightarrow{B} \cdot \overrightarrow{A}$$

$$\overrightarrow{A} \cdot (\overrightarrow{B} + \overrightarrow{C}) = \overrightarrow{A} \cdot \overrightarrow{B} + \overrightarrow{A} \cdot \overrightarrow{C}$$

If

$$
\begin{aligned}
r_1 &= (x_1\hat{x} + y_1\hat{y} + z_1\hat{z}) \\
r_2 &= (x_2\hat{x} + y_2\hat{y} + z_2\hat{z})
\end{aligned}
$$

$$
\begin{aligned}
\overrightarrow{r_1} \cdot \overrightarrow{r_2} &= (x_1\hat{x} + y_1\hat{y} + z_1\hat{z}) \cdot (x_2\hat{x} + y_2\hat{y} + z_2\hat{z}) \\
&= x_1x_2 + y_1y_2 + z_1z_2
\end{aligned}
$$

1.3.2 Projection of Vectors

The projection of $\overrightarrow{B}$ on $\overrightarrow{A}$ is denoted by $\overrightarrow{B}_{\overrightarrow{A}}$

$$
\begin{aligned}
\overrightarrow{B}_{\overrightarrow{A}} &= \left(\frac{\overrightarrow{B} \cdot \overrightarrow{A}}{A}\right) \cdot \hat{A} \\
&= \left(\frac{\overrightarrow{B} \cdot \overrightarrow{A}}{A}\right) \cdot \frac{\overrightarrow{A}}{A} \\
&= \left(\frac{\overrightarrow{B} \cdot \overrightarrow{A}}{A^2}\right) \cdot \overrightarrow{A}
\end{aligned}
$$

1.4 Vector Product/ Cross Product

$\overrightarrow{A} \times \overrightarrow{B} = AB\sin\theta\hat{r}; \hat{r} \perp \overrightarrow{A}, \hat{r} \perp \overrightarrow{B}$

$\left\|\overrightarrow{A} \times \overrightarrow{B}\right\| = AB\sin\theta =$ Area of parallelogram formed by $\overrightarrow{A}$ and $\overrightarrow{B}$

1.4.1 Properties

$$
\begin{aligned}
\overrightarrow{A} \times \overrightarrow{B} &= -\overrightarrow{B} \times \overrightarrow{A} \\
\overrightarrow{A} \times (\overrightarrow{B} + \overrightarrow{C}) &= \overrightarrow{A} \times \overrightarrow{B} + \overrightarrow{A} \times \overrightarrow{C}
\end{aligned}
$$

If

$$
\begin{aligned}
r_1 &= (x_1\hat{x} + y_1\hat{y} + z_1\hat{z}) \\
r_2 &= (x_2\hat{x} + y_2\hat{y} + z_2\hat{z})
\end{aligned}
$$

$$
\begin{aligned}
\overrightarrow{r_1} \times \overrightarrow{r_2} &= (x_1\hat{x} + y_1\hat{y} + z_1\hat{z}) \times (x_2\hat{x} + y_2\hat{y} + z_2\hat{z}) \\
&= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}
\end{aligned}
$$

2 Derivatives of Vectors

$$\begin{aligned}
\overrightarrow{v} &= \frac{\mathrm{d}\overrightarrow{r}}{\mathrm{d}t} \\
&= \lim_{\Delta t \to 0} \frac{\overrightarrow{r}(t+\Delta t) - \overrightarrow{r}(t)}{\Delta t} \\
\overrightarrow{a} &= \frac{\mathrm{d}\overrightarrow{v}}{\mathrm{d}t} = \dot{\overrightarrow{v}} = \ddot{\overrightarrow{r}}
\end{aligned}$$

2.1 Properties

$$\frac{\mathrm{d}}{\mathrm{d}t}(\overrightarrow{A} \times \overrightarrow{B}) = \frac{\mathrm{d}\overrightarrow{A}}{\mathrm{d}t} \times \overrightarrow{B} + \overrightarrow{A} \times \frac{\mathrm{d}\overrightarrow{B}}{\mathrm{d}t}$$