IE 613: Assignment 1

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Question 1:
FILE: ex1.py

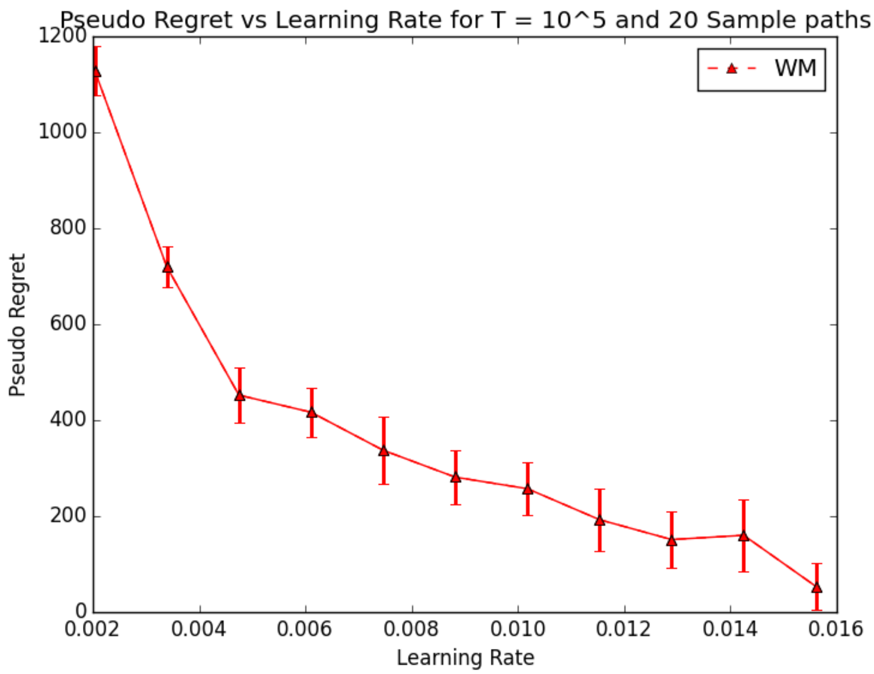

Figure 1: Pseudo Regret vs Learning Rate for Weighted Majority Algorithm

From the plot above, we can conclude that the Pseudo Regret of the weighted majority algorithm is a decreasing function of the Learning rate η. We found confidence intervals by running the algorithm for 20 sample paths for each value of η and assuming the underlying distribution is t.

Question 2:
FILE: ex2.py

We can observe from the plot that the pseudo regret decreases with η for all the 3 algorithms. Further we also observe that the pseudo regret values attains negative values for EXP3 and EXP3-IX algorithms for values of learning rate 0.002 onwards. On the other and the pseudo regret never reaches zero for EXP3.P algorithm.

Question 3:
We can conclude from the above plots that the EXP3-IX algorithm performs better than EXP3 and EXP3.P algorithms across all values of η in the sense that the pseudo regret is lowest for EXP3-IX as compared to EXP3 and EXP3.P. Moreover we can also see that the pseudo regret of the EXP3.P algorithm never attains zero or crosses it. In this sense we can claim that the EXP3.P

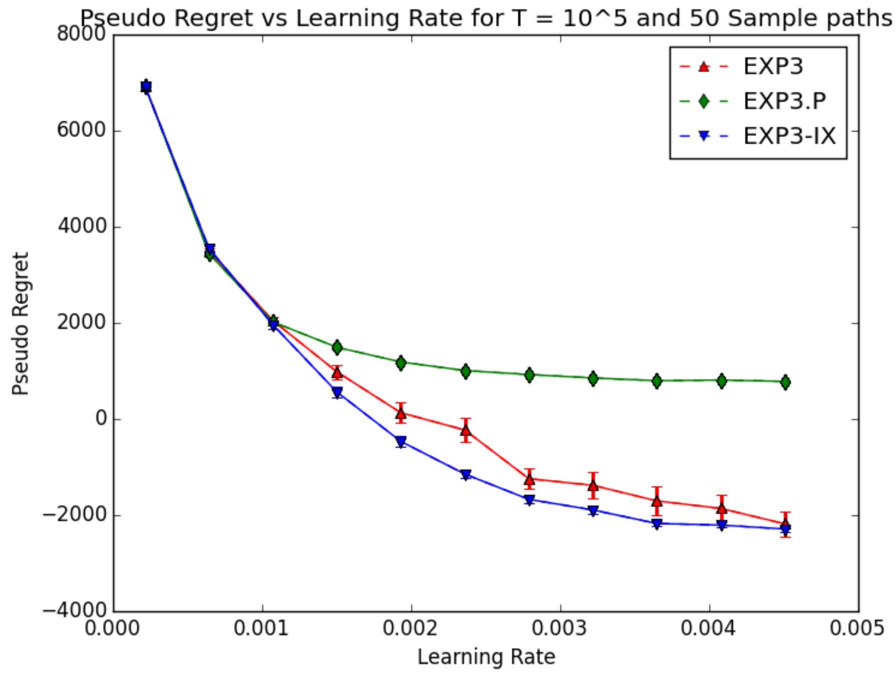

Figure 2: Pseudo Regret vs Learning Rate for EXP3, EXP3.P and EXP3-IX Algorithm

algorithm fares worse amongst the three. Note that it is not necessary for the actual regrets of the algorithms to actually attain this lower bound. It is also to be noted that all the three algorithms works reasonably well in the sense of pseudo regret till the value η=0.001 after which we can see stark differences in the pseudo regrets of the three algorithms.

Question 4:
Algorithm:
Input: Finite Hypothesis Class $\mathbb{H}$: Learning Rate $\eta \geq 0$
initialize: $w^0 = (0, \ldots, 0) \in \mathbb{R}^{|\mathbb{H}|}$: $Z_0 = |\mathbb{H}|$
for t=1,2,...,T, do,

Receive sample x_t and correspondingly compute $h_1(x_t), h_2(x_t), \ldots, h_{|\mathbb{H}|}(x_t)$
Environment determines y_t without revealing it to the learner
define $\hat{p}_t = \frac{1}{Z_{t-1}} \sum_{i:h_i(x_t)=1} w_i^{t-1}$
Predict $\hat{y}_t = 1$ with probability $\hat{p}_t$
Receive the noised label z_t
Update: $w_i^t = w_i^{t-1} exp(-\eta|h_i(x_t) - z_t|)$; $\quad Z_t = \sum_{i=1}^{|\mathbb{H}|} w_i^t$

Bounds:
Here, we came up with a stochastic algorithm(due to the incorporation of a noise factor ν which follows Bernoulli(ν)), hence we would be giving an upper bound on the total no. of mistakes in terms of expectations.

The Expected no. of mistakes is given as:

$$\mathcal{M}(A) = \mathbb{E}\Big[\sum_{t=1}^{T} |\hat{y}_t - h^*(x_t)|\Big]$$

Now, for a finite hypothesis class $\mathbb{H}$ and a fixed upper bound $\gamma < \frac{1}{2}$ on the noise rate, the following paper shows that the expected no. of mistakes is upper bounded by $\mathcal{O}(log|\mathbb{H}|)$.

Paper: "Agnostic Online Learning" by Shai Ben-David, David Pal and Shai Shalev-Schwartz.

Question 5:
Consider an online algorithm that enjoys a regret of the form $\alpha\sqrt{T}$, but its parameters require the knowledge of T. Hence we introduce the doubling trick as given in the question.
To Show: if regret of A on each period of 2m rounds is at most $\alpha\sqrt{2^m}$, then the total regret is at most:

$$\frac{\sqrt{2}}{\sqrt{2}-1}\alpha\sqrt{T}$$

Proof. Let us assume that the total no. of rounds that the algorithm runs for is T. Also lets assume $2^k \leq T \leq 2^{k+1} - 1$ for some value of $k \in \mathbb{N}$.

Now, the time points which belong to different rounds and the upper bound of regret for each of these rounds are given as:
m=0: 1 Regret Bound= $\alpha\sqrt{2^0}$
m=1: 2,3 Regret Upper Bound= $\alpha\sqrt{2^1}$
m=2: 4,5,6,7 Regret Upper Bound= $\alpha\sqrt{2^2}$
m=3: 8,9,10,11,12,13,14,15 Regret Upper Bound= $\alpha\sqrt{2^3}$
$\vdots$ m=k: $2^k, \ldots, 2^{k+1} - 1$ Regret Upper Bound= $\alpha\sqrt{2^k}$

$\therefore$ The Upper bound of the total regret considering all the rounds is given as:

$$\begin{aligned}
&\sum_{m=0}^{k} \alpha\sqrt{2^m} \\
&= \alpha\big[\sqrt{2^0} + \sqrt{2^1} + \sqrt{2^2} + \cdots + \sqrt{2^k}\big] \\
&= \alpha\frac{2^{\frac{k+1}{2}} - 1}{\sqrt{2}-1} \\
&\leq \alpha\frac{2^{\frac{k+1}{2}}}{\sqrt{2}-1} \\
&= \alpha\sqrt{2}\frac{2^{\frac{k}{2}}}{\sqrt{2}-1} \\
&\leq \frac{\sqrt{2}}{\sqrt{2}-1}\alpha\sqrt{T} \quad \big[since, 2^k \leq T \leq 2^{k+1} - 1\big]
\end{aligned}$$

$\square$