Oscillatory Motion

Oscillatory Motion

Oscillatory motion is defined as the to and fro motion of an object from its mean position.

Examples of Oscillatory Motion

- Oscillation of simple pendulum
- Vibrating strings of musical instruments
- Movement of spring
- Alternating current
- Cosmological model

Simple Harmonic Motion

Simple harmonic motion (SHM) is a type of oscillatory motion which is defined for the particle moving along the straight line with an acceleration which is moving towards a fixed point on the line such that the magnitude is proportional to the distance from the fixed point.

A simple harmonic motion can be represented by the relations

$$y = a \sin(\omega t + \Phi) \;\ldots\ldots\ldots\; 1$$

or

$$y = a \cos(\omega t + \Phi) \;\ldots\ldots\ldots\; 2$$

Simple harmonic oscillator.

The system when displaced from its equilibrium position experiences a restoring force F proportional to the displacement y is known as harmonic oscillator.

This restoring force acts in the direction opposite the displacement from the equilibrium position.

From Newton's second law of motion,

F = ma

where F is restoring force of oscillator = - ky.

$$-k\,y = m\frac{d^2y}{dt^2}$$

$$\frac{d^2y}{dt^2} + \frac{ky}{m} = 0$$

$$\frac{d^2y}{dt^2} + \omega^2 y = 0 \qquad \text{where } \omega^2 = \frac{k}{m}$$

This is a second order, linear differential equation.

Multiplying on both sides by $2\frac{dy}{dt}$

$$2\frac{dy}{dt}\frac{d^2y}{dt^2} + 2\,\omega^2 y\,\frac{dy}{dt} = 0$$

Integrating on both sides, we get

$$\left(\frac{dy}{dt}\right)^2 + \omega^2 y^2 = C$$

When the displacement is maximum i.e. at $y = a$ and $\frac{dy}{dt} = 0$

$$0 + \omega^2 a^2 = C \quad \text{or} \quad C = \omega^2 a^2$$

$$\left(\frac{dy}{dt}\right)^2 + \omega^2 y^2 = \omega^2 a^2$$

$$v = \frac{dy}{dt} = \omega\sqrt{(a^2 - y^2)}$$

this is the expression for **velocity** of a particle executing simple harmonic motion at a time t.

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again $\frac{dy}{\sqrt{(a^2 - y^2)}} = \omega dt$

integrating $\quad \sin^{-1}\frac{y}{a} = \omega t + \Phi \qquad \frac{y}{a} = \sin(\omega t + \Phi)$

$y = a\sin(\omega t + \Phi)$ where Φ is known as **phase constant**.

Acceleration $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d}{dt}[a\omega\cos(\omega t + \Phi)] = -a\omega^2\sin(\omega t + \Phi)$

$a = -\omega^2 y$

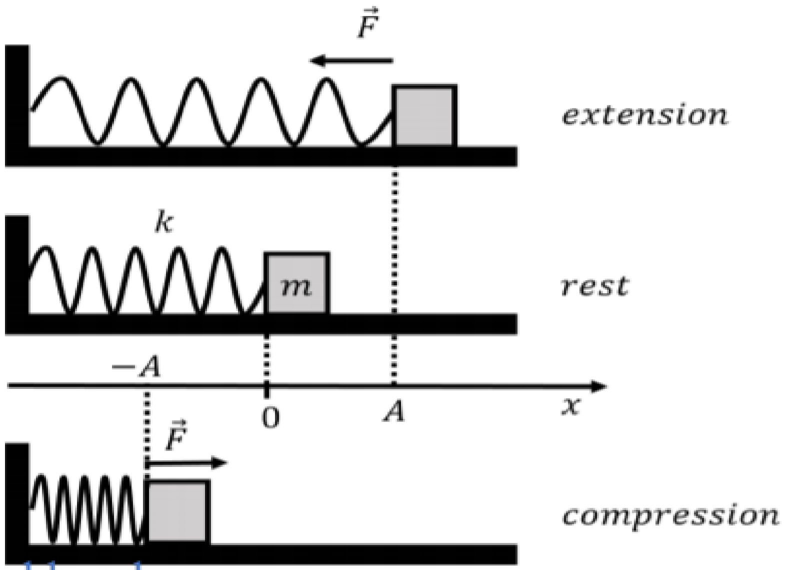
Time period $\quad T = \frac{2\pi}{\omega} = 2\pi\frac{1}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}}$

Therefore the **frequency** of oscillator Physics class by Peshal Pokharel

Mass Spring System

A spring-mass system consists of suspending or attaching a mass to the free end of the spring, while the other end of the spring is connected to a fixed point. The spring-mass system can usually be used to find the period of any object that performs simple harmonic motion. For example, the spring-mass system can be used to simulate the movement of human tendons using computer graphics, as well as the deformation of the skin of the feet.

It is assumed that the mass 'm' and the spring are on a smooth horizontal surface. When the force is applied to stretch the spring and then released, the spring regains the position of equilibrium.



Due to this force, the mass begins to vibrate back and forth. If x is displacement of mass from its equilibrium position at any instant of time t, then from Hoke's law the restoring force acting on the mass is given by

Force α extension

$$F = -k\,x \qquad \ldots\ldots\ldots\ldots\ldots \qquad 1$$

Here –ve sign shows that force and displacement are in opposite direction , and $k = \frac{F}{x}$ be the proportionality constant and called force constant or stiffness of spring

From Newtons second law of motion

$$F = \frac{dP}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} = m\,\frac{d^2X}{dt^2} \qquad \ldots\ldots\ldots\ldots \qquad 2$$

From equation 1 and 2

$$m\frac{d^2X}{dt^2} = -kx$$

$$\frac{d^2X}{dt^2} + \frac{k}{m}\ x = 0$$

$$\frac{d^2X}{dt^2} + \omega^2\ x = 0 \qquad \text{.............} \qquad 3$$

This is differential equation of mass attached to the spring, as the acceleration is proportional to displacement, the motion of mass on the spring is simple harmonic and time period of oscillation

$$T = \frac{2\pi}{\omega} = 2\pi\ \frac{1}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}} \qquad \text{..........}4$$

The equation shows that the period of oscillation is independent of both the amplitude and acceleration due to gravity. The above equation is also valid in the case when a constant force is being applied on the mass, i.e. a constant force can not change the period of oscillation.

Energy In Simple Harmonic Motion

A particle executing SHM is moving under the action of restoring force so this particle possesses both potential energy due to restoring force and kinetic energy due to its motion. Total mechanical energy is the sum of kinetic energy and potential energy. Consider a particle with mass m performing simple harmonic motion the instantaneous velocity of the particle performing S.H.M. at a distance x from the mean position is given by

$$v = \omega \sqrt{(a^2 - x^2)}$$

$$\therefore\ v^2 = \omega^2 (a^2 - x^2)$$

$\therefore$ Kinetic energy

$$K.E. = \frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 (a^2 - x^2) \quad \ldots\ldots\ldots \quad 1$$

Consider a particle of mass m performing simple harmonic motion at a distance x from its mean position. Since the restoring force acting on the particle is

F= -kx where k is the force constant.

Now, the particle is given further infinitesimal displacement dx against the restoring force F. Let the work done to displace the particle be dw. Therefore, The work done dw during the displacement is

$$dw = -F \cdot dx = -(-k.\ x)dx = k.x.dx$$

Therefore, the total work done to displace the particle now from 0 to x is

$$= \int_0^x dw = \int_0^x kxdx = k \int_0^x x\,dx$$

Hence Total work done $= \frac{1}{2}kx^2 = \frac{1}{2}m.\omega^2\ x^2$

The total work done here is stored in the form of potential energy.

$\therefore$ Potential energy P.E. $= \frac{1}{2}m\omega^2\ x^2$

Total energy = Kinetic energy + Potential energy

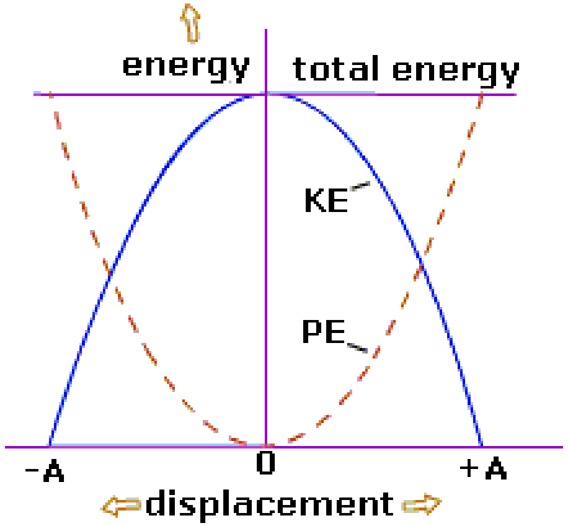
$$E = \frac{1}{2}\, m\,\omega^2\,(a^2 - x^2) + \frac{1}{2} m\omega^2\, x^2$$

$$E = \frac{1}{2} m\omega^2\, a^2$$

The law of conservation of energy states that energy can neither be created nor destroyed. Therefore, the total energy in simple harmonic motion will always be constant. However, kinetic energy and potential energy are interchangeable. Given below is the graph of kinetic and potential energy vs instantaneous displacement.

At the **mean position**, the total energy in simple harmonic motion is purely kinetic and at the **extreme position**, the total energy in simple harmonic motion is purely potential energy.

At **other positions**, kinetic and potential energies are interconvertible and their sum is equal to $\frac{1}{2} k\omega^2\, a^2$.



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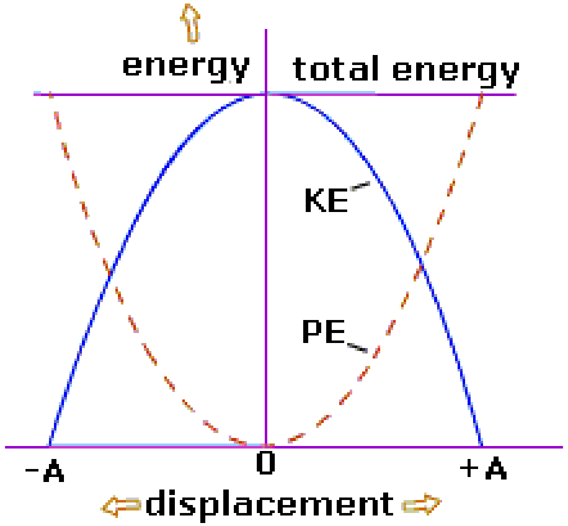

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Electric and Magnetic Fields

- Electric charge
- The electric force
- Coulombs law

According to Coulomb's law, the force of attraction or repulsion between two charged bodies is directly proportional to the product of their charges and inversely proportional to the square of the distance between them. It acts along the line joining the two charges considered to be point charges.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Where ε_0 be the permittivity of free space.

Permeability ε

It is ability of a material to store electrical potential energy under the influence of an electric field measured by the ratio of the capacitance of a capacitor with the material as dielectric to its capacitance with vacuum . It is also called also dielectric constant.

Permittivity is the measure of how easy or difficult it is to form an electric field inside a medium.

Permittivity of free space(ε_0) is the physical quantity, is the capability of a vacuum to allow electric fields to pass through it. It is the ability of the free space or vacuum to permit electric field lines to pass through. It is generally equal to 8.85×10^{-12} F/m.

Permittivity plays a **significant** role in electrostatics. It determines the capacitance of the capacitor. Permittivity depends on the frequency, magnitude, and direction of the applied field.

The **electric field intensity** at a point is the force experienced by a unit positive charge placed at that point.

Electric Field Intensity is a vector quantity.

It is denoted by ‘E’.

Formula: Electric Field = F/q.

Unit of E is NC^{-1} or Vm^{-1}.

The electric field intensity due to a positive charge is always directed away from the charge and the intensity due to a negative charge is always directed towards the charge.

Due to a point charge q, the intensity of the electric field at a point d units away from it is given by the expression:

Electric Field Intensity (E) == $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ NC^{-1}

The intensity of the electric field at any point due to a number of charges is equal to the vector sum of the intensities produced by the separate charges.

Electric potential Energy

Electric potential energy is the energy that is needed to move a charge against an electric field. The magnitude of electric potential depends on the amount of work done in moving the object from one point to another against the electric field.

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$$U= -W = - \int_{\infty}^{0} F\,.dr = - \int_{\infty}^{0} \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\,.dr =- \frac{q_1 q_2}{4\pi\varepsilon_0}\int_{\infty}^{0}\frac{1}{r^2}\,.dr = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

For a system of charges q_1,q_2,q_3,……. separated by a distance r_{12},r_{13},r_{23},……, the electric potential energy is given by

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} + \ldots\ldots\ldots\right]$$

$$U= \frac{1}{4\pi\varepsilon_0}\sum_{i\neq j}\frac{q_i q_j}{r_{ij}}$$

Electric potential

Electric potential energy per unit test charge at a point in side electric field is called electric potential.

$$V = \frac{U}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_0}{r . q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad \text{unit} \quad \frac{J}{C} = \text{volt}$$

Potential difference

potential difference is the amount of work done in moving the unit positive test charge from one point to another against the electric field.

$$V_{AB} = -W = -\int_{r_A}^{r_B} F \, . dr = -\int_{r_A}^{r_B} \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2} . dx$$

$$= -\frac{q}{4\pi\varepsilon_0}\int_{r_A}^{r_B}\frac{1}{x^2} . dx \;=\; \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

Force on a Moving Charge

Magnetic fields can exert a force on electric charge only if it is moving, just as a moving charge produces a magnetic field. This force increases with both an increase in charge and magnetic field strength. Moreover, the force is greater when charges have higher velocities.

Experimentally it is found that, magnitude of magnetic force

i. is directly proportional to amount of charge.

ii. is directly proportional to velocity of charge particle.

iii. is directly proportional to strength of magnetic field.

iv. is directly proportional to sine of angle between velocity and magnetic field.

$F = q\ v\ B \sin\theta$

The magnetic force, however, always acts perpendicular to the velocity.

$F = q\ (v \times B)$

The direction of the force can be determined using Fleming's Right-hand Rule.

Lorentz Force

When a charge travels through both an electric and magnetic field, the net force on the charge is called the Lorentz force. It is simply the sum of the magnetic and electric forces:

$$F = F_e + F_m = q\,E + q\,(v \times B)$$

$$F = q\,(E + v \times B)$$

Magnetic Permeability

The magnetic permeability is defined as the property exhibited by the material where the material allows the magnetic line of force to pass through it. The SI unit of magnetic permeability is Henry per meter.

Magnetic permeability is defined as the ratio of flux density to the magnetic force which is given as:

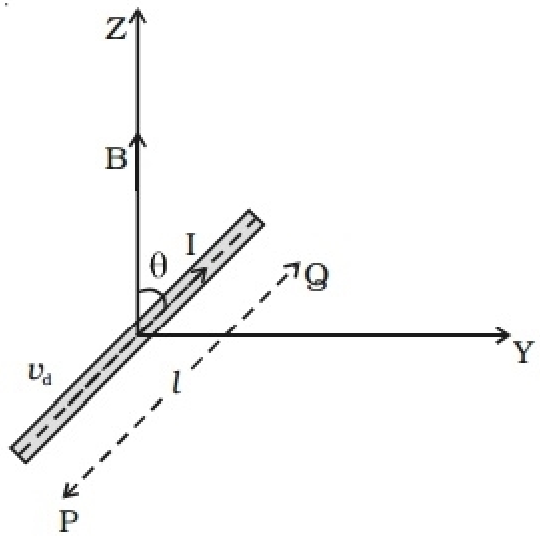
$$\mu = \frac{B}{H}$$

Absolute permeability is related to the permeability of free space and is a constant value which is given as: $\mu_0 = 4\pi \times 10^{-7}$ Wb $A^{-1}m^{-1}$.

Force on a current carrying conductor placed in a magnetic field

Let us consider a conductor PQ of length l and area of cross section A. The conductor is placed in a uniform magnetic field of induction B making an angle θ with the field

A current I flows along PQ. Hence, the electrons are drifted along QP with drift velocity vd. If n is the number of free electrons per unit volume in the conductor, then the current is



$I = nAev_d$

Multiplying both sides by the length l of the conductor,

$I.l = l.\ nAev_d$

Since the electrons move under the influence of magnetic field, the magnetic lorentz force on a moving electron.

$f = -\ e\ (v_d \times B)$ The negative sign indicates that the charge of the electron is negative.

The number of free electrons in the conductor

$$N = n A l$$

The magnetic Lorentz force on all the moving free electrons

$$F = N f$$

$$= n A l [-e (v_d \times B)]$$

$$= -n A l e (v_d \times B)$$

$$F = I (l \times B)$$

Direction of force

The direction of the force on a current carrying conductor placed in a magnetic field is given by Fleming’s Left Hand Rule. The forefinger, the middle finger and the thumb of the left hand are stretched in mutually perpendicular directions. If the forefinger points in the direction of the magnetic field, the middle finger points in the direction of the current, then the thumb points in the direction of the force on the conductor.

Torque experienced by a current loop in uniform magnetic field

Let us consider a rectangular loop PQRS of length l and breadth b (Fig). It carries a current of I along PQRS. The loop is placed in a uniform magnetic field of induction B. Let θ be the angle between the normal to the plane of the loop and the direction of the magnetic field.

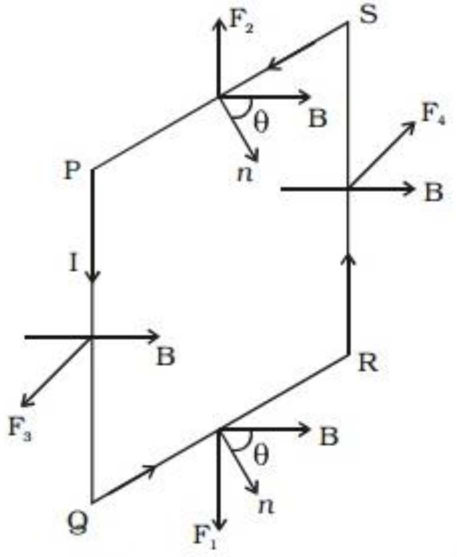


Fig Torque on a current loop placed in a magnetic field

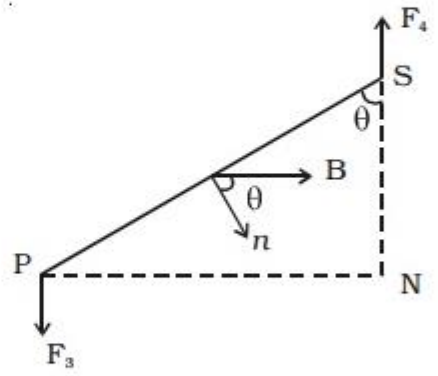


Fig Torque

Force on arm QR $F_1 = I\,B\,b\,\sin(90^0 - \theta) = I\,B\,b\,\cos\theta$

Force on arm SP $F_2 = I\,B\,b\,\sin(90^0 + \theta) = I\,B\,b\,\cos\theta$

The force F1 and F2 are equal in magnitude, opposite in direction and have the same line of action.

Hence their resultant effect on the loop is equal to zero.

Force on arm PQ F3= I B L sin900 = I B L

F3 acts perpendicular to the plane of the paper and outwards

Force on arm RS F4= I B L sin900 = I B L

F4 acts perpendicular to the plane of the paper and inwards.

The forces F3 and F4 are equal in magnitude, opposite in direction and have different lines of action.

So, they constitute a couple.

Hence, Torque $\tau = BIL \times PN = BIl \times PS \times \sin\theta = BIL \times b \sin\theta = BIA \sin\theta$

If the coil contains n turns, $\tau = nIBA \sin\theta$

So, the torque is maximum when the coil is parallel to the magnetic field and zero when the coil is perpendicular to the magnetic field.

The Magnetic moment

The magnetic moment of a current loop can be defined as the product of the current flowing in the loop and the area of the rectangular loop. Mathematically,

$\mu = I.\, A$

Here, A is a vector quantity with the magnitude equal to the area of the rectangular loop and the direction is given by the right-hand thumb rule. In the equation written earlier, we can see that the torque exerted on a current-carrying coil placed in a magnetic field can be given by the vector product of the magnetic moment and the magnetic field.

$\tau = \mu \times B = \mu B \sin\theta$

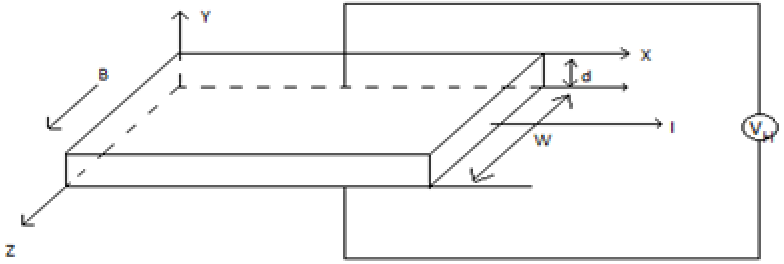
Since a magnetic dipole experiences a torque when placed in an external field, work must be done to change its orientation. This work done is also be referred as energy of dipole

$$U = \int_{\theta_i}^{\theta_f} \tau \,.\, d\theta = \int_{\theta_i}^{\theta_f} \mu B \sin\theta . d\theta = \int_{90^0}^{\theta_f} \mu B \sin\theta . d\theta = \mu B \int_{90^0}^{\theta_f} \sin\theta . d\theta$$

$U = - \mu B \cos\theta = - \mu . B$

Hall effect

When a current-carrying conductor is perpendicular to a magnetic field, a voltage generated is measured at right angles to the current path. Hall effect is defined as the production of a voltage difference across an electrical conductor which is transverse to an electric current and with respect to an applied magnetic field it is perpendicular to the current.



The above figure shows a conductor placed in a magnetic field (B) along the z-axis. The current (I) flows through it along the x-axis

Hall voltage (V_H) is developed along y-axis with electric field intensity E_H.

At Equilibrium, Force due to Hall voltage on charge carriers = Force due to magnetic field

$e.E_H = B.e.v$ 1

Where, v = Drift velocity.

Sience current density J = -nev or $v = \frac{-J}{ne}$

$$e.E_H = B.e. \frac{-J}{ne}$$

$$\frac{E_H}{JB} = \frac{-1}{ne}$$

The term $\frac{E_H}{JB}$ is called Hall coefficient R_H

$$R_H = \frac{E_H}{JB} = \frac{-1}{ne} \qquad 2$$

Let V_H be the hall voltage and d be the width of the strip then

$$E_H = \frac{V_H}{d} \qquad 3$$

From eq 1, 2 & 3

$$e. \frac{V_H}{d} = B.e. \frac{I}{A.ne}$$

$$V_H = \frac{I.B}{A.n.e}.d = \frac{I.B}{thickness.width.n.e}.d == \frac{I.B}{x.n.e}$$ is Hall voltage

$$\frac{V_H}{I} = \frac{B}{x.n.e}$$ is Hall resistance

Since

mobility of the carrier is defined as the ratio of drift velocity acquired in unit applied electric field.

$$\mu = \frac{v}{E}$$

$$\mu = \frac{1}{E} \cdot \frac{J}{ne} = \frac{1}{E} \cdot \frac{\sigma E}{ne}$$

$$= \frac{\sigma}{ne} = \sigma . R_H$$

$$Since\ \sigma = \frac{1}{\rho}$$

$$\mu = \frac{R_H}{\rho}$$

Where ρ is resistivity

Applications of Hall effect

- It is used to determine if the given material is a superconductor, semiconductor or insulator.
- It is used to measure the magnetic field and current mobility.
- It is used to determine the polarity of current.
- The number of free electrons per unit volume of conductors from Hall coefficient R_H.
- They find applications in position sensing as they are immune to water, mud, dust, and dirt.
- They are used in integrated circuits as Hall effect sensors.
- Some of the examples for the application of Hall Effect sensors are the current transformers, Position sensing, Galaxy S4 Accessories, Keyboard switch, computers, Proximity sensing, speed detection, current sensing applications, tachometers, anti-lock braking systems, magnetometers, DC motors, disk drives etc…

Properties of Electromagnetic Waves.

- Electromagnetic waves are transverse in nature as they propagate by varying the electric and magnetic fields such that the two fields are perpendicular to each other.
- Accelerated charges are responsible to produce electromagnetic waves.
- Electromagnetic waves have constant velocity in vacuum

$$C = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3.0 \text{ x } 10^8 \text{ m/sec}$$

- Electromagnetic wave propagation does not require any material medium to travel.
- The inherent characteristic of an electromagnetic wave is its frequency. Their frequencies remain unchanged but its wavelength changes when the wave travels from one medium to another.
- The refractive index of a material is given by:

$$= \sqrt{\varepsilon_r \mu_r}$$

- Electromagnetic wave follows the principle of superposition

- The light vector (also known as the electric vector) is the reason for the optical effects due to an electromagnetic wave.
- In an electromagnetic wave, the oscillating electric and magnetic fields are in the same phase and their magnitudes have a constant ratio. The ratio of the amplitudes of electric and magnetic fields is equal to the velocity of the electromagnetic wave. $C = \frac{E_0}{B_0}$
- The energy is carried by the electric and magnetic fields of electromagnetic waves are equal, i.e. the electric energy (U_e) and the magnetic energy (U_m) are equal;
- There is a vector quantity S called the Poynting vector which represents the energy transferred by electromagnetic waves per second per unit area.

$$S = \frac{1}{\mu_0} [E \times B]$$

- Electromagnetic radiation from outer space has given us so much information about the universe, its existence and other celestial bodies.