Note : The formulation for source panel method is based on the book "Fundamentals of Aerodynamics" by JD Anderson (Page 284-294)

Source Panel Method

Source panel method is one of the numerical techniques which are used for solving incompressible, potential and nonlifting flows over arbitrary bodies. The basic ideas of the source panel method are described here.

Let us assume the source sheet as shown in the Figure 1.Let s be the distance measured along the source sheet in the edge view. Here $\lambda = \lambda(s)$ is the source strength per unit length along s . Hence, the strength of an elemental portion ds of the sheet is λds . The source sheet is the combination of line sources and line sinks. So $\lambda(s)$ can change from positive to negative along the sheet.

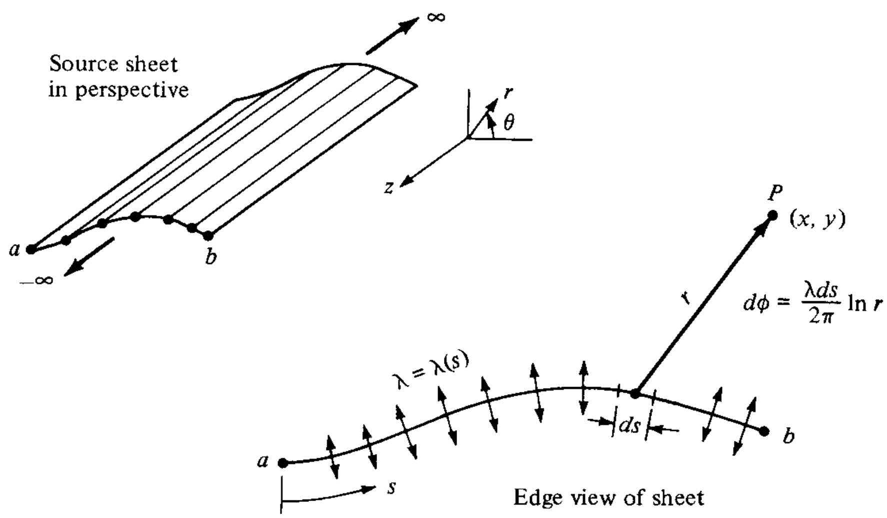


Figure 1 Source Sheet

Let's assume point $P(x, y)$ in the flow which is situated at a distance r from ds . The elemental section of source sheet of strength λds induces an infinitesimally small potential $d\phi$ at point P .For source flow, we know that velocity potential $d\phi$ is given by

$$d\phi = \frac{\lambda ds}{2\pi} \ln r \tag{1}$$

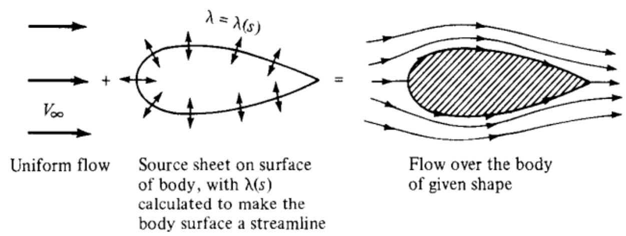


Figure 2 Superposition of a uniform flow and a source sheet on a body of given shape to produce the flow over a body

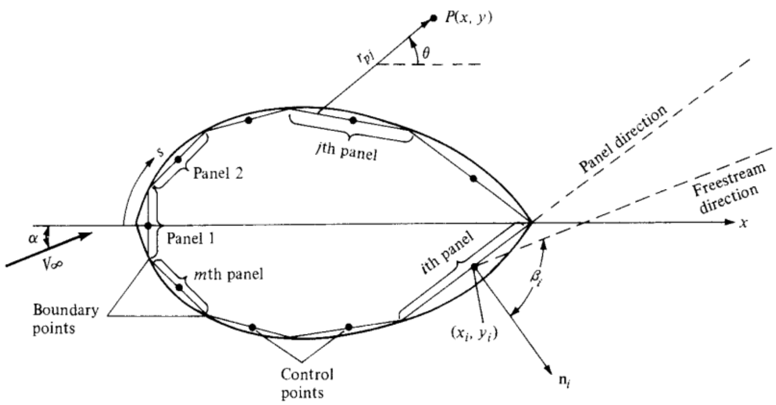


Figure 3 Source panel distribution over the surface of a body of arbitrary shape

The complete velocity potential induced at point P by entire source sheet from a to b, is obtained by integrating equation 1.

$$\phi(x, y) = \int_a^b \frac{\lambda ds}{2\pi} \ln r \tag{2}$$

Now, consider a body of arbitrary shape in a flow with freestream velocity V_∞ as shown in Figure 2. Suppose the surface of the prescribed body is covered with a source sheet where the strength $\lambda(s)$ varies in a such a way that the combined action of uniform flow and source sheet makes the airfoil surface a streamline of the flow.

Let us approximate the source sheet by a series of straight panels, as shown in Figure 3 and let the source strength λ per unit length be constant over a given panel, but assume that it varies from one panel to other. If there are a total of n panels, the source panel strengths per unit length will be $\lambda_1, \lambda_2, \lambda_3 \ldots.., \lambda_j \ldots, \lambda_n$. Let midpoint of each panel be a control point in which the boundary condition can be imposed numerically.

Let P be a point located at (x, y) in the flow, and let r_{pj} be the distance from any point on the jth panel to P ,as shown in Figure 3. The velocity potential induced at P due to the jth panel is given by

$$\Delta\phi_j = \frac{\lambda_j}{2\pi}\int_j \ln r_{pj} ds_j \quad \text{(from equation (2))} \tag{3}$$

λ_j is constant over the jth panel and the integral is taken over the jth panel only. Then, the velocity potential at P due to all the panels is obtained by summing up equation (3) over all the panels.

$$\phi(P) = \sum_{j=1}^{n} \Delta\phi_j = \sum_{j=1}^{n} \frac{\lambda_j}{2\pi}\int_j \ln r_{pj} ds_j \tag{4}$$

Where,

$$r_{Pj} = \sqrt{(x - x_j)^2 + (y - y_j)^2} \tag{5}$$

In equation (5), (x_j, y_j) are the coordinates along the surface of the $j\text{th}$ panel. As point P is just an arbitrary point in the flow, we can put P at the control point of the $i\text{th}$ panel. Let (x_i, y_i) be the coordinates of this control point as shown in Figure 3. Hence, the equations (4) and (5) become

$$\phi(x_i, y_i) = \sum_{j=1}^{n} \frac{\lambda_j}{2\pi}\int_j \ln r_{ij} ds_j \tag{6}$$

and
$$r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{7}$$

The contribution of all the panels to the potential at the control point of the $i\text{th}$ panel is hence given by equation (6).

Let $\mathbf{n}_i$ be the unit vector normal to the $i\text{th}$ panel, directed outward as shown in Figure 3 . The component of $\mathbf{V}_\infty$ to the $i\text{th}$ panel is

$$V_{\infty,n} = \mathbf{V}_\infty \cdot \mathbf{n}_i = V_\infty \cos\beta_i \tag{8}$$

Where β_i is the angle between $\mathbf{V}_\infty$ and $\mathbf{n}_i$.

The normal component of velocity induced at (x_i, y_i) by the source panels is obtained by differentiating equation (6) w.r.t $\mathbf{n}_i$.

$$V_n = \frac{\partial}{\partial n_i}\left[\phi(x_i, y_i)\right] \tag{9}$$

$$V_n = \frac{\partial}{\partial n_i}\left[\sum_{j=1}^{n}\frac{\lambda_j}{2\pi}\int_j \ln r_{ij} ds_j\right] \quad (10)$$

When $j = i$, the contribution to the derivative is $\frac{\lambda_i}{2}$.Therefore, the equation (10) becomes

$$V_n = \frac{\lambda_i}{2} + \sum_{\substack{j=1 \\ (j\neq 1)}}^{n}\frac{\lambda_j}{2\pi}\int_j \frac{\partial}{\partial}\left(\ln r_{ij}\right) ds_j$$

(11)

Here, $\frac{\lambda_i}{2}$ is the normal velocity induced at the ith control point by ith panel itself and the summation is the normal velocity induced at the ith control panel by all the other panels.

The normal component of the flow velocity at the ith control point is the sum of that due to the freestream and that due to the source panels. According to the boundary condition, the normal velocity should be zero.

$$V_{\infty,n} + V_n = 0 \quad (12)$$

Replacing equations (8) and (11) into (12),we get

$$\frac{\lambda_i}{2} + \sum_{\substack{j=1 \\ (j\neq 1)}}^{n}\frac{\lambda_j}{2\pi}\int_j \frac{\partial}{\partial n_i}\left(\ln r_{ij}\right) ds_j + V_\infty \cos\beta_i = 0$$

(13)

Let $\int_j \frac{\partial}{\partial n_i}\left(\ln r_{ij}\right) ds_j = I_{i,j}$ when the control point is on the ith panel and integral is over the jth panel.

Hence equation (13) becomes

$$\frac{\lambda_i}{2} + \sum_{\substack{j=1 \\ (j\neq 1)}}^{n}\frac{\lambda_j}{2\pi} I_{i,j} + V_\infty \cos\beta_i = 0 \quad (14)$$

Equation (14) is a linear algebraic equation with n unknowns , $\lambda_1, \lambda_2, \lambda_3 ....., \lambda_j ..., \lambda_n$. When this boundary condition is applied to the control points of all the panels , $i = 1, 2, ..., n$,there will be a system of n linear algebraic equations with n unknowns. After solving the set of equations, we will obtain the value of the strength λ of each panel.

The component of freestream velocity tangent to the surface can be obtained as

$$V_{\infty,s} = V_\infty \sin \beta_i \tag{15}$$

The tangential velocity V_s at the control point of the ith panel induced by all the panels is obtained by differentiating equation (6) w.r.t *s*.

$$V_s = \frac{\partial \phi}{\partial s} = \sum_{j=1}^{n} \frac{\lambda_j}{2\pi} \int_j \frac{\partial}{\partial s}\left(\ln r_{ij}\right) ds_j \tag{16}$$

The tangential velocity on a panel induced by the panel itself is zero and so the term corresponding to $j = i$ is zero. The total surface velocity at the ith control point V_i is the sum of the contribution from the freestream (equation (15)) and from the source panels(equation (16)).

$$V_i = V_{\infty,s} + V_s = V_\infty \sin \beta_i + \sum_{\substack{j=1 \\ (j\neq 1)}}^{n} \frac{\lambda_j}{2\pi} \int_j \frac{\partial}{\partial s}\left(\ln r_{ij}\right) ds_j \tag{17}$$

The pressure coefficient at the ith control point is obtained using following equation,

$$C_{P,i} = 1 - \left(\frac{V_i}{V_\infty}\right)^2 \tag{18}$$

Now, let’ s simplify the above equation as follows.

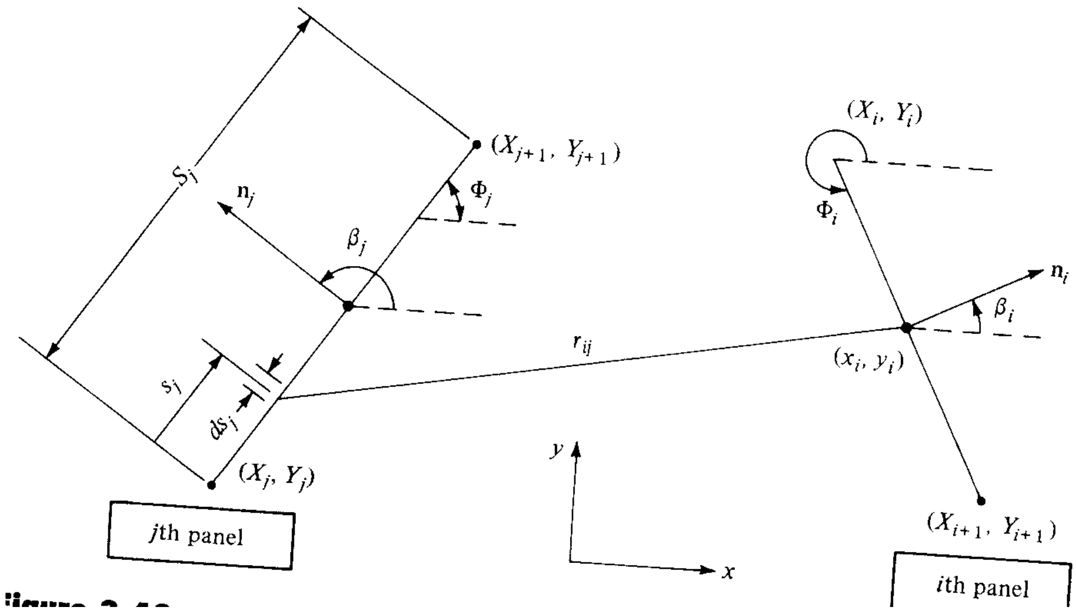


Figure 4 Geometry required for the evaluation of $I_{i,j}$

Nodes of airfoil are generated as follows:

```
for i= n:-1:1

    theta = (i-n)*2*pi/n;
    x = 0.5*c*(1+cos(theta));
if(x/c)<p
    yc(n+1-i)  = m*c/p^2*(2*p*(x/c)-(x/c)^2);
    dydx(n+1-i)  = (2*m/p^2)*  (p-x/c);
    beta(n+1-i)  = atan(dydx(n+1-i));
else
    yc(n+1-i)  = m*c/(1-p)^2 *  ((1-2*p)+2*p*(x/c)-(x/c)^2);
    dydx(n+1-i)  = (2*m/(1-p)^2)*  (p-x/c);
    beta(n+1-i)  = atan(dydx(n+1-i));
end
yt=5*t*c*(0.2969*sqrt(x/c)-0.1260*(x/c)...
    -0.3516*(x/c)^2+0.2843*(x/c)^3-0.1036*(x/c)^4);

if(i<(0.5*n+1))
    xa(n+1-i)=x - yt*sin(beta(n+1-i));
    ya(n+1-i)=yc(n+1-i)+yt*cos(beta(n+1-i));
else
    xa(n+1-i)=x + yt*sin(beta(n+1-i));
    ya(n+1-i)=yc(n+1-i)-yt*cos(beta(n+1-i));
end
```

```
end
xa(n+1)= c ;
ya(n+1) = 0;
yc(n+1) = 0;  % trailing edge
```

Nodes of cylinder are generated as follows:

```
r = 1; % radius of cylinder
for i= n:-1:1
    if i>=n/2
    theta = (i-3*n/2)*2*pi/n;
    else
        theta = (i-n/2)*2*pi/n;
    end

    x(n+1-i) = r*cos(theta);
    y(n+1-i) = r*sin(theta);
end

x(n+1)= -r ;
y(n+1) = 0;

a = 1;
xa(1)=x(n);
ya(1)=y(n);

for i = 1:n
    if mod(i,2)==0
        a = a+1;
        xa(a)=x(i);
        ya(a)=y(i);
    end
end

% computing control points and panel length
for i = 1:n
    xmid(i) = (xa(i)+xa(i+1))/2;
    ymid(i) = (ya(i)+ya(i+1))/2;
    Sj(i) = sqrt((xa(i+1)-xa(i))^2+(ya(i+1)-ya(i))^2);
end
```

$$I_{i,j} = \int_j \frac{\partial}{\partial n_i}\left(\ln r_{ij}\right) ds_j \tag{19}$$

and $$r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

then, $$\frac{\partial}{\partial}\left(\ln r_{ij}\right) = \frac{1}{r_{ij}}\frac{\partial r_{ij}}{\partial n_i} = \frac{1}{r_{ij}} \times \frac{1}{2}\left[\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2\right]^{\frac{-1}{2}} \times \left[2\left(x_i - x_j\right)\frac{dx_i}{dn_i} + 2\left(y_i - y_j\right)\frac{dy_i}{dn_i}\right]$$

from geometry,

$$\frac{dx_i}{dn_i} = \cos\beta_i \quad \text{and} \quad \frac{dy_i}{dn_i} = \sin\beta_i$$

$$x_j = X_j + s_j \cos\Phi_j$$

(20)

$$y_j = Y_j + s_j \sin\Phi_j$$

(21)

or, $$\frac{\partial}{\partial}\left(\ln r_{ij}\right) = \frac{\left(x_i - x_j\right)\cos\beta_i + \left(y_i - y_j\right)\sin\beta_i}{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2} \qquad (22)$$

$$\beta_i = \Phi_i + \frac{\pi}{2}$$

$$\therefore \quad \sin\beta_i = \cos\Phi_i$$

$$\text{and } \cos\beta_i = -\sin\Phi_i$$

Substituting equations (20) and (22) into (19),we obtain

$$I_{i,j} = \int_0^{S_j} \frac{Cs_j + D}{s_j{}^2 + 2As_j + B} ds_j \qquad (15)$$

Where,

$$A = -\left(x_{i-}X_j\right)\cos\Phi_j - \left(y_i - Y_j\right)\sin\Phi_j$$

$$B = \left(x_i - X_j\right)^2 + \left(y_i - Y_j\right)^2$$

$$C = \sin(\Phi_i - \Phi_j)$$

$$D = \left(y_{i-}Y_j\right)\cos\Phi_i - \left(x_i - X_j\right)\sin\Phi_i$$

$$S_j = \sqrt{\left(X_{j+1} - X_j\right)^2 + \left(Y_{j+1} - Y_j\right)^2}$$

$$E = \sqrt{B - A^2}$$

We get ,

$$I_{i,j} = \frac{C}{2}\ln\left(\frac{S_j{}^2 + 2AS_j + B}{B}\right) + \frac{D - AC}{E}\left(\tan^{-1}\frac{S_j + A}{E} - \tan^{-1}\frac{A}{E}\right) \qquad (16)$$

```
% Calculating angles and integrals

 for i = 1:n
      phi(i) = atan2((ya(i+1)-ya(i)),(xa(i+1)-xa(i)));
      eta(i) = phi(i)+pi/2;


 end




for i = 1:n
    for j = 1:n
         if i~=j

        A=-(xmid(i)-xa(j))*cos(phi(j))-(ymid(i)-ya(j))*sin(phi(j));
        B = (xmid(i)-xa(j))^2+(ymid(i)-ya(j))^2;
        C = sin(phi(i)-phi(j));
        D = (ymid(i)-ya(j))*cos(phi(i))-(xmid(i)-xa(j))*sin(phi(i));
        E = sqrt(B-A^2);
        I(i,j) = C/2*log((Sj(j)^2+2*A*Sj(j)+B)/B)+(D-
A*C)/E*(atan2((Sj(j)+A),E)-atan2(A,E));
                 end
       end
end
```

Substituting the value of $I_{i,j}$ for different *i*th panel in equation (3.153),we get n linear algebraic equations with n unknowns, $\lambda_1, \lambda_2, \lambda_3 \ldots\ldots\ldots, \lambda_n$.

$$\begin{array}{l}
\frac{\lambda_1}{2} + \frac{\lambda_2}{2\pi} \times I_{12} + \frac{\lambda_3}{2\pi} \times I_{13} + \ldots\ldots\ldots + \frac{\lambda_n}{2\pi} \times I_{1n} = -V_\infty \cos\beta_1 \\
\frac{\lambda_1}{2\pi} \times I_{21} + \frac{\lambda_2}{2} + \frac{\lambda_3}{2\pi} \times I_{23} + \ldots\ldots\ldots + \frac{\lambda_n}{2\pi} \times I_{2n} = -V_\infty \cos\beta_2 \\
\frac{\lambda_1}{2\pi} \times I_{31} + \frac{\lambda_2}{2\pi} \times I_{32} + \frac{\lambda_3}{2} + \ldots\ldots\ldots + \frac{\lambda_n}{2\pi} \times I_{3n} = -V_\infty \cos\beta_3 \\
\cdot \quad \cdot \quad \cdot \quad \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
\cdot \quad \cdot \quad \cdot \quad \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
\cdot \quad \cdot \quad \cdot \quad \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
\frac{\lambda_1}{2\pi} \times I_{n1} + \frac{\lambda_2}{2\pi} \times I_{n2} + \frac{\lambda_3}{2\pi} \times I_{n3} + \ldots\ldots\ldots + \frac{\lambda_n}{2\pi} = -V_\infty \cos\beta_n
\end{array}$$

Converting above equation in matrix form, we get

$$\begin{bmatrix} 1/2 & \mathrm{I}_{12}/2\pi & \mathrm{I}_{13}/2\pi & . & . & . & \mathrm{I}_{1n}/2\pi \\ \mathrm{I}_{12}/2\pi & 1/2 & \mathrm{I}_{23}/2\pi & . & . & . & \mathrm{I}_{2n}/2\pi \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ \mathrm{I}_{n1}/2\pi & \mathrm{I}_{n2}/2\pi & \mathrm{I}_{n3}/2\pi & . & . & . & 1/2 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_1 \\ . \\ . \\ . \\ . \\ \lambda_n \end{bmatrix} = \begin{bmatrix} -V_\infty \cos\beta_1 \\ -V_\infty \cos\beta_2 \\ . \\ . \\ . \\ . \\ -V_\infty \cos\beta_n \end{bmatrix}$$

By solving above matrix,we get values of strength of each panel i.e. $\lambda_1, \lambda_2, \lambda_3 ....., \lambda_j ..., \lambda_n$

It can be written as

$$P\lambda = Q \tag{17}$$

```
% solving matrix equation
P = zeros(n,n);

for i = 1:n
    for j = 1:n
        if i==j
        P(i,j) = pi;
        else
        P(i,j) = I(i,j);
        end
    end
end


Q = zeros(n,1);
U = 1;

for i = 1:n
    Q(i,1) = -2*pi*U*cos(eta(i));
end

lamda = zeros(n,1);
lamda = mldivide(P,Q);
```

The velocity at the control point of the ith panel can be obtained from the equation (17) .The integral over the jth panel in that equation can be obtained in a similar manner as before. The result is

$$J_{i,j} = \int_j \frac{\partial}{\partial s}\left(\ln r_{ij}\right) ds_j = \frac{D - AC}{2E} \ln\left(\frac{S_j{}^2 + 2AS_j + B}{B}\right) - C\left(\tan^{-1}\frac{S_j + A}{E} - \tan^{-1}\frac{A}{E}\right) \tag{18}$$

```
for i = 1:n
    for j = 1:n
         if i~=j
```

```
        A=-(xmid(i)-xa(j))*cos(phi(j))-(ymid(i)-ya(j))*sin(phi(j));
        B = (xmid(i)-xa(j))^2+(ymid(i)-ya(j))^2;
        C = sin(phi(i)-phi(j));
        D = (ymid(i)-ya(j))*cos(phi(i))-(xmid(i)-xa(j))*sin(phi(i));
        E = sqrt(B-A^2);
        J(i,j) = (D-A*C)/(2*E)*log((Sj(j)^2+2*A*Sj(j)+B)/B)-
C*(atan2((Sj(j)+A),E)-atan2(A,E));
        end
       end
end

% calculating tangential velocity accordng to the equation (17)
for i = 1:n
b(i) = 0.0;
end
for i = 1:n
    for j = 1:n
        if i~=j
        b(i) = b(i)+lamda(j,1)/2/pi*J(i,j);
        end
    end
end
```

Substituting the obtained values of λ in equation (17),we get the velocities $V_1, V_2, V_3 ...., V_n$ .In turn, the pressure coefficients $C_{p,1}, C_{p,2}, C_{p,3} ....., C_{p,n}$ are obtained directly from

$$C_{p,i} = 1 - \left(\frac{V_i}{V_\infty}\right)^2 \tag{19}$$

```
for i = 1:n
        v(i) = U*sin(eta(i))+b(i);
      ut = v(i)/U;
  cp(i) = 1-ut^2; % calculating cp at each control points

  end
```

The matlab program to find pressure distribution over an airfoil using source panel method is given as follows :

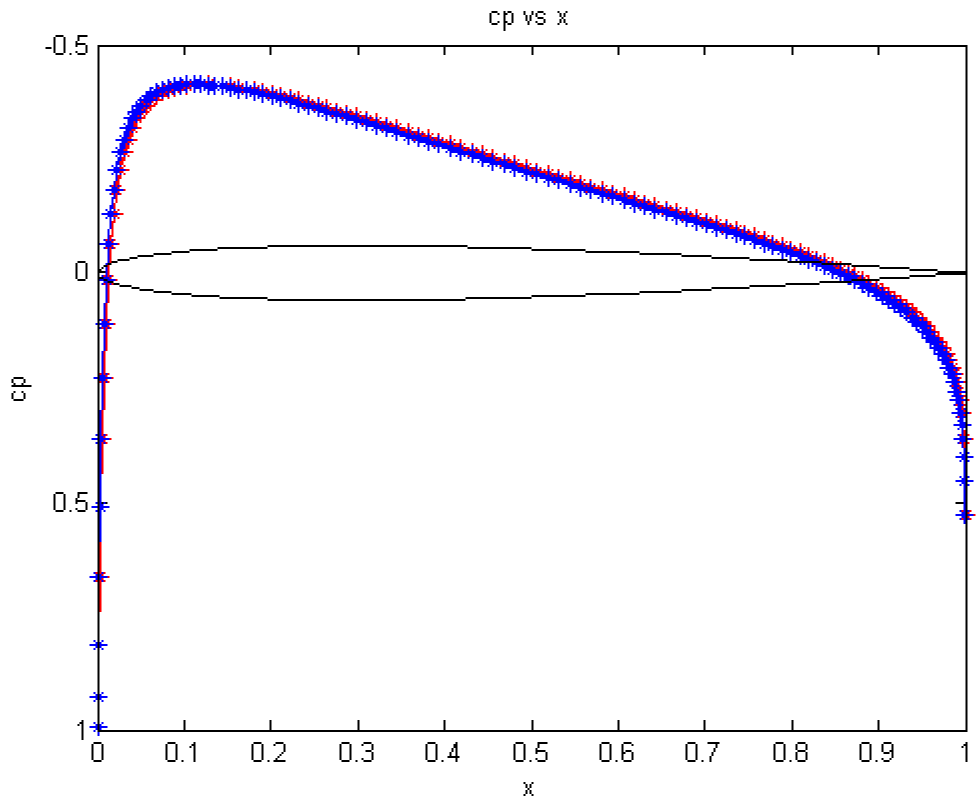
```
clc
clear all
close all
nacaseries = input('Enter the 4-digit naca series = ');
 c = input('Enter the chord length = ');
 s = num2str(nacaseries);

 % creating points on airfoil
if numel(s)==2
    s1 = str2double(s(1));
 s2 = str2double(s(2));
```

```
 m=0;p=0;t=(10*s1+s2)*0.01;
else
    s1 = str2double(s(1));
 s2 = str2double(s(2));
 s3 = str2double(s(3));
 s4 = str2double(s(4));
    m = s1*0.01; p = s2*0.1 ; t = (10*s3+s4)*0.01;
end

n = 250;
for i= n:-1:1

    theta = (i-n)*2*pi/n;
    %theta = 2*pi-theta;
   % angle = theta*180/pi
    x = 0.5*c*(1+cos(theta));
if(x/c)<p
    yc(n+1-i) = m*c/p^2*(2*p*(x/c)-(x/c)^2);
    dydx(n+1-i) = (2*m/p^2)* (p-x/c);
    beta(n+1-i) = atan(dydx(n+1-i));
else
    yc(n+1-i) = m*c/(1-p)^2 * ((1-2*p)+2*p*(x/c)-(x/c)^2);
    dydx(n+1-i) = (2*m/(1-p)^2)* (p-x/c);
    beta(n+1-i) = atan(dydx(n+1-i));
end
yt=5*t*c*(0.2969*sqrt(x/c)-0.1260*(x/c)...
    -0.3516*(x/c)^2+0.2843*(x/c)^3-0.1036*(x/c)^4);

if(i<(0.5*n+1))
    xa(n+1-i)=x - yt*sin(beta(n+1-i));
    ya(n+1-i)=yc(n+1-i)+yt*cos(beta(n+1-i));
else
    xa(n+1-i)=x + yt*sin(beta(n+1-i));
    ya(n+1-i)=yc(n+1-i)-yt*cos(beta(n+1-i));
end

end
xa(n+1)= c ;
ya(n+1) = 0;
yc(n+1) = 0;  % trailing edge

% computing control points and panel length
for i = 1:n
    xmid(i) = (xa(i)+xa(i+1))/2;
    ymid(i) = (ya(i)+ya(i+1))/2;
    Sj(i) = sqrt((xa(i+1)-xa(i))^2+(ya(i+1)-ya(i))^2);
end
%a = 0;
%alpha = pi/180*a;
 % Calculating angles and integrals

 for i = 1:n
      phi(i) = atan2((ya(i+1)-ya(i)),(xa(i+1)-xa(i)));
      eta(i) = phi(i)+pi/2;
         %phii(i) = phii(i)*180/pi;
```

```
 end

for i = 1:n
    for j = 1:n
         if i~=j

        A=-(xmid(i)-xa(j))*cos(phi(j))-(ymid(i)-ya(j))*sin(phi(j));
        B = (xmid(i)-xa(j))^2+(ymid(i)-ya(j))^2;
        C = sin(phi(i)-phi(j));
        D = (ymid(i)-ya(j))*cos(phi(i))-(xmid(i)-xa(j))*sin(phi(i));
        E = sqrt(B-A^2);
        I(i,j) = C/2*log((Sj(j)^2+2*A*Sj(j)+B)/B)+(D-
A*C)/E*(atan2((Sj(j)+A),E)-atan2(A,E));
        J(i,j) = (D-A*C)/(2*E)*log((Sj(j)^2+2*A*Sj(j)+B)/B)-
C*(atan2((Sj(j)+A),E)-atan2(A,E));
        end
       end
end

% solving matrix equation
P = zeros(n,n);

for i = 1:n
    for j = 1:n
        if i==j
        P(i,j) = pi;
        else
        P(i,j) = I(i,j);
        end
    end
end

Q = zeros(n,1);
U = 1;

for i = 1:n
    Q(i,1) = -2*pi*U*cos(eta(i));
end

lamda = zeros(n,1);
lamda = mldivide(P,Q);

% calculating tangential velocity accordng to the equation (17)
for i = 1:n
b(i) = 0.0;
end
for i = 1:n
    for j = 1:n
        if i~=j
```

```
        b(i) = b(i)+lamda(j,1)/2/pi*J(i,j);
        end
    end
end

  for i = 1:n
        v(i) = U*sin(eta(i))+b(i);
      ut = v(i)/U;
  cp(i) = 1-ut^2; % calculating cp at each control points

  end


    plot(xa(1:n/2),cp(1:n/2),'-*r')
    set(gca,'Ydir','reverse')
    hold on
    plot(xa(n/2+1:n),cp(n/2+1:n),'-*b')

    hold on
    plot(xa,ya,'-k')
    xlabel('x')
    ylabel('cp')
    title('cp vs x')
```

The plot of cp vs x of NACA0012 airfoil at 0^o AOA using panel method



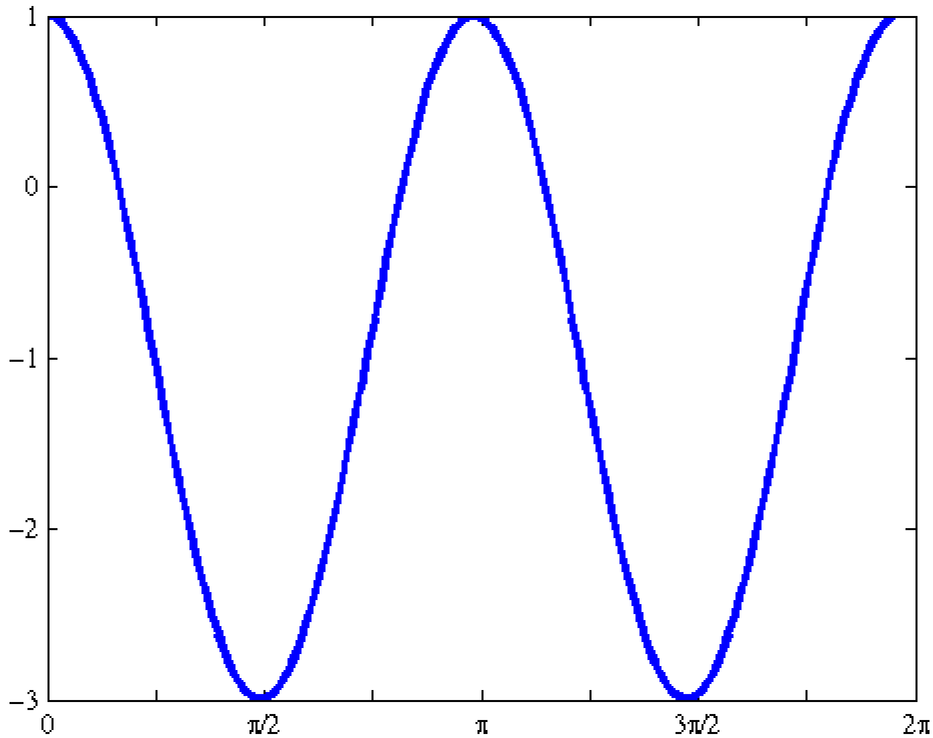
The matlab program to find pressure distribution over an airfoil using source panel method is given as follows :

```
clc
clear all
close all

 % creating points on airfoil


n = 250;
r = 1; % radius of cylinder
for i= n:-1:1
    if i>=n/2
    theta = (i-3*n/2)*2*pi/n;
    else
        theta = (i-n/2)*2*pi/n;
    end

    x(n+1-i) = r*cos(theta);
    y(n+1-i) = r*sin(theta);
end

x(n+1)= -r ;
y(n+1) = 0;
```

```
a = 1;
xa(1)=x(n);
ya(1)=y(n);

for i = 1:n
    if mod(i,2)==0
        a = a+1;
        xa(a)=x(i);
        ya(a)=y(i);
    end
end

% computing control points and panel length
for i = 1:n/2
    xmid(i) = (xa(i)+xa(i+1))/2;
    ymid(i) = (ya(i)+ya(i+1))/2;
    Sj(i) = sqrt((xa(i+1)-xa(i))^2+(ya(i+1)-ya(i))^2);
end
%a = 0;
%alpha = pi/180*a;
 % Calculating angles and integrals

 for i = 1:n/2
      phi(i) = atan2((ya(i+1)-ya(i)),(xa(i+1)-xa(i)));
      eta(i) = phi(i)+pi/2;
         %phii(i) = phii(i)*180/pi;

 end




for i = 1:n/2
    for j = 1:n /2
         if i~=j

        A=-(xmid(i)-xa(j))*cos(phi(j))-(ymid(i)-ya(j))*sin(phi(j));
        B = (xmid(i)-xa(j))^2+(ymid(i)-ya(j))^2;
        C = sin(phi(i)-phi(j));
        D = (ymid(i)-ya(j))*cos(phi(i))-(xmid(i)-xa(j))*sin(phi(i));
        E = sqrt(B-A^2);
        I(i,j) = C/2*log((Sj(j)^2+2*A*Sj(j)+B)/B)+(D-
A*C)/E*(atan2((Sj(j)+A),E)-atan2(A,E));
        J(i,j) = (D-A*C)/(2*E)*log((Sj(j)^2+2*A*Sj(j)+B)/B)-
C*(atan2((Sj(j)+A),E)-atan2(A,E));
        end
       end
end

% solving matrix equation
P = zeros(n/2,n/2);

for i = 1:n /2
    for j = 1:n/2
        if i==j
        P(i,j) = pi;
```

```
        else
        P(i,j) = I(i,j);
        end
    end
end

P
Q = zeros(n/2,1);
U = 1;

for i = 1:n/2
    Q(i,1) = -2*pi*U*cos(eta(i));
end

lamda = zeros(n/2,1);
lamda = mldivide(P,Q);


% calculating tangential velocity accordng to the equation (17)
for i = 1:n/2
b(i) = 0.0;
end
for i = 1:n/2
    for j = 1:n/2
        if i~=j
        b(i) = b(i)+lamda(j,1)/2/pi*J(i,j);
        end
    end
end

  for i = 1:n/2
        v(i) = U*sin(eta(i))+b(i);
      ut = v(i)/U;
  cp(i) = 1-ut^2; % calculating cp at each control points

  end

  for i = 1:n/2
      theta(i) = (i-1)*2*pi/n/2;
  end



plot(theta(1:n/2), cp(1:n/2),'LineWidth', 3);
set(gca,'FontName','Symbol')
set(gca,'XTickLabel','0|  |p/2|  |p|  |3p/2|  |2p')
set(gca,'YTick',-3:1:1)
```

The plot of cp vs θ of cylinder using panel method is given below.



1
0
−1
−2
−3
0
π/2
π
3π/2
2π