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HW 2

CSCI 576

Problem : 01 Colour Theory

$$C(X,Y,Z) = \alpha_1 P_1(X_1, Y_1, Z_1) + \alpha_2 P_2(X_2, Y_2, Z_2) + \alpha_3 P_3(X_3, Y_3, Z_3)$$

Part 1

* Normalised Chromaticity co-ordinates of Primaries

$$P_1 \Rightarrow x_1 = \frac{X_1}{X_1+Y_1+Z_1} \; ; \; y_1 = \frac{Y_1}{X_1+Y_1+Z_1} \; ; \; z_1 = \frac{Z_1}{X_1+Y_1 Z_1}$$

$$P_2 \Rightarrow x_2 = \frac{X_2}{X_2+Y_2+Z_2} \; ; \; y_2 = \frac{Y_2}{X_2+Y_2+Z_2} \; ; \; z_2 = \frac{Z_2}{X_2+Y_2+Z_2}$$

$$P_3 \Rightarrow x_3 = \frac{X_3}{X_3+Y_3+Z_3} \; ; \; y_3 = \frac{Y_3}{Y_3+X_3+Z_3} \; ; \; z_3 = \frac{Z_3}{X_3+Y_3+Z_3}$$

* Normalised Chromaticity co-ordinates of Colour C

$$C \Rightarrow x = \frac{X}{X+Y+Z} \; ; \; y = \frac{Y}{X+Y+Z} \; ; \; z = \frac{Z}{X+Y+Z}$$

Part 2

Normalised chromaticity co-ordinates of the Colour C in terms of normalised chromaticity co-ordinates of P_1, P_2, P_3 is given by:-

$*\ X = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3$

$Y = \alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3$ — Individual components

$Z = \alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3$

$$x = \frac{\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3}{\alpha_1 (X_1 + Y_1 + Z_1) + \alpha_2 (X_2 + Y_2 + Z_2) + \alpha_3 (X_3 + Y_3 + Z_3)}$$

$$y = \frac{\alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3}{\alpha_1 (X_1 + Y_1 + Z_1) + \alpha_2 (X_2 + Y_2 + Z_2) + \alpha_3 (X_3 + Y_3 + Z_3)}$$

$$z = \frac{\alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3}{\alpha_1 (X_1 + Y_1 + Z_1) + \alpha_2 (X_2 + Y_2 + Z_2) + \alpha_3 (X_3 + Y_3 + Z_3)}$$

Part 3

Chromaticity co-ordinates of any color C can also be represented as a linear combination of the chromaticity co-ordinates of the respective primaries.

Hence we need to show that the normalised values are a linear combination of the normalised primaries

i.e
$$x = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$
$$y = \beta_1 y_1 + \beta_2 y_2 + \beta_3 y_3$$
$$z = \beta_1 z_1 + \beta_2 z_2 + \beta_3 z_3$$

$$\therefore \quad x_1 = \frac{X_1}{X_1+Y_1+Z_1}$$

$$\therefore \quad x = \frac{X}{X+Y+Z}$$

and $\quad X = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3$

$$\therefore \quad x(X+Y+Z) = \alpha_1 \, x_1 (X_1+Y_1+Z_1)$$

$$+ \alpha_2 \, x_2 (X_2+Y_2+Z_2) + \alpha_3 x_3 (X_3+Y_3+Z_3)$$

But $\quad (X+Y+Z) = \alpha_1 (X_1+Y_1+Z_1) + \alpha_2 (X_2+Y_2+Z_2)$

$$+ \alpha_3 (X_3+Y_3+Z_3) \quad \hookrightarrow (2)$$

$$\therefore \quad x = \frac{\alpha_1 x_1 (X_1+Y_1+Z_1)}{X+Y+Z} + \frac{\alpha_2 x_2 (X_2+Y_2+Z_2)}{(X+Y+Z)}$$

$$+ \alpha_3 \, x_3 \frac{(X_3+Y_3+Z_3)}{(X+Y+Z)}$$

$$\therefore \quad \beta_1 = \frac{\alpha_1 (X_1+Y_1+Z_1)}{X+Y+Z}$$

$$\therefore \quad \beta_2 = \frac{\alpha_2 (X_2+Y_2+Z_2)}{X+Y+Z}$$

$$\therefore \quad \beta_3 = \frac{\alpha_3 (X_3+Y_3+Z_3)}{X+Y+Z}$$

$$\therefore \beta_1 = \frac{\alpha_1(x_1+y_1+z_1)}{\alpha_1(x_1+y_1+z_1)+\alpha_2(x_2+y_2+z_2)+\alpha_3(x_3+y_3+z_3)}$$

$$\therefore \beta_2 = \frac{\alpha_2(x_2+y_2+z_2)}{\alpha_1(x_1+y_1+z_1)+\alpha_2(x_2+y_2+z_2)+\alpha_3(x_3+y_3+z_3)}$$

$$\therefore \beta_3 = \frac{\alpha_3(x_3+y_3+z_3)}{\alpha_1(x_1+y_1+z_1)+\alpha_2(x_2+y_2+z_2)+\alpha_3(x_3+y_3+z_3)}$$

Similarly

$Y = \alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3$ & $y_1 = \frac{Y_1}{X_1+Y_1+Z_1}$ & $y = \frac{Y}{X+Y+Z}$

$\alpha(X+Y+Z) = \alpha_1(X_1+Y_1+Z_1) + \alpha_2(X_2+Y_2+Z_2) + \alpha_3(X_3+Y_3+Z_3)$

$\therefore y = \beta_1 y_1 + \beta_2 y_2 + \beta_3 y_3 \longrightarrow$ Similarly as above

$\therefore z = \beta_1 z_1 + \beta_2 z_2 + \beta_3 z_3 \longrightarrow$ Same as above.

A] Problem 2 : Entropy Coding

- $P(X) = x^2$
- $P(Y) = (1-x^2)$

Part 1

$$\therefore H(X) = -\sum P_i \log P_i$$

$$= -x^2\log_2(x^2) + (1-x^2)\log_2(1-x^2)$$

$$= -x^2\log_2(x^2) - (1-x^2)\log_2(1-x^2)$$

$$H(X) = -2x^2\log_2 x - (1-x^2)\log_2(1-x^2)$$

*] Minimum entropy can occur in the case when only 1 symbol is present in the entire system or

$P(X) = 1$ or $P(Y) = 1$

$\therefore x^2 = 1$ or $1 - x^2 = 1$

$x = \pm 1$ $\quad \therefore x^2 = 0$

$\therefore x = 0$

$\therefore$ Entropy will be minimum at $x = \pm 1, 0$

Part 3]

Entropy will be maximum when both the probabilities are equal

$$P(X) = P(Y)$$

$$\therefore (1 - x^2) = x^2$$

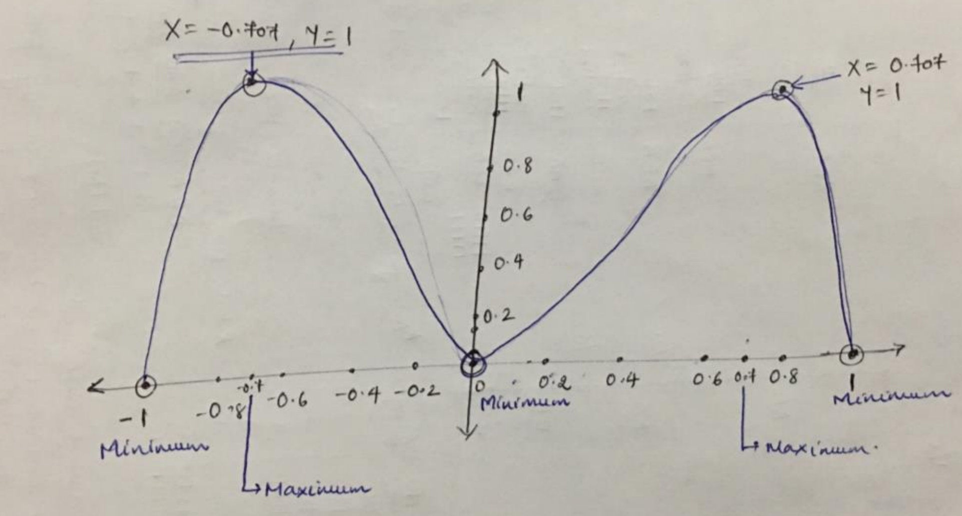
$$\therefore x^2 = \frac{1}{2}$$

$$\therefore x = \pm \frac{1}{\sqrt{2}}$$

$$\therefore x = \pm 0.707$$

Plot of Entropy v/s X

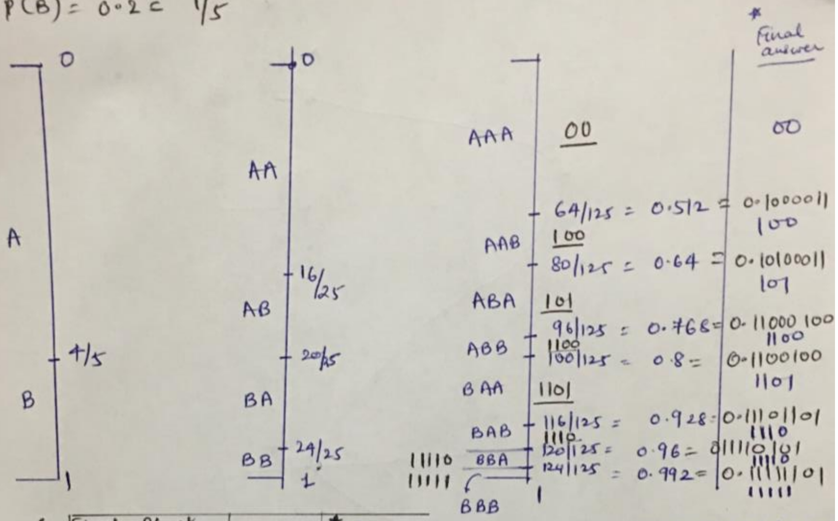
$$H = -\left[x^2 \log_2 x^2 + (1 - x^2)\log_2 (1 - x^2)\right]$$



Problem 3] Symbols used A, B

$P(A) = 0.8 = 4/5$

$P(B) = 0.2 = 1/5$



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Final Block	(Length)
AAA – 00	2
AAB – 100	3
ABA → 101	3
ABB – 1100	4
BAA – 1101	4
BAB – 1110	4
BBA → 11110	5
BBB → 11111	5

* Average Code length $= \frac{2+3+3+4+4+4+5+5}{8}$

$= 3.375$

This is the optimum value

* Bits for code of the message

ABA BBA ABB AAA BBB

$= 3 + 5 + 4 + 2 + 5$

$= 19$ bits

* Better the above Code word

→ The above codeword can be improved by preventing the shrinking when the interval bounds get too close.