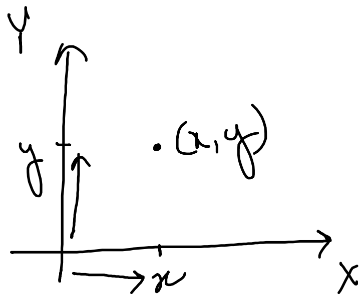
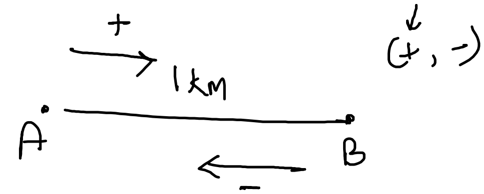
Linear Algebra



Quantities

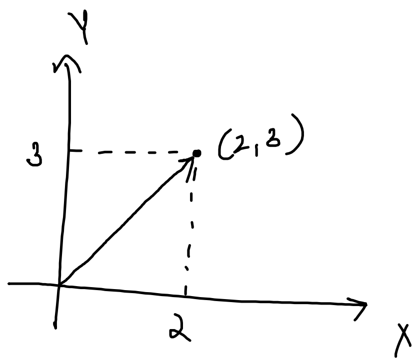
Scalar → Magnitude

Vector → Magnitude & direction → (+, −)

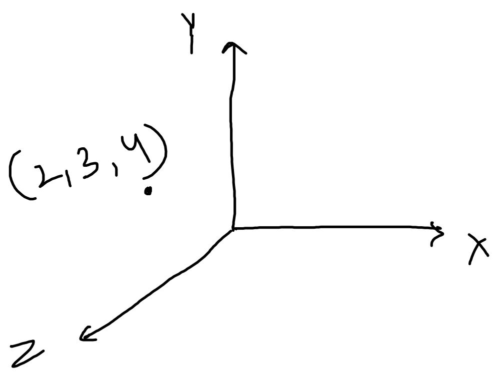


distance = 1 + 1 = 2km

displacement = 1 − 1 = 0 km



Vectors = [2 3] ⇒ 2d



Vector = [2 3 4] ⇒ 3d

↓

vector = [2 3 4 1 5 6] ⇒ 6 dimensions

↓

vector in nd = $[2 \quad 3 \quad 4 \quad \text{-----} \; n] \Rightarrow n$ dimensione.

MATRIX ⇒ it is a table of numbers.

Cols

$$\text{Rows} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}_{4\times4}$$

$$i/p \longrightarrow \boxed{\text{Transformation (linear)}} \longrightarrow o/p$$

Addition

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} \Rightarrow \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

Q

$$\begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 8 \\ 9 & 10 & 11 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 11 \\ 14 & 16 & 18 \end{bmatrix}$$

Multiplication

$$\underset{1\times2}{\begin{bmatrix} 1 & 2 \end{bmatrix}} \underset{2\times1}{\begin{bmatrix} 2 \\ 1 \end{bmatrix}} \Rightarrow \underset{1\times1}{\begin{bmatrix} 1\times2+2\times1 \end{bmatrix}} \Rightarrow [4]$$

$1\times\cancel{2}$ $\quad$ 1 $\quad$ $\cancel{2}\times 1$ $\quad$ 1×1

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times\cancel{2}} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix}_{\cancel{2}\times 2} \Rightarrow \begin{bmatrix} ae+gb & af+bh \\ ce+dg & cf+dh \end{bmatrix}_{2\times 2}$$

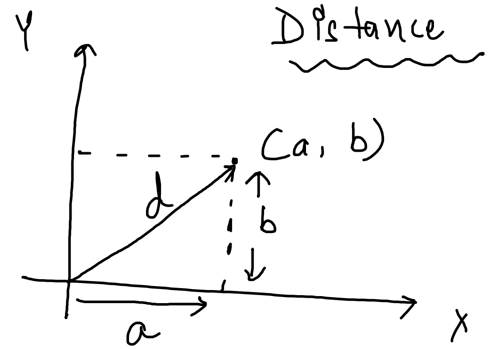
In order to perform matrix multiplication,

no. of columns in first matrix = no. of rows in second Matrix

a) $a_{m\times n} \times b_{p\times q} \Rightarrow$ No

b) $a_{m\times n} \times b_{n\times q} \Rightarrow C_{m\times q}$

Distance



Distance of a point from origin

By Pythagoras Theorem,

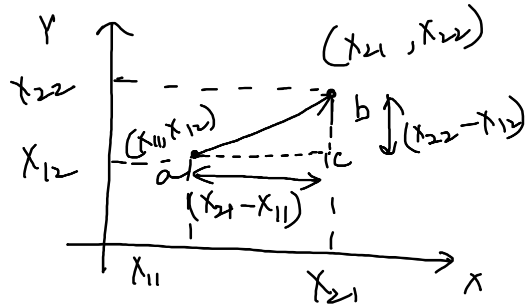
$$d^2 = a^2 + b^2$$

$$d = \sqrt{a^2+b^2}$$

lets extend this idea to n dimension

distance, $d = \sqrt{a^2+b^2+c^2+d^2+ \cdots + n^2}$

Distance b/w two points:



$a = [x_{11} \quad x_{12}]$

$b = [x_{21} \quad x_{22}]$

By Pythagoras Theorem,

$$d^2 = (x_{21}-x_{11})^2 + (x_{22}-x_{12})^2$$

$$d = \sqrt{(x_{21}-x_{11})^2 + (x_{22}-x_{12})^2} = \sqrt{(x_{11}-x_{21})^2 + (x_{12}-x_{22})^2}$$

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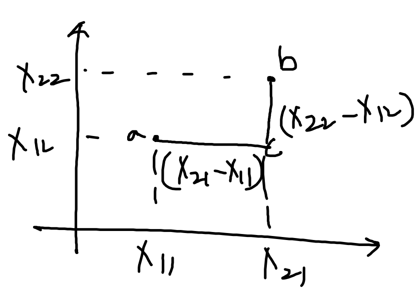
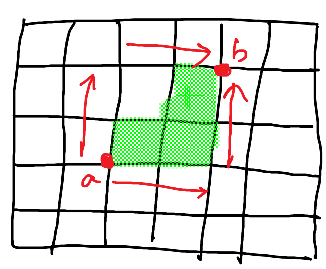
$$\boxed{\text{Euclidean distance} = \left[\sum_{i=1}^{n} |x_{1i} - x_{2i}|^2\right]^{1/2}}$$ $\rightarrow$ L2 Norm

Manhattan Distance





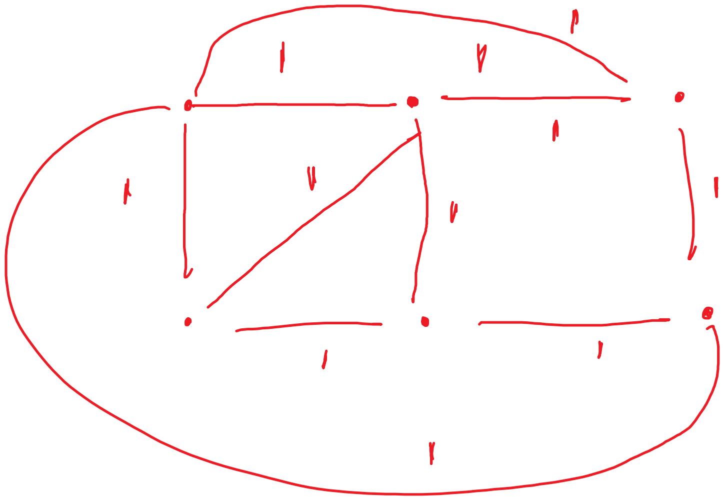
$$d = |x_{21} - x_{11}| + |x_{22} - x_{12}|$$

$$d = |x_{11} - x_{21}| + |x_{12} - x_{22}|$$

***

n

$$\boxed{d = \sum_{i=1}^{n} |x_{1i} - x_{2i}|} \rightarrow L1 \text{ Norm}$$



when you have high dimensional data, use Manhattan distance.

Minkowski distance

$$d = \left[\sum_{i=1}^{n} |x_{1i} - x_{2i}|^{p} \right]^{1/p} \quad L_p \text{ Norm}$$

$$p = 1, 2, 3 \ldots$$

lets put $p=1$, $d = \sum_{i=1}^{n} |x_{1i} - x_{2i}| \Rightarrow$ Manhattan distance

lets put $p=2$, $d = \left[\sum_{i} |x_{1i} - x_{2i}]^{2} \right]^{1/2} \Rightarrow$ Euclidean distance lets put $p=2$, $d = \left[\sum_{i=1}^{n} |X_{1i} - X_{2i}| \right] \Rightarrow$ Euclidean distance

Vector Multiflication

✓ dot product $\rightarrow$ scalar / Number

$\times$ cross-product $\rightarrow$ direction

dot product in linear algebra,

$$a = [a_1, a_2, a_3 ---- a_n]_{1\times n} = a^T$$

$$b = [b_1, b_2, b_3 --- b_n]_{1\times n}$$

$$a \cdot b = [a_1b_1 + a_2b_2 + a_3b_3 + --- + a_nb_n]_{1\times 1}$$

Vectors Representation

default $\leftarrow$ Column vector

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

Row vector

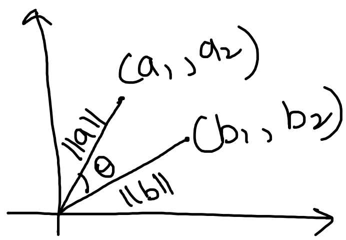
$$[a_1 --- a_n]$$

$$a^T = [a_1 \; a_2 \; a_3 ---- a_n]_{1\times n} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b \end{bmatrix}$$

$$a' = \begin{bmatrix} a_1 & a_2 & a_3 & \cdots & a_n \end{bmatrix}_{1\times n} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix}_{n\times 1}$$

$$a^T \cdot b = \begin{bmatrix} a_1 b_1 + a_2 b_2 + a_3 b_3 + \cdots + a_n b_n \end{bmatrix}_{1\times 1}$$

Angle b/w vectors



(Geometric)
dot product

$$a \cdot b = \|a\| \cdot \|b\| \cos\theta$$

$$a^T \cdot b = \begin{bmatrix} a_1 b_1 + a_2 b_2 \end{bmatrix} = a \cdot b$$

(linear algebra way)

$$\|a\| \cdot \|b\| \cos\theta = \begin{bmatrix} a_1 b_1 + a_2 b_2 \end{bmatrix}$$

$$\cos\theta = \frac{\begin{bmatrix} a_1 b_1 + a_2 b_2 \end{bmatrix}}{\|a\| \cdot \|b\|}$$

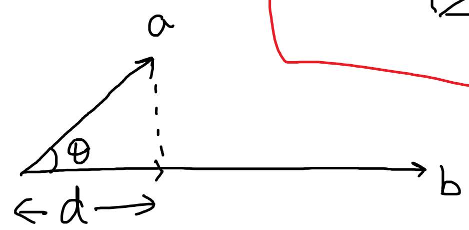
*

$$\boxed{\theta = \cos^{-1} \frac{\begin{bmatrix} a_1 b_1 + a_2 b_2 \end{bmatrix}}{\|a\| \cdot \|b\|}}$$

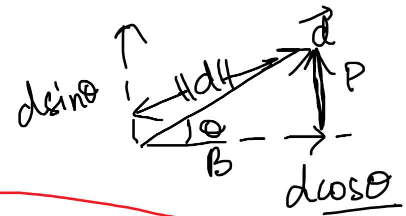
where, $\|a\| = \sqrt{a_1^2 + a_2^2 + \cdots a_n^2}$

$$\|b\| = \sqrt{b_1^2 + b_2^2 + \dots + b_n^2}$$

Projection



Basic Trigonometry



$\sin\theta = \frac{P}{H}$

$P = H\sin\theta$

$P = \|d\|\sin\theta$

$\cos\theta = \frac{B}{H}$

$B = H\cos\theta$

$= \|d\|\cos\theta$

$d \Rightarrow \|a\|\cos\theta$

$a \cdot b = \|a\| \, \|b\| \cos\theta$

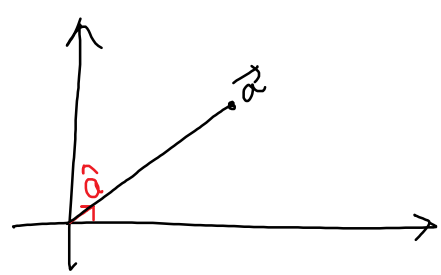
$a \cdot b = d \, \|b\|$

$$\boxed{d = \frac{a \cdot b}{\|b\|}} \rightarrow \text{Projection of vector } a \text{ on } b$$

Unit Vector → vector with magnitude 1

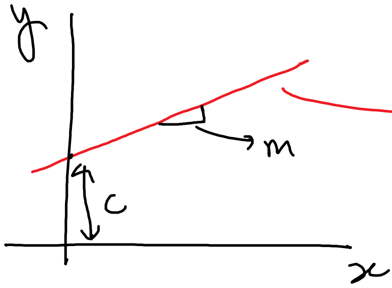
→ gives information about direction

$$\hat{a} = \frac{\vec{a}}{\|a\|}$$

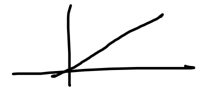


Lines & Planes

Line



$y = mx + c$



$c=0$ (line passing through origin)

simple geometry   trigonometrical geometry   Calculus

$$\text{Slope}(m) = \frac{y_2 - y_1}{x_2 - x_1} = \tan\theta = \frac{dy}{dx}$$

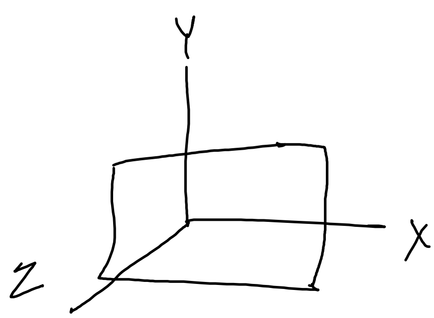
General Equation of line,

$$= \boxed{ax + by + c = 0}$$

$$by = -ax - c$$

$$\boxed{y = -\frac{a}{b}x - \frac{c}{b}}$$

Plane:



General Equation is: $ax + by + cz + d = 0$

↓ change coeff

$$w_1 x + w_2 y + w_3 z + w_0 = 0$$

$\downarrow$ change axis name

$$w_1 x_1 + w_2 x_2 + w_3 x_3 + w_0 = 0$$

Above 3d: Hyperplane: $w_0 + [w_1 x_1 + w_2 x_2 - - - - - + w_n x_n] = 0$

$w_0 + w^T \cdot x = 0$ (linear algebra way)

lets say hyperplane is passing through origin,

$$\boxed{w^T x = 0} \quad {}^* w_0 = 0$$

Eigen Value & eigen vector → vectors that do not rotate when linear transformation is applied on them

Eigen Value ↓ value by which the original vector gets scaled up

$$A\vec{x} = \lambda \vec{x}$$

A ↓ Matrix that applies LT on vector

$\vec{x}$ ↓ vector

λ ↓ eigen value

$\vec{x}$ → eigen vector