

# PHYS6017 – Computer techniques for physics

Assignment two:

## Early Detection of Mechanical Wear in Rotating Machinery Using Vibration Signal Analysis

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### Project Objective

This project investigates the detection of early-stage mechanical wear in rotating machinery through vibration signal analysis using computational signal-processing techniques. Fourier analysis and digital filtering are applied to vibration data to identify wear-related changes in the frequency content of the signal.

### Background Theory

#### Vibration in rotating systems

In an ideal rotating system, vibration is smooth and periodic and is dominated by a small number of frequencies related to the rotation rate. Mechanical wear, imbalance, or surface defects introduce additional forces that act periodically or impulsively, leading to the appearance of new frequency components and increased noise in the vibration signal.

Vibration arises from time-dependent forces acting on a mechanical system, which produce accelerations measurable as vibration. Because all vibration components are combined into a single waveform, wear-related features cannot be clearly identified from the time-domain signal alone.

#### Frequency-domain analysis

Vibration signals are analysed in the frequency domain using the discrete Fourier transform (DFT). For a sampled signal  $x_n$  with  $n = 0, \dots, N - 1$ , the DFT is defined as

$$X_k = \sum_{n=0}^{N-1} x_n e^{-2\pi i kn/N},$$

where  $X_k$  represents the contribution of frequency

$$f_k = \frac{k f_s}{N},$$

and  $f_s$  is the sampling frequency.

The magnitude spectrum  $|X_k|$  is used to identify dominant frequency components. For healthy operation, the spectrum is dominated by a peak at the fundamental rotation frequency. As wear increases, additional spectral components appear, including harmonics at integer multiples of the rotation frequency and increased broadband energy at higher frequencies.

## Digital filtering

Digital filters are used to isolate frequency ranges of interest. For example, low-pass filters suppress high-frequency noise, high-pass filters emphasise isolate short-duration impact-like forces, band-pass filters isolate specific frequency bands associated with mechanical irregularities [2].

In the frequency domain, digital filtering is implemented by multiplying the Fourier spectrum by a filter transfer function:

$$Y_k = H_k X_k,$$

where  $X_k$  is the Fourier transform of the original signal,  $H_k$  is the frequency response of the filter, and  $Y_k$  is the filtered spectrum.

Filtering can be implemented numerically either in the frequency domain or through time-domain difference equations. For an ideal band-pass filter,

$$H_k = \begin{cases} 1, & f_1 \leq f_k \leq f_2, \\ 0, & \text{otherwise,} \end{cases}$$

where  $f_1$  and  $f_2$  define the passband limits.

## Method

### Signal model and input parameters

The vibration signal is modelled as

$$x(t) = A_0 \sin(2\pi f_0 t) + \sum_k A_k \sin(2\pi f_k t) + \eta(t),$$

where  $f_0$  is the rotation frequency,  $A_0$  its amplitude,  $f_k$  and  $A_k$  represent additional frequency components associated with non-ideal behaviour, and  $\eta(t)$  is a stochastic noise term.

The main input parameters are:

- sampling frequency  $f_s$ ,

- total signal duration  $T$  (and hence the number of samples  $N$ ),
- fundamental rotation frequency  $f_0$ ,
- noise amplitude and statistical properties,
- amplitudes and frequencies of additional components representing mechanical wear,
- digital filter parameters such as cutoff frequencies.

## Modelling of mechanical defects

Mechanical wear is modelled by superimposing additional deterministic components onto an ideal periodic vibration signal. The baseline signal corresponds to smooth rotation at a fundamental frequency of  $f_0 = 25$  Hz with constant amplitude and the addition of Gaussian noise.

Wear severity is controlled by a dimensionless parameter  $\lambda \in [0, 1]$ , which scales the amplitudes of all defect-related components. The wear model introduces harmonics at  $2f_0 = 50$  Hz and  $3f_0 = 75$  Hz, a sub-harmonic at  $0.5f_0 = 12.5$  Hz, and modulation sidebands at  $f_0 \pm 10$  Hz. Broadband high-frequency noise is increased to represent impact-related vibration.

## Results

Wear-related components are identified through changes in the frequency-domain representation of the vibration signal. As the wear parameter  $\lambda$  increases, harmonic and sub-harmonic spectral power increases relative to the fundamental component, together with a rise in broadband high-frequency energy. These effects are examined using time-domain inspection, frequency-domain analysis, digital filtering, direct spectral comparison, quantitative spectral indicators, and parameter sensitivity analysis.

### Time-domain vibration signals

Figure 1 shows time-domain vibration signals for healthy operation and increasing wear severity. In the healthy case, the signal is dominated by a near-sinusoidal component at the fundamental rotation frequency with low-amplitude noise. As wear increases, waveform distortion and amplitude variability increase, and impulsive features become visible at moderate and severe wear levels.

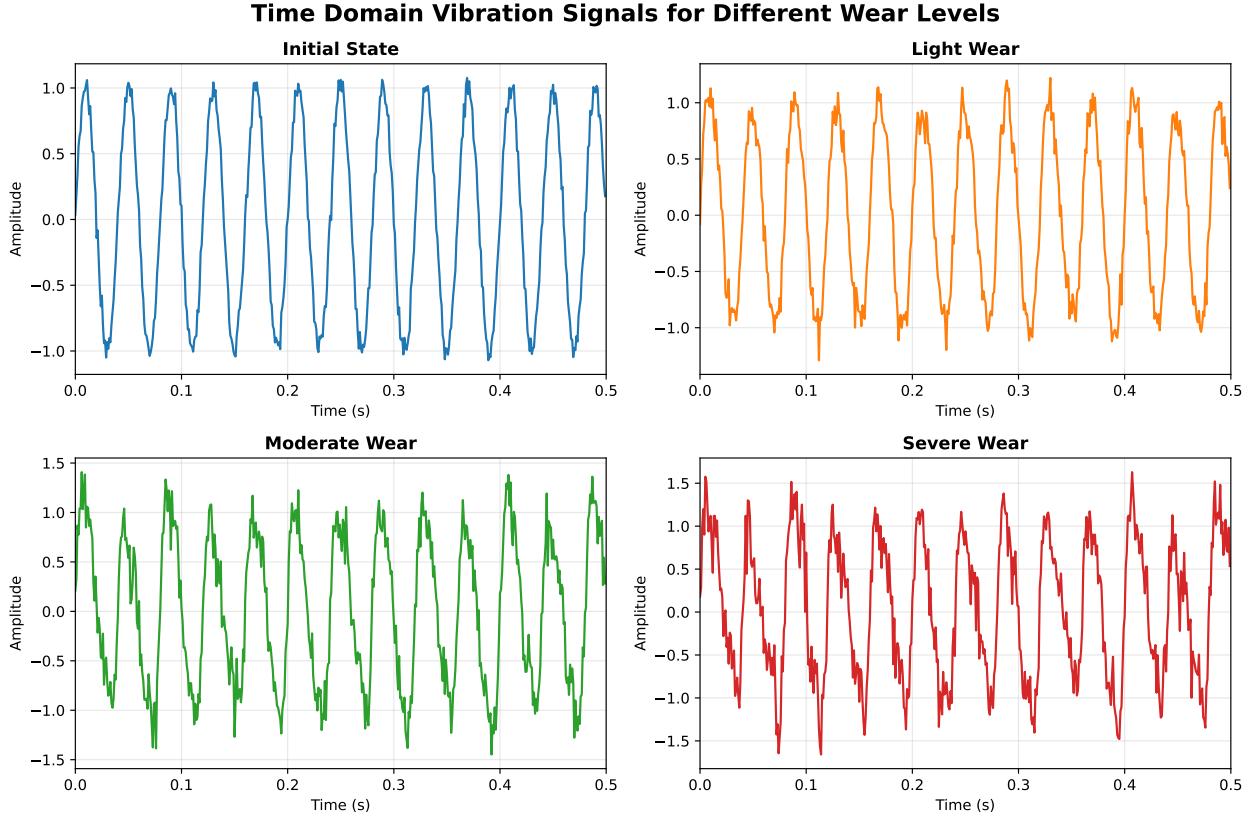


Figure 1: Time-domain vibration signals for increasing levels of mechanical wear.

### Frequency-domain analysis

The frequency content of the vibration signals was analysed using the discrete Fourier transform. Figure 2 shows the magnitude spectra corresponding to each wear condition.

For the healthy system, the spectrum is dominated by a sharp peak at the fundamental rotation frequency  $f_0$ , with negligible power at higher harmonics. As wear increases, additional peaks emerge at integer multiples of  $f_0$ , indicating nonlinear excitation of the system.

In the moderate and severe wear cases, the spectrum also exhibits a significant rise in high-frequency energy distributed across a wide frequency range. This behaviour is consistent with repeated impulsive forces generated by surface defects or mechanical looseness.

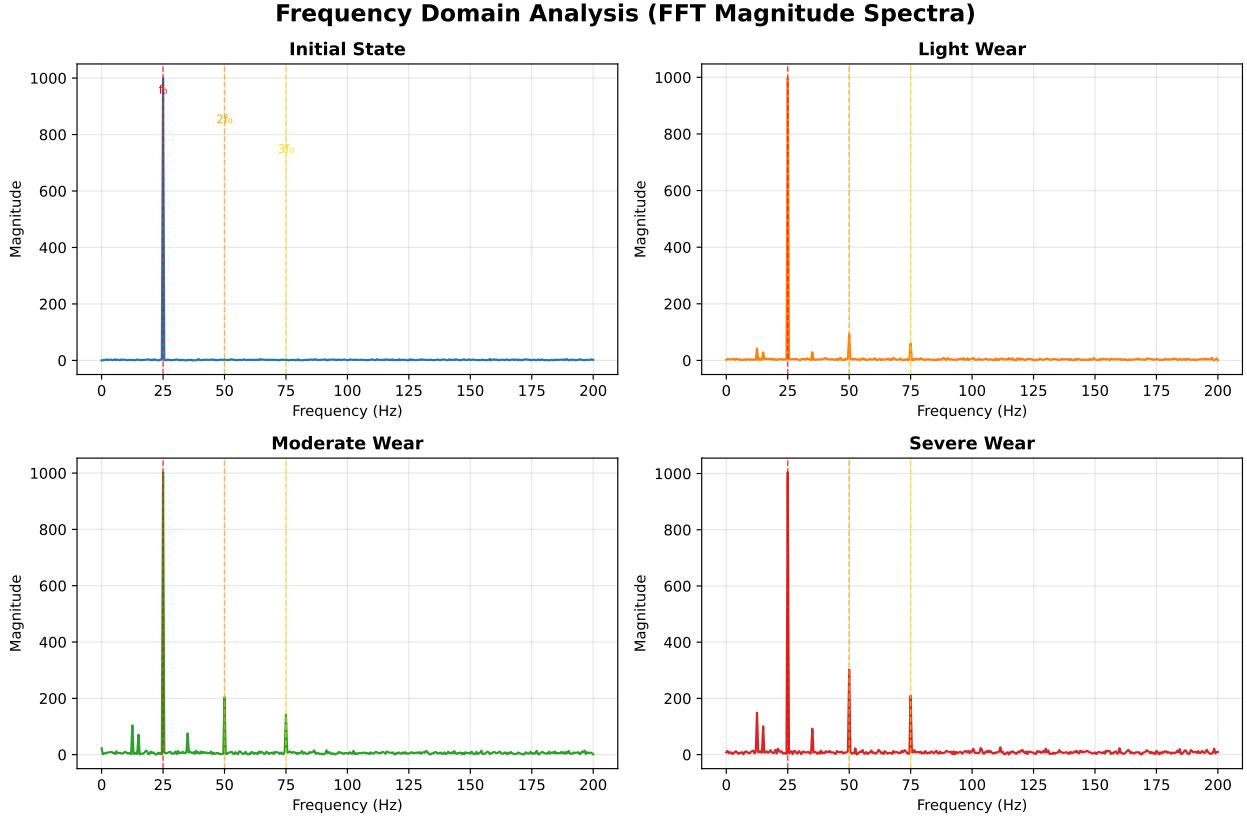


Figure 2: Frequency-domain magnitude spectra for increasing levels of mechanical wear.

## Digital filtering

Digital filters are applied to isolate physically relevant features of the vibration signal. Figure 3 shows the effect of low-pass, high-pass, and band-pass filtering for a representative wear case.

Low-pass filtering isolates the dominant rotational component by suppressing high-frequency noise. High-pass filtering suppresses the fundamental frequency and emphasises short-duration, impact-related vibration. Band-pass filtering isolates selected frequency ranges associated with wear-related periodic components.

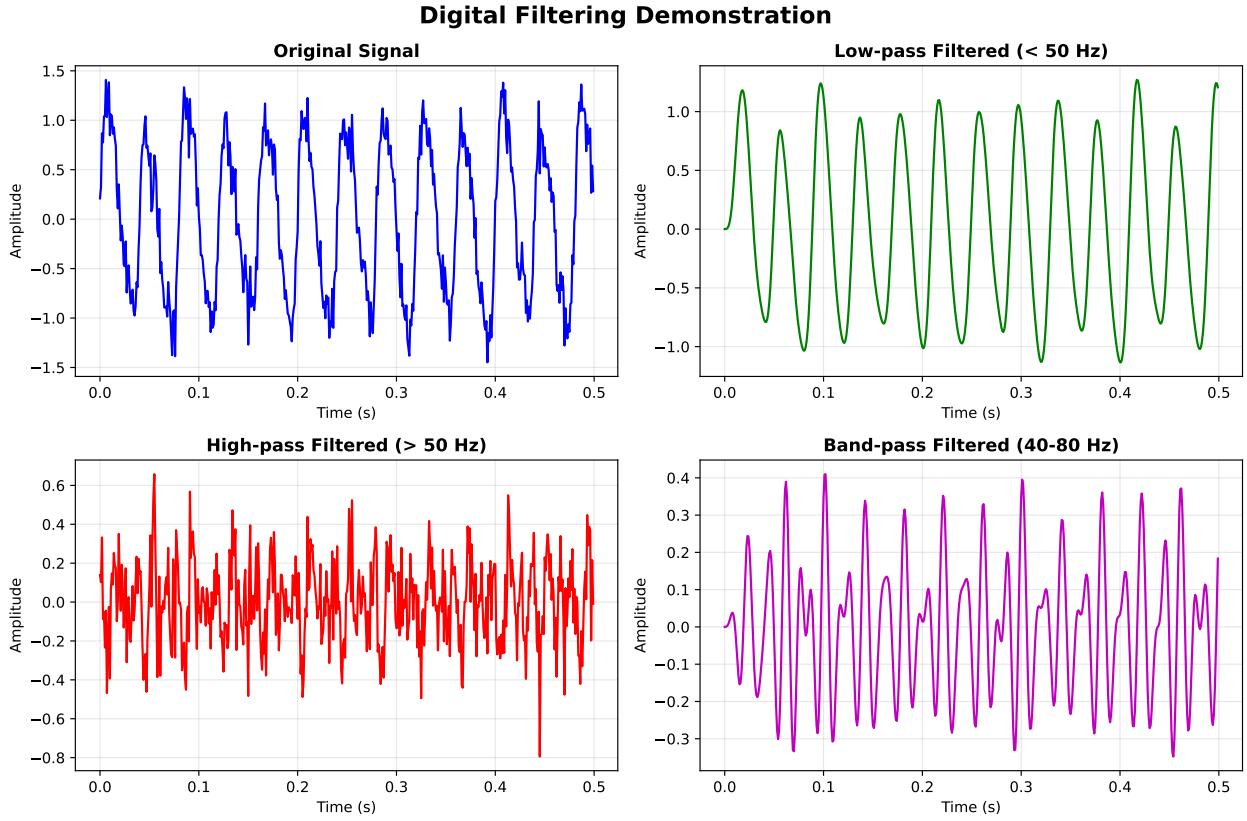


Figure 3: Effect of digital filtering on a representative vibration signal.

### Detailed spectral comparison

Figure 4 compares the frequency spectra of healthy and severely worn systems. On a linear scale, the worn spectrum exhibits strong harmonic peaks at multiples of the fundamental frequency.

On a logarithmic scale, the worn spectrum shows an increase in broadband noise across a wide frequency range, which is not evident in the time-domain signal.

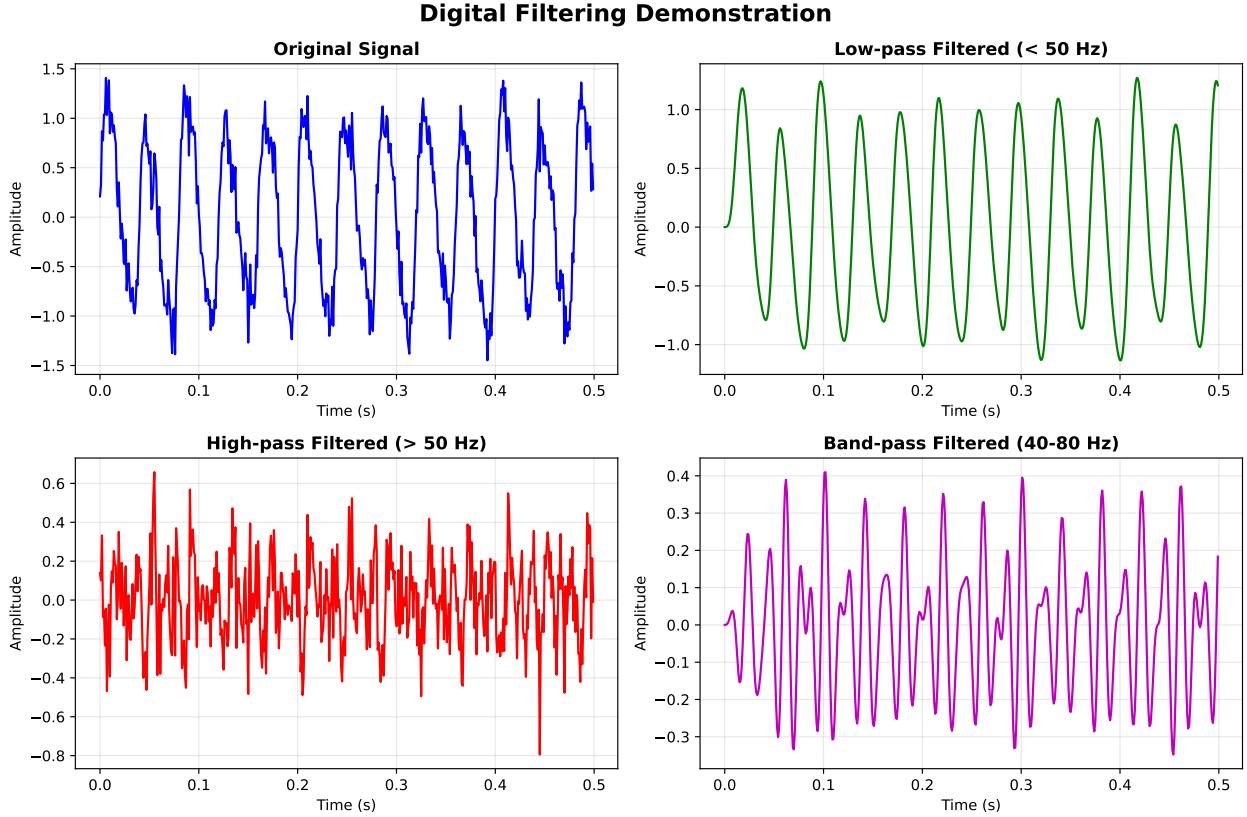


Figure 4: Detailed comparison of healthy and severe wear frequency spectra (linear and log scales).

### Quantitative wear indicators

Several quantitative indicators are extracted from the vibration data to characterise wear progression. Figure 5 shows the variation of these metrics with increasing wear level.

The root-mean-square (RMS) vibration amplitude increases monotonically with wear. The harmonic-to-fundamental power ratio and the relative power in high-frequency bands also increase, indicating a growing contribution from non-fundamental spectral components. In addition, the number of distinct spectral peaks increases between light and moderate wear.

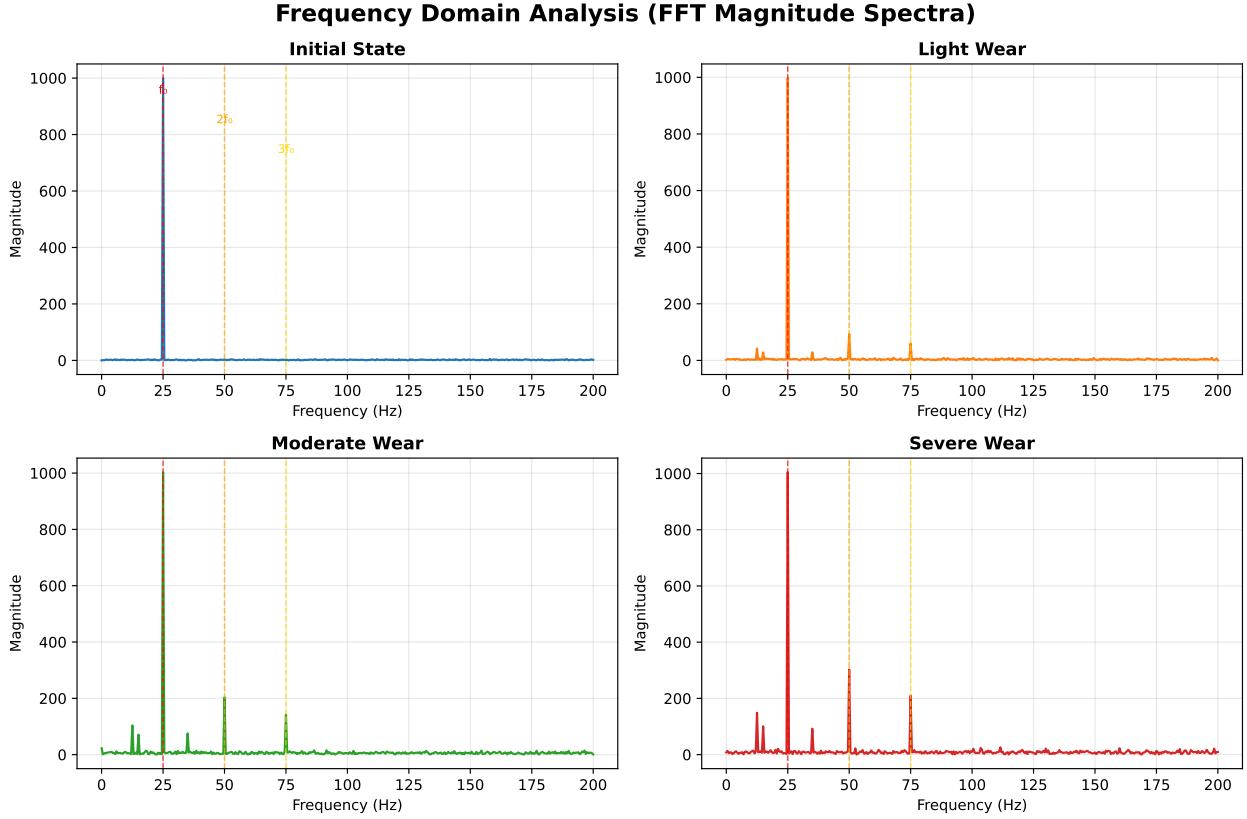


Figure 5: Quantitative wear indicators as a function of wear level.

### Parameter sensitivity analysis

Figure 6 shows the variation of several quantitative indicators as a function of wear level. The root-mean-square (RMS) vibration amplitude increases monotonically with wear. The harmonic content ratio and high-frequency spectral power also increase, indicating a growing contribution from non-fundamental frequency components. In addition, the number of distinct spectral peaks increases between light and moderate wear levels.

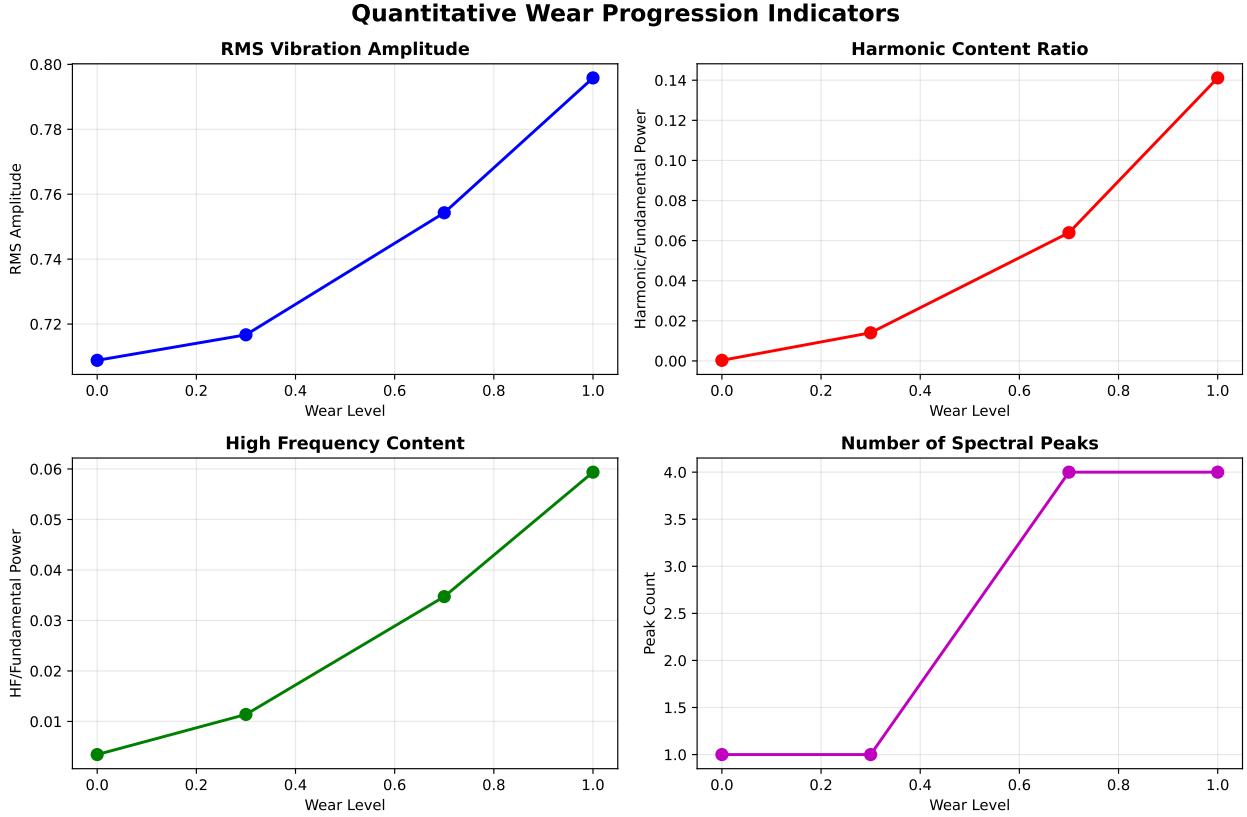


Figure 6: Parameter sensitivity analysis: effect of wear and noise on the spectrum.

## Discussion

The results show that frequency-domain analysis provides a sensitive method for detecting mechanical wear. While time-domain signals exhibit visible changes only at higher wear levels, spectral features such as harmonic growth and increased high-frequency content emerge at earlier stages of degradation.

In practical condition-monitoring applications, vibration analysis is typically combined with quantitative indicators and threshold-based decision rules. As discussed by R. Raghuwanshi and M. Patil [1], spectral features are commonly used to support early fault detection in rotating machinery.

The model employed here captures the key physical effect of mechanical wear through the introduction of additional periodic and impulsive forces that produce identifiable spectral signatures. Further refinement could include detailed bearing geometry, load-dependent dynamics, or time-evolving defect growth to extend the applicability of the analysis to more complex operating conditions.

## Conclusion

This project demonstrated the use of computational signal-processing techniques to detect mechanical wear in rotating machinery using vibration data. Fourier analysis and digital filtering were applied to extract diagnostic features from noisy time-dependent signals.

The results show that increasing wear produces systematic increases in harmonic content, high-frequency spectral power, and vibration amplitude, providing quantitative indicators that are not easily identified in the time domain. The study highlights the importance of numerical methods for vibration-based condition monitoring in realistic engineering systems.

## Appendix A: Signal parameters

Table 1: Input parameters used in the vibration signal simulations.

Parameter	Description
$f_s$	Sampling frequency of the vibration signal
$T$	Total signal duration
$N = f_s T$	Number of samples in the time series
$f_0$	Fundamental rotation frequency of the system
$A_0$	Amplitude of the ideal periodic vibration component
$\sigma$	Noise amplitude (Gaussian white noise)
$A_k$	Amplitudes of wear-related vibration components
$f_k$	Frequencies of wear-related vibration components
$f_c$	Filter cutoff frequencies

## Appendix B: Quantitative wear dynamics

Table 2: Wear-related signal components introduced in the vibration model and their physical interpretation.

Component Type	Frequency Content	Physical Interpretation
Fundamental	$f_0$	Nominal smooth rotation
2nd harmonic	$2f_0$	Nonlinear bearing response
3rd harmonic	$3f_0$	Higher-order contact nonlinearity
Sub-harmonic	$0.5f_0$	Intermittent contact or looseness
Sidebands	$f_0 \pm \Delta f$	Modulation due to wear
High-frequency broadband	$> 100$ Hz	Impacts and surface degradation

Quantitative wear progression is characterised using spectral power metrics and root-mean-square (RMS) vibration amplitude. At zero wear ( $\lambda = 0$ ), the signal is dominated by the fundamental component, with negligible harmonic power and an RMS amplitude of approximately 0.708. At  $\lambda = 0.3$ , the second harmonic power increases to approximately  $8.1 \times 10^3$  and the RMS amplitude to 0.718.

At higher wear levels, harmonic growth becomes pronounced. For  $\lambda = 0.7$ , the combined harmonic power exceeds  $6.8 \times 10^4$ , increasing to above  $1.4 \times 10^5$  at  $\lambda = 1.0$ . Over the same range, the harmonic-to-fundamental power ratio increases from  $4 \times 10^{-4}$  to approximately 0.15, and the number of detectable spectral peaks increases from one to four.

Table 3: Summary of vibration-based indicators used to detect mechanical defects.

Spectral Feature	Diagnostic Interpretation
Increase in high-frequency power	Repeated short-duration impacts caused by surface defects
Growth of harmonic peaks	Non-sinusoidal forcing due to wear or looseness
Broadening of frequency spectrum	Increased variability and impulsive excitation
Elevated broadband noise floor	Frequent impacts or degraded contact conditions
Increase in RMS vibration amplitude	Overall growth in vibrational energy with wear severity

## References

- [1] R. Raghuvanshi and M. Patil, *Vibration Analysis for Detection of Bearing Faults in Rotating Machinery*, International Journal of Engineering Research and Applications, vol. 7, no. 4, pp. 21–25, 2017.
- [2] R. B. Randall, *Vibration-based Condition Monitoring*, Wiley, 2011.