Time-averaged intensity of interference from multiple sources

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ABSTRACT

This document describes the calculation of time-averaged (over one period) intensity of multiple sources. Used in the course FYS1004 Säteilykentät ja fotonit, University of Helsinki.

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Amplitude of source i with amplitude E_i and at distance r_i and at time t

$$A_i(r_i,t) = E_i \cos(2\pi r_i + 2\pi t) \tag{1}$$

Here we have set wavelength and frequency to unity: $\lambda = f = 1$.

We need to sum over the sources to get the total amplitude and integrate the square of the total amplitude over one period ($P = 1/f = 1$) to obtain the time-averaged intensity:

$$I(\mathbf{r}) = \int_0^1 \left[\sum_{i=1}^{n} A_i(r_i,t)\right]^2 dt = \int_0^1 \left[\sum_{i=1}^{n} E_i \cos(2\pi r_i + 2\pi t)\right]^2 dt \tag{2}$$

Here r_i is now the distance between point $\mathbf{r}$ and the location of source i.

Plugging this in MAPLE the integral can be calculated exactly:

$$I(\mathbf{r}) = 2(S_1^2 + S_2^2 - S_2 S_3) + \frac{1}{2} S_3^2 \tag{3}$$

where

$$S_1 = \sum_{i=1}^{n} E_i \sin(\pi r_i)\cos(\pi r_i) \tag{4}$$

$$S_2 = \sum_{i=1}^{n} E_i \cos^2(\pi r_i) \tag{5}$$

$$S_3 = \sum_{i=1}^{n} E_i \tag{6}$$

$$\tag{7}$$

As an example set $n = 2$. This gives us

$$I(\mathbf{r}) = \frac{E_1^2}{2} + \frac{E_2^2}{2} + E_1 E_2 \cos(2\pi r_1 - 2\pi r_2). \tag{8}$$

Assuming $E_1 = E_2 = 1$ we get

$$I(\mathbf{r}) = 1 + \cos(2\pi\Delta r), \tag{9}$$

where $\Delta r = r_2 - r_1$. (This is excatly the expression in `doublesource_2dmap.f90`.)

Calculation of the interference pattern is implemented as a Fortran subroutine in source code file `multisource_2dmap.f90` where you can set up multiple sources in a 2D square lattice. The file can be compiled as a Python module (for instructions, see the comments in the file) and an usage example is given in `multisource.py`.