QUANTUM AND SOLID STATE PHYSICS : ASSIGNMENT 7

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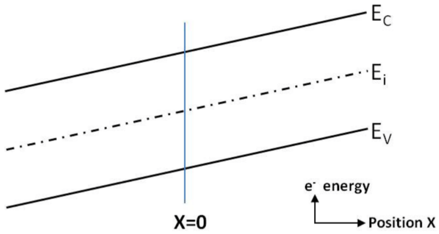
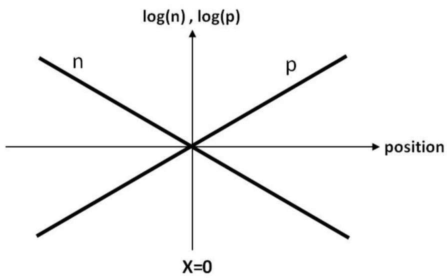
Exercise 1.

Consider a semiconductor sample with both an electric field $\overrightarrow{E}$ and carrier concentration gradient. Shown below is the equilibrium band diagram across the length of the sample, and also the carrier concentrations, $\log n$, $\log p$, as functions of position across the sample.





(1) What is the direction of the electric field? Is $\overrightarrow{E}$ constant, with respect to x, or dependent on x? Choose the correct answer below.
   (a) $\rightarrow$, constant
   (b) $\rightarrow$, position dependent
   (c) $\leftarrow$, constant
   (d) $\leftarrow$, position dependent

(2) Indicate with arrows, the correct direction of electron and hole flow in the sample due to both diffusion and drift, and the correct direction of the corresponding current densities.

Date: Thursday 3rd December, 2015.

Solution 1.

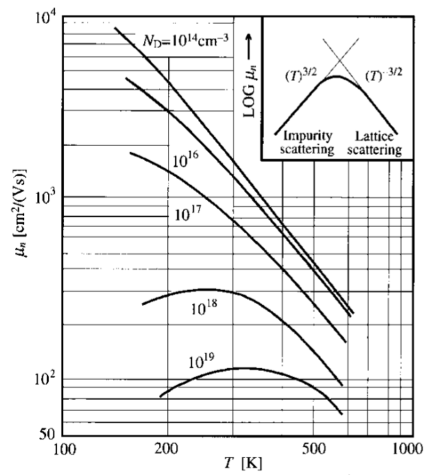
(1) The electric field is directed $\rightarrow$, and is constant.

Direction of electron drift	$\leftarrow$	J_{drift_n}	$\rightarrow$
Direction of electron diffusion	$\rightarrow$	$J_{\text{diffusion}_n}$	$\leftarrow$
Direction of hole drift	$\rightarrow$	J_{drift_p}	$\rightarrow$
Direction of hole diffusion	$\leftarrow$	$J_{\text{diffusion}_p}$	$\leftarrow$

(2)

Exercise 2.

Shown below is the electron mobility dependence on temperature for a sample of Silicon.



Calculate the resistivity ρ of this sample of Silicon at 300 K, when it is doped with 10^{16} Phosphorus atoms per cm^3, and 10^{14} Boron atoms per cm^3.

Solution 2.

As the concentration of Phosphorus atoms is more than that of Boron atoms, the material is N-type.
Therefore,

$$
\begin{aligned}
\rho &= \frac{1}{q\mu_n n} \\
&= \frac{1}{q\mu_n} \\
&= \frac{1}{\left(1.6 \times 10^{19}\,\mathrm{C}\right)\left(10^3 \frac{\mathrm{cm}^2}{\mathrm{V\,s}}\right)\left(10^{16} \frac{1}{\mathrm{cm}^3}\right)} \\
&= 0.625\Omega\,\mathrm{cm}
\end{aligned}
$$

Exercise 3.
A free particle has the following initial wave function, i.e. at $t = 0$,

$$\psi(x, 0) = Ae^{-a|x|}$$

where A and a are positive and real constants.

(1) Find A.
(2) Find $\widetilde{\psi}(k)$. Remember that

$$\psi(x, 0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \widetilde{\psi}(k) e^{ikx} \, \mathrm{d}k$$

Also, use your knowledge of even and odd functions to help you solve the integral.
(3) Construct $\psi(x, t)$. Your answer should simply show the integral.
(4) Consider a case where a is very small. What can you say about the uncertainty in position and momentum for the particle at $t = 0$, described by the wave function $\psi(x, 0)$?
(5) Consider a case where a is very large. What can you say about the uncertainty in position and momentum for the particle at $t = 0$, described by the wave function $\psi(x, 0)$?

Solution 3.
(1)

$$\begin{aligned}
1 &= \int_{-\infty}^{\infty} \left|\psi(x, 0)\right|^2 \mathrm{d}x \\
&= \int_{-\infty}^{\infty} A^2 e^{-2a|x|} \, \mathrm{d}x \\
&= 2A^2 \int_{0}^{\infty} e^{-2ax} \, \mathrm{d}x \\
&= 2A^2 \left. \frac{e^{-2ax}}{-2a} \right|_{0}^{\infty} \\
&= 2A^2 \left(0 - \frac{1}{2a}\right) \\
&= \frac{2A^2}{2a} \\
\therefore A &= \sqrt{a}
\end{aligned}$$

(2)

$$\widetilde{\psi}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x,0) e^{-ikx} \, \mathrm{d}x$$
$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sqrt{a} e^{-a|x|} e^{-ikx} \, \mathrm{d}x$$
$$= \sqrt{\frac{2a}{\pi}} \frac{a}{a^2 + k^2}$$

(3)

$$\psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \widetilde{\psi}(k) e^{ikx} e^{-\frac{iEt}{\hbar}} \, \mathrm{d}k$$
$$= \frac{a^{\frac{3}{2}}}{\pi} \int_{-\infty}^{\infty} \frac{e^{ikx} e^{-\frac{ik^2\hbar}{2m} t}}{a^2 + k^2} \, \mathrm{d}k$$

(4) If a is very small, $\psi(x,0)$ is almost constant. Hence, σ_x is high, and σ_p is low.
(5) If a is very large, $\psi(x,0)$ is large. Hence, σ_x is low, and σ_p is high.

Exercise 4.
Consider a case of even potential, such that $V(-x) = V(x)$.

(1) If $\psi(x)$ is a solution to the time independent Schrödinger equation, what can you say about $\psi(-x)$? Hint: Replace x by $-x$ in the Schrödinger equation.
(2) Based on the principle of superposition, suggest a way of constructing an even and an odd solution from $\psi(x)$ and $\psi(-x)$. Don't forget to normalize.

Solution 4.
(1)

$$-\frac{\hbar^2}{2m} \frac{\mathrm{d}^2\psi(x)}{\mathrm{d}x^2} + V(x)\psi(x) = E\psi(x)$$

Therefore, replacing x by $-x$,

$$-\frac{\hbar^2}{2m} \frac{\mathrm{d}^2\psi(-x)}{\mathrm{d}x^2} + V(-x)\psi(-x) = E\psi(-x)$$

Therefore, as $V(-x) = V(x)$,

$$-\frac{\hbar^2}{2m} \frac{\mathrm{d}^2\psi(-x)}{\mathrm{d}x^2} + V(x)\psi(-x) = E\psi(-x)$$

Therefore, $\psi(-x)$ is also a solution to the time independent Schrödinger equation.

Exercise 5.

Repeat the derivation of the solution to the finite well potential, this time looking only at the odd solutions. The general solution in this case is

$$\psi(x) = \begin{cases} Ae^{kx} & ; \quad x < -a \\ F\sin(\beta x) & ; \quad -a \leq x \leq a \\ Ce^{-kx} & ; \quad a < x \end{cases}$$

where the potential is defined as

$$V(x) = \begin{cases} 0 & ; \quad x < -a \\ -V_0 & ; \quad -a \leq x \leq a \\ 0 & ; \quad a < x \end{cases}$$

(1) Find the equation linking k and β.
(2) Define $z = \beta a$ and $y = ka$, and draw the two sets of curves.
(3) Sketch how we find the allowed states using your results from above.
(4) Is there always an allowed odd state?
(5) When V_0 approaches infinity, describe the allowed energy states in terms of allowed values of β. Verify that your result combined with the similar result we got in class for the even solutions describes the allowed states in an infinite potential well.

Solution 5.

(1)

$$\psi(x) = \begin{cases} Ae^{kx} & ; \quad x < -a \\ F\sin(\beta x) & ; \quad -a \leq x \leq a \\ Ce^{-kx} & ; \quad a < x \end{cases}$$

As ψ is continuous at the boundaries,

$$\begin{aligned} Ae^{-ka} &= F\sin(-\beta a) \\ F\sin(\beta a) &= Ce^{-ka} \end{aligned}$$

Therefore,

$$A = -C$$

As ψ' is continuous at the boundaries,

$$\begin{aligned} Ake^{-ka} &= F\beta\cos(-\beta a) \\ F\beta\cos(\beta a) &= -Cke^{-ka} \end{aligned}$$

Therefore,

$$-k = \beta\cot(\beta a)$$

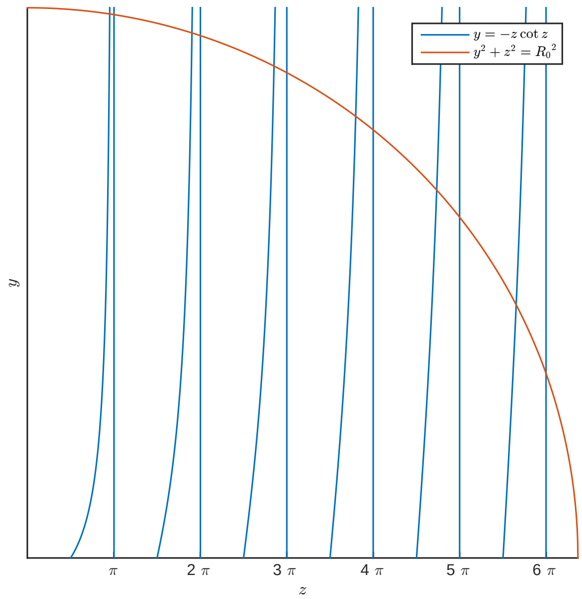
(2) Let

$$\begin{aligned} z &= \beta a \\ y &= ka \end{aligned}$$

Therefore,

$$y = -z\cot z$$
$$y^2 + z^2 = {R_0}^2$$

Therefore,



(3)
(4) If $R_0 < \frac{\pi}{2}$, the curves do not intersect, and there is no allowed odd state.
(5) If $V_0 \to \infty$, $R_0 \to \infty$,

$$z = n\pi$$

where $n \in \mathbb{N}$.
Therefore,

$$\beta_n = \frac{n\pi}{2a}$$

For an infinite potential well,

$$k_n = \frac{n\pi}{2a}$$

Therefore, this is consistent with the result from class.