

Physics 2

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1 Lecturer Information

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2 Textbooks

1. D. Halliday, R. Resnick, and K. S. Krane: *Physics*, 5th edition, vol. 2 (Wiley)
2. D.J. Griffiths: *Introduction to Electrodynamics*

Part I

Electrostatics

1 Coulomb's Law

The force between two charged particles is directly proportional to the product of the charges of the particles, and inversely proportional to the square of the distance between them.

$$\begin{aligned} F &\propto \frac{q_1 q_2}{r^2} \\ F &= k \frac{q_1 q_2}{r^2} \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \end{aligned}$$

The constant of proportionality is $k = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$.

$\epsilon_0 = 8.8541878162 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ is called the permittivity of free space.

In vector notation,

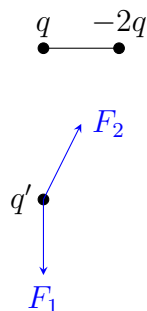
$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

Charge is defined according to this law.

Exercise 1.

A charge q is placed at the origin. A charge $-2q$ is placed at 1 m from it, in the x direction. Find a point on the y -axis where the total force acting on a charge q' will be parallel to the x -axis.

Solution 1.



For the net force to be in the x direction, the components of F_1 and F_2 in the y direction must cancel each other out.

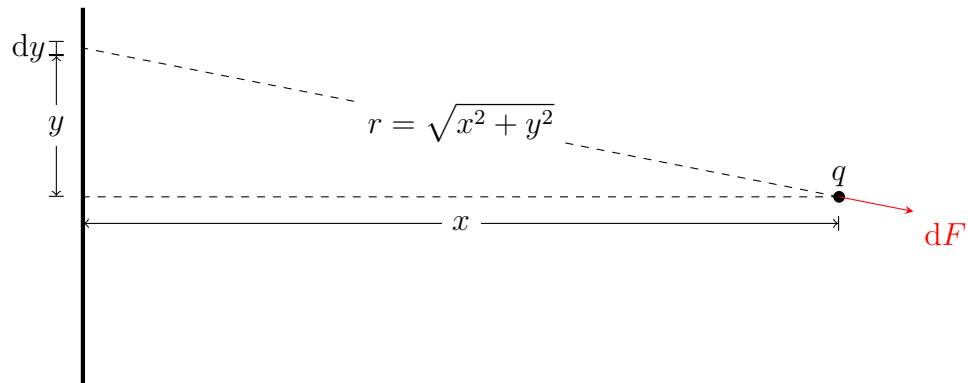
$$\begin{aligned}
 F_1 &= F_2 \sin \theta \\
 \therefore k \cdot \frac{(q')(-2q)}{y^2 + 1} \cdot \frac{y}{\sqrt{y^2 + 1}} &= k \cdot \frac{(q)(q')}{y^2} \\
 \therefore \frac{-2y}{(y^2 + 1)^{3/2}} &= \frac{1}{y^2} \\
 \therefore y &= \pm \sqrt{\frac{1}{2^{2/3} - 1}}
 \end{aligned}$$

Exercise 2.

A rod of length L has a uniformly distributed charge Q , with line charge density $\lambda = \frac{Q}{L}$. A point charge q is kept at a distance x as shown.



Solution 2.



The y components of the forces of the elemental charges at y and $-y$ on q are cancelled out. Therefore, the net force is in the x direction only.

$$\begin{aligned}
 dF &= k \frac{(dQ)(q)}{r^2} \\
 dF_x &= dF \cos \theta \\
 &= k \frac{(dQ)(q)}{r^2} \cos \theta \\
 &= k \frac{(\lambda dy)(q)}{x^2 + y^2} \frac{x}{\sqrt{x^2 + y^2}} \\
 &= k \lambda q x \frac{dy}{(x^2 + y^2)^{3/2}} \\
 \therefore \vec{F} &= \hat{x} \int dF_x \\
 &= \hat{x} \lambda q x \int_{-L/2}^{L/2} \frac{dy}{(x^2 + y^2)^{3/2}}
 \end{aligned}$$

Substituting $y = x \tan \theta$ and $dy = x \sec^2 \theta d\theta$

$$\begin{aligned}
 \vec{F} &= \hat{x} \lambda q k x \int_{-\theta_0}^{\theta_0} \frac{1}{x^2} \cos \theta d\theta \\
 &= \hat{x} \frac{\lambda q k}{x} \int_{-\theta_0}^{\theta_0} \cos \theta d\theta
 \end{aligned}$$

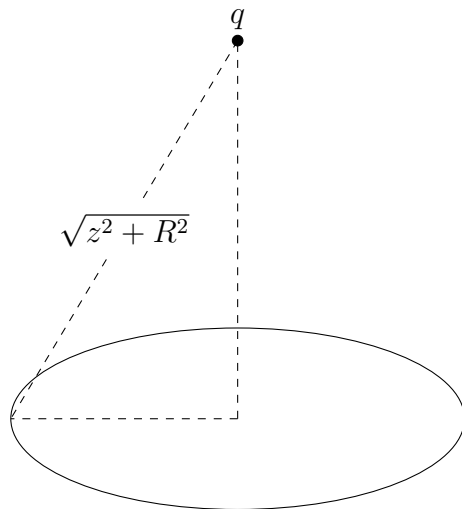
Therefore,

$$\begin{aligned}
 \vec{F} &= \hat{x} \frac{2\lambda q k}{x} \sin \theta_0 \\
 &= \hat{x} \frac{2\lambda q k}{x} \frac{\frac{L}{2}}{\left(\left(\frac{L}{2} \right)^2 + x^2 \right)^{1/2}} \\
 &= \hat{x} \frac{2 \left(\frac{Q}{L} \right) q k}{x} \cdot \frac{\frac{L}{2}}{\left(\left(\frac{L}{2} \right)^2 + x^2 \right)^{1/2}} \\
 &= k \frac{Qq}{x \left(\left(\frac{L}{2} \right)^2 + x^2 \right)^{1/2}} \hat{x}
 \end{aligned}$$

Exercise 3.

A point charge q is kept at a distance z above a ring of radius R charged with $Q = 2\pi R\lambda$, where λ is the linear charge density. Find the force acting on q .

Solution 3.



Due to the symmetry of the ring, the net force acting on q is in the z direction only.

$$\begin{aligned}
 dF_z &= dF \cos \theta \\
 &= k \frac{(dQ)(q)}{z^2 + R^2} \cos \theta \\
 &= k \frac{(dQ)(q)}{z^2 + R^2} \frac{z}{\sqrt{z^2 + R^2}} \\
 &= kqz \frac{dQ}{(z^2 + R^2)^{3/2}} \\
 \therefore \vec{F} &= \hat{z} \int dF_z \\
 &= \hat{z} kqz \frac{1}{(z^2 + R^2)^{3/2}} \int_0^Q dQ \\
 &= k \frac{Qqz}{(z^2 + R^2)^{3/2}} \vec{z}
 \end{aligned}$$

Exercise 4.

A point charge q is kept at a distance z above a disk of radius R charged with $Q = \pi R^2 \sigma$, where σ is the surface charge density. Find the force acting on q .

Solution 4.

The disk can be considered to be made up of elemental rings, with radii varying from 0 to R .

Therefore,

$$\begin{aligned}
 d\vec{F} &= k \frac{qQ_{\text{ring}}}{(z^2 + R^2)^{3/2}} \hat{z} \\
 &= k \frac{q(\sigma \cdot 2\pi r \cdot dr)}{(z^2 + R^2)^{3/2}} z \hat{z}
 \end{aligned}$$

Hence,

$$\begin{aligned}\vec{F} &= \int \mathrm{d}\vec{F} \\ &= \hat{z} \int_0^R k \frac{q\sigma \cdot 2\pi r z \cdot \mathrm{d}r}{(z^2 + R^2)^{3/2}} \\ &= 2kzq\sigma\pi \left(\frac{1}{|z|} - \frac{1}{\sqrt{z^2 + R^2}} \right) \hat{z}\end{aligned}$$

If $z \ll R$, i.e. for an infinite sheet,

$$F = 2q\sigma\pi k$$