Wear Signal Modeling: Theory and Implementation

Mathematical Model:

The complete vibration signal with wear is modeled as:

$$x(t) = A_0 \sin(2\pi f_0 t) + \sum_k A_k(\lambda) \sin(2\pi f_k t + \phi_k) + \eta(t)$$

Where:

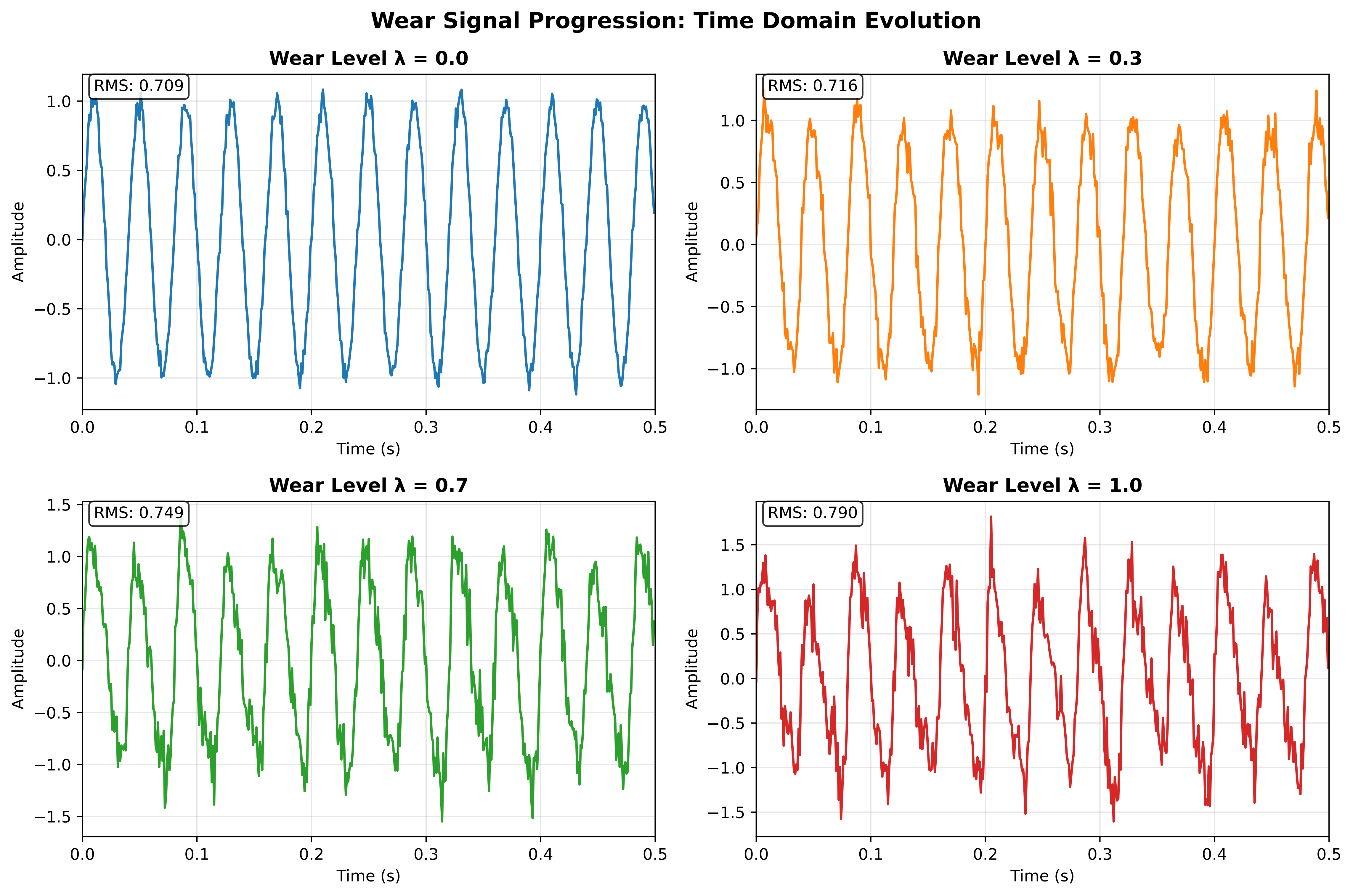
- A_0 = fundamental amplitude (healthy baseline)
- f_0 = rotation frequency (25 Hz = 1500 RPM)
- $A_k(\lambda)$ = wear-dependent amplitude scaling
- λ = dimensionless wear parameter [0, 1]
- $\eta(t)$ = additive Gaussian noise

Table 1: Wear Signal Components and Physical Interpretation

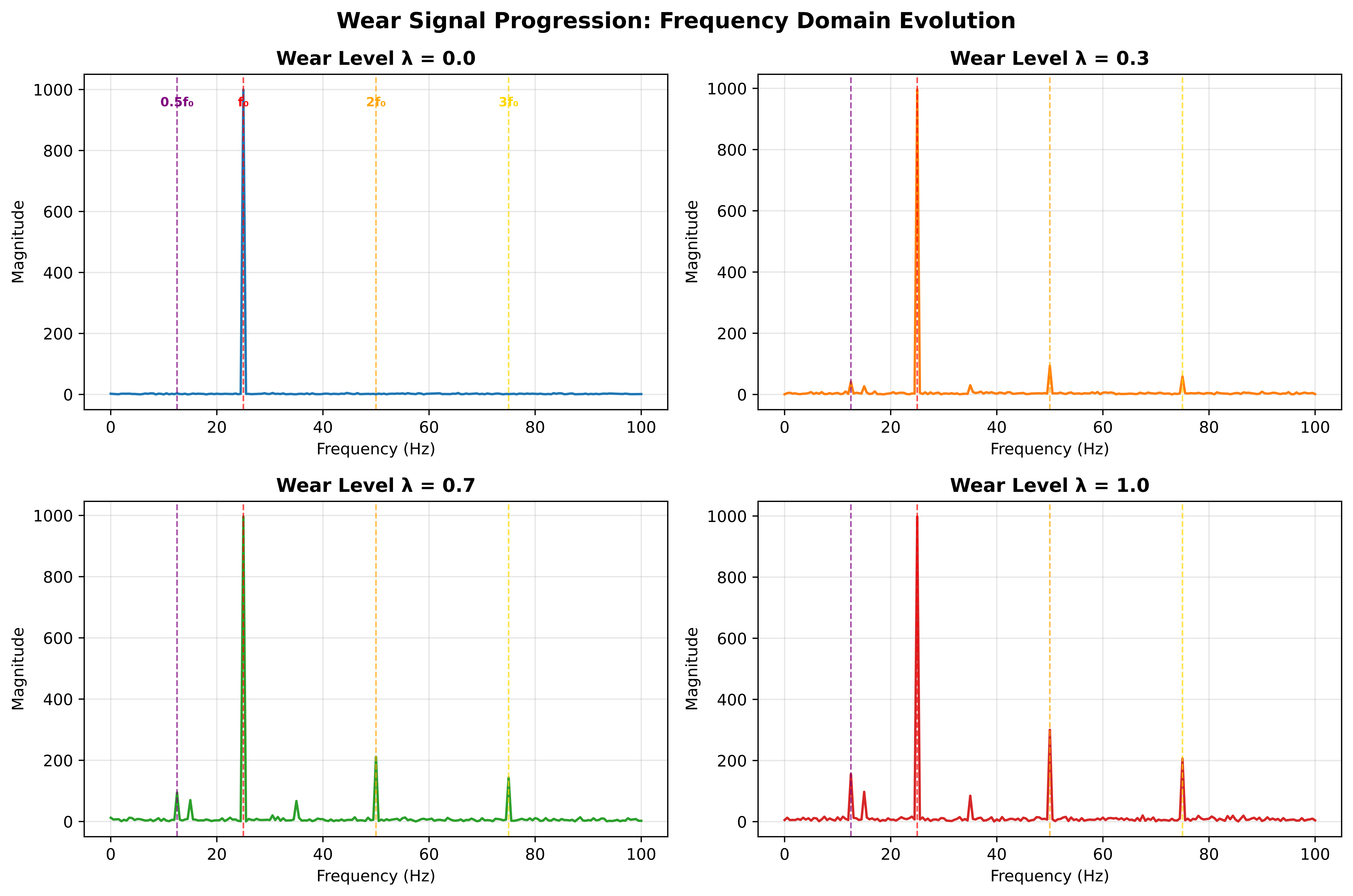
Component Type	Frequency	Amplitude Scaling	Physical Origin	Diagnostic Significance
Fundamental	f_0 (25 Hz)	A_0 = 1.0 (constant)	Ideal rotation	Baseline reference
2nd Harmonic	$2f_0$ (50 Hz)	0.3λ	Nonlinear contact forces	Primary wear indicator
3rd Harmonic	$3f_0$ (75 Hz)	0.2λ	Higher-order nonlinearity	Advanced degradation
Sub-harmonic	$0.5f_0$ (12.5 Hz)	0.15λ	Intermittent contact	Early fault detection
Upper Sideband	f_0+10 (35 Hz)	0.1λ	Amplitude modulation	Modulation effects
Lower Sideband	f_0-10 (15 Hz)	0.1λ	Phase modulation	Bearing cage effects
Broadband Noise	All frequencies	0.05+0.15λ	Surface roughness	Overall degradation

Implementation Notes:

- Wear parameter λ scales all defect amplitudes simultaneously
- Frequency selection based on established rotating machinery diagnostics
- Amplitude ratios chosen to reflect typical bearing fault signatures



Wear Signal Progression: Time Domain Evolution
Wear Level λ = 0.0
RMS: 0.709
Wear Level λ = 0.3
RMS: 0.716
Wear Level λ = 0.7
RMS: 0.749
Wear Level λ = 1.0
RMS: 0.790
Amplitude
Time (s)



Wear Signal Progression: Frequency Domain Evolution
Wear Level λ = 0.0
Wear Level λ = 0.3
Wear Level λ = 0.7
Wear Level λ = 1.0
0.5f₀
f₀
2f₀
3f₀
Magnitude
Frequency (Hz)
0
200
400
600
800
1000
0
20
40
60
80
100

Quantitative Analysis of Wear Signal Components

Wear Level λ	RMS	Fund. Power	2nd Harm.	3rd Harm.	Sub-harm.	Total Harm.	Harm. Ratio	Peaks
0.0	0.7088	999877.0	15.54	12.66	23.09	28.20	0.0004	1
0.3	0.7161	992332.2	8864.78	3542.17	1875.72	12406.95	0.0130	1
0.7	0.7492	992727.8	44648.43	20797.54	9098.95	65445.98	0.0662	3
1.0	0.7901	996877.3	90497.07	42939.33	24598.68	133436.41	0.1383	4

Table 2: Measured Power Distribution Across Frequency Components

Physical Interpretation of Results:

Observation	Physical Mechanism	Diagnostic Implication
Fundamental power remains stable	Rotation frequency unchanged	Baseline reference maintained
2nd harmonic grows with λ	Nonlinear contact stiffness	Primary wear detection
3rd harmonic appears at λ > 0.3	Higher-order nonlinearity	Advanced degradation marker
Sub-harmonic emerges early	Intermittent contact events	Early fault indicator
Peak count increases	Spectral complexity growth	Overall system degradation
Harmonic ratio scales with λ²	Quadratic nonlinear effects	Sensitive wear metric