Physics 2 : Recitations

Aakash Jog

2014-15

Contents

1 Instructor Information

Dr. Richard Spitzberg

Office: Ma Aabadot 119
E-mail: rms9999@gmail.com

Part I
Electrostatics

1 Gravitation and Electromagnetism

Gravitation	Electromagnetism
$F_G = G\frac{m_1 m_2}{r^2}$	$F_E = k\frac{q_1 q_2}{r^2}$
$G = 6.7 \times 10^{11} \mathrm{N\,m^2\,kg^{-2}}$	$8.99 \times 10^{9} \mathrm{N\,m^2\,C^{-2}}$

2 Coulomb's Law

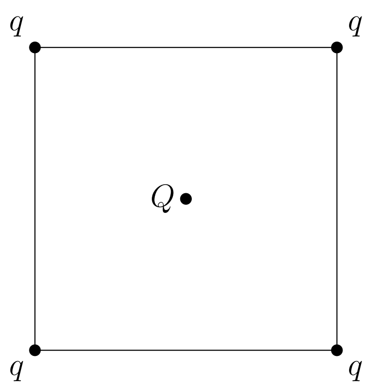
Recitation 1 – Exercise 1.

Four identical charges q are placed in the corners of a square of length a. A fifth charge Q is free to move along the straight line perpendicular to the square plane and passing through its centre. When the charge Q is in the same plane as the other charges, all the forces in the system cancel out.

1. Calculate Q for a given q and a.

2. Find the force $\overrightarrow{F(z)}$ acting on the charge Q when it is at height z above the square.

Recitation 1 – Solution 1.



Consider q on the top right corner of the square. The total force acting on it is 0. Therefore

$$
\begin{aligned}
0 &= \frac{kq^2}{a^2}\cos\frac{\pi}{4} + \frac{kq^2}{a^2}\cos\frac{\pi}{4} + \frac{kq^2}{2a^2} + \frac{2kQq}{a^2} \\
\therefore Q &= -\frac{1+2\sqrt{2}}{4}q
\end{aligned}
$$

If Q is at a height z from the plane, the distance between each q and Q is

$$
r = \sqrt{z^2 + \left(\frac{a}{\sqrt{2}}\right)^2}
$$

Therefore the force of each q on Q is $\dfrac{kQq}{r^2}$.
Due to symmetry, the components of the forces in the z direction will add up, and all other components will cancel out.
Let the angle between the z direction and the line joining q and Q be φ.
Therefore, the net force is

$$
\begin{aligned}
F &= 4\frac{kQq}{r^2}\cos\varphi \\
&= 4\frac{kQq}{r^2}\frac{z}{r} \\
&= 4\frac{kQq}{z^2\left(1+\dfrac{a^2}{2z^2}\right)^{3/2}}
\end{aligned}
$$

Recitation 1 – Exercise 2.

1. A wire of length 3 metre is charged with 2 $\mathrm{C\,m^{-1}}$. What is the wire's total charge?

Recitation 1 – Solution 2.

$$
\begin{aligned}
\lambda &= \frac{Q}{L} \\
\therefore Q &= L\lambda \\
&= 6\mathrm{C}
\end{aligned}
$$

Recitation 1 – Exercise 3.

A wire of length L has the following charge distribution: $\lambda = \lambda_0 \cos \frac{\pi x}{L}$, where x is the distance from the wire's edge. What is the wire's total charge?

Recitation 1 – Solution 3.

$$\begin{aligned} \lambda &= \frac{\mathrm{d}q}{\mathrm{d}x} \\ \therefore \frac{\mathrm{d}q}{\mathrm{d}x} &= \lambda_0 \cos \frac{\pi x}{L} \\ \therefore q &= \int_0^L \lambda_0 \cos \frac{\pi x}{L} \, \mathrm{d}x \\ &= 0 \end{aligned}$$

Recitation 1 – Exercise 4.

A hollow sphere of radius R is uniformly charged with a charge Q. Calculate the charge distribution on the surface of the sphere.

Recitation 1 – Solution 4.

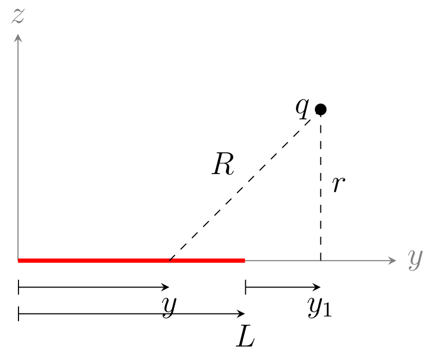
$$\begin{aligned} \sigma &= \frac{Q}{A} \\ &= \frac{Q}{4\pi R^2} \end{aligned}$$

Recitation 1 – Exercise 5.

A straight thin wire is uniformly charged with distribution λ. A charge q is positioned at distance y_1 beneath the wire and r away form it.

1. Find the force acting on the charge q.
2. Show that when the charge is positioned in front of the centre of the wire the $\hat{y}$ component of the force is cancelled.
3. Calculate the force an infinite straight wire will exert on the charge q.

Recitation 1 – Solution 5.



Consider an elemental charge $\mathrm{d}Q$ of length $\mathrm{d}y$, at distance y as shown. Let the angle between the line joining $\mathrm{d}Q$ and q and the y direction be θ.

$$F_y = F\cos\theta$$
$$F_z = F\sin\theta$$

Let

$$a = L + y_1$$
$$\therefore R = \sqrt{r^2 + (a-y)^2}$$

Therefore,

$$\cos\theta = \frac{a-y}{R}$$
$$\sin\theta = \frac{r}{R}$$

Therefore,

$$\begin{aligned} F_y &= kq\int_0^L \frac{\lambda\,\mathrm{d}y}{R^2}\frac{(a-y)}{R} \\ &= kq\lambda\int_0^L \frac{\mathrm{d}y(a-y)}{\left((a-y)^2+r^2\right)^{3/2}} \\ &= kq\lambda\left(\frac{1}{\sqrt{{y_1}^2+r^2}} - \frac{1}{\sqrt{a^2+r^2}}\right) \end{aligned}$$

$$F_z = kq\int_0^L \frac{\lambda\,\mathrm{d}y}{R^2}\frac{r}{R}$$

$$= kq\lambda\int_0^L \frac{r\,\mathrm{d}y}{\left((a-y)^2+r^2\right)^{3/2}}$$

$$= \frac{kq\lambda}{r}\left(\frac{a}{\sqrt{r^2+a^2}} - \frac{y_1}{\sqrt{r^2+{y_1}^2}}\right)$$

When the charge is positioned above the centre of the wire,

$$y_1 = -\frac{L}{2}$$

$$\therefore a = \frac{L}{2}$$

Therefore,

$$F_y = kq\lambda\left(\frac{1}{\sqrt{{y_1}^2+r^2}} - \frac{1}{\sqrt{a^2+r^2}}\right)$$

$$= kq\lambda\left(\frac{1}{\sqrt{-\frac{L}{2}^2+r^2}} - \frac{1}{\sqrt{\frac{L}{2}^2+r^2}}\right)$$

$$= 0$$

$$
\begin{aligned}
F_z &= \frac{kq\lambda}{r}\left(\frac{a}{\sqrt{r^2+a^2}} - \frac{y_1}{\sqrt{r^2+{y_1}^2}}\right) \\
&= \frac{kq\lambda}{r}\left(\frac{\frac{L}{2}}{\sqrt{r^2+\frac{L}{2}^2}} - \frac{-\frac{L}{2}}{\sqrt{r^2+-\frac{L}{2}^2}}\right) \\
&= \frac{kq\lambda}{r}\left(\frac{L}{\sqrt{r^2+\frac{L}{2}^2}}\right) \\
&= \frac{kq\lambda}{r}\left(\frac{1}{\sqrt{\frac{1}{4}+\left(\frac{r}{L}\right)^2}}\right)
\end{aligned}
$$

If the line is infinite, $L \to \infty$. Therefore

$$
\begin{aligned}
F_z &= \frac{kq\lambda}{r}\left(\frac{1}{\sqrt{\frac{1}{4}+\left(\frac{r}{L}\right)^2}}\right) \\
&= \frac{2kq\lambda}{r}
\end{aligned}
$$

3 Gauss' Law

Recitation 2 – Exercise 2.

A ball of radius a is charged with distribution $\rho = \rho_0 \frac{r}{a}$. Find the electric field everywhere.

Recitation 2 – Solution 2.
Consider a spherical Gaussian surface of radius r.
If $r \le a$, the charge in the interior of the Gaussian surface is

$$\begin{aligned} q(r) &= \int_0^r \frac{\rho_0 r}{a} \cdot 4\pi r^2 \, \mathrm{d}r \\ &= \frac{\rho_0}{a} \pi r^4 \end{aligned}$$

Therefore, by Gauss' Law,

$$\begin{aligned} E \cdot 4\pi r^2 &= \frac{q(r)}{\varepsilon_0} \\ \therefore E &= \frac{\rho_0 \pi r^4}{4\pi a r^2} \\ &= \frac{\rho_0 r^2}{4a\varepsilon_0} \end{aligned}$$

If $r \ge a$, the entire ball of charge is in the interior of the Gaussian surface. Therefore,

$$\begin{aligned} Q &= q(a) \\ &= \frac{\rho_0}{a} \cdot \pi a^4 \\ &= \rho_0 \pi a^3 \end{aligned}$$

Therefore, by Gauss' Law,

$$\begin{aligned} E \cdot 4\pi r^2 &= \frac{Q}{\varepsilon_0} \\ \therefore E &= \frac{Q}{4\pi r^2 \varepsilon_0} \\ &= \frac{\rho_0 a^3}{4r^2\varepsilon_0} \end{aligned}$$

Therefore,

$$E = \begin{cases} \dfrac{\rho_0 r^2}{4a\varepsilon_0} & ; \quad r \le a \\ \dfrac{\rho_0 a^3}{4r^2\varepsilon} & ; \quad r \ge a \end{cases}$$

Recitation 2 – Exercise 3.

An infinitely long cylinder of radius a is charged with distribution $\rho = \rho_0 \dfrac{r}{a}$. Find the electric field everywhere.

Recitation 2 – Solution 3.

Consider a infinite cylindrical Gaussian surface with radius r.
If $r \leq a$, the charge in the interior of the Gaussian surface is

$$\begin{aligned} q(r) &= \int_0^r \frac{\rho_0 r}{a} \pi r^2 \,\mathrm{d}r \\ &= \frac{2\pi\rho_0 L r^3}{3a} \end{aligned}$$

Therefore, by Gauss' Law,

$$\begin{aligned} E \cdot 2\pi r L &= \frac{2\pi\rho_0 L r^3}{3a\varepsilon_0} \\ \therefore E &= \frac{\rho_0 r^2}{3a\varepsilon_0} \end{aligned}$$

If $r \geq a$, the entire cylinder of charge is in the interior of the Gaussian surface. Therefore,

$$\begin{aligned} Q &= q(a) \\ &= \frac{2\pi\rho_0 L a^3}{3a} \\ &= \frac{2\pi\rho_0 L a^2}{3} \end{aligned}$$

Therefore, by Gauss' Law,

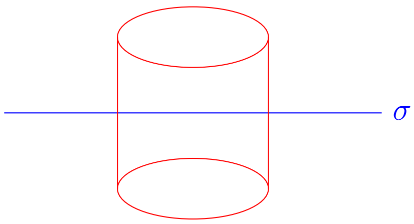
$$\begin{aligned} E \cdot 2\pi r L &= \frac{2\pi\rho_0 L a^2}{3\varepsilon_0} \\ \therefore E &= \frac{\rho_0 a^2}{3\varepsilon_0 r} \end{aligned}$$

Therefore,

$$E = \begin{cases} \dfrac{\rho_0 r^2}{3a\varepsilon_0} & ; \quad r \leq a \\ \dfrac{\rho_0 a^2}{3\varepsilon_0 r} & ; \quad r \geq a \end{cases}$$

Recitation 2 – Exercise 4.
Find the electric field due to a thin infinite plane of uniform charge distribution σ.

Recitation 2 – Solution 4.
Consider a cylindrical Gaussian surface, with ends of area A, as shown.



The charge in the interior of the surface is

$$\mathrm{d}q = A\sigma$$

Therefore, by Gauss' Law,

$$\begin{aligned} E_1 \cdot A_1 + E_2 \cdot A_2 &= \frac{A\sigma}{\varepsilon_0} \\ \therefore 2EA &= \frac{A\sigma}{\varepsilon_0} \\ \therefore E &= \frac{\sigma}{2\varepsilon_0} \end{aligned}$$