Note : The Vortex Lattice Method presented here is based on the book "Aerodynamics for Engineers" by Bertin and Cummings (Page 389-404)

Vortex Lattice Method

In Vortex Lattice Method (VLM), the wing is represented as surface on which grids of horseshoe vortices are superimposed. The velocities induced by each horseshoe vortex at a specified control point are calculated using *Biot-Savart* law. Then a summation is performed for all control points on the wing to produce a set of linear algebraic equations for the horseshoe vortex strengths that satisfy the boundary condition of no flow through the wing. The vortex strengths are related to the wing circulation.

To solve the governing equations, the continuous distribution of bound vorticity over the wing surface is approximated by a finite number of discrete horseshoe vortices as shown in Fig. 1. The individual horseshoe vortices are placed in trapezoidal panels (also called finite elements or lattices). This procedure for obtaining a numerical solution to the flow is termed as vortex lattice method.

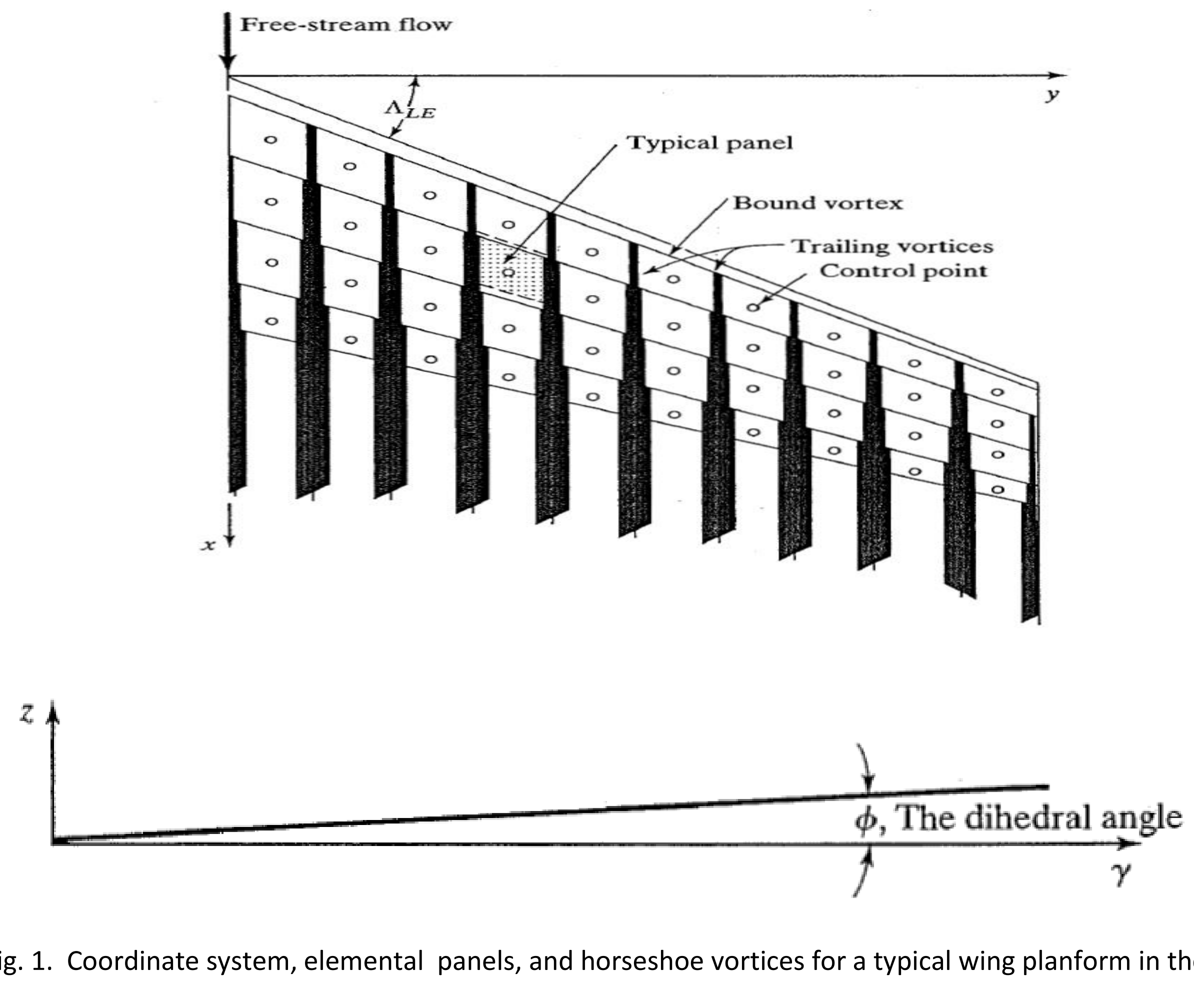
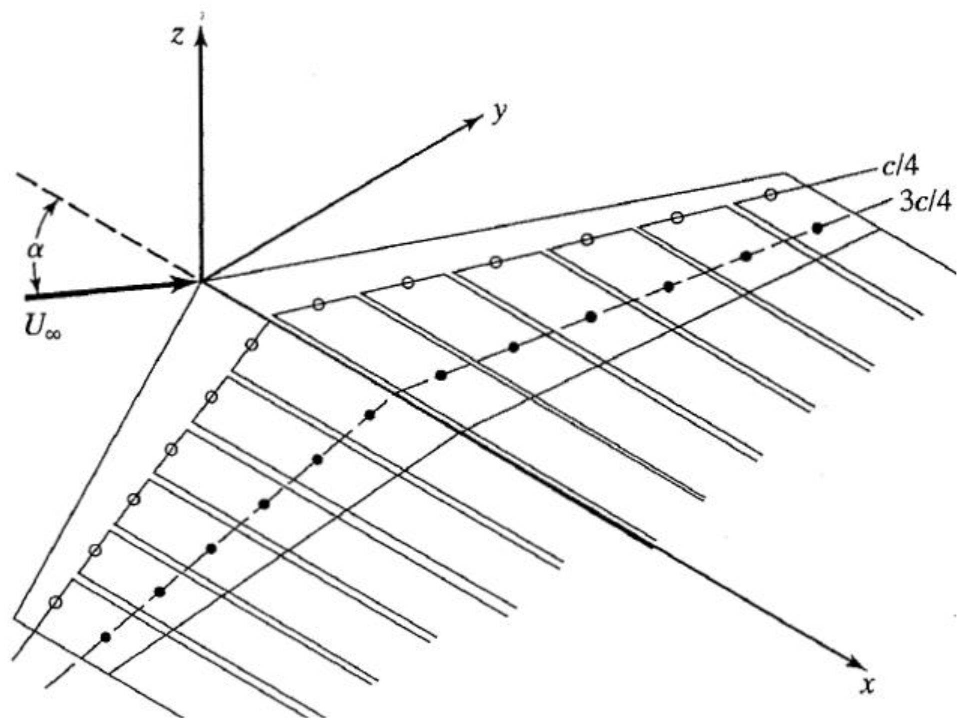


Fig. 1. Coordinate system, elemental panels, and horseshoe vortices for a typical wing planform in the vortex panel method

The bound vortex coincides with the quarter-chord line of panel(or element) and hence they are aligned with the local sweepback angle. In a rigorous theoretical analysis, the vortex lattice panels are located on the mean camber surface of the wing and, when the trailing vortices leave the wing, they follow a curved path. But, for many engineering applications, suitable accuracy can be obtained using linearized theory in which straight-line trailing vortices extend downstream to infinity. In the linearized approach, the trailing vortices are aligned either parallel to the free stream or parallel to the vehicle axis. Both orientations provide similar accuracy within the assumptions of linearized theory. Here, we assume that the trailing vortices are parallel to the axis of the vehicle.as shown in Fig. 2. The geometric coefficients do not change as the angle of attack is changed. Application of the boundary condition that the flow is tangent to the wing surface at the control points of each of the 2 N panels (i.e. there is no flow through the surface) provides a set of simultaneous equations in the unknown vortex circulation strengths. The control point of each panel is centered spanwise on the three-quarter-chord line midway between the trailing vortex legs.



Filled circles represent the control points

Fig.2. Distributed horseshoe vortices representing the lifting flow field over a swept wing

Why is the three-quarter-chord location used as the control point ?

A vortex filament whose strength Γ represents the lifting character of the section is placed at the quarter-chord location. It induces a velocity $U = \dfrac{\Gamma}{2\pi r}$ at point c, the control point which is at distance r from the vortex filament as shown in Fig.3.

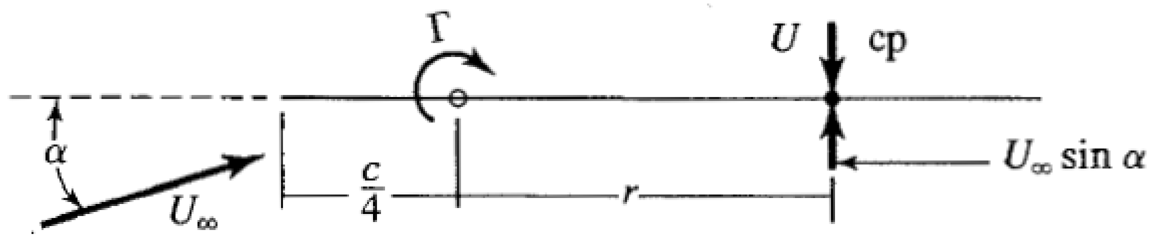


Fig.3. Planar airfoil section indicating location of control point where flow is parallel to the surface

If the flow is to be parallel to the surface at the control point, the incidence of the surface relative to the free stream is given by

$$\alpha \approx \sin\alpha = \frac{U}{U_\infty} = \frac{\Gamma}{2\pi r U_\infty}$$

We know,

$$l = \pi \rho_\infty U_\infty^2 \alpha c = \rho_\infty U_\infty \Gamma$$

Substituting α in the above equation, we get

$$\rho_\infty U_\infty^2 c\pi \frac{\Gamma}{2\pi r U_\infty} = \rho_\infty U_\infty \Gamma$$

Solving for r yields,

$$r = \frac{c}{2}$$

The velocity induced by a vortex filament of strength Γ_n and a length of dl is given by the *Biot-Savart* law,

$$\overrightarrow{dV} = \frac{\Gamma_n (d\vec{l} \times \vec{r})}{4\pi r^3} \tag{1}$$

Referring to the sketch of Fig. 4, the magnitude of the induced velocity is

$$dV = \frac{\Gamma_n (dl.r.\sin\theta)}{4\pi r^3} = \frac{\Gamma_n \sin\theta dl}{4\pi r^2} \tag{2}$$

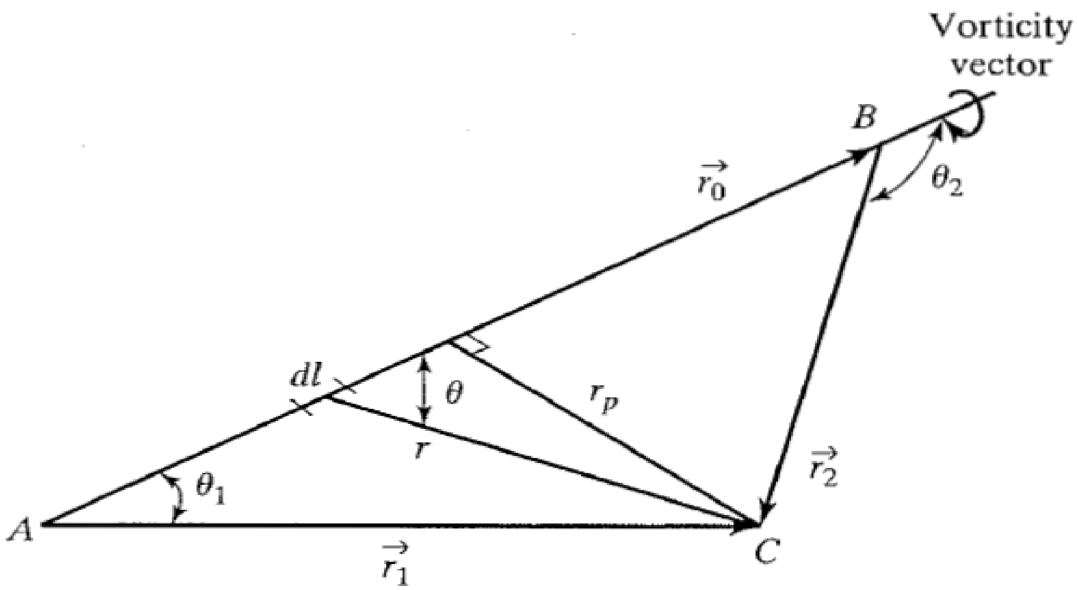


Fig.4. Nomenclature for calculating the velocity induced by a finite-length vortex segment

Since we are interested in the flow field induced by a horseshoe vortex which consists of three straight segments, let us use equation (1) to calculate the effect of each segment separately. Referring Fig. 4. , let AB be such a segment, with the vorticity vector directed from A to B and let C be a point in space whose normal distance from the line AB is r_p. We can integrate between A and B to find the magnitude of the induced velocity :

$$V = \frac{\Gamma_n}{4\pi r_p} \int_{\theta_1}^{\theta_2} \sin\theta d\theta = \frac{\Gamma_n}{4\pi r_p} \left(\cos\theta_1 - \cos\theta_2\right) \tag{3}$$

Note that, if vortex filament extends to infinity in both directions, then $\theta_1 = 0$ and $\theta_1 = \pi$. In this case

$$V = \frac{\Gamma_n}{2\pi r_p}$$

Which is the result used for infinite-span airfoils. Let $\vec{r}_0$, $\vec{r}_1$ and $\vec{r}_2$ designate the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$ respectively as shown in Fig.4. Then

$$r_p = \frac{|\vec{r}_1 \times \vec{r}_2|}{r_0} \qquad \cos\theta_1 = \frac{\vec{r}_0.\vec{r}_1}{r_0 r_1} \qquad \cos\theta_1 = \frac{\vec{r}_0.\vec{r}_2}{r_0 r_2}$$

In these equations, if a vector quantity (such as $\vec{r}_0$) is written without a superscript arrow, the symbol represents the magnitude of the parameter. Thus, r_0 is the magnitude of the vector $\vec{r}_0$. Also note that $|\vec{r}_1 \times \vec{r}_2|$ represents the magnitude of the vector cross product. Substituting these expressions into equation (3) and noting that the direction of the induced velocity is given by the unit vector

$$\frac{\vec{r}_1 \times \vec{r}_2}{|\vec{r}_1 \times \vec{r}_2|}$$

yields

$$\vec{V} = \frac{\Gamma_n}{4\pi\left(\frac{|\vec{r}_1 \times \vec{r}_2|}{r_0}\right)}\left(\frac{\vec{r}_0.\vec{r}_1}{r_0 r_1} - \frac{\vec{r}_0.\vec{r}_2}{r_0 r_2}\right) = \frac{\Gamma_n}{4\pi}\frac{\vec{r}_1 \times \vec{r}_2}{|\vec{r}_1 \times \vec{r}_2|^2}\left[\vec{r}_0.\left(\frac{\vec{r}_1}{r_1} - \frac{\vec{r}_2}{r_2}\right)\right] \tag{4}$$

This is the basic expression for the calculation of the induced velocity by the horseshoe vortices in the VLM. It can be used regardless of the assumed orientation of the vortices.

We can now use equation (4) to calculate the velocity that is induced at a general point in space (x, y, z) by the horseshoe vortex shown in Fig.5. The horseshoe vortex may be assumed to represent that for a typical wing panel (say $n\text{th}$ panel)in Fig.1. Segment AB represents the bound vortex portion of the horseshoe system and coincides with the quarter-chord line of the panel element. The trailing vortices are parallel to the x axis. The resultant induced velocity vector will be calculated by considering the influence of each of the elements.

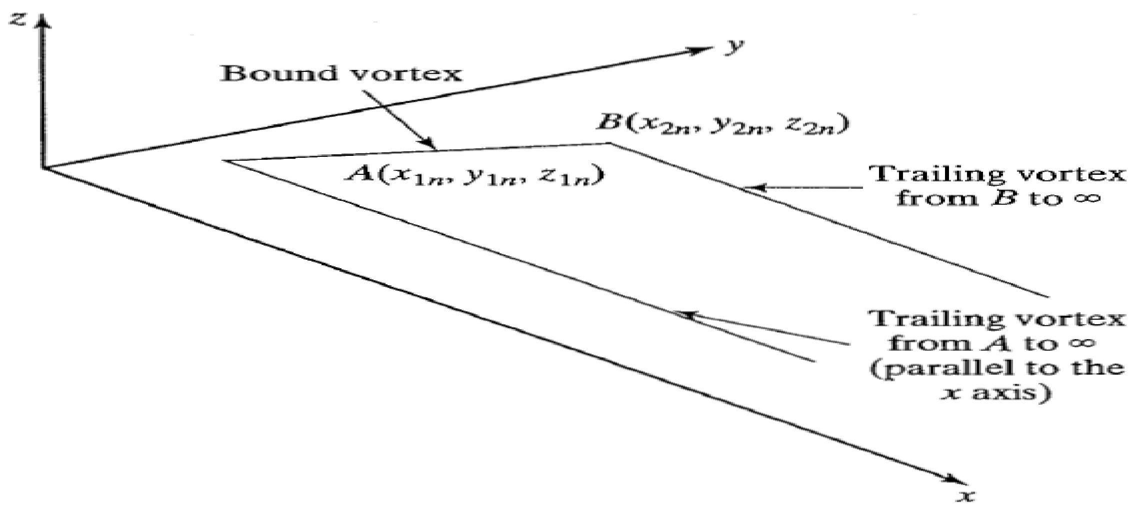


Fig.5. Typical horseshoe vortex

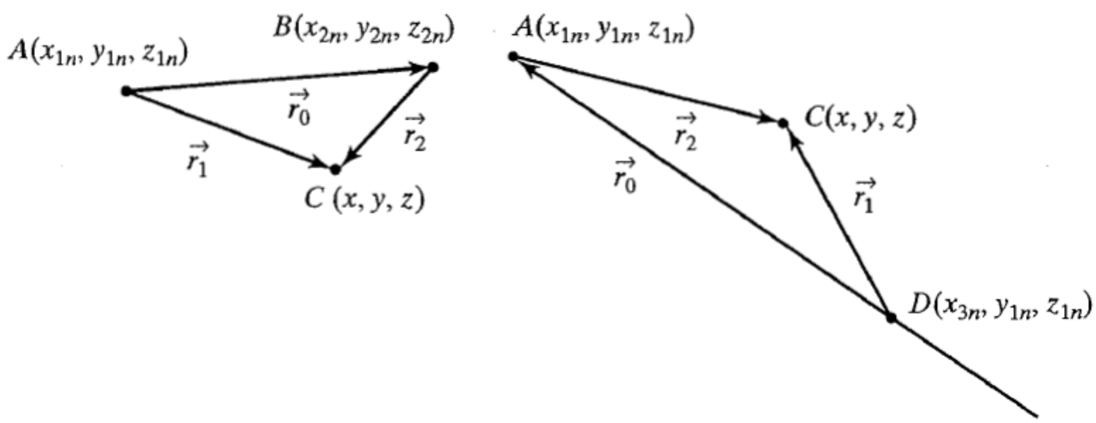


Fig.6. Vector elements for the calculation of the induced velocities.

For the bound vortex, segment $\overrightarrow{AB}$,

$$\vec{r}_0 = \overrightarrow{AB} = \left(x_{2n} - x_{1n}\right)\hat{i} + \left(y_{2n} - y_{1n}\right) j + \left(z_{2n} - z_{1n}\right) k$$

$$\vec{r}_1 = \left(x - x_{1n}\right)\hat{i} + \left(y - y_{1n}\right) j + \left(z - z_{1n}\right) k$$

$$\vec{r}_2 = \left(x - x_{2n}\right)\hat{i} + \left(y - y_{2n}\right) j + \left(z - z_{2n}\right) k$$

Using equation (4) to calculate the velocity induced at some point $C(x, y, z)$ by the vortex filament AB (shown in Fig.5 and Fig.6) ,

$$\vec{V}_{AB} = \frac{\Gamma_n}{4\pi}\{\text{Fac1}_{AB}\}\{\text{Fac2}_{AB}\} \tag{5a}$$

where,

$$\begin{aligned}\{\text{Fac1}_{AB}\} &= \frac{\vec{r}_1 \times \vec{r}_2}{\left|\vec{r}_1 \times \vec{r}_2\right|^2} \\ &= \Big\{\big[(y-y_{1n})(z-z_{2n})-(y-y_{2n})(z-z_{1n})\big]\hat{i} \\ &\quad -\big[(x-x_{1n})(z-z_{2n})-(x-x_{2n})(z-z_{1n})\big]j \\ &\quad +\big[(x-x_{1n})(y-y_{2n})-(x-x_{2n})(y-y_{1n})\big]k\Big\} / \\ &\quad \Big\{\big[(y-y_{1n})(z-z_{2n})-(y-y_{2n})(z-z_{1n})\big]^2 \\ &\quad +\big[(x-x_{1n})(z-z_{2n})-(x-x_{2n})(z-z_{1n})\big]^2 \\ &\quad +\big[(x-x_{1n})(y-y_{2n})-(x-x_{2n})(y-y_{1n})\big]^2\Big\}\end{aligned}$$

and

$$\begin{aligned}\{\text{Fac2}_{AB}\} &= \left(\vec{r}_0 . \frac{\vec{r}_1}{r_1} - \vec{r}_0 . \frac{\vec{r}_2}{r_2}\right) \\ &= \Big\{\big[(x_{2n}-x_{1n})(x-x_{1n})+(y_{2n}-y_{1n})(y-y_{1n})+(z_{2n}-z_{1n})(z-z_{1n})\big] / \\ &\quad \sqrt{(x-x_{1n})^2+(y-y_{1n})^2+(z-z_{1n})^2} \\ &\quad -\big[(x_{2n}-x_{1n})(x-x_{2n})+(y_{2n}-y_{1n})(y-y_{2n})+(z_{2n}-z_{1n})(z-z_{2n})\big] / \\ &\quad \sqrt{(x-x_{1n})^2+(y-y_{1n})^2+(z-z_{1n})^2}\end{aligned}$$

To calculate the velocity induced by the filament that extends from A to ∞, let us first calculate the velocity induced by the collinear, finite-length filament that extends from A to D. Since, $\overrightarrow{r_0}$ is in the direction of the vorticity vector,

$$\begin{aligned}\overrightarrow{r_0} &= \overrightarrow{DA} = (x_{1n}-x_{3n})\hat{i} \\ \vec{r}_1 &= (x-x_{3n})\hat{i}+(y-y_{1n})j+(z-z_{1n})k \\ \vec{r}_2 &= (x-x_{2n})\hat{i}+(y-y_{2n})j+(z-z_{2n})k\end{aligned}$$

as shown in Fig. 6 . Thus, the induced velocity is

$$\vec{V}_{AD} = \frac{\Gamma_n}{4\pi}\left\{\text{Fac1}_{AD}\right\}\left\{\text{Fac2}_{AD}\right\}$$

where,

$$\left\{\text{Fac1}_{AD}\right\} = \frac{(z - z_{1n})\, j + (y_{1n} - y)\, k}{\sqrt{(x - x_{3n})^2 + (y - y_{1n})^2 + (z - z_{1n})^2}}$$

and

$$\left\{\text{Fac2}_{AD}\right\} = (x_{3n} - x_{1n})\left\{\frac{x_{3n} - x}{\sqrt{(x - x_{3n})^2 + (y - y_{1n})^2 + (z - z_{1n})^2}} + \frac{x - x_{1n}}{\sqrt{(x - x_{1n})^2 + (y - y_{1n})^2 + (z - z_{1n})^2}}\right\}$$

Letting x_3 go to ∞, the first term of $\left\{\text{Fac2}_{AD}\right\}$ goes to 1.0. Therefore, the velocity induced by the vortex filament that extends from A to ∞ in a positive direction parallel to the x axis is given by

$$\overrightarrow{V_{A\infty}} = \frac{\Gamma_n}{4\pi}\left\{\frac{(z - z_{1n})\, j + (y_{1n} - y)\, k}{\left[(z - z_{1n})^2 + (y_{1n} - y)^2\right]}\right\}\left[1 + \frac{x - x_{1n}}{\sqrt{(x - x_{1n})^2 + (y - y_{1n})^2 + (z - z_{1n})^2}}\right] \tag{5b}$$

Similarly, the velocity induced by the vortex filament that extends from B to ∞ in a positive direction parallel to the x axis is given by

$$\overrightarrow{V_{B\infty}} = -\frac{\Gamma_n}{4\pi}\left\{\frac{(z - z_{2n})\, j + (y_{2n} - y)\, k}{\left[(z - z_{2n})^2 + (y_{2n} - y)^2\right]}\right\}\left[1 + \frac{x - x_{2n}}{\sqrt{(x - x_{2n})^2 + (y - y_{2n})^2 + (z - z_{2n})^2}}\right] \tag{5c}$$

The total velocity induced at some point (x, y, z) by the horseshoe vortex representing one of the surface elements (i.e., that for the nth panel) is the sum of the components given in equation (5). Let the point (x, y, z) be the control point of nth panel which is designated by the coordinates (x_m, y_m, z_m). The velocity induced at the mth control point by the vortex representing the nth panel will be designated as $\overrightarrow{V_{m,n}}$. From equation (5),we see that

$$\overrightarrow{V_{m,n}} = \overrightarrow{C_{m,n}}\Gamma_n \tag{6}$$

where the influence coefficient $\overrightarrow{C_{m,n}}$ depends on the geometry of the $n\mathrm{th}$ horseshoe vortex and its distance from the control point of the $m\mathrm{th}$ panel . Since the governing equation is linear, the velocities induced by the $2N$ vortices are added together to obtain an expression for the total induced velocity at the $m\mathrm{th}$ point.

$$\overrightarrow{V_{m,n}} = \sum_{n=1}^{2N} \overrightarrow{C_{m,n}} \Gamma_n \tag{7}$$

Application of Boundary Conditions

The boundary condition states that the resultant flow is tangent to wing at each and every control point which is located at the midspan of of three-quarter-chord line of each elemental point . If the flow is tangent to the wing, the component of the induced velocity normal to the wing at the control point balances the normal component of the free stream velocity. To evaluate the induced velocity components, we must introduce at this point our convention that the trailing vortices are parallel to the x-axis.

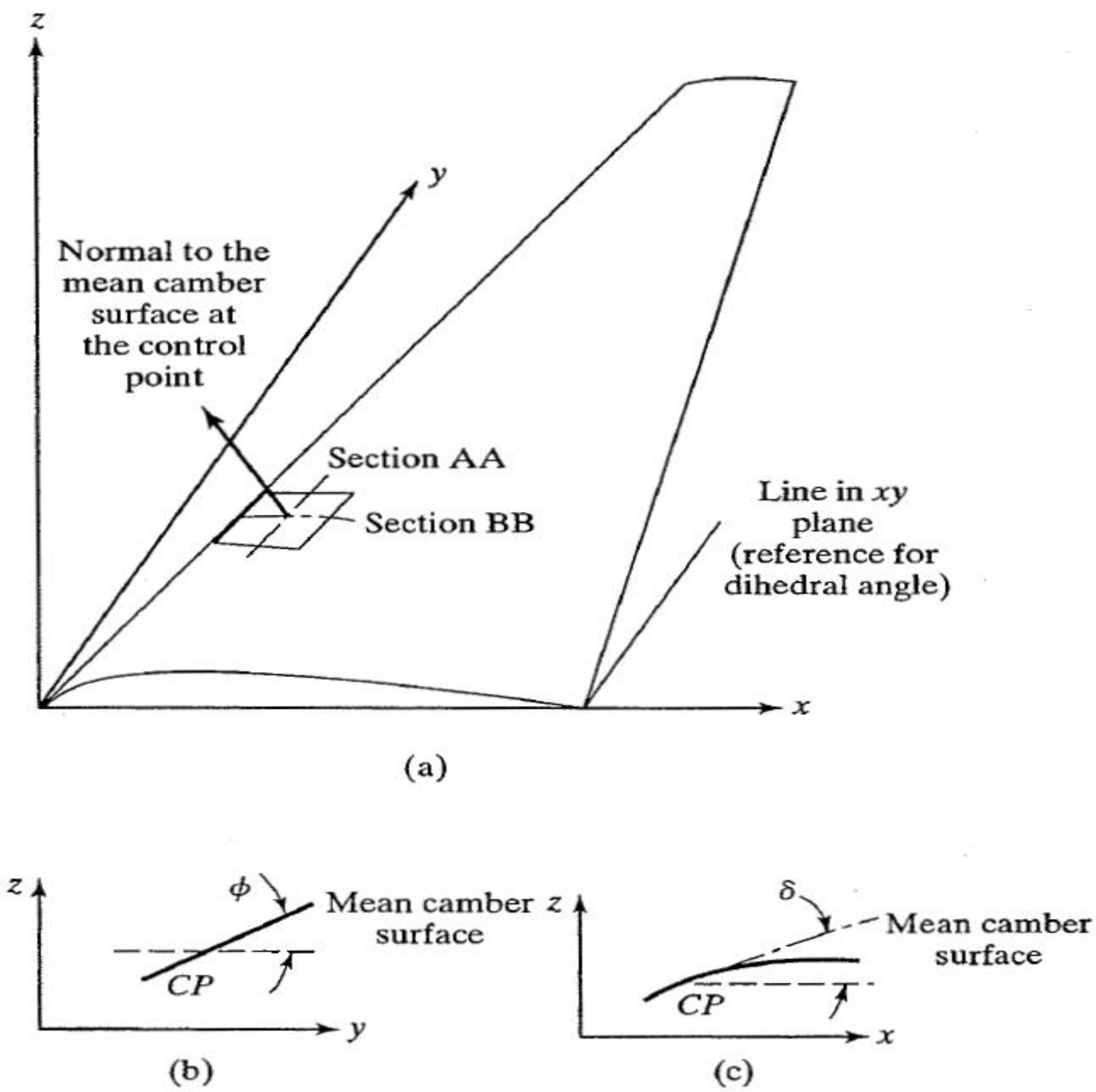


Fig. 7. Nomenclature for the tangency requirement: (a) normal to element of the mean camber surface (b) section AA (c) Section BB.

Referring to Fig.7, the tangency requirements yields the relation

$$-u_m \sin\delta\cos\phi - v_m \cos\delta\sin\phi + w_m \cos\phi\cos\delta + U_\infty \sin(\alpha-\delta)\cos\phi = 0 \quad (8)$$

where ϕ is the dihedral angle,as shown in Fig.7. and δ is the slope of the mean camber line at the control point. Thus,

$$\delta = \tan^{-1}\left(\frac{dz}{dx}\right)_m$$

For wings where the slope of the mean camber line is small and which are at small angles of attack, equation (8) can be written as

$$w_m - v_m \tan\phi + U_\infty \left[\alpha - \left(\frac{dz}{dx} \right)_m \right] = 0 \tag{9}$$

Relations for a Planar Wing

Equations (5) through (9) ar those for the VLM where the trailing vortices are parallel to the x axis. As such, they can be solved to determine the lifting flow for a twisted wing with dihedral .Let us apply these equations to a relatively simple geometry such as planar wing. For a planar wing, $z_{1n} = z_{2n} = 0$ for all the bound vortices. Furthermore, $z_m = 0$ for all the control points. Thus, for our planar wing,

$$\begin{aligned} \vec{V}_{AB} = \frac{\Gamma_n}{4\pi} & \frac{k}{(x_m - x_{1n})(y_m - y_{2n}) - (x_m - x_{2n})(y_m - y_{1n})} \\ & + \left[\frac{(x_{2n} - x_{1n})(x_m - x_{1n}) + (y_{2n} - y_{1n})(y_m - y_{1n})}{\sqrt{(x_m - x_{1n})^2 + (y_m - y_{1n})^2}} \right. \\ & \left. - \frac{(x_{2n} - x_{1n})(x_m - x_{2n}) + (y_{2n} - y_{1n})(y_m - y_{2n})}{\sqrt{(x_m - x_{2n})^2 + (y_m - y_{2n})^2}} \right] \end{aligned} \tag{10a}$$

$$\overrightarrow{V_{A\infty}} = \frac{\Gamma_n}{4\pi} \frac{k}{(y_{1n} - y_m)} \left[1 + \frac{x_m - x_{1n}}{\sqrt{(x_m - x_{1n})^2 + (y_m - y_{1n})^2}} \right] \tag{10b}$$

$$\overrightarrow{V_{B\infty}} = \frac{\Gamma_n}{4\pi} \frac{k}{(y_{2n} - y_m)} \left[1 + \frac{x_m - x_{2n}}{\sqrt{(x_m - x_{2n})^2 + (y_m - y_{2n})^2}} \right] \tag{10c}$$

Note that, for the planar wing, all three components of the vortex representing $n\text{th}$ panel induce a velocity at the control point of the $m\text{th}$ panel which is in z direction (i.e., downwash). Therefore,we can simplify equation (10) by combining the components into one expression

$$
\begin{aligned}
w_{m,n} = \frac{\Gamma_n}{4\pi}\Bigg\{ & \frac{1}{(x_m - x_{1n})(y_m - y_{2n}) - (x_m - x_{2n})(y_m - y_{1n})} \\
& + \left[\frac{(x_{2n} - x_{1n})(x_m - x_{1n}) + (y_{2n} - y_{1n})(y_m - y_{1n})}{\sqrt{(x_m - x_{1n})^2 + (y_m - y_{1n})^2}} \right. \\
& \left. - \frac{(x_{2n} - x_{1n})(x_m - x_{2n}) + (y_{2n} - y_{1n})(y_m - y_{2n})}{\sqrt{(x_m - x_{2n})^2 + (y_m - y_{2n})^2}} \right] \\
& + \frac{1}{(y_{1n} - y_m)}\left[1 + \frac{x_m - x_{1n}}{\sqrt{(x_m - x_{1n})^2 + (y_m - y_{1n})^2}} \right] \\
& - \frac{1}{(y_{2n} - y_m)}\left[1 + \frac{x_m - x_{2n}}{\sqrt{(x_m - x_{2n})^2 + (y_m - y_{2n})^2}} \right]
\end{aligned} \tag{11}
$$

Summing the contributions of all the vortices to the downwash at the control point of the mth panel ,

$$
w_m = \sum_{n=1}^{2N} w_{m,n} \tag{12}
$$

```
% Calculate downwash velocity at mth panel of starboard wing induced by horse
shoe vortex of n panels of the starboard wing


for m = 1:num
    for n = 1:num
        x_m1n = x_ctrl_rt(m)-xtop_rt(n);
        y_m1n = y_ctrl_rt(m)-ytop_rt(n);
        x_m2n = x_ctrl_rt(m)-xtop_rt(n+1);
        y_m2n = y_ctrl_rt(m)-ytop_rt(n+1);
        x_2n1n = xtop_rt(n+1)-xtop_rt(n);
        y_2n1n = ytop_rt(n+1)-ytop_rt(n);
        d_m1n = sqrt(x_m1n^2+y_m1n^2);
        d_m2n = sqrt(x_m2n^2+y_m2n^2);
        first_first_term = 1/(x_m1n*y_m2n-x_m2n*y_m1n);
        first_sec_term = ((x_2n1n*x_m1n+y_2n1n*y_m1n)/d_m1n)-
((x_2n1n*x_m2n+y_2n1n*y_m2n)/d_m2n);
        first_term = first_first_term*first_sec_term;
        sec_term = (-1/y_m1n)*(1+(x_m1n/d_m1n));
        third_term = (-1/y_m2n)*(1+(x_m2n/d_m2n));
        w_s(m,n) = first_term+sec_term-third_term;     % downwash at
starboard wing
    end
end
```

```
% Calculate downwash velocity at mth panel of starboard wing induced by horse
shoe vortex of n panels of the ports wing


for m = 1:num
    for n = 1:num
        x_m1n = x_ctrl_rt(m)-xtop_lt(n+1);
        y_m1n = y_ctrl_rt(m)-ytop_lt(n+1);
        x_m2n = x_ctrl_rt(m)-xtop_lt(n);
        y_m2n = y_ctrl_rt(m)-ytop_lt(n);
        x_2n1n = xtop_lt(n)-xtop_lt(n+1);
        y_2n1n = ytop_lt(n)-ytop_lt(n+1);
        d_m1n = sqrt(x_m1n^2+y_m1n^2);
        d_m2n = sqrt(x_m2n^2+y_m2n^2);
        first_first_term = 1/(x_m1n*y_m2n-x_m2n*y_m1n);
        first_sec_term = ((x_2n1n*x_m1n+y_2n1n*y_m1n)/d_m1n)-
((x_2n1n*x_m2n+y_2n1n*y_m2n)/d_m2n);
        first_term = first_first_term*first_sec_term;
        sec_term = (-1/y_m1n)*(1+(x_m1n/d_m1n));
        third_term = (-1/y_m2n)*(1+(x_m2n/d_m2n));
        w_p(m,n) = first_term+sec_term-third_term;     % downwash at
starboard wing
    end
end
```

Let's apply tangency defined by equations (8) and (9). Since, we are considering a planar wing in this section, $(dz/dx)_m = 0$ and $\phi = 0$. The component of the free-stream velocity perpendicular to the wing is $U_\infty \sin\alpha$ at any point on the wing. Thus, the resultant flow will be tangent to the wing if the total vortex-induced downwash at the control point of the mth panel ,which is calculated using equation (12) balances the normal component of the free-stream velocity :

$$w_m + U_\infty \sin\alpha = 0 \tag{13}$$

For small angles of attack,

$$w_m = -U_\infty \alpha \tag{14}$$

```
alpha = a*pi/180;


for i = 1:num
    rhs(i) = -U*sin(alpha);
end
```

Let us consider a wing that has a relatively simple geometry as shown in Fig. 8. The wing has an aspect ratio of 5, a taper ration of unity and an uncambered section . Since the taper ratio is unity, the leading edge,the quarter-chord line,the three-quarter-chord line, and the trailing edge all have the same sweep,45^0 .

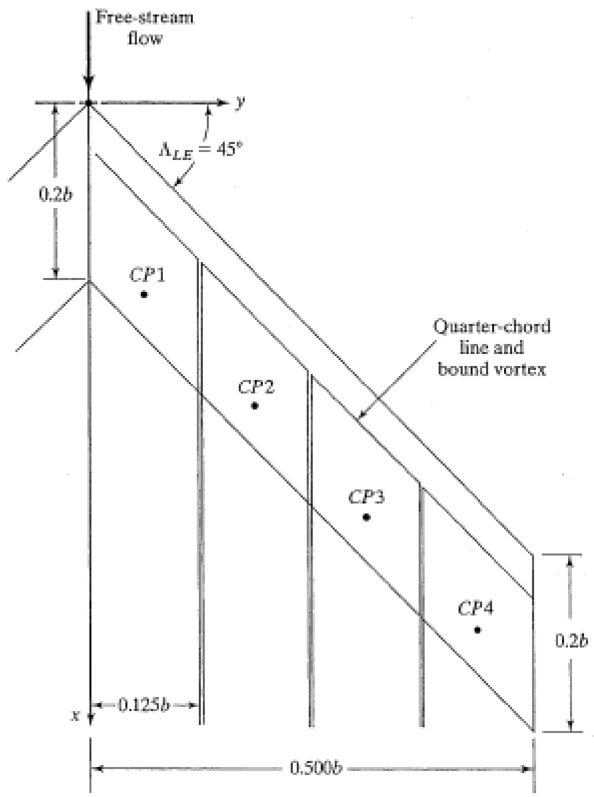


Fig.8. Four panel representation of a swept wing, taper ratio unity, AR=5, $\Lambda = 45^0$

The lift force acting at a point on the star-board wing (+y) is equal to that at the corresponding point on the port wing (-y). Because of symmetry, we need only to solve for the strengths of the vortices of the startboard wing. Furthermore, we need to apply the tangency condition only at the control points of the star-board wing . But,we need to include the contributions of the horseshoe vortices of the port wing to the velocities induced at these control points of the star-board wing. Thus,for thus planar symmetric flow,equation (12) becomes

$$w_m = \sum_{n=1}^{N} w_{m,ns} + \sum_{n=1}^{N} w_{m,np}$$

Here, s and p denotes starboard and port wings respectively.

```
for m = 1:num
    for n = 1:num
```

```
        w(m,n) = (w_s(m,n)+w_p(m,n))/(4*pi); % net downwash at starboard wing
    end
end
```

The planform of the starboard wing is divided into four panels,each panel extending from the leading edge to the trailing edge.

Matlab code for creation of panels at starboard wing :

```
xa_rt = 0;
ya_rt = 0;

xb_rt = 0.2*b;
yb_rt = 0;

xc_rt = 0.7*b;
yc_rt = 0.5*b;

xd_rt = 0.5*b;
yd_rt = 0.5*b;


plot(xa_rt,ya_rt,'ro',xb_rt,yb_rt,'ro',xc_rt,yc_rt,'ro',xd_rt,yd_rt,'ro')
v1 = [xa_rt,xb_rt];
v2 = [ya_rt,yb_rt];
line(v1,v2)

v1 = [xb_rt,xc_rt];
v2 = [yb_rt,yc_rt];
line(v1,v2)

v1 = [xc_rt,xd_rt];
v2 = [yc_rt,yd_rt];
line(v1,v2)

v1 = [xd_rt,xa_rt];
v2 = [yd_rt,ya_rt];
line(v1,v2)

hold on

dx_rt = (xd_rt-xa_rt)/num;
dy_rt = (yd_rt-ya_rt)/num;

for i = 1:num
    xtop_rt(i) = (i-1)*dx_rt+0.2*b/4;
    ytop_rt(i) = (i-1)*dy_rt;

end
xtop_rt(num+1) = xd_rt+0.2*b/4;
ytop_rt(num+1) = yd_rt;
```

```
plot(xtop_rt,ytop_rt,'*r')
```

Matlab code for creation of panels at port wing:

```
xa_lt = xa_rt;
ya_lt = -ya_rt;

xb_lt = xb_rt;
yb_lt = -yb_rt;

xc_lt = xc_rt;
yc_lt =-yc_rt;

xd_lt = xd_rt;
yd_lt = -yd_rt;


plot(xa_lt,ya_lt,'ro',xb_lt,yb_lt,'ro',xc_lt,yc_lt,'ro',xd_lt,yd_lt,'ro')
v1 = [xa_lt,xb_lt];
v2 = [ya_lt,yb_lt];
line(v1,v2)

v1 = [xb_lt,xc_lt];
v2 = [yb_lt,yc_lt];
line(v1,v2)

v1 = [xc_lt,xd_lt];
v2 = [yc_lt,yd_lt];
line(v1,v2)

v1 = [xd_lt,xa_lt];
v2 = [yd_lt,ya_lt];
line(v1,v2)

hold on
%
% dx_lt = (xd_lt-xa_lt)/num;
% dy_lt = (yd_lt-ya_lt)/num;

for i = 1:num
    xtop_lt(i) = xtop_rt(i);
    ytop_lt(i) = -ytop_rt(i);

end
xtop_lt(num+1) = xtop_rt(num+1);
ytop_lt(num+1) = -ytop_rt(num+1);

plot(xtop_lt,ytop_lt,'*r')

hold on
```

Matlab code to compute control points at starboard wing :

```
for i = 1:num
  x_ctrl_rt(i) = (xtop_rt(i)+xtop_rt(i+1))/2+0.2*b/2;
  y_ctrl_rt(i) = (ytop_rt(i)+ytop_rt(i+1))/2;
  end
  plot(x_ctrl_rt,y_ctrl_rt,'*b')

  hold on

for i = 1:num
    v1 = [xtop_rt(i),xtop_rt(i+1)];
    v2 = [ytop_rt(i),ytop_rt(i+1)];
line(v1,v2)
end
hold on

for i = 1:num
    v1 = [xtop_rt(i),xbottom_rt(i)];
    v2 = [ytop_rt(i),ybottom_rt(i)];
line(v1,v2)
end
```

Matlab code to compute control points at port wing :

```
for i = 1:num
  x_ctrl_lt(i) = x_ctrl_rt(i);
  y_ctrl_lt(i) = -y_ctrl_rt(i);
  end
  plot(x_ctrl_lt,y_ctrl_lt,'*b')
```

The strength of each of the vortices are determined by solving set of linear equations in matrix form.

```
% calculating circulation by solving linear equations

g = mldivide(w,rhs');
```

Since,the panels extend from leading edge to trailing edge ,the lift acting on the nth panel is

$$l_n = \rho_\infty U_\infty \Gamma_n$$

which is also the lift per span . Since the flow is symmetric ,the total lift for the wing is

$$L = 2\int_0^{0.5b} \rho_\infty U_\infty \Gamma(y)dy$$

or for finite element panels

$$L = 2\rho_\infty U_\infty \sum_{n=1}^{num} \Gamma_n \Delta y_n$$

Hence,

$$C_L = \frac{L}{q_\infty S}$$

```
gamma = 0;
for i = 1:num
    gamma = gamma + g(i)*dy_rt;
end

L = 2*rho*U*gamma;

c = 0.2;

q = 0.5*rho*U^2;
S = b*c;
cl = L/(q*S)
```

The matlab code to calculate the C_L of a planar swept wing with panels divided in spanwise direction only is given below.

```
;% This program calculates the cl of a wing using vortex lattice method

clc
clear all
close all

a = input('Enter the angle of attack = ');
U = input('Enter the freestream velocity = ');
num = input('Enter the number of panels = ');
b = 1;

xa_rt = 0;
ya_rt = 0;

xb_rt = 0.2*b;
yb_rt = 0;

xc_rt = 0.7*b;
yc_rt = 0.5*b;

xd_rt = 0.5*b;
yd_rt = 0.5*b;
```

```
plot(xa_rt,ya_rt,'ro',xb_rt,yb_rt,'ro',xc_rt,yc_rt,'ro',xd_rt,yd_rt,'ro')
v1 = [xa_rt,xb_rt];
v2 = [ya_rt,yb_rt];
line(v1,v2)

v1 = [xb_rt,xc_rt];
v2 = [yb_rt,yc_rt];
line(v1,v2)

v1 = [xc_rt,xd_rt];
v2 = [yc_rt,yd_rt];
line(v1,v2)

v1 = [xd_rt,xa_rt];
v2 = [yd_rt,ya_rt];
line(v1,v2)

hold on

dx_rt = (xd_rt-xa_rt)/num;
dy_rt = (yd_rt-ya_rt)/num;

for i = 1:num
    xtop_rt(i) = (i-1)*dx_rt+0.2*b/4;
    ytop_rt(i) = (i-1)*dy_rt;

end
xtop_rt(num+1) = xd_rt+0.2*b/4;
ytop_rt(num+1) = yd_rt;

plot(xtop_rt,ytop_rt,'*r')

hold on

for i = 1:num
    xbottom_rt(i) = (i-1)*dx_rt+0.2*b;
    ybottom_rt(i) = (i-1)*dy_rt;
end
xbottom_rt(num+1) = xd_rt+0.2*b;
ybottom_rt(num+1) = yd_rt;


  plot(xbottom_rt,ybottom_rt,'r*')
  hold on

  for i = 1:num
  x_ctrl_rt(i) = (xtop_rt(i)+xtop_rt(i+1))/2+0.2*b/2;
  y_ctrl_rt(i) = (ytop_rt(i)+ytop_rt(i+1))/2;
  end
  plot(x_ctrl_rt,y_ctrl_rt,'*b')

  hold on
```

```
for i = 1:num
    v1 = [xtop_rt(i),xtop_rt(i+1)];
    v2 = [ytop_rt(i),ytop_rt(i+1)];
line(v1,v2)
end
hold on

for i = 1:num
    v1 = [xtop_rt(i),xbottom_rt(i)];
    v2 = [ytop_rt(i),ybottom_rt(i)];
line(v1,v2)
end

% Calculate downwash velocity at mth panel induced by horse shoe vortex of
% n panels of the starboard wing


for m = 1:num
    for n = 1:num
        x_m1n = x_ctrl_rt(m)-xtop_rt(n);
        y_m1n = y_ctrl_rt(m)-ytop_rt(n);
        x_m2n = x_ctrl_rt(m)-xtop_rt(n+1);
        y_m2n = y_ctrl_rt(m)-ytop_rt(n+1);
        x_2n1n = xtop_rt(n+1)-xtop_rt(n);
        y_2n1n = ytop_rt(n+1)-ytop_rt(n);
        d_m1n = sqrt(x_m1n^2+y_m1n^2);
        d_m2n = sqrt(x_m2n^2+y_m2n^2);
        first_first_term = 1/(x_m1n*y_m2n-x_m2n*y_m1n);
        first_sec_term = ((x_2n1n*x_m1n+y_2n1n*y_m1n)/d_m1n)-
((x_2n1n*x_m2n+y_2n1n*y_m2n)/d_m2n);
        first_term = first_first_term*first_sec_term;
        sec_term = (-1/y_m1n)*(1+(x_m1n/d_m1n));
        third_term = (-1/y_m2n)*(1+(x_m2n/d_m2n));
        w_s(m,n) = first_term+sec_term-third_term;     % downwash at
starboard wing
    end
end

% Calculate downwash velocity at mth panelof startboard wing induced by horse
% shoe vortex of nth panels of the port wing

xa_lt = xa_rt;
ya_lt = -ya_rt;

xb_lt = xb_rt;
yb_lt = -yb_rt;

xc_lt = xc_rt;
yc_lt =-yc_rt;

xd_lt = xd_rt;
yd_lt = -yd_rt;
```

```
plot(xa_lt,ya_lt,'ro',xb_lt,yb_lt,'ro',xc_lt,yc_lt,'ro',xd_lt,yd_lt,'ro')
v1 = [xa_lt,xb_lt];
v2 = [ya_lt,yb_lt];
line(v1,v2)

v1 = [xb_lt,xc_lt];
v2 = [yb_lt,yc_lt];
line(v1,v2)

v1 = [xc_lt,xd_lt];
v2 = [yc_lt,yd_lt];
line(v1,v2)

v1 = [xd_lt,xa_lt];
v2 = [yd_lt,ya_lt];
line(v1,v2)

hold on
%
% dx_lt = (xd_lt-xa_lt)/num;
% dy_lt = (yd_lt-ya_lt)/num;

for i = 1:num
    xtop_lt(i) = xtop_rt(i);
    ytop_lt(i) = -ytop_rt(i);

end
xtop_lt(num+1) = xtop_rt(num+1);
ytop_lt(num+1) = -ytop_rt(num+1);

plot(xtop_lt,ytop_lt,'*r')

hold on

for i = 1:num
    xbottom_lt(i) = xbottom_rt(i);
    ybottom_lt(i) = -ybottom_rt(i);
end
xbottom_lt(num+1) = xbottom_rt(num+1);
ybottom_lt(num+1) = -ybottom_rt(num+1);


  plot(xbottom_lt,ybottom_lt,'r*')
  hold on

  for i = 1:num
  x_ctrl_lt(i) = x_ctrl_rt(i);
  y_ctrl_lt(i) = -y_ctrl_rt(i);
  end
  plot(x_ctrl_lt,y_ctrl_lt,'*b')

  hold on
```

```
for i = 1:num
    v1 = [xtop_lt(i),xtop_lt(i+1)];
    v2 = [ytop_lt(i),ytop_lt(i+1)];
line(v1,v2)
end
hold on

for i = 1:num
    v1 = [xtop_lt(i),xbottom_lt(i)];
    v2 = [ytop_lt(i),ybottom_lt(i)];
line(v1,v2)
end




for m = 1:num
    for n = 1:num
        x_m1n = x_ctrl_rt(m)-xtop_lt(n+1);
        y_m1n = y_ctrl_rt(m)-ytop_lt(n+1);
        x_m2n = x_ctrl_rt(m)-xtop_lt(n);
        y_m2n = y_ctrl_rt(m)-ytop_lt(n);
        x_2n1n = xtop_lt(n)-xtop_lt(n+1);
        y_2n1n = ytop_lt(n)-ytop_lt(n+1);
        d_m1n = sqrt(x_m1n^2+y_m1n^2);
        d_m2n = sqrt(x_m2n^2+y_m2n^2);
        first_first_term = 1/(x_m1n*y_m2n-x_m2n*y_m1n);
        first_sec_term = ((x_2n1n*x_m1n+y_2n1n*y_m1n)/d_m1n)-
((x_2n1n*x_m2n+y_2n1n*y_m2n)/d_m2n);
        first_term = first_first_term*first_sec_term;
        sec_term = (-1/y_m1n)*(1+(x_m1n/d_m1n));
        third_term = (-1/y_m2n)*(1+(x_m2n/d_m2n));
        w_p(m,n) = first_term+sec_term-third_term;     % downwash at
starboard wing
    end
end


for m = 1:num
    for n = 1:num
        w(m,n) = (w_s(m,n)+w_p(m,n))/(4*pi); % net downwash at starboard wing
    end
end

alpha = a*pi/180;


for i = 1:num
    rhs(i) = -U*sin(alpha);
end


rho = 1.225;
% calculating circulation by solving linear equations
```

```
g = mldivide(w,rhs');


gamma = 0;
for i = 1:num
    gamma = gamma + g(i)*dy_rt;
end

L = 2*rho*U*gamma;

c = 0.2;

q = 0.5*rho*U^2;
S = b*c;
cl = L/(q*S)
```

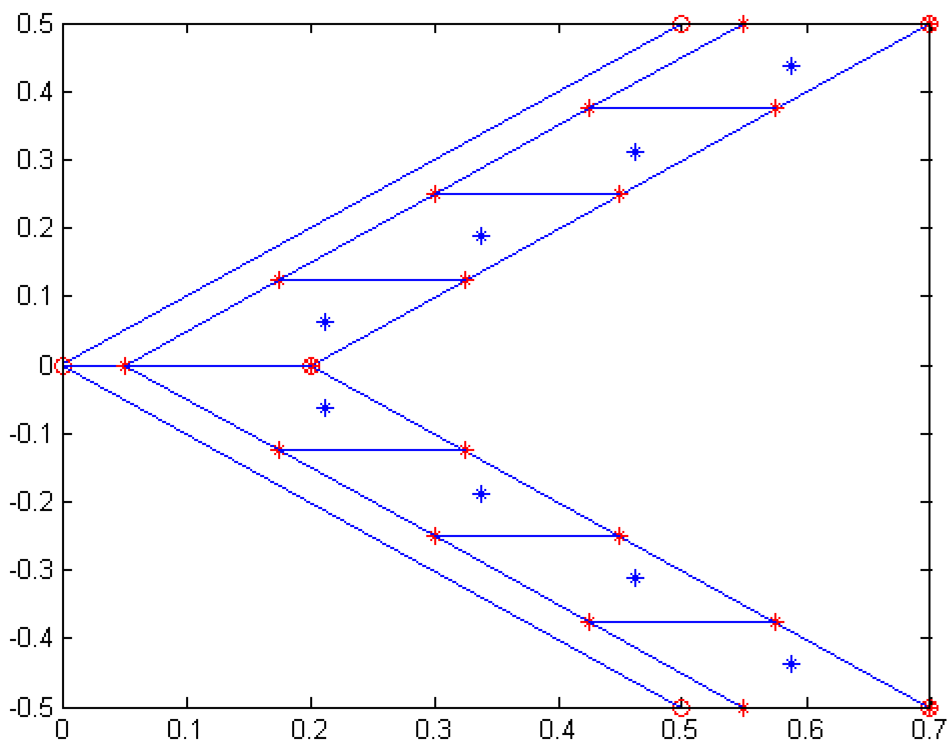


Fig.9. Planar swept wing with 8 panels in total

The coefficient of lift at 2^0 angle of attack is calculated to be 0.1202.

The matlab code for calculating C_L of planar swept wing with panels divided in chordwise and spanwise directions is given below.

```
clc
clear all
```

```
close all

a = input('Enter the angle of attack = ');
U = input('Enter the freestream velocity = ');
N = input('Enter the number of panels in spanwise direction = ');
M = input('Enter the number of panels in chordwise direction = ');


b = 1;
LX=0.2*b;
LY=0.5*b;

dx=LX/M;
dy=LY/N;

for i=1:M+1;
for j=1:N+1;
x0_rt(i,j)=(i-1)*dx;
y_rt(i,j)=(j-1)*dy;
x_rt(i,j)= x0_rt(i,j)+y_rt(i,j);

end
end



for i = 1:M
    for j = 1:N+1
    xtop_rt(i,j)  = x_rt(i,j)+LX/4/M;
    ytop_rt(i,j)  =  y_rt(i,j);
    end
end

k = 0;
  for i = 1:M
      for j = 1:N
          k = k+1;
  xctrl_rt(k)  =  (xtop_rt(i,j)+xtop_rt(i,j+1))/2+LX*0.5/M;
  yctrl_rt(k)  =  (y_rt(i,j)+y_rt(i,j+1))*0.5;
      end
  end

  for i = 1:M+1
      for j = 1:N+1
          x_lt(i,j)  = x_rt(i,j);
          y_lt(i,j)  = -y_rt(i,j);

      end
  end

  for i = 1:M
      for j = 1:N+1
    xtop_lt(i,j)  = xtop_rt(i,j);
    ytop_lt(i,j)  =  -ytop_rt(i,j);
    end
```

```
  end

    for i = 1:k

  xctrl_lt(i) = xctrl_rt(i);
  yctrl_lt(i) = -yctrl_rt(i);
      end


 for l = 1:k
     m = 0;
     for i = 1:M
         for j = 1:N
             m = m+1;
        x_m1n = xctrl_rt(l)-xtop_rt(i,j);
        y_m1n = yctrl_rt(l)-ytop_rt(i,j);
        x_m2n = xctrl_rt(l)-xtop_rt(i,j+1);
        y_m2n = yctrl_rt(l)-ytop_rt(i,j+1);
        x_2n1n = xtop_rt(i,j+1)-xtop_rt(i,j);
        y_2n1n = ytop_rt(i,j+1)-ytop_rt(i,j);
        d_m1n = sqrt(x_m1n^2+y_m1n^2);
        d_m2n = sqrt(x_m2n^2+y_m2n^2);
        first_first_term = 1/(x_m1n*y_m2n-x_m2n*y_m1n);
        first_sec_term = ((x_2n1n*x_m1n+y_2n1n*y_m1n)/d_m1n)-
((x_2n1n*x_m2n+y_2n1n*y_m2n)/d_m2n);
        first_term = first_first_term*first_sec_term;
        sec_term = (-1/y_m1n)*(1+(x_m1n/d_m1n));
        third_term = (-1/y_m2n)*(1+(x_m2n/d_m2n));
        w_s(l,m) = first_term+sec_term-third_term;     % downwash at
starboard wing
         end
     end
 end




 for l = 1:k
     m = 0;
    for i = 1:M
       for j = 1:N
           m = m+1;
        x_m1n = xctrl_rt(l)-xtop_lt(i,j+1);
        y_m1n = yctrl_rt(l)-ytop_lt(i,j+1);
        x_m2n = xctrl_rt(l)-xtop_lt(i,j);
        y_m2n = yctrl_rt(l)-ytop_lt(i,j);
        x_2n1n = xtop_lt(i,j)-xtop_lt(i,j+1);
        y_2n1n = ytop_lt(i,j)-ytop_lt(i,j+1);
        d_m1n = sqrt(x_m1n^2+y_m1n^2);
        d_m2n = sqrt(x_m2n^2+y_m2n^2);
        first_first_term = 1/(x_m1n*y_m2n-x_m2n*y_m1n);
        first_sec_term = ((x_2n1n*x_m1n+y_2n1n*y_m1n)/d_m1n)-
((x_2n1n*x_m2n+y_2n1n*y_m2n)/d_m2n);
        first_term = first_first_term*first_sec_term;
        sec_term = (-1/y_m1n)*(1+(x_m1n/d_m1n));
```

```
        third_term = (-1/y_m2n)*(1+(x_m2n/d_m2n));
        w_p(l,m) = first_term+sec_term-third_term;     % downwash at
starboard wing
       end
    end
 end


for i = 1:k
    for  j= 1:m
        w(i,j) = (w_s(i,j)+w_p(i,j))/(4*pi); % net downwash at starboard wing
    end
end

alpha = a*pi/180;


for i = 1:k
    rhs(i) = -U*sin(alpha);
end


rho = 1.225;
% calculating circulation by solving linear equations

g = mldivide(w,rhs');


gamma = 0;
for i = 1:k
    gamma = gamma + g(i)*dy;
end

L = 2*rho*U*gamma;

c = 0.2;

q = 0.5*rho*U^2;
S = b*c;
cl = L/(q*S)

 plot(x_rt,y_rt,'*-r',x_rt',y_rt','-*r')

hold on

plot(xtop_rt',ytop_rt','*-b')

hold on
plot(xctrl_rt',yctrl_rt','*g')
hold on
plot(x_lt,y_lt,'*-r',x_lt',y_lt','-*r')

hold on
```

```
plot(xtop_lt',ytop_lt','*-b')

hold on
plot(xctrl_lt',yctrl_lt','*g')

grid on
xlabel('x')
ylabel('y')
```

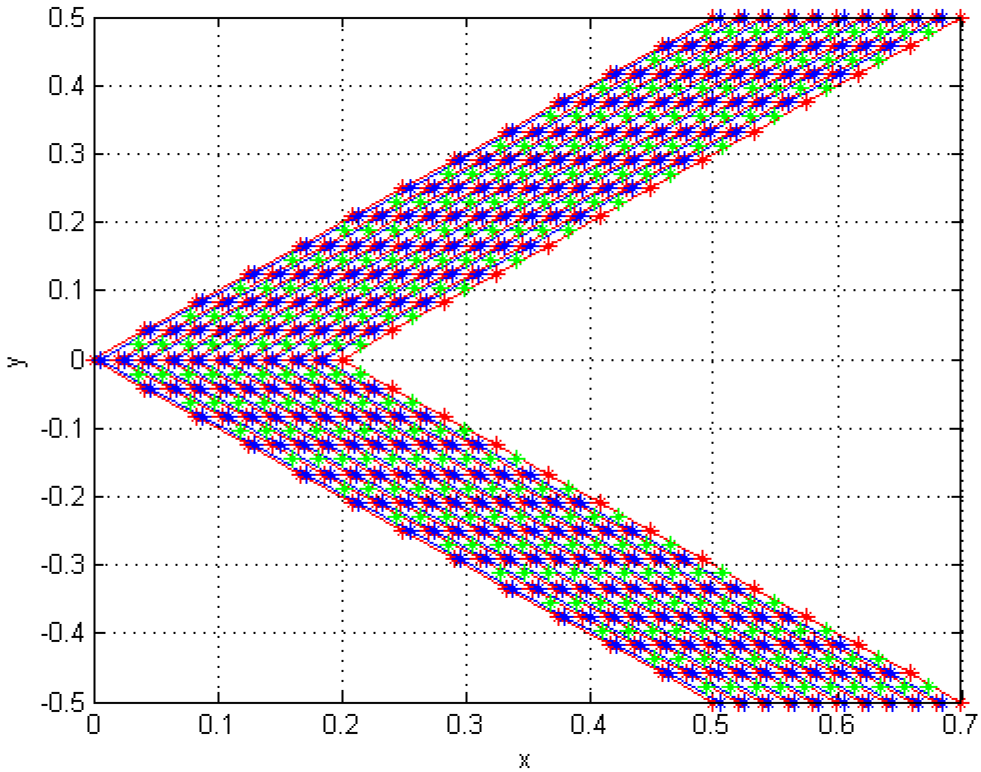


Fig. 10. Planar swept wing with 240 panels in total

The coefficient of lift at 2^0 angle of attack is calculated to be 0.1142.