QUANTUM AND SOLID STATE PHYSICS : ASSIGNMENT 4

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Exercise 1.
True or False:

(1) Semiconductor materials at 0 K have the same structure as metals - bands either overlap or are only partially filled.
(2) In a solid, many atoms are brought close together, so that the split energy levels form essentially continuous bands.
(3) Column V donor levels usually lie approximately 0.95 eVbelow the conduction band in Si.

Solution 1.
(1) False, at 0 K, semiconductors have the same structure as insulators.
(2) True
(3) False, donor levels are closer to the E_C and not closer to E_V, as suggested by the given energy difference.

Exercise 2.
The following diagrams illustrate, for 3 different materials, an electron in the conduction band recombining with a hole in the valence band and emitting a photon of energy $E_{\text{photon}} = E_g$, where E_g is the band gap energy.

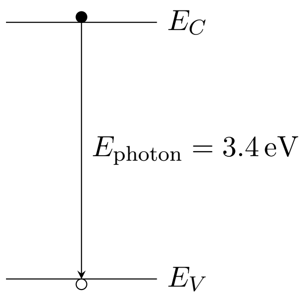


FIGURE 1. GaN : $E_g = 3.4\,\text{eV}$

Date: Thursday 12$^{\text{th}}$ November, 2015.

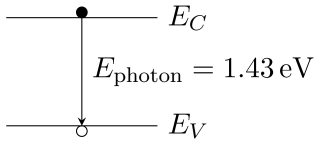


FIGURE 2. GaAs : $E_g = 1.43\,\text{eV}$

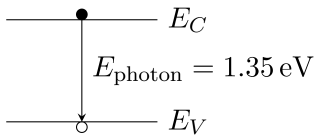


FIGURE 3. GaAs : $E_g = 1.35\,\text{eV}$

(1) For which materials will the emitted photon have the largest frequency?
   (a) GaN
   (b) GaAs
   (c) InP
(2) For which materials will the emitted photon have the largest wavelength?
   (a) GaN
   (b) GaAs
   (c) InP
(3) At room temperature, which material will have the maximum intrinsic carrier concentration n_i?
   (a) GaN
   (b) GaAs
   (c) InP

Solution 2.

(1) The emitted photon will have the largest frequency for GaN.
(2) The emitted photon will have the largest wavelength for InP.
(3) At room temperature, InP will have the maximum intrinsic carrier concentration.

Exercise 3.

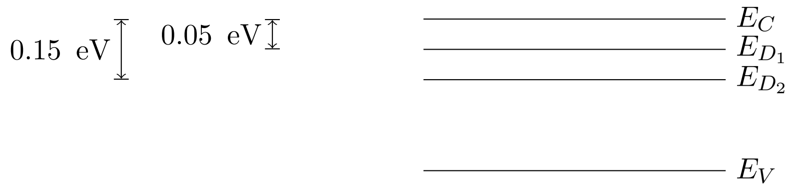
Consider a sample of silicon that is doped with two donor levels N_{D_1} donors per cm^3 at donor energy E_{D_1} with ionization energy 0.05 eV, and N_{D_2} donors per cm^3 at donor energy E_{D_2} (deeper donor) with ionization energy 0.15 eV.

(1) Draw the energy diagram for this material.
(2) Draw a graph of the electron concentration on a log scale as a function of $\frac{1}{T}$.

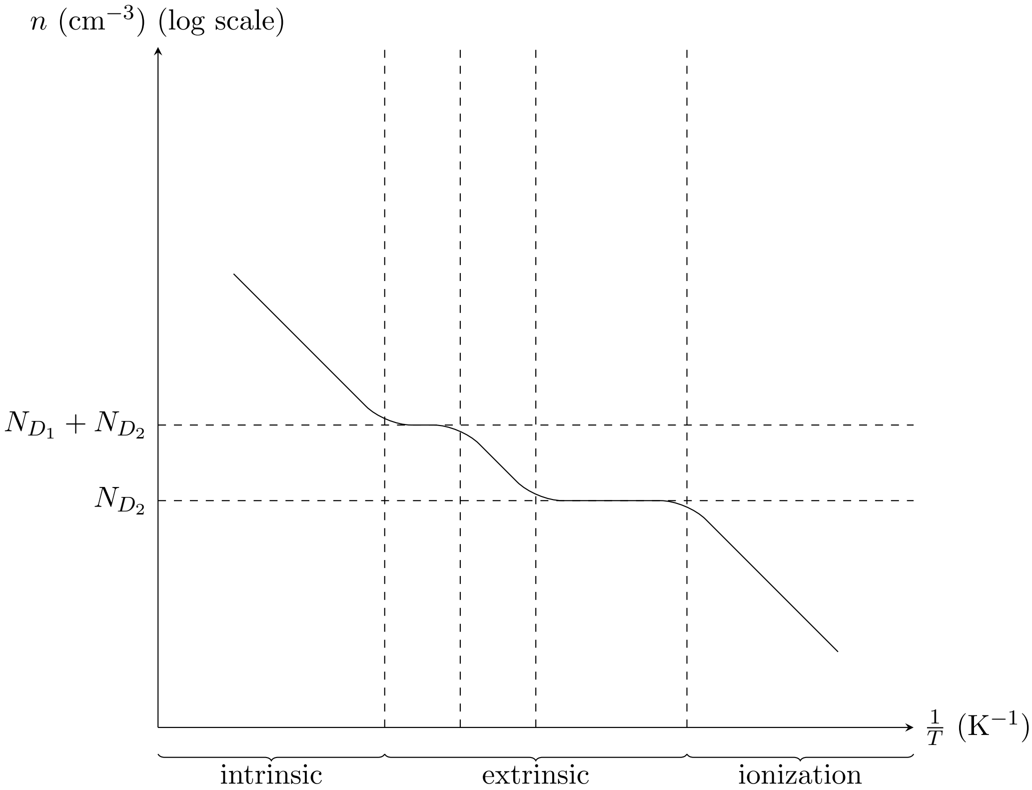
Label the three regions of the plot, ionization, extrinsic, and intrinsic. On the graph, write the values of electron concentration in the extrinsic region.

Solution 3.

(1)



(2)



Exercise 4.

(1) ψ_1 and ψ_2 are momentum eigenfunctions corresponding to different momentum eigenvalues, $p_1 \neq p_2$. Is $\psi = \psi_1 + \psi_2$ also a momentum eigenfunction?
  (a) Yes.
  (b) No.
  (c) It is impossible to answer based on the provided information.

(2) Two particles, 1 and 2, are described by plane waves of the form e^{ikx}. Particle 1 is described by a smaller wavelength than particle 2, $\lambda_1 < \lambda_2$. Which particle has a larger momentum?
  (a) Particle 1.

(b) Particle 2.
(c) Both have the same momentum.
(d) It is impossible to answer based on the provided information.

(3) Consider the eigenvalue equation

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2} f(x) = cf(x)$$

How many of the following give an eigenfunction and a corresponding eigenvalue to the above equation?
(a) $f(x) = sin(kx)$, $c = k^2$
(b) $f(x) = e^{-x}$, $c = 1$
(c) $f(x) = e^{ikx}$, $c = -k^2$
(d) $f(x) = x^3$, $c = 6$
(a) 1
(b) 2
(c) 3
(d) 4
(e) none

Solution 4.

(1) Let

$$\psi_1 = A_1 e^{ik_1 x}$$
$$\psi_2 = A_2 e^{ik_2 x}$$

Therefore,

$$\begin{aligned}\frac{\hbar}{i}\frac{\partial}{\partial x}(\psi_1 + \psi_2) &= \frac{\hbar}{i}\frac{\partial}{\partial x}\left(A_1 e^{ik_1 x} + A_2 e^{ik_2 x}\right)\\ &= \frac{\hbar}{i} ik_1\psi_1 + ik_2\psi_2\\ &\neq c(\psi_1 + \psi_2)\end{aligned}$$

Therefore, $\underline{\psi = \psi_1 + \psi_2 \text{ is not a momentum eigenfunction}}$.

(2) According to the de Broglie hypothesis,

$$p = \frac{h}{\lambda}$$

Therefore, as $\lambda_1 < \lambda_2$, $p_1 > p_2$. Hence, <u>particle 1</u> has larger momentum.

(3)

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2} f(x) = cf(x)$$

Therefore,

$$\frac{\mathrm{d}^2 \sin(kx)}{\mathrm{d}x^2} = c\sin(kx)$$
$$-\sin(k^2 x) = c\sin(kx)$$

Therefore, $\underline{c = k^2 \text{ is not an eigenvalue}}$ of the function.

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2} f(x) = cf(x)$$

Therefore,

$$\begin{aligned} \frac{\mathrm{d}^2 e^{-x}{}^{-x}}{\mathrm{d}e^2} &= ce^{-x} \\ \therefore e^{-x} &= ce^{-x} \end{aligned}$$

Therefore, $\underline{c = 1 \text{ is an eigenvalue}}$ of the function.

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2} f(x) = cf(x)$$

Therefore,

$$\begin{aligned} \frac{\mathrm{d}^2}{\mathrm{d}x^2} e^{ikx} &= ce^{ikx} \\ \therefore -k^2 e^{ikx} &= ce^{ikx} \end{aligned}$$

Therefore, $\underline{c = -k^2 \text{ is an eigenvalue}}$ of the function.

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2} f(x) = cf(x)$$

Therefore,

$$\begin{aligned} \frac{\mathrm{d}^2 x^3}{\mathrm{d}x^2} &= cx^3 \\ 6x &= cx^3 \end{aligned}$$

Therefore, $\underline{c = 6 \text{ is not an eigenvalue}}$ of the function.

Hence, $\underline{2}$ of the pairs are pairs of eigenvalues and eigenfunctions.

Exercise 5.

A particle is represented, at time $t = 0$ by the following wave function:

$$\psi(x,0) = \begin{cases} A\left(a^2 - x^2\right) & ; \quad -a \leq x \leq a \\ 0 & ; \quad \text{otherwise} \end{cases}$$

(1) Find σ_x, the uncertainty in x.
(2) Find σ_p, the uncertainty in p.
(3) Check if your results are consistent with the uncertainty principle.

Solution 5.
(1)

$$\begin{aligned}\sigma_x &= \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \\ &= \sqrt{\frac{1}{7}a^2} \\ &= \frac{a}{\sqrt{7}}\end{aligned}$$

(2)

$$\begin{aligned}\sigma_p &= \sqrt{\langle p^2 \rangle - \langle p \rangle^2} \\ &= \sqrt{\frac{5h^2}{2a^2}} \\ &= \sqrt{\frac{5}{2}}\frac{h}{a}\end{aligned}$$

(3)

$$\begin{aligned}\sigma_x \sigma_p &= \frac{a}{\sqrt{7}}\sqrt{\frac{5}{2}}\frac{h}{a} \\ &= \sqrt{\frac{5}{14}}h \\ &= \frac{h}{2}\sqrt{\frac{10}{7}} \\ &> \frac{h}{2}\end{aligned}$$

Therefore, the results are consistent with the uncertainty principle.

Exercise 6.
Consider a particle described by a certain wave function $\psi(x)$, and an observable A, associated with operator $\hat{A}$.
Determine whether the following statements are true or false, and explain your answer.

(1) The two following actions are identical and give the same result - measuring observable A for this state $\psi(x)$ and applying the operator $\hat{A}$ on $\psi(x)$.
(2) Measuring observable A and applying the operator $\hat{A}$ on $\psi(x)$ both cause a collapse of the wave function.
(3) Measuring observable A always returns an eigenvalue of operator $\hat{A}$.

Solution 6.
(1) False. When an observable is measured, the result is an eigenvalue of the operator corresponding to the observable. When an operator is applied to the wave function, the result is a function.

(2) False, as the wave function collapses only on physical measurement of the particle, only the process of measuring A causes the collapse, while the process of applying the operator in the wave function does not.
(3) True