Report: Lab 06

Submitted by: Aakash Banik (16i190010)

Question PREP:
np.random.randn function generates samples from a Standard Normal distn.

Question 1:
FILE : ex1a.py and ex1b.py

Subpart 2:
There hasn't been any difficulty in computing the Hessian as such because the dimension of x is 2. Moreover the Hessian is independent of any x-terms.

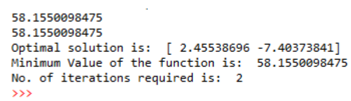

Figure 1: Optimal Solution, Minimum Value and No. of iterations when starting point is [0,0] for Newton's Method

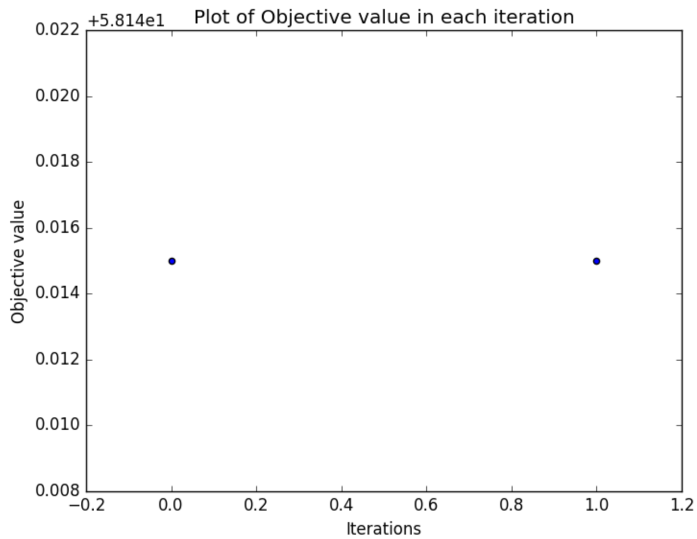

Figure 2: Plot of objective value in each iteration for Newton's Method

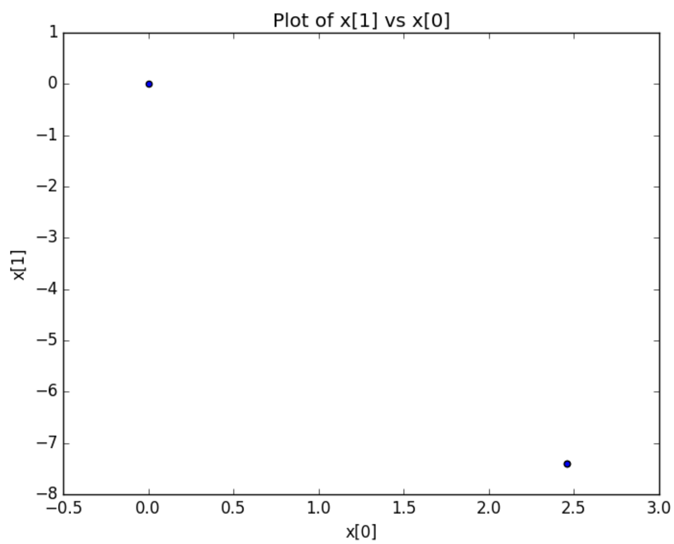

Figure 3: Plot of iterates across all iterations for Newton's Method

The function value converges to 58.1550098475 in just two iterations. We cant say much about the convergence rate as there are only two iterations. We can say though that the function value as well as the iterates converges Q-quadratically.

Subpart 3:

```
58.2249945982
58.1844917935
58.1555401444
58.1555281049
58.1555277763
58.1555277464
58.1555277374
58.1555277347
58.1555277347
Optimal Solution is:  [ 2.05460934 -5.33336993]
Minimum Value of the function is:  58.1555277347
No. of iterations:  9
>>>
```

Figure 4: Optimal Solution, Minimum Value and No. of iterations when starting point is [0,0] for BFGS Method

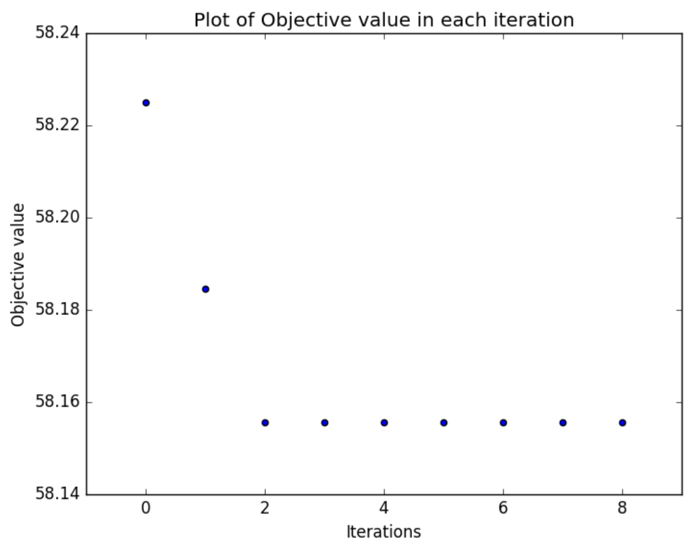

Figure 5: Plot of objective value in each iteration for BFGS Method

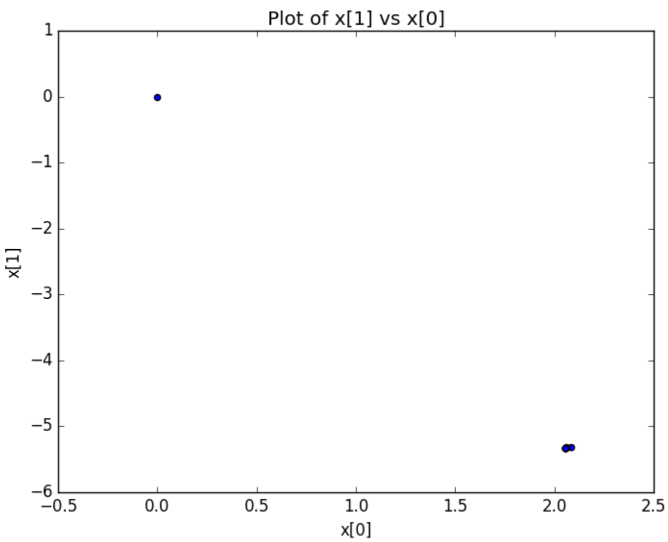

Figure 6: Plot of iterates across all iterations for BFGS Method

The function value converges to 58.1555277347 in 9 iterations. From the plots in figures 5, we can see that the objective value converges slowly to 58.1555277347. The iterates can also be seen to be converging to a point in figure 6.

Subpart 4:
From the above observations, we can conclude that the Newton's Method is much faster than the BFGS method as the no. of iterations required to converge is lower in Newton's method than in case of BFGS method. The minimum value computed is almost same for both the algorithms.

Subpart 6:

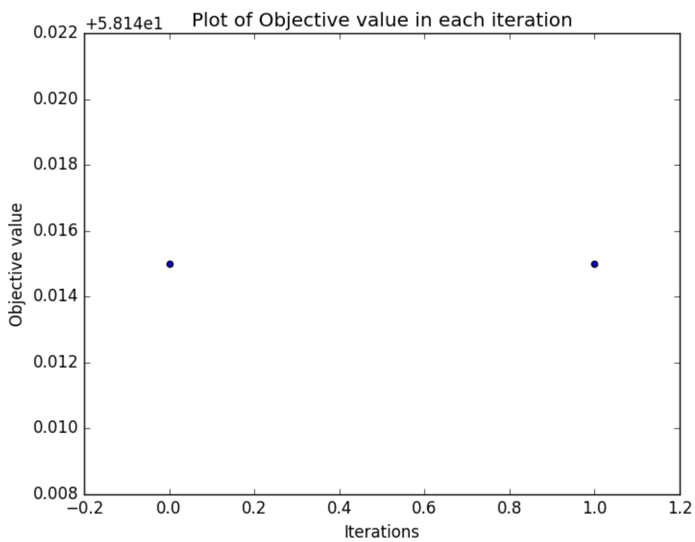

Figure 7: Plot of objective value in each iteration for Newton's Method when starting point is [1,1]

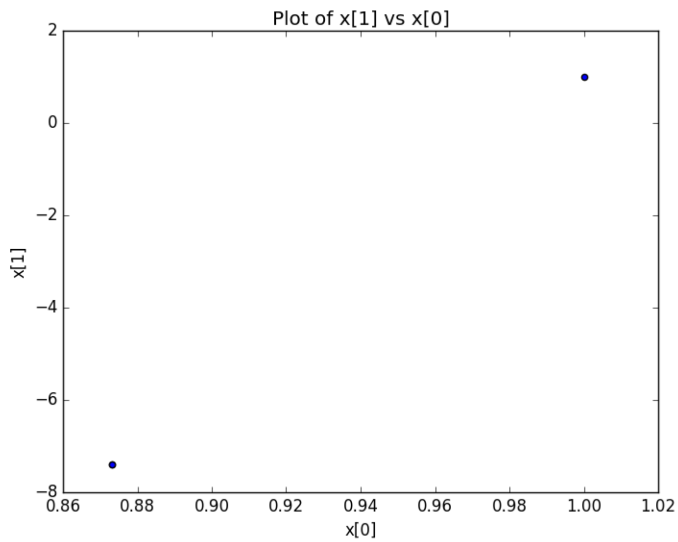

Figure 8: Plot of iterates across all iterations for Newton's Method when starting point is [1,1]

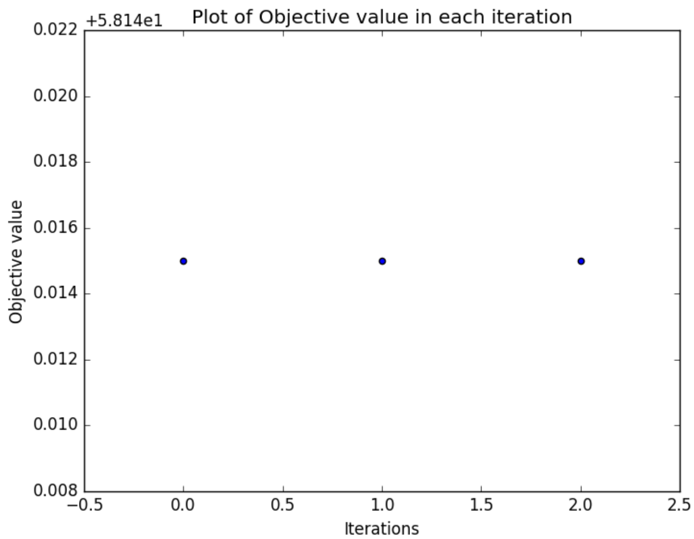

Figure 9: Plot of objective value in each iteration for Newton's Method when starting point is [-1,-1]

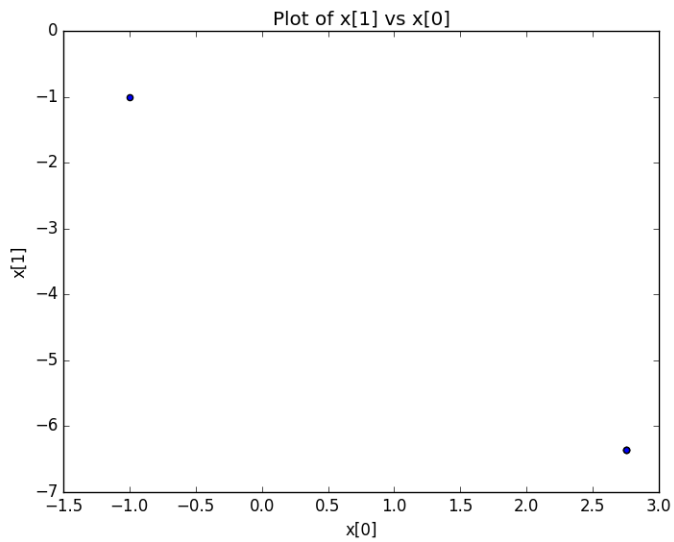

Figure 10: Plot of iterates across all iterations for Newton's Method when starting point is [-1,-1]

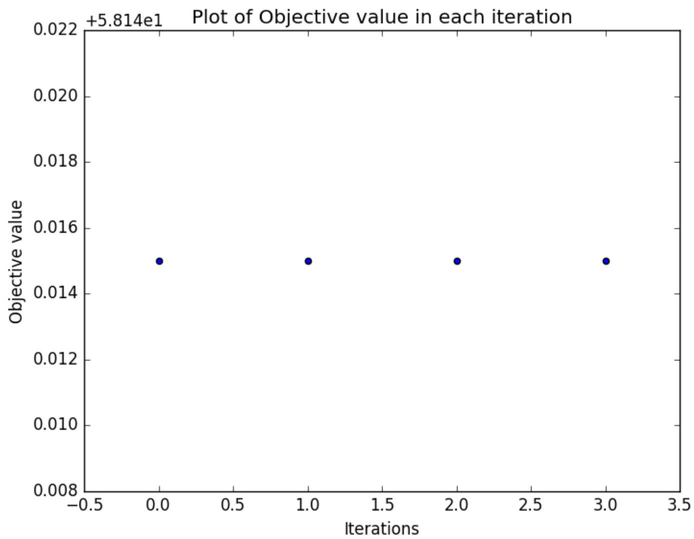

Figure 11: Plot of objective value in each iteration for Newton's Method when starting point is [5,5]

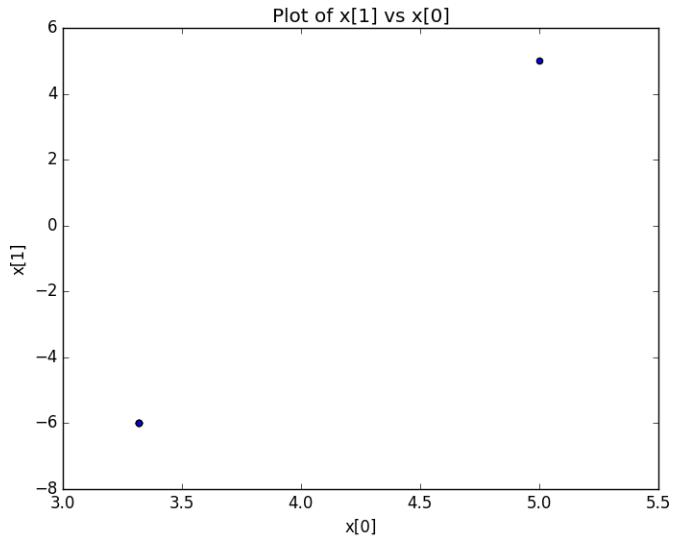

Figure 12: Plot of iterates across all iterations for Newton's Method when starting point is [5,5]

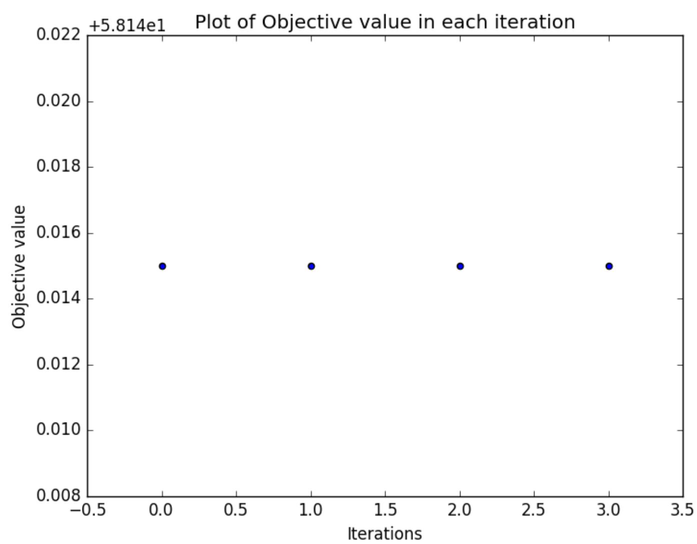

Figure 13: Plot of objective value in each iteration for Newton's Method when starting point is [-5,-5]

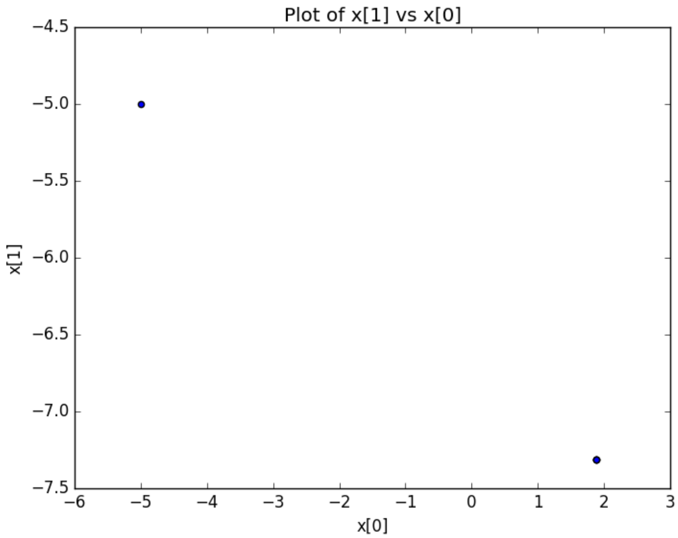

Figure 14: Plot of iterates across all iterations for Newton's Method when starting point is [-5,-5]

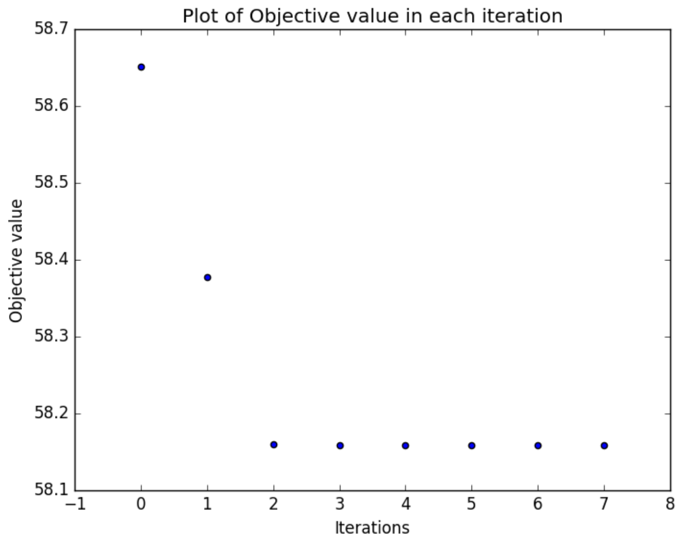

Figure 15: Plot of objective value in each iteration for BFGS Method when starting point is [1,1]

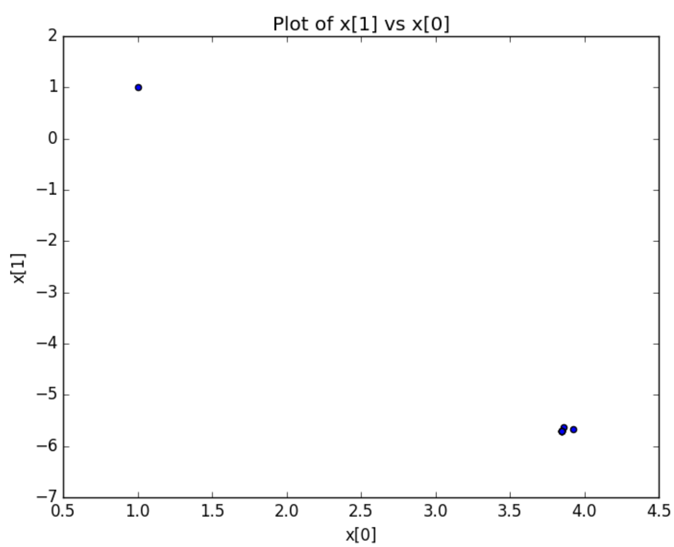

Figure 16: Plot of iterates across all iterations for BFGS Method when starting point is [1,1]

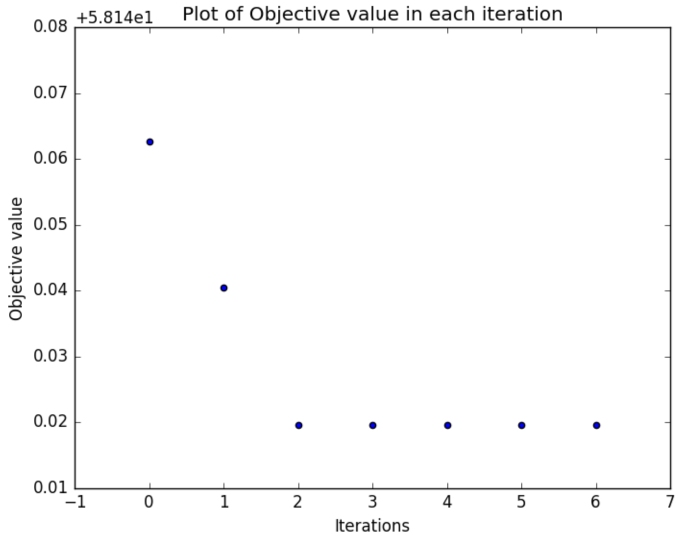

Figure 17: Plot of objective value in each iteration for BFGS Method when starting point is [-1,-1]

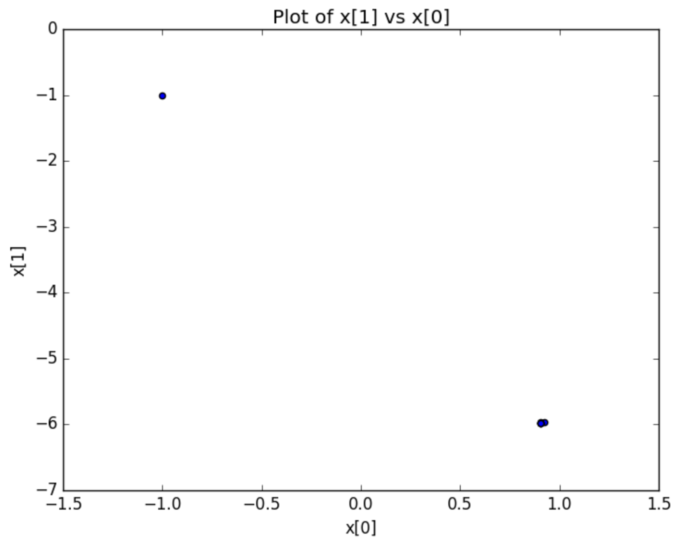

Figure 18: Plot of iterates across all iterations for BFGS Method when starting point is [-1,-1]

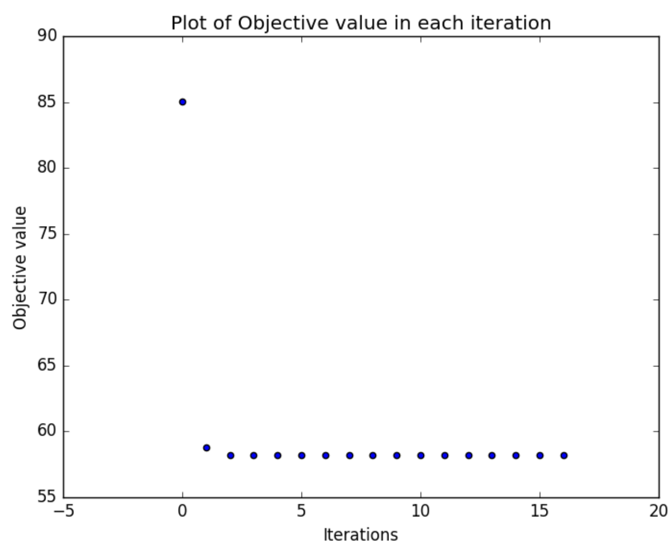

Figure 19: Plot of objective value in each iteration for BFGS Method when starting point is [5,5]

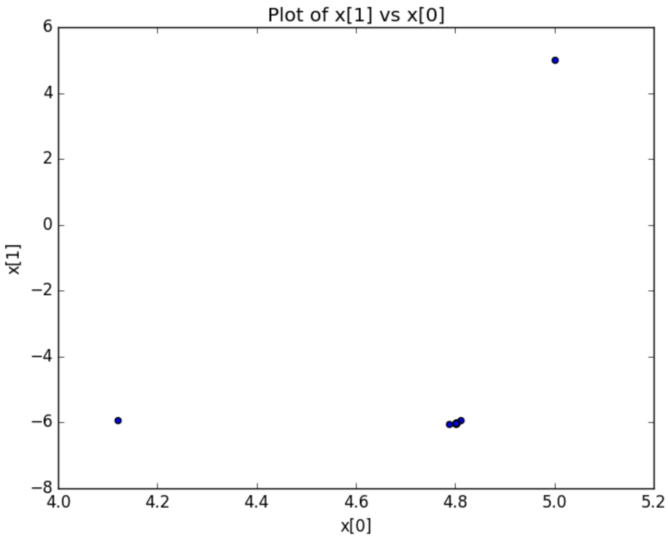

Figure 20: Plot of iterates across all iterations for BFGS Method when starting point is [5,5]

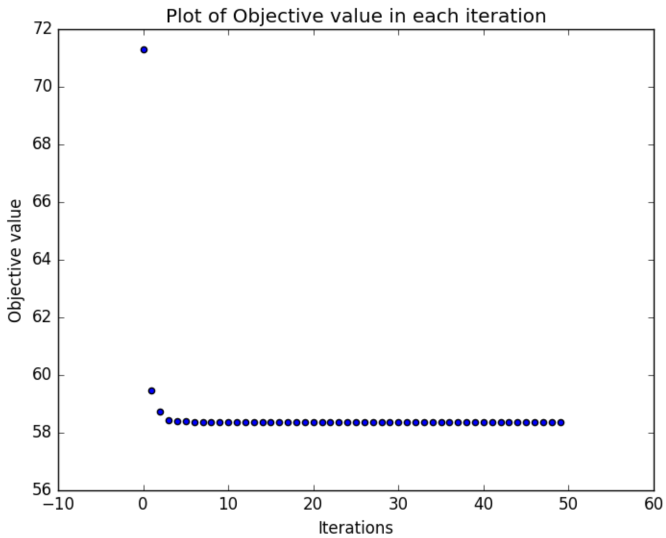

Figure 21: Plot of objective value in each iteration for BFGS Method when starting point is [-5,-5]

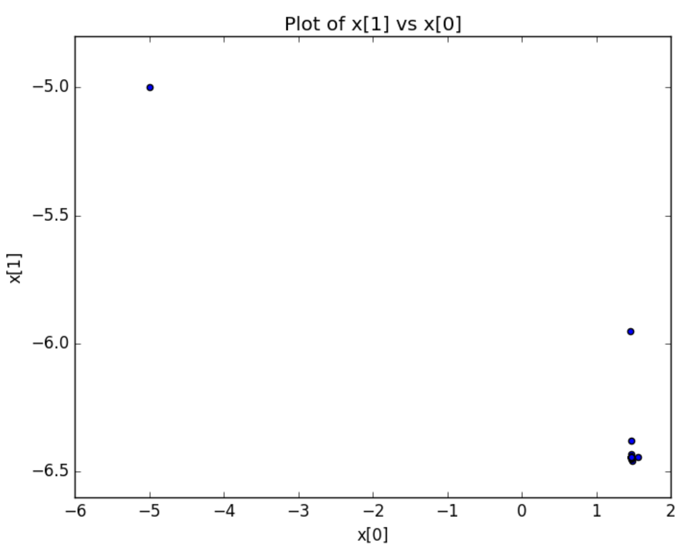

Figure 22: Plot of iterates across all iterations for BFGS Method when starting point is [-5,-5]

The rate of convergence is faster for Newton's method as compared to BFGS method as is visible from all the above plots. It can also be seen from the plots that the objective values are almost same for both the algorithms in all the 4 cases.
We can say from the plots that the iterates converge Q-quadratically for both Newton's and BFGS method across all 4 cases. And the objective value converges Q-linearly for both Newton's and BFGS algorithm across all the cases.

Question 2:
ex2c_bfgs.py, ex2c_newton.py, ex2d_bfgs.py, ex2d_newton.py, ex2e_bfgs.py, ex2e_newton.py

Subpart 1:
The newly added regularizer term adds a bound on the value that x_i's can take. Here the objective value is penalised for large values of x_i's as per the regularizing parameter λ.

Subpart 3:

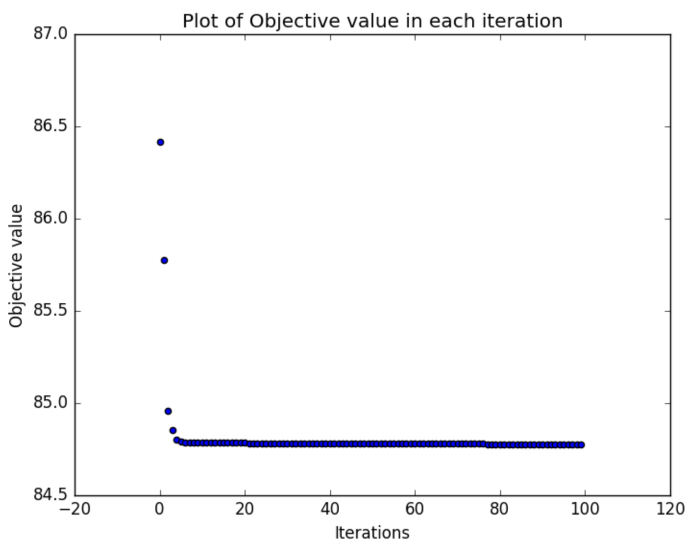

Figure 23: Plot of objective value in each iteration for BFGS Method when starting point is [0,0]

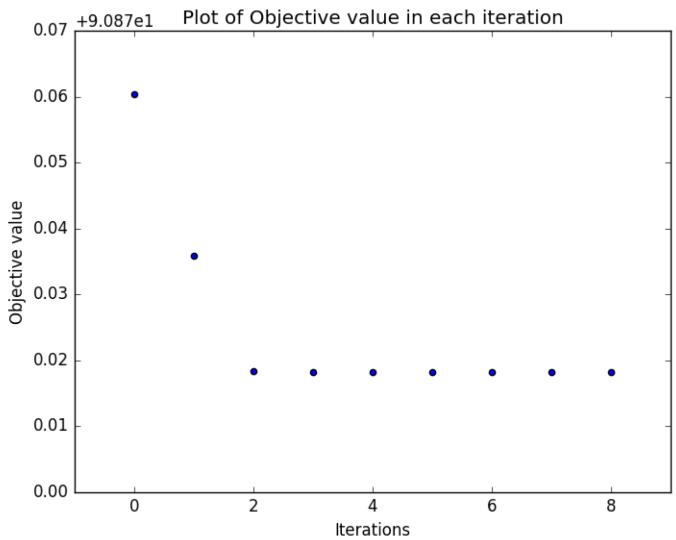

Figure 24: Plot of iterates across all iterations for BFGS Method when starting point is [1,1]

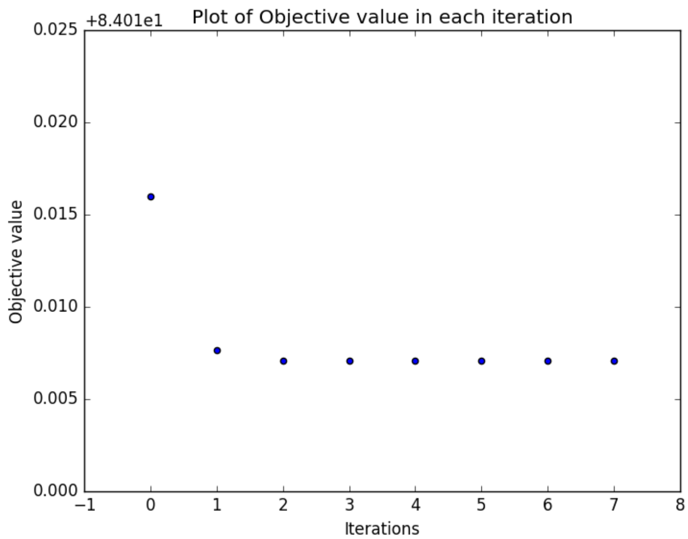

Figure 25: Plot of objective value in each iteration for BFGS Method when starting point is [-1,-1]

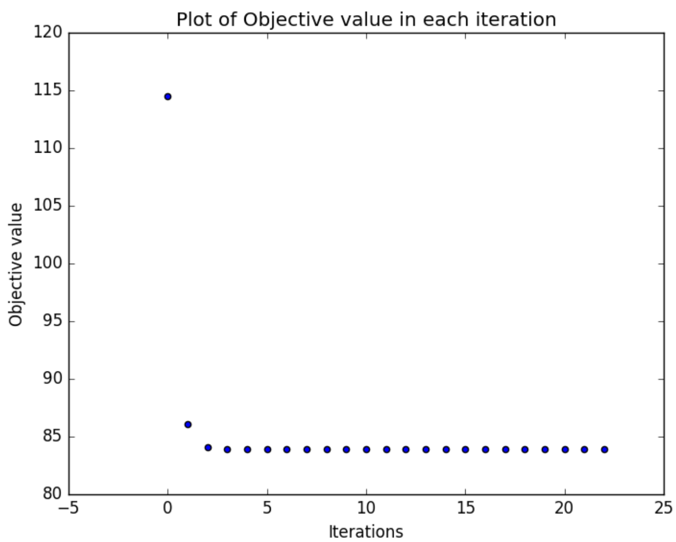

Figure 26: Plot of iterates across all iterations for BFGS Method when starting point is [5,5]

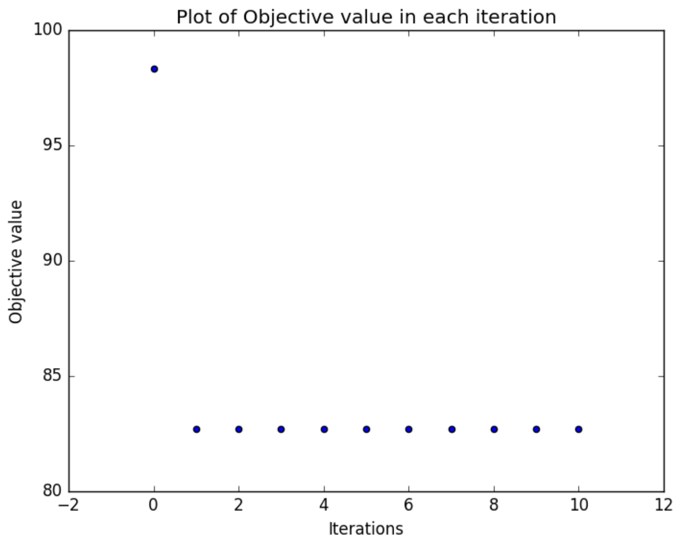

Figure 27: Plot of objective value in each iteration for BFGS Method when starting point is [-5,-5]

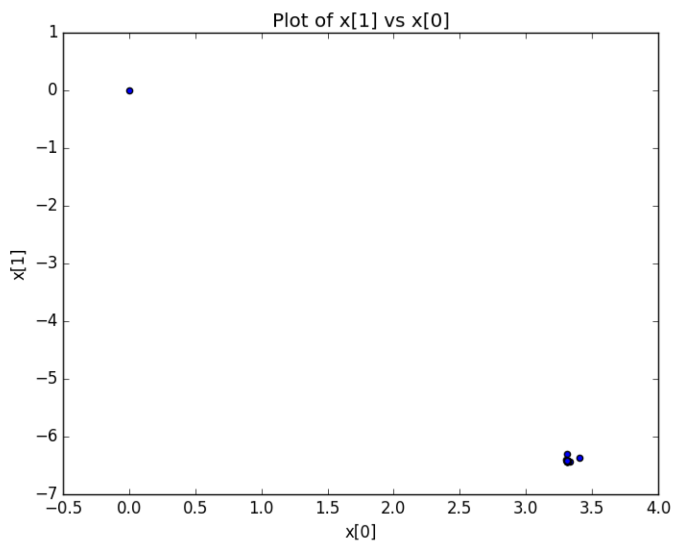

Figure 28: Plot of iterates across all iterations for BFGS Method when starting point is [0,0]

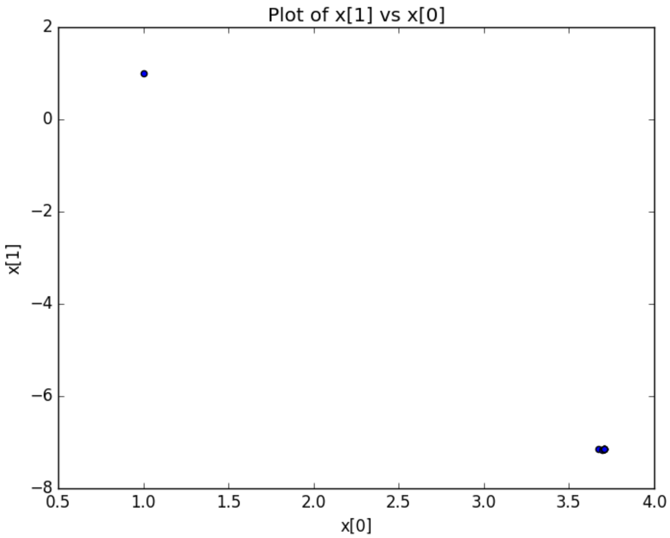

Figure 29: Plot of iterates across all iterations for BFGS Method when starting point is [1,1]

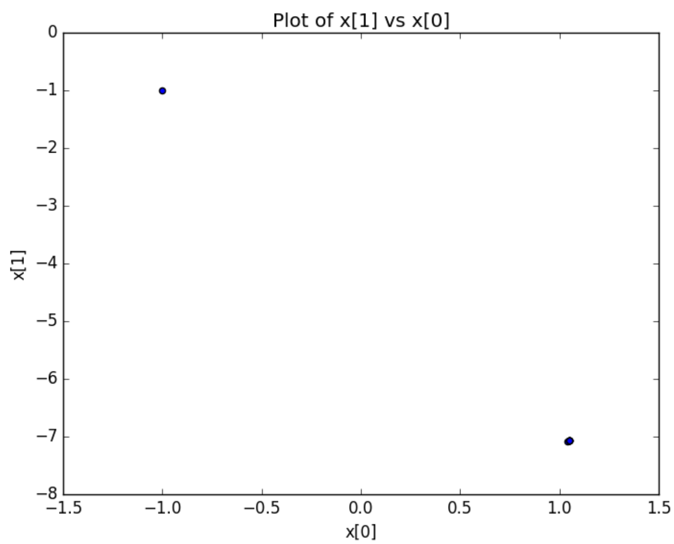

Figure 30: Plot of iterates across all iterations for BFGS Method when starting point is [-1,-1]

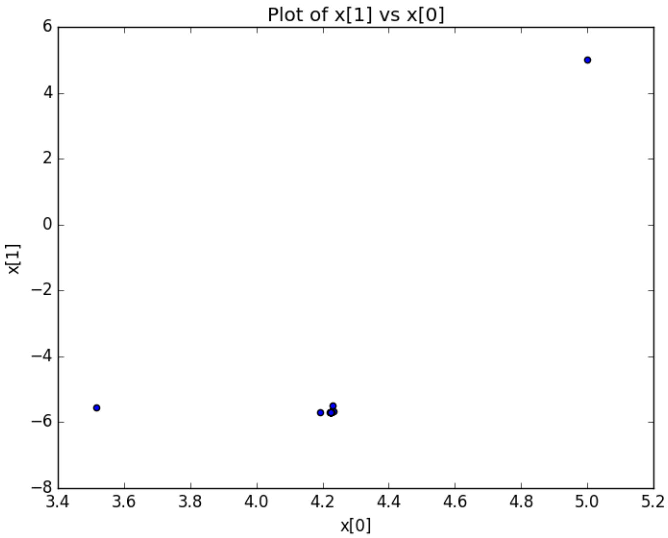

Figure 31: Plot of iterates across all iterations for BFGS Method when starting point is [5,5]

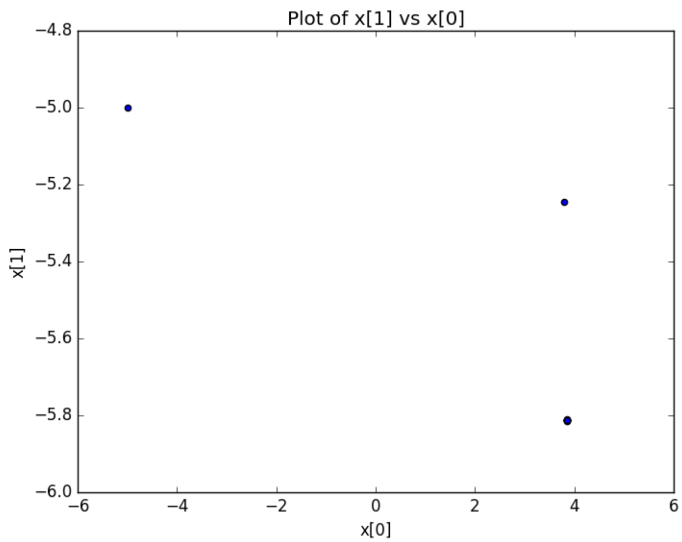

Figure 32: Plot of iterates across all iterations for BFGS Method when starting point is [-5,-5]

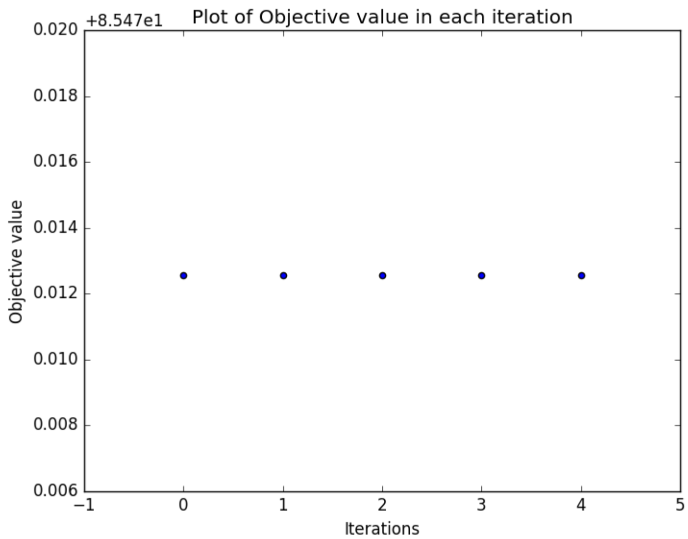

Figure 33: Plot of objective value in each iteration for Newton's Method when starting point is [0,0]

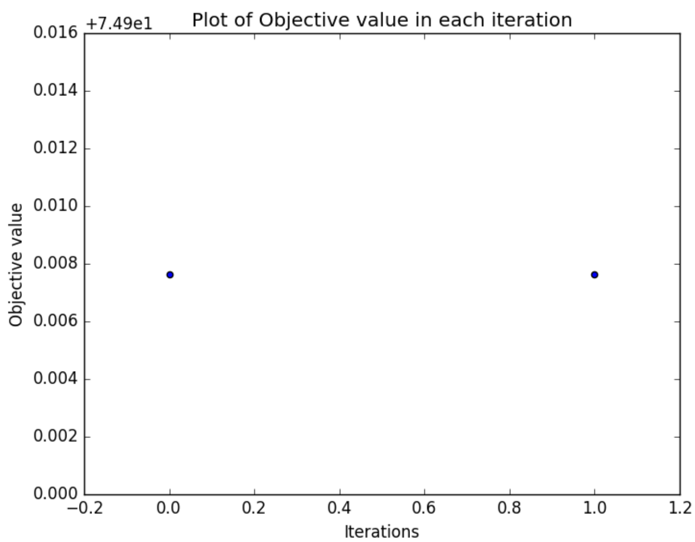

Figure 34: Plot of objective value in each iteration for Newton's Method when starting point is [1,1]

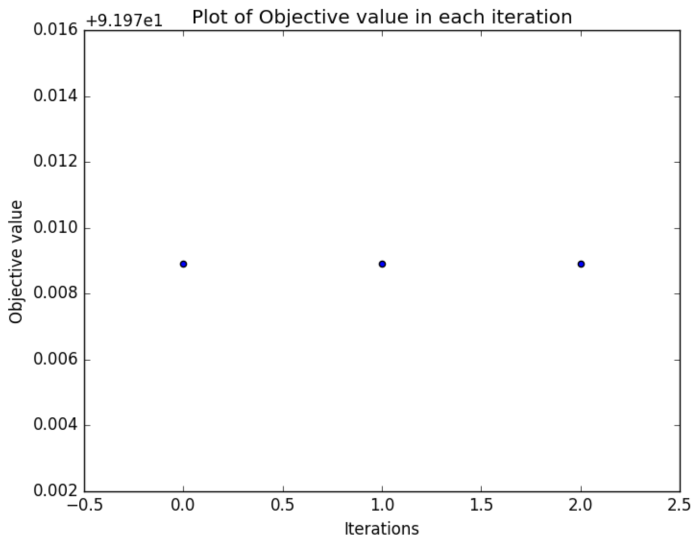

Figure 35: Plot of objective value in each iteration for Newton's Method when starting point is [-1,-1]

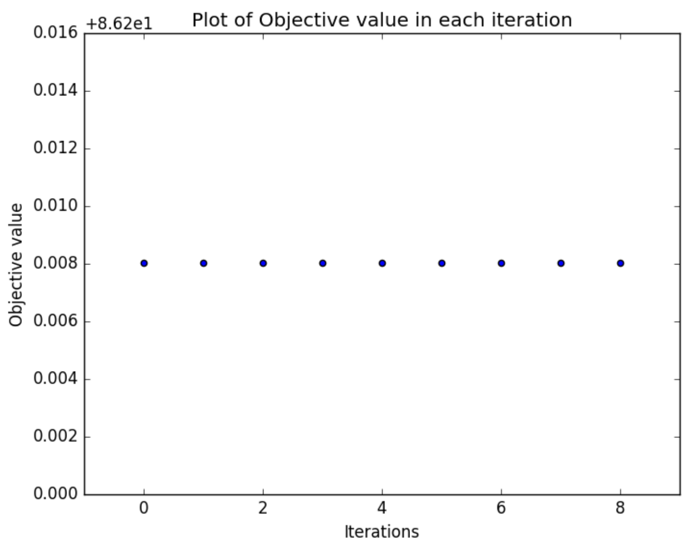

Figure 36: Plot of objective value in each iteration for Newton's Method when starting point is [5,5]

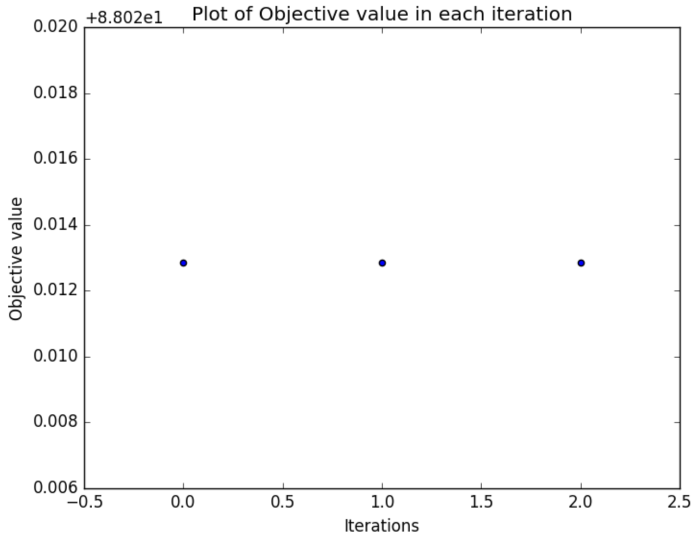

Figure 37: Plot of objective value in each iteration for Newton's Method when starting point is [-5,-5]

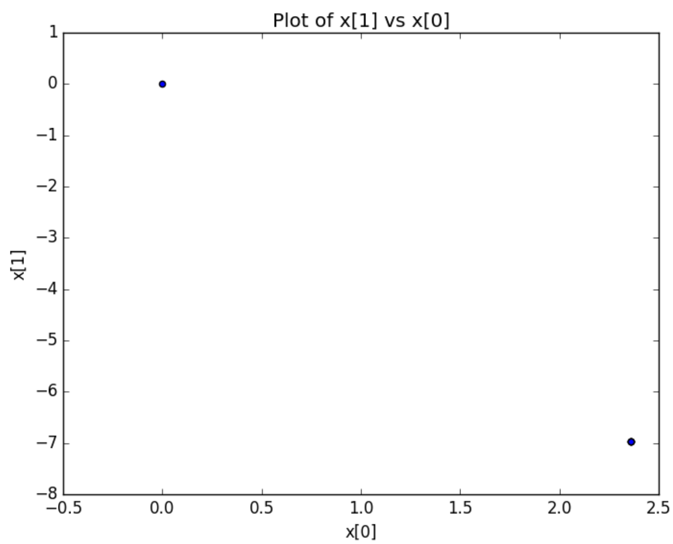

Figure 38: Plot of iterates across all iterations for Newton's Method when starting point is [0,0]

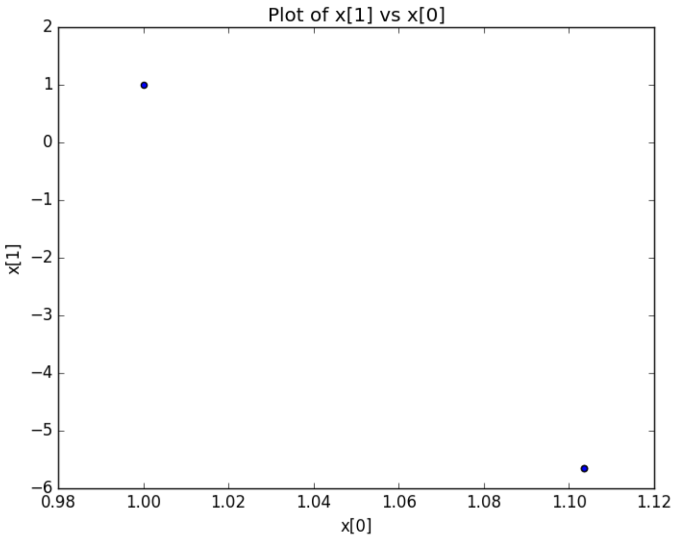

Figure 39: Plot of iterates across all iterations for Newton's Method when starting point is [1,1]

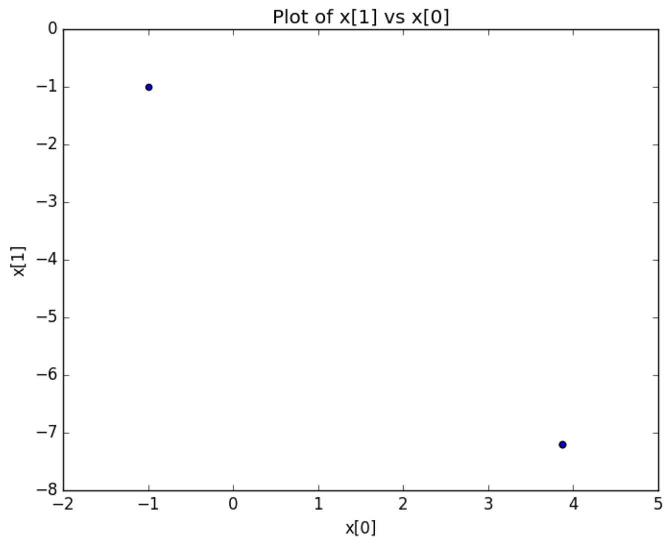

Figure 40: Plot of iterates across all iterations for Newton's Method when starting point is [-1,-1]

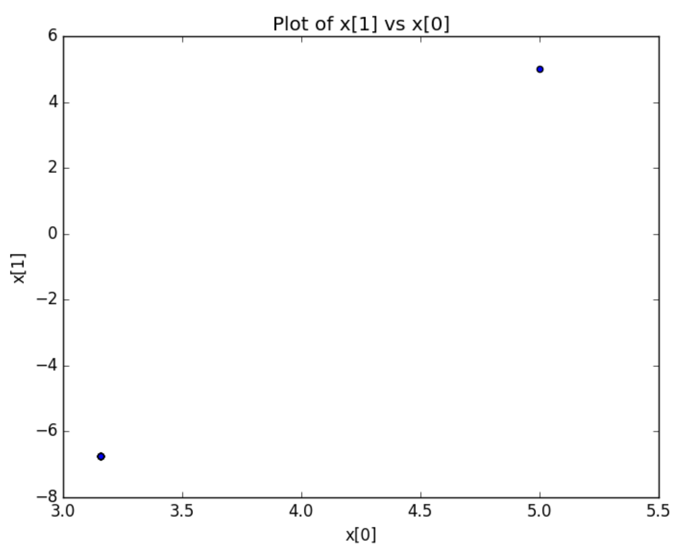

Figure 41: Plot of iterates across all iterations for Newton's Method when starting point is [5,5]

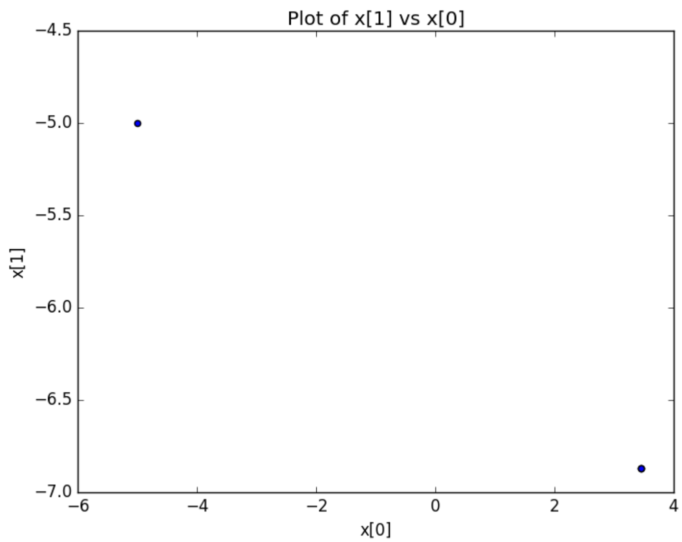

Figure 42: Plot of iterates across all iterations for Newton's Method when starting point is [-5,-5]

From the above plots, we can conclude that the rate of convergence is faster for Newton's method as compared to BFGS method as is visible from all the above plots. It can also be seen from the plots that the objective values are almost same for both the algorithms in all the 4 cases.
We can say from the plots that the iterates converge Q-quadratically for both Newton's and BFGS method across all 4 cases. And the objective value converges Q-linearly for both Newton's and BFGS algorithm across all the cases.

Subpart 4:

```
No. of iterations reqd for each lambda:  [5, 4, 2, 2, 2, 2, 4]
Minimum objective Value is:  58.173595153  corresponding to the value of lambda:  0.001
>>>
```

Figure 43: No. of iterations for each lambda for Newton's method when starting point is [0,0]and best value of lambda

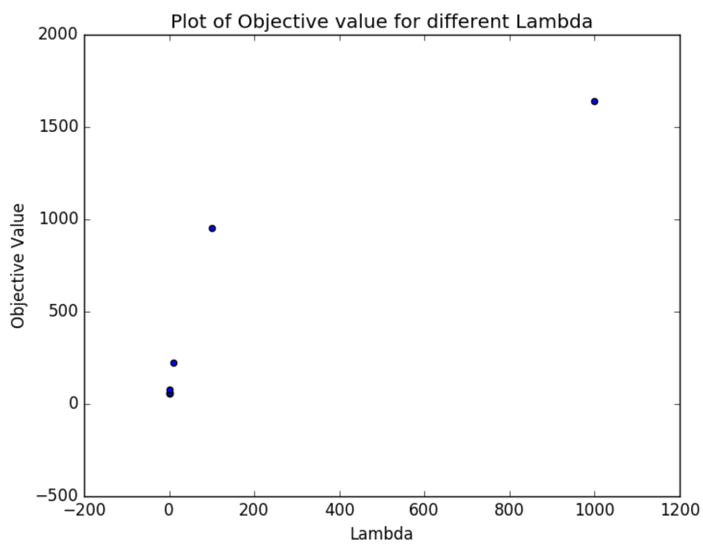

Figure 44: Plot of objective value for different lambdas in Newton's method when starting point is [0,0]

0.001 is the value of λ which achieves the lowest objective function value and it makes complete sense as the new objective function adds an extra penalising factor on the original objective function. So as the value of λ increases the optimal objective value also increases. This can be observed clearly from figure 44.
The running times though does not follow any particular pattern as in evident from the number of iterations observed for each λ in figure 43.

```
No. of iterations reqd. for each lambda:  [83, 22, 31, 25, 11, 9, 64]
Minimum objective Value is:  58.2988921508  corresponding to the value of lambda:  0.001
>>>
```

Figure 45: No. of iterations for each lambda for BFGS method when starting point is [0,0] and best value of lambda

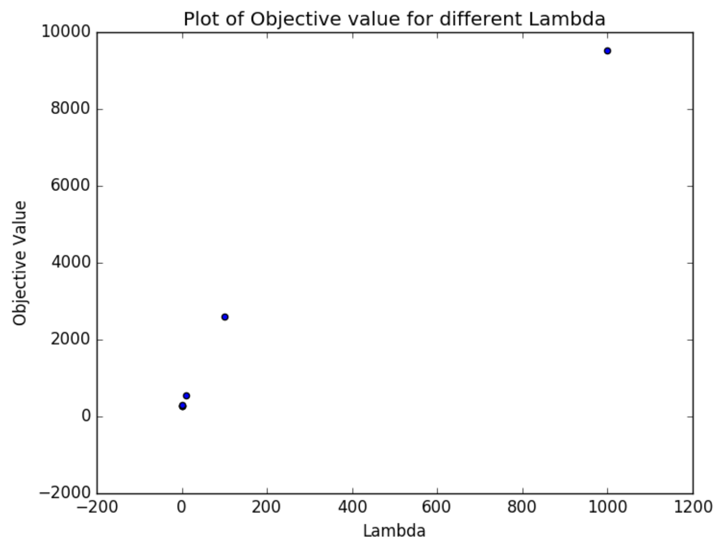

Figure 46: Plot of objective value for different lambdas in BFGS method when starting point is [0,0]

0.001 is the value of λ which achieves the lowest objective function value and it makes complete sense as the new objective function adds an extra penalising factor on the original objective function. So as the value of λ increases the optimal objective value also increases. This can be observed clearly from figure 46.
The running times though does not follow any particular pattern as in evident from the number of iterations observed for each λ in figure 45.

Subpart 5:

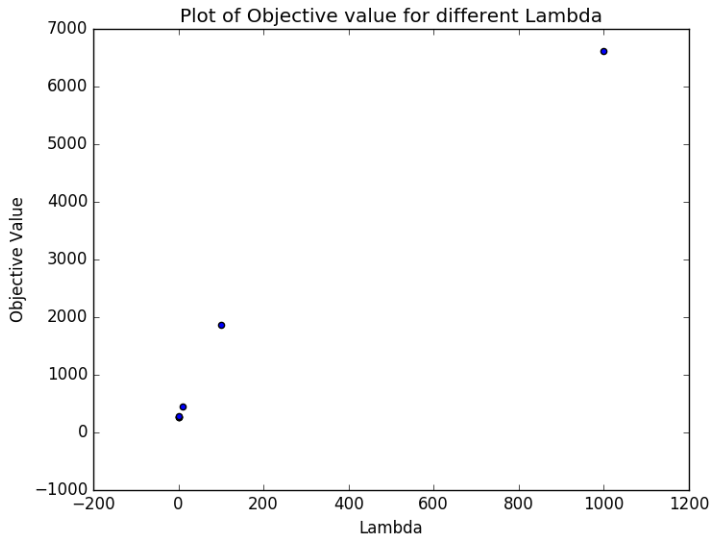

Figure 47: Plot of optimal objective value for different lambdas in Newton's method when A is 500x1 and starting point is [0,0]

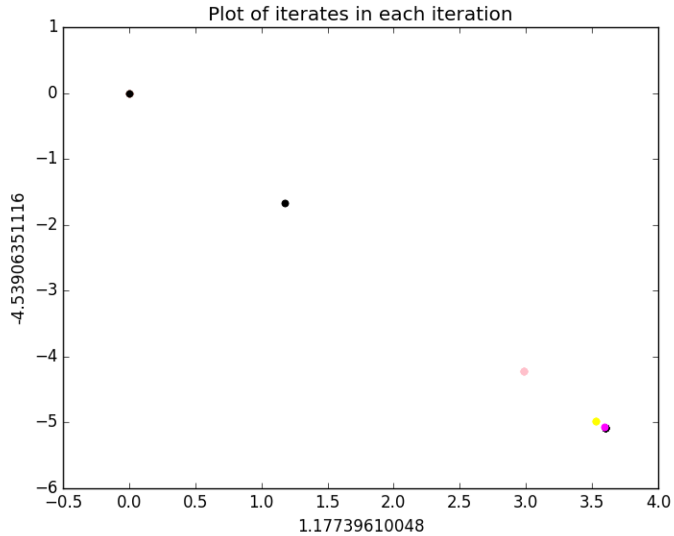

Figure 48: Plot of iterates in each iteration for different lambdas in Newton's method when A is of 500x2 and starting point is [0,0]

```
No. of iterations reqd for each lambda:  [4, 2, 3, 3, 6, 2, 2]
Minimum objective Value is:  58.1702193105  corresponding to the value of lambda:  0.001
>>>
```

Figure 49: Plot of optimal objective value for different lambdas in BFGS method when A is 500x1 and starting point is [0,0]

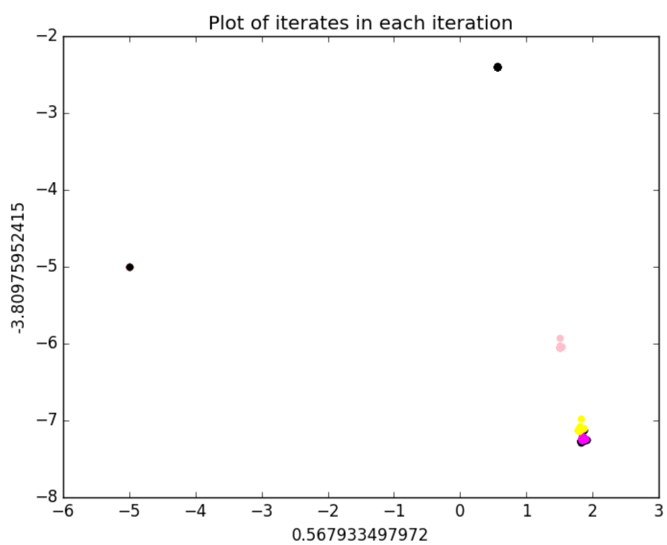

Figure 50: Plot of iterates in each iteration for different lambdas in BFGS method when A is of 500x2 and starting point is [0,0]

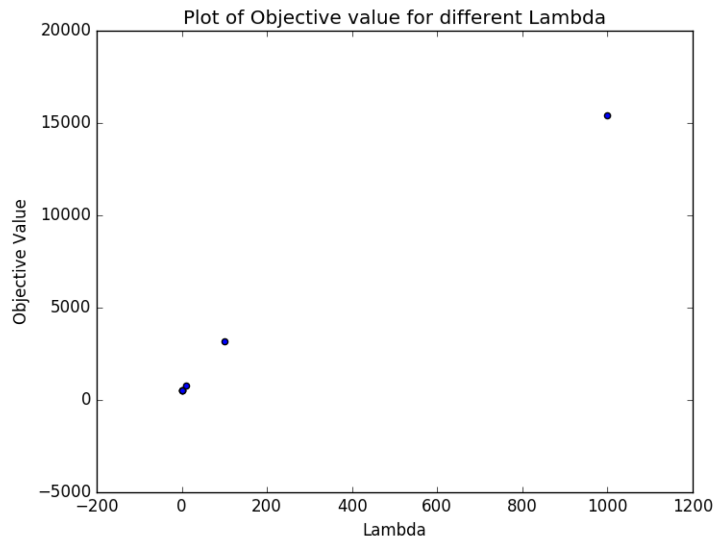

Figure 51: Plot of optimal objective value for different lambdas in Newton's method when A is 1000x1 and starting point is [0,0]

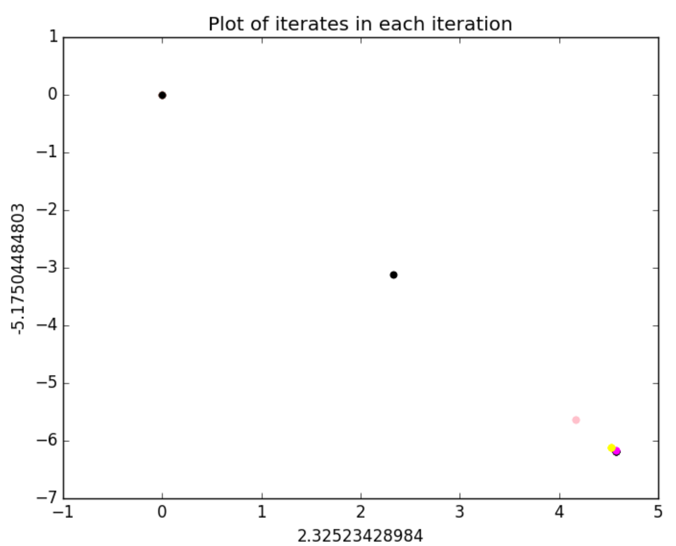

Figure 52: Plot of iterates in each iteration for different lambdas in Newton's method when A is of 1000x2 and starting point is [0,0]

The table is submitted offline to the TA.