QUANTUM AND SOLID STATE PHYSICS : ASSIGNMENT 13

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Exercise 1.
Suppose we have a PN junction at equilibrium, which means we bring in contact a P-type material and an N-type material, with contacts at both ends attached to ground. The net current across the device is zero, i.e.

$$J_{\text{total}} = 0$$

Explain why we expect to have a built-in electric field in this device.

Solution 1.
As the two parts of the junction are P-type and N-type, they have excess holes and electrons, respectively. Therefore, due to the carrier gradients, the electrons from the N-type part, and the holes from the P-type part drift towards the respective other regions. Therefore the N-type part is positively charged, and the P-type part is negatively charged. Therefore, an electric field must be generated.

Exercise 2.
At $t = 0$, consider the electron in an Hydrogen atom, described by the wave function

$$\psi = \frac{1}{6}\left(4\psi_{100} + 3\psi_{211} - \psi_{210} + \sqrt{10}\psi_{21(-1)}\right)$$

where ψ_{nlm} are normalized eigenfunctions of the energy operator for the Coulomb potential.

(1) Suppose we measure the energy of the electron, what energies can we measure, in eV and with what probabilities?
(2) Suppose we measure the energy corresponding to the lowest probability from above. Write $\psi(r, \theta, \varphi, t)$ after the measurement, in the form ψ_{nlm}. Is the electron in a stationary state?

Consider the original electron described by

$$\psi = \frac{1}{6}\left(4\psi_{100} + 3\psi_{211} - \psi_{210} + \sqrt{10}\psi_{21(-1)}\right)$$

(3) Suppose we measure the total angular momentum L^2 of the electron, what values can we measure, in terms of $\hbar$, and with what probabilities?
(4) Suppose we measure the total angular momentum L^2 of the electron, and the result is $2\hbar^2$. Write down $\psi(r, \theta, \varphi)$ after the measurement,

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in the form ψ_{nlm}. Hint: You should have three unknown parameters in your answer.

(5) Suppose that now we measure the angular momentum in the x-direction, L_x, of the electron above, right after the measurement done above. The result is $\hbar$. Find the three unknown parameters above. In your solution, assume that $\psi(r, \theta, \varphi)$ you found above is an eigenstate of L_x.

Solution 2.

(1) The energy level corresponding to ψ_{100} is

$$E_1 = -13.6\text{eV}$$

The energy level corresponding to ψ_{211} is

$$E_2 = -\frac{13.6}{4}\text{eV}$$

The energy level corresponding to ψ_{210} is

$$E_2 = -\frac{13.6}{4}\text{eV}$$

The energy level corresponding to $\psi_{21(-1)}$ is

$$E_2 = -\frac{13.6}{4}\text{eV}$$

Therefore, the probability of the measured energy being E_1 is

$$\begin{aligned} P(E_1) &= \left(\frac{4}{6}\right)^2 \\ &= \frac{16}{36} \\ &= \frac{4}{9} \end{aligned}$$

Therefore, the probability of the measured energy being E_2 is

$$\begin{aligned} P(E_2) &= \left(\frac{3}{6}\right)^2 + \left(\frac{-1}{6}\right)^2 + \left(\frac{\sqrt{10}}{6}\right)^2 \\ &= \frac{5}{9} \end{aligned}$$

(2) If the energy corresponding to the lowest probability, i.e. E_1 is measured,

$$\begin{aligned} \psi(r, \theta, \varphi, t) &= \psi(r, \theta, \varphi, 0)e^{-\frac{iE_1 t}{\hbar}} \\ &= \frac{2}{3}\psi_{100}e^{-\frac{iE_1 t}{\hbar}} \end{aligned}$$

Therefore, the electron is in a stationary state.

(3) The value of L^2 corresponding to ψ_{100} is

$$l(l+1)\hbar^2 = 0$$

The value of L^2 corresponding to ψ_{211} is

$$l(l+1)\hbar^2 = 2\hbar^2$$

The value of L^2 corresponding to ψ_{210} is

$$l(l+1)\hbar^2 = 2\hbar^2$$

The value of L^2 corresponding to $\psi_{21(-1)}$ is

$$l(l+1)\hbar^2 = 2\hbar^2$$

Therefore, the probability of the measured value of L^2 being 0 is

$$\begin{aligned} P(0) &= \left(\frac{4}{6}\right)^2 \\ &= \frac{4}{9} \end{aligned}$$

Therefore, the probability of the measured value of L^2 being $2\hbar^2$ is

$$\begin{aligned} P\left(2\hbar^2\right) &= \left(\frac{3}{6}\right)^2 + \left(\frac{-1}{6}\right)^2 + \left(\frac{\sqrt{10}}{6}\right)^2 \\ &= \frac{5}{9} \end{aligned}$$

(4)

$$\psi(r,\theta,\varphi) = \frac{1}{6}\left(A\psi_{211} + B\psi_{210} + C\psi_{21(-1)}\right)$$

(5)

$$\hat{L}_x = \frac{\hat{L}_+ + \hat{L}_-}{2}$$

Therefore,

$$\begin{aligned}
\hat{L}_x\psi = & \frac{1}{2}\hat{L}_+\psi + \frac{1}{2}\hat{L}_-\psi \\
= & \frac{1}{2}\frac{1}{6}\hat{L}_+\left(A\psi_{211} + B\psi_{210} + C\psi_{21(-1)}\right) \\
& + \frac{1}{2}\frac{1}{6}\hat{L}_-\left(A\psi_{211} + B\psi_{210} + C\psi_{21(-1)}\right) \\
= & \frac{1}{12}AA_{l(m+1)}\psi_{212} + BB_{l(m+1)}\psi_{211} + CC_{l(m+1)}\psi_{210} \\
& + \frac{1}{12}AA_{l(m-1)}\psi_{210} + BB_{l(m-1)}\psi_{21-1} + CC_{l(m-1)}\psi_{21(-2)} \\
= & \frac{1}{12}A\hbar\sqrt{l(l+1) - m(m+1)}\psi_{212} \\
& + \frac{1}{12}B\hbar\sqrt{l(l+1) - m(m+1)}\psi_{211} \\
& + \frac{1}{12}C\hbar\sqrt{l(l+1) - m(m+1)}\psi_{210} \\
& + \frac{1}{12}A\hbar\sqrt{l(l+1) - m(m-1)}\psi_{210} \\
& + \frac{1}{12}B\hbar\sqrt{l(l+1) - m(m-1)}\psi_{21(-1)} \\
& + \frac{1}{12}C\hbar\sqrt{l(l+1) - m(m-1)}\psi_{21(-2)} \\
= & \frac{1}{12}A\hbar\sqrt{2-2}\psi_{212} + \frac{1}{12}B\hbar\sqrt{2-0}\psi_{211} + \frac{1}{12}C\hbar\sqrt{2-0}\psi_{210} \\
& + \frac{1}{12}A\hbar\sqrt{2-0}\psi_{210} + \frac{1}{12}B\hbar\sqrt{2-0}\psi_{21(-1)} + \frac{1}{12}C\hbar\sqrt{2-2}\psi_{21(-2)} \\
= & \frac{1}{12}B\hbar\sqrt{2}\psi_{211} + \frac{1}{12}C\hbar\sqrt{2}\psi_{210} \\
& + \frac{1}{12}A\hbar\sqrt{2}\psi_{210} + \frac{1}{12}B\hbar\sqrt{2}\psi_{21(-1)}
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{1}{6}\left(\hat{L}_xA\psi_{211} + \hat{L}_xB\psi_{210} + \hat{L}_xC\psi_{21(-1)}\right) = & \frac{1}{12}B\hbar\sqrt{2}\psi_{211} + \frac{1}{12}C\hbar\sqrt{2}\psi_{210} \\
& + \frac{1}{12}A\hbar\sqrt{2}\psi_{210} + \frac{1}{12}B\hbar\sqrt{2}\psi_{21(-1)} \\
= & \frac{1}{6}\left(\frac{1}{\sqrt{2}}\hbar B\psi_{211}\right) \\
& + \frac{1}{6}\left(\frac{1}{\sqrt{2}}(C\hbar + A\hbar)\psi_{210}\right) \\
& + \frac{1}{6}\left(\frac{1}{\sqrt{2}}B\hbar\psi_{21(-1)}\right)
\end{aligned}$$

Therefore, as the measured value is $\hbar$,

$$\hbar = \frac{\hbar B}{\sqrt{2}A}$$
$$\hbar = \frac{\hbar(C + A)}{\sqrt{2}B}$$
$$\hbar = \frac{\hbar B}{\sqrt{2}C}$$

Therefore,

$$B = \sqrt{2}A$$
$$C + A = \sqrt{2}B$$
$$B = \sqrt{2}C$$

Therefore,

$$C = A$$
$$B = \sqrt{2}A$$

Therefore,

$$\psi(r, \theta, \varphi) = \frac{1}{6}\left(A\psi_{211} + \sqrt{2}A\psi_{210} + A\psi_{21(-1)}\right)$$

Therefore, normalizing,

$$1 = \frac{|A|^2}{36}(1 + 2 + 1)$$
$$= \frac{|A|^2}{9}$$

Therefore,

$$|A|^2 = 9$$

Therefore,

$$A = 3$$