The Creativity Decline: Evidence from US Patents *

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Abstract

Economists have long struggled to understand why aggregate productivity growth has dropped in recent decades while the number of new patents filed has steadily increased. I offer an explanation for this puzzling divergence: the creativity embodied in US patents has dropped dramatically over time. To separate creative from derivative patents, I develop a novel, text-based, measure of patent creativity: the share of technical terminology that did not appear in previous patents. I show that only creative and not derivative patents are associated with significant improvements in firm level productivity. Using the measure, I show that inventors on average file creative patents upon entry, and file derivative patents with more experience. I embed this life-cycle of creativity in a growth model with endogenous creation and imitation of technologies. In this model, falling population growth explains 27% of the observed decline in patent creativity, 30% of the slowdown in productivity growth, and 64% of the increase in patenting.

Keywords: Creativity, innovation, productivity, patents, text-as-data, demographics.

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1. Introduction

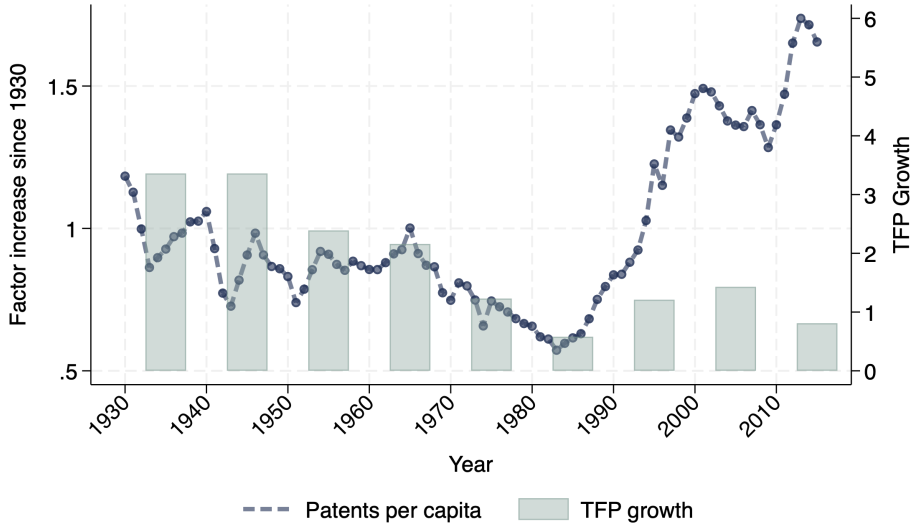
Patents are a commonly used and granularly available measure of intensity of innovation in an economy. Since 1980, US R&D expenditures, number of patents and citation-weighted patents have increased exponentially year-over-year.[1] In sharp contrast, productivity growth has either stagnated or slowed down (Bloom et al., 2020; Gordon, 2012; Syverson, 2017)(figure 1). Previous studies often attribute the slowdown in productivity growth along with the rise in R&D spending to a decline in research productivity. However, the origins of declining aggregate research productivity are still largely unknown because of a lack of widely-available measure of innovation output which tracks productivity growth at the micro and the macro level. In this paper, I reconcile the rise in patents with the slowdown in aggregate productivity growth by documenting a decline in creativity of patents, and I show that these patterns are partly driven by a lack of younger inventors.

This paper makes three main contributions to the literature on innovation and growth. First, I develop a novel text-based measure of patent creativity to distinguish creative from derivative patenting. Patent creativity uses the share of previously unused technical terminology in a patent to quantify the extent to which an invention comprises of novel features, products or processes. Second, I show that productivity growth at the firm and industry-level is only associated with creative and not derivative patenting. In the aggregate, creative patents have been declining over time in line with the pattern of aggregate productivity growth. Third, equipped with this measure, I propose a novel explanation for the rise in innovative activity and fall in creativity and productivity growth: the changing composition of inventors. I document that inventors tend to file creative patents when they first begin patenting and file more derivative patents as time goes on. This life-cycle is embedded in a growth model with creation and imitation of ideas. In this model, a decline in population growth and the subsequent decline in the rate of entry of new inventors alters the composition of inventors, shifting from creative to derivative. Through the lens of my model, I explain a significant chunk of all the aggregate patterns.

I now describe the three contributions.

[1] Appendix figure 1 shows the number of cite-weighted and influential patents per capita.

Figure 1: Productivity growth and patenting



Notes: This figure plots the total number of patents per capita (by year) along with aggregate productivity growth (by decade). Productivity growth after 1950 is calculated using multi-factor productivity series from the Bureau of Labor Statistics (BLS), and before 1950 from Gordon (2010).

I begin by developing the measure of patent creativity. A patent describes in detail the working or features of an invention, and to do so uses a range of technical terminology. To construct my measure, I decompose the text of each patent (beginning in 1930) into one-word (unigrams), two-word (bigrams) and three-word (trigrams) combinations, and subsequently remove those that are commonly-used in everyday English language to obtain a list of technical terms. I then classify these technical terms into ones that were previously unused in the five years before the patent was filed. This process yields the share of new technical terms in a patent that is my measure of patent creativity. For the baseline version, I consider the measure with bigrams, and I show that my empirical results are unchanged for a measure that uses all three - unigrams, bigrams and trigrams.

Before documenting the main empirical findings, I validate that my measure indeed captures the degree to which a patent comprises novel innovations. Through examination of top scoring creative patents, I observe that creative terminology in these patents captures the description of new products, processes and features. I undertake a series of validation exercises to further bolster this observation. First, I show that top scoring creative patents tend to cite recent academic research rather than previously filed patents. Second, top scoring patents also score higher on ex-post measures of patent quality. These patents receive more citations and higher valuation. Third, I show that creative patents are costlier investments for a firm, and that a creative patent is associated with about 24.9% higher R&D expenditure than a derivative patent. These findings together suggest that creative patents are costly investments that tend to originate from recent academic research and generate higher ex-post value and follow-on innovation than derivative patents.

Having validated my measure, I use it to learn about the relationship between the nature of patenting and productivity growth for firms and industries. For a panel of publicly-listed manufacturing firms between 1951 and 2015,[2] I find that only creative, and not derivative, patents are associated with higher productivity growth.[3] This relationship also holds within the firm (controlling for firm and year fixed effects). In other words, when firms file more creative patents, they tend to experience higher productivity growth. The estimates imply that one additional creative patent is associated with a 0.151 percentage point increase in productivity growth. Additionally, I find that this estimate is remarkably stable over time, when I calculate it for three separate episodes of US productivity growth (1950-1973, 1974-1993, 1994-2015). This relationship between creative and derivative patents, and productivity growth also holds at the industry level.

In the aggregate, I find a sharp contrast in aggregate patterns before and after 1980. During 1930-1980, the creativity of an average patent was largely unchanged as the number of new patents per capita filed every year declined by 44.5%. During 1981-2015, the number of patents per capita doubled and the average patent creativity declined by 47.4%. This decline in creativity of patents is large enough such that the increase in patenting during 1981-2015 is entirely driven by the increase in derivative patents. These time series patterns result in a correlation of 75.7% between creative patents per capita and productivity growth, while the correlation between patents per capita and productivity growth is -23.4%. This decline in creativity is also in contrast to existing measures of influence - such as citations or influence á la Kelly et al. (2021) - where the average influence of patents has been increasing

[2] Compustat accounts are only available starting in 1951.

[3] Productivity is calculated as revenue-based labor and total factor productivity (LPR, TFPR).

over time.

Using the measure, I examine a potential explanation for the decline in creativity and aggregate productivity growth. I use patents matched to inventors to show that inventors are about three-times (3.2x) as likely to file a creative patent on entry compared to later on in their career. This number falls to 1.52x for the second patent, 1.25x for the third patent, and so on. In the aggregate, I find that percentage of patents filed by first time inventors have dropped from about 50% in 1980 to 27% in 2016. This drop in share of patents by new inventors reflects the overall demographic shifts in the US. The share of 20-35 year olds in the US workforce dropped from about 47.6% to 28.5% during the same time period.

To evaluate how the demographic shift in the US affects aggregate creativity and productivity growth, I write down an endogenous growth model which combines two empirically motivated features - creativity is associated with technological improvements and the creativity life-cycle of inventors - with a structure of spillovers based on imitation. To incorporate these features, my model has two key ingredients. First, there are two types of inventors: creative inventors and derivative inventors. Derivative inventors make an imitation choice á la Perla and Tonetti (2014). They can either stick with their current technology or choose to imitate. When imitating, they are assigned a random technology from the economy. Meanwhile, creative inventors make continuous technology improvements. These improvements additionally expand the existing set of technologies available for imitation by derivative inventors. Second, the life-cycle of an inventor starts with entry. Upon entry, inventors are likely to be creative with their technology. At some point, these creative inventors lose their creativity, and become derivative. Derivative inventors can choose to imitate, and they can again be creative or derivative with their imitated technology.

I solve for the Balanced Growth Path (BGP) equilibrium of this economic environment. Along the BGP, the free entry condition - private gain from buying a creative entry draw versus cost of entry - determines the number of inventors per capita, while the rate of entry is determined by the population growth rate. As entry is a source of creative inventors, a fall in population growth therefore decreases the share of creative inventors by reducing the rate of entry. This reduction in creativity further worsens the set of technologies available for imitation by derivative inventors, and therefore decreases the rate of imitation. Through these two forces, a fall in population growth results in a shift in the composition of inventors towards fewer creative and more derivative inventors and leads to a fall in aggregate productivity growth. However, the number of inventors per capita is determined separately and depend on the value of buying a creative draw. As the economy slows down, the value of a creative draw relative to the cost increases, which results in an increase in the number of inventors per capita.

The model is calibrated to match two key moments: productivity growth differences between creative and derivative innovations, and the life-cycle of inventor creativity. Falling population growth in the model explains 27% of the decrease creativity in the economy, 30% of the decrease in productivity growth, and 64% of the increase in patenting or inventors per capita.

Related Literature. My efforts contribute to several different strands of literature within innovation and economic growth. A large set of studies use a patent's influence on follow-on innovation as their measure of its quality and contribution to the degree of technological change. Lerner and Seru (2022) summarize the method to use year-normalized citations as a measure of patent quality. Kelly et al. (2021) use text-similarity based on words to identify patents with the most influence on following patents.[4] These measures all suggest that the average influence of a patent has been increasing over time. Different from these, a handful of studies have developed measures of novelty/originality, e.g. Hall et al. (2001), Watzinger and Schnitzer (2019) and Arts et al. (2021).[5] Similar to these, my measure uses the share of new technical terminology to identify creative and derivative patents, and uses this distinction to show that only creative and not derivative patents are associated with

[4] My measurement approach has two differences from Kelly et al. (2021). First, Kelly et al. (2021) evaluate intellectual impact of a patent on follow-on innovation, and use data 5 years before and after patent filing, while I use only data prior to the patent to evaluate creativity. So, for example, I can evaluate the creativity of patent published in 2022 while they would need to wait until 2027 to observe all subsequently filed patents. Second, I use the share of new technical bigrams to define creativity of patents while they use word similarity where a patent is mapped into groups of around four thousand words.

[5] For example, Hall et al. (2001) use past citations, Watzinger and Schnitzer (2019) use words to capture novelty in a patent, Lanjouw and Schankerman (2004) use number of claims in a patent as a measure of quality. In especially related work, Arts et al. (2021) identify novel patents using total number of unique new words, bigrams and trigrams, and show that these patents are more likely to be linked with awards, such as nobel prizes.

productivity growth at the firm, industry and aggregate level.

A second strand of literature in economic growth, pioneered by Gordon (2012) and Bloom et al. (2020), documents that over the last fifty years the rise in patents and research spending was not accompanied by an increase in aggregate productivity growth. Syverson (2017) and Byrne et al. (2016) argue that more recently productivity growth declined. Several studies have proposed explanations for the slowdown in productivity growth (Aghion et al., 2019; De Ridder, 2019; Corhay et al., 2020; Akcigit and Kerr, 2018; Akcigit and Ates, 2021; Jones, 2020; Peters and Walsh, 2021; Hopenhayn et al., 2018; Karahan et al., 2019). I propose a novel channel which rationalizes both aggregate facts - the decline in productivity growth and the increase in innovations - through a decline in creativity of innovations driven by a lack of creative younger inventors.

A third strand of literature consists of models of endogenous and semi-endogenous growth (Jones, 1995; Peters and Walsh, 2021; Inokuma and Sanchez, 2023), which study the relationship between population growth and productivity growth. I construct an endogenous growth model where population growth slows down productivity growth by the changing composition of inventors towards less creative incumbent inventors. My framework builds on models of ideas and knowledge diffusion in Perla and Tonetti (2014) and Perla et al. (2021)[6], and adds creative innovations and population growth to this framework.

Finally, a fourth strand of literature studying science and innovation documents a significant role for age in innovation, e.g. Galenson and Weinberg (2000), Jones and Weinberg (2011), Koffi (2021) and Jones (2010). Others emphasize the role of age, and other characteristics in making executive and investment decisions (e.g. Adams and Ferreira (2009), Faccio et al. (2016), Weber and Zulehner (2010), Hirshleifer et al. (2012), Acemoglu et al. (2014)). This paper adds to these studies by documenting the role of age and new inventors in generating creative innovations.

The rest of the article is organized as follows: Section 1 introduces the measure of creativity, section 2 discusses the empirical findings, and section 3 builds the quantitative framework.

[6]Similar to Benhabib et al. (2021), Luttmer (2012), and Lucas Jr and Moll (2014).

2. Measuring Creativity in US Patents

My goal is to measure the share of previously unused or newly introduced technical terminology in a patent: patent creativity. Terminology in a patent details the functioning or features of the invention, and thus, with this classification exercise I aim to capture the degree to which an invention comprises of novel features, products or processes. My primary text-to-data source is a total of 5,018,824 patents granted by the US patent office (USPTO) to US inventors and filed between 1930 and 2016.[7,8,9] Out of these patents, I only consider 4,907,959 patents (dropping 110,865 patents - 2.21%) which have at least a total of 1000 words. I calculate patent creativity for each patent in two steps, first I decompose patent text into technical bigrams,[10] and then classify these technical bigrams as creative and derivative.

Identifying technical bigrams

For these 4,907,959 patents, I collect and parse title, abstract, brief and detailed description of the invention, and claim of invention. I then decompose each patent into two-word combinations or bigrams (e.g. 'cloud computing'),[11,12] which is the unit of my analysis. For computational ease and to focus on bigrams which are representative of the content of a patent, I only consider bigrams which appear at least twice in a patent, and those which do not contain filler words ('a', 'the', 'of', etc.).[13]

[7] All inventors who report filing from locations within the US are classified as US inventors.

[8] To avoid selection issues due to publication lags while maximizing coverage, I only keep patents filed during or before 2016. There are frequently large gaps between filing and granting dates of patents (Lerner and Seru (2022)) which leads to a tail off in patents towards the end of the sample period.

[9] Patents granted before 1976 are published as images with text translated from them using Optical Character Recognition Technology. As highlighted in Kelly et al. (2021), this text contains spelling mistakes. Appendix Figure 2 shows that these spelling mistakes are larger before 1920 than after. Therefore, I exclude patents granted before 1920.

[10] Later on, I re-calculate patent creativity using unigrams and trigrams, and check the robustness of my results.

[11] Recent studies using text-to-data approaches in economics have used bigrams as their unit of analysis (e.g. Hassan et al. (2019), Bloom et al. (2021)) guided by computation linguistics (e.g. Bekkerman and Allan (2004), Tan et al. (2002)) research which suggests that accuracy of text classification usually improves when bigrams ('machine learning') are used instead of words ('machine').

[12] In section D, I check the robustness of my empirical results to including unigrams and trigrams.

[13] These filler words are also called stop words and are the most frequent words in most collections of English language text.

To separately identify technical bigrams from non-technical bigrams, I use the Corpus of Historical American English (COHA) which is a decade by decade sample of English text sourced from magazines, articles, books and newspapers. I decompose all publications in COHA dated before 1900 into bigrams, and call these 'non-technical' bigrams. I then remove this list of non-technical bigrams from each patent. The remaining bigrams in a patent are technical bigrams.

Measuring Patent Creativity

Having decomposed the full text of a patent (p) into a list of technical bigrams ($b = 1, ...B_p$), I now describe construction of measure - patent creativity. For a patent p filed in year t, I classify each bigram as creative at time t if the bigram does not appear in any patent filed in the previous five years. Formally,

$$\text{Creative bigram}_{b,t} = 1\{b \notin \mathbb{B}_{t'=t-5,..,t-1}\}$$

where b is a bigram at time t, and $\mathbb{B}_{t'=t-5,..,t-1}$ is the set of bigrams which appear in patents filed between time $t-5, ...t-1$.

For each patent p, I measure patent creativity using the share of creative technical bigrams out of the total number of technical bigrams. Formally,

$$\text{Patent Creativity}_p = \frac{1}{|B_p|} \sum_{b=1}^{B_p} 1\{b \notin \mathbb{B}_{t'=t-5,..,t-1}\}$$

where b = 1,2...B_p are bigrams in patent p, and $\mathbb{B}_{t'=t-5,..,t-1}$ is the collection of all bigrams used in patents filed 5 years before the patent p. For example, if a patent contains two technical bigrams 'a' and 'b', 'a' is creative and appears 3 times, while 'b' is derivative and appears 7 times, then the creativity of this patent is 0.3.

To aid interpretation and account for level differences in use of new terminology across domains, I standardize patent creativity by the average in a technology class throughout the sample. Therefore, the average patent creativity score is 1 by definition. Appendix table 1 Panel A shows patent-level summary stats. On an average, a patent contains 5,744 bigrams out of which roughly 10.72% (or 616) are technical bigrams. Out of these, 7.62% (or 47) are classified as creative bigrams and the rest as derivative bigrams.

When aggregating patent creativity up to the firm or sector level, I define a dummy variable which denotes a patent as creative if the patent is in the top 10 percentile by patent creativity.[14] Therefore, creative and derivative patenting at the firm level is defined as:

$$\text{(1)} \qquad \begin{aligned} \text{Creative Patenting}_{i,t} &= \sum_{p \in P_{i,t}} 1_p\{\text{Patent Creativity}_p \text{ in top } 10\%\} \\ \text{Derivative Patenting}_{i,t} &= \sum_{p \in P_{i,t}} 1_p\{\text{Patent Creativity}_p \text{ in bottom } 10\%\} \end{aligned}$$

where $P_{i,t}$ is the set of patents applied in year t by firm i. Appendix Table 1 Panel C shows summary statistics for firm-year level observations.[15] On an average in my sample, firms in my sample file 19.73 patents per year out of which 1.52 or 13.83% are creative and rest are derivative.

Validation of measurement

I now describe the output of my measure and provide evidence to validate that patent creativity captures the degree to which an invention comprises of new features, products or processes.

The measure relies on identifying features of an invention using technical bigrams in a patent, and then classifying these technical bigrams into creative and derivative. To examine this through examples, Table 2 reports the list of 20 most creative patents which contain at least one creative bigram in their title to illustrate the usage of these bigrams in context. Creative technical bigrams are highlighted in bold. One of the top creative patent is about a 'grounding coupling' product, applied in 1951 by Monson Abraham Owen. In this case the title in entirety is a creative technical bigram and refers to a product which the inventor describes as the following:

> A device for bonding an *electric ground wire* to an *electric outlet box* or switch

[14] In section 3.D, conclusions remain unchanged when I perform robustness exercises with top 20 percentile instead of top 10 percentile.

[15] Patents are matched to publicly listed firms with a GVKEY identifier in Compustat.

box, and has for a general object the provision of a **grounding coupling**, or wedge, which is efficient as well as simple and economical to manufacture.

Examining the rest of the patents in this list, all creative technical bigrams identify labels for products, or features of products/processes described in these patents. By definition of a creative technical bigram, these products or features are the first to be labeled in such a way.

Having examined the inner workings of my measurement approach on a case by case basis, I probe the properties of my measure in the cross-section to reaffirm that the variation identifies patents containing new products, processes or features. In Figure 2, I show binned scatter plots and regression estimates from the following regression equation:

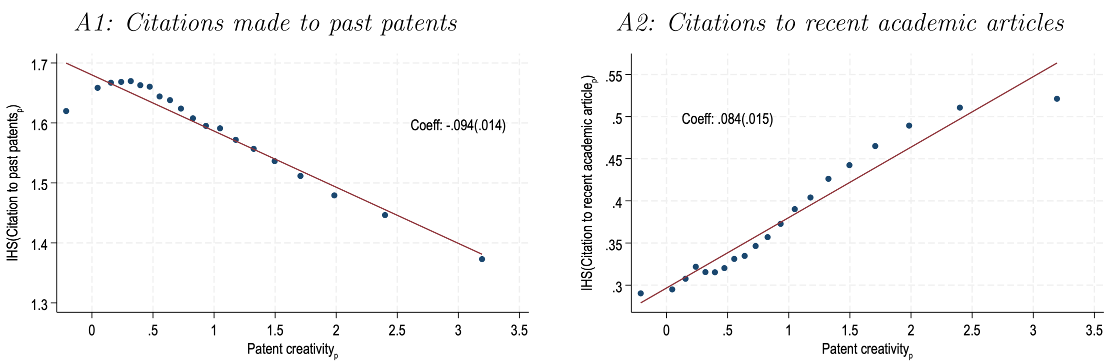
$$Y_p = \alpha + \beta \text{Patent Creativity}_p + \delta_{c_p,t_p} + \epsilon_t$$

where Y_p denotes various ex-ante and ex-post patent level metrics, δ_{c_p,t_p} denote 3-digit technology class and filing year fixed effects. These estimates use cross-sectional variation comparing patents within technology class and filing year cells. To begin with, one might expect patents with higher creativity to reference fewer previously filed patents and to rely on other non-patent or academic sources for their inventions. Figure 2 Panel A1 shows a binned scatter plot of citations made to previously filed patents against patent creativity. Panel A2 replicates the same binned scatter plot with citations made to recently published academic articles - articles which are published within the previous 5 years of patent filing. Both binned scatter plots focus on variation within technology class and year - control for technology class x year fixed effects. Panels A1 and A2 show that a patent with creativity equal to the sample average (recall that creativity is standardized in such a way that the sample average is 1) cites 9.4% fewer other previous patents and 8.4% more academic papers than a patent with zero creativity. These results emphasize that patent creativity identifies innovations which build more on outside sources (such as academic papers) and less on previous patents.

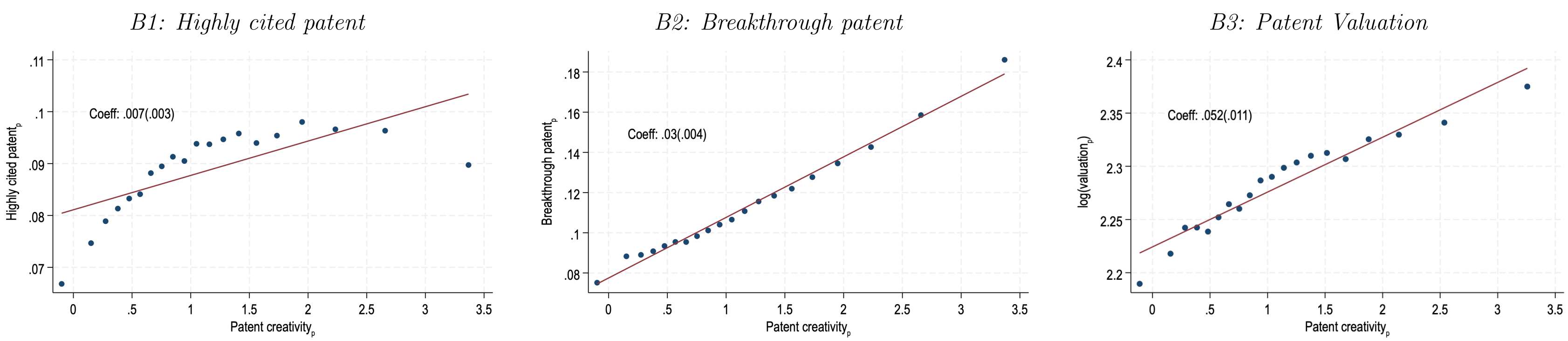
I next examine the association of patent creativity with other measures of ex-post patent quality - citations, influence and valuation. Figure 2 Panels B1 and B2 show binned scatter

Figure 2: Validation: Patent creativity, citations, influence and valuation

Panel A: Citations made to previous patents and academic papers (observed before filing)



Panel B: Patent outcomes (observed after filing)



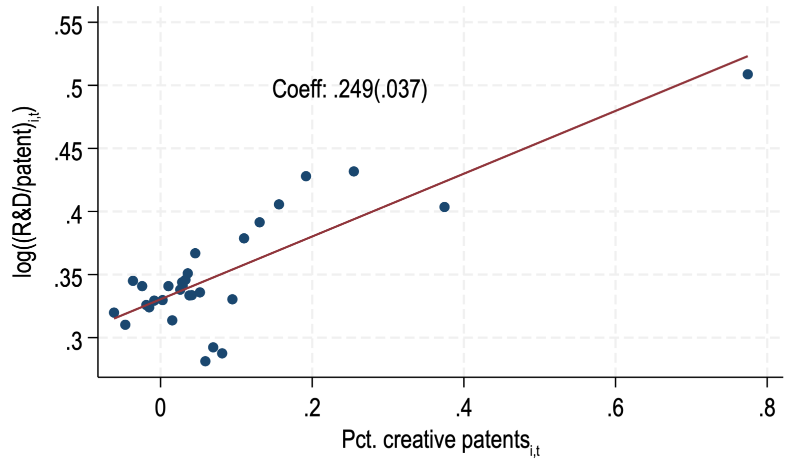
Notes: This figure plots binned scatter plots for citations made by the patent to previous patents and academic papers (in Panel A), and citations, influence, and valuation attributed to the patent (in Panel B) against patent creativity. Each binned scatter plots a regression line estimated at the patent level along with estimated coefficients and standard errors. Panel A shows plots of inverse hyperbolic sine (to account for positive values along with zeros) of citations made to previously filed patents and citations made to recent academic articles against patent creativity. Panel B shows plots of a dummy for highly cited patent (in B1) and breakthrough patent (in B2) along with log of patent valuation (in B3) against patent creativity. Highly cited patents are those which are in the top 10 percentile by number of citations received 10 years after filing. Breakthrough patent is a dummy for each patent using Kelly et al. (2021). Patent valuation for each patent is from Kogan et al. (2017). The binned scatter plot and estimates control for technology class and year fixed effects. Standard errors are clustered by technology class.

plots of a dummy which denotes whether a patent is in the top 10 percentile of (technology-class x year normalized) citations and a dummy which denotes breakthrough patent against patent creativity. As before, I control for technology class x year fixed effects. The coefficient in these binned scatter plots signifies that a one unit increase in patent creativity is associated with 0.7 percentage point increase (7.5 percent relative to the average) in probability that a patent is in top 10 percent of citations, 3 percentage point (37.5 percent relative to the average) increase in probability that a patent is classified as a breakthrough patent. In figure 2 panel B3, I find a positive and statistically significant relationship between patent creativity and patent valuation. A unit increase in patent creativity is associated with 5.2% increase in patent valuation. A creative patent - patent in the top 10 percentile of patent creativity - is associated with a 7.78% increase in patent valuation.

One might expect these quality benefits from creative patents to come at a higher cost to firms. To probe this further, I inspect the relationship between R&D spending, and creative and derivative patenting at the firm level. For firm i at time t, to calculate R&D expenditures per patent, I divide R&D expenditures in year t by the number of patents filed in year t. Figure 3 plots a binned scatter plot of R&D expediture per patent for a firm i in year t against the percentage of creative patents filed in that year. The figure includes additional controls for industry and year fixed effects. Including year fixed effects leverages cross-sectional variation, comparing firms in a year that file a higher share of creative patents versus others. I restrict my sample to a panel of manufacturing firms which file at least 10 patents. The coefficient 0.249 (s.e.=0.037) indicates that creative patents are associated with 24.9% higher R&D spending per patent than derivative patents.

In summary from these validation exercises, creative patents draw less from past patents and more from external sources. These patents also consistently score higher on measures of ex-post quality and value, and are linked to higher R&D costs. This evidence reaffirms that cross-sectional variation in patent creativity identifies patents with novel features, products, or processes.

Figure 3: Validation: R&D expenditure and creative patenting



Notes: The figure plots a binned scatter plot of log of R&D expenditure per patent against percentage of creative patents filed by firm i at time t. R&D expenditure per patent is calculated by dividing yearly R&D expenditure for firm i at time t recorded in Compustat by the number of patents filed by firm i in year t. The binned scatter plot controls for 3-digit NAICS industry and year fixed effects. The sample of firms includes publicly listed manufacturing firms which file at least ten patents between 1950 and 2015. The coefficient of the regression corresponding to the binned scatter plot is reported. Standard errors are clustered by firm.

3. Empirical Findings

This section presents the three main empirical results, the robustness of these empirical results, and comparison of these results with other measures of patent quality.

A. Creative patenting and productivity growth at the firm level

Next, I examine the relationship between productivity growth and creative patents at the firm level.

To this end, I use two measures of revenue-based productivity for firm i in year t: first, as my baseline, using sales per employee to calculate revenue-based labor productivity, and second as robustness using the Olley and Pakes (1996) control function approach to calculate revenue-based total factor productivity. While calculating the two measures of productivity, I exclude the years of two major recessions (1981-82 and 2007-08). To examine the relationship between productivity growth, and creative and derivative creative patenting, I use the following regression equation:

$$\Delta^5 log(\text{LaborProductivity})_{i,t} = \alpha + \beta_1 IHS(\text{Creative Patents})_{i,t} + \chi_{i,t} + \delta_i + \delta_t + \epsilon_{i,t}$$

where $\Delta^5 log(\text{LaborProductivity})_{i,t}$ is calculated as $(log(\text{Sales/Emp})_{i,t} - log(\text{Sales/Emp})_{i,t-5})/5$ and expressed in percentage terms. $IHS(\text{Creative Patents})_{i,t}$ is the inverse hyperbolic sine of creative patents filed by firm i in year t.[16] As IHS is functionally similar to logs, β_1 is interpreted as the semi-elasticity of productivity growth to creative patenting. $\chi_{i,t}$ denotes controls for past R&D expenditures between years $t-5$ and t, lags of overall industry sales[17], and polynomials of firm age[18]. All specifications include time fixed effects δ_t and build up to firm fixed effects δ_i. The sample consists of publicly listed manufacturing firms - in NAICS industries 31-33, which file at least ten patents between 1951 and 2015. In all, these firms account for 1.28 million patents out of 4.15 million patents filed by US inventors during this time period.

Table 1 presents the results. Panel A Column (1) shows a regression of productivity growth on the inverse hyperbolic sine of the number of patents filed by firm i in year t, while controlling for industry and year fixed effects. The coefficient is positive (coeff.= 0.054, s.e.=0.035), which implies that a 1% increase in patents is associated with 0.054 percentage point increase in productivity growth. However, columns 2 and 3 show that this positive relationship between patenting and productivity growth is driven by creative patenting. Column 2 replaces patents with creative patents and shows that the coefficient of interest increases from 0.054 (for patents, s.e.=0.035) to 0.180 (for creative patents, s.e.=0.057, statistically significant at 1%). Column 3 adds derivative patents to the specification and finds a coefficient of -0.015 (s.e.=0.043), while the coefficient of creative patenting is largely unchanged (coeff.= 0.194, s.e.=0.069). Columns 4, 5 and 6 show a more restrictive specification by adding firm fixed effects to the regression specification in columns 1, 2 and 3. When adding

[16] I use Inverse Hyperbolic Sine instead of logs to accommodate firms x year observations with zero patents. Appendix table 6 replicates the these results with $log(1 + \text{Creative Patents}_{i,t})$ in place of IHS.

[17] Calculated as the log of sum of sales of all Compustat firms in the industry of firm i at time t and time $t-5$.

[18] Firm age is calculated as year since initial public offering.

Table 1: Creative patenting and firm-level productivity growth

	LP Growth$_{i,t}$ (5-year log changes, in pct.)							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ihs(creative patents$_{i,t}$)		0.180***	0.194***		0.152**	0.229***	0.207***	0.189**
		(0.057)	(0.069)		(0.076)	(0.077)	(0.076)	(0.075)
ihs(derivative patents$_{i,t}$)			–0.015			–0.133**	–0.129**	
			(0.043)			(0.052)	(0.057)	
ihs(patents$_{i,t}$)	0.054			–0.067				
	(0.035)			(0.051)				
ihs(derivative patents - cite wt.$_{i,t}$)								–0.096**
								(0.049)
R^2	0.088	0.088	0.088	0.259	0.259	0.259	0.327	0.326
N	40,100	40,100	40,100	39,984	39,984	39,984	39,437	39,437
Controls	Y	Y	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Industry FE	Y	Y	Y	N/A	N/A	N/A	N/A	N/A
Firm FE	N	N	N	Y	Y	Y	Y	Y
Industry x Year FE	N	N	N	N	N	N	Y	Y

Notes: Table reports results from a regression of LP Growth$_{i,t}$, calculated using 5-year changes in log(Sale/Emp) ($log(LP_{i,t}) - log(LP_{i,t-5})$), on inverse hyperbolic sine (IHS) of yearly creative and derivative patenting. The sample is a yearly panel of manufacturing firms in Compustat, which file at least 10 patent during 1951-2015. Creative and derivative patenting is as defined in section 2. All specifications control for IHS of R&D spending by firm i between time $t-1$ and $t-5$, polynomials of year since initial public offering, and logs of industry level sales. Standard errors are clustered by firm.

firm fixed effects, the association between patents and productivity growth is negative but statistically insignificant (coeff.= -0.067, s.e.= 0.051). However, the association between creative patents and productivity growth continues to be positive (coeff.= 0.152, s.e.=0.076) and statistically significant at 5%. Column 6 shows my primary baseline specification with firm and year fixed effects with both creative and derivative patenting as independent variables. The coefficient of creative patenting in this specification (coeff.= 0.229, s.e= 0.077) implies that doubling the number of creative patents on average is associated with 0.229 percentage point increase in productivity growth. In other words, an additional creative patent is associated with 0.151 percentage point increase in productivity growth (0.229/1.521). On the other hand, an increase in derivative patents is associated with a drop in productivity growth (coeff.= -0.133, s.e.=0.052).[19,20] These results suggest that the association between creative patents and productivity growth is not driven by selection of firms, rather within-firm and across-time variation in creative patenting and productivity growth. In column 7, I find that these results are also not driven by industry wide trends in productivity growth and creative patenting by adding industry x year fixed effects along with firm and year fixed effects. Conclusions remain unchanged in column 8 which replaces derivative patenting with cite-weighted derivative patenting.

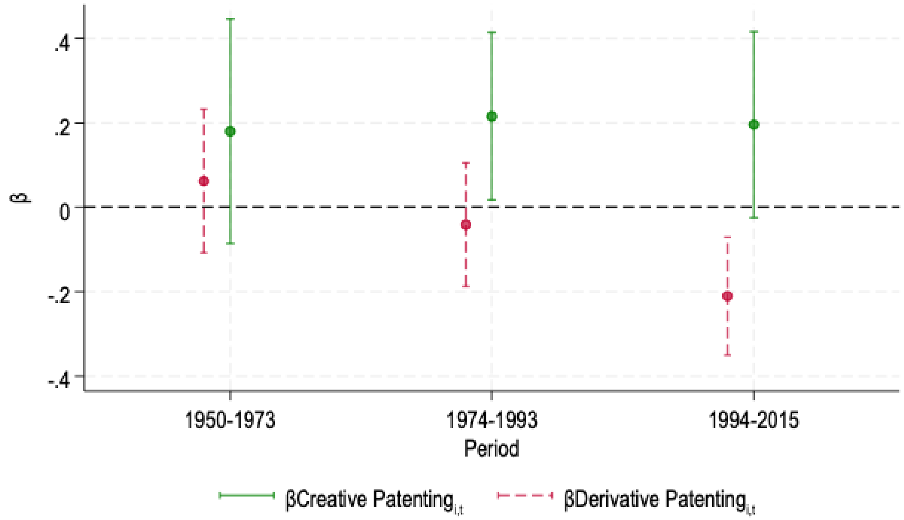
To understand the variation in the coefficients of creative patenting and derivative patenting over time, I interact the coefficients of creative and derivative patenting in my baseline specification in table 1 column 6 with dummies for three periods of US productivity growth á la Syverson (2017) - 1950-73, 1974-93, and 1994-2015. Figure 4 shows that the three estimated coefficients of creative patenting for these three periods are remarkably stable over time - 0.187 (s.e. = 0.134), 0.213 (s.e.=0.101), and 0.197 (s.e.=0.112), respectively. On the other hand, the estimated coefficients for derivative patenting are 0.059 (s.e.=0.086), -0.040 (s.e.=0.075), -0.211 (s.e.=0.071). Appendix table 3 replicates columns 6 and 7 in table 1 with three other firm level outcomes - total factor productivity growth, sales growth and employment growth. These outcomes are calculated in line with labor productivity growth in Table 1 as 5 year log changes. Results with total factor productivity growth are in nearly identical compared to the baseline coefficient in table 1 column 6. For sales growth and employment growth, I find that an additional creative patent is associated with 5.5 (0.238/0.043) times higher sales growth and 3.2 (=0.135/0.042) times higher employment growth than an additional derivative patent.

To compare these results against other metrics of novelty (Watzinger and Schnitzer, 2019; Lanjouw and Schankerman, 2004), Appendix table 4 shows results from a regression which replicates table 1 and adds patents in the top 10 percentile according to these metrics. Among these measures, only creative patenting is associated with firm-level productivity growth.

[19] Appendix Figure 3 shows binned scatter plots for coefficients of creative and derivative patenting for this specification.

[20] Appendix Figure 4 shows the differences in coefficients with leads and lags of productivity growth, and finds that the association is strongest at the time of filing the creative patent.

Figure 4: Patent creativity and firm level productivity growth, by period



Notes: The figure plots period by period coefficients for a regression of growth in labor productivity, and inverse hyperbolic sine of creative and derivative patenting at the firm level. These coefficients are calculated by interacting the coefficient of creative patenting and derivative patenting in Table 1 column 6 by a dummy for each of the three periods: 1950-73, 1974-1993, and 1994-2005. These periods are chosen as three separate episodes of US productivity á la Syverson (2017).

Industry level. One concern in interpreting the relationship between creative patenting and growth in revenue-based productivity at the firm level is that it could be driven by markups associated with new products rather than improvements in productivity. To address this, I examine the relationship at the industry level using multi-factor productivity data from the Bureau of Labor Statistics (BLS) between 1950 and 2015 for Standard Industry Classification (SIC) 2-digit industries, which is based on indexes of real quantity and cost measures of sectoral output and capital, labor, energy, materials, and purchased business services inputs[21].

To this end, I map patents into industries using mapping using mapping from Goldschlag et al. (2020). To examine the correlation between productivity growth and creative and derivative patenting, I use the following specification:

$$\Delta^5 log(TFP_{n,t}) = \alpha + \beta_C log(\text{creative patents}_{n,t}) + \delta_n + \delta_t + \epsilon_{n,t}$$

[21]For more details, refer to Gullickson and Harper (1987).

where $\Delta^5 log(TFP_{n,t})$ denotes 5-year changes in log of TFP for industry n in year t, $log(\text{creative patents}_{n,t})$ is the log of creative patents filed by firms in industry n in year t. Unlike firm level regressions, an industry-year level panel does not contain frequent zero patenting observations, allowing me to use logs for independent variables.

Table 2: Creative patenting and industry level productivity growth

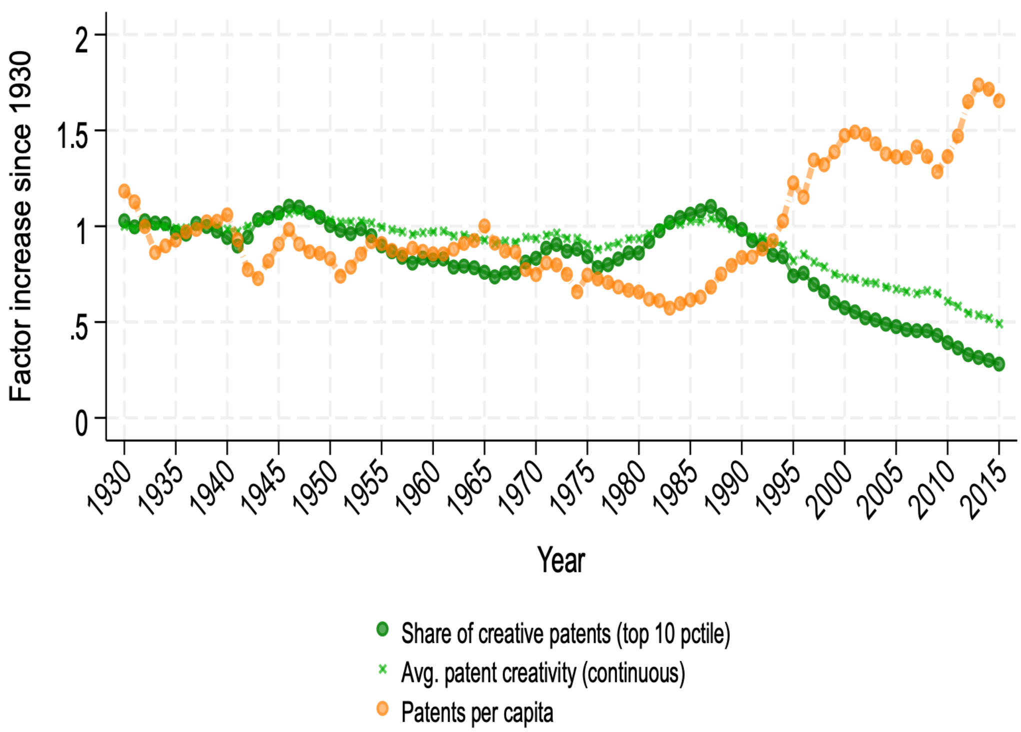
	TFP Growth$_{n,t}$ (5-year log differences, in pct.)					
	(1)	(2)	(3)	(4)	(5)	(6)
log(patents$_{n,t}$)	0.947***			0.140		
	(0.121)			(0.414)		
log(creative patents$_{n,t}$)		0.927***	1.020***		0.955***	2.195***
		(0.109)	(0.222)		(0.323)	(0.529)
log(derivative patents$_{n,t}$)			–0.110			–2.057***
			(0.249)			(0.677)
R^2	0.118	0.136	0.136	0.456	0.464	0.476
N	864	862	862	864	862	862
Year FE	N	N	N	Y	Y	Y
Industry FE	N	N	N	Y	Y	Y

Notes: Table reports results from a regression of MFP Growth$_{i,t}$, calculated using 5-year changes in log(MFP) ($log(MFP_{n,t}) - log(MFP_{n,t-5})$), on log of yearly creative and derivative patenting. Multi-factor productivity is as calculated by Bureau of Labor Statistics. The sample is a yearly panel of 2-digit SIC manufacturing industries which account for at least 500 patents between 1950 and 2015. Creative and derivative patenting is as defined in section 2. Standard errors are clustered by industry.

Table 2 shows the results. Columns 1, 2 and 3 present results without any controls, and columns 4, 5, and 6 replicate the first three columns with industry and year fixed effects. Column 1 shows that without any controls patents are positively associated with productivity growth at the industry level (coeff=0.947, s.e.=0.121). Columns 2 replaces patents with creative patents and finds a nearly identical coefficient (coeff.=0.927, s.e.=0.109). Column 3, which adds derivative patents along with creative patents shows that the association between patents and productivity growth is entirely driven by creative patents with a positive and statistically significant coefficient (coeff=1.020, s.e.=0.222). While derivative patents are negatively - with an insignificant coefficient - associated with productivity growth (coeff=-0.110, s.e.=0.249). Columns 4, 5, and 6 replicate the first three columns and add controls for industry and year fixed effects. Focusing on narrower variation within industry and time, I find that patents are not significantly associated with productivity growth (coeff=0.140, s.e.=0.414) and the coefficient drops to about one-fifth compared to column 1. However, the coefficient of creative patents in column 5 (coeff=0.955, s.e.=0.323) is nearly identical to the coefficient of creative patents without controls in column 2 (coeff.=0.927, s.e.=0.109). This coefficient suggests that doubling the number of creative patents on average increases productivity growth at the industry level by 0.927 percentage point.

B. The recent creativity decline

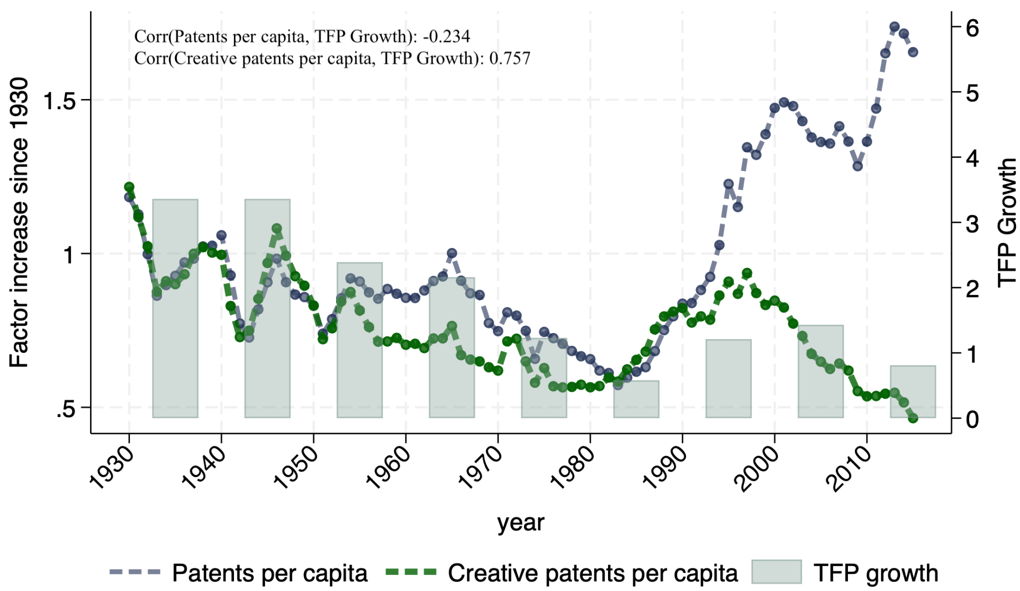
Figure 5: Patent creativity and number of patents, by year



Notes: This figure plots the share of creative patents, the average creativity of patents by filing year along with the number of patents per capita. All numbers are in normalized such that the average for 1930s is 1.

Having documented that only creative (and not derivative) patents are associated with firm and industry-level improvements in productivity, I examine the patterns in creative patenting at the aggregate level. Figure 5 plots the average patent creativity of patents by year of application along with the number of patents. The figure shows a clear difference patterns before and after 1980. During 1930-1980, the number of new patents filed per year per capita by US inventors decreased by 46.1% (from 0.310 in 1931 to 0.167 patents per capita in 1980). Meanwhile, the share of creative patents was largely constant - decreasing by 7.7% (from 14.30 percent in 1931 to 13.20 percent). However, after 1981, as the overall number of patents per capita have more than doubled (from 0.167 patents per capita to 0.421 patents per capita), the share of creative patents decreased by 69.7% (from 13.2 percent to 4.0 percent).[22]

Figure 6: Productivity growth, patenting and creative patenting



Notes: This figure plots patents per capita and creative patents per capita by year, and productivity growth by decade for the US. Productivity growth after 1950 is calculated using multi-factor productivity series from the Bureau of Labor Statistics (BLS), and before 1950 from Gordon (2010). The correlation between number of patents capita and productivity growth is -23.4%, and the correlation between creative patents per capita and productivity growth is 75.7%.

Figure 6 plots the overall volume of new creative patents per capita per year – top 10% of patents are classified as creative. Between 1930 and 1980, creative patents per capita decreased by 33.4% mirroring the pattern of total patenting per capita. From 1980 to 2015, the number of creative patents per capita follows a boom and a bust pattern[23],

[22] Average creativity of these patents decreased by from 1.168 in 1981 to 0.614 in 2015.

[23] Consistent with De Ridder (2019), Appendix Figure 6 shows that this boom and bust in creative patenting is entirely driven by computer manufacturing.

increasing by 60% between 1980 and 1995, and then decreasing by 48.4% - overall a decrease of 18.9% during 1980 and 2015. Productivity growth follows a boom and bust pattern as well - increasing by 0.49 percentage points between 1980s and 1990s, and then falling by 0.66 percentage points between 1990s and 2015. The correlation between the time series of creative patenting per capita and productivity growth is 75.7%, while the correlation between total patenting per capita and productivity growth is -23.4%.

In section A, I estimated a reduced form relationship between creative patenting at the firm and the industry level. Using the firm level estimates, the overall decrease in creative patenting of 58.5% between 1930 and 2015 can predicts a 0.134 percentage point (0.5845 * 0.230 = 0.134 p.p.) decline in productivity growth, which explains 5.3% of the overall decline in TFP growth (3.36-0.81 = 2.55 p.p.). Using the industry estimates, this decrease predicts a 0.558 percentage point decrease in productivity growth, which explains about 21.9% of the overall decline. In section 4, I link the reduced form relationship at the firm level to the aggregate productivity growth through the lens of a model.

In comparison to patterns of creativity, appendix figure 5 plots the average number of citations received and the average influence of a patent on follow-on patenting Kelly et al. (2021). The average patent in 2000s receives about four times as many citations as in 1980, and is about twice as influential than a patent in 1980s.

C. Inventor entry and creativity

Having observed a recent decline in creativity of patents, I examine a factor responsible for this decline. Using data on patents[24] matched to inventors, I use the following specification to understand the relationship between creativity of a patent against order of invention of the patent:

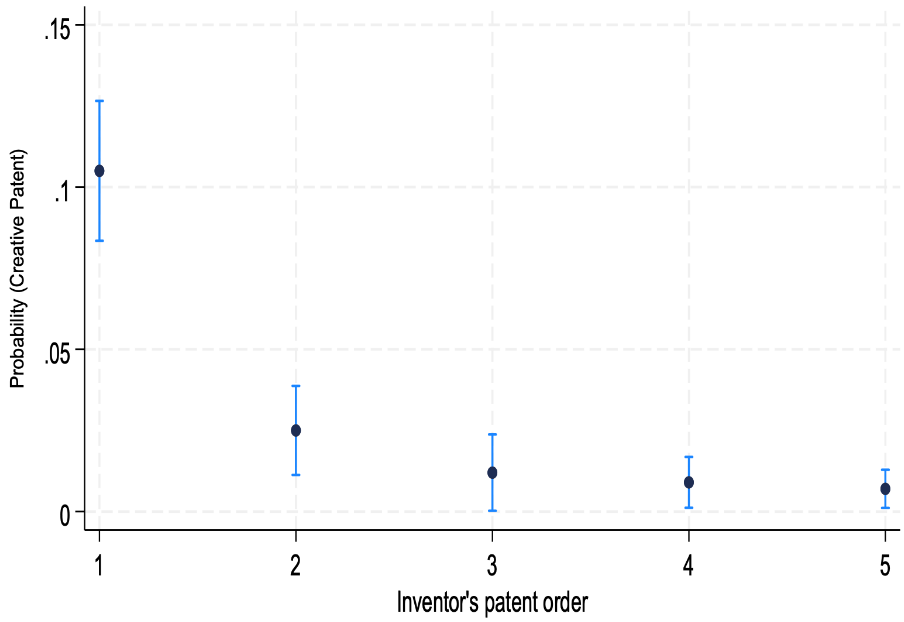
$$\text{Patent creativity}_p = \alpha + \sum_{i=1,\ldots,5} \beta_i \{\text{Pct. of inventors with patent order i}\} + \epsilon_p$$

Patent creativity of a patent p is regressed against the percentage of coauthors of the patent, who are first-time inventors, second-time inventors, and so forth, up to a maximum of five

[24]This data is only available for patents filed between 1981 and 2015.

instances. I restrict the sample to patents which have majority authors who file more than five patents over their lifetime to calculate the estimates separately from selection effects. In Figure 7, I plot these coefficients along with the confidence intervals. I find that an

Figure 7: Patent creativity by inventor's patenting order



Notes: The figure plots coefficients from a regression of Patent creativity$_p$ on the share of authors of the patent which are first-time inventors, second-time inventors, and so on. The figure also plots bands which denote 95% confidence intervals. The sample includes patents filed between 1981 and 2015. Standard errors are clustered by technology class.

inventor's first patent is on average their most creative one, and creativity falls as inventors file subsequent patents. In particular, the coefficient of the first patent is 0.105 (s.e.=0.011), which implies that the first patent by inventors is on average 2.2-times (or 0.105/0.0.048 = 2.18) more creative than the patents filed after the first five patents. Coefficient of the second patent is 0.0.025 (s.e.=0.007), the third patent is 0.012 (s.e.=0.006), the fourth patent is 0.009 (s.e.=0.004) and the fifth patent is 0.007 (s.e.=0.003). The constant, 0.048 (s.e.=0.002), denotes the average creativity of patents filed by inventors who have filed more than five patents. Appendix table 5 shows that this life-cycle of creative patenting within inventors is a consistent feature when adding controls for year and technology-class fixed effects. The table also shows that this is a consistent feature across the sample when splitting the sample between patents filed before and after 1995.

In the aggregate, appendix figure 8 plots the percentage of first-time inventors as a share of all active inventors by year of invention. Active inventor is one who files at least one patent in that year. The share of these inventors declined from 49.8% in 1981 to 26.3% in 2016. This trend is reflective of the trend in the overall drop in young people as a share of all college educated people in the US. To show this, the figure plots 20-35 year old college educated people as a share of all college educated people. This number dropped from 47.6% in the 1980 census to 28.5% in the 2016 census.

So far, I have shown three facts about creative patenting. First, at the firm level, only creative patents are associated with productivity growth. Second, patents have become less creative over time. Third, new inventors are more likely to file creative patents than incumbents. To quantify the changing composition of inventors on aggregate creativity and productivity growth, I use the structure of the growth model in the next section (section 4). Before describing the model, I check for robustness of my empirical results.

D. Robustness of empirical results

D.1. Phrase length - unigrams, bigrams and trigrams.

To understand how patterns of creativity change in response to changing length of phrase (bigrams, unigrams or trigrams), I re-calculate patent creativity using technical unigrams, technical bigrams, technical trigrams. Appendix table 1 shows summary statistics. Intuitively, for an average patent, the share of creative technical unigrams is 2% of all technical unigrams and the share of creative technical trigrams is 46% of all technical trigrams. Compared to bigrams (8.0% of technical bigrams are classified as creative), unigrams and trigrams tend to over and under assign creativity in a patent, respectively. Table 3 shows robustness of the three main empirical results against the patent creativity calculated using share of new technical unigrams, bigrams (baseline), and trigrams. The three panels correspond to the three main results - creative patenting and productivity growth at the firm level, creativity decline on the aggregate, and decline in inventor creativity. Panel A column 1 (coeff.=0.229, s.e.=0.077) shows baseline coefficients with creative patents calculated using bigrams, the same as in table 1 column 6. Column 2 shows the coefficient (coeff.=0.306, s.e.=0.077) with creative patents calculated using unigrams, bigrams and trigrams. In columns 3 and

4, I add creative patents calculated using unigrams and trigrams to my baseline calculated using bigrams, respectively. Creative patents calculated using bigrams continue to be significantly associated with firm level productivity growth, while those calculated using unigrams (coeff.=0.128, s.e.=0.078) and trigrams (coeff.=-0.046, s.e.=0.077) are not statistically significant and smaller in magnitude.

Panel B columns 1 and 2 show that the combined measure calculated using unigrams, bigrams and trigrams declines at 1.9% per year (coeff= -0.019, s.e.= 0.001), similar to the bigram-only based measure which declines at 2.2% per year (coeff= -0.022, s.e.= 0.001). Columns 3 and 4 show that measures of creativity calculated using unigrams and trigrams decline significantly over time as well. Panel C shows that the new entrant creativity premium is also significantly higher for all three measures of creativity.

D.2. Changing length of patents over time

One concern while interpreting the aggregate creativity decline as a meaningful decline in creativity of innovations is the rise in patent length over time. This is less of a concern for the other two findings which control for time fixed effects. The average patent filed in 1950 contained 3,303 words. In comparison, the average patent filed in 2015 contained 9,851 words. To check whether the decline in creativity is driven by increasing patent length, I recalculate creativity for each patent separately using title, abstract, detailed description and claims. These distinctions in sections of a patent are only available after 1981. During 1980-2015, the length of average detailed description section increased from 2,420 to 8,112 words, while the length of title and abstract changed from 9 words to 10 words and 102 to 98 words, respectively. Table 4 shows the average decline in creativity through a regression of average patent creativity calculated across patents for each year against the filing year. The decline in patent creativity is largest for abstracts (3.1% per year) and claims (2.5% per year), while the decline for titles (2.0% per year) and descriptions (1.9% per year) is slightly slower than the overall decline (2.2% per year).

Appendix table 6 shows robustness checks for firm level results with changing cut-offs for denoting creative patents from top 10 percentile to top 20 percentile, and changing inverse hyperbolic sine to log of one plus.

Table 3: Robustness: Phrase length

Panel A: Firm-level productivity growth				
	LP Growth$_{i,t}$ (5-year log changes, in pct.)			
	(1)	(2)	(3)	(4)
IHS(creative patents$_{i,t}$ - bigram, baseline)	0.229***		0.175**	0.251***
	(0.077)		(0.073)	(0.076)
IHS(creative patents$_{i,t}$ - all)		0.306***		
		(0.077)		
IHS(creative patents$_{i,t}$ - unigram)			0.128	
			(0.078)	
IHS(creative patents$_{i,t}$ - trigram)				–0.046
				(0.077)
R^2	0.259	0.260	0.259	0.259
N	39,984	39,984	39,984	39,984
Controls	Y	Y	Y	Y
Year FE	Y	Y	Y	Y
Firm FE	Y	Y	Y	Y
Panel B: Creativity decline between 1950-2015				
	Average Patent Creativity$_t$			
	(1)	(2)	(3)	(4)
	Bigram	All	Unigram	Trigram
Year	–0.022***	–0.019***	–0.018***	–0.005***
	(0.001)	(0.001)	(0.003)	(0.001)
R^2	0.947	0.911	0.679	0.692
N	35	35	35	35
Panel C: New entrant creativity				
	Creative Patent$_p$			
	(1)	(2)	(3)	(4)
	Bigram	All	Unigram	Trigram
Pct. (order = 1)$_p$	0.105***	0.107***	0.061***	0.078***
	(0.011)	(0.013)	(0.012)	(0.008)
Pct. (order = 2)$_p$	0.025***	0.029***	0.021***	0.011*
	(0.007)	(0.009)	(0.007)	(0.006)
Constant	0.048***	0.050***	0.027***	0.073***
	(0.002)	(0.002)	(0.003)	(0.003)
R^2	0.014	0.014	0.008	0.006
N	2,270,207	2,109,810	2,270,177	2,109,810

Notes: Table reports robustness of main empirical findings to varying phrase length with unigrams, bigrams and trigrams. Panel A replicates labor productivity regressions from table 1 column 6. Panel B shows the coefficient from a regression of average patent creativity by year against year of filing. Panel C shows the robustness for figure 7. Column 1 shows the baseline coefficients with bigrams. Column 2 replicates column 1 by re-calculating patent creativity with unigrams, bigrams and trigrams. Column 3 and column 4 separately add or show coefficients for creative patents calculated using unigrams and trigrams, separately.

Table 4: Robustness: Creativity decline and patent length

	Creativity decling during 1981-2015				
	Average Patent Creativity$_t$				
	(1)	(2)	(3)	(4)	(5)
	All text	Title	Abstract	Description	Claims
Year	–0.022***	–0.020***	–0.031***	–0.019***	–0.025***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
R^2	0.947	0.926	0.966	0.931	0.949
N	35	35	35	35	35

Notes: Table reports robustness of the second empirical finding - creativity decline - against increase in the length of patents. Column 1 shows the baseline decrease in creativity by regressing average patent creativity by year against year of patent filing. Column 2-5 replicates column 1 by calculating creativity of patents using title, abstract, description and claims, respectively. Standard errors are robust.

4. Creativity in a Model

In this section, I develop an endogenous growth model with creativity and subsequent spillovers of creative innovations. The model builds on the structure of spillovers in Perla and Tonetti (2014), and the model structure with continuous time is similar to Perla et al. (2021).

A. Preferences and production

Preferences. A representative household is endowed with labor L, which exogenously grows at g_L. Time is continuous and infinite horizon. Utility of the representative consumer is:

$$\bar{U}(t) = \int_t^\infty U(C(t)) \exp^{-\rho(\tilde{t}-t)} \, d\tilde{t}$$
$$s.t. \qquad C(t) \leq \frac{W(t)}{P(t)} \left(L_p(t) + L_E(t) + L_\chi(t)\right) + \Pi_t$$

Consumption good (C) is aggregated over micro varieties by a competitive final goods producer. ρ is the discount factor, and instantaneous utility is CRRA power utility.

The representative household maximizes the infinite stream of utilities subject to labor income from production in the form of real wage $\frac{W(t)}{P(t)}$, from labor supplied to production $L_p(t)$, entry $L_E(t)$, and imitation $L_\chi(t)$.

Production. The final good $Y(t)$ is produced by a competitive producer by aggregating a set of varieties $I(t)$, where each variety is produced by an inventor acting as an entrepreneur.

$$Y(t) = \left(\int_{v \in I(t)} y(v)^{\sigma-1} dv \right)^{\frac{1}{1-\sigma}}$$

Inventors produce using labor $y(v) = vl(v)$ as the only input in production and differ in their productivities Z. These productivities are distributed according to a distribution $\Phi(Z)$. Henceforth, I refer to a variety v by its productivity Z, as v and Z are indistinguishable.

While producing inventors earn the following profits as a result of optimally chosen quantities under monopolistic competition:

$$\Pi(Z) = \frac{1}{\sigma} \left(\frac{P(Z)}{P} \right)^{1-\sigma} \frac{Y}{P}$$

where $P = \left(\int p(Z)^{1-\sigma} d\Phi(Z) \right)^{\frac{1}{1-\sigma}}$ denotes the aggregate price index, and $p(Z)$ is the price charged by each inventor for their variety.

B. Innovation

At each instant, a mass of $I(t)$ inventors hold their respective technologies (Z) and earn profits from producing with Z, and an infinite mass of inventors make entry decisions. Out of the total mass of inventors $I(t)$, some are in creative state C $(\Omega_C(t))$, and the rest of them are in the derivative state D $(\Omega_D(t) = 1 - \Omega_C(t))$. The total mass of inventors along with the distribution across states are endogenously determined under equilibrium. The following discussion describes their path and choices in creative and derivative states.

In the derivative state, inventors choose either to continue with their current technology or imitate and sample another technology from the distribution of technologies in the economy.

Their flow value of holding technology Z at time t in state D is:

(2)
$$r(t)V_D(t,Z) = \underbrace{\Pi(t,Z)}_{\text{flow profits}} + \underbrace{\max\left(V_N(t) - V_D(t,Z) - \eta\frac{W(t)}{P(t)}, 0\right)}_{\text{abandon and search}} - \delta V_D(t,Z) + \underbrace{\partial_t V_D(t,Z)}_{\text{continuation value}}$$

where $r(t)$ is the interest rate, $V_N(t)$ is the expected value of searching for another technology and η is the search cost in terms of labor units. With probability δ, they exit the economy. When inventors choose to imitate, they are either assigned another derivative technology with probability $(1-\tau_C)$ and continue to be in the derivative state, or they are assigned a creative technology with probability (τ_C) and jump to the creative state.

$$V_N(t) = \tau_C \int_{M(t)}^{\infty} V_C(t,Z)d\Phi_C(Z) + (1-\tau_C)\int_{M(t)}^{\infty} V_D(t,Z)d\Phi_D(Z)$$

where Φ_C denotes the distribution of technologies over Z for creative inventors and Φ_D denotes the distribution of technologies over Z for derivative inventors at time t. Both of these distributions are endogenous and are determined under equilibrium, described in detail in following discussion.

This choice structure implies that at every instant a derivative inventor is evaluating their current valuation of holding technology Z, $V_D(t,Z)$, against the net present value of choosing a new technology $V_N(t) - \eta\frac{W(t)}{P(t)}$. Assuming $V_D(t,Z)$ is increasing in Z, the structure of this decision implies that there exists a cut-off productivity $M(t)$ below which all inventors choose to abandon their technology and search another one. This choice structure is also the same as in Perla and Tonetti (2014) except that I add the formulation of a creative state ($\tau_C > 0$). For convenience, I define the rate of growth of this threshold $M(t)$ as $g_m(t) = \frac{M^{'}(t)}{M(t)}$.

In the creative state, inventors make improvements on their productivity, and I assume that inventor's productivity in the creative state evolves according to a reflective-Geometric

Brownian Motion (GBM):

$$\frac{dZ_t}{Z_t} = \left(\mu + \frac{\nu^2}{2}\right) dt + \nu dW_t \qquad \text{if } Z > M(t)$$
$$\frac{dZ_t}{Z_t} = \max\left\{\left(\mu + \frac{\nu^2}{2}\right) dt + \nu dW_t, 0\right\} \qquad \text{if } Z = M(t)$$

where μ_t is the drift, ν_t is the volatility, W_t is Brownian motion, and $M(t)$ is the continuously evolving lower bound of the distribution of productivities. Reflective Brownian motion prevents productivity to be pushed below $M(t)$[25]. Along the BGP, this GBM will result in a power-tail distribution of productivities.

With some exogenous probability (α), a creative inventor jumps to the derivative state while holding on to their technology Z. The resulting law of motion of firm's valuation in the creative state evolves according to the following Hamilton Jacobi Bellman (HJB) equations in the region where $Z > M(t)$:

$$\text{(3)} \qquad rV_C(t,Z) = \underbrace{\Pi(t,Z)}_{\text{flow profits}} + \underbrace{(\mu + \frac{\nu^2}{2})Z\partial_Z V_C(t,Z) + \frac{\nu^2}{2}Z^2\partial_Z^2 V_C(t,Z)}_{\text{GBM}} + \underbrace{\alpha(V_D(Z,t) - V_C(t,Z))}_{\text{Derivative shock}} - \underbrace{\delta V_D(Z,t)}_{\text{Exit}} + \underbrace{\partial_t V_C(t,Z)}_{\text{Continuation value}}$$

Entry. At each instant, an infinite mass of inventors is waiting to enter. These entrants make a decision similar to derivative inventors. They pay a fixed cost $\eta_E \frac{W(t)}{P(t)}$ to enter, and then $\eta \frac{W(t)}{P(t)}$ to imitate a technology to produce with. The only difference for entrants versus derivative inventors is that they realize the creative state with probability τ_C^E, which is allowed to be different than the probability with which derivative inventors join the creative state τ_C.

$$\text{(4)} \qquad \text{Enter if} \qquad V_N^E - (\eta_E + \eta)\frac{W}{P} \geq 0$$
$$\text{where } V_N^E = \tau_C^E \int_{M(t)}^{\infty} V_C(Z) d\Phi_C(Z) + (1 - \tau_C^E) \int_{M(t)}^{\infty} V_D(Z) d\Phi_D(Z)$$

[25]Intuitively, this structure of creativity assumes that it is impossible to be creative on redundant technologies or those which are not operational in the economy.

Along the BGP, the number of inventors per capita (I/L) are constant and this free entry condition pins down the number of inventors per capita. On the other hand, the rate of entry depends on the rate of population growth.

C. Productivity distributions

The final part of the environment is to determine the evolution of productivity distributions. At each instant, inventors in the creative (derivative) state, $\Omega_C(t)I(t) = C(t)$ inventors, are distributed across technologies Z as $\Phi_C(t,Z)$ ($\Phi_D(t,Z)$). The structure of my economy results in the following Kolmogorov forward equations, which describe the evolution of the creative (Φ_C) and derivative productivity distributions (Φ_D) by summarizing the inflow and outflow of inventors at each point in the productivity distribution:

$$\begin{aligned}\partial_t \Phi_D(t,Z) = {} & \underbrace{(1-\tau_C^E)E(t)\frac{\Phi_D(t,Z)}{D(t)} + (1-\tau_C^I)S(t)\frac{\Phi_D(t,Z)}{D(t)}}_{\text{Entry and abandoning existing}} \\ & + \underbrace{\alpha\frac{C(t)}{D(t)}\Phi_C(t,Z)}_{\text{From creative state}} - \underbrace{\frac{S(t)}{N(t)}}_{\text{Abandoning}} - \underbrace{\delta\Phi_D(t,Z)}_{\text{Exit}}\end{aligned} \tag{5}$$

$$\begin{aligned}\partial_t \Phi_C(t,Z) = {} & \underbrace{\frac{\Phi_C(t,Z)}{C(t)}(\tau_C^E E(t) + \tau_C^I S(t))}_{\text{Entry and abandoning existing}} - \underbrace{\alpha\Phi_C(t,Z)}_{\text{Switch to derivative}} \\ & - \underbrace{(\mu - \frac{\nu^2}{2})Z\partial_Z\Phi_C(t,Z) + \frac{\nu^2}{2}Z^2\partial_Z^2\Phi_C(t,Z)}_{\text{GBM}} - \underbrace{\delta\Phi_C(t,Z)}_{\text{Exit}}\end{aligned} \tag{6}$$

The left hand side of each equation is the time evolution of derivative and creative distributions at productivity Z and time t. The evolution of derivative inventors is a combination of four terms. First term denotes two sources of additions to the inventors in the derivative state: $1-\tau_C^E$ share of new entry inventors ($E(t)$) and $1-\tau_C$ share of imitating inventors ($S(t)$). The second term reflects the third source, which is incoming inventors who get the derivative shock in the creative state. The third and last term reflects subtractions which is the set of imitating inventors at the threshold ($M(t)$). Evolution of the creative distribution is given by three terms. First denotes two sources of additions to the inventors in the creative state: τ_C^E share of new entry inventors and τ_C share of imitating inventors. The second term denotes subtractions in the form of inventors receiving a shock and moving to derivative state. The final term denotes the set of changes in the set of inventors at Z following GBM.

D. Computing a balanced growth path equilibrium (BGP)

Having described the environment, I now define and summarize the computation of the balanced growth path (BGP) equilibrium.

Assumption 1 *To compute the BGP, I assume that the initial derivative distribution at $t = 0$ is a Pareto Distribution.*

$$\Phi_D(t = 0) = 1 - M(0)Z^{-\alpha_{D,0}}$$

where $\alpha_{D,0}$ is a free parameter.

Definition of BGP. *A balanced growth path equilibrium consists of initial distributions for creative and derivative inventors: $\Phi_D(0), \Phi_C(0)$ with support $[M(0), \infty)$. A sequence of distributions $\{\Phi(t, Z)\}_{t>0}$, inventor imitation policies $M(t)$, variety prices $p(t, Z)$, labor allocation $l(t, Z)$, wages $\{W(t)\}_{t\geq 0}$, endogenous measure of varieties $\{\Omega_N(t), \Omega_C(t)\}_{t\geq 0}$, and a growth rate $g_m > 0$, such that:*

- *Given aggregate prices, and distributions:*
  - *Inventor valuations and imitation choices are given by equations 3 and 2 (HJB Block).*
  - *$M(t)$ evolves at a constant rate $g_m = \frac{M'(t)}{M(t)}$*
  - *$p(t, Z)$ and $l(t, Z)$ are optimal static choices.*
  - *The number of inventors per capita are consistent with free entry.*
- *Product and labor markets clear at each instant.*
- *The distribution of productivities (given by 6 and 5 - KFE Block) for creative and derivative inventors are stationary when scaled.*

E. Theoretical results

Before I solve for all endogenous variables in general equilibrium through computation, I solve the HJB block and KFE block separately. Using the solution for the KFE block, I prove three propositions to provide intuition to show how population growth and the entry rate determine aggregate creativity and growth. Throughout these propositions, I assume that exit probability $\delta = 0$. Full proof of these propositions is deferred to the model appendix. I discuss the results and intuition.

Lemma 1 *Along the balanced growth path, the rate of entry is determined by the rate of population growth.*

$$\frac{E(t)}{I(t)} = g_L$$

Proof: Along the BGP, I/L is constant, which implies that $\dot{I}/I = \dot{L}/L = g_L$.

Lemma 1 shows that the rate of entry is directly determined by population growth. However, the level of entry is determined separately through the free entry condition in equation 4.

Proposition 1 *Share of creative inventors (Ω_C) is determined by the rate of population growth (g_L) and the rate at which incumbents search for new technologies (S_N), and is given by the following expression:*

$$\Omega_C = \frac{\tau_C^E g_L + \tau_C S_N(g_L)}{g_L + \alpha}$$

where τ_C^E (τ_C^I) is the probability with which entering (existing) inventors realize the creative state. g_L is the population growth rate, which is also the rate of entry along the balanced growth path, S_N is the rate at which existing inventors abandon their technology and search for a new one.

Proof: See model appendix M. ·

Proposition 1 shows the direct and indirect effect of population growth on the share of creative inventors in the economy. Intuitively, along the balanced growth path, there are two sources of creative entry - entering inventors and adopting inventors. Therefore, the share of creative inventors is a weighted average of the rate of entry (g_L) and rate of imitation (S_N) multiplied by the probability with which they tend to enter the creative state (τ_C^E and τ_C). The denominator is the rate of entry (g_L) and the rate of exiting the creative state (α).

Proposition 2 *Along the balanced growth path, technologies of creative inventors are distributed as:*

$$\Phi_C(Z) = 1 - M(t)Z^{-\alpha_C}$$

where $\alpha_C = 1 - 2\frac{\mu - g_m}{\nu^2}$.

Technologies of derivative inventors are distributed as:

$$\Phi_D(\infty, z) = \zeta(\Omega_C)\ \Phi_C(\infty, z) + (1 - \zeta(\Omega_C))\ \Phi_D(0, z)$$

where

$$\zeta(\Omega_C) = \frac{\alpha_D \alpha \tau_C \Omega_C}{(\alpha_D - \alpha_C)(g_L(1 - \tau_C^E) + (1 - \tau_C)\Omega_C \alpha)}$$

Given that $\tau_C < 1$, $\zeta(\Omega_C)$ *is an increasing function in* Ω_C.

Proof: See model appendix M. ·

Proposition 2 shows that the distribution of technologies (Z) in the economy is more heavy tailed - weighted towards technologies of creative inventors - if there are more creative inventors in the economy. Intuitively, at any point in time α percentage of creative inventors become derivative while holding on to their technology. This process leads to a derivative distribution which is a mixture of initial derivative distribution with tail parameter (α_D) and the creative distribution with tail parameter (α_C).

Proposition 3 *Along the balanced growth path, aggregate productivity growth is given by:*

$$g_{Y/PL_p} = \underbrace{\frac{1}{\sigma - 1} g_L}_{\text{Growth due to increase in \# varieties}} + \underbrace{g_m}_{\text{Growth in productivity per variety}} \tag{7}$$

where g_m *is given by:*

$$g_m = \frac{g_L(1 - \tau_C^E) + \alpha(1 - \tau_C)\Omega_C}{\alpha_D \tau_C (1 - \Omega_C)}$$

Proof: See model appendix M. ·

Finally, proposition 3 shows that along the balanced growth path, productivity growth of the economy increases monotonically in the share of creative inventors. Intuitively, a higher share of creative inventors encourages more derivative inventors to abandon their current less productive technologies, thereby, accelerating the shift in the lower bound of technologies ($M(t)$) and accelerating aggregate productivity growth.

Computing equilibrium. To compute the full general equilibrium through computation, I use an iterative search algorithm to look for a value of labor in production (L_p) and a productivity growth rate (g_m) which solves together the Kolmogorov forward equations (5 and 6) and HJB equations (2 and 3), along with balancing total labor supply in the economy. More details about the solution process are in the model appendix.

F. Calibration

In this section, I describe calibration of the model to fit a combination of moments from US macroeconomic data, and moments highlighted in the empirical results in sections A and C. I fit the model to data points of the US economy in the early 1980s. The following table highlights calibrated moments:

Key parameters

Parameter	Value
Discount factor per instant (ρ)	0.01
Elasticity of substitution (σ)	3.15
Initial derivative tail parameter (α_D)	4.99
Drift of creative GBM ($\mu + \frac{\nu^2}{2}$)	0.0023
Volatility of creative GBM (ν_C)	0.12

Parameter	Value
Updating cost (η_S)	4.44
Entry cost (η_E)	10.89
Imitator creativity probability (τ_c^I)	0.13
Entry creativity probability (τ_c^E)	0.44
Creative-derivative transition probability (α)	0.18

This calibration is a combination of substituting values from the literature and moments from creativity and macroeconomic data. I calibrate the discount factor (ρ), elasticity of substitution (σ), and tail parameter of starting derivative distribution (α_D) exactly as specified in Perla et al. (2021). Along with these, I choose exit probability $\delta = 0.02$. The remaining seven parameters are calibrated to exactly match the following seven moments.

Aggregate productivity growth. To calculate aggregate US productivity, I use growth in multi-factor productivity provided by the Bureau of Labor Statistics (BLS) and take averages by decade. The average multi-factor productivity growth between 1951-60 was 2.39%.

Percentage of inventors/entrepreneurs in the Economy. I calibrate this moment using the percentage of business owners/managers out of total employed workforce in the US, which is 12.5%.

Creative inventors's/firm's productivity growth. To calculate this, I use the estimated relationship between a creative patent and productivity growth in table 1. I set the value of μ to exactly match 0.0023. Derivative inventors by assumption do not experience any productivity improvements.

Inventor's creativity life-cycle dynamics. This moment is calculated using life-cyle dynamics of creativity in figure 7. I use two moments from this figure. First, the entry creativity premium or the probability that an entrant files a creative patent (25.7%). Second, with each additional patent, the probability of filing a creative patent decreases by 18%. These two moments pin down the value of the value of p_C^E and α.

Percentage of creative patents. The percentage of creative patents in 1950 plotted in figure 5. In the model, I assume that creative and derivative inventors each file one patent in each period. Therefore, the share of creative patents in the model is $\frac{C}{C+D}$. This moment is used to pin down p_C^U

Creative versus derivative patent valuation. To calculate this moment, I use coefficients from the relationship between creative and derivative patenting and stock returns in section 2. The coefficients in the OLS estimates imply that on an average creative patents are worth 7.8% more than derivative patents. This moment helps pin down the tail parameter of the creative state. Having calibrated the drift parameter of creative entpreneurs, this moment pins down the volatility of GBM of the creative entpreneurs.

Having calibrated the ten parameters, I set population growth to 1.7%, as in the 1950s. At the end of this calibration exercise, I obtain a creative tail parameter of 3.96 ($\leq \alpha_{D,0} =$ 4.99), and a derivative distribution of productivities with weight of 41.1% on the creative distribution.

Table 5: Model Results - Falling population growth

	(1) 1950*	(2) 2010	(3) Chg. 1950-2010	(4) Chg. in data	(5) Pct. Explained
Population growth rate	1.72%	0.79%	(in pct.)	(in pct.)	(in pct.)
Productivity Growth (g_m)	2.39%	1.92%	-19.67	-66.11	29.8
Pct. Creative Inventors (Ω_C)	13.09%	10.80%	-17.47	-64.79	27.0
Innovators/Capita (I/L)	12.50%	19.44%	55.54	86.43	64.3
Adopters (S_N)	9.49%	8.00%	-15.68		
Avg. Creative Valuation (rel to $V(Z=M)$)	1.52	1.68	10.65		
Avg. Derivative Valuation (rel to $V(Z=M)$)	1.41	1.41	-0.03		

Notes: *** denotes matched cross-section**. The table reports results from decreasing population growth described in section 4. The model is calibrated to match productivity growth, percentage creative entrepreneurs and entrepreneurs per capita during 1950s (col 2), when population growth rate was 1.72%. In column 2, the results are shown for changing population growth to 0.79% (population growth during 2010s).

G. Counterfactuals

I now discuss how the economy, presented and calibrated as above, responds to declining population growth. To answer this, I compute the stationary BGP equilibrium of this economy for changing values of US population growth decade by decade (from 1950 to 2010). The BGP equilibrium for the 1950s is exactly matched. For these different equilibria corresponding to decade by decade values of population growth, I calculate share of creative inventors, productivity growth and inventors per capita. Share of creative inventors and inventors per capita are model analogs for average creativity and total number of inventors in the economy. Table 5 presents the results from a declining population growth in the model while keeping all other parameters constant. US population growth declined from 1.7% in the 1950s to 0.8% in the 2010s. This change in the economy results in a 17.5% (from 13.1% to 10.8%) decline in share of creative inventors, which explains 27.0% of the creativity decline in the data. Given the decline in creativity and the relationship between creativity and productivity growth (proposition 3), productivity growth slows down by 19.7% (2.4% to 1.9%), which explains about 29.8% of the overall productivity growth slowdown in the data. Finally, the model explains the third data pattern as well - there is an increase in the overall number of inventors per capita by 55.5% (from 12.5% to 19.4%), which explains about 64.3% of the overall increase in inventors per capita in the data. To understand why there is an increase in the number of inventors per capita, the table shows the average creative and average derivative valuation. As productivity growth in the model slows down, the creative tail becomes fatter (recall $\alpha_C = 1 - 2\frac{\mu - g_m}{\nu^2}$). Therefore, the average valuation of an inventor's firm in the creative state increases - by 10.7% (from 1.5% to 1.7%). As entrants are more likely to enter the creative state, the level of entry increases. In this economy, pushing the level of entry higher does not guarantee an increase in the share of creative inventors. The share of creative inventors depends on the rate of entry and churn, which in turn is determined by population growth and the rate of imitation - both of which decrease. The percentage of inventors who adopt declines by 15.7% (from 9.5% to 8.0%).

5. Conclusion

In this paper, I argue that the recent increase in patenting, that is accompanied by a decrease in productivity growth, is largely driven by a decline in patent creativity. To do this, I develop a novel measure of patent creativity, which captures the extent to which a patent contains new products, processes or features. Using this measure, I document that only creative patents are associated with improvements in productivity growth at the firm, the industry and the aggregate level. Equipped with this measure and using moments in a growth model, I show that composition of inventors - entrants vs incumbents - is a key determinant of aggregate creativity.

This paper proposes a novel measure and using this measure argues that the decline in aggregate research productivity is due to changing composition of inventors over time. This opens a range of avenues for further research into effective policies to improve the composition of researchers towards more creative ones.

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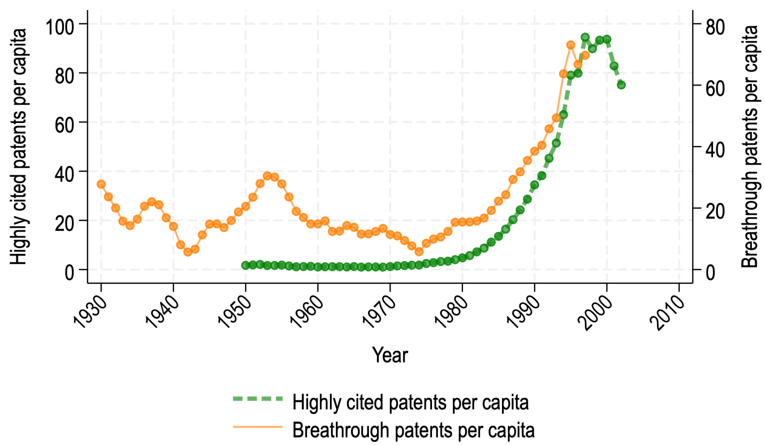
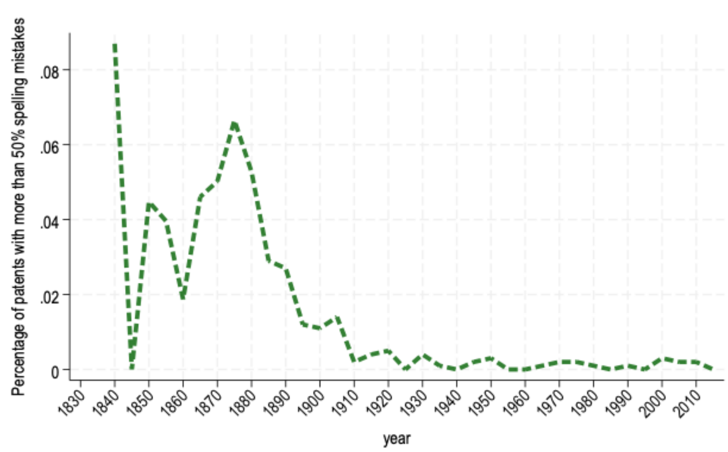
Appendix tables and figures

Appendix Figure 1: Highly cited and breakthrough patents per capita



Notes: This figure plots the total number of breakthrough patents and highly cited patents per capita (by year). Highly cited patents are those which are in the top 10 percentile by number of citations received 10 years after filing. Breakthrough patents per year are calculated using Kelly et al. (2021).

Appendix Figure 2: Percentage of patents with major spelling mistakes



Notes: The figure plots the percentage of patents with mistakes in technical bigrams by year. To construct this figure, I randomly sample patents by year and run all words in technical bigrams through python spell checker. The figure then plots the percentage of randomly sampled patents where at least 50% of technical bigrams contain spelling mistakes by year.

Appendix Table 1: Summary Statistics

	Mean	SD	p1	p50	p99	N
Panel A: Patent Level						
Technical bigrams$_p$	616	1,151	47	370	4258	4,907,959
Creative technical bigrams$_p$	47	241	0	20	408	4,907,959
Share creative technical bigrams$_p$	0.08	0.08	0.00	0.06	0.36	4,907,959
Patent creativity$_p$	1.00	0.89	0.00	0.79	3.64	4,907,959
Share creative technical unigrams$_p$	0.02	0.05	0.00	0.00	0.26	4,907,574
Share creative technical trigrams$_p$	0.46	0.25	0.00	0.47	1.00	4,465,695
Length$_p$	5,744	7,237	1,120	3,899	33,025	4,907,959
Valuation$_p$(KPSS)	15	41	0	6	150	1,986,014
Panel B: Firm Level						
Patenting$_{i,t}$	19.73	78.34	0.00	2.00	321.00	65,104
Creative patenting$_{i,t}$	1.52	6.19	0.00	0.00	29.00	65,104
Derivative patenting$_{i,t}$	18.20	73.50	0.00	2.00	298.00	65,104
LP Growth$_{i,t}$ (in pct.)	5.38	6.90	-9.67	5.06	21.47	41,499
Sales Growth$_{i,t}$ (in pct.)	9.71	14.30	-29.95	8.40	55.44	44,940
Emp Growth$_{i,t}$ (in pct.)	3.69	10.06	-16.64	2.66	26.26	41,499
TFP Growth$_{i,t}$ (in pct.)	4.07	6.04	-8.90	3.78	17.86	39,493
R&D per patent$_{i,t}$	41.63	988.59	0.04	2.22	266.90	37,451

Notes: This table shows summary statistics - mean, standard deviation, 1st percentile, median, 99th percentile and number of observations - for the patent level (in panel A) and firm level (in panel B) data used in empirical results.

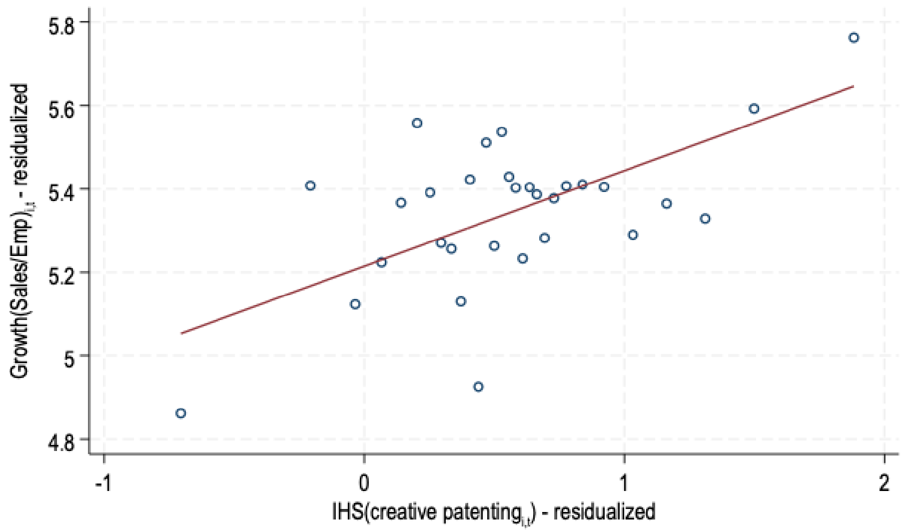
Appendix Table 2: Examples of the most creative patents

Application year	Patent ID	Title (creative bigrams in bold)	Assignee	Creativity
1951	2698226	**gascarburetting apparatus**	Reynold A Peduzzi (Individual)	11.94
1954	2710381	**grounding coupling**	Monson Abraham Owen (Individual)	11.54
2003	7873161	small hardware implementation of the **subbyte function** of rijndael	NXP B.V.	11.48
1944	2424318	**equalized singlephase** traction motor	Westinghouse Electric Corp	11.33
1954	2876587	**candlestick flower** arranger	Candlestick Flower Arranger	11.22
2008	7791894	compact **rackmount storage** server	Oracle America, Inc	10.95
1941	2324646	**lymphogranuloma venereum antigen** and method of preparing it	ER Squibb and Sons LLC	10.92
1966	3445820	overlapped and **underlapped accessing** of **multi-speed storage** devices		10.88
1937	2136131	**distemper vaccine** and method of preparing the same	Robert Green (Individual)	10.74
1967	3715754	**tethered chaff strand countermeasure** with trailing end kite	J Parry (US Air Force)	10.55
1960	3101960	**closet ring**	Ryan R Danescu (Individual)	10.53
1979	4230334	**cantilevered medial trailer**	Vern D. Mabry, Jr. (Individual)	10.51
1950	2626374	**locomotive regeneration** control	Westinghouse Electric Corp	10.49
1987	4733424	retractable and **slidable doormat housing**	David E. Gurkin (Individual)	10.48
1990	5251129	method for **automated morphological** analysis of word structure	General Electric Company	10.37
2000	6535593	system and method for billing communications services provisioned on demand in **converging telecommunications** network	Simplified Development Corp.	10.36
2003	7034007	low **adenosine antisense** oligonucleotide compositions kit method for treatment of airway disorders associated with bronchoconstriction lung inflammation allergyies **surfactant depletion**	East Carolina University	10.34
1943	2383325	**opposedangle brush** holder	Westinghouse Electric Corp	10.33
1938	2204064	**equine encephalomyelitis vaccine**	Joseph W Beard (Individual)	10.27
1947	2508198	**busdifferential relay**	Westinghouse Electric Corp	10.26
1950	2753663	production of **hybrid seed** corn	Research Corp	10.13
2010	8335699	method of **financing unfunded liabilities**	Retirement Benefit Solutions, LLC	10.12
1981	4355306	**dynamic stack** data compression and decompression system	International Business Machines Corporation	10.12
1997	6105064	system for **placing packets** on network for transmission from **sending endnode** to **receiving endnode** at times which are determined by window size and metering interval	Novell, Inc.	10.10
1993	5628517	**contracting-expanding selfsealing cryogenic** tube seals	Florida Atlantic University	10.00

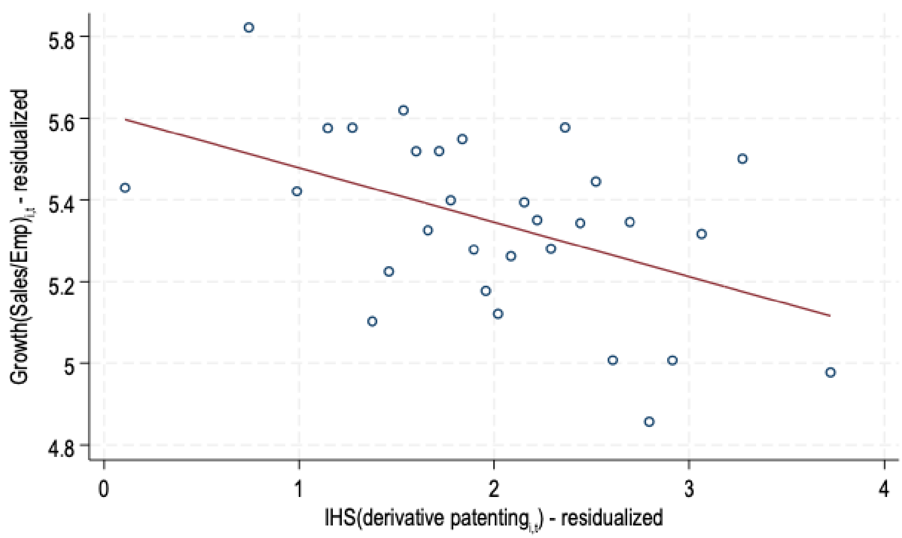
Notes: The table shows filing year (in column 1), patent id (in column 2), title (in column 3), the assignee entity (column 4) and patent creativity (in column 4) for the top 25 patents by patent creativity which have at least one creative technical bigram in their title. Patent creativity is normalized to the average in a 3-digit CPC technology class.

Appendix Figure 3: Binned scatter plot: Creative and derivative patenting, and firm level productivity growth

Panel A: Creative patenting and firm-level productivity Growth



Panel B: Derivative patenting and firm-level productivity Growth



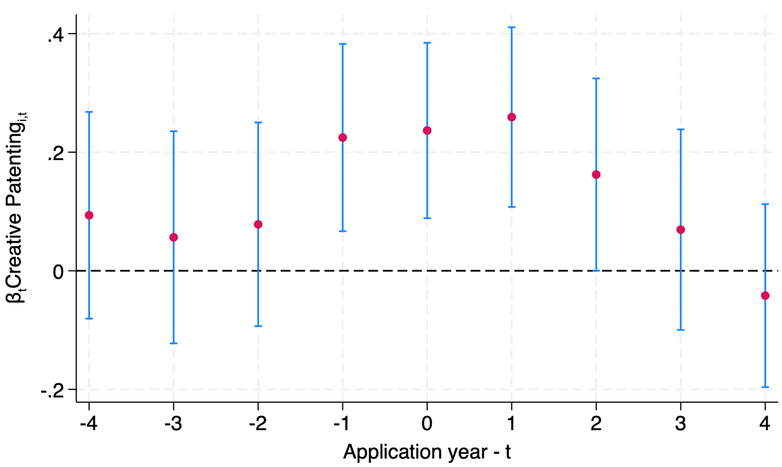
Notes: This figure shows binned scatter plots of growth in labor productivity$_{i,t}$ against inverse hyperbolic sine of creative and derivative patenting. These binned scatter plots correspond to coefficients of creative and derivative patenting in table 1 column 6.

Appendix Table 3: Creative patenting, TFP Growth, Sales Growth and Employment Growth

	TFPGrowth$_{i,t}$ (in pct.)		SalesGrowth$_{i,t}$ (in pct.)		EmpGrowth$_{i,t}$ (in pct.)	
	(1)	(2)	(3)	(4)	(5)	(6)
ihs(creative patents$_{i,t}$)	0.232***	0.197***	0.358**	0.339**	0.205*	0.228**
	(0.064)	(0.063)	(0.140)	(0.149)	(0.113)	(0.115)
ihs(derivative patents$_{i,t}$)	–0.043	–0.049	0.790***	0.867***	0.763***	0.848***
	(0.046)	(0.050)	(0.113)	(0.124)	(0.085)	(0.091)
R^2	0.263	0.337	0.402	0.453	0.406	0.470
N	38,087	37,635	43,347	42,873	39,984	39,437
β(1 creative patent)	0.152	0.130	0.235	0.222	0.135	0.150
β(1 derivative patent)	–0.002	–0.003	0.043	0.048	0.042	0.047

Notes: Table reports results from a regression of TFPGrowth$_{i,t}$ (in columns 1 and 2), SalesGrowth$_{i,t}$ (in columns 3 and 4), and EmpGrowth$_{i,t}$ (in column 5 and 6) on inverse hyperbolic sine (IHS) of yearly creative and derivative patenting. TFP growth, sales growth and employment growth are calculated using 5-year log changes in TFP, Sales and Employment. TFP is calculated using Olley and Pakes (1996) control function approach. The sample is a yearly panel of manufacturing firms in Compustat, which file at least 10 patent during 1951-2015. Regression specification is same as table 1 columns 6 and 7. Standard errors are clustered by firm.

Appendix Figure 4: Patent creativity and firm level productivity growth: timing of response



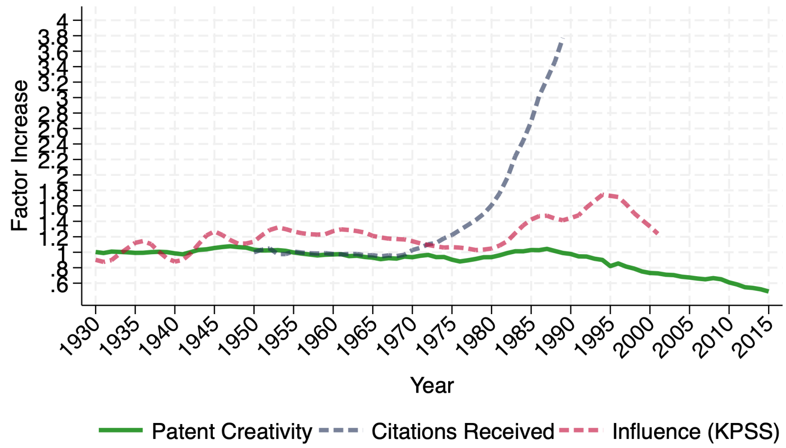
Notes: This figure shows coefficients corresponding to a regression of labor productivity$_{i,t}$ against inverse hyperbolic sine of creative and derivative patenting with varying leads and lags of labor productivity$_{i,t+k}$, where k is the lag denoted on the x-axis. These coefficients and confidence intervals correspond to the specification in table 1 column 6.

Appendix Table 4: Comparison: Other measures of novelty

	(Sales/Emp) Growth$_{i,t}$ (5-year differences)			
	(1)	(2)	(3)	(4)
ihs(creative patents$_{i,t}$)	0.307***	0.282**	0.217**	0.272***
	(0.108)	(0.110)	(0.097)	(0.098)
ihs(top 10 pct.$_{\cdot i,t}$ - KSW (2021))	−0.039			
	(0.113)			
ihs(top 10 pct.$_{\cdot i,t}$ - bck sim. KPST (2021))		0.019		
		(0.118)		
ihs(top 10 pct.$_{\cdot i,t}$ - new unigrams)			0.117	
			(0.106)	
ihs(top 10 pct.$_{\cdot i,t}$ - # of claims)				0.018
				(0.089)
R^2	0.259	0.259	0.216	0.216
N	20,414	20,414	24,803	24,803
Controls	Y	Y	Y	Y
Year FE	Y	Y	Y	Y
Firm FE	Y	Y	Y	Y

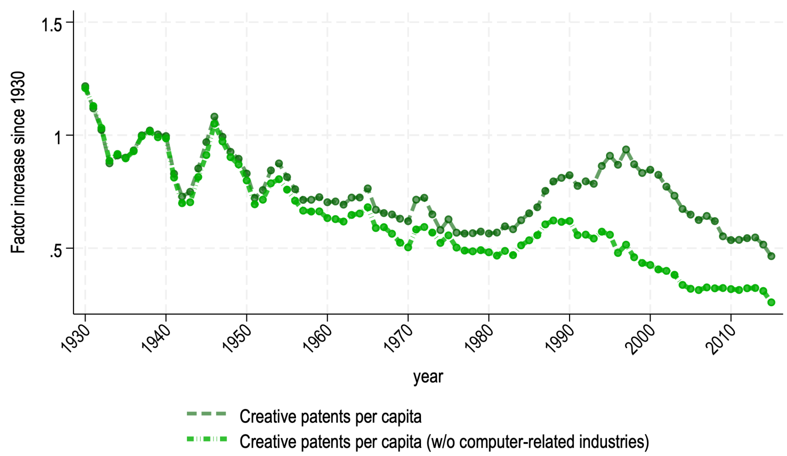
Notes: Table reports results from a regression of LPGrowth$_{i,t}$, calculated using 5-year changes in log(LP) ($log(LP_{i,t}) - log(LP_{i,t-5})$), against inverse hyperbolic sine (IHS) of yearly creative and patenting, and other measures of patent originality. In col 1, number of academic citations Watzinger and Schnitzer (2019); in col 2, backward looking text similarity Kelly et al. (2021); in col 3, percentage of new unigrams in a patent; in col 7-8, number of claims of invention in a patent Lanjouw and Schankerman (2004). Similar to a creative patent, for a continuous measure of originality, an original patent is defined as one which has originality in the top 10 percentile by technology class. Samples differ in these regressions because of differences in availability of these measures.

Appendix Figure 5: Comparison of patent creativity against citations and influence



Notes: This figure plots average patent creativity along with average citations and average influence. Citations are calculated as the number of citations received within 10-years of filing by patents, and the average influence as calculated by Kelly et al. (2021).

Appendix Figure 6: Creative patents per capita with and without Computer and IT related industries



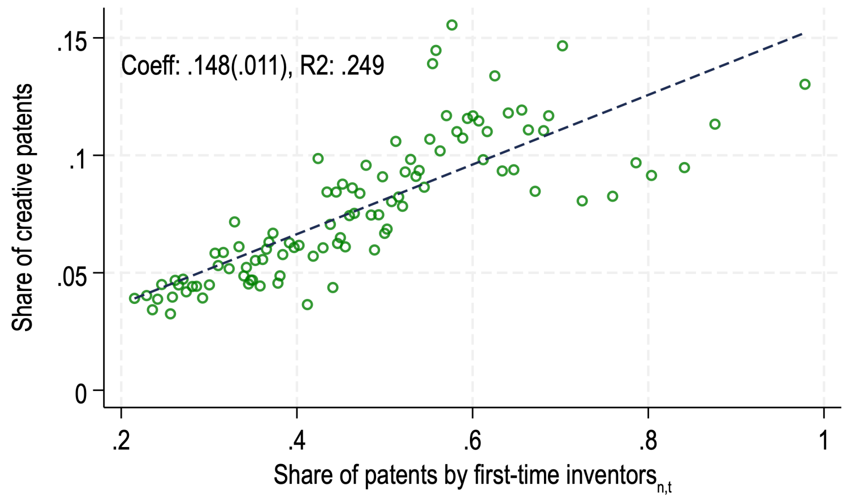
Notes: This figure plots the overall number of creative patents per capita with and without patents filed under CPC 3-digit technolology classes, which are part of the Computer and IT related manufacturing industries.

Appendix Table 5: Patent creativity by inventor's patenting order

	Creative Patent$_p$				
Sample:		All		1981-1995	1996-2015
	(1)	(2)	(3)	(4)	(5)
Pct. (order = 1)$_p$	0.105***	0.087***	0.073***	0.096***	0.063***
	(0.011)	(0.009)	(0.006)	(0.011)	(0.005)
Pct. (order = 2)$_p$	0.025***	0.011*	0.002	–0.007	0.007**
	(0.007)	(0.006)	(0.003)	(0.005)	(0.003)
Pct. (order = 3)$_p$	0.012*	–0.001	–0.008***	–0.018***	–0.003
	(0.006)	(0.005)	(0.003)	(0.003)	(0.003)
Pct. (order = 4)$_p$	0.009**	–0.001	–0.008***	–0.010***	–0.007***
	(0.004)	(0.003)	(0.002)	(0.003)	(0.002)
Pct. (order = 5)$_p$	0.007***	–0.001	–0.008***	–0.006***	–0.009***
	(0.003)	(0.002)	(0.001)	(0.002)	(0.001)
Constant	0.048***	0.053***	0.057***	0.100***	0.048***
	(0.002)	(0.004)	(0.001)	(0.003)	(0.001)
R^2	0.014	0.025	0.047	0.055	0.027
N	2,270,207	2,270,207	2,270,141	481,402	1,603,677
Filing year FE	N	Y	Y	Y	Y
Technology Class FE	N	N	Y	Y	Y

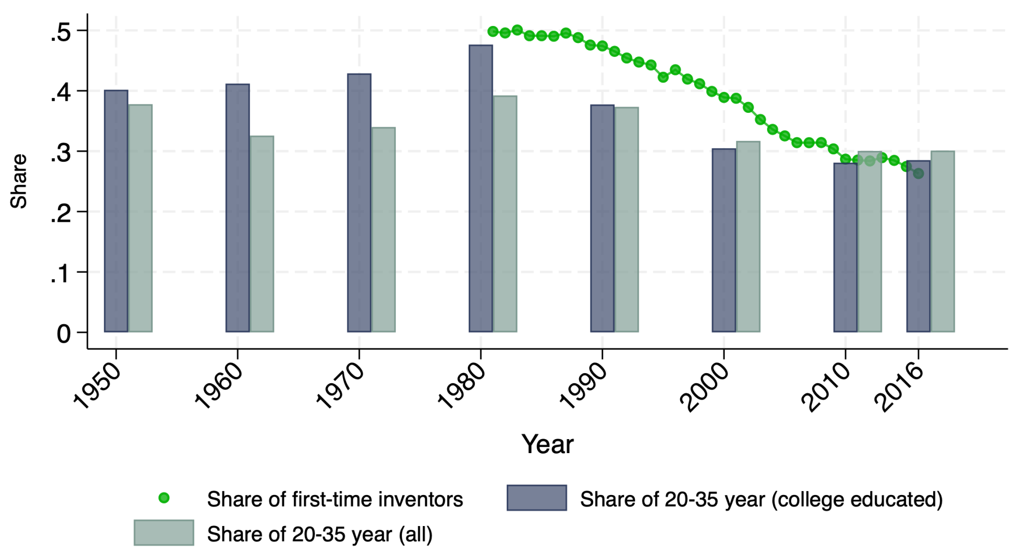
Notes: The table shows results from a regression of Patent creativity$_p$ on the share of authors of the patent which are first-time inventors, second-time inventors, and so on. Column 1-3 includes patents filed between 1981 and 2015. Columns 4and 5 split the sample into two periods: 1981-1995 and 1996-2015. Standard errors are clustered by technology class.

Appendix Figure 7: Share of first time inventors and share of creative inventors



Notes: This figure shows a binned scatter plot of share of creative patents against the share of patents by majority first-time author for a yearly panel of 2-digit SIC manufacturing industries from 1981 to 2015. Share of patents by majority first-time authors at the industry level is calculated by aggregating over patents which have a majority of authors as new authors, which have never patented before.

Appendix Figure 8: Share of first-time inventors and young persons



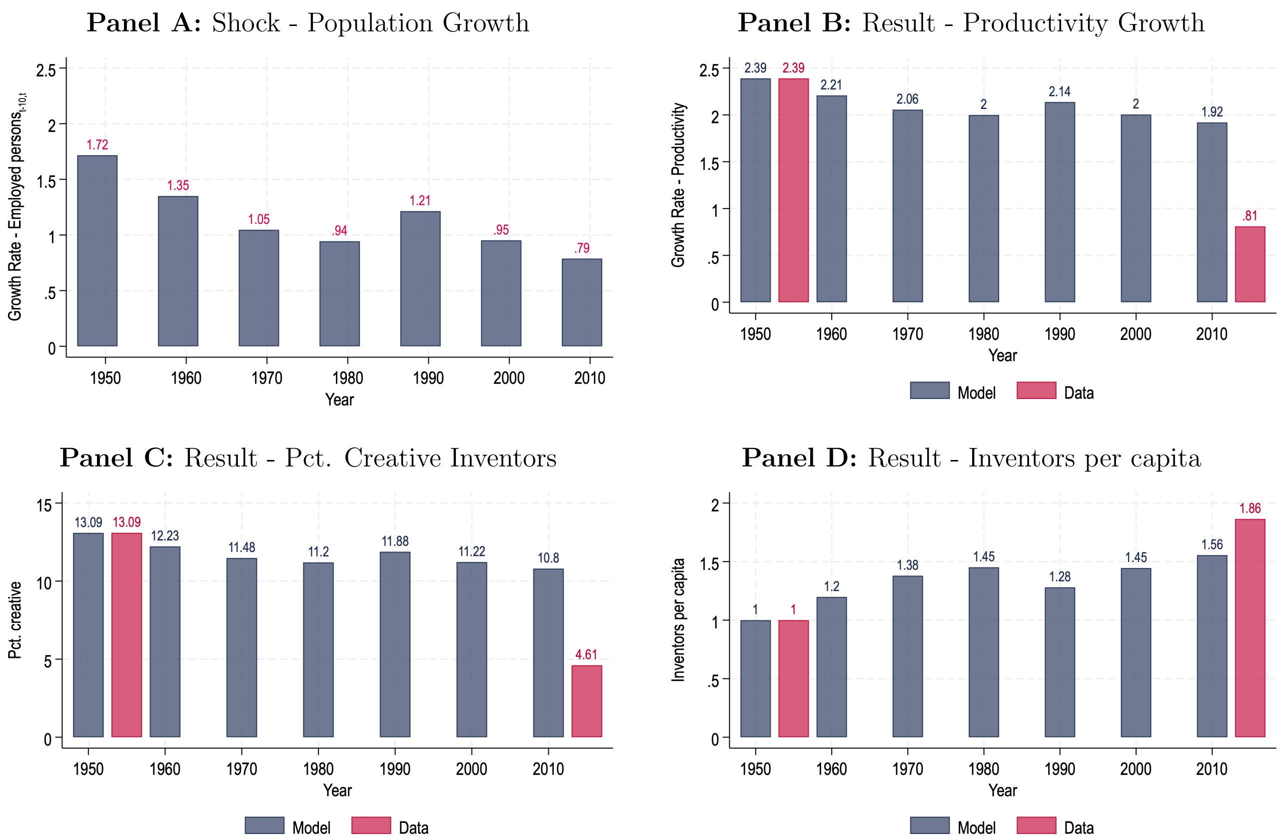
Notes: This figure shows the share of patents filed by first-time inventors, the share of 20-35 year olds in US workforce, the share of 20-35 year olds in overall US college educated workforce. The share of 20-35 year olds in US workforce and the share of 20-35 year olds in overall US college educated workforce are calculated using decade by decade samples of the American Communities Survey.

Appendix Table 6: Robustness: Cut-off and functional form

	LP Growth$_{i,t}$ (5-year log changes, in pct.)		
	(1)	(2)	(3)
IHS(creative patents$_{i,t}$ - bigram, top10% baseline)	0.229***		
	(0.077)		
IHS(creative patents$_{i,t}$ - bigram, bot90% baseline)	–0.133**		
	(0.052)		
IHS(creative patents$_{i,t}$ - bigram, top20%)		0.139**	
		(0.066)	
IHS(creative patents$_{i,t}$ - bigram, bot80%)		–0.129**	
		(0.054)	
log(1 + creative patents$_{i,t}$ - bigram, top10%)			0.316***
			(0.102)
log(1 + creative patents$_{i,t}$ - bigram, bot90%)			–0.172***
			(0.063)
R^2	0.259	0.259	0.259
N	39,984	39,984	39,984
Controls	Y	Y	Y
Year FE	Y	Y	Y
Firm FE	Y	Y	Y

Notes: Table reports results from a regression of labor productivity growth on creative and derivative patents, defined for the purpose of robustness using top 10% cutoff as baseline in columns 1 and 3, and top 20% cut-off in column 2. Column 3 replaces inverse hyperbolic sine with log (1+x). All specifications control for IHS of R&D spending by firm i between time $t-1$ and $t-5$, and polynomials of year since initial public offering. Standard errors are clustered by firm.

Appendix Figure 9: Model Counterfactual



Notes: The figure plots the shock or parameter change fed exogenously to the model (g_L) in panel A. In panels B, C, and D, the figure shows the change in the economy to productivity growth, percentage of creative inventors, and the number of inventors per capita, respectively in response to the change in population growth.

Data Appendix

I download full text of 3,171,775 USPTO patents from the USPTO bulk downloads website in the digital format granted post 1976. To these I append patents before 1976 provided by Kelly et al. (2021). These are patents which have at least one inventor who reporting their location within the US. To begin with, I remove all punctuation, numbers and text which contains tables.

A. Mapping patents to firms (GVKEYs) and industries

I use a combination of patent to GVKEY (Compustat firm identifier) mapping provided by by Kogan et al. (2017), Dorn et al. (2020) and Arora et al. (2021). I given priority to Kogan et al. (2017) where possible because Dorn et al. (2020) use web search techniques and rely on recent firm information for the match (rather than during the time of patent filing).

B. Other patent characteristics

I download the following patent characteristics from PatentsView (USPTO patent data website): citations, technology class, filing date, grant date, number of claims, inventor name, inventor location, disambiguated inventor id, inventor gender, and continuation status. I download the citation file which contains patent by patent pairwise citations. From this data, I calculate all citations received by a patent within 10 years of filing, and normalized these citations by the average in technology class x year. Technology class is 4 digit CPC technology class.

C. Calculating TFP from Compustat accounts

I calculate total factor productivity (TFP) using Olley and Pakes (1996) method. I use the OP method option under PRODEST STATA package. To calculate TFP, I use COMPUSTAT accounting variables: Sales for revenue, Plants property and gross equipment stock as Capital (state), Employment as labor (free), and capital expenditures as investment (proxy). To calculate labor productivity (LP), I divide Sales by number of Employees reported in Compustat. The correlation between TFP and LP is 0.88.

Model Appendix

D. Static demand equations and profits

CES demand structure implies:

$$Y(Z) = \frac{P(Z)}{P}^{-\sigma} \frac{Y}{P}$$

where $P^{1-\sigma} = N(t) \int_Z P(Z)^{1-\sigma} d\Phi(Z)$ and $N(t)$ is the number of innovators/entrepreneurs. From the CES structure, price $P(Z) = \frac{\sigma}{\sigma-1} \frac{W}{Z}$. Thus, I can re-write the price index as:

$$P^{1-\sigma} = N(t) \int_Z \left(\frac{\sigma}{\sigma-1} \frac{W}{Z} \right)^{(1-\sigma)} d\Phi(Z)$$

$$\text{or} \qquad \frac{W}{P} = N(t)^{\frac{1}{(\sigma-1)}} \frac{1}{\bar{\sigma}} \left(\int_Z Z^{(\sigma-1)} d\Phi(Z) \right)^{\frac{1}{\sigma-1}}$$

$$\text{or} \qquad \frac{W}{P} = N(t)^{\frac{1}{(\sigma-1)}} \frac{1}{\bar{\sigma}} \left(\int_Z Z^{(\sigma-1)} d\Phi(Z) \right)^{\frac{1}{\sigma-1}}$$

Profits of a firm with productivity Z are:

$$\Pi(Z) = \frac{1}{\sigma} \left(\frac{P(Z)}{P} \right)^{1-\sigma} \frac{Y}{P}$$

$$\text{or} \qquad \Pi(Z) = \frac{1}{\sigma} \left(\frac{Z}{\left(\int_Z Z^{\sigma-1} d\Phi(Z)\right)^{\frac{1}{\sigma-1}}} \right)^{\sigma-1} \frac{Y}{P}$$

E. Labor market clearing and resource constraint

Labor is engaged in production, entry, and searching for new ideas is equal to the total labor supply at time t.

$$L(t) = N(t) \int l(t, Z) d\Phi(t, Z) + \eta_E E(t) + \eta_S S(t)$$

Total goods produced is equal to total goods consumed.

$$\frac{Y(t)}{P(t)} = C(t)$$

F. Normalization

In order to analyze the economy along the balance growth path, I normalize the following:

$$\begin{aligned}
z &= \frac{Z}{M(t)} \\
\pi(t, Z) &= \Pi(t, Z)\frac{I(t)}{L^p(t)M(t)W(t)} \\
w(t) &= \frac{W(t)}{M(t)P(t)} \\
v(t, z) &= V(t, Z)\frac{I(t)}{L^p(t)M(t)w(t)} \\
n(t) &= \frac{I(t)}{L^p(t)} \\
F_D(t, z) &= \frac{1}{D(t)}\Phi_D(t, Z) \\
F_C(t, z) &= \frac{1}{C(t)}\Phi_C(t, Z) \\
y(t) &= Y(t)\frac{I(t)}{L^p(t)M(t)P(t)}
\end{aligned}$$

where $M(t)$ is minimum productivity threshold, $C(t)$ and $D(t)$ denote mass of creative and derivative innovators/entrepreneurs (and thus varieties) in the economy and $n(t)$ denotes the mass of entrepreneurs/varieties per capita.

G. Normalized HJBs

The above normalization implies that the equations of motion of the value function and firm's dynamic problem take the following form:

(8)

Derivative HJB: $(r + g_N(t) - g_L(t) - g_M(t) - g_w(t))v_D(t,z) = \pi(z) - g_m z\partial_z v(z) + \partial_t v_D(t,z)$

(9)

Creative HJB: $(r + g_N(t) - g_L(t) - g_M(t) - g_w(t))v_C(t,z) = \chi_C\Omega + (\mu_C - g_m)z\partial_z v(z) +$
$(\mu_C + \frac{\nu^2}{2})\partial_z^2 v(z) + \alpha(v_D(z) - v_C(z)) + \partial_t v_C(t,z)$

(10)

Entry: $\int_z (p_C^E v(z)dF_C(z) + (1 - p_C^E)v(z)dF_D(z)) - (\eta_E + \eta_S)\Omega = 0$

(11)

Abandon cut-off: $\int_z (p_C v(z)dF_C(z) + (1 - p_C)v(z)dF_D(z)) - \eta_S\Omega = 0$

H. Normalized KFEs

Similarly, I write the normalized KFE equations as the following:

$$(12)\quad \partial_t F_C(t,Z) = \left(\tau_C^E\frac{E}{C} + \tau_C\frac{S}{C} - g_L\right)F_c(z) - \alpha F_c(z) - (\mu - g_m)z\partial_z F(z) + \frac{\nu^2}{2}z^2\partial^2 F(z)$$

$$(13)\quad \partial_t F_D(t,Z) = \frac{C}{N}F_C(z) + (1 - \tau_C^E)\frac{E}{N} + (1 - \tau_C)\frac{S}{N})F_D(z) - \tilde{S}$$

Both F_C and F_D are determined in equilibrium along the BGP. F_D is determined by the constant inflow of entrepreneurs from the creative state, and the constant outflow of entrepreneurs choosing to adopt new technology.

I. Household optimization and the real interest rate

I suppose that the household utility takes the standard CRRA form:

$$\bar{U}(t) = \int_t^\infty \frac{C(t)^{1-\zeta} - 1}{1-\zeta} \exp^{-\rho(\tilde{t}-t)} d\tilde{t}$$

The consumers own the firm, which implies that the the effective changes in valuation are a return on their assets ($A(t)$):

$$\dot{A}(t) = r(t)A(t) + W(t)L_p(t) - C(t) - \chi_E(t)$$

The number of firms grow with the population, therefore replacing total assets ($A(t)$) with normalized assets ($a(t)$):

$$\dot{a}(t) = (r(t) - g_L)a(t) + W(t)\tilde{L}_p(t) - c(t) - \chi_E(t)$$

Setting up the current value Hamiltonian:

$$\hat{H}(a, c, \mu) = u(c(t)) + \mu(t)[w(t) + (r(t) - n)a(t) - c(t)]$$

Which implies that:

$$\zeta \frac{\dot{c}(t)}{c(t)} = r(t) - \rho$$
$$\implies r(t) = \rho + \zeta g_c(t)$$

J. Normalized budget constraint

Justifying the normalization for profits and aggregate expenditure:

$$\Pi(Z) = \frac{1}{\sigma}\left(\frac{P(Z)}{P}\right)^{1-\sigma}\frac{Y}{P}$$

Simplifying:

$$\Pi(Z) = \frac{1}{\sigma}\left(\frac{z}{\bar{z}}\right)^{\sigma-1}\frac{Y}{NP}$$
$$\pi(z) = \frac{\Pi(Z)N}{wML} = \frac{1}{\sigma}\left(\frac{z}{\bar{z}}\right)^{\sigma-1}\frac{Y}{PwML} = \frac{1}{\sigma}\left(\frac{z}{\bar{z}}\right)^{\sigma-1}\frac{y}{w}$$

Now for the budget constraint, assuming $\tilde{L} = c + d + \eta_E e + \eta S$:

$$1 = \int_M^\infty \tilde{l}(Z)d\Phi(Z) + c + d + \eta_E e + \eta S$$
$$1 - \tilde{L} = \int_1^\infty \bar{\sigma}w^{-\sigma}z^{\sigma-1}yNf(z)dz$$
$$1 - \tilde{L} = \bar{\sigma}w^{-\sigma}\bar{z}^{\sigma-1}Ny$$

Note that from the price index condition, we get: $N^{1/\sigma-1}\bar{z} = \bar{\sigma}w$. Therefore,

$$y = \bar{\sigma}w\tilde{L}_p$$

K. Productivity and Consumption Growth

Recall the non-normalized labor market constraint:

$$L(t) = N(t)\int l(t,z)d\Phi(Z) + \eta_E E + \eta_S S$$

Substituting $l(t,z)$ individual firm labor employment decisions:

$$\begin{aligned} L^p(t) = L(t) - \eta_E E - \eta_S S &= N(t)\int \frac{y(t,Z)}{Z}d\Phi(Z) \\ &= N(t)\left(\int \frac{P(t,Z)}{P(t)}\right)^{-\sigma}\frac{Y(t)}{P(t)}d\Phi(Z) \end{aligned}$$

Substituting for prices:

$$L^p(t) = N(t)\frac{\int Z^{\sigma-1}d\Phi(Z)}{\bar{Z}^{\sigma}}\frac{Y}{P}N(t)^{-\frac{\sigma}{\sigma-1}}$$
$$L^p(t) = N(t)^{-\frac{1}{\sigma-1}}\bar{Z}^{\frac{-1}{\sigma-1}}\frac{Y(t)}{P(t)}$$

Which implies that aggregate productivity is:

$$\frac{Y(t)}{L^p(t)P(t)} = N^{\frac{1}{\sigma-1}}\bar{Z} = N^{\frac{1}{\sigma-1}}M\bar{z}$$

Aggregate productivity growth is therefore given by:

$$\frac{1}{dt}\left(\frac{\dot{Y(t)}}{L^p(t)P(t)}\right) = \frac{1}{\sigma-1}\frac{\dot{N(t)}}{N(t)} + \frac{\dot{M(t)}}{M(t)}$$

Therefore, along the balanced growth path, productivity growth is given by:

$$\frac{1}{dt}\left(\frac{\dot{Y}}{L^pP}\right) = \frac{1}{\sigma-1}\frac{\dot{N}}{N} + \frac{\dot{M}}{M} = \frac{1}{\sigma-1}g_N + g_M$$

Therefore, $g_y = g_w = \frac{1}{\sigma-1}g_L$, and $g_{Y/LP} = g_m + \frac{1}{\sigma-1}g_L$. This implies that the interest rate:

$$r(t) = \rho + \zeta(g_m + \frac{1}{\sigma-1}g_L)$$

L. The Balanced Growth Path equilibrium

Following the BGP equilibrium defined in the paper, the following equations summarize the BGP. More importantly, this should be unconditional on time t. So, I remove time subscripts and time derivatives.

1. Following HJB equations should be satisfied:

(14) Derivative HJB: $(r+g_N-g_L-g_M-g_w)v_D(z)=\pi(z)-g_m z\partial_z v(z)$

(15) Creative HJB: $(r+g_N-g_L-g_M-g_w)v_C(z)=\chi_C\Omega+(\mu_C-g_m)z\partial_z v(z)+$
$(\mu_C+\frac{\nu^2}{2})\partial_z^2 v(z)+\alpha(v_D(z)-v_C(z))$

(16) Entry: $\int_z(p_C^E v(z)dF_C(z)+(1-p_C^E)v(z)dF_D(z))-(\eta_E+\eta_S)\Omega=0$

(17) Abandon cut-off: $\int_z(p_C v(z)dF_C(z)+(1-p_C)v(z)dF_D(z))-v(1)-\eta_S\Omega=0$

(18) Smooth pasting at abandon cut-off: $v^{'}(1)=0$

2. Following KFE equations should be satisfied:

$$0=\left(\tau_C^E\frac{E}{C}+p_C\frac{S}{C}-g_L\right)F_c(z)-\alpha F_c(z)-(\mu-g_m)z\partial_z F(z)+\frac{\nu^2}{2}z^2\partial^2F(z) \tag{19}$$

$$0=\frac{C}{N}F_C(z)+(1-p_C^E)\frac{E}{N}+(1-p_C)\frac{S}{N})F_D(z)-\tilde{S} \tag{20}$$

3. Labor market clears and the interest rate is calculated using household preferences.

M. Proofs of propositions

To solve the KFE block, I assume the initial derivative distribution is $F_D(t=0,z)=1-z^{-\alpha_D}$. To solve for the creative and derivative distribution along the BGP, I guess that creative distribution is a Pareto Distribution $1-z^{-\alpha_C}$, and the derivative distribution is a mixture of the initial derivative distribution and the creative distribution: $F_D(t=\infty,z)=\zeta(1-z^{-\alpha_D})+(1-\zeta)(1-z^{-\alpha_C})$. α_C and ζ are endogenous. Substituting these guesses in the two

KFE equations, results in the following three equations:

$$
\begin{aligned}
0 &= \frac{(\tau_C^E(g_N + \delta))}{\Omega_C} + \frac{(\tau_C^I S_N)}{\Omega_C} - (g_N + \delta) - \alpha \\
0 &= \frac{(1-\tau_C^E)(g_N + \delta)}{(1-\Omega_C)} + \frac{(1-\tau_C^I)S_N}{(1-\Omega_C)} - (g_N + \delta) - S_N/(1-\Omega_C) + \alpha(\Omega_C/(1-\Omega_C)); \\
0 &= 1 - 2\frac{(\mu_C - g_m)}{\nu^2} - \alpha_C \\
0 &= -\frac{((1-\tau_C^E)(gN + \Omega_C))}{(1-\Omega_C)} + \frac{(1-\tau_C^I)S_N)}{(1-\Omega_C)} + g_m \alpha_D \\
0 &= S_N/(1-\Omega_C) - (g_m)(w\alpha_C + (1-w)\alpha_D);
\end{aligned}
$$

Using these equations and solving for w, Ω_C, α_C and S_N, I prove propositions 1, 2 and 3.

N. Computing the Balanced Growth Path equilibrium

To complete solve the Balanced Growth Path computationally, I use a combination of guess and verify, and searching for fixed points. In particular, I follow the following algorithm:

First, to derive a value function which solves appendix equations 13-17, I guess form of the value functions for creative and derivative state as $V_J = a_J \pi(z=1) + b_j \frac{\sigma-1}{\theta_1} z^{\theta_1} + \frac{\sigma-1}{\theta_2} z^{\theta_2}$, where J denotes one of creative or derivative states. Substituting this form in the equations 13-17, I get the following expressions for a_J and b_J.

$$
\begin{aligned}
a_D &= \frac{\pi_{Min}}{r - g_m - g_w - (g_m)(\sigma - 1) + \delta} \\
b_D &= 0 \\
a_C &= \frac{\pi_{Min} + \alpha_0 a I}{r - g_m - g_w - (\sigma-1)(\mu_C - g_m) - \frac{\nu^2}{2}(\sigma-1)(\sigma-2) + \alpha_0 + \delta} \\
b_C &= 0
\end{aligned}
$$

Having solved for the value function in terms of aggregate prices and the productivity distribution. I follow the following algorithm to compute the equilibrium.

1. Guess $\tilde{L}_p(0)$, share of labor in production.

(a) Guess $g_m(0)$, productivity growth rate.

   i. Guess n, innovators per capita.

   ii. Given L_p, g_m and Ω, calculate pi and A.

   iii. Using pi, solve for v which satisfies Creative and Derivative HJB. (Guess and verify solution)

   iv. Calculate residual of the value matching problem at the Abandon cut-off.

   v. If $residual < tol$, move to (b); otherwise $\Omega(1) = \Omega(0) + (residual) * \Delta_\Omega$, and jump back to (i).

(b) Using the value function calculated in $i - v$, and using the division of entry and exit conditions, calculate the entry residual.

(c) If $entry\ residual < tol$, move to (2); otherwise $g_m(1) = g_m(0) + (entry\ residual) * \Delta_{g_m}$, and jump back to (a).

2. Use the value of g_m to resolve the KFE block and obtain equilibrium values of c (Creative innovators), d (Derivative innovators), and s abandoning derivative innovators.

3. Substitute these values in the labor market equalization condition to obtain a new value of L_p.

$$\tilde{L}_p(1) = 1 - \underbrace{\eta_E \tilde{E}}_{\text{Entry Costs}} - \underbrace{\eta \tilde{S}}_{\text{Search Costs}}$$

4. Finally, if $\tilde{L}_p(1) - \tilde{L}_p(0) \leq tol$, then equilibrium is reached, otherwise start at step (1) with $\tilde{L}_p(1)$.