

Q A principal of school claims that students have above average IQ. A random sample is taken with a mean of 112.5. The mean & std dev of population is 100 & 15. Test your hypothesis.

Sol. (i) $H_0 : \mu \leq 100$
 $H_A : \mu > 100$

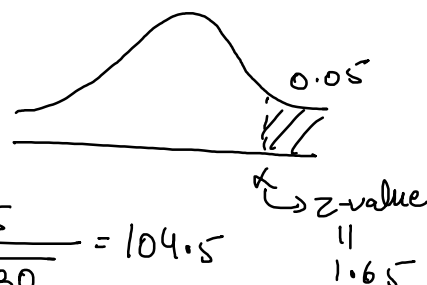
(ii) Need to check whether test is one tailed or two tailed



(iii) $\mu = 100$, $\sigma = 15$, $\bar{x} = 112.5$

AR/TR

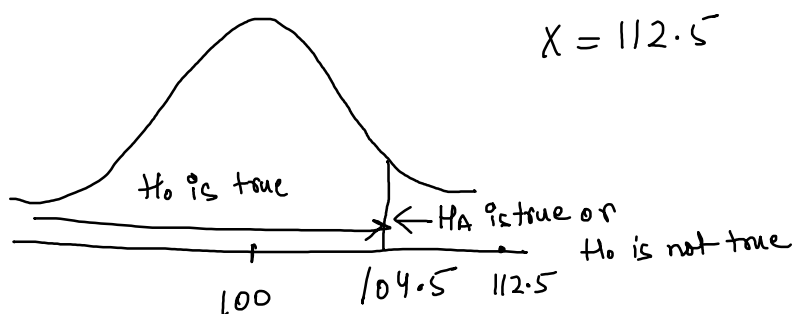
$\alpha = 0.05$



$$UL = \mu + z \times \frac{\sigma}{\sqrt{n}} = 100 + 1.65 \times \frac{15}{\sqrt{30}} = 104.5$$

4.5

$$LL = \mu - z \times \frac{\sigma}{\sqrt{n}} = 100 - 1.65 \times \frac{15}{\sqrt{30}} = 100 - 4.5 = 95.5$$



lets compare 112.5 with 104.5

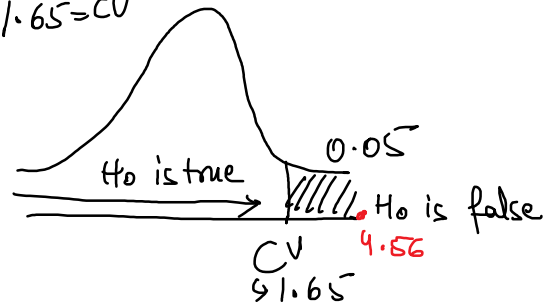
$$112.5 > 104.5$$

hence, Reject H_0

2) Critical Value Method

$\alpha = 0.05$, test is right tailed

$$Z_{cal} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}, \quad Z_{tab}(\alpha = 0.05) = 1.65 = CV$$



$$Z_{cal} = \frac{112.5 - 100}{\frac{15}{\sqrt{30}}} = \frac{12.5}{15/\sqrt{30}} = 4.56$$

$$\therefore Z_{cal} > Z_{tab}$$

\therefore Reject H_0

3) Pvalue Method: $\alpha = 0.05$, $Z_{cal} = \frac{112.5 - 100}{\frac{15}{\sqrt{30}}} = 4.56$



$$P(Z_{cal} = 4.56) = 1 - A_L = 0.0000034$$

compare pvalue with significance level

$$\begin{aligned} & \text{pvalue} <^{\text{(less than)}} \alpha \\ & (0.0000034) < 0.05 \end{aligned}$$

Hence, Reject H_0

$$\mu = 0, \sigma = 0.1\%, \sigma = 0.25$$

Q A researcher has agreed upon a data of daily return of portfolio of call option over a recent 250 days period. The mean of daily return is 0.1% & std dev is 0.25% . The researcher claims the mean daily portfolio is not 0. Construct CI at 95%. & test the belief.

Sol. $H_0 \Rightarrow \mu = 0$ test is two tailed. $CI + SL^{(\alpha)} = 1$

$$H_A \Rightarrow \mu \neq 0$$

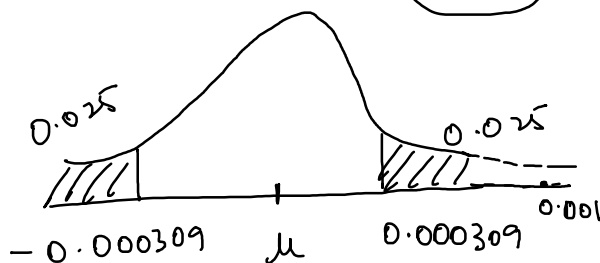
$$CI = 0.95, \alpha = 1 - 0.95 = 0.05$$

AR

$$\sigma = 0.1\%, \mu = 0, \sigma = 0.0025$$

7.96

$$UL = 0 + 1.96 \times \frac{0.0025}{\sqrt{250}} = 0.000309$$



$$LL = 0 - 1.96 \times \frac{0.0025}{\sqrt{250}}$$

$$= -0.000309$$

$$\begin{aligned} -0.000309 & < 0.1\% > 0.000309 \\ & 0.001\% \end{aligned}$$

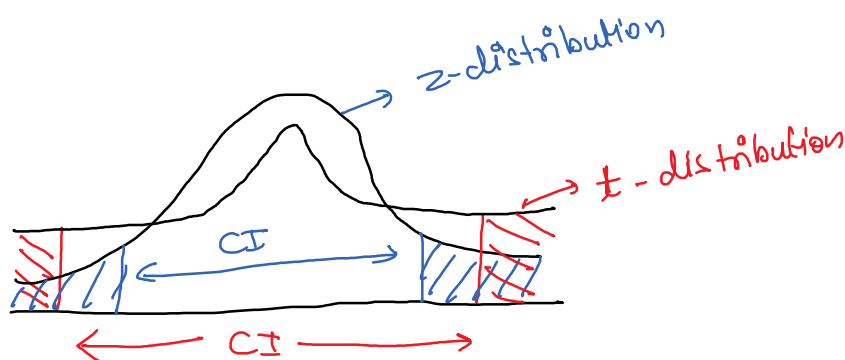
Reject H_0 .

CV Method & P-value is your assignment!

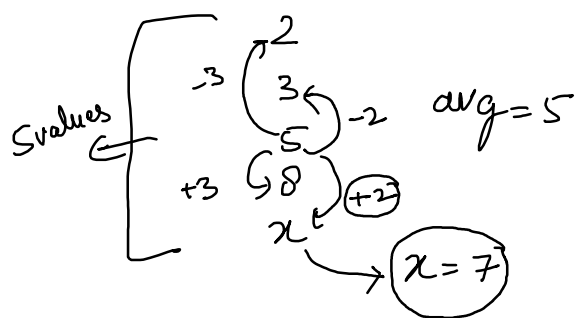
T-distribution

low confidence
samples are very less
($n < 30$)

$$t = \frac{x - \mu}{s/\sqrt{n}}$$



Degrees of freedom: logically independent values



9 have 4 logically independent values!

for n values, degrees of freedom = $(n-1)$

↳ logically independent values

t_{cal} , t_{tab} , x , df

t_{tab} compare it with t_{cal}

Q A company manufactures car batteries with average life span of 2 years or more. An engineer believes this value to be less. Using 10 samples, he measured the life span & found it be 1.8 years with a std dev of 0.15.

At 99% CI, is there enough evidence to reject H_0 .

Sol.

$$H_0: \mu \geq 2$$

$$H_A: \mu < 2$$



$$CI = 99\% = 0.99$$

$$\alpha = 1 - 0.99 = 0.01$$

$$n = 10, \quad df = 10 - 1 = 9 \quad t_{tab} = 2.821 \quad (0.01)$$

$$t_{cal} = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{1.8 - 2}{\frac{0.15}{\sqrt{10}}} = -4.25$$

$$t_{cal} < t_{tab}$$

Reject H_0