Physics 2 : Recitations

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Contents

1 Instructor Information

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Part I

Electrostatics

1 Gravitation and Electromagnetism

Gravitation	Electromagnetism
$F_G = G\dfrac{m_1 m_2}{r^2}$	$F_E = k\dfrac{q_1 q_2}{r^2}$
$G = 6.7 \times 10^{11}\mathrm{N\,m^2\,kg^{-2}}$	$8.99 \times 10^{9}\mathrm{N\,m^2\,C^{-2}}$

2 Coulomb's Law

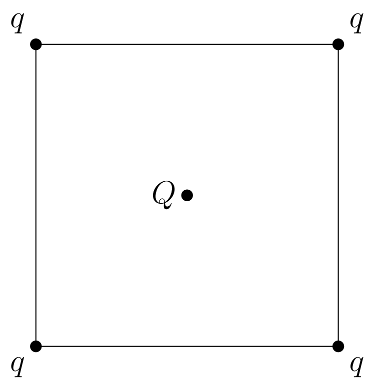
Exercise 1.

Four identical charges q are placed in the corners of a square of length a. A fifth charge Q is free to move along the straight line perpendicular to the square plane and passing through its centre. When the charge Q is in the same plane as the other charges, all the forces in the system cancel out.

1. Calculate Q for a given q and a.

2. Find the force $\overrightarrow{F(z)}$ acting on the charge Q when it is at height z above the square.

Solution 1.



Consider q on the top right corner of the square. The total force acting on it is 0. Therefore

$$\begin{aligned} 0 &= \frac{kq^2}{a^2}\cos\frac{\pi}{4} + \frac{kq^2}{a^2}\cos\frac{\pi}{4} + \frac{kq^2}{2a^2} + \frac{2kQq}{a^2} \\ \therefore Q &= -\frac{1+2\sqrt{2}}{4}q \end{aligned}$$

If Q is at a height z from the plane, the distance between each q and Q is

$$r = \sqrt{z^2 + \left(\frac{a}{\sqrt{2}}\right)^2}$$

Therefore the force of each q on Q is $\dfrac{kQq}{r^2}$.
Due to symmetry, the components of the forces in the z direction will add up, and all other components will cancel out.
Let the angle between the z direction and the line joining q and Q be φ.
Therefore, the net force is

$$\begin{aligned} F &= 4\frac{kQq}{r^2}\cos\varphi \\ &= 4\frac{kQq}{r^2}\frac{z}{r} \\ &= 4\frac{kQq}{z^2\left(1+\dfrac{a^2}{2z^2}\right)^{3/2}} \end{aligned}$$

Exercise 2.

1. A wire of length 3 metre is charged with $2\ \mathrm{C\,m^{-1}}$. What is the wire's total charge?

Solution 2.

$$\begin{aligned} \lambda &= \frac{Q}{L} \\ \therefore Q &= L\lambda \\ &= 6\mathrm{C} \end{aligned}$$

Exercise 3.
A wire of length L has the following charge distribution: $\lambda = \lambda_0 \cos \dfrac{\pi x}{L}$, where x is the distance from the wire's edge. What is the wire's total charge?

Solution 3.

$$\begin{aligned}
\lambda &= \frac{\mathrm{d}q}{\mathrm{d}x} \\
\therefore \frac{\mathrm{d}q}{\mathrm{d}x} &= \lambda_0 \cos \frac{\pi x}{L} \\
\therefore q &= \int\limits_0^L \lambda_0 \cos \frac{\pi x}{L} \, \mathrm{d}x \\
&= 0
\end{aligned}$$

Exercise 4.
A hollow sphere of radius R is uniformly charged with a charge Q. Calculate the charge distribution on the surface of the sphere.

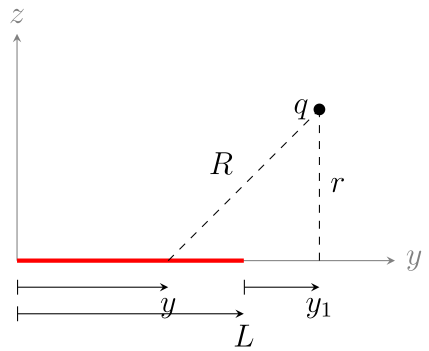
Solution 4.

$$\begin{aligned}
\sigma &= \frac{Q}{A} \\
&= \frac{Q}{4\pi R^2}
\end{aligned}$$

Exercise 5.
A straight thin wire is uniformly charged with distribution λ. A charge q is positioned at distance y_1 beneath the wire and r away form it.

1. Find the force acting on the charge q.
2. Show that when the charge is positioned in front of the centre of the wire the $\hat{y}$ component of the force is cancelled.
3. Calculate the force an infinite straight wire will exert on the charge q.

Solution 5.



Consider an elemental charge $\mathrm{d}Q$ of length $\mathrm{d}y$, at distance y as shown. Let the angle between the line joining $\mathrm{d}Q$ and q and the y direction be θ.

$$F_y = F\cos\theta$$
$$F_z = F\sin\theta$$

Let

$$a = L + y_1$$
$$\therefore R = \sqrt{r^2 + (a-y)^2}$$

Therefore,

$$\cos\theta = \frac{a-y}{R}$$
$$\sin\theta = \frac{r}{R}$$

Therefore,

$$\begin{aligned} F_y &= kq\int_0^L \frac{\lambda\,\mathrm{d}y}{R^2}\frac{(a-y)}{R} \\ &= kq\lambda\int_0^L \frac{\mathrm{d}y(a-y)}{\left((a-y)^2+r^2\right)^{3/2}} \\ &= kq\lambda\left(\frac{1}{\sqrt{{y_1}^2+r^2}} - \frac{1}{\sqrt{a^2+r^2}}\right) \end{aligned}$$

$$
\begin{aligned}
F_z &= kq \int_0^L \frac{\lambda \,\mathrm{d}y}{R^2} \frac{r}{R} \\
&= kq\lambda \int_0^L \frac{r \,\mathrm{d}y}{\left((a-y)^2 + r^2\right)^{3/2}} \\
&= \frac{kq\lambda}{r} \left(\frac{a}{\sqrt{r^2 + a^2}} - \frac{y_1}{\sqrt{r^2 + {y_1}^2}} \right)
\end{aligned}
$$

When the charge is positioned above the centre of the wire,

$$
\begin{aligned}
y_1 &= -\frac{L}{2} \\
\therefore a &= \frac{L}{2}
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
F_y &= kq\lambda \left(\frac{1}{\sqrt{{y_1}^2 + r^2}} - \frac{1}{\sqrt{a^2 + r^2}} \right) \\
&= kq\lambda \left(\frac{1}{\sqrt{-\frac{L}{2}^2 + r^2}} - \frac{1}{\sqrt{\frac{L}{2}^2 + r^2}} \right) \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
F_z &= \frac{kq\lambda}{r}\left(\frac{a}{\sqrt{r^2+a^2}} - \frac{y_1}{\sqrt{r^2+{y_1}^2}}\right) \\
&= \frac{kq\lambda}{r}\left(\frac{\frac{L}{2}}{\sqrt{r^2+\frac{L}{2}^2}} - \frac{-\frac{L}{2}}{\sqrt{r^2+-\frac{L}{2}^2}}\right) \\
&= \frac{kq\lambda}{r}\left(\frac{L}{\sqrt{r^2+\frac{L}{2}^2}}\right) \\
&= \frac{kq\lambda}{r}\left(\frac{1}{\sqrt{\frac{1}{4}+\left(\frac{r}{L}\right)^2}}\right)
\end{aligned}
$$

If the line is infinite, $L \to \infty$. Therefore

$$
\begin{aligned}
F_z &= \frac{kq\lambda}{r}\left(\frac{1}{\sqrt{\frac{1}{4}+\left(\frac{r}{L}\right)^2}}\right) \\
&= \frac{2kq\lambda}{r}
\end{aligned}
$$