QUANTUM AND SOLID STATE PHYSICS : ASSIGNMENT 2

AAKASH JOG
ID : 989323563

Exercise 1.
If we require a silicon sample to have an electron concentration 4 times higher than its hole concentration, i.e. $n = 4p$, which of the following dopant concentration would you choose, expressed in terms of intrinsic carrier concentration n_i?

(1) $N_D = 0.5n_i$
(2) $N_D = 2n_i$
(3) $N_D = 1.5n_i$
(4) $N_A = 0.5n_i$
(5) $N_A = 2n_i$
(6) $N_A = 1.5n_i$

Solution 1.
As the requirement is to have an electron concentration higher than the original sample, the dopant must be a donor.
For the original sample,

$$np = {n_i}^2$$

Therefore, for the doped sample,

$$4pn = {N_D}^2$$
$$\therefore 4{n_i}^2 = {N_D}^2$$

Therefore,

$$N_D = 2n_i$$

Hence, the dopant concentration should be $\underline{(2)\ N_D = 2n_i}$.

Exercise 2.
Let's examine 3 wafers, all of which are doped with a concentration of 10^{15} dopants cm^{-3}.

(1) Wafer A: Silicon, in which the dopant energy level is 1.095 eVabove the valence energy level E_V.
(2) Wafer B: Silicon, in which the dopant energy level is 1.08 eVabove the valence energy level E_V.
(3) Wafer C: GaAs, in which the dopant energy level is 1.095 eVabove the valence energy level E_V.

Date: Thursday 5$^{\text{th}}$ November, 2015.

(1) Do we expect the dopants in wafers A, B, and C to be donors or acceptors? Draw the energy diagrams for each wafer, and clearly label all relevant energy levels and spacing between them.
(2) As the temperature is increased, which wafer will be the first to transition from the ionization region to the extrinsic region?
(3) For which wafer will intrinsic carriers start to dominate at the lowest temperature?

Solution 2.

(1) As the dopant energy in the wafers is closer to E_C, than E_V, the dopants will be donors.

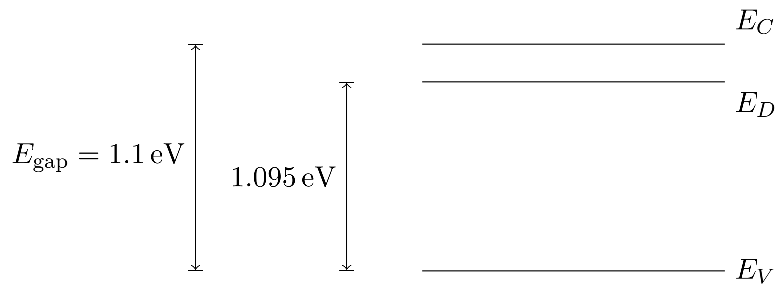


FIGURE 1. Energy Diagram: Wafer A

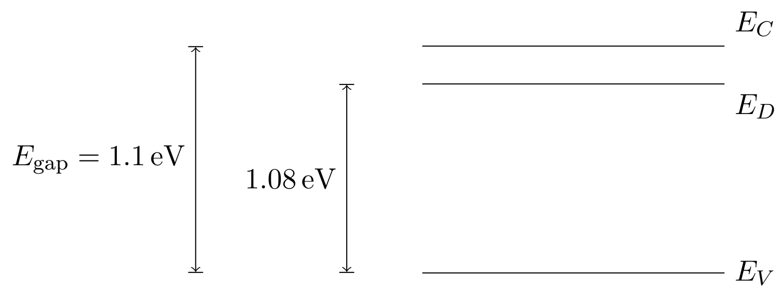


FIGURE 2. Energy Diagram: Wafer B

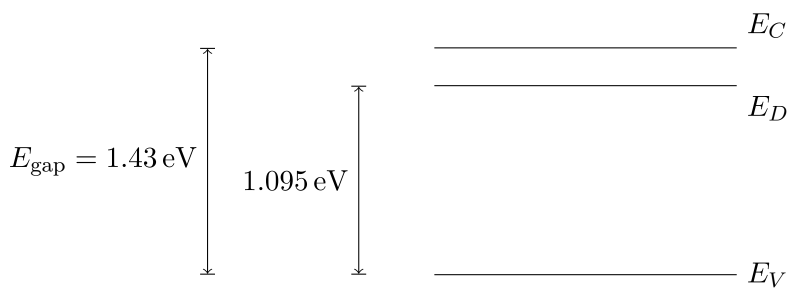


FIGURE 3. Energy Diagram: Wafer C

(2) When the temperature is increased, the wafer for which the gap between E_D and E_C is the least will be the first to transition into the extrinsic region. Therefore, wafer A will be the first to transition.
(3) The energy band gap of Si is less than that of GaAs. However, in wafer B, the dopant charge carriers are at a lower energy level than that of those in wafer A. Hence, the intrinsic carriers in wafer B will start to dominate at the lowest temperature.

Exercise 3.
Two wafers of silicon are doped with the following concentration

$$N_{D1} = 10^{15}\text{cm}^{-3}$$
$$N_{D2} = 10^{16}\text{cm}^{-3}$$

Is it possible that the electron concentration n in both wafers is identical, i.e. $n_1 = n_2$?

(1) No
(2) Yes, in the extrinsic region
(3) Yes, in the intrinsic region
(4) Yes, in both the extrinsic and intrinsic regions

Solution 3.

$$n = N_D + \text{number of thermally generated EHPs}$$

Therefore,

$$n_1 = N_{D1} + \text{number of thermally generated EHPs}$$
$$n_2 = N_{D2} + \text{number of thermally generated EHPs}$$

Therefore,

$$n_1 \neq n_2$$

Therefore, it is not possible for the electron concentration in both wafers to be identical.

Exercise 4.
A particle is represented, at time $t = 0$, by the following wave function.

$$\psi(x,0) = \begin{cases} A\frac{x}{a} & ; \quad 0 \leq x \leq a \\ A\frac{b-x}{b-a} & ; \quad a \leq x \leq b \\ 0 & ; \quad \text{otherwise} \end{cases}$$

(1) Determine the normalization constant A.
(2) Sketch $\psi(x,0)$ as a function of x.
(3) Where is the particle most likely to be found at $t = 0$?
(4) What is the probability of finding the particle to the left of a?
(5) What is the expectation value of x, at time $t = 0$?

Solution 4.
(1)

$$
\begin{aligned}
1 &= \int_{-\infty}^{\infty} \left|\psi(x,0)\right|^2 \mathrm{d}x \\
&= \int_0^a A^2 \frac{x^2}{a^2} \mathrm{d}x + \int_a^b A^2 \frac{(b-x)^2}{(b-a)^2} \mathrm{d}x \\
&= \frac{A^2}{a^2} \left.\frac{x^3}{3}\right|_0^a + \frac{A^2}{(b-a)^2} \left. b^2x - bx^2 + \frac{x^3}{3}\right|_a^b \\
&= \frac{A^2}{a^2}\frac{a^3}{3} + \frac{A^2}{(b-a)^2}\left(b^3 - b^3 + \frac{b^3}{3} - ab^2 + a^2b - \frac{a^3}{3}\right) \\
&= \frac{A^2 a}{3} + \frac{A^2}{(b-a)^2}\left(\frac{b^3}{3} - ab^2 + a^2b - \frac{a^3}{3}\right) \\
&= A^2\left(\frac{a}{3} + \frac{b^3}{3(b-a)^2} - \frac{ab^2}{(b-a)^2} + \frac{a^2b}{(b-a)^2} - \frac{a^3}{3(b-a)^2}\right) \\
&= A^2\left(\frac{a(b-a)^2 + b^3 - 3ab^2 + 3a^2b - a^3}{3(b-a)^2}\right) \\
&= A^2\left(\frac{b(b-a)^2}{3(b-a)^2}\right) \\
&= \frac{A^2 b}{3}
\end{aligned}
$$

Therefore,

$$A = \sqrt{\frac{3}{b}}$$

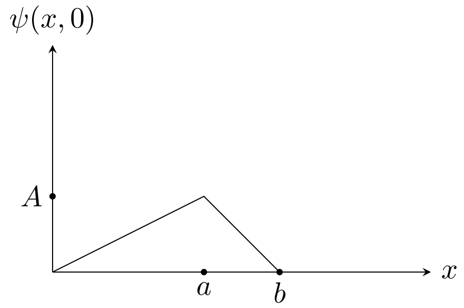
(2)

$$A = \sqrt{\frac{3}{b}}$$

Therefore,

$$
\psi(x,0) = \begin{cases} \sqrt{\frac{3}{b}}\frac{x}{a} & ; \quad 0 \le x \le a \\ \sqrt{\frac{3}{b}}\frac{b-x}{b-a} & ; \quad a \le x \le b \\ 0 & ; \quad \text{otherwise} \end{cases}
$$

Therefore,



(3) As the value of $|\psi(x)|^2$ is maximum at $x = a$, the particle is most likely to be found at $\underline{x = a}$.

(4) The probability of finding the particle at to the left of a is

$$\int_0^a \left|\psi(x,0)\right|^2 \mathrm{d}x$$

$$= \int_0^a \frac{3x^2}{ba^2}\,\mathrm{d}x$$

$$= \left.\frac{x^3}{a^2 b}\right|_0^a$$

$$= \frac{a}{b}$$

(5)

$$\langle x\rangle = \int_{-\infty}^{\infty} x\left|\psi(x)\right|^2 \mathrm{d}x$$

$$= \int_0^a x\left(\frac{A^2x^2}{a^2}\right)\mathrm{d}x + \int_a^b x\frac{A^2(b-x)^2}{(b-a)^2}\,\mathrm{d}x$$

$$= \frac{1}{4b}\left(6a^2 - 2ab - b^2\right)$$

Exercise 5.

A particle is represented, at time $t = 0$, by the following wave function.

$$\psi(x,0) = \begin{cases} A\left(a^2 - x^2\right) & ; \quad -a \le x \le a \\ 0 & ; \quad \text{otherwise} \end{cases}$$

(1) Determine the normalization constant A.
(2) What is the expectation value of x, at time $t = 0$?
(3) What is the expectation value of p, at time $t = 0$? Can you answer this part based on Ehrenfest's theorem? Explain your answer
(4) What is the expectation value of x^2, at time $t = 0$?
(5) What is the expectation value of p^2, at time $t = 0$?

Solution 5.
(1)

$$\begin{aligned}1 &= \int\limits_{-\infty}^{\infty} \left|\psi(x,0)\right|^2 \mathrm{d}x \\ &= \int\limits_{-a}^{q} A^2 \left(a^2 - x^2\right) \mathrm{d}x \\ &= \frac{16a^2}{15A^2}\end{aligned}$$

Therefore,

$$A^2 = \frac{15}{16a^2}$$

Therefore,

$$A = \sqrt{\frac{15}{16a^2}}$$

(2)

$$\begin{aligned}\langle x\rangle &= \int\limits_{-a}^{a} x \left|\psi(x,0)\right|^2 \mathrm{d}x \\ &= \int\limits_{-a}^{a} xA^2 \left(a^2 - x^2\right)^2 \mathrm{d}x\end{aligned}$$

As the function is odd, and the interval is symmetric across the origin, the integral is zero.
Therefore,

$$\langle x\rangle = 0$$

(3) By Ehrenfest's theorem,

$$\begin{aligned}\langle p\rangle &= m\langle v\rangle \\ &= m\,\frac{\mathrm{d}\langle x\rangle}{\mathrm{d}t} \\ &= 0\end{aligned}$$

(4)

$$
\begin{aligned}
\left\langle x^2 \right\rangle &= \int_{-a}^{a} x^2 \left|\psi(x,0)\right|^2 \mathrm{d}x \\
&= \int_{-a}^{a} x^2 A^2 \left(a^2 - x^2\right)^2 \mathrm{d}x \\
&= 2\int_{0}^{a} x^2 A^2 \left(a^2 - x^2\right)^2 \mathrm{d}x \\
&= \frac{a^2}{7}
\end{aligned}
$$

(5)

$$
\begin{aligned}
\left\langle p^2 \right\rangle &= \int_{-a}^{a} \psi^*(x,0) \left(-\hbar^2\right) \frac{\partial^2}{\partial x^2} \psi(x,0) \,\mathrm{d}x \\
&= \int_{-a}^{a} A \left(a^2 - x^2\right) \left(-\hbar^2\right) \frac{\partial^2}{\partial x^2} A \left(a^2 - x^2\right) \mathrm{d}x \\
&= \frac{5\hbar^2}{2a^2}
\end{aligned}
$$