CSCI 6342: Linear Algebra
Assignment II

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Part I: Pen-and Paper

1. Suppose that the inverse of a square matrix is defined using only left multiplication: define $\mathbf{A}^{-1}$ as the matrix with the property that $\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$. In this exercise, you will prove that this implies that $\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$ using two steps:

- **First show that the identity matrix I is unique. That is, if there's any other matrix J such that AJ=A=JA, then I=J.**
- **Use the above fact and the associativity of matrix multiplication to establish $\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$**

Solution. The inverse of a square matrix is written as $\mathbf{A}^{-1}$ which follows the property that $\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$ We have to prove the associativity of this equation. We accomplish this in two steps:
We know that any matrix multiplied by the identity matrix **I** is the matrix itself.
$\mathbf{AI} = \mathbf{A}$
Consider another matrix **J** such that $\mathbf{AJ} = \mathbf{A}$

$$\mathbf{AJ} = \mathbf{A} = \mathbf{JA}$$

$$\mathbf{AI} = \mathbf{A} = \mathbf{IA}$$

If we compare the two equations we get,

$$\mathbf{AI} = \mathbf{AJ}$$

$$\mathbf{I} = \mathbf{J}$$

This shows that an identity matrix **I** is a unique matrix.
Our property $\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$ holds true.As we proved above, multiplying by identity matrix results in associativity.

$$\mathbf{AI} = \mathbf{A}$$

$$\mathbf{I} = \mathbf{A}.\mathbf{A^{-1}} \tag{1}$$

By associativity

$$\mathbf{A}.\mathbf{A^{-1}} = \mathbf{A^{-1}}\mathbf{A}$$

From equation (1)

$$\mathbf{A^{-1}}\mathbf{A} = \mathbf{I}$$

$$\mathbf{A^{-1}} = \mathbf{A^{-1}}\mathbf{I} \quad \square$$