PHYSICS 2 : ASSIGNMENT 1

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Exercise 1.

1. Twelve equal charges, q, are situated at the comers of a regular 12- sided polygon (for instance, one on each numeral of a clock face). What is the net force on a test charge Q at the centre?

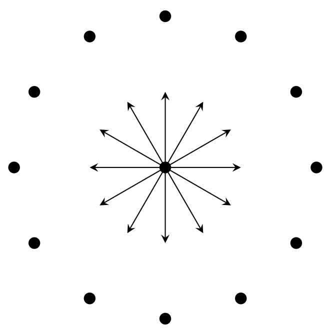
2. Suppose one of the 12 qs is removed (the one at "6 o'clock"). What is the force on Q? Explain your reasoning carefully.

3. Now 13 equal charges, q, are placed at the comers of a regular 13-sided polygon. What is the force on a test charge Q at the centre?

4. If one of the 13 qs is removed, what is the force on Q? Explain your reasoning.

Solution 1.

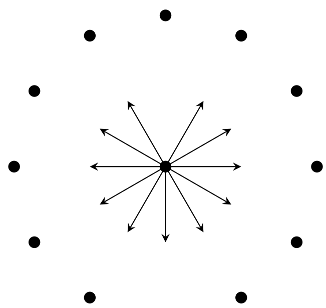
1.



The vector sum of the forces shown above is zero. Therefore, the net force acting on Q is zero.

Date: Wednesday 18^{th} March, 2015.

2.

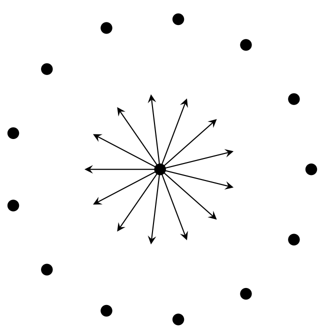


All of the forces, except for force due to the charge at 12 o'clock, are cancelled out. Hence the net force is

$$\overrightarrow{F} = k\frac{Qq}{R^2}$$

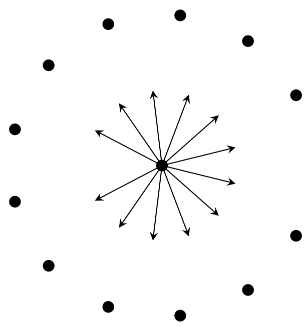
in the 6 o'clock direction.

3.



The vector sum of the forces shown above is zero. Therefore, the net force acting on Q is zero.

4.



If one of the 13 charges is removed, the net force on Q will be equal to the force due to the removed q, which was previously acting on it.
Therefore,

$$F = \frac{kQq}{r^2}$$

in the direction of the removed charge.

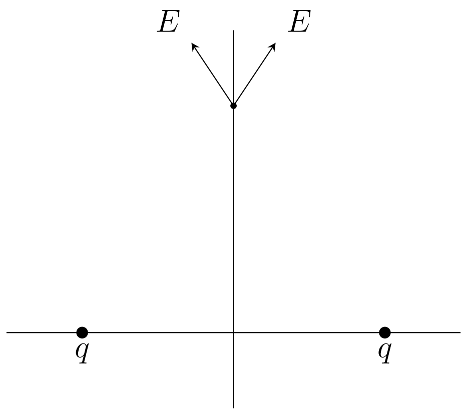
Exercise 2.

1. Find the electric field (magnitude and direction) a distance z above the midpoint between two equal charges, q, a distance d apart. Check that your result is consistent with what you'd expect when $z >> d$.

2. Repeat part 1., only this time make the right-hand charge $-q$ instead of $+q$.

Solution 2.

1.



Let the angle between E and the vertical direction be θ.
By symmetry, the horizontal components of the fields will cancel out and the vertical components will add up.
Therefore,

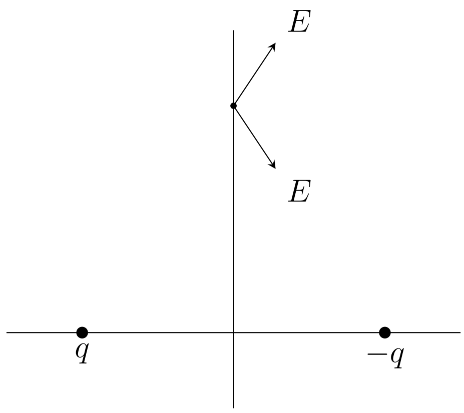
$$\begin{aligned}\overrightarrow{E_{\text{net}}} &= 2E\cos\theta\hat{z}\\ &= \frac{2kq}{\frac{d^2}{4}+z^2}\cdot\frac{z}{\sqrt{\frac{d^2}{4}+z^2}}\\ &= \frac{2kqz}{\left(\frac{d^2}{4}+z^2\right)^{3/2}}\end{aligned}$$

If $z >> d$,

$$\begin{aligned}\overrightarrow{E_{\text{net}}} &= \frac{2kqz}{z^3}\\ &= \frac{2kq}{z^2}\end{aligned}$$

This is equivalent to the field at z due to a point charge $2q$. Thus, this result is consistent with Coulomb's Law for point charges.

2.



Let the angle between E and the vertical direction be θ.
By symmetry, the vertical components of the fields will cancel out and the horizontal components will add up.
Therefore,

$$\begin{aligned}\overrightarrow{E_{\text{net}}} &= 2E\sin\theta\hat{z}\\ &= \frac{2kq}{\frac{d^2}{4}+z^2}\cdot\frac{\frac{d}{2}}{\sqrt{\frac{d^2}{4}+z^2}}\\ &= \frac{2kqd}{\left(\frac{d^2}{4}+z^2\right)^{3/2}}\end{aligned}$$

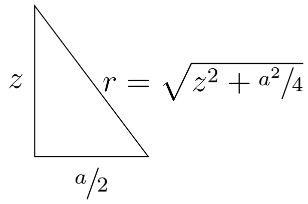
If $z >> d$,

$$\overrightarrow{E_{\text{net}}} = 0$$

This is equivalent to a field at z due to a neutral point charge. Thus, this result is consistent with Coulomb's Law for point charges.

Exercise 3.
Find the electric field a distance z above the centre of a square loop of side a carrying uniform line charge λ.

Solution 3.
Let F be the field at P due to one side. Let the angle between $\overrightarrow{F}$ and $\hat{z}$ be θ. Therefore, due to symmetry, the net field at P due to all four sides will be $4F\cos\theta$.



Therefore,

$$
\begin{aligned}
F &= k\frac{\lambda a}{r\left({}^{a^2}\!/_4 + r^2\right)^{1/2}} \\
\therefore F_{\text{net}} &= 4k\frac{\lambda a}{r\left({}^{a^2}\!/_4 + r^2\right)^{1/2}} \cdot \frac{{}^{a}\!/_2}{r} \\
&= 2k\frac{\lambda a^2}{r^2\left({}^{a^2}\!/_4 + r^2\right)^{1/2}}
\end{aligned}
$$

Exercise 4.
Find the electric field a distance z above the centre of a flat circular disk of radius R, which carries a uniform surface charge σ. What does your formula give in the limit $R \to \infty$? Also check the case $z >> R$.

Solution 4.
Consider an elemental ring of radius r and thickness $\mathrm{d}r$. Therefore,

$$
\begin{aligned}
\mathrm{d}q &= \sigma \cdot 2\pi r\,\mathrm{d}r \\
\therefore \mathrm{d}E &= \frac{kz\,\mathrm{d}q}{\left(z^2 + r^2\right)^{3/2}} \\
&= \frac{2\sigma\pi z r\,\mathrm{d}r}{4\pi\varepsilon_0\left(z^2 + r^2\right)^{3/2}} \\
&= \frac{z\sigma r\,\mathrm{d}r}{2\varepsilon_0\left(z^2 + r^2\right)^{3/2}} \\
\therefore E &= \frac{z\sigma}{2\varepsilon_0}\int_0^R \frac{r\,\mathrm{d}r}{\left(z^2 + r^2\right)^{3/2}} \\
&= \frac{z\sigma}{2\varepsilon_0}\left[\frac{-1}{\sqrt{z^2 + r^2}}\right]_0^R \\
&= \frac{z\sigma}{2\varepsilon_0}\left(\frac{-1}{\sqrt{z^2 + R^2}} + \frac{1}{z}\right) \\
&= \frac{z\sigma}{2\varepsilon_0}\left(\frac{1}{z} - \frac{1}{\sqrt{z^2 + R^2}}\right)
\end{aligned}
$$

If $R \to \infty$,

$$E = \frac{z\sigma}{2\varepsilon_0}\left(\frac{1}{z}\right)$$
$$= \frac{\sigma}{2\varepsilon_0}$$

This is consistent with the value of electric field due to an infinite plane of charge.

Exercise 5.
Find the electric field a distance z from the centre of a spherical surface of radius R, which carries a uniform charge density σ.

Solution 5.

$$\mathrm{d}q = \sigma R^2 \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi$$
$$\therefore \mathrm{d}E = k \frac{\mathrm{d}q}{R^2 + z^2 - 2Rz\cos\theta}$$

Due to symmetry, only the components of all $\mathrm{d}E$ in the z direction will add up, and all other components will cancel out.
Let the angle between $\overrightarrow{\mathrm{d}E}$ and the z-axis be α.
Therefore,

$$\cos\alpha = \frac{z - R\cos\theta}{\sqrt{R^2 + z^2 - 2Rz\cos\theta}}$$

Therefore,

$$E = \int \mathrm{d}E \cos\alpha$$
$$= \int_0^{2\pi}\int_0^{\pi} \frac{k\sigma R^2 \sin\theta (z - R\cos\theta)}{\left(R^2 + z^2 - 2Rz\cos\theta\right)^{3/2}} \, \mathrm{d}\theta \, \mathrm{d}\varphi$$

Integrating,
If $z = R$,

$$E = \frac{kq}{R^2}$$

Else,

$$E = \frac{kq}{4Rz^2}\left((z+R) - |z-R| - (z^2+R^2)\left(\frac{1}{z+R} - \frac{1}{|z-R|}\right)\right)$$

Therefore,

$$E = \begin{cases} 0 & ; \quad z < R \\ \dfrac{kq}{z^2} & ; \quad z \geq R \end{cases}$$

Exercise 6.
Find the field inside and outside a sphere of radius R, which carries a uniform volume charge density ρ.

Solution 6.
From the previous result, if $z < R$, only the part of the sphere on the inside, i.e. the smaller sphere of radius z will have an effect on the field at the point. The part of the sphere outside will have no field at the point.

$$\begin{aligned} E &= \int \frac{k\,\mathrm{d}q}{z^2} \\ &= \int_0^z \frac{k\rho \cdot 4\pi}{R^2} \cdot r^2\,\mathrm{d}r \\ &= \frac{kqz}{R^3} \end{aligned}$$

If $z \geq R$, the whole sphere will affect the field.
Therefore, by the previous result,

$$E = \frac{kq}{R^2}$$

Therefore,

$$E = \begin{cases} \dfrac{kq}{z^2} & ; \quad z \geq R \\ \dfrac{kqz}{R^2} & ; \quad z \leq R \end{cases}$$