Ch 2.2: Assessing Model Accuracy

Lecture 3 - CMSE 381

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Announcements

Last time:

- Ch 2.1, Vocab day!

Announcements:

- Get on slack!
  - +1 point on the first homework if you post a gif in the thread
- First homework due TODAY. Covers:
  - Weds 8/31 lecture
  - Fri 9/2 Lecture
- Office hours
  - Tuesdays, Dr Munch (EGR 1511) 9:30-10:30am
  - Wednesdays, Emily (EGR 1508A) 10:30-noon
  - Thursdays, Dr Munch (EGR 1511) 3:30-4:30pm
  - Fridays, Emily (EGR 1508A) 1-2:30pm

Covered in this lecture

- Mean Squared Error (regression)
- Train vs Test
- Bias Variance Trade off

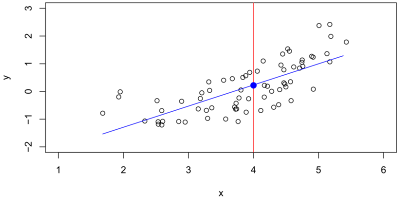
Quick review of notation

Section 1

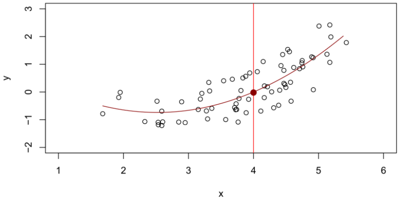
Mean Squared Error

Which is better?

A linear model $\hat{f}_L(X) = \hat{\beta}_0 + \hat{\beta}_1 X$ gives a reasonable fit here



A quadratic model $\hat{f}_Q(X) = \hat{\beta}_0 + \hat{\beta}_1 X + \hat{\beta}_2 X^2$ fits slightly better.

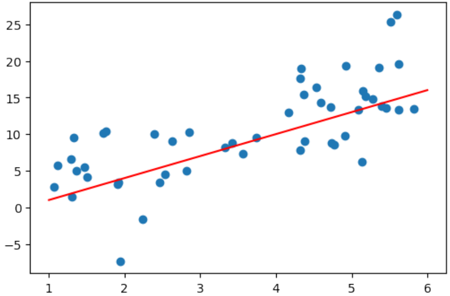


No free lunch

Mean Squared Error

Error in the regression setting

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{f}(x_i))^2$$



Group Work

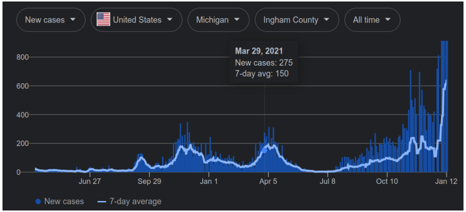
Given the following data, you decide to use the model

$$\hat{f}(X_1, X_2) = 1 - 3X_1 + 2X_2.$$

What is the MSE?

X_1	X_2	Y
0	7	14
1	-3	-6
5	2	-10
-1	1	7

Training MSE



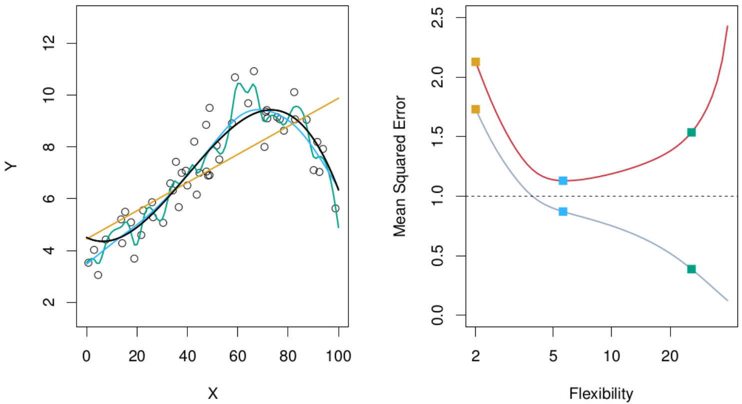
Train vs test

Training set:
The observations $\{(x_1, y_1), \cdots, (x_n, y_n)\}$ used to get the estimate $\hat{f}$

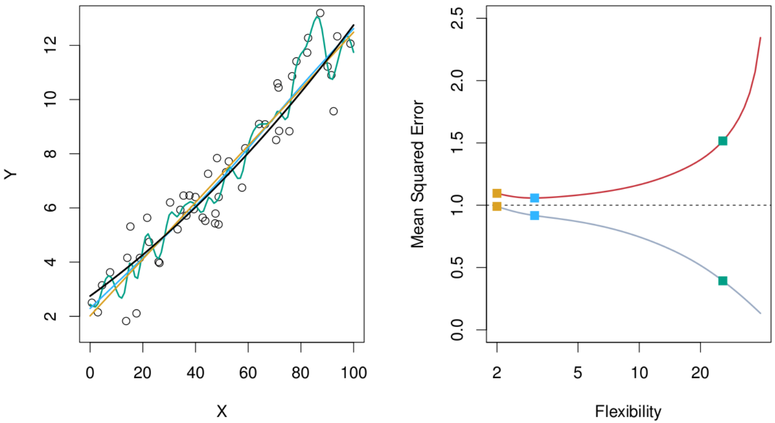
Test set:
The observations $\{(x'_1, y'_1), \cdots, (x'_{n'}, y'_{n'})\}$ used to compute the average squared prediction error

$$\frac{1}{n'} \sum_i (y'_i - \hat{f}(x'_i))^2$$

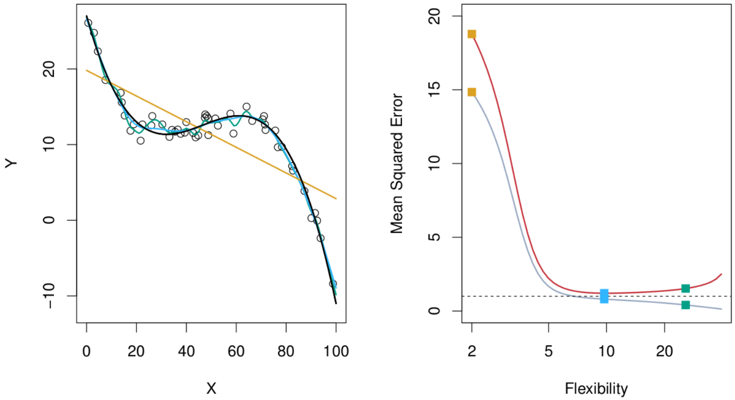
Why not just get the best model for the training data?



A more linear example



A more non-linear example



A simple solution: Train/test split

More on this in Ch 5

Section 2

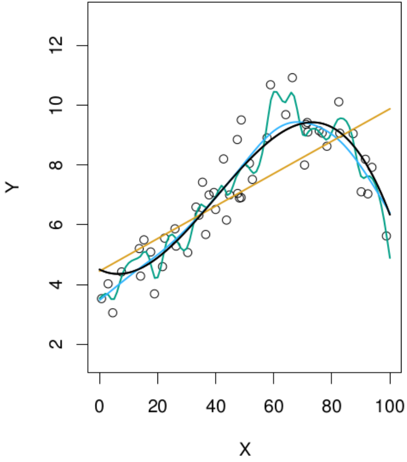
Bias-Variance Trade-Off

Bias-variance

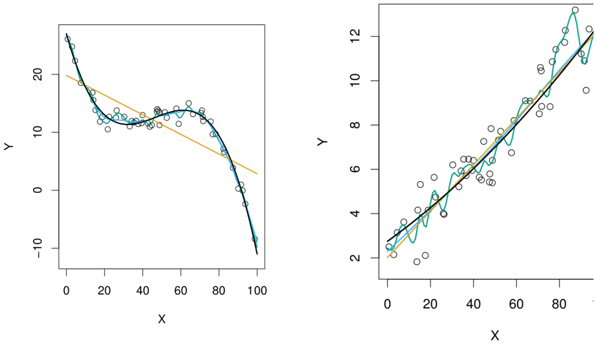
$$E(y_0 - \hat{f}(x_0))^2 = \mathrm{Var}(\hat{f}(x_0)) + \left[\mathrm{Bias}(\hat{f}(x_0))\right]^2 + \mathrm{Var}(\varepsilon)$$

Variance

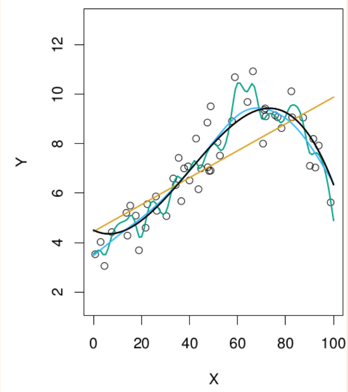
Variance: the amount by which $\hat{f}$ would change if we estimated it using a different training data set.

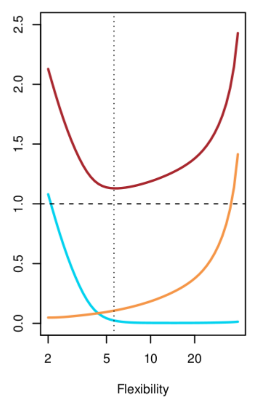


Bias: the error that is introduced by approximating a (complicated) real-life problem by a much simpler model.



Group work



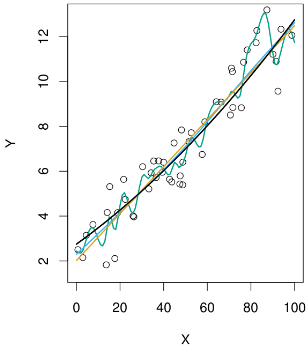


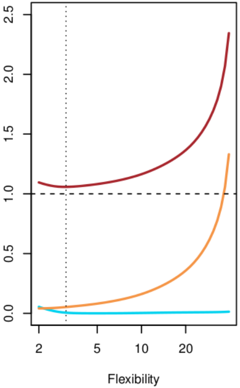
Label the line corresponding to each of the following:

- MSE
- Bias
- Variance of $\hat{f}(x_0)$
- Variance of ε

$$E(y_0 - \hat{f}(x_0))^2 = \mathrm{Var}(\hat{f}(x_0)) + \left[\mathrm{Bias}(\hat{f}(x_0))\right]^2 + \mathrm{Var}(\varepsilon)$$

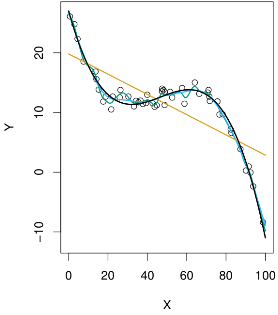
Another example

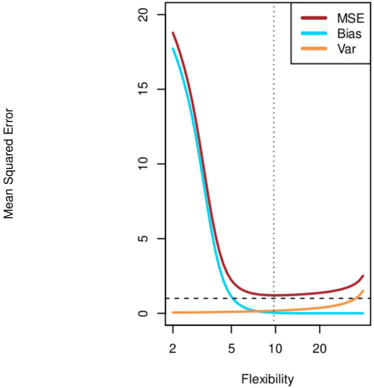




$$E(y_0 - \hat{f}(x_0))^2 = \mathrm{Var}(\hat{f}(x_0)) + \left[\mathrm{Bias}(\hat{f}(x_0))\right]^2 + \mathrm{Var}(\varepsilon)$$

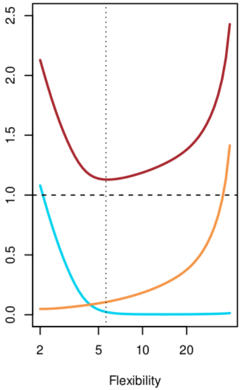
Yet another example





$$E(y_0 - \hat{f}(x_0))^2 = \mathrm{Var}(\hat{f}(x_0)) + \left[\mathrm{Bias}(\hat{f}(x_0))\right]^2 + \mathrm{Var}(\varepsilon)$$

Bias-variance trade off



$$E(y_0 - \hat{f}(x_0))^2 = \mathrm{Var}(\hat{f}(x_0)) + \left[\mathrm{Bias}(\hat{f}(x_0))\right]^2 + \mathrm{Var}(\varepsilon)$$

Group work: coding

See jupyter notebook

Next time

- Wednesday:
  - Homework due midnight on D2L
- Friday:
  - 3.1 Linear Regression