Assignment 3

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1 Pen-and-paper

1.1

Read the proof of Theorem 7.2 given in Module 8. (The theorem was stated in Module 7 but the proof is in Module 8.) What can you conclude about nullspace(A) and nullspace(A^TA)? Then, use this and Theorem 8.3 to prove that rank(A)=rank(A^TA) even if rank(A) is less than the number of columns.

The proof of Theorem 7.2 in module 8 proves that nullspace(A) = nullspace(A^TA)
$\therefore$ this shows that nullspace(A) is a subset of nullspace(A^TA).
We can say that A and A^TA will have the same column space and row space.

Assume, $x \subset null(A)$, then AX = 0. Multiplying by A^T on both sides

we have $A^TAx = 0$ which means $x \subset null(A^TA)$

$\therefore$ null(A) $\subset$ null(A^TA)

Proof:

Let x $\in$ N(A) where N(A) is the nullspace of A.

A x = 0
$\Rightarrow A^TA$ x = 0
$\Rightarrow x \in N(A^TA)$

Hence N(A) $\subset$ N(A^TA)
Again let, $x \in N(A^TA)$

$A^TAx = 0$
$x^TA^TAx = 0$
$(Ax^T)(Ax) = 0$
$(Ax) = 0$
$x \in N(A)$

Hence N(A^TA)N(A) $\subset$ N(A)
Therefore,

$N(A^TA) = N(A)$
$\dim(N(A^TA)) = \dim(N(A))$
$\text{rank}(A^TA) = \text{rank}(N(A))$