PHYSICS 2 : ASSIGNMENT 4

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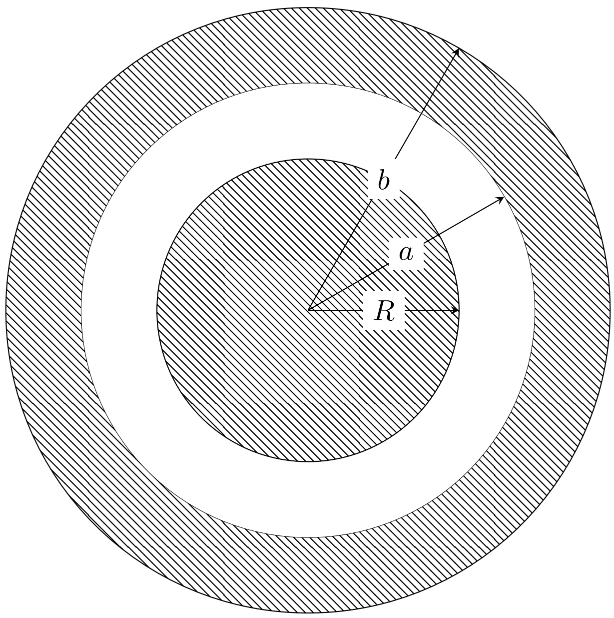
Exercise 1.
A metal sphere of radius R, carrying charge q, is surrounded by a thick concentric metal shell (inner radius a, outer radius b). The shell carries no net charge.

(1) Find the surface charge density a at R, at a, and at b.
(2) Find the potential at the center, using infinity as the reference point.
(3) Now the outer surface is touched to a grounding wire, which lowers its potential to zero (same as at infinity). How do your answers to the previous sub-questions change?

Solution 1.



(1) Consider a spherical Gaussian surface with radius r, with $a < r < b$. By Gauss' Law, and as the electric field inside the thick metallic shell must be 0, the net charge inside the Gaussian surface must be 0.
Therefore, the sum of the charge on the surface of the sphere and the charge on the inner surface of the shell must be 0. Hence, the charge on the inner surface of the shell must be $-q$.
As the shell is electrically neutral, the sum of the charges on the inner and outer surfaces of the shell must be 0. Hence, the charge on the outer

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surface of the shell must be q.
Therefore,

$$\sigma_R = \frac{q}{4\pi R^2}$$
$$\sigma_a = \frac{-q}{4\pi a^2}$$
$$\sigma_b = \frac{q}{4\pi b^2}$$

(2)

$$\begin{aligned}\varphi_{\text{centre}} &= \varphi \text{ due to charge at } R + \varphi \text{ due to charge at } a + \varphi \text{ due to charge at } b \\ &= \frac{q}{4\pi\varepsilon_0 R} + \frac{-q}{4\pi\varepsilon_0 a} + \frac{q}{4\pi\varepsilon_0 b}\end{aligned}$$

(3)

$$\begin{aligned}\varphi_b &= \varphi \text{ due to charge at } R + \varphi \text{ due to charge at } a + \varphi \text{ due to charge at } b \\ &= \frac{q}{4\pi\varepsilon_0 b} + \frac{-q}{4\pi\varepsilon_0 b} + \frac{q'}{4\pi\varepsilon_0 b} \\ &= \frac{q}{4\pi\varepsilon_0 b}\end{aligned}$$

As the outer surface is grounded, its potential will be zero.
Therefore

$$0 = \frac{q'}{4\pi\varepsilon_0 b}$$
$$\therefore q' = 0$$

Therefore,

$$\sigma_R = \frac{q}{4\pi R^2}$$
$$\sigma_a = \frac{-q}{4\pi a^2}$$
$$\begin{aligned}\sigma_b &= \frac{q'}{4\pi b^2} \\ &= 0\end{aligned}$$

$$\begin{aligned}\varphi_{\text{centre}} &= \varphi \text{ due to charge at } R + \varphi \text{ due to charge at } a + \varphi \text{ due to charge at } b \\ &= \frac{q}{4\pi\varepsilon_0 R} + \frac{-q}{4\pi\varepsilon_0 a} + \frac{q'}{4\pi\varepsilon_0 b} \\ &= \frac{q}{4\pi\varepsilon_0 R} + \frac{-q}{4\pi\varepsilon_0 a}\end{aligned}$$

Exercise 2.
A metal sphere of radius R carries a total charge Q. What is the force of repulsion between the "northern" hemisphere and the "southern" hemisphere?

Solution 2.

$$E_{\text{northern hemisphere}} = \frac{1}{2}\frac{Q}{4\pi\varepsilon_0 R^2}$$

Due to the symmetry of the sphere, the components of the force of all elemental charges, in the direction of the north-south axis add up, and all other components cancel out.
Therefore,

$$\begin{aligned}
F &= \int_0^{\frac{\pi}{2}}\int_0^{2\pi} E_{\text{northern hemisphere}}\sigma R^2 \,\mathrm{d}\varphi \sin(\theta)\cos(\theta)\,\mathrm{d}\theta \\
&= \int_0^{\frac{\pi}{2}}\int_0^{2\pi} \frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R^2}\cdot\frac{Q}{4\pi R^2}R^2\sin(\theta)\cos(\theta)\,\mathrm{d}\varphi\,\mathrm{d}\theta \\
&= \int_0^{\frac{\pi}{2}}\int_0^{2\pi} \frac{1}{2\pi\varepsilon_0}\left(\frac{Q}{4\pi R}\right)^2 \sin(\theta)\cos(\theta)\,\mathrm{d}\varphi\,\mathrm{d}\theta \\
&= \int_0^{2\pi} \frac{1}{2\pi\varepsilon_0}\left(\frac{Q}{4\pi R}\right)^2 \\
&= \frac{1}{2\pi\varepsilon_0}\left(\frac{Q}{4\pi R}\right)^2 \\
&= \frac{Q^2}{32\pi^2\varepsilon_0 R^2}
\end{aligned}$$

Exercise 3.

Two spherical cavities, of radii a and b, are hollowed out from the interior of a (neutral) conducting sphere of radius R. At the center of each cavity a point charge is placed - call these charges q_a and q_b.

(1) Find the surface charges σ_a, σ_b, and σ_R.
(2) What is the field outside the conductor?
(3) What is the field within each cavity?
(4) What is the force on q_a and q_b?
(5) Which of these answers would change if a third charge, q_c, were brought near the conductor?

Solution 3.

(1) Consider a spherical Gaussian surface which surrounds the cavity of radius a. By Gauss' Law, and as the electric field inside the metallic sphere must be 0, the net charge inside the Gaussian surface must be 0. Therefore, the charge on the surface of the cavity must be $-q_a$.
Therefore,

$$\sigma_a = \frac{-q_a}{4\pi a^2}$$

Similarly, for the cavity with radius b,

$$\sigma_b = \frac{-q_b}{4\pi b^2}$$

As the metallic sphere is electrically neutral,

$$q_R = q_a + q_b$$
$$\therefore \sigma_R = q_a + q_b$$
$$\therefore \sigma_R = \frac{q_a + q_b}{4\pi R^2}$$

(2) Consider a spherical Gaussian surface with radius r.
Therefore, by Gauss' Law,

$$EA = \frac{q_{\text{inside}}}{\varepsilon_0}$$
$$= \frac{q_a + q_b}{\varepsilon_0}$$
$$\therefore E = \frac{q_a + q_b}{4\pi\varepsilon_0 r^2}$$

(3) Consider a spherical Gaussian surface with centre at the centre of the cavity with radius a, and radius r'. Therefore, by Gauss' Law,

$$EA = \frac{q_a}{\varepsilon_0}$$
$$\therefore E = \frac{q_a}{4\pi\varepsilon_0 (r')^2}$$

Similarly, inside the cavity with radius b,

$$E = \frac{q_b}{4\pi\varepsilon_0 (r')^2}$$

where r' is measured from the centre of the cavity with radius b.

(4) The field at the position of q_a due to the cavity with radius a is 0, as the field inside a shell of charge is 0.
Similarly, the field due to the outer surface is also 0.
The net field due to the other charge and the other cavity is 0, as the net charge on them is 0.
Therefore, the net field on q_a is zero. Hence there is no force on q_a.
Similarly, there is no force on q_b.

(5) None of the above answers will change if a charge q_c is brought near the conductor.