PHYSICS 2 : ASSIGNMENT 6

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Exercise 1.
A sphere of linear dielectric material has embedded in it a uniform free charge density ρ. Find the potential at the center of the sphere (relative to infinity), if its radius R and its dielectric constant is ε_r.

Solution 1.
Consider a spherical Gaussian surface just outside the surface of the sphere. By Gauss' Law in dielectrics,

$$\begin{aligned}\int \varepsilon_r E \, \mathrm{d}A &= Q \\ \therefore \varepsilon_r E \cdot 4\pi R^2 &= \rho \cdot \frac{4}{3}\pi R^3 \\ \therefore E &= \frac{\rho R}{3\varepsilon_r}\end{aligned}$$

Outside the sphere,

$$\begin{aligned}E &= \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi R^3 \rho}{r^2} \\ &= \frac{\rho R^3}{3\varepsilon_0 r^2}\end{aligned}$$

Therefore,

$$\begin{aligned}\varphi_{\text{centre}} &= -\int\limits_{\infty}^{0} E \cdot \mathrm{d}r \\ &= -\int\limits_{\infty}^{R} \frac{\rho R^2}{3\varepsilon_0 r^2} \, \mathrm{d}r - \int\limits_{R}^{0} \frac{\rho R}{3\varepsilon_r} \, \mathrm{d}r \\ &= \frac{\rho R^2}{3\varepsilon_0}\left(1 + \frac{1}{2\varepsilon_r}\right)\end{aligned}$$

Exercise 2.
A certain coaxial cable consists of a copper wire, radius a, surrounded by a concentric copper tube of inner radius b and outer radius c, as shown. The space between is partially filled (from b to c), with material of dielectric constant ε_r, as shown. Find the capacitance per unit length of this cable.

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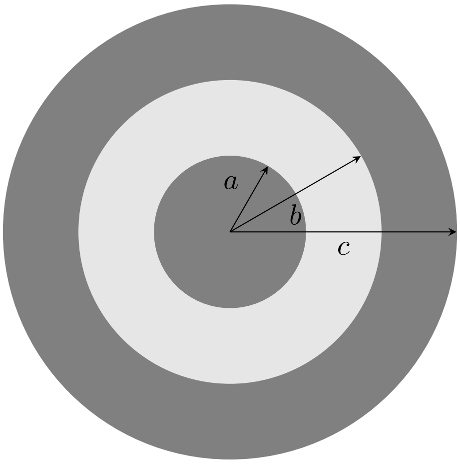


Solution 2.

Let the charge on the surfaces of radii a and b be Q and $-Q$ respectively.
Consider a cylindrical Gaussian surface with radius r and length l.
Therefore, by Gauss' Law,
If $a < r < b$,

$$E = \frac{Q}{2\pi\varepsilon_0 r l}$$

If $b < r < c$,

$$E = \frac{Q}{2\pi\varepsilon_r r l}$$

Therefore,

$$\begin{aligned} V &= -\int_c^a E \,\mathrm{d}r \\ &= -\int_c^b \frac{Q}{2\pi\varepsilon_r r l}\,\mathrm{d}r - \int_b^a \frac{Q}{2\pi\varepsilon_0 r l}\,\mathrm{d}r \\ &= \frac{Q}{2\pi l}\left(\frac{1}{\varepsilon_0}\ln\left(\frac{b}{a}\right) + \frac{1}{\varepsilon_r}\ln\left(\frac{c}{b}\right)\right) \end{aligned}$$

Therefore,

$$\begin{aligned} C &= \frac{Q}{V} \\ \therefore \frac{C}{l} &= \frac{Q}{Vl} \\ &= \frac{2\pi}{\frac{1}{\varepsilon_0}\ln\left(\frac{b}{a}\right) + \frac{1}{\varepsilon_r}\ln\left(\frac{b}{a}\right)} \end{aligned}$$

Exercise 3.
Calculate the attraction force between the plates of a capacitor carrying charge q. Using this result show that the force per unit area acting on either plate is given by $\frac{1}{2}\varepsilon_0 E^2$. Note: this result is true for a conductor of any shape with electric field $\overrightarrow{E}$ at its surface.

Solution 3.
Let the area of the capacitor plates be A, the charge of each of them be $\pm Q$, and the distance between them be l.
Therefore,

$$\begin{aligned} U &= \frac{Q^2}{2C} \\ &= \frac{Q^2}{2A\frac{\varepsilon_0}{l}} \\ &= \frac{Q^2 l}{2A\varepsilon_0} \end{aligned}$$

Therefore,

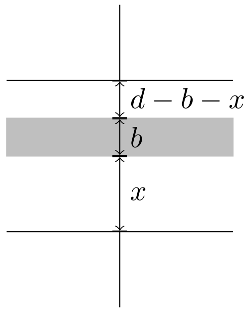
$$\begin{aligned} F &= \frac{\mathrm{d}U}{\mathrm{d}l} \\ &= \frac{Q^2}{2A\varepsilon_0} \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{F}{A} &= \frac{Q^2}{2A^2\varepsilon_0} \\ &= \frac{\sigma^2}{2\varepsilon_0} \\ &= \frac{1}{2}\left(\frac{\sigma}{\varepsilon_0}\right)^2 \varepsilon_0 \\ &= \frac{1}{2}\varepsilon_0 E^2 \end{aligned}$$

Exercise 4.
A dielectric slab of thickness b is inserted between the plates of a parallel-plate capacitor of plate separation d. Calculate the capacitance.

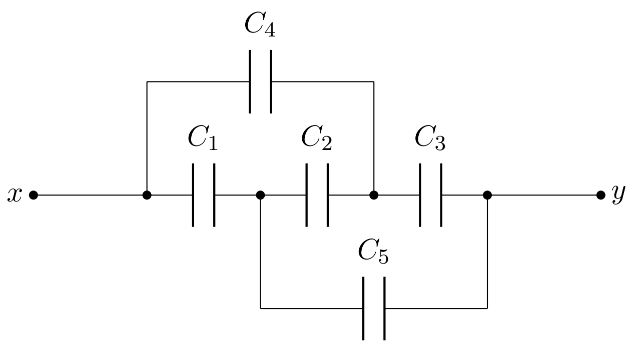
Solution 4.
Let the permittivity of the dielectric material be ε.
Let the distances between the plates and the edges of the slab be as shown.



Therefore, the arrangement is equivalent to three capacitors in series.
Let the capacitances of the three be C_1, C_2, C_3, respectively.
Therefore,

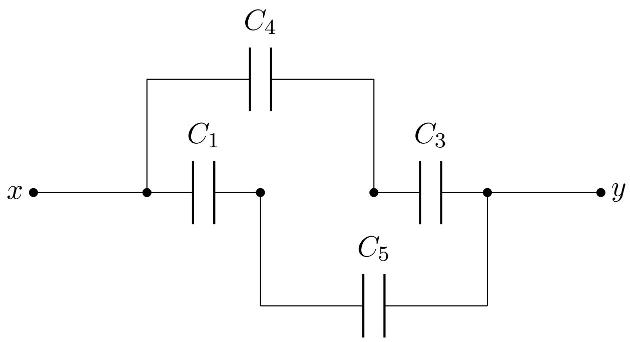
$$\begin{aligned}\frac{1}{C} &= \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\\ &= \frac{d-b-x}{A\varepsilon_0}+\frac{b}{A\varepsilon}+\frac{x}{A\varepsilon_0}\\ &= \frac{d-b}{A\varepsilon_0}+\frac{b}{A\varepsilon}\\ \therefore C &= -\frac{A\varepsilon\varepsilon_0}{\varepsilon(b-d)-b\varepsilon_0}\end{aligned}$$

Exercise 5.
Find the equivalent capacitance between points x and y in



Assume that $C_2 = 10\mu\text{F}$ and that the other capacitors are all $4\mu\text{F}$.

Solution 5.
As all of C_1, C_3, C_4, C_5 are equal, the Wheatstone bridge is balanced.
Therefore the current through it is zero.
Hence, the circuit is equivalent to



Therefore,

$$\begin{aligned} C &= \frac{C_1C_5}{C_1+C_5} + \frac{C_3C_4}{C_3+C_4} \\ &= \frac{16}{8} + \frac{16}{8} \\ &= 4\mu\text{F} \end{aligned}$$