Report: Lab 09

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0.1 Model:

As we have already discussed in the problem statement, we have a source of energy P, which can emit energy across K different non overlapping channels over a band of width W. Now, there are N energy harvesting devices (nodes) that can harvest energy. Let, μ_{nk} denote the mean gain of energy by the n^{th} node when the source P emits energy through the k^{th} channel. $n \in [N] \quad k \in [K]$. Time is divided into slots. In each slot, the source selects a channel and emits energy. The nodes harvest energy in each slot and the amount of energy harvested depends on the gains in that round.

We can approach this problem with the following two objective functions:

- We might want to know the channel which maximises the total energy consumption across all energy harvestors. i.e. we would like to find:

$$arg\ \underset{k}{max} \quad \sum_{n \in [N]} \mu_{kn}$$

- In the previous approach to the problem, it might so happen that the optima is achieved for a value of k for which one or more nodes might die out. In that case, we can frame our objective function as the maximum energy consumed across all nodes such that none of the nodes die out. i.e. we would like to find:

$$arg\ \underset{k}{max}\ \underset{n}{min}\ \mu_{nk}$$

In the next section we present the algorithms for getting the best channel as per the above two models.

0.2 Model 1

Algorithm for Model 1:

Input: No. of channels K; No. of nodes N.
Step 1: for t=1,2,...,K, do:
Source emits energy through channel t. Observe the energy harvested by each

node and calculate the estimated gain of all nodes $\hat{\mu_t}$ for channel t.

Step 2: for t=K+1, K+2, ..., do:

$$K_t = arg\ \max_k \{\hat{\mu_k} + \sqrt{\frac{\alpha \log(t)}{Total\ No.\ of\ times\ channel\ k\ has\ been\ used}}\}$$

Source emits energy through channel K_t. Observe the energy harvested by each node and calculate the estimated gain of all nodes $\hat{\mu_{K_t}}$ for channel t.

Step 3: At whichever round we decide to stop, the channel K_t found at that last round would give us the optimum value to our problem.

Note:

- α is the exploration rate. A large value of α would imply we focus more on exploration than exploitation.
- The estimated gain for channel k till round T is given as:

$$\hat{\mu_k} = \frac{\sum_{t=1}^{T} \sum_{n \in N} X_{nkt}}{Total\ No.\ of\ times\ channel\ k\ has\ been\ selected\ till\ T\ rounds}$$

Algorithm for Model 2:

Input: No. of channels K; No. of nodes N.

Step 1: for t=1,2,...,K, do:

Source emits energy through channel t. Observe the energy harvested by each node and calculate its estimated gain $\hat{\mu_{tn}}$ for channel t.

Step 2: for t=K+1,K+2, ..., do:

for each channel k, compute n_k as:

$$n_k = arg\min_n \{\hat{\mu_{kn}} - \sqrt{\frac{\alpha \log(t)}{Total\ No.\ of\ times\ channel\ k\ has\ been\ used}}\}$$

Now, determine k_t^* as:

$$k_t^* = arg\max_k \{\hat{\mu_{kn_k}} - \sqrt{\frac{\alpha \log(t)}{Total\ No.\ of\ times\ channel\ i\ has\ been\ used}}\}$$

Source emits energy through channel k_t^*. We observe the energy harvested by each node and calculate the estimated gain of them $\hat{\mu_{K_t}}$ for channel t.

Step 3: At whichever round we decide to stop, the channel k_t^* found at that last round would give us the optimum value to our problem.

0.3 Results and Analysis

Model 1:

We conducted the simulation for K=10, N=50, T=25000 and sample paths=20. We also fixed the parameters μ_{nk} as $\mu_{nk} = \frac{nk}{(K+1)(N+1)^2}$ for n=1(1)50 and k=1(1)10.
We considered two different cases:

- Gains follow Bernoulli distribution with the above parameters.
- Gains follow Exponential distribution with the above parameters.

The plot of pseudo cumulative regret vs T when the gains followed bernoulli distribution is given as:

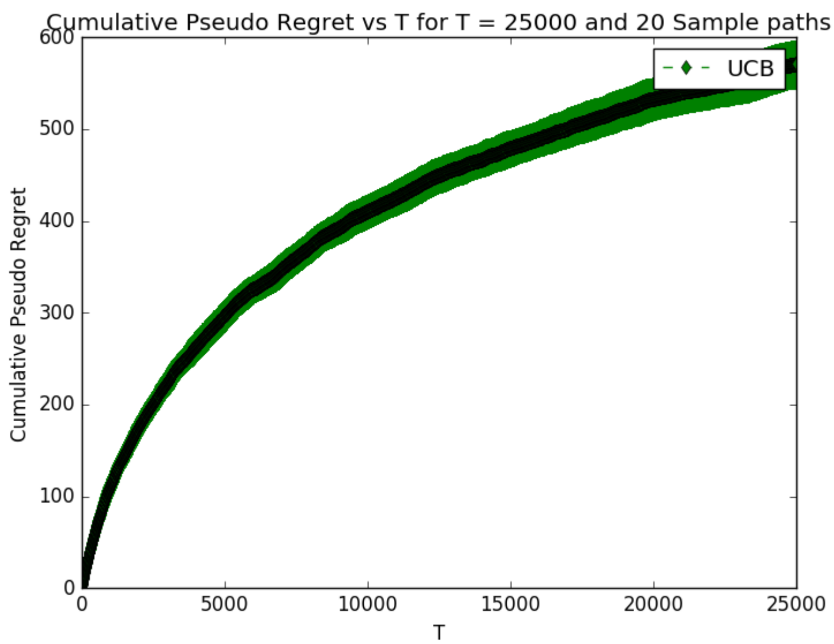

Figure 1: Pseudo Cumulative Regret vs T [Gains∼ Benoulli]

The plot of pseudo cumulative regret vs T when the gains followed exponential distribution is given as:

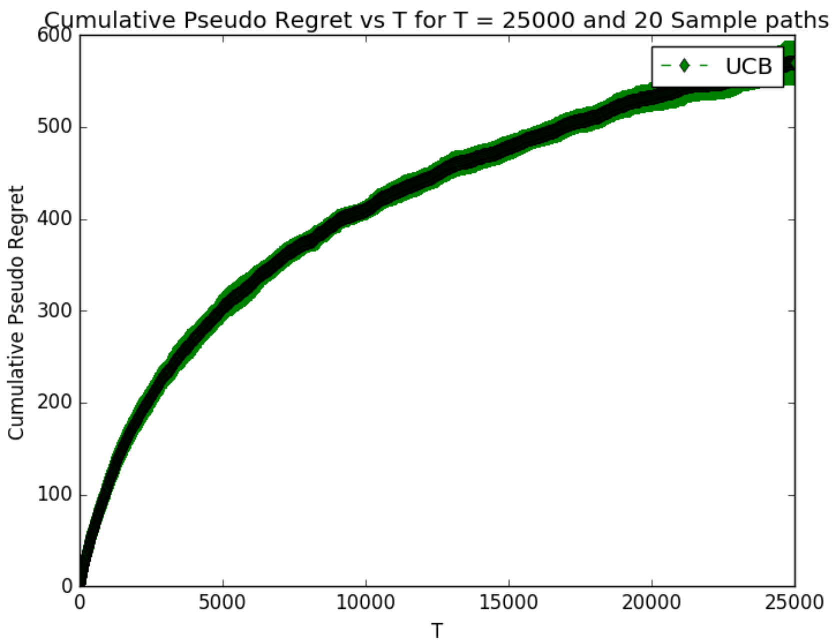

Figure 2: Pseudo Cumulative Regret vs T [Gains∼ Exponential]

We can conclude from the above two figures that the Cumulative Regret is sublinear for both the cases. Hence the algorithm is learning.

The plot of average frequency against the channels when the Gains $\sim$ Bernoulli is:

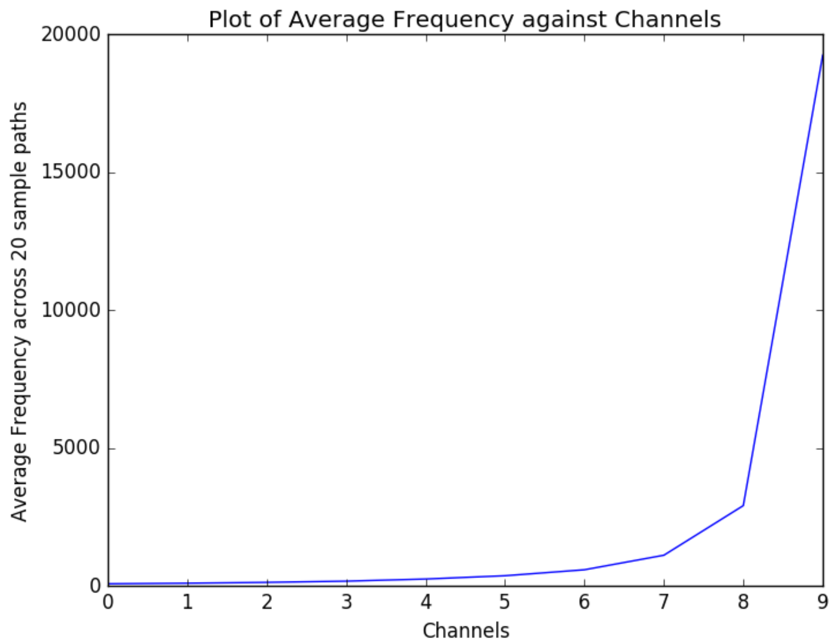

Figure 3: Avg. Frequency vs Channels [Gains$\sim$ Bernoulli]

The plot of average frequency against the channels when the Gains $\sim$ Exponential is:

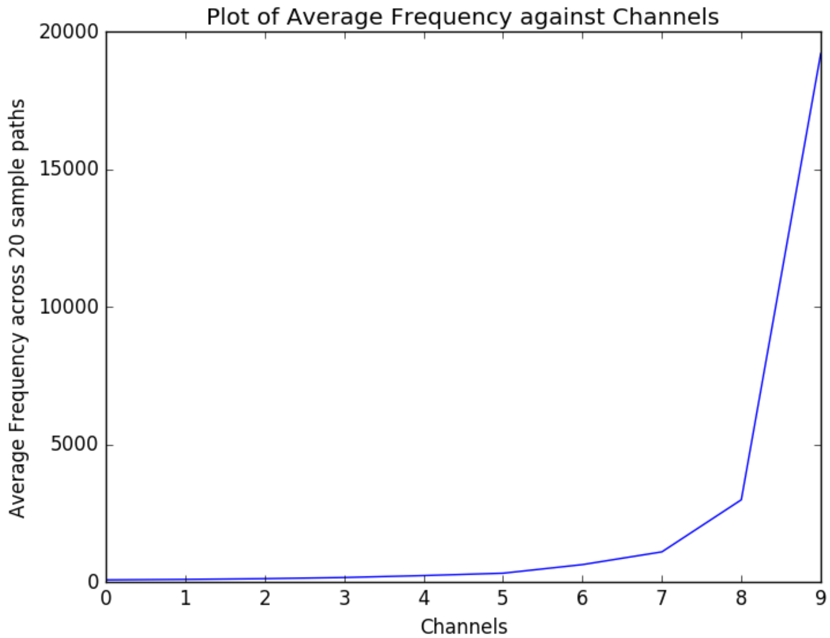

Figure 4: Avg. Frequency vs Channels [Gains$\sim$ Exponential]

Since the last channel has the greatest mean gain, so the frequency of the last channel being chosen by the algorithm is maximum.

Model 2:

We conducted the simulation for K=8, N=15, T=25000 and sample paths=20. We also fixed the parameters μ_{nk} as $\mu_{nk} = \frac{nk}{(K+1)(N+1)^2}$ for n=1(1)15 and k=1(1)8.
We considered two different cases:

- Gains follow Bernoulli distribution with the above parameters.
- Gains follow Exponential distribution with the above parameters.

The plot of pseudo cumulative regret vs T when the gains followed bernoulli distribution is given as:

The plot of pseudo cumulative regret vs T when the gains followed exponential distribution is given as:

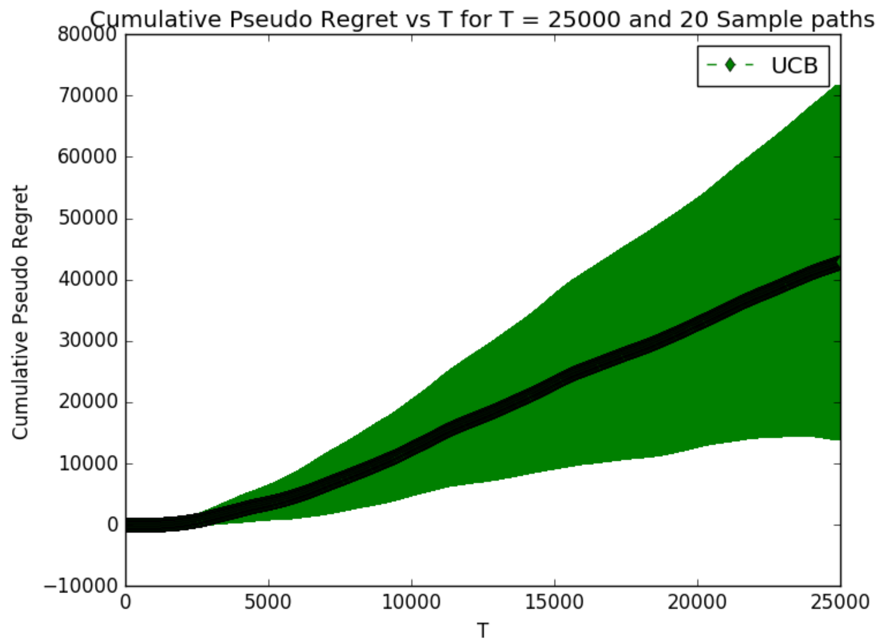

Figure 5: Pseudo Cumulative Regret vs T [Gains∼ Benoulli]

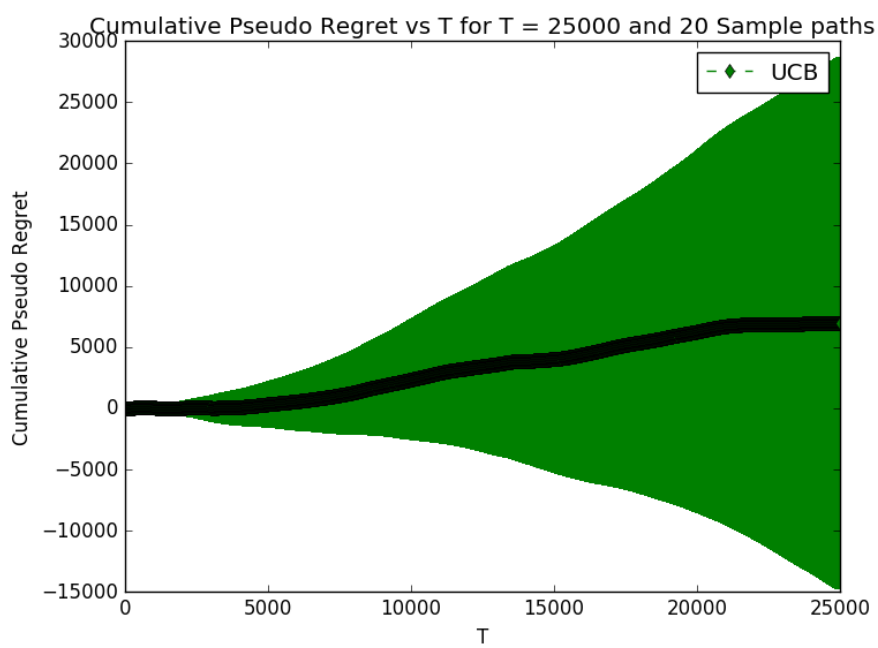

Figure 6: Pseudo Cumulative Regret vs T [Gains∼ Exponential]

We can conclude from the above two figures that the Cumulative Regret is sublinear for both the cases. Hence the algorithm is learning.

The plot of average frequency against the channels when the Gains $\sim$ Bernoulli is:

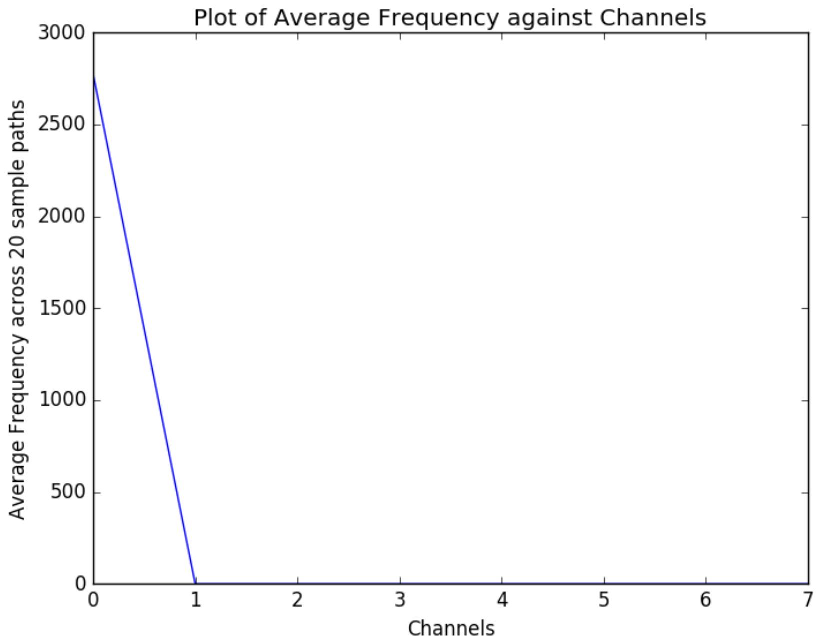

Figure 7: Avg. Frequency vs Channels [Gains$\sim$ Bernoulli]

The plot of average frequency against the channels when the Gains $\sim$ Exponential is:

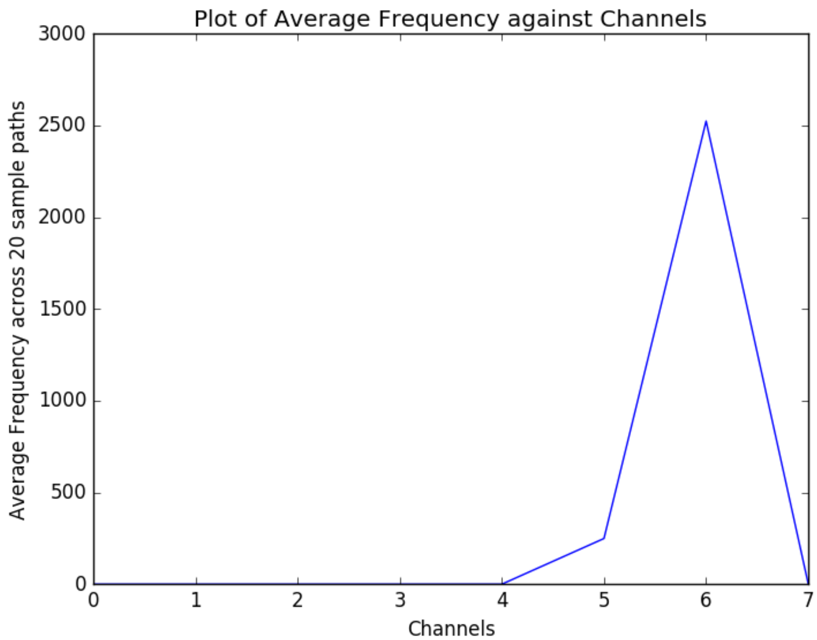

Figure 8: Avg. Frequency vs Channels [Gains$\sim$ Exponential]

Since the last channel has the greatest mean gain, so the frequency of the last channel being chosen by the algorithm is maximum.