PHYSICS 2 : ASSIGNMENT 2

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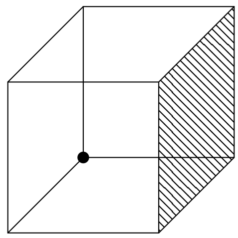
Exercise 1.
A charge q sits at the back comer of a cube, as shown. What is the flux of $\overrightarrow{E}$ through the shaded side?



Solution 1.
Let the area each surface of the box be A.
If the charge was kept in the centre of a box with faces of area $4A$, the total flux through all surfaces, i.e. through $24A$ will be $\oint \overrightarrow{E} \, \mathrm{d}\overrightarrow{A}$.
Therefore, the flux through the shaded area is

$$\begin{aligned}\phi &= \frac{1}{24} \oint \overrightarrow{E} \, \mathrm{d}\overrightarrow{A} \\ &= \frac{1}{24} \frac{q}{\varepsilon_0}\end{aligned}$$

Exercise 2.
Find the electric field a distance s from an infinitely long straight wire, which carries a uniform line charge λ.

Solution 2.
Consider a cylindrical Gaussian surface, with radius r and length L such that the wire passes through its axis.

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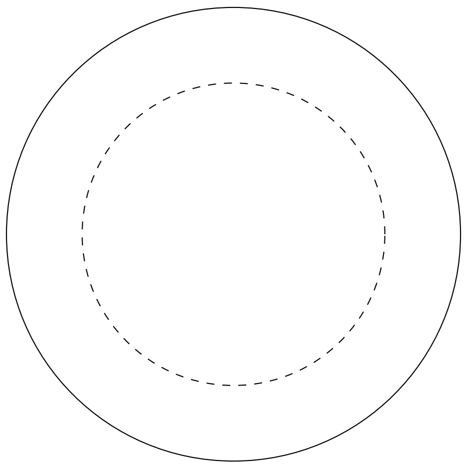
Therefore, by Gauss' Law,

$$\oint \overrightarrow{E} \cdot \mathrm{d}\overrightarrow{A} = \frac{q_{\text{inside}}}{\varepsilon_0}$$
$$\therefore E(2\pi r L) = \frac{\lambda L}{\varepsilon_0}$$
$$\therefore E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Exercise 3.
Find the electric field inside a sphere which carries a charge density proportional to the distance from the origin, $\rho = kr$, for some constant k.

Solution 3.



Consider a spherical Gaussian surface of radius a as shown.
The charge inside the Gaussian surface is

$$\begin{aligned} q_{\text{inside}} &= \int_0^a \rho \cdot 4\pi r^2 \,\mathrm{d}r \\ &= \int_0^a 4\pi k r^3 \,\mathrm{d}r \\ &= \pi k r^4 \Big|_0^a \\ &= \pi k a^4 \end{aligned}$$

Therefore, by Gauss' Law,

$$\oint \overrightarrow{E} \cdot \mathrm{d}\overrightarrow{A} = \frac{q_{\text{inside}}}{\varepsilon_0}$$
$$\therefore E(4\pi a^2) = \frac{\pi k a^4}{\varepsilon_0}$$
$$\therefore E = \frac{k a^2}{4\varepsilon_0}$$

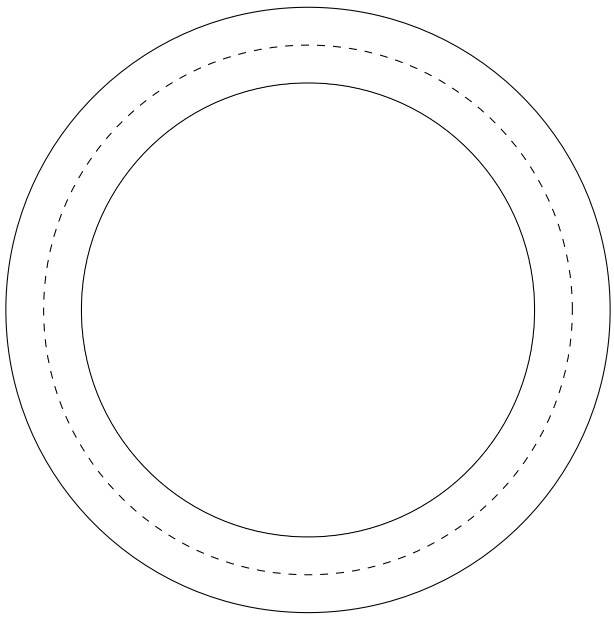
Exercise 4.

A hollow spherical shell carries charge density $\rho = \dfrac{k}{r^2}$ in the region $a \leq r \leq b$. Find the electric field in three regions:

(i) $r < a$
(ii) $a < r < b$
(iii) $b < r$

Plot $|E|$ as a function of r.

Solution 4.



Consider a spherical Gaussian surface of radius r.

If $r < a$, the total charge inside the Gaussian surface is zero. Therefore, by Gauss' Law,

$$E(4\pi r^2) = \frac{0}{\varepsilon_0}$$
$$\therefore E = 0$$

If $a < r < b$,

$$\begin{aligned} q_{\text{inside}} &= \int_a^r \rho \cdot 4\pi\tilde{r}^2 \,\mathrm{d}\tilde{r} \\ &= \int_a^r \frac{k}{\tilde{r}^2} \cdot 4\pi\tilde{r}^2 \,\mathrm{d}\tilde{r} \\ &= \int_a^r 4k\pi \,\mathrm{d}\tilde{r} \\ &= 4k\pi(r-a) \end{aligned}$$

Therefore, by Gauss' Law,

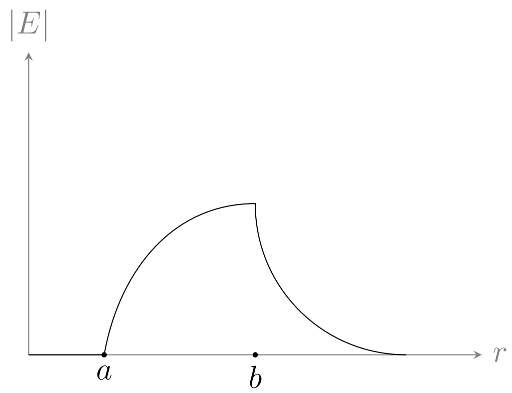
$$\begin{aligned} E(4\pi r^2) &= \frac{4k\pi(r-a)}{\varepsilon_0} \\ \therefore E &= \frac{k(r-a)}{\varepsilon_0 r^2} \end{aligned}$$

If $b < r$,

$$q_{\text{inside}} = 4k\pi(b-a)$$

Therefore, by Gauss' Law,

$$\begin{aligned} E(4\pi r^2) &= \frac{4k\pi(b-a)}{\varepsilon_0} \\ \therefore E &= \frac{k(b-a)}{\varepsilon_0 r^2} \end{aligned}$$



Exercise 5.
A long coaxial cable carries a uniform volume charge density ρ on the inner cylinder (radius a), and a uniform surface charge density on the outer cylindrical shell (radius b). This surface charge is negative and of just the right magnitude so that the cable as a whole is electrically neutral. Find the electric field in each of the three regions:

(i) inside the inner cylinder ($s < a$)
(ii) between the cylinders($a < s < b$)
(iii) outside the cable ($b < s$) Plot $|E|$ as a function of s.

Solution 5.
Consider a cylindrical Gaussian surface with radius s and length L, concentric to the cable.

If $s < a$,

$$q_{\text{inside}} = \rho \cdot \pi s^2 L$$

Therefore, by Gauss' Law,

$$E(2\pi sL) = \frac{\pi\rho s^2 L}{\varepsilon_0}$$
$$\therefore E = \frac{\rho s}{2\varepsilon_0}$$

If $a < s < b$,

$$q_{\text{inside}} = \rho \cdot \pi a^2 L$$

Therefore, by Gauss' Law,

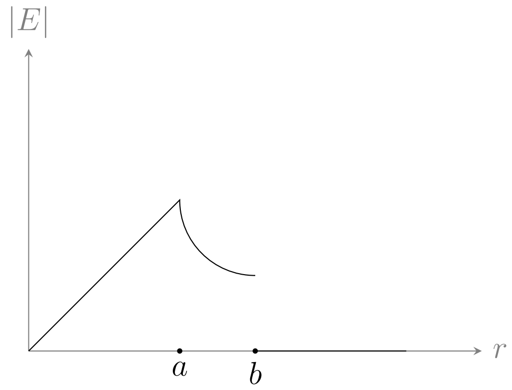
$$E(2\pi sL) = \frac{\pi\rho a^2 L}{\varepsilon_0}$$
$$\therefore E = \frac{\rho a^2}{2s\varepsilon_0}$$

If $b < s$,

$$q_{\text{inside}} = 0$$

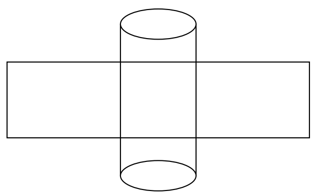
Therefore, by Gauss' Law,

$$E = 0$$



Exercise 6.
An infinite plane slab, of thickness $2d$, carries a uniform volume charge density ρ. Find the electric field, as a function of y, where $y = 0$ at the centre. Plot $\overrightarrow{E}$ versus y, calling E positive when it points in the positive $+y$ direction and negative when it points in the $-y$ direction.

Solution 6.
Consider a cylindrical Gaussian surface with base area A, as shown.



If $|y| < d$,

$$q_{\text{inside}} = \rho \cdot 2yA$$

Therefore, by Gauss' Law,

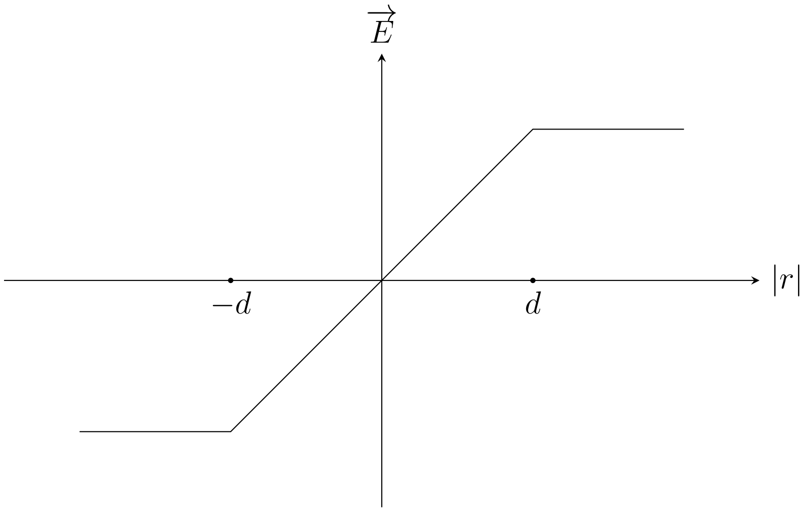
$$E \cdot 2A = \frac{2\rho y A}{\varepsilon_0}$$
$$\therefore E = \frac{\rho y}{\varepsilon_0}$$

If $|y| > d$,

$$q_{\text{inside}} = \rho \cdot 2dA$$

Therefore, by Gauss' Law,

$$E \cdot 2A = \frac{2\rho A d}{\varepsilon_0}$$
$$\therefore E = \frac{\rho d}{\varepsilon_0}$$


$\overrightarrow{E}$
$|r|$
$-d$
d