QUANTUM AND SOLID STATE PHYSICS : ASSIGNMENT 2

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Exercise 1.
Treating atoms as hard spheres that are as densely packed as possible, calculate the fraction of the unit cell volume filled with hard spheres, i.e. the atomic packing factor, for

(1) SC
(2) BCC
(3) FCC
(4) Diamond

Draw the unit cell for each of the above structures.

Solution 1.

(1) Let the lattice constant be a.
Therefore the radius of each atom is $\frac{a}{2}$.
The total number of atoms in a unit cell of SC is 1.
Therefore,

$$\begin{aligned}\text{APF} &= \frac{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}{a^3} \\ &= \frac{\frac{4}{3}\pi\frac{a^3}{8}}{a^3} \\ &= \frac{\pi}{6}\end{aligned}$$

(2) Let the lattice constant be a.
Therefore the radius of each atom is $\frac{a\sqrt{3}}{4}$.
The total number of atoms in a unit cell of BCC is 2.
Therefore,

$$\begin{aligned}\text{APF} &= \frac{2\cdot\frac{4}{3}\pi\left(\frac{a\sqrt{3}}{4}\right)^3}{a^3} \\ &= \frac{\frac{8}{3}\pi\frac{3\sqrt{3}a^3}{16}}{a^3} \\ &= \frac{\sqrt{3}\pi}{2}\end{aligned}$$

(3) Let the lattice constant be a.
Therefore the radius of each atom is $\frac{a\sqrt{2}}{4}$.

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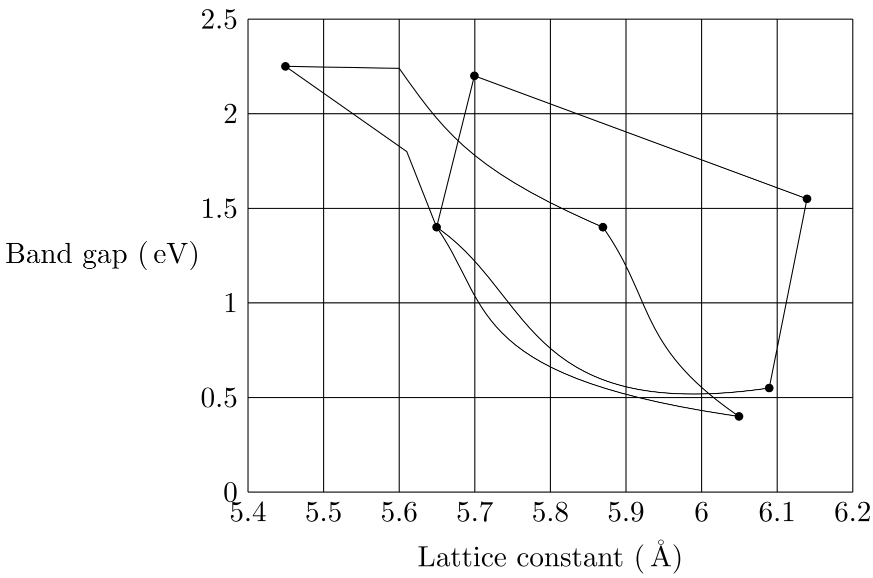
The total number of atoms in a unit cell of FCC is 2.
Therefore,

$$\begin{aligned}
\text{APF} &= \frac{4 \cdot \frac{4}{3}\pi \left(\frac{a\sqrt{2}}{4}\right)^3}{a^3} \\
&= \frac{\frac{16}{3}\pi \frac{2\sqrt{2}a^3}{16}}{a^3} \\
&= \frac{2\sqrt{2}\pi}{3}
\end{aligned}$$

(4) Let the lattice constant be a.
Therefore the radius of each atom is $\frac{a\sqrt{3}}{8}$.
The total number of atoms in a unit cell of diamond lattice is 8.
Therefore,

$$\begin{aligned}
\text{APF} &= \frac{8 \cdot \frac{4}{3}\pi \left(\frac{a\sqrt{3}}{8}\right)^3}{a^3} \\
&= \frac{\frac{32}{3}\pi \frac{3\sqrt{3}a^3}{512}}{a^3} \\
&= \frac{\sqrt{3}\pi}{16}
\end{aligned}$$

Exercise 2.
Please refer to the following figure, which shows the relationship between the energy band gap, and lattice constants for several materials.



(1) Consider growing an epitaxial later of InAs on the following substrates
 (a) InP
 (b) AlAs

(c) GaAs
(d) GaP

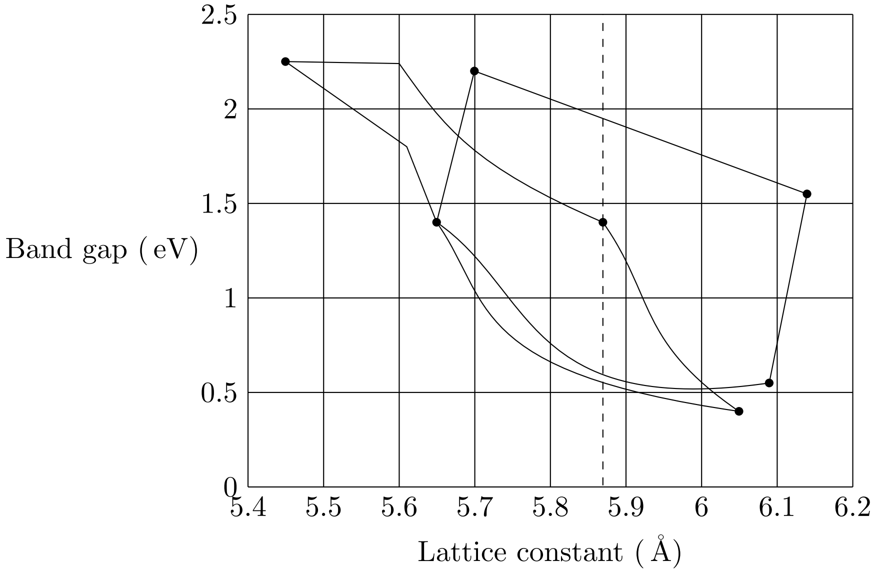
For which substrate would there be minimum strain in the growth structure, and why?

(a) InP
(b) AlAs
(c) GaAs
(d) GaP

(2) Draw a line on the figure above, such that all materials along this line could be grown as lattice-matched epitaxial layers to an underlying InP substrate.

Solution 2.

(1) There would be minimum strain in the growth structure if the substrate used is InP. This is because the difference between the lattice constants of InAs and InP, as compared to the other alternatives.



(2)

Exercise 3.

Determine whether the following statements are true or false, and explain your answer.

(1) The wavelength of the electron is given by the de Broglie equation.
(2) The wave associated with a moving electron is an electromagnetic wave.
(3) The kinetic energy of the electron is given by the equation $E = hf$.
(4) A photograph of a person is created with only a few incident photons, and the output image collected on a screen. The result of this experiment is a very faint image of the person.
(5) The solution to the Schrödinger equation is the wave function, which is a probability amplitude of a quantum system in space and time.

Solution 3.

(1) True, according to the de Broglie hypothesis, the wavelength of a wave corresponding to a particle is given by the de Broglie equation.
(2) False, the wave associated with a moving electron is not an electromagnetic wave, but is an hypothetical wave which describes the wave function ψ.
(3) False, the equation $E = hf$ gives the total, i.e. kinetic and potential energy of the electron.
(4) False, the result of this experiment is a few dots on the screen, as the interaction of the photons with the person forces them to behave like particles.
(5) False, the solution is the wave function, which is a complex number, whose square is the probability amplitude of a quantum system in space and time.

Exercise 4.

Suppose you drop a rock off a cliff of a height h. As it falls, a million photographs are taken, at random intervals. On each picture, the distance of the rock has fallen is being measured.

(1) Show that the probability density is given by

$$\rho(x) = \frac{1}{2\sqrt{hx}}$$

where $0 \leq x \leq h$.
(2) What is the average of all measured distances?
(3) what is the standard deviation for the measured distances?
(4) What is the probability that a photograph, selected at random, would show a distance x more than one standard deviation away from the average?

Solution 4.

(1)

$$x(t) = \frac{1}{2}gt^2$$

Let the total time of the free fall be T.
Therefore,

$$T = \sqrt{\frac{2h}{g}}$$

Therefore,

$$\begin{aligned} P\frac{\mathrm{d}t}{T} &= \frac{\mathrm{d}x}{\sqrt{\frac{2h}{g}}g\sqrt{\frac{2x}{g}}} \\ &= \frac{\mathrm{d}x}{2\sqrt{hx}} \end{aligned}$$

As the rock is falling under constanct acceleration,

$$\frac{\mathrm{d}x}{\mathrm{d}t} = gt$$

Therefore,

$$\rho(x) = \frac{1}{2\sqrt{hx}}$$

(2)

$$\begin{aligned}
\langle x \rangle &= \int_0^h x\rho(x) \\
&= \int_0^h \frac{x\,\mathrm{d}x}{2\sqrt{hx}} \\
&= \int_0^h \frac{\sqrt{x}}{2\sqrt{h}}\,\mathrm{d}x \\
&= \left.\frac{x^{\frac{3}{2}}}{3\sqrt{h}}\right|_0^h \\
&= \frac{h^{\frac{3}{2}}}{3h^{\frac{1}{2}}} \\
&= \frac{h}{3}
\end{aligned}$$

(3)

$$\begin{aligned}
\sigma^2 &= \int_0^h x^2\rho(x)\,\mathrm{d}x - \left(\int_0^h x\rho(x)\,\mathrm{d}x\right) \\
&= \int_0^h \frac{x^2}{2\sqrt{hx}}\,\mathrm{d}x - \frac{h^2}{9} \\
&= \frac{1}{2\sqrt{h}}\left.\left(\frac{2}{5}x^{\frac{5}{2}}\right)\right|_0^h - \frac{h^2}{9} \\
&= \frac{h^2}{5} - \frac{h^2}{9} \\
&= \frac{4h^2}{45} \\
\therefore \sigma &= \frac{2h}{3\sqrt{5}}
\end{aligned}$$

(4)

$$\sigma = \frac{2h}{3\sqrt{5}}$$

Therefore,

$$\begin{aligned}
\langle x\rangle - \sigma + x &= \frac{h}{3} - \frac{2h}{3\sqrt{5}} + x \\
&= \frac{h(\sqrt{5}-2)}{3\sqrt{5}} + x \\
\langle x\rangle + \sigma + x &= \frac{h}{3} - \frac{2h}{3\sqrt{5}} + x \\
&= \frac{h(\sqrt{5}+2)}{3\sqrt{5}} + x
\end{aligned}$$

Therefore,

$$\begin{aligned}
\rho\big(\langle x\rangle - \sigma + x\big) &= \rho\left(\frac{h(\sqrt{5}+2)}{3\sqrt{5}} + hx\right) \\
&= \frac{1}{2\sqrt{h\frac{h(\sqrt{5}-2)}{3\sqrt{5}} + hx}} \\
&= \frac{1}{2\sqrt{\frac{h^2(\sqrt{5}-2)}{3\sqrt{5}} + hx}} \\
\rho\big(\langle x\rangle + \sigma + x\big) &= \rho\left(\frac{h(\sqrt{5}+2)}{3\sqrt{5}} + hx\right) \\
&= \frac{1}{2\sqrt{h\frac{h(\sqrt{5}+2)}{3\sqrt{5}} + hx}} \\
&= \frac{1}{2\sqrt{\frac{h^2(\sqrt{5}+2)}{3\sqrt{5}} + hx}}
\end{aligned}$$