Ch 3.3: Even More Linear Regression

Lecture 7 - CMSE 381

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Announcements

Last time:

- 3.2 Multiple Linear Regression

Announcements:

- Third homework posted on github
- Office hours

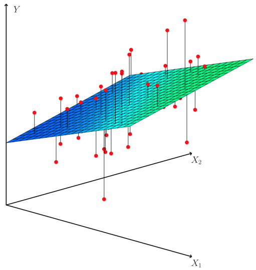
Covered in this lecture

- Qualitative predictors
- Extending the linear model with interaction terms
- Hierarchy principle
- Polynomial regression

Section 1

Review from last time

Linear Regression with Multiple Variables



- Predict Y on a multiple variables X

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots \beta_p x_p + \varepsilon$$

- Find good guesses for $\hat{\beta}_0$, $\hat{\beta}_1, \cdots$.
- $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i + \cdots + \hat{\beta}_p x_p$

- $e_i = y_i - \hat{y}_i$ is the ith residual
- RSS $= \sum_i e_i^2$
- RSS is minimized at *least squares coefficient estimates*

Questions to ask of your model

1. Is at least one of the predictors $X_1, \cdots, X_p$ useful in predicting the response?
2. Do all the predictors help to explain Y, or is only a subset of the predictors useful?
3. How well does the model fit the data?
4. Given a set of predictor values, what response value should we predict, and how accurate is our prediction?

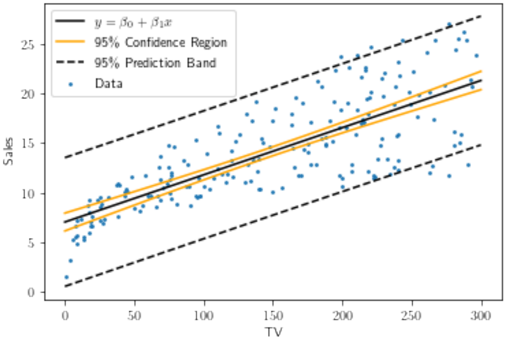
Confidence vs Prediction Model

Confidence Interval

The range likely to contain the population parameter (mean, standard deviation) of interest.

Prediction Interval

The range that likely contains the value of the dependent variable for a single new observation given specific values of the independent variables.



Specific to the Advertising Data

Confidence interval: quantify the uncertainty surrounding the average sales over a large number of cities.

Advertising example:
If \$100K is spent on TV, and \$20K on radio, **in each of *n* cities**

95% CI for sales:
[10,985, 11,528].

Prediction Interval: quantify the uncertainty in sales for a particular city.

Advertising example:
Given that \$100,000 is spent on TV advertising and \$20,000 is spent on radio advertising in **Gotham City**

95% prediction interval for Gotham:
[7,930, 14,580].

Section 2

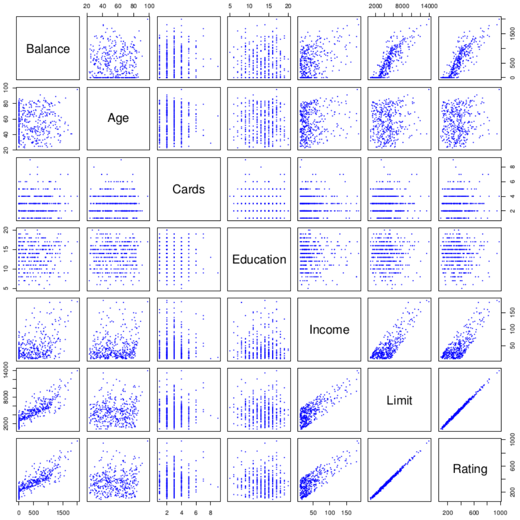
Qualitative Predictors

Reminder: Qualitative vs Quantitative predictors

Quantitative:

Qualitative/Categorical:

New data set! Credit card balance



- `own`: house ownership
- `student`: student status
- `status`: marital status
- `region`: East, West, or South

What if....

... your variables aren't quantitative?

- Home ownership
- Student status
- Major
- Gender
- Ethnicity
- Country of origin

Example

Investigate differences in credit card balance between people who own a house and those who don't, ignoring the other variables.

One-hot encoding

Create a new variable

$$x_i = \begin{cases} 1 & \text{if } i\text{th person is a student} \\ 0 & \text{if } i\text{th person is not a student} \end{cases}$$

Model:

$$\begin{aligned} y_i &= \beta_0 + \beta_1 x_i + \varepsilon_i \\ &= \begin{cases} \beta_0 + \beta_1 + \varepsilon_i & \text{if } i\text{th person is student} \\ \beta_0 + \varepsilon_i & \text{if } i\text{th person isn't} \end{cases} \end{aligned}$$

Interpretation

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	480.3694	23.434	20.499	0.000	434.300	526.439
Student[T.Yes]	396.4556	74.104	5.350	0.000	250.771	542.140

Model:

$$y = 480.36 + 396.46 \cdot x_{student}$$

Who cares about 0/1?

Old version: 0/1

$$x_i = \begin{cases} 1 & \text{if } i\text{th person is a student} \\ 0 & \text{if } i\text{th person is not a student} \end{cases}$$

Model:

$$\begin{aligned} y_i &= \beta_0 + \beta_1 x_i + \varepsilon_i \\ &= \begin{cases} \beta_0 + \beta_1 + \varepsilon_i & \text{if } i\text{th person is student} \\ \beta_0 + \varepsilon_i & \text{if } i\text{th person isn't} \end{cases} \end{aligned}$$

Alternative version: ± 1

$$x_i = \begin{cases} 1 & \text{if } i\text{th person is a student} \\ -1 & \text{if } i\text{th person is not a student} \end{cases}$$

Model:

$$\begin{aligned} y_i &= \beta_0 + \beta_1 x_i + \varepsilon_i \\ &= \begin{cases} \beta_0 + \beta_1 + \varepsilon_i & \text{if } i\text{th person is student} \\ \beta_0 - \beta_1 + \varepsilon_i & \text{if } i\text{th person isn't} \end{cases} \end{aligned}$$

Qualitiative Predictor with More than Two Levels

`Region`:

	x_{i1}	x_{i2}
South		
West		
East		

Create spare dummy variables:

$$x_{i1} = \begin{cases} 1 & \text{if } i\text{th person from South} \\ 0 & \text{if } i\text{th person not from South} \end{cases}$$

$$x_{i2} = \begin{cases} 1 & \text{if } i\text{th person from West} \\ 0 & \text{if } i\text{th person not from West} \end{cases}$$

$$\begin{aligned} y_i &= \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \varepsilon_i \\ &= \begin{cases} \beta_0 + \beta_1 x_{i1} + \varepsilon_i & \text{if } i\text{th person from South} \\ \beta_0 + \beta_2 x_{i2} + \varepsilon_i & \text{if } i\text{th person from West} \\ \beta_0 + \varepsilon_i & \text{if } i\text{th person from East} \end{cases} \end{aligned}$$

More on multiple levels

	Coefficient	Std. error	t-statistic	p-value
`Intercept`	531.00	46.32	11.464	< 0.0001
`region[South]`	-18.69	65.02	-0.287	0.7740
`region[West]`	-12.50	56.68	-0.221	0.8260

Do code section on "Playing with multi-level variables"

Section 3

Extending the linear model

Assumptions so far

Back to our Advertising data set

$$\hat{Y}_{sales} = \beta_0 + \beta_1 \cdot X_{TV} + \beta_2 \cdot X_{radio} + \beta_3 \cdot X_{newspaper}$$

Assumed (implicitly) that the effect on sales by increasing one medium is independent of the others.

What if spending money on radio advertising increases the effectiveness of TV advertising? How do we model it?

Interaction Term

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \varepsilon$$

$$\begin{aligned} Y_{sales} &= \beta_0 + \beta_1 X_{TV} + \beta_2 X_{radio} + \beta_3 X_{radio} X_{TV} + \varepsilon \\ &= \beta_0 + (\beta_1 + \beta_3 X_{radio}) X_{TV} + \beta_2 X_{radio} + \varepsilon \end{aligned}$$

Interaction term

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \varepsilon$$

	Coefficient	Std. error	t-statistic	p-value
`Intercept`	6.7502	0.248	27.23	< 0.0001
`TV`	0.0191	0.002	12.70	< 0.0001
`radio`	0.0289	0.009	3.24	0.0014
`TV`×`radio`	0.0011	0.000	20.73	< 0.0001

$$\begin{aligned} Y_{sales} &= \beta_0 + \beta_1 X_{TV} + \beta_2 X_{radio} + \beta_3 X_{radio} X_{TV} + \varepsilon \\ &= \beta_0 + (\beta_1 + \beta_3 X_{radio}) X_{TV} + \beta_2 X_{radio} + \varepsilon \end{aligned}$$

Interpretation

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Do the section on “Interaction Terms”

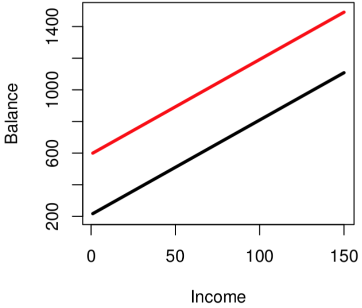
Hierarchy principle

Sometimes p-value for interaction term is very small, but associated main effects are not.

The hierarchy principle:

For credit data set:
Predict balance using income (quantitative) and student (qualitative)

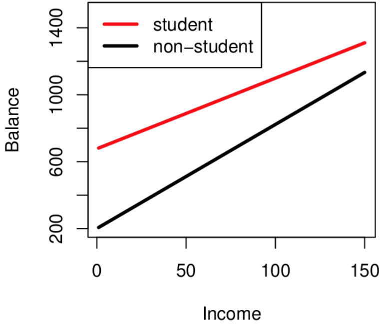
$$\begin{aligned}\texttt{balance}_i &\approx \beta_0 + \beta_1 \cdot \texttt{income}_i + \begin{cases}\beta_2 & \text{if student}\\ 0 & \text{if not}\end{cases}\\ &\approx \beta_1 \cdot \texttt{income}_i + \begin{cases}\beta_0 + \beta_2 & \text{if student}\\ \beta_0 & \text{if not}\end{cases}\end{aligned}$$



For credit data set:
Predict balance using income (quantitative) and student (qualitative)

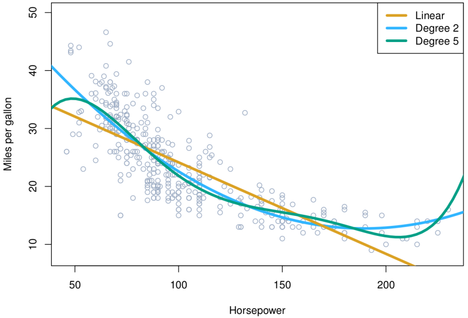
$$\texttt{balance}_i \approx \beta_0 + \beta_1 \cdot \texttt{income}_i + \begin{cases} \beta_2 + \beta_3 \cdot \texttt{income}_i & \text{if student} \\ 0 & \text{if not} \end{cases}$$

$$\approx \begin{cases} (\beta_0 + \beta_2) + (\beta_1 + \beta_3) \cdot \texttt{income}_i & \text{if student} \\ \beta_0 + \beta_1 \cdot \texttt{income}_i & \text{if not} \end{cases}$$



Nonlinear relationships

$$\texttt{mpg} = \beta_0 + \beta_1 \cdot \texttt{horsepower} + \beta_2 \cdot \texttt{horsepower}^2 + \varepsilon$$



	Coefficient	Std. error	t-statistic	p-value
`Intercept`	56.9001	1.8004	31.6	< 0.0001
`horsepower`	-0.4662	0.0311	-15.0	< 0.0001
`horsepower`^{2}	0.0012	0.0001	10.1	< 0.0001

Next time

Lec #	Date		Topic	Reading	Homeworks
1	W	Aug 31	Intro / First day stuff / Python Review Pt 1	1	
2	F	Sep 2	What is statistical learning?	2.1	
	M	Sep 5	No class - Labor day		
3	W	Sep 7	Assessing Model Accuracy	2.2.1, 2.2.2	HW #1 Due
4	F	Sep 9	Linear Regression	3.1	
5	M	Sep 12	More Linear Regression	3.1/3.2	
6	W	Sep 14	Even more linear regression	3.2.2	HW #2 Due
7	F	Sep 16	Probably more linear regression	3.3	
8	M	Sep 19	Intro to classification, Logisitic Regression	2.2.3, 4.1, 4.2, 4.3	
9	W	Sep 21	More logistic regression		HW #3 Due
10	F	Sep 23	Review		
11	M	Sep 26	***Midterm #1***		
12	W	Sep 28	[No class, Dr Munch out of town]		
13	F	Sep 30	[No class, Dr Munch out of town]		
14	M	Oct 3	Leave one out CV	5.1.1, 5.1.2	
15	W	Oct 5	k-fold CV	5.1.3	
16	F	Oct 7	More k-fold CV	5.1.4	
17	M	Oct 10	CV for classification	5.1.5	HW #4 Due
18	W	Oct 12	Resampling methods: Bootstrap	5.2	
19	F	Oct 14	Subset selection	6.1	
20	M	Oct 17	Shrinkage: Ridge	6.2.1	HW #5 Due
21	W	Oct 19	Shrinkage: Lasso	6.2.2	
22	F	Oct 21	Dimension Reduction	6.3	