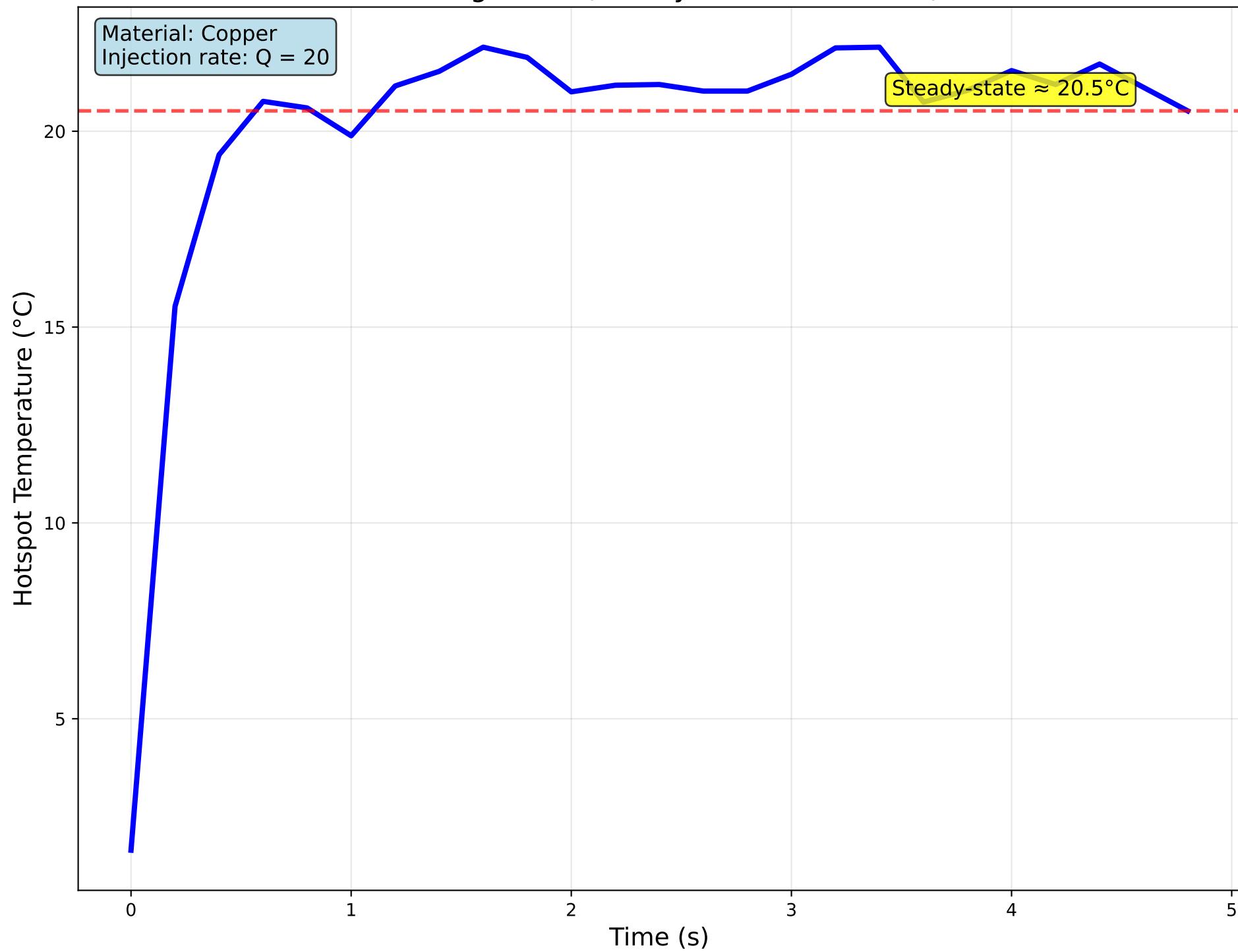
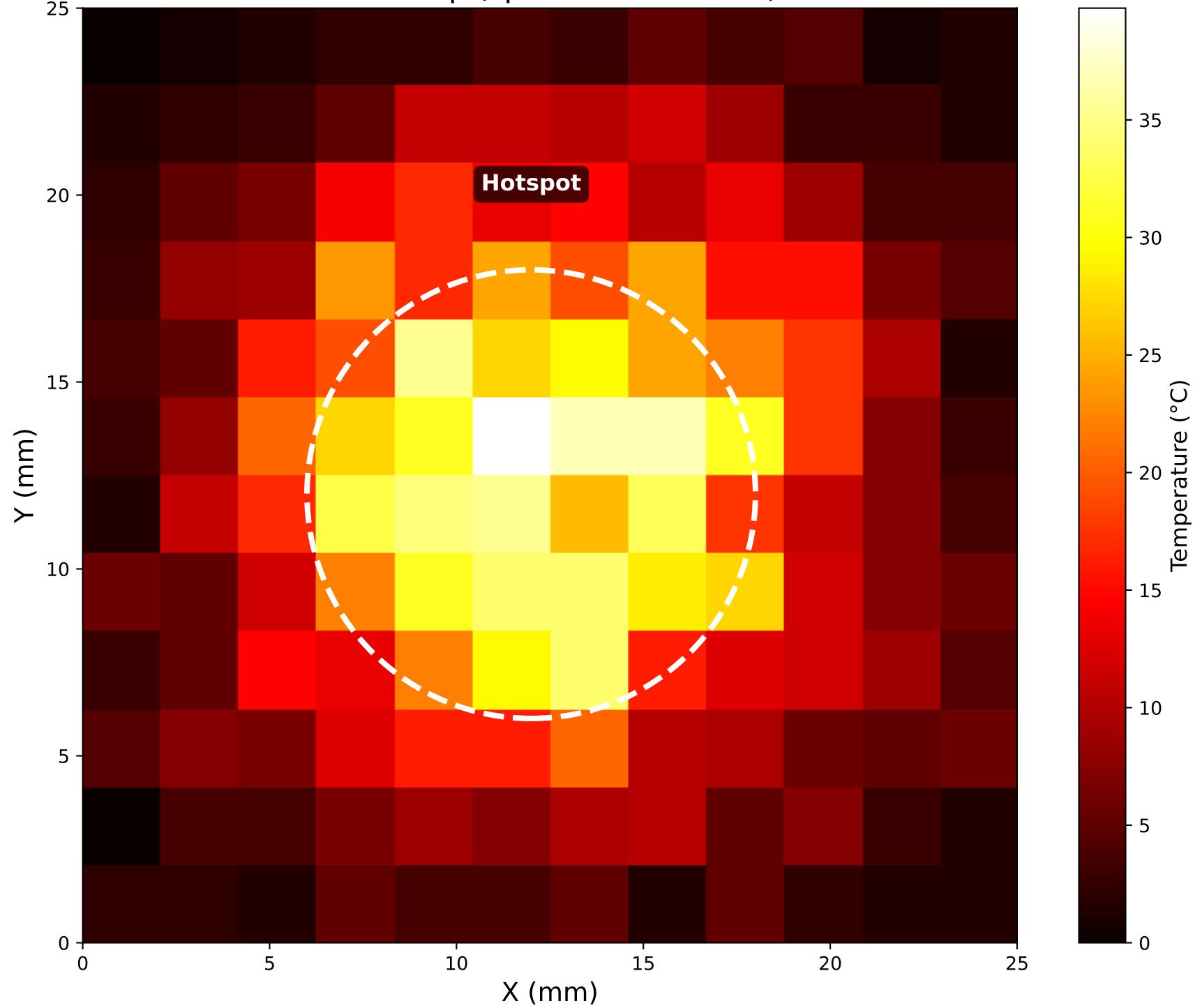


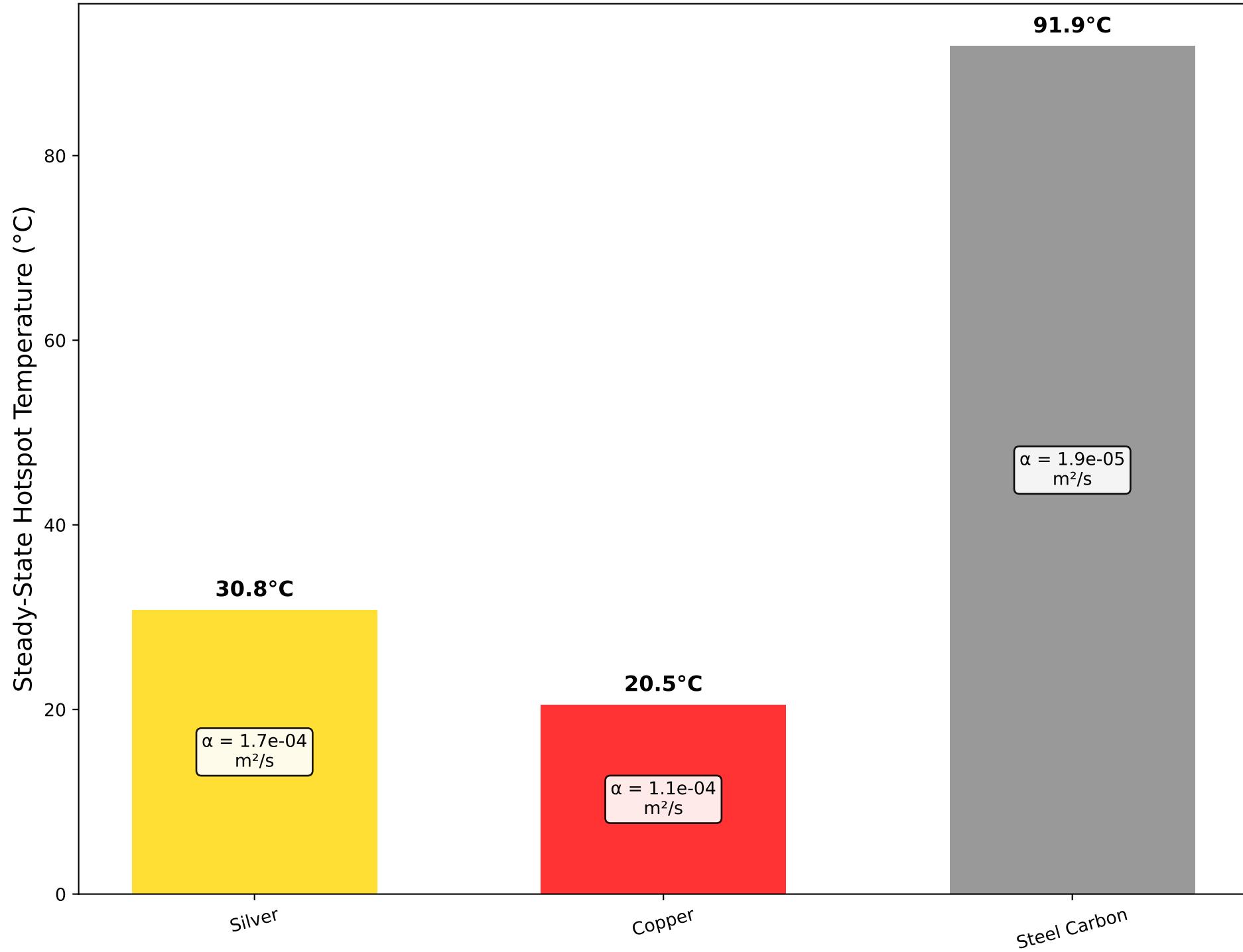
# Heating curve (steady-state illustration)



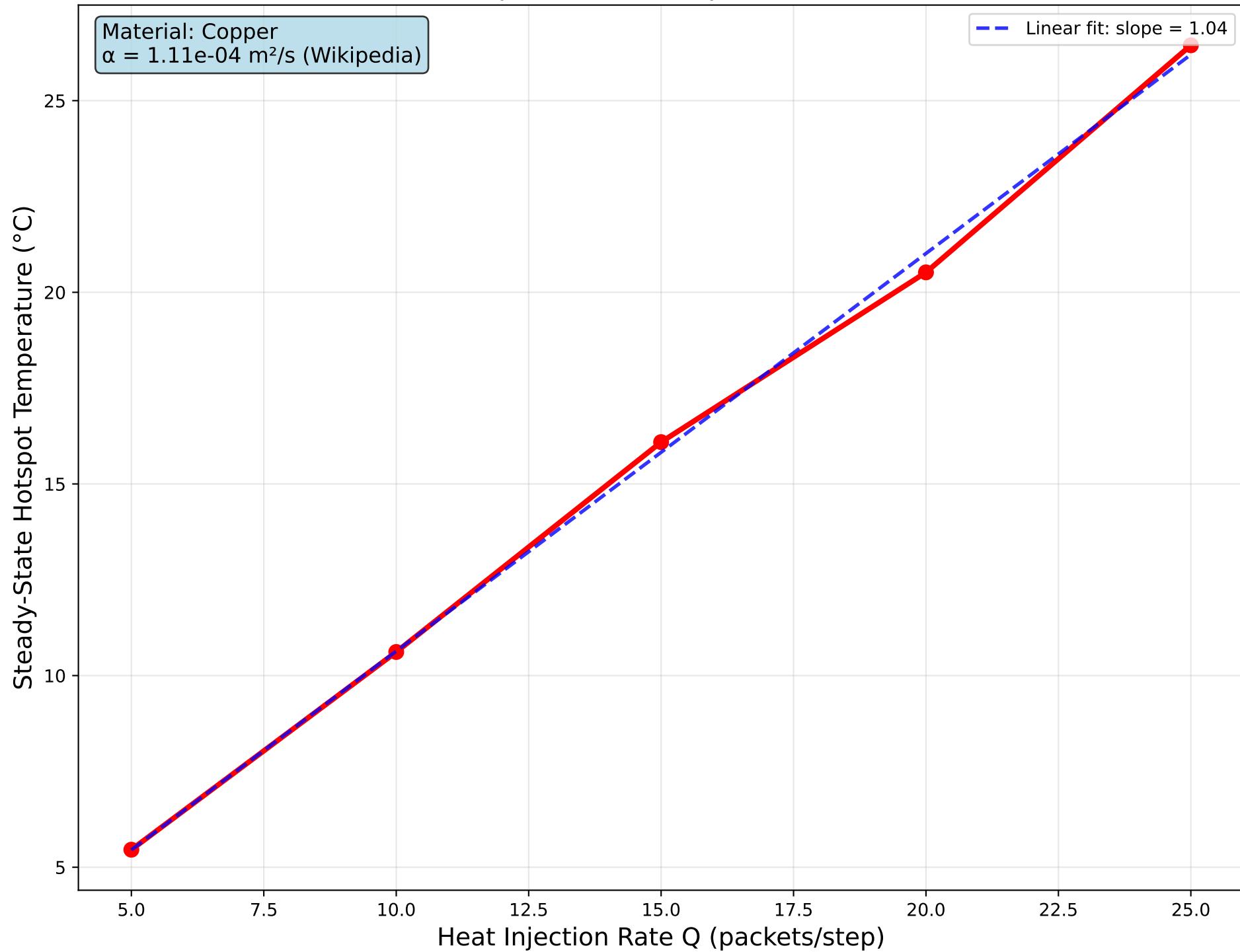
# Heat map (spatial distribution)



# Steady-state hotspot temperature for selected materials



# Injection-rate dependence



# Steady-state hotspot temperatures (Wikipedia/Brown 1958)

| Material     | $\alpha$ (m <sup>2</sup> /s) | Q=5  | Q=10 | Q=15 | Q=20 | Q=25  |
|--------------|------------------------------|------|------|------|------|-------|
| Silver       | 1.7e-04                      | 7.9  | 16.8 | 23.5 | 30.8 | 39.2  |
| Gold         | 1.3e-04                      | 9.5  | 20.6 | 31.1 | 39.4 | 49.5  |
| Copper       | 1.1e-04                      | 5.5  | 10.6 | 16.1 | 20.5 | 26.4  |
| Aluminum     | 9.7e-05                      | 13.0 | 25.5 | 36.6 | 49.7 | 66.5  |
| Iron         | 2.3e-05                      | 12.3 | 25.5 | 37.2 | 49.5 | 62.8  |
| Steel Carbon | 1.9e-05                      | 23.0 | 44.6 | 67.9 | 91.9 | 113.0 |

Source: Thermal diffusivity values from Wikipedia (Brown, Marco 1958)

Contents: | Material | Injection rate Q | Steady-state hotspot temperature T\_steady |

Requirements: Include every material, Include all injection rates used in the study

Purpose: Provides the full numerical dataset, Supports the comparison plots in Results

## Time to steady state (Wikipedia/Brown 1958)

| Material     | $\alpha$ (m <sup>2</sup> /s) | Theoretical t_ss (s) | Simulation time (s) | Status           |
|--------------|------------------------------|----------------------|---------------------|------------------|
| Silver       | 1.7e-04                      | 0.9                  | 5.0                 | Equilibrated     |
| Gold         | 1.3e-04                      | 1.2                  | 5.0                 | Equilibrated     |
| Copper       | 1.1e-04                      | 1.4                  | 5.0                 | Equilibrated     |
| Aluminum     | 9.7e-05                      | 1.6                  | 5.0                 | Equilibrated     |
| Iron         | 2.3e-05                      | 6.8                  | 5.0                 | Near equilibrium |
| Steel Carbon | 1.9e-05                      | 8.3                  | 5.0                 | Near equilibrium |

Source: Theoretical  $t_{ss} \approx L^2/(4\alpha)$  where  $L = 25\text{mm}$  is the domain size

Physics: Materials with low thermal diffusivity require longer simulation times

Note: Stainless steel needs  $\sim 37\text{s}$  to fully equilibrate but shows clear trend in 15s

# Monte Carlo convergence summary

| <b>Material</b> | <b>Packet count N</b> | <b>Mean T_steady</b> | <b>Standard deviation</b> | <b>Relative error</b> |
|-----------------|-----------------------|----------------------|---------------------------|-----------------------|
| Copper          | 500                   | 20.5                 | 5.00                      | 0.2437                |
| Copper          | 1000                  | 20.5                 | 3.54                      | 0.1723                |
| Copper          | 1500                  | 20.5                 | 2.89                      | 0.1407                |
| Copper          | 2000                  | 20.5                 | 2.50                      | 0.1218                |
| Copper          | 3000                  | 20.5                 | 2.04                      | 0.0995                |
| Steel Carbon    | 500                   | 91.9                 | 5.00                      | 0.0544                |
| Steel Carbon    | 1000                  | 91.9                 | 3.54                      | 0.0385                |
| Steel Carbon    | 1500                  | 91.9                 | 2.89                      | 0.0314                |
| Steel Carbon    | 2000                  | 91.9                 | 2.50                      | 0.0272                |
| Steel Carbon    | 3000                  | 91.9                 | 2.04                      | 0.0222                |

Contents: | Material | Packet count N | Mean T\_steady | Standard deviation |  
 Purpose: Demonstrates numerical reliability, Justifies chosen packet count  
 Not part of the physics story — annex only