Physics 2

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Contents

1 Lecturer Information

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2 Textbooks

1. D. Halliday, R. Resnick, and K. S. Krane: *Physics*, 5th edition, vol. 2 (Wiley)
2. D.J. Griffiths: *Introduction to Electrodynamics*

Part I
Electrostatics

1 Coulomb's Law

The force between two charged particles is directly proportional to the product of the charges of the particles, and inversely proportional to the square of the distance between them.

$$
\begin{aligned}
F &\propto \frac{q_1 q_2}{r^2} \\
F &= k\frac{q_1 q_2}{r^2} \\
&= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}
\end{aligned}
$$

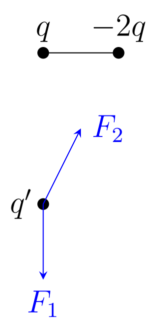
The constant of proportionality is $k = 8.99 \times 10^9 \mathrm{N\,m^2\,C^{-2}}$.
$\varepsilon_0 = 8.8541878162 \times 10^{-12} \mathrm{C^2\,N^{-1}\,m^{-2}}$ is called the permittivity of free space.
In vector notation,

$$\overrightarrow{F_{21}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{{r_{12}}^2}\hat{r_{12}}$$

Charge is defined according to this law.

Exercise 1.
A charge q is placed at the origin. A charge $-2q$ is placed at 1 m from it, in the x direction. Find a point on the y-axis where the total force acting on a charge q' will be parallel to the x-axis.

Solution 1.



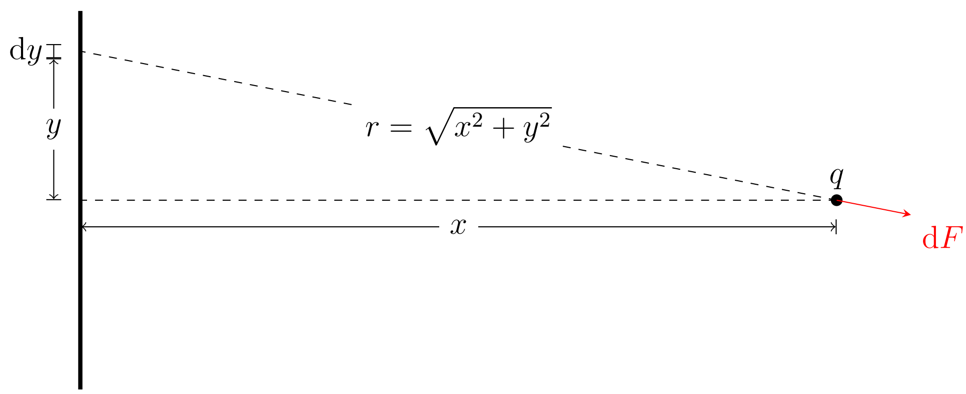
For the net force to be in the x direction, the components of F_1 and F_2 in the y direction must cancel each other out.

$$
\begin{aligned}
F_1 &= F_2 \sin\theta \\
\therefore \cancel{k} \cdot \frac{(\cancel{q})(-2\cancel{q})}{y^2+1} \cdot \frac{y}{\sqrt{y^2+1}} &= \cancel{k} \cdot \frac{(\cancel{q})(\cancel{q})}{y^2} \\
\therefore \frac{-2y}{(y^2+1)^{3/2}} &= \frac{1}{y^2} \\
\therefore y &= \pm\sqrt{\frac{1}{2^{2/3}-1}}
\end{aligned}
$$

Exercise 2.

A rod of length L has a uniformly distributed charge Q, with line charge density $\lambda = \dfrac{Q}{L}$. A point charge q is kept at a distance x as shown.

Solution 2.



The y components of the forces of the elemental charges at y and $-y$ on q are cancelled out. Therefore, the net force is in the x direction only.

$$\begin{aligned}
\mathrm{d}F &= k\frac{(\mathrm{d}Q)(q)}{r^2} \\
\mathrm{d}F_x &= \mathrm{d}F\cos\theta \\
&= k\frac{(\mathrm{d}Q)(q)}{r^2}\cos\theta \\
&= k\frac{(\lambda\,\mathrm{d}y)(q)}{x^2+y^2}\frac{x}{\sqrt{x^2+y^2}} \\
&= k\lambda qx\frac{\mathrm{d}y}{(x^2+y^2)^{3/2}} \\
\therefore \overrightarrow{F} &= \hat{x}\int \mathrm{d}F_x \\
&= \hat{k}\lambda qx\int_{-L/2}^{L/2}\frac{\mathrm{d}y}{(x^2+y^2)^{3/2}}
\end{aligned}$$

Substituting $y = x\tan\theta$ and $\mathrm{d}y = x\sec^2\theta\,\mathrm{d}\theta$

$$\begin{aligned}
\overrightarrow{F} &= \hat{x}\lambda qkx\int_{-\theta_0}^{\theta_0}\frac{1}{x^2}\cos\theta\,\mathrm{d}\theta \\
&= \hat{x}\frac{\lambda qk}{x}\int_{-\theta_0}^{\theta_0}\cos\theta\,\mathrm{d}\theta
\end{aligned}$$

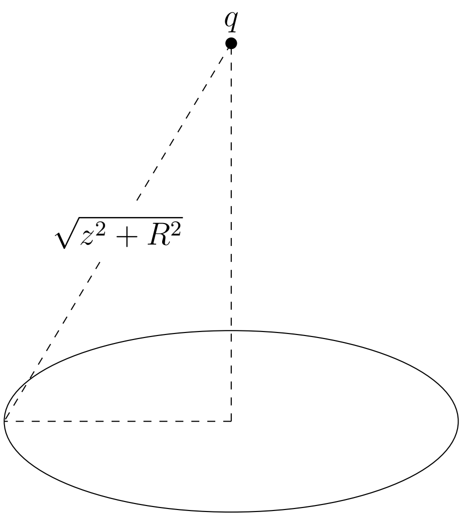
Therefore,

$$\begin{aligned}
\overrightarrow{F} &= \hat{x}\frac{2\lambda qk}{x}\sin\theta_0 \\
&= \hat{x}\frac{2\lambda qk}{x}\frac{\frac{L}{2}}{\left(\left(\frac{L}{2}\right)^2 + x^2\right)^{1/2}} \\
&= \hat{x}\frac{2\left(\frac{Q}{L}\right)qk}{x}\cdot\frac{\frac{L}{2}}{\left(\left(\frac{L}{2}\right)^2 + x^2\right)^{1/2}} \\
&= k\frac{Qq}{x\left(\left(\frac{L}{2}\right)^2 + x^2\right)^{1/2}}\hat{x}
\end{aligned}$$

Exercise 3.

A point charge q is kept at a distance z above a ring of radius R charged with $Q = 2\pi R\lambda$, where λ is the linear charge density. Find the force acting on q.

Solution 3.



Due to the symmetry of the ring, the net force acting on q is in the z direction only.

$$\begin{aligned}
\mathrm{d}F_z &= \mathrm{d}F\cos\theta \\
&= k\frac{(\mathrm{d}Q)(q)}{z^2+R^2}\cos\theta \\
&= k\frac{(\mathrm{d}Q)(q)}{z^2+R^2}\frac{z}{\sqrt{z^2+R^2}} \\
&= kqz\frac{\mathrm{d}Q}{\left(z^2+R^2\right)^{3/2}} \\
\therefore \overrightarrow{F} &= \hat{z}\int \mathrm{d}F_z \\
&= \hat{z}kqz\frac{1}{\left(z^2+R^2\right)^{3/2}}\int\limits_0^Q \mathrm{d}Q \\
&= k\frac{Qq}{\left(z^2+R^2\right)^{3/2}}\overrightarrow{z}
\end{aligned}$$

Exercise 4.

A point charge q is kept at a distance z above a disk of radius R charged with $Q = \pi R^2\sigma$, where σ is the surface charge density. Find the force acting on q.

Solution 4.

The disk can be considered to be made up of elemental rings, with radii varying from 0 to R.
Therefore,

$$\begin{aligned}
\mathrm{d}\overrightarrow{F} &= k\frac{qQ_{\text{ring}}}{\left(z^2+R^2\right)^{3/2}}\hat{z} \\
&= k\frac{q(\sigma\cdot 2\pi r\cdot \mathrm{d}r)}{\left(z^2+R^2\right)^{3/2}}z\hat{z}
\end{aligned}$$

Hence,

$$\begin{aligned}
\overrightarrow{F} &= \int \mathrm{d}\overrightarrow{F} \\
&= \hat{z} \int\limits_0^R k \frac{q\sigma \cdot 2\pi r z \cdot \mathrm{d}r}{\left(z^2 + R^2\right)^{3/2}} \\
&= 2kzq\sigma\pi \left(\frac{1}{|z|} - \frac{1}{\sqrt{z^2 + R^2}} \right) \hat{z}
\end{aligned}$$

If $z << R$, i.e. for an infinite sheet,

$$F = 2q\sigma\pi k$$