

wave function

A wave function in quantum physics is a mathematical description of the quantum state of an isolated quantum system. The wave function is a complex-valued probability amplitude, and the probabilities for the possible results of measurements made on the system can be derived from it. It describes the quantum state of the particle as a function of momentum, spin, position, and time. It is denoted by Ψ .

Properties of Wave Function

- Ψ must have only one value at any point.
- The value of Ψ at any point must not be infinite, it must be finite value.
- The value of ψ should vary continuously from one point to another and only single-valued.
- The total probability of electron in the space around nucleus must be one.
- Using the wave function the probability distribution in three-dimensional is established.
- If a particle exists, then the probability of finding a particle is 1.

Significance of Wave Function

- In the physical sense, Ψ gives the amplitude of the wave associated with the electron.
- In the case of light or sound, the square of the amplitude of the wave at any point will give us the intensity of the sound or light at that particular point.
- Ψ^2 determines the probability of finding the moving electron in a given region. i.e. a large value of Ψ^2 means a high probability of finding the electron at that place and a small value of Ψ^2 means low probability.
- If Ψ^2 is almost zero at a particular point, it means that the probability of finding the electron at that point is negligible.
- Since electrons can have any wave function, there are many atomic orbitals in an atom.
- The probability will provide the best possible description of any such situation which we will be unable to describe with certainty.
- Most of the time electrons are used to spend in the volume of the sphere which is bounded by that distance and for the rest of the time, the electrons can be found outside the volume of the sphere.

Operators

An operator \hat{A} is a “mathematical object” that maps one state vector, $|\psi\rangle$ into another, $|\phi\rangle$, i.e. $\hat{A} |\psi\rangle = |\phi\rangle$. If $\hat{A} |\psi\rangle = a|\psi\rangle$, with a real, then $|\psi\rangle$ is said to be an eigenstate (or eigenfunction) of \hat{A} with eigenvalue a .

For example, the plane wave state

$\psi_p(x) = \langle x|\psi_p\rangle = A e^{-ipx/\hbar}$ is an eigenstate of the momentum operator, differentiating ψ with respect to x

$$\frac{\partial \psi}{\partial x} = \frac{-i}{\hbar} (-P) A e^{-ipx/\hbar} = \frac{-i}{\hbar} (-P) \psi = \frac{iP}{\hbar} \psi$$

$$P.\psi = \frac{\hbar}{i} \frac{\partial \psi}{\partial x} = -i\hbar \frac{\partial \psi}{\partial x}$$

$\hat{P} = -i\hbar \frac{\partial}{\partial x}$ is called momentum operator.

The expectation value (the long-term average value of the variable) of P for a particle moving along x-axis is given by

$$\begin{aligned}\bar{P} &= \int_{-\infty}^{\infty} \psi^* \hat{P} \psi dx = \int_{-\infty}^{\infty} \psi^* \left(-i\hbar \frac{\partial}{\partial x}\right) \psi dx = \int_{-\infty}^{\infty} \psi^* \left(-i\hbar \frac{\partial}{\partial x} A e^{-ipx/\hbar}\right) dx \\ &= -i\hbar \frac{iP}{\hbar} \int_{-\infty}^{\infty} \psi^* \psi dx = P = \hbar k\end{aligned}$$

i.e the expectation value of something has definite value is that value itself.

Again differentiating ψ with respect to t , we get

$$\frac{\partial \psi}{\partial t} = \frac{-i}{\hbar} E \psi$$

$$E.\psi = \frac{-\hbar}{i} \frac{\partial \psi}{\partial t} = i\hbar \frac{\partial \psi}{\partial t}$$

$\hat{E} = -i\hbar \frac{\partial}{\partial t}$ is called Energy operator.

The expectation value of E for a particle moving along x-axis is given by

$$\bar{E} = \int_{-\infty}^{\infty} \psi^* \hat{E} \psi dx = \int_{-\infty}^{\infty} \psi^* \left(i\hbar \frac{\partial}{\partial t} \right) \psi dx$$

$$= \int_{-\infty}^{\infty} \psi^* \left(i\hbar \frac{\partial}{\partial t} A e^{-ipx/\hbar} \right) dx$$

$$= i\hbar \frac{-iE}{\hbar} \int_{-\infty}^{\infty} \psi^* \psi dx$$

$$= E = \hbar \omega \text{ is the average value of energy itself.}$$

Similarly, $\hat{x} = x$ is called position operator.

Schrödinger Equation

Schrödinger Equation is a mathematical expression which describes the change of a physical quantity over time in which the quantum effects like wave-particle duality are significant. The Schrödinger Equation has two forms the time-dependent Schrödinger Equation and the time-independent Schrödinger Equation.

Time-independent Schrödinger Equation

Considering a complex plane wave:

$$\Psi(x, t) = A e^{i(kx - \omega t)} \quad 1$$

Now the Hamiltonian of a system is

$$H = T + V$$

Where 'V' is the potential energy and 'T' is the kinetic energy. As we already know that 'H' is the total energy, we can rewrite the equation as:

$$E = \frac{p^2}{2m} + V(x) \quad 2$$

Now taking the derivatives,

$$\frac{\partial \psi}{\partial t} = -i\omega A e^{i(kx - \omega t)} = -i\omega \psi(x, t) \quad 3$$

$$\frac{\partial^2 \psi}{\partial t^2} = -k^2 A e^{i(kx - \omega t)} = -k^2 \psi(x, t) \quad 4$$

We know that,

$$P = \frac{2\pi\hbar}{\lambda} \text{ and } k = \frac{2\pi}{\lambda} \quad 5$$

where ‘ λ ’ is the wavelength and ‘ k ’ is the wavenumber.

We have

$$k = \frac{P}{\hbar}$$

Therefore,

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{p^2}{\hbar^2} \psi(x, t) \quad 6$$

Now multiplying Ψ to the eq 2. we get,

$$E \psi = \frac{p^2}{2m} \psi + V(x) \psi \quad 7$$

The above expression can be written as:

$$E = \frac{-\hbar^2}{2m} \frac{d^2 \psi}{dx^2} + V(x) \quad 8$$

$$\frac{d^2 \psi}{dx^2} + \frac{2m[E-V]}{\hbar^2} \psi = 0 \quad 9$$

This is the time independent Schrödinger Wave Equation

The time-dependent Schrodinger equation

The wave function associated with the motion of free particle is given by

$$\psi = Ae^{-i[\omega t - kx]} = Ae^{-i/\hbar[Et - Px]}$$

differentiating ψ with respect to x

$$\frac{d\psi}{dx} = \frac{-i}{\hbar} (-P) Ae^{-i/\hbar[Et - Px]} = \frac{iP}{\hbar} \psi$$

Again differentiating with respect to x .

$$\frac{d^2\psi}{dx^2} = \left(\frac{iP}{\hbar}\right)^2 \psi = \frac{-P^2}{\hbar^2} \psi$$

$$P^2\psi = -\hbar^2 \frac{d^2\psi}{dx^2}$$

differentiating ψ with respect to t ,

$$\frac{\partial\psi}{\partial t} = \frac{-iE}{\hbar} \psi \quad \text{or} \quad E\psi = \frac{-\hbar}{i} \frac{\partial\psi}{\partial t} = i\hbar \frac{\partial\psi}{\partial t}$$

total energy of a particle is given by

$$E = \frac{1}{2} m v^2 + V = \frac{p^2}{2m} + V$$

Multiplying both sides by ψ

$$\left[\frac{p^2}{2m} + V \right] \psi = E \psi$$

$$\frac{p^2 \psi}{2m} + V \psi = E \psi$$

using eq 1

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} + V \psi = i \hbar \frac{\partial \psi}{\partial t}$$

this is time dependent Schrodinger wave equation

Application of Schrodinger theory

Particle in a box

Consider one dimensional closed box of width L. A particle of mass 'm' is moving in a one-dimensional region along X-axis specified by the limits $x=0$ and $x=L$ as shown in fig. The potential energy of particle inside the box is zero and infinity elsewhere.

I.e Potential energy $V(x)$ is of the form

$$V(x) = 0; \text{ if } 0 < x < L$$

$$= \infty; \text{ if } x < 0 \text{ and } x > L$$

The one-dimensional [time independent Schrodinger wave equation](#) is given by

$$\frac{d^2\psi}{dx^2} + \frac{2m[E-V]}{\hbar^2} \psi = 0 \quad (1)$$

Here we have changed partial derivatives in to exact because equation now contains only one variable i.e x-Co-ordinate. Inside the box $V(x) = 0$

Therefore, the Schrodinger equation in this region becomes

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2} \psi = 0$$

$$\text{Or } \frac{d^2\psi}{dx^2} + K^2 \psi = 0 \quad (2) \quad \text{Where } k = \sqrt{\frac{2mE}{\hbar^2}} \quad (3)$$

K is called the Propagation constant of the wave associated with particle and it has dimensions reciprocal of length.

The general solution of eq (2) is

$$\Psi = A \sin Kx + B \cos Kx \quad (4)$$

Where A and B are arbitrary conditions and these will be determined by the boundary conditions.

The particle will always remain inside the box because of infinite potential barrier at the walls. So the probability of finding the particle outside the box is zero i.e. $\psi(x)=0$ outside the box. We know that the wave function must be continuous at the boundaries of potential well at $x=0$ and $x=L$, i.e.

$$\psi(x)=0 \text{ at } x=0 \quad (5)$$

$$\Psi(x)=0 \text{ at } x=L \quad (6)$$

These equations are known as Boundary conditions.

Apply Boundary condition of eq.(5) to eq.(4)

$$0=A \sin (K*0) +B \cos (K*0)$$

$$0= 0+B*1 \quad \text{or} \quad B=0 \quad (7)$$

Therefore eq. (4) becomes

$$\Psi(x) = A \sin K.x \quad (8)$$

Applying the boundary condition of eq. (6) to eq. (8), we have

$$0=A \sin KL$$

$$\sin KL=0= \sin (n\pi)$$

$$KL=n\pi$$

$$K=\frac{n\pi}{L} \quad (9)$$

Where $n=1, 2, 3, \dots$

A Cannot be zero in eq. (9) because then both A and B would be zero. This will give a zero wave function everywhere which means particle is not inside the box.

Substitute the value of K from eq. (9) in eq. (8) to get

$$\Psi(x)=A \sin \left(\frac{n\pi}{L} x\right)$$

As the wave function depends on quantum number n so we write it ψ_n .

$$\text{Thus } \Psi_n=A \sin \left(\frac{n\pi}{L} x\right) \quad 0 < x < L$$

This is the wave function or eigen function of the particle in a box.

And

$$\Psi_n=0 \quad \text{outside the box}$$

Energy value or Eigen value of particle in a box:

Put this value of K from equation (9) in eq. (3)

$$\frac{n\pi}{L} = \sqrt{\frac{2m E}{\hbar^2}}$$

$$E = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

Where $n = 1, 2, 3 \dots$ Is called the Quantum number.

As E depends on n, we shall denote the energy of particle

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \quad (10)$$

This means the energy of particle in potential well is quantized. Each value of energy given by eqⁿ 10, is called eigen value and corresponding function ψ_n are called eigen function.

The coefficient A is called Normalizing constant and can be determined using Normalizing condition.

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$$\int_0^L \psi^* \psi dx = 1$$

$$A^2 \int_0^L \sin^2 \left(\frac{n\pi x}{L} \right) dx = 1$$

$$A^2 \int_0^L \frac{1}{2} [1 - \cos 2 \left(\frac{n\pi x}{L} \right)] dx = 1$$

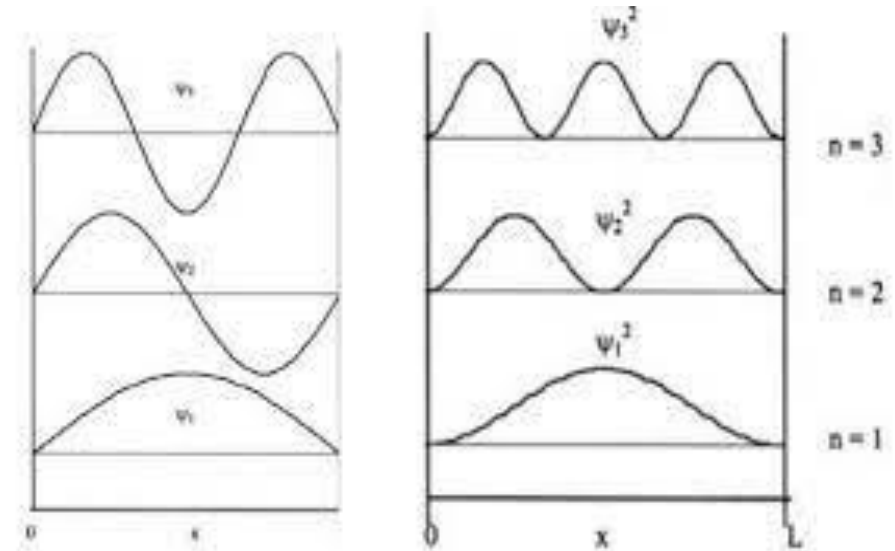
$$A^2 \frac{1}{2} \left[\int_0^L dx - \int_0^L \cos 2 \left(\frac{n\pi x}{L} \right) dx \right] = 1$$

$$A^2 \left[\frac{L}{2} - 0 \right] = 1$$

$$A^2 \frac{L}{2} = 1$$

$$A = \sqrt{\frac{2}{L}} \quad \text{Hence the normalized wave function of the electron is}$$

$$\Psi(x) = \sqrt{\frac{2}{L}} \sin \left(\frac{n\pi}{L} x \right)$$



The Schrödinger Equation for the hydrogen atom

The hydrogen atom, consisting of an electron and a proton, is a two-particle system, and the internal motion of two particles around their center of mass is equivalent to the motion of a single particle with a reduced mass. This reduced particle is located at \hat{r} , where \hat{r} is the vector specifying the position of the electron relative to the position of the proton. The length of \hat{r} is the distance between the proton and the electron, and the direction of \hat{r} is given by the orientation of the vector pointing from the proton to the electron. Since the proton is much more massive than the electron, we will assume throughout this chapter that the reduced mass equals the electron mass and the proton is located at the center of mass.

$$\hat{H}(r,\theta,\varphi) \cdot \psi(r,\theta,\varphi) = E\psi(r,\theta,\varphi) \quad 1$$

For the hydrogen atom, however, the distance, r , between the two particles can vary, unlike the diatomic molecule where the bond length was fixed, and the rigid rotor model was used. The hydrogen atom Hamiltonian also contains a potential energy term, V , to describe the attraction between the proton and the electron. This term is the Coulomb potential energy,

$$V(r) = \frac{1}{4\pi\epsilon_0 r} e \cdot (-e) \quad 2$$

where r is the distance between the electron and the proton. The Coulomb potential energy depends inversely on the distance between the electron and the nucleus and does not depend on any angles. Such a potential is called a central potential.

The time-independent Schrödinger equation is

$$\nabla^2 \psi + \frac{2m[E-V]}{\hbar^2} \psi = 0$$

Ψ is the wave function and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is Laplacian in cartesian coordinate.

Schrödinger equation in spherical coordinates

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi}{\partial \varphi^2} + \frac{2m}{\hbar^2} \left[E - \frac{e^2}{4\pi\epsilon_0 r} \right] \psi = 0 \quad 3$$

Since the angular momentum operator does not involve the radial variable, r , we can separate variables in Equation 3

$$\psi(r, \theta, \varphi) = R(r) \cdot \Theta(\theta) \cdot \Phi(\varphi) \quad 4$$

So eq 3 can be written as

$$\frac{\Theta \Phi}{r^2} \frac{d}{dr} \left(\frac{r^2 dR}{dr} \right) + \frac{R\Phi}{r^2 \sin\theta} \frac{d}{d\theta} \left(\sin\theta \frac{d\Theta}{d\theta} \right) + \frac{R\Theta}{r^2 \sin^2\theta} \frac{d^2\Phi}{d\varphi^2} + \frac{2m}{\hbar^2} \left[E - \frac{e^2}{4\pi\epsilon_0 r} \right] R\Theta\Phi = 0 \quad 5$$

Now multiplying eq 5 by the term $\frac{r^2 \sin^2\theta}{R\Theta\Phi}$, we get

$$\frac{\sin^2\theta}{R} \frac{d}{dr} \left(\frac{r^2 dR}{dr} \right) + \frac{\sin\theta}{\Theta} \frac{d}{d\theta} \left(\sin\theta \frac{d\Theta}{d\theta} \right) + \frac{1}{\Phi} \frac{d^2\Phi}{d\varphi^2} + \frac{2m}{\hbar^2} \left[E - \frac{e^2}{4\pi\epsilon_0 r} \right] r^2 \sin^2\theta = 0$$

$$\frac{\sin^2\theta}{R} \frac{d}{dr} \left(\frac{r^2 dR}{dr} \right) + \frac{\sin\theta}{\Theta} \frac{d}{d\theta} \left(\sin\theta \frac{d\Theta}{d\theta} \right) + \frac{2m}{\hbar^2} \left[E - \frac{e^2}{4\pi\epsilon_0 r} \right] r^2 \sin^2\theta = - \frac{1}{\Phi} \frac{d^2\Phi}{d\varphi^2} \quad 6$$

The LHS of the eq 6 is a function of r and θ while the RHS is the function of ϕ alone.

$$\text{Put } - \frac{1}{\Phi} \frac{d^2\Phi}{d\varphi^2} = m_l^2$$

$$\text{So, } \frac{d^2\Phi}{d\varphi^2} + m_l^2 \Phi = 0 \quad 7$$

Substituting m_l^2

$$\frac{1}{R} \frac{d}{dr} \left(\frac{r^2 dR}{dr} \right) + \frac{1}{\Theta \sin\theta} \frac{d}{d\theta} \left(\sin\theta \frac{d\Theta}{d\theta} \right) + \frac{2m}{\hbar^2} \left[E - \frac{e^2}{4\pi\epsilon_0 r} \right] r^2 = \frac{m_l^2}{\sin^2\theta}$$

$$\frac{1}{R} \frac{d}{dr} \left(\frac{r^2 dR}{dr} \right) + \frac{2m}{\hbar^2} \left[E - \frac{e^2}{4\pi\epsilon_0 r} \right] r^2 = \frac{m_l^2}{\sin^2\theta} - \frac{1}{\Theta \sin\theta} \frac{d}{d\theta} \left(\sin\theta \frac{d\Theta}{d\theta} \right)$$

$$\frac{m_l^2}{\sin^2\theta} - \frac{1}{\Theta \sin\theta} \frac{d}{d\theta} \left(\sin\theta \frac{d\Theta}{d\theta} \right) = l(l+1) \quad \text{This is called polar equation} \quad 8$$

$$\frac{1}{R} \frac{d}{dr} \left(\frac{r^2 dR}{dr} \right) + \frac{2mr^2}{\hbar^2} \left[E - \frac{e^2}{4\pi\epsilon_0 r} \right] = l(l+1) \quad \text{This is called radial equation.} \quad 9$$

The solution of equation 7, the solution for ϕ is $\Phi(\phi) = A e^{(im_l\phi)}$

For this function is single valued. The permitted value of m_l are

$m_l = 0, \pm 1, \pm 2, \pm 3, \pm 4, \dots$ and known as magnetic orbital quantum number of hydrogen atom.

The solution of equation 8, the solution for Θ are finite everywhere if $l = 0, 1, 2, \dots$ and known as orbital quantum number of hydrogen atom.

The solution of equation 9, the solution for R are finite everywhere are those for which

$$E = -\frac{me^4}{8\epsilon_0^2 n^2 \hbar^2} = -\frac{13.6 \text{ eV}}{n^2}, \quad n = 1, 2, 3, \dots \text{ and known as total quantum number of hydrogen atom.}$$

Significance of quantum number

Schrödinger's approach requires three quantum numbers (n , l , and m_l) to specify a wavefunction for the electron. The quantum numbers provide information about the spatial distribution of an electron. of quantum numbers:

- i. The principal quantum number (n): The principal quantum number helps in defining the size and energy of the orbit. n can be any positive integer (NOT zero). $n=1,2,3,4,\dots$
- ii. The orbital quantum number l describes the shape of the region of space occupied by the electron. The allowed values of l depend on the value of n and can range from 0 to $n - 1$: i.e. $l=0,1,2,3,\dots(n-1)$.
- iii. The magnetic quantum number (m_l) describes the orientation of the region of space occupied by an electron with respect to an applied magnetic field. The allowed values of m_l depend on the value of l : m_l can range from $-l$ to l in integral steps:

$$m_l = -l, -l+1, \dots, 0, \dots, l-1, l$$

For example, if $l=0$, m_l can be only 0; if $l = 1$, m_l can be -1 , 0, or $+1$;

and if $l = 2$, m_l can be -2 , -1 , 0, $+1$, or $+2$.

Degeneracy

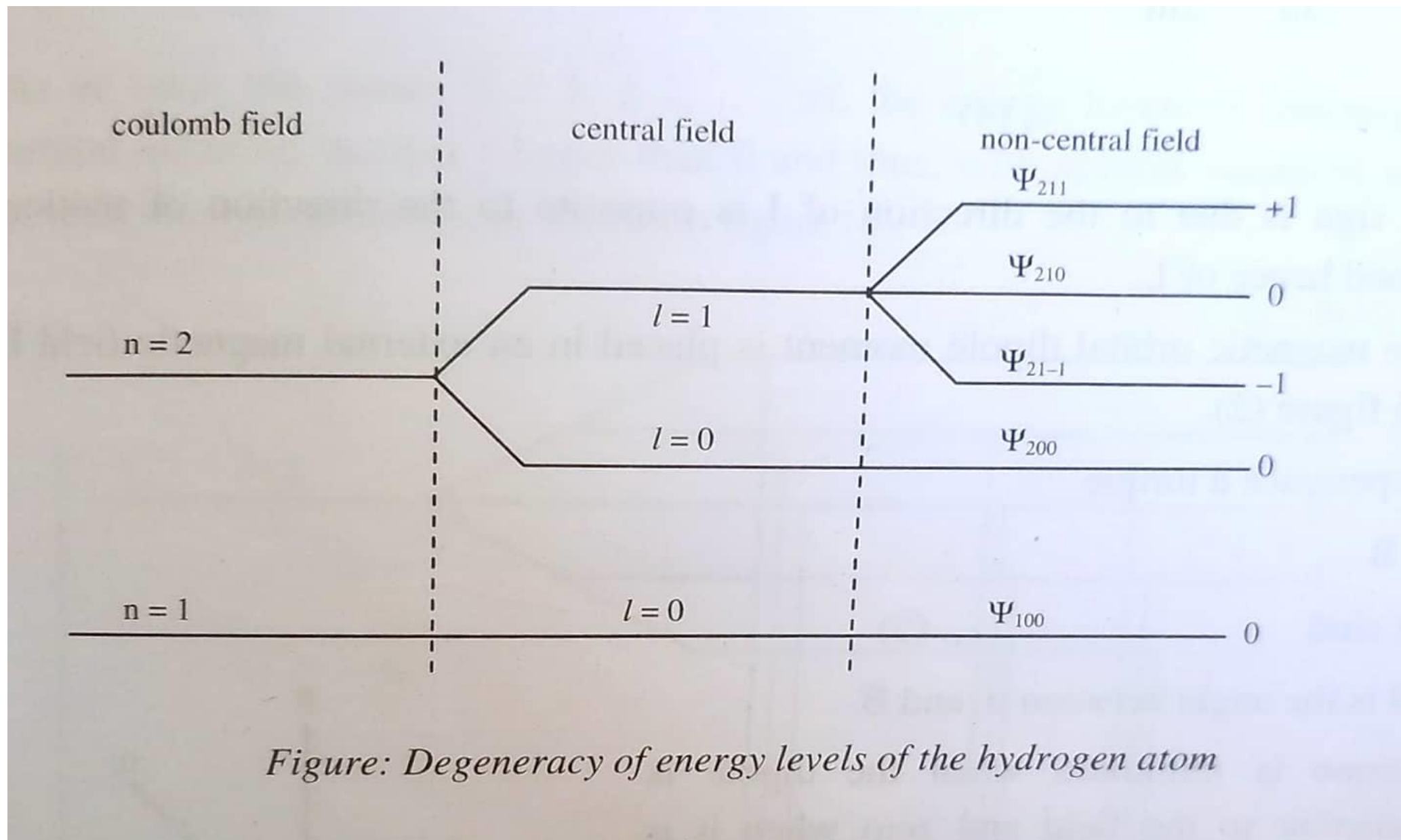
Each quantum state of the hydrogen atom is specified with three quantum numbers: n , l and m . Two or more different states of a quantum mechanical system are said to be degenerate if they give the same value

of energy upon measurement. Since
$$E = -\frac{me^4}{8\epsilon_0^2 n^2 h^2}$$

That means the E is independent of l and m . So how many states, $|n, l, m\rangle$, have the same energy for a particular value of n , for a particular value of n , l can range from 0 to $n - 1$. And each l can have different values of m , so the total degeneracy is $(2l + 1)$.

The degeneracy in m is the number of states with different values of m that have the same value of l . For any particular value of l , you can have m values of $-l, -l + 1, \dots, 0, \dots, l - 1, l$. And that's $(2l + 1)$ possible m states for a particular value of l . So you can plug in $(2l + 1)$ for the degeneracy in m :

$$\text{Degeneracy} = \sum_{l=0}^{n-1} (2l + 1)$$



Zeeman Effect

Zeeman effect is the effect of the splitting of a spectral line into two or more components of slightly different frequency when the light source is placed in a magnetic field. It is named after the Dutch physicist Pieter Zeeman, who discovered it in 1896 and received a Nobel prize for this discovery. Magnetic splitting or nuclear Zeeman effect is caused by the magnetic dipole interaction between the nuclear spin moment and the internal magnetic field at the nucleus.

The Zeeman effect has helped physicists determine the energy levels in atoms and identify them in terms of angular momenta. Zeeman effect is also applicable to know the magnetic field of space particles and various stars.

Normal Zeeman effect is the phenomenon that explains the splitting of a spectral line into three components in a magnetic field when observed in a direction perpendicular to the applied magnetic field. This effect is explained by the basis of classical physics. In normal Zeeman effect, only orbital angular momentum is considered. The spin angular momentum, in this case, is zero. The elements like He, Zn, Cd, Hg, etc give the normal Zeeman effect.

Anomalous Zeeman effect is the phenomenon that explains the splitting of a spectral line into four or more components in a magnetic field when viewed in a direction perpendicular to the magnetic field. This effect is more complex unlike normal Zeeman effect; thus, it can be explained by basis of quantum mechanics. The atoms with spin angular momentum show the anomalous Zeeman effect. The elements like Na, Cr, etc give the anomalous effect.

Let an electron of mass m be rotating with speed v in a circular orbit of radius r around the nucleus of an atom. So, it has a magnetic dipole moment associated with it, is called an orbital magnetic dipole moment μ .

$$\mu = I A$$

where I is the conventional current in the direction opposite to v ,

Since $I = \text{Charge of electron} / \text{time period}$

$$I = \frac{e}{T}$$

$$\mu = \frac{e.v}{2\pi r} \cdot A = \frac{e.v}{2\pi r} \cdot \pi r^2 = \frac{e.v.r}{2}$$

$$\mu = \frac{e.m.v.r}{2.m} = \frac{e}{2.m} L$$

when the magnetic dipole is placed in an external magnetic field B , then it experience a torque.

$$\tau = \mu \times B$$

$$\tau = \mu \times B$$

$$\tau = \mu B \sin\theta$$

the potential energy associated with it

$$E_B = \mu \cdot B = \mu B \cos\theta$$

$$= \frac{e}{2m} L B \cos\theta$$

$$E_B = \frac{e}{2m} B L_z$$

According to quantum mechanics $L_z = m_l \hbar$. Therefore the total energy is

$$E = E_0 + E_B$$

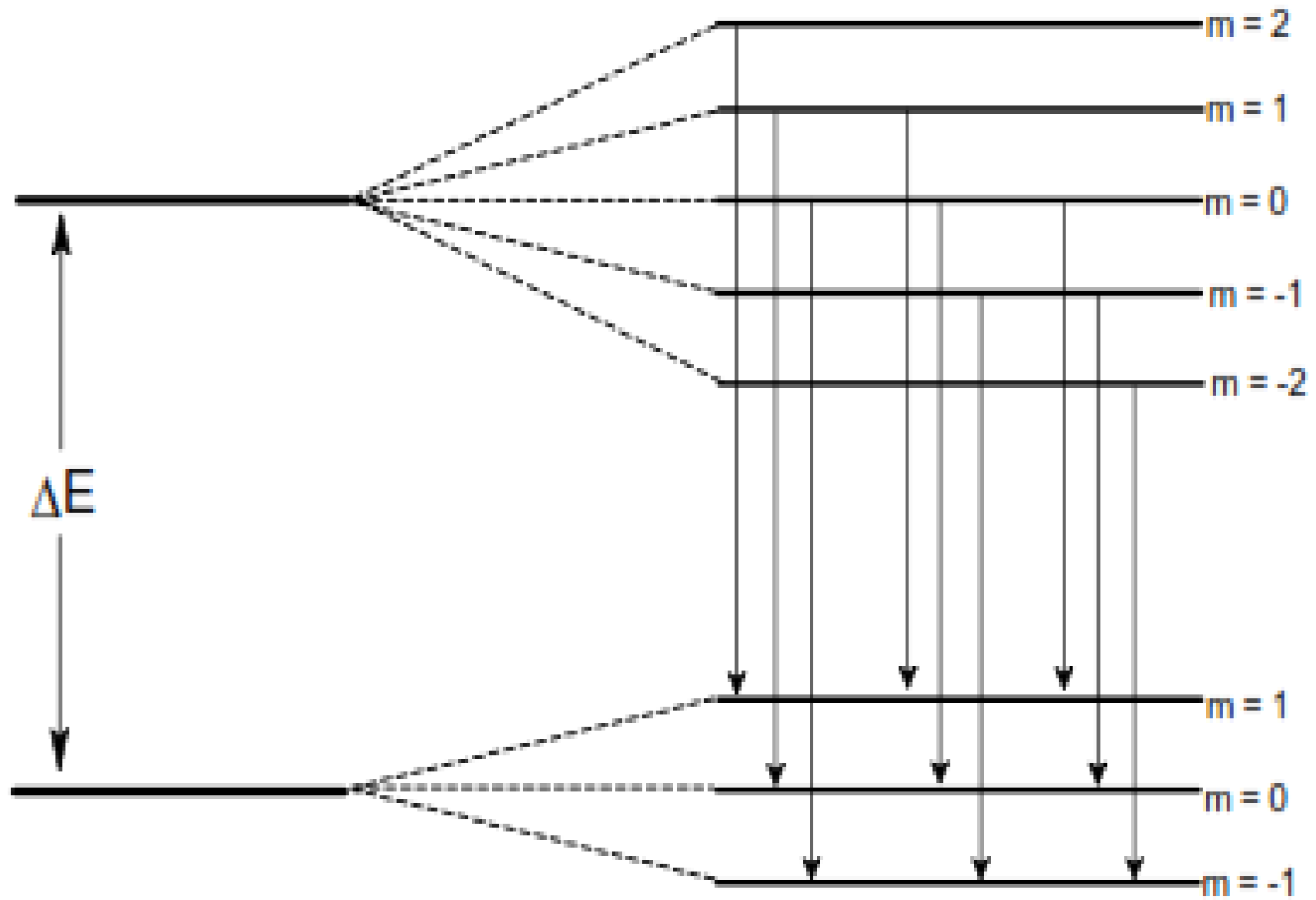
$$= E_0 + \frac{e}{2m} B L_z$$

$$= E_0 + \frac{e \hbar}{2m} B m_l$$

$$= E_0 + m_l \mu_B B$$

E_0 is the energy in the absence of the field and $\mu_B = \frac{e \hbar}{2m}$ is called the Bohr magneton, having dimensions the same as the dipole moment.

Since $m_l = -l$ to $+l$ in integral steps, thus, each energy level splits into $2l + 1$ levels. Using selection rule $\Delta m_l = 0, \pm 1$ nine transitions are possible from $l=2$, to $l=1$ as shown in figure.



Stern-Gerlach experiment

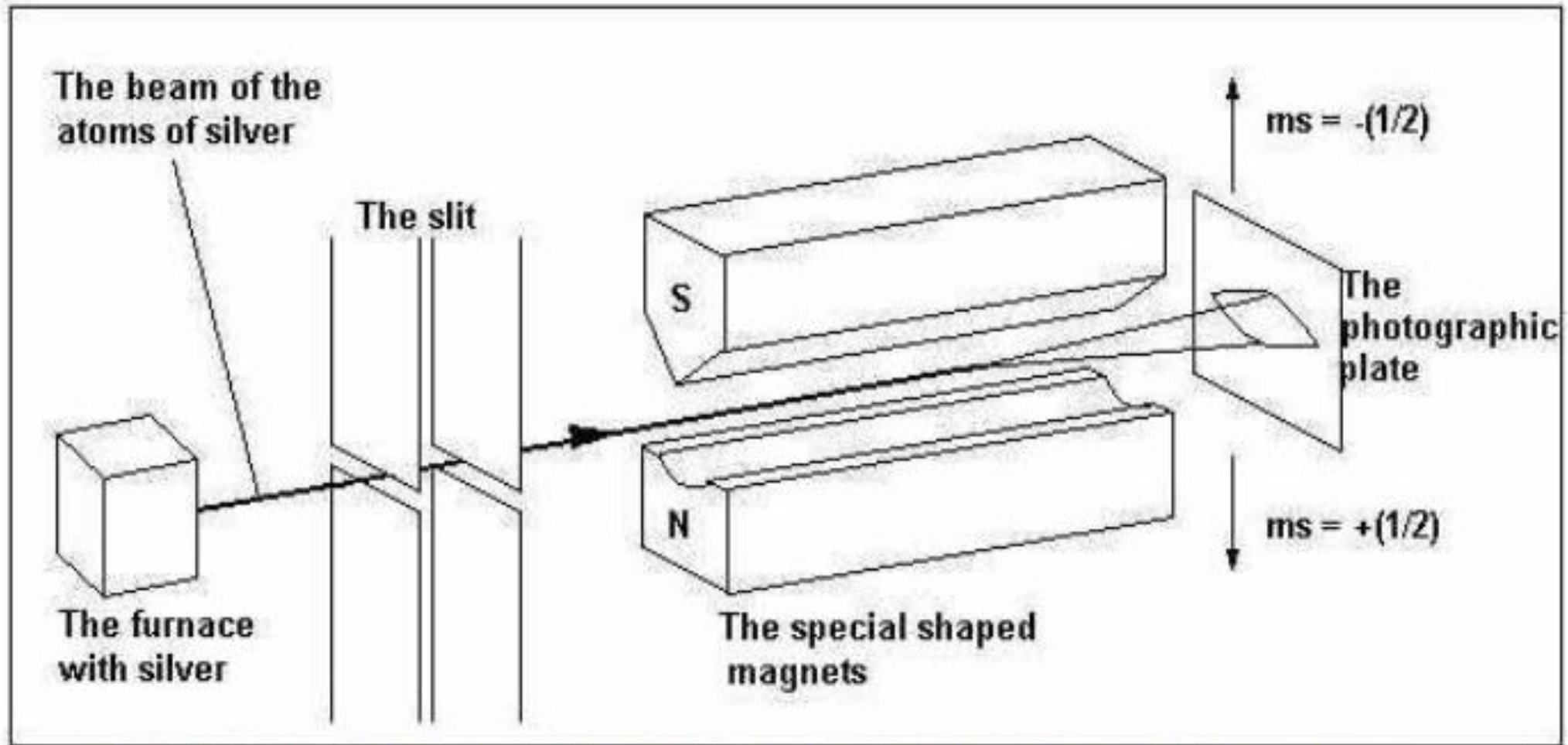


Fig. Experimental Setup for Stern and Gerlach Exp.

Consider a beam of Ag (silver) atom which was passed through an in homogenous magnetic field. To produce the beam , Ag (silver) was vaporized by heating it in a small electric oven ,which was furnished with a slit shaped exit aperture. Two further slits also exist. Then a thin beam of Ag which travel along X axis passed between the magnet pole pieces N and S. To avoid deflection of silver atoms the arrangement was enclosed in a highly evacuated glass vessel. The magnetic field between specially pole pieces had a large space rate of vibration provided by having one pole in the form of a knife edge and other in the form of channel. The magnetic field has much intensity near the knife edge than anywhere in the gap.

The beam strikes on glass plate P after emerging from the field. Two cases exist:

- In homogenous magnetic field the lines are equidistance from each other so there is no transverse displacement.
- In non-homogenous magnetic field transverse displacement is in +Z direction. When field off a thin straight line was obtained on P. When field is on the trace was divided into two lines except at the ends where Ag passes through the field outside the region of strongest field near the knife edge shaped pole piece. In silver atom the outermost electron is in a $2S_{1/2}$ state. For an s state of an electron, the orbital quantum number $l=0$. Therefore, the orbital magnetic dipole moment of the electron is zero. This result confirms the existence of electron spin and the postulates of Space Quantization.