Model Questions Sets For Practice

MODEL SET 1

Bachelor Level/First Year/Second Semester/Science — Full Marks: 80
Computer Science and Information Technology [MTH. 163] — Pass Marks: 32
(Mathematics II) — Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Group 'A'

Attempt any three questions: **($3 \times 10 = 30$)**

1. Let $a_1 = (1, -2, -5)$, $a_2 = (2, 5, 6)$ and $b = (7, 4, -3)$ are three vectors. Determine whether b can be generated as a linear combination of a_1 and a_2. That is, determine whether x_1 and x_2 exist such that $b = a_1x_1 + a_2x_2$ has solution. Find it.

2. Let $\begin{bmatrix} 3 & 0 & -1 & 5 & 9 & -2 \\ -5 & 2 & 4 & 0 & -3 & 1 \\ -8 & -6 & 3 & 1 & 7 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 2 & 3 \\ 1 & 5 \\ 4 & 1 \\ -1 & 2 \\ 2 & 3 \end{bmatrix}$

   Compute AB if possible. **Ans:** $\begin{bmatrix} 15 & 18 \\ -2 & 13 \\ -44 & -36 \end{bmatrix}$

3. Find the bases for the row space, column space and the null space of the matrix,

$$\begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$$

[**Ans:** Row space: {(1, 3, −5, 1, 5), (0, 1, −2, 2, −7), (0, 0, 0, −4, 20)}

Column space: {(−2, 1, 3, 1), (−5, 3, 11, 7), (0, 1, 7, 5)}

Null: {(−1, 2, 1, 0, 0), (−1, −3, 0, 5, 1)}]

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4. Find QR factorization of $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Ans: $Q = \begin{bmatrix} \frac{1}{2} & \frac{-3}{\sqrt{12}} & 0 \\ \frac{1}{2} & \frac{1}{\sqrt{12}} & \frac{-2}{\sqrt{6}} \\ \frac{1}{2} & \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{6}} \\ \frac{1}{2} & \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{6}} \end{bmatrix}$ $R = \begin{bmatrix} 2 & 3/2 & 1 \\ 0 & 3/\sqrt{12} & 2/\sqrt{12} \\ 0 & 0 & 2/\sqrt{6} \end{bmatrix}$

Group 'B'

Attempt any ten questions: **(10×5 = 50)**

5. Define a system of linear equations and to solution. When a system is consistent and inconsistent? Give the graphical representation of consistency of linear equations.

6. Define the standard matrix for a linear transformation T. find the standard matrix A for the linear transaction $T(x) = 4x$ for $x \in \mathbb{R}^2$.

Ans: $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

7. By using inverse matrix method, solve the system:

$3x_1 + 4x_2 = 3$

$5x_1 + 6x_2 = 7$

Ans: $(x_1, x_2) = (5, -3)$

8. Using determinant, determine whether the vectors v_1, v_2, v_3, are linearly independent or not where

$v_1 = \begin{bmatrix} 5 \\ -7 \\ 9 \end{bmatrix}$, $v_2 = \begin{bmatrix} 3 \\ -3 \\ 5 \end{bmatrix}$, $v_3 = \begin{bmatrix} 2 \\ -7 \\ 5 \end{bmatrix}$

Ans: Independent

9. Let $B = \{1 + t^2, t + t^2, 1 + 2t + t^2\}$ is a basis for P_2. Find the coordinate vector $P(t) = 1 + 4t + 7t^2$ relative to B.

Ans: (2, 6, 1)

10. Find the solution of the difference equation,

$y_{k+3} - 2y_{k+2} - 5y_{k+1} + 6y_k = 0$ for all k.

Ans: $(1^k, -2^k, 3^k)$

1. Determine the eigen values and eigen vectors of $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ in complex number.

Ans: $i, -i$; $\begin{bmatrix} i \\ 1 \end{bmatrix}$, $\begin{bmatrix} -i \\ 1 \end{bmatrix}$

12. Is the set of vectors $\{(1, 0, 1), (0, 1, 0), (-1, 0, 1)\}$ orthogonal? Obtain the corresponding orthonormal set is $\mathbb{R}^3$.

Ans: $\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$ $(0, 1, 0)$ $\left(\frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

13. Let $(Z, +)$ and $(2Z, +)$ are two binary structures where Z is the set of all integers then show that $\phi: Z \to 2Z$, is defined by $\phi(n) = 2n$ is an isomorphism.

14. Solve the equation $x^2 + 2x + 4 = 0$ in Z_4. Ans: 2

15. Let $A = \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix}$, $b_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $b_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $B = \{b_1, b_2\}$. Find the B–matrix of the transformation $x \to Ax$ with $P = [b_1, b_2]$. Ans: $\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$

MODEL SET 2

Group 'A'

Attempt any three questions: **(3×10 = 30)**

1. Determine if the following homogeneous system has a non-trivial solution. Then describe the solution set.

$3x_1 + 5x_2 - 4x_3 = 0$

$-3x_1 - 2x_2 + 4x_3 = 0$

$6x_1 + x_2 - 8x_3 = 0$

Ans: $x_3\left(\frac{4}{3}, 0, 1\right)$

2. Let A is $n \times n$ matrix, is invertible if and only if A is row equivalent to in. Use this statement to find A^{-1} if exists where

$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & 3 & 8 \end{bmatrix}$.

Ans: $\begin{bmatrix} -9/2 & 7 & -3/2 \\ -2 & 4 & -1 \\ 3/2 & -2 & 1/2 \end{bmatrix}$

3. Define basis of a subspace of a vector space. Let $v_1 = (0, 2, -1)$, $v_2 = (2, 2, 0)$, $v_3 = (6, 16, -5)$ where $v_4 = 5v_1 + 3v_2$ and let H = Span $\{v_1, v_2, v_3\}$, show that Span $\{v_1, v_2, v_3\}$ = Span (v_1, v_2) and find a basis for a subspace H.

4. Find the equation $y = \beta_0 + \beta_1 x$ of the least squares line that best fits the given data. Points: (–1, 0), (0, 1), (1, 2) and (2, 4). Ans: $y = 1.1 + 1.3x$

Group 'B'

Attempt any ten questions: **(10×5 = 50)**

5. If $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ show that $\begin{bmatrix} h \\ k \end{bmatrix}$ is the Span $\{u, v\}$ for all h and k.

Ans: $h = 4, k = 0$

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If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and define $T = \mathbb{R}^2 \to \mathbb{R}^2$ by $T(x) = Ax$, then find the image under T of $u = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ and $v = \begin{bmatrix} a \\ b \end{bmatrix}$. **Ans:** $\begin{bmatrix} -1 \\ -2 \end{bmatrix}, \begin{bmatrix} a \\ b \end{bmatrix}$

Write the algorithm for finding the inverse of A. Using this find the inverse of $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$ **Ans:** $\begin{bmatrix} \frac{-9}{2} & 7 & \frac{-3}{2} \\ -2 & 4 & -1 \\ \frac{3}{2} & -2 & \frac{1}{2} \end{bmatrix}$

Let S be parallelogram determined by vectors $b_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $b_2 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ and let $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$. Compute the area of image of S under the mapping $x \to Ax$. **Ans:** 28

9. Determine, the set of vectors from a basis for $\mathbb{R}^3$ or not: (1, 4, 3), (0, 3, 1), (3, –5, 4), (0, 2, – 2) **Ans:** No.

10. Find the basis and dimension of $\left\{ \begin{bmatrix} 2c \\ a-b \\ b-3c \\ a+2b \end{bmatrix}, a, b, c \in \mathbb{R} \right\}$.

Ans: basis = $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -3 \\ 0 \end{bmatrix} \right\}$, dim = 3

11. Find the eigenvalue of $\begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$. **Ans:** 1, 2, 3

12. Define inner product space. Let $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are in $\mathbb{R}^2$ and defined as $\langle u, v \rangle = 4u_1v_1 + 5u_2v_2$ then show that $\langle u, v \rangle$ defines and inner product.

13. Define subgroup. Let $\mathbb{Z}_4 = \{0, 1, 2, 3\}$ is a group under addition. Determine whether $H = \{0, 1, 3\}$ is a subgroup of $\mathbb{Z}_4$ or not. **Ans:** not

14. Solve the equation $x^2 - 5x + 6 = 0$ in $\mathbb{Z}_{12}$. **Ans:** 2, 3, 6, 11

15. Define rank of a matrix and state the rank theorem. If A is 7×9 matrix with two-dimensional null space, find the rank of A.

MODEL SET 3

Group 'A'

Attempt any three questions: **(3×10 = 30)**

1. Determine if the following system is consistent if consistent solve the system.

$$-2x_1 - 3x_2 + 4x_3 = 5$$
$$x_1 - 2x_3 = 4$$
$$x_1 + 3x_2 - x_3 = 2$$

Ans: $\left(26, \frac{-13}{3}, 26\right)$

2. Let $v_1 = (3, 6, 2)$, $v_2 = (-1, 0, 1)$, $x = (3, 12, 7)$ and $B = \{v_1, v_2\}$.then B is a basis for $H = \text{Span}\{v_1, v_2\}$. Determine if x is in H and if it is, find the coordinate vector of x relative to B. **Ans:** $[x]_B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

3. Find the LU factorization of the matrix, $\begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -1 & 7 & 0 \end{bmatrix}$.

Ans: $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ \frac{-1}{3} & 1 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 0 & 5 \end{bmatrix}$

4. Find the last squares line $y = \beta_0 + \beta_1 x$ that best fits the data (–2, 3), (–1, 5), (0, 5), (1, 4) and (2, 3). Suppose the errors in measuring the y–values of the last two data points are greater than for the other points. Weight these data half as much as the rest of the data. **Ans:** $y = 4.0 + (0.1)x$

Group 'B'

Attempt any ten questions: **(10×5 = 50)**

5. Define a subset spanned by vectors. Give the geometrical description of span {u, v} in $\mathbb{R}^3$.

6. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such the $T(x_1, x_2) = (x_1 + x_2, 4x_1 + 5x_2)$. Find the x such that $T(x) = (3, 8)$. **Ans:** (7, – 4)

7. Consider the production model $x = Cx + d$ for an economy with two sectors where,

$C = \begin{bmatrix} 0 & 0.5 \\ 0.6 & 0.2 \end{bmatrix}$, $d = \begin{bmatrix} 50 \\ 30 \end{bmatrix}$ **Ans:** $\begin{bmatrix} 110 \\ 120 \end{bmatrix}$

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8. Using cofactor expansion, compute the determinant of

$$\begin{bmatrix} 4 & 0 & -7 & 3 & -5 \\ 0 & 0 & 2 & 0 & 0 \\ 7 & 3 & -6 & 4 & -8 \\ 5 & 0 & 5 & 2 & -3 \\ 0 & 0 & 9 & -1 & 2 \end{bmatrix}$$

Ans: 6

9. Find the coordinate vector $[x]_B$ of x relative to the given basis $B = \{b_1, b_2\}$ where $b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $b_2 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$ and $x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$.

Ans: $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

10. Let $b_1 = (1, -3)$, $b_2 = (-2, 4)$, $c_1 = (-7, 9)$, $c_2 = (-5, 7)$ are the bases of $\mathbb{R}^2$ given by $B = \{b_1, b_2\}$ and $c = \{c_1, c_2\}$ then
   i. Find the change of coordinate matrix from C to B.
   ii. Find the change of coordinate matrix from B to C.

Ans: $\begin{bmatrix} 5 & 3 \\ 6 & 4 \end{bmatrix}, \begin{bmatrix} 2 & \frac{-3}{2} \\ -3 & \frac{5}{2} \end{bmatrix}$

11. Define characteristics equation of a matrix. Find the characteristics equation of

$$\begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ans: 5, 3, 1

12. Let u and v are non-zero vectors in $\mathbb{R}^3$ and θ be angle between them. Then prove that $u \cdot v = \| u \| \| v \| \cos \theta$ where the symbols have their usual meaning.

13. Let $*$ is defined on $\mathbb{R}^+$ by $a * b = \sqrt{ab}$. Then show $\mathbb{R}^+$ is not a group.

14. Find the additive and multiplicative inverse of (3, 2) in ring $Z_4 \times Z_7$.

Ans: (1, 5) (3, 4)

15. Find the basis for eigenspace corresponding to the eigenvalue of $\begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}$

Ans: $\left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}$

MODEL SET 4

Group 'A'

Attempt any three questions: **(3×10 = 30)**

1. Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation. T is one-to-one if and only if the equation $T(x) = 0$ has only the trivial solution, prove the statement.

2. What do you understand by Gram-Schmidt process for orthogonal vectors? Let $x_1 = (1, 1, 1, 1, 1)$, $x_2 = (0, 1, 1, 1)$ and $x_3 = (0, 0, 1, 1)$. Then $\{x_1, x_2, x_3\}$ is linearly independent and thus is a basis for a subspace W of $\mathbb{R}^4$. Using Gram-Schmidt process construct an orthogonal basis for W.

Ans: $(1, 1, 1, 1, 1) \left(\frac{-3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right), \left(0, \frac{-2}{3}, \frac{1}{3}, \frac{1}{3}\right)$

3. Find the eigenvalue of $A = \begin{bmatrix} 0.50 & -0.60 \\ 0.75 & 1.1 \end{bmatrix}$ and find the basis for each eigenspace.

Ans: $0.8 + 0.6i, 0.8 - 0.6i \begin{bmatrix} -2+4i \\ 5 \end{bmatrix} \begin{bmatrix} -2-4i \\ 5 \end{bmatrix}$

4. Find the equation $y = \beta_0 + \beta_1 x$ for the least squares line that best fits the data points (2, 3), (3, 2), (5, 1), (6, 0).

Ans: $y = 4.3 - 0.7x$

Group 'B'

Attempt any ten questions: **(10×5 = 50)**

5. Determine for what value of h, the set of vectors $\{v_1, v_2, v_3\}$ is linearly dependent, $v_1 = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$, $v_2 = \begin{bmatrix} 3 \\ -5 \\ 7 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ -5 \\ h \end{bmatrix}$.

Ans: −6

6. Define one-to-one and onto transformation. If a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^4$ is defined as $T(x_1, x_2, x_3, x_4) = (0, x_1 + x_2, x_2 + x_3, x_3 + x_4)$ then check T is one-to-one and onto.

Ans: not one-to-one, not onto

7. Define singular and non-singular matrix. Examine the matrix is singular or non singular, $\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$.

Ans: non-singular

8. Find the determinant by row reduction to echelon form,

$$\begin{bmatrix} 1 & 5 & -3 \\ -3 & -3 & 3 \\ 2 & 13 & 7 \end{bmatrix}$$

Ans: −18

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9. Let $A = \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}$ and $w = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. Determine if w is in Col(A). Is w in Nul (A)?

Ans: $w \in$ Col(A), $w \in$ Nul (A

10. Find the dimension of null space of $\begin{bmatrix} 2 & -1 & 1 & -6 & 8 \\ 1 & -2 & -4 & 3 & -2 \\ -7 & 8 & 10 & 3 & -10 \\ 4 & -5 & -7 & 0 & 4 \end{bmatrix}$.

Ans: 3

11. Find the basis for the eigenspace corresponding to the eigenvalue $\lambda = 3$ where,

$$A = \begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}$$

Ans: $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$

12. Show that $\{v_1, v_2, v_3\}$ is an orthogonal basis for $\mathbb{R}^3$ where $v_1 = \left(\frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$, $v_2 = \left(\frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$, $v_3 = \left(\frac{-1}{\sqrt{66}}, \frac{-4}{\sqrt{66}}, \frac{7}{\sqrt{66}}\right)$.

13. Let $G = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix}, ab - bc \neq 0; a, b, c, d \right\}$. Then show that G is a group under multiplication.

14. Define zero divisors in ring. Find zero divisor of a ring Z_{10}.

Ans: 2, 4, 5, 6, 8

15. Why the system $x_1 - 3x_2 = 4$, $-3x_1 + 9x_2 = 8$ inconsistent? Give graphical representation.

MODEL SET 5

Group 'A'

Attempt any three questions: **(3×10 = 30)**

1. Define echelon form of a given matrix A with example. Given matrix $\begin{bmatrix} 0 & 3 & -6 & 6 \\ 3 & -7 & 8 & -5 \\ 3 & -9 & 12 & -9 \end{bmatrix}$ apply the row operation to transform the echelon and then reduced echelon form.

Ans: $\begin{bmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

2. Complete AB of the partitioned matrices

$$A = \left[\begin{array}{ccc|cc} 2 & -3 & 1 & 0 & -4 \\ 1 & 5 & -2 & 3 & -1 \\ \hline 0 & -4 & -2 & 7 & -1 \end{array}\right], B = \left[\begin{array}{cc} 6 & 4 \\ -2 & 1 \\ -3 & 7 \\ \hline -1 & 3 \\ 5 & 2 \end{array}\right]$$

Ans: $\begin{bmatrix} -5 & 4 \\ -6 & 2 \\ 2 & 1 \end{bmatrix}$

3. Let $b_1 = (1, 0, 0)$, $b_2 = (-3, 4, 0)$, $b_3 = (3, -6, 3)$ and $x = (-8, 2, 3)$ then
   i. Show that $B = \{a_1, b_2, b_3\}$ is a basis of $\mathbb{R}^3$.
   ii. Find the change of coordinate matrix from B to the standard basis.
   iii. Find $[x]_B$.

4. Find the equation $y = a_0 + a_1x$ for the least squares line that best fits the data points (2, 1), (5, 2), (7, 3), (8, 3).

Ans: $y = \frac{2}{7} + \frac{5x}{14}$

Group 'B'

Attempt any ten questions: **(10×5 = 50)**

5. Define linear dependence of a set of vectors. Are the following sets of vectors linearly dependent?

$$\begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 5 \end{bmatrix}.$$

Ans: dependent

6. Let $A = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$, $u = \begin{bmatrix} 1 \\ 0 \\ -4 \end{bmatrix}$ and $v = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Define $T(x) = Ax$. Then find T(u) and T(v).

Ans: (0.5, 0, −2), (0.5a, 0.5b, 0.5c)

7. If $A = \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}$ and $x = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$, compute $(Ax)^T$, x^TA^T and xx^T. Can we compute A^Tx^T?

Ans: $(Ax)^T = x^TA^T = [-4 \quad 2]$, $xx^T = \begin{bmatrix} 25 & 15 \\ 15 & 9 \end{bmatrix}$

8. State the Cramer's rule to finding solution of a system of linear equations and justify it.

9. Determine $w \in$ Nul(A) where $w = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$

Ans: Yes, $w \in$ Nul A

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0. If 4×7 matrix A has rank 3. Find dim (Nul A), dim (Row A) and rank of A^T.

Ans: (4, 3, 3)

1. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be linear transformation defined by $T(x, y) = (x - y, x + y)$ and $B = \{b_1, b_2\}$ be basis where $b_1 = (1, 1)$ and $b_2 = (-1, 0)$. Then find B matrix for T be. $[T]_B$.

Ans: $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$

2. Find the least square solution of $Ax = b$ for

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}, b = \begin{bmatrix} 7 \\ 2 \\ 3 \\ 6 \\ 5 \\ 4 \end{bmatrix}$$

Ans: $\begin{bmatrix} 5 \\ -1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

13. Define subgroup of a group G. Show that $\mathbb{R}$ is a group.

14. Find zero divisors of the rings: (i) Z_{16} (ii) Z_{11}

Ans: (i) 2, 4, 6, 8, 10, 12, 14 (ii) no zero divisors

15. Let $A = PDP^{-1}$. Compute A^4 if $P = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$.

Ans: $\begin{bmatrix} 226 & -525 \\ 90 & -209 \end{bmatrix}$

MODEL SET 6

Group 'A'

Attempt any three questions: **(3×10 = 30)**

1. Determine, if the following homogeneous system has a non-trivial solution

$$x_1 + 3x_2 - 5x_3 = 0$$
$$x_1 + 4x_2 - 8x_3 = 0$$
$$-3x_1 - 7x_2 + 9x_3 = 0$$

Ans: $x = x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$

2. Solve the Leontief production equation for an economy with three sectors given by

$$\begin{bmatrix} 0.5 & 0.4 & 0.2 & 0.2 \\ 0.3 & 0.1 & 0.1 & 0.3 \end{bmatrix} \text{and } d = \begin{bmatrix} 50 \\ 30 \\ 20 \end{bmatrix}$$

Ans: (226, 119, 78)

3. Define coordinate vector of x relative to the basis B. Prove that, such vector has unique linear combination with the basis for x.

4. Diagonalizable the matrix, if possible

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 1 & 4 & -3 & 0 \\ -1 & -2 & 0 & -3 \end{bmatrix}$$

Ans: $P = \begin{bmatrix} 0 & 0 & -8 & -16 \\ 0 & 0 & 4 & -4 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} -3 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

Group 'B'

Attempt any ten questions: **(10×5 = 50)**

5. Define the consistency of a system of linear equations. Show that the following system is inconsistent;

$$2x - 3y + 7z = 5$$
$$3x + y - 3z = 13$$
$$2x + 19y - 47z = 32.$$

6. Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is a linear transformation. Then T is one-to-one if and only if $T(x) = 0$ has only the trivial solution.

7. Let $A = \begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}$. Show that $AB \neq BA$.

8. By choosing suitable example, show that
   (i) det (AB) = det (A) . det (B).
   (ii) det (A + B) ≠ det (A) + det (B).

9. Let W be the set of all vectors of the form $\begin{bmatrix} 2s + 4t \\ 3s \\ 2s - 3t \\ 3t \end{bmatrix}$. Show that W is a subspace of $\mathbb{R}^4$.

10. Let $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$ when $b_1 = \begin{bmatrix} -9 \\ 0 \end{bmatrix}$, $b_2 = \begin{bmatrix} -5 \\ -1 \end{bmatrix}$, $c_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$, $c_2 = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$ are two bases for $\mathbb{R}^2$ then
   i. Find the change of coordinate matrix from B to C.
   ii. Find the change of coordinate matrix from C to B.

Ans: $\begin{bmatrix} 6 & 4 \\ -5 & 3 \end{bmatrix}$, $\begin{bmatrix} -\frac{3}{2} & -2 \\ \frac{5}{2} & 3 \end{bmatrix}$

11. Find the eigenvalue of $A = \begin{bmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{bmatrix}$.

Ans: 0, 2, 3

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12. Show that $\{v_1, v_2, v_3\}$ is an orthogonal set of vectors where $v_1 = (3, 1, 1)$, $v_2 = (-1, 2, 1)$, $v_3 = \left(\frac{-1}{3}, -2, \frac{7}{2}\right)$.

13. Define group with binary operation. Let $*$ is defined on Q^+ by $a * b = \frac{ab}{2}$ then show that Q^+ is a group under the binary operation $*$.

14. When we called a ring is a field? Prove that a ring Z_{11} is a field.

15. Let $y = (-1, -5, -10)$, $u_1 = (5, -2, 1)$ and $u_2 = (1, 2, -1)$. Find the nearest point is W to y and the distance between y and the nearest point where $W = \text{Span}\{u_1, u_2\}$ **Ans:** $(-1, -8, 4)$, $\sqrt{45}$

MODEL SET 7

Group 'A'

Attempt any three questions: **(3×10 = 30)**

1. Determine if the following system is consistent,

$$x_2 - 4x_3 = 8$$
$$2x_1 - 3x_1 + 2x_3 = 1$$
$$5x_1 - 8x_2 + 7x_3 = 1.$$

Ans: Consistent

2. Diagonalizable: $\begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$.

Ans: $P = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

3. Discuss, when two vectors are orthogonal to each other. Let u and v are vectors, prove that $[\text{dist}(u, -v)]^2 = [\text{Dist}(u, v)]^2$ iff $u \cdot v = 0$.

4. Find the equation $y = \beta_0 + \beta_1 x$ of the least squares line that best fits the given data: (2, 3), (3, 2) (5, 1) and (6, 0). **Ans:** $y = 4.3 - 0.7x$

Group 'B'

Attempt any ten questions: **(10×5 = 50)**

5. Determine if the given set is linearly dependent:

i. $\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 9 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix}$

ii. $\begin{bmatrix} -2 \\ 4 \\ 6 \\ 10 \end{bmatrix}, \begin{bmatrix} 3 \\ -6 \\ -9 \\ 15 \end{bmatrix}$

Ans: (i) Dependent (ii) Independent

6. Let $A = \begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix}$ and $b = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$. Define T by $Tx = Ax$. Find a vector x whose image under T is b. **Ans:** (3, 1, 0)

7. Let $\begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$ where A_{11} is $p \times p$, A_{22} is $q \times q$ and A is invertible. Find a formula for A^{-1}. **Ans:** $\begin{bmatrix} A_{11}^{-1} & -A_{11}^{-1} A_{12} A_{22}^{-1} \\ 0 & A_{22}^{-1} \end{bmatrix}$

8. Compute the determinant of $\begin{vmatrix} 2 & -4 & 3 \\ 3 & 1 & 2 \\ 1 & 4 & -1 \end{vmatrix}$ **Ans:** -5

9. If $B = \{(1, -2), (-3, 5)\}$ and $x = (2, -5)$ then find $[x]_B$. **Ans:** (5, 1).

10. Consider two bases $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$ for V such that $b_1 = 4c_1 + c_2$ and $b_2 = -6c_1 + c_2$. Suppose that $x = 3b_1 + b_2$ then find $[x]_C$. **Ans:** (6, 4)

11. What do you mean by eigenvalues, eigenvectors and characteristics polynomial of a matrix? Explain with suitable example.

12. Define Gram-Schmidt process. Let $W = \text{Span}\{x_1, x_2\}$ where $x_1 = \{3, 6, 0\}$ and $x_2 = (1, 2, 2)$. Then construct an orthogonal basis $\{v_1, v_2\}$ for W.

13. Determine whether $C = \{x + iy : x, y \in \mathbb{R}\}$ is a group under addition or not. **Ans:** Group

14. Prove that every field F is an integral domain.

15. Find rank of A, dimension of null space of A,

$$A = \begin{bmatrix} 2 & -1 & 1 & -6 & 8 \\ 1 & -2 & -4 & 3 & -2 \\ -7 & 8 & 10 & 3 & -10 \\ 4 & -5 & -7 & 0 & 4 \end{bmatrix}$$

Ans: rank = 2, dim Nul (A) = 3

MODEL SET 8

Group 'A'

Attempt any three questions: **(3×10 = 30)**

1. Prove that the transformation T is linear. Also, find the matrix that implements the mapping: $T(x_1, x_2, x_3) = (x_1 - 5x_2 + 4x_3, x_2 - 6x_3)$. Also, check whether T is one to - one and onto or not. **Ans:** $\begin{bmatrix} 1 & 5 & -4 \\ 0 & 1 & 6 \end{bmatrix}$, not one to one but onto

2. The set of matrices of the form $\begin{bmatrix} 0 & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}$ is a subspace of the vector space of 3×3 matrices. Verify it.

3. Let V and W are vectors space over a field F or real numbers. Let dim (V) = n, dim (W) = m. Let $\{e_1, e_2, \ldots, e_n\}$ be a basis for V and $\{f_1, \ldots, f_m\}$ be a basis for W. Then prove that each linear transformation T: $V \to W$ can be represented by $m \times n$ matrix A with elements from F such that Y = AX where $X = (x_1, x_2, \ldots x_n)$ and $Y = (y_1, y_2, \ldots, y_m)$ are column matrices of coordinates of $v \in N$ relative to its basis and coordinates of $w \in W$ relative to its basis, respectively.

4. What is the least squares solution? Find a least squares solution of $Ax = b$ where $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$.

Ans: $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Group 'B'

Attempt any ten questions: **(10×5 = 50)**

5. Prove that any set $\{v_1, \ldots, v_n\}$ in $\mathbb{R}^n$, is linearly dependent if p> n.

6. Prove that the transformation $T(x_1, x_2, x_3) = (x_1 - 5x_2 + 4x_3, x_2 - 6x_3)$ is linear.

7. State the Column-Row Expansion Theorem for two matrix. Let $A = \begin{bmatrix} -3 & 1 & 2 \\ 1 & -4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 4 & 5 \\ 6 & 3 \end{bmatrix}$. Find AB by Column–Row expansion.

Ans: $\begin{bmatrix} 10 & 8 \\ 16 & -4 \end{bmatrix}$

8. Using determinant, show that the matrix A is invertible where $A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix}$.

9. Let $v_1 = (1, -2, 3)$, $v_2 = (-2, 7, -9)$. Determine if $\{v_1, v_2\}$ is a basis for $\mathbb{R}^3$. Is $\{v_1, v_2\}$ a basis for $\mathbb{R}^2$?

Ans: not, not

10. Let $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$ are bases for $\mathbb{R}^2$. If $b_1 = (6, -12)$, $b_2 = (4, 2)$, $c_1 = (4, 2)$ and $c_2 = (3, 9)$. Then find the coordinate matrix from B to C and also from C to B.

Ans: $\begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & \frac{-1}{2} \\ 1 & \frac{3}{2} \end{bmatrix}$

11. Let $A = \begin{bmatrix} 4 & -9 \\ 4 & -8 \end{bmatrix}$, $b_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $b_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $B = \{b_1, b_2\}$. Find the B–matrix for the transformation $x \to Ax$ with $P = [b_1, b_2]$.

Ans: $\begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix}$

12. Find the orthogonal projection of y onto u where y = (7, 6) and u = (4, 2).

Ans: (8, 4)

13. Check whether G is group or not where $G = \{1, \omega, \omega^2\}$ under multiplication. With ω is an imaginary cube root of unity.

Ans: group.

14. Define integral domain with example. Show that the ring Z_{10} is not an integral domain.

15. Let a and b are two positive numbers. Find the area of the region bounded by the ellipse $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 1$.

Ans: πab

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