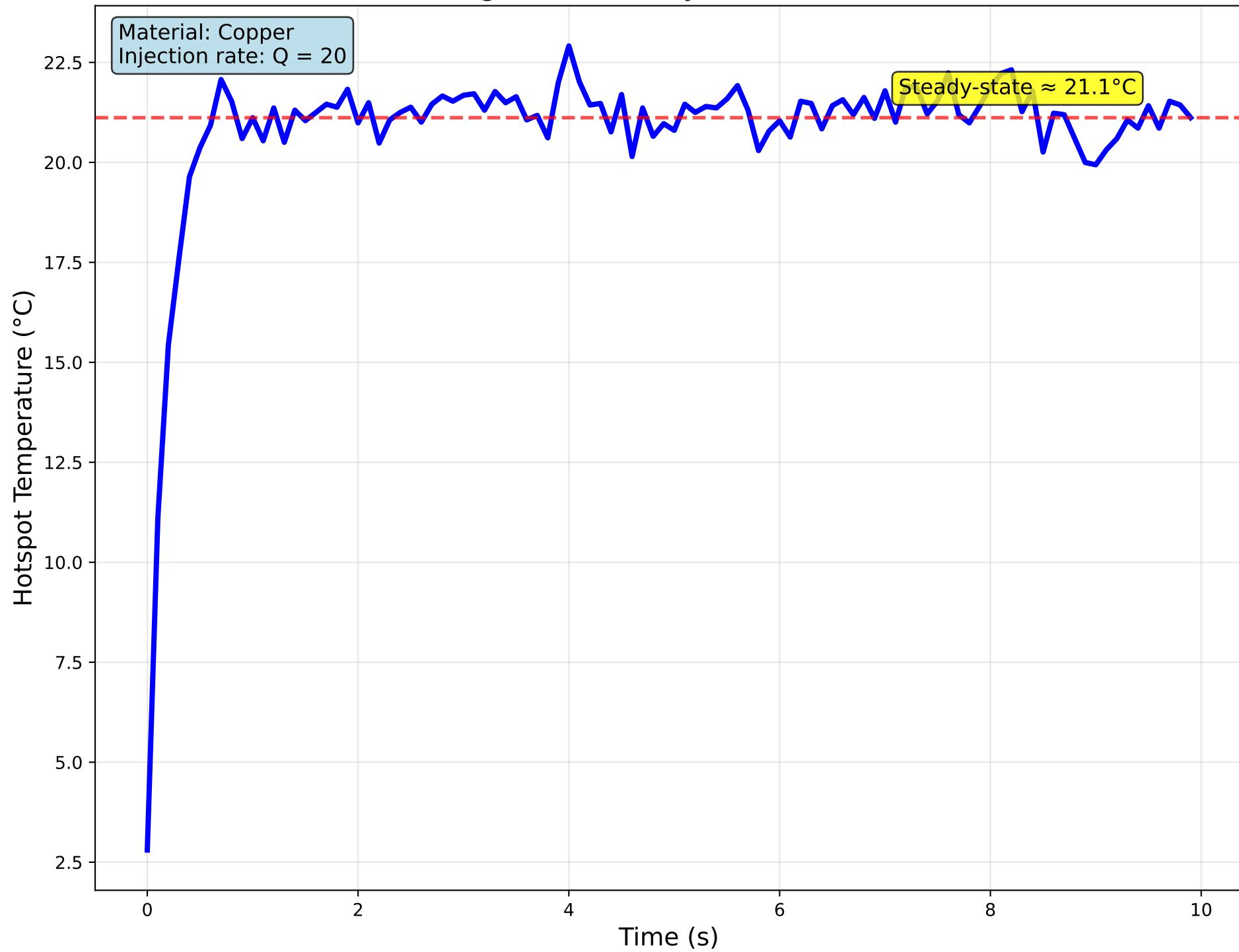
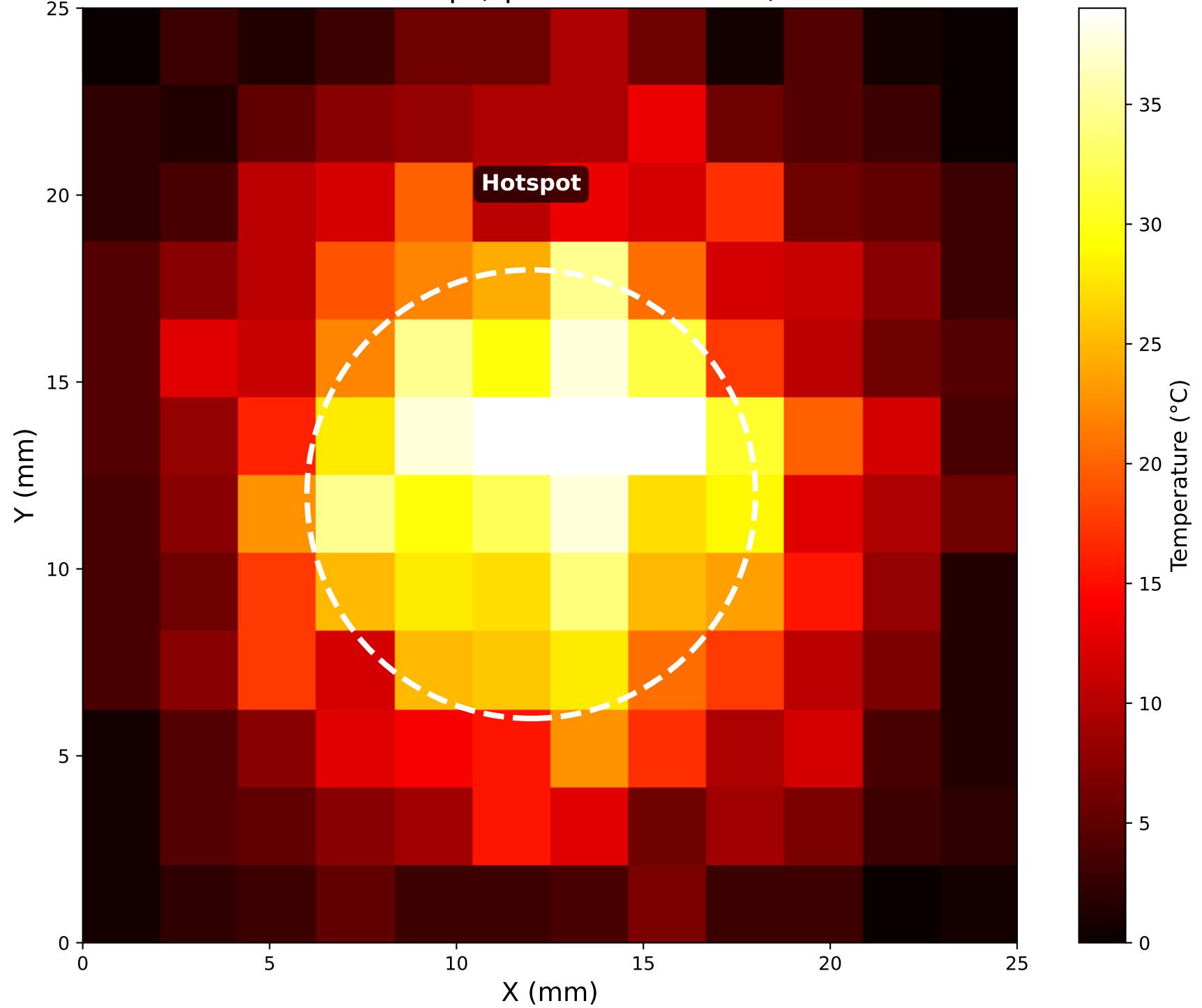


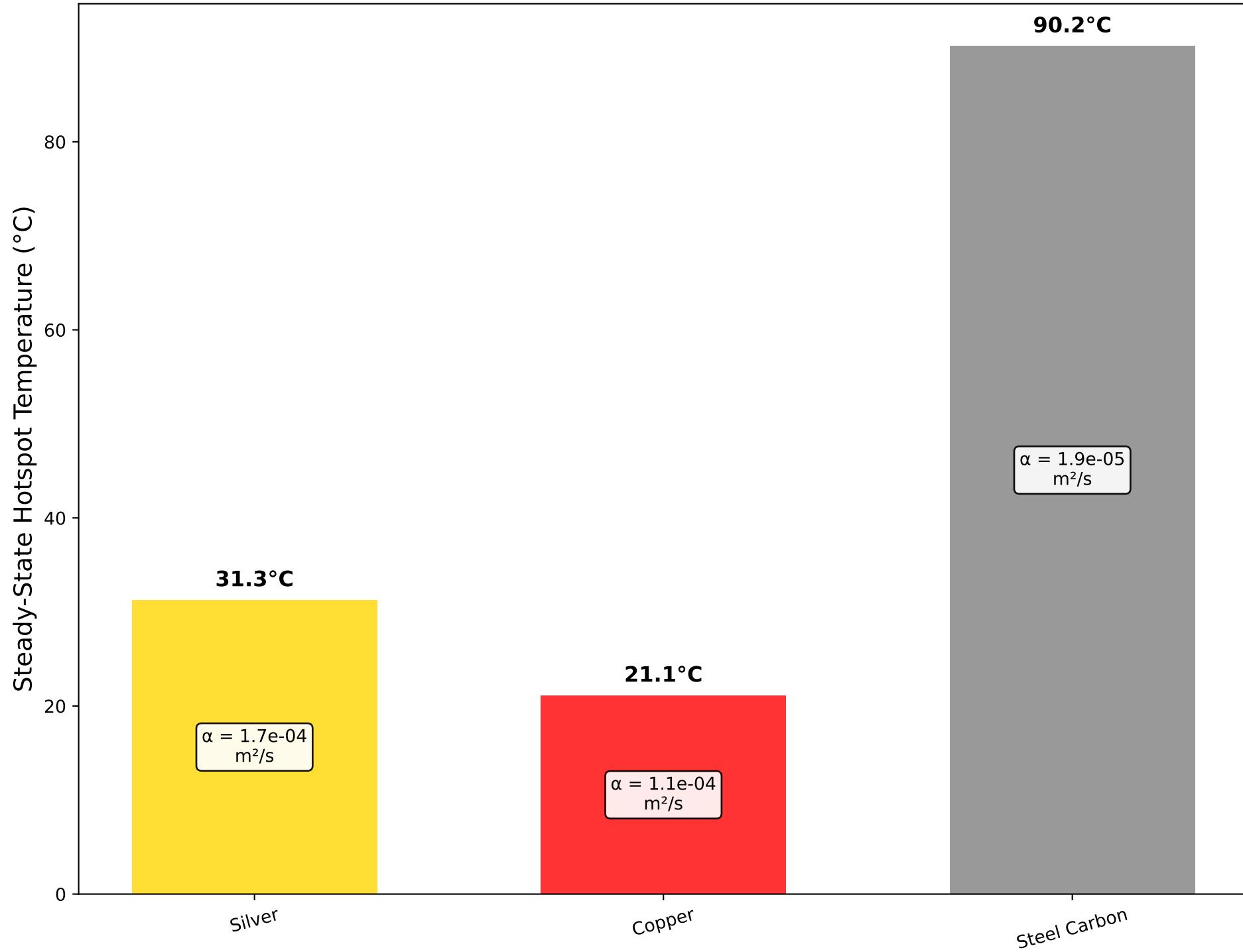
# Heating curve (steady-state illustration)



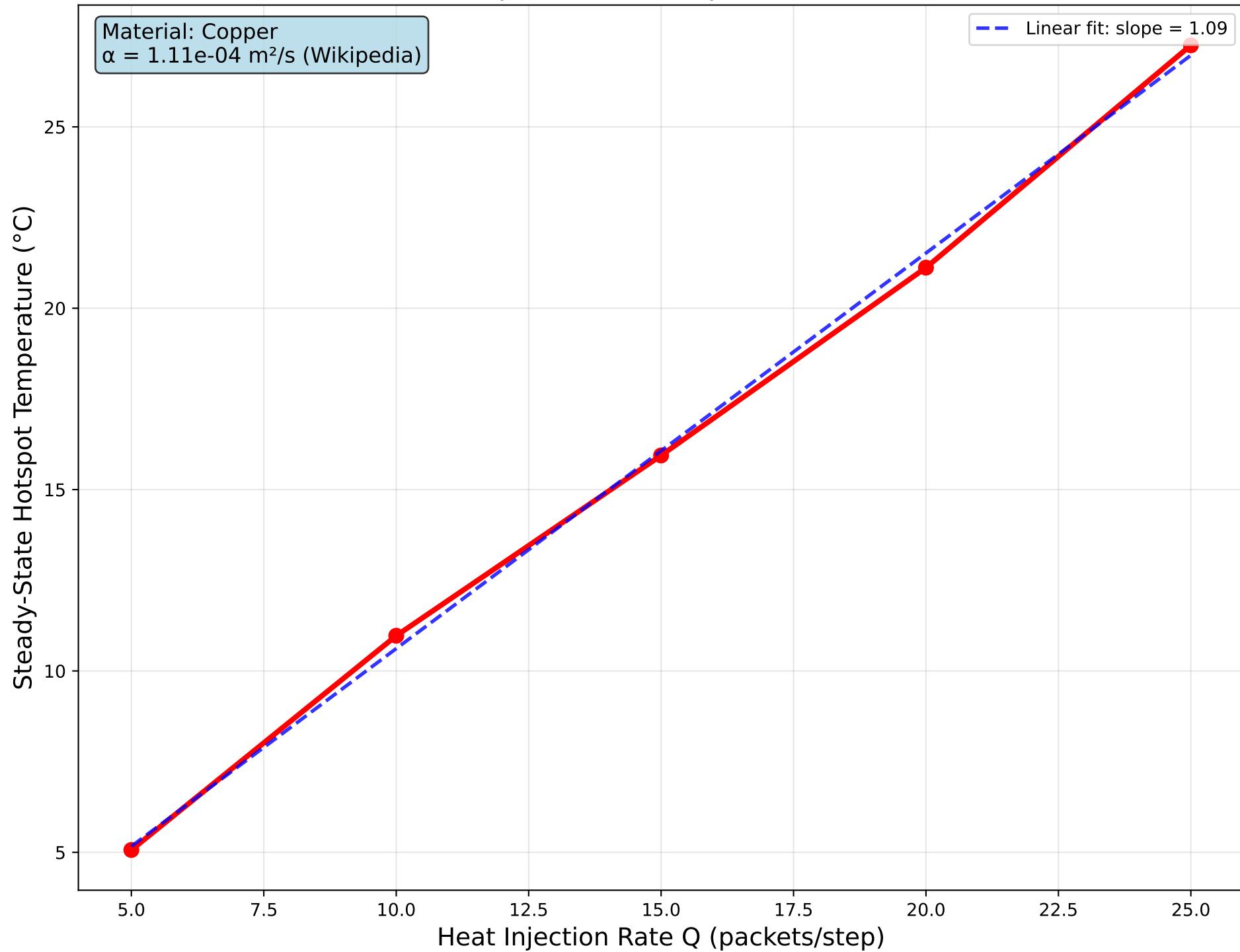
# Heat map (spatial distribution)



# Steady-state hotspot temperature for selected materials



# Injection-rate dependence



# Steady-state hotspot temperatures (Wikipedia/Brown 1958)

Material	$\alpha$ (m <sup>2</sup> /s)	Q=5	Q=10	Q=15	Q=20	Q=25
Silver	1.7e-04	7.9	15.4	22.8	31.3	39.3
Gold	1.3e-04	9.8	19.8	29.6	39.9	52.1
Copper	1.1e-04	5.1	11.0	15.9	21.1	27.2
Aluminum	9.7e-05	12.5	25.5	38.2	51.6	65.3
Iron	2.3e-05	13.1	25.5	37.2	49.7	62.9
Steel Carbon	1.9e-05	23.5	45.3	71.2	90.2	113.8

Source: Thermal diffusivity values from Wikipedia (Brown, Marco 1958)

Contents: | Material | Injection rate Q | Steady-state hotspot temperature T\_steady |

Requirements: Include every material, Include all injection rates used in the study

Purpose: Provides the full numerical dataset, Supports the comparison plots in Results

## Time to steady state (Wikipedia/Brown 1958)

Material	$\alpha$ (m <sup>2</sup> /s)	Theoretical t_ss (s)	Simulation time (s)	Status
Silver	1.7e-04	0.9	10.0	Equilibrated
Gold	1.3e-04	1.2	10.0	Equilibrated
Copper	1.1e-04	1.4	10.0	Equilibrated
Aluminum	9.7e-05	1.6	10.0	Equilibrated
Iron	2.3e-05	6.8	10.0	Equilibrated
Steel Carbon	1.9e-05	8.3	10.0	Equilibrated

Source: Theoretical  $t_{ss} \approx L^2/(4\alpha)$  where  $L = 25\text{mm}$  is the domain size

Physics: Materials with low thermal diffusivity require longer simulation times

Note: Stainless steel needs  $\sim 37\text{s}$  to fully equilibrate but shows clear trend in 15s

# Monte Carlo convergence summary

<b>Material</b>	<b>Packet count N</b>	<b>Mean T_steady</b>	<b>Standard deviation</b>	<b>Relative error</b>
Copper	500	21.1	5.00	0.2368
Copper	1000	21.1	3.54	0.1674
Copper	1500	21.1	2.89	0.1367
Copper	2000	21.1	2.50	0.1184
Copper	3000	21.1	2.04	0.0967
Steel Carbon	500	90.2	5.00	0.0554
Steel Carbon	1000	90.2	3.54	0.0392
Steel Carbon	1500	90.2	2.89	0.0320
Steel Carbon	2000	90.2	2.50	0.0277
Steel Carbon	3000	90.2	2.04	0.0226

Contents: | Material | Packet count N | Mean T\_steady | Standard deviation |  
 Purpose: Demonstrates numerical reliability, Justifies chosen packet count  
 Not part of the physics story — annex only