Nptel Online Certification Course
Indian Institute of Technology Kharagpur
Computer Vision
Assignment - Week 0

Number of questions: 9 **Total marks: Not Applicable**

QUESTION 1: **Type: Equal answer**

Consider the following system of linear equations.

$$\begin{array}{ccccccccc} 2x & + & 7y & + & 3z & + & w & = & 6 \\ 3x & + & 5y & + & 2z & + & 2w & = & 4 \\ 9x & + & 4y & + & z & + & 7w & = & 2 \end{array}$$

By solving the above mentioned system of linear equations, mention the number of free variables in the provided space below.

Correct Answer: 2

Detailed Solution: A system is consistent if there exists at least one solution. First, prepare an augmented matrix using the coefficients of variables and the constants. It looks like

$$\begin{bmatrix} 2 & 7 & 3 & 1 & 6 \\ 3 & 5 & 2 & 2 & 4 \\ 9 & 4 & 1 & 7 & 2 \end{bmatrix}$$

The three rows of the augmented matrix are referred as R_1, R_2, and R_3. Follow the steps mentioned here to bring the matrix into the reduced row-echelon form.

1. $R_2 - R_1$. Then, swap R_1 and R_2.
2. $R_2 - 2R_1$. Then, $R_3 - 9R_1$
3. $R_3 - 2R_2$. Then, $R_2/11$
4. $R_1 + 2R_2$.

The reduced row-echelon form looks like

$$\begin{bmatrix} 1 & 0 & -1/11 & 9/11 & -2/11 \\ 0 & 1 & 5/11 & -1/11 & 10/11 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

It can be seen that the system is consistent and z,w are two free variables.

QUESTION 2: **Type:MCQ**

Find out the eigen values of a matrix $A = \begin{bmatrix} -2 & 1 \\ 12 & -3 \end{bmatrix}$ and their corresponding eigen vectors

a) $-1, -2$ and $k_1(1/3, -1)$, $k_2(-1/4, -1)$ respectively. k_1, and k_2 are two constants for free variables.

b) $1, -6$ and $k_1(1/3, 1)$, $k_2(-1/4, 1)$ respectively. k_1, and k_2 are two constants for free variables.

c) $1, 6$ and $k_1(-1/3, 1)$, $k_2(-1/4, 1)$ respectively. k_1, and k_2 are two constants for free variables.

d) $-1, 6$ and $k_1(-1/3, -1)$, $k_2(-1/4, -1)$ respectively. k_1, and k_2 are two constants for free variables.

Correct Answer: b

Detailed Solution: Assume that λ is a variable of eigen values. Determinant of the matrix $A - \lambda I = 0$ gives both the eigen values and the solution of linear equations given by $A - \lambda I = 0$ gives their corresponding eigen vectors.

QUESTION 3: **Type:MCQ**

Consider the data given below.

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}, v_3 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, y = \begin{bmatrix} -4 \\ 3 \\ h \end{bmatrix}$$

Which of the following options can be a value for h so that y lies in Span$\{v_1, v_2, v_3\}$.

a) 5

b) -5

c) -8

d) 8

Correct Answer: a

Detailed Solution:

y belongs to Span$\{v_1, v_2, v_3\}$ if there exist scalars a, b, c such that $av_1 + bv_2 + cv_3 = y$

$$a\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} + b\begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix} + c\begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \\ h \end{bmatrix}$$

This can be seen as a system of linear equations with three variables. The constant h can be computed by solving this system of linear equations by reducing the augmented matrix to row-echelon form. It is shown below.

$$\begin{bmatrix} 1 & 5 & -3 & -4 \\ -1 & -4 & 1 & 3 \\ -2 & -7 & 0 & h \end{bmatrix} \sim \begin{bmatrix} 1 & 5 & -3 & -4 \\ 0 & 1 & -2 & -1 \\ 0 & 3 & -6 & h-8 \end{bmatrix} \sim \begin{bmatrix} 1 & 5 & -3 & -4 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & h-5 \end{bmatrix}$$

The system is said to be consistent if there is no pivot in the fourth column. Hence, $h - 5 = 0$ and y is in Span$\{v_1, v_2, v_3\}$ if $h = 5$.

QUESTION 4: **Type:MSQ**

Choose the correct options from the following.

a) The set of vectors

$$\begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

are linearly dependent

b) The set of vectors

$$\begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 0 \end{bmatrix}$$

are linearly independent

c) Any set of vectors from R^n that contains the zero vector is dependent.

d) Any set of vectors from R^n that contains the zero vector is independent.

Correct Answer: a,b,c

Detailed Solution: A set of vectors is linearly dependent if any of them can be represented as a linear combination of the others. Otherwise, they are dependent.

The linear dependency or independency can also be verified by considering the three vectors as three columns of a matrix. If the reduced row echelon form of that matrix contains a zero row, then the given set of vectors is linearly dependent. Otherwise, linearly independent.

QUESTION 5: **Type:MCQ**

Compute A^{10} given that $A = PDP^{-1}$, where

$$A = \begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix}, P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}, D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

a)

$$\begin{bmatrix} 2047 & 1023 \\ -2046 & -1022 \end{bmatrix}$$

b)

$$\begin{bmatrix} 2046 & 1023 \\ -2047 & -1022 \end{bmatrix}$$

c)

$$\begin{bmatrix} 2047 & 1024 \\ -2046 & -1026 \end{bmatrix}$$

d)

$$\begin{bmatrix} 2048 & 1023 \\ -2042 & -1021 \end{bmatrix}$$

Correct Answer: a

Detailed Solution:

$$P^{-1} = \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}, A^{10} = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 2^{10} & 0 \\ 0 & 1^{10} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$$

Since matrix A is diagonal, A^{10} is just the $PD^{10}P^{-1}$. And, D^{10} is its diagonal elements raised to the power 10.

<u>QUESTION 6:</u> **Type:MCQ**

Consider the following vectors, and compute the orthogonal projection of y onto u.

$$y = \begin{bmatrix} 7 \\ 6 \end{bmatrix}, u = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

a)

$$\begin{bmatrix} 8 \\ 4 \end{bmatrix}$$

b)

$$\begin{bmatrix} -8 \\ 4 \end{bmatrix}$$

c)

$$\begin{bmatrix} 8 \\ -4 \end{bmatrix}$$

d)

$$\begin{bmatrix} -8 \\ -4 \end{bmatrix}$$

Correct Answer: a

Detailed Solution: The orthogonal projection of y onto u is

$$\hat{y} = \frac{y.u}{u.u} u$$

QUESTION 7: **Type:MCQ**

A single die is rolled once. Assume that the die is fair. Find the probability that the result of the roll is a number less than two and even.

a) 0.1

b) 0.01

c) 0

d) 0.001

Correct Answer: c

Detailed Solution:

The all-possible result when a die is rolled for one time is, Sample space = [1,2,3,4,5,6]. The all-possible result of the roll with number less than two is, Possible result = [1]. In the above shown possible result, there is no even number less than two. So, the probability that the result of the roll is a number less than two and even is, Probability = $\frac{0}{6} = 0$

QUESTION 8: **Type:MCQ**

Consider Z as a standard normal random variable, then another normal random variable $X = \sigma Z + \mu$ can be expressed as

a) $X \sim N(\mu/2, \sigma^2/2)$

b) $X \sim N(\mu, \sigma^2/2)$

c) $X \sim N(\mu/2, \sigma^2)$

d) $X \sim N(\mu, \sigma^2)$

Correct Answer: d

Detailed Solution: If Z is a standard normal random variable, i.e., $Z \sim N(0, 1)$, and $X = \sigma Z + \mu$, then X is a normal random variable with mean μ and variance σ^2, i.e., $X \sim N(\mu, \sigma^2)$.

QUESTION 9: **Type:MSQ**

Which of the following are the properties of a Poisson experiment?

a) The experiment results in outcomes that can be classified as successes or failures.

b) The probability that a success will occur is proportional to the size of the region.

c) The probability that a success will occur in an extremely small region is virtually zero.

d) The average number of successes (μ) that occurs in a specified region is unknown.

Correct Answer: a,b,c
