FIITJEE

Solutions to JEE(Main)-2020

Test Date: 3rd September 2020 (Second Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of **3 hours** duration.
2. This **Test Paper** consists of **75** questions. The maximum marks are **300**.
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 25 questions in each part of equal weightage out of which 20 questions are MCQs and 5 questions are numerical value based. Each question is allotted **4 (four)** marks for correct response.
4. **(Q. No. 01 – 20, 26 – 45, 51 – 70)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
5. **(Q. No. 21 – 25, 46 – 50, 71 – 75)** contains 15 Numerical based questions with answer as numerical value. Each question carries **+4 marks** for correct answer. There is no negative marking.
6. Candidates will be awarded marks as stated above in **instruction No.3** for correct response of each question. One mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer box.
7. There is only one correct response for each question. Marked up more than one response in any question will be treated as wrong response and marked up for wrong response will be deducted accordingly as per **instruction 6** above.

PART –A (PHYSICS)

1. The electric field of a plane electromagnetic wave propagating along the x direction in vacuum is $\vec{E} = E_0\hat{j}\cos(\omega t - kx)$. The magnetic field $\vec{B}$, at the moment t = 0 is:

(A) $\vec{B} = E_0\sqrt{\mu_0 \epsilon_0}\cos(kx)\hat{j}$
(B) $\vec{B} = E_0\sqrt{\mu_0 \epsilon_0}\cos(kx)\hat{k}$
(C) $\vec{B} = \frac{E_0}{\sqrt{\mu_0 \epsilon_0}}\cos(kx)\hat{k}$
(D) $\vec{B} = \frac{E_0}{\sqrt{\mu_0 \epsilon_0}}\cos(kx)\hat{j}$

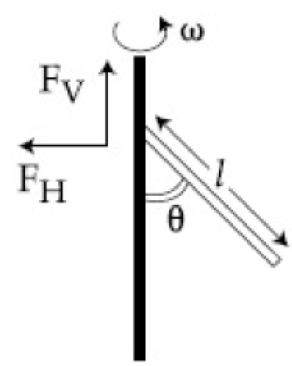
2. A block of mass 1.9 kg is at rest at the edge of a table, of height 1 m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 m/s in the horizontal direction just before the combined system strikes the floor, is
[Take g = 10 m/s^2. Assume there is no rotational motion and loss of energy after the collision is negligible.]

(A) 21 J
(B) 20 J
(C) 23 J
(D) 19 J

3. Hydrogen ion and singly ionized helium atom are accelerated, from rest, through the same potential difference. The ratio of final speeds of hydrogen and helium ions is close to:

(A) 1: 2
(B) 10 : 7
(C) 5 : 7
(D) 2 : 1

4. A uniform rod of length 'l' is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω the rod makes an angle θ with it (see figure). To find θ equate the rate of change of angular momentum (direction going into the paper) $\frac{ml^2}{12}\omega^2\sin\theta\cos\theta$ about the centre of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of θ is then such that:



(A) $\cos\theta = \frac{g}{l\omega^2}$
(B) $\cos\theta = \frac{2g}{3l\omega^2}$
(C) $\cos\theta = \frac{3g}{2l\omega^2}$
(D) $\cos\theta = \frac{g}{2l\omega^2}$

5. Amount of solar energy received on the earth's surface per unit area per unit time is defined a solar constant. Dimension of solar constant is:

(A) $M^2L^0T^{-1}$
(B) ML^2T^{-2}
(C) ML^0T^{-3}
(D) MLT^{-2}

6. A calorimeter of water equivalent 20 g contains 180 g of water at $25^{o}C$, 'm' grams of steam at $100^{o}C$ is mixed in it till the temperature of the mixture is 31^{o}C. The value of 'm' is close to (Latent heat of water = 540 cal g^{-1}, specific heat of water $= 1 cal g^{-1}\,{}^{o}C^{-1}$)
(A) 2.6 (B) 4 (C) 2 (D) 3.2

7. Which of the following will NOT be observed when a multimeter (operating in resistance measuring mode) probes connected across a component, are just reversed?
(A) Multimeter shows a deflection, accompanied by a splash light out of connected component in one direction and NO deflection on reversing the probes if the chosen component is LED.
(B) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is metal wire.
(C) Multimeter shows NO deflection in both case i.e. before and after reversing the probes if the chosen component is capacitor.
(D) Multimeter shows an equal deflection in both cases i.e. before and after reversing the probes if the chosen component is resistor.

8. Two sources of light emit X – rays of wavelength 1 nm and visible light of wavelength 500 nm, respectively. Both the sources emit light of the same power 200 W. The ratio of the number density of photons of X – rays to the number density of photons of the visible light of the given wavelengths is:
(A) $\frac{1}{500}$ (B) 500 (C) 250 (D) $\frac{1}{250}$

9. Concentric metallic hollow spheres of radii R and 4R hold charges Q_1 and Q_2 respectively. Given that surface charge densities of the concentric spheres are equal, the potential difference V (R) – V (4R) is:
(A) $\frac{3Q_2}{4\pi\epsilon_0 R}$ (B) $\frac{Q_2}{4\pi\epsilon_0 R}$ (C) $\frac{3Q_1}{16\pi\epsilon_0 R}$ (D) $\frac{3Q_1}{4\pi\epsilon_0 R}$

10. A block of mass m attached to a massless spring is performing oscillatory motion of amplitude 'A' on a frictionless horizontal plane. If half of the mass of the block breaks off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system become fA. The value of f is:
(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\sqrt{2}$ (D) 1

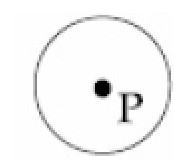
11. Two light waves having the same wavelength λ in vacuum are in phase initially. Then the first wave travels path L_1 through a medium of refractive index n_1 while the second wave travels a path of length L_2 through a medium of refractive index n_2. After this the phase difference between the two waves is:
(A) $\frac{2\pi}{\lambda}\left(\frac{L_2}{n_1}-\frac{L_1}{n_2}\right)$ (B) $\frac{2\pi}{\lambda}\left(\frac{L_1}{n_1}-\frac{L_2}{n_2}\right)$
(C) $\frac{2\pi}{\lambda}\left(n_1L_1-n_2L_2\right)$ (D) $\frac{2\pi}{\lambda}\left(n_2L_1-n_1L_2\right)$

12. A metallic sphere cools from $50^{o}C$ to 40^{o} in 300s. If atmospheric temperature around is $20^{o}C$, then the sphere's temperature after the next 5 minutes will be close to:
(A) $35^{o}C$
(B) $31^{o}C$
(C) $33^{o}C$
D) $28^{o}C$

13. If a semiconductor photodiode can detect a photon with a maximum wavelength of 400 nm, then its band gap energy is:
Planck's constant $h = 6.63 \times 10^{-34}$ J.s.
Speed of light $c = 3 \times 10^{8}\ m/s$
(A) 3.1eV
(B) 1.5eV
(C) 2.0eV
(D) 1.1eV

14. To raise the temperature of a certain mass of gas by 50^{o} C at a constant pressure, 160 calories of heat is required. When the same mass of gas is cooled by $100^{o}C$ at constant volume, 240 calories of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal)?
(A) 7
(B) 5
(C) 3
(D) 6

15. A perfectly diamagnetic sphere has a small spherical cavity at its centre, which is filled with a paramagnetic substance. The whole system is placed in a uniform magnetic field $\vec{B}$. Then the field inside the paramagnetic substance is:



(A) zero
(B) much large than $|\vec{B}|$ but opposite to $\vec{B}$
(C) $\vec{B}$
(D) much large than $|\vec{B}|$ and parallel to $\vec{B}$

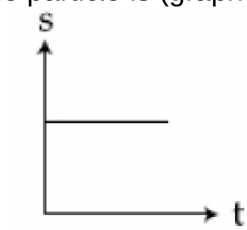
16. The radius R of a nucleus of mass number A can be estimated by the formula $R = (1.3 \times 10^{-15}) A^{1/3} m$. It follows that the mass density of a nucleus is of the order of:
$(M_{prot} = M_{neut.} \simeq 1.67 \times 10^{-27}\ kg)$
(A) $10^{3}\ kg\ m^{-3}$
(B) $10^{17}\ kg\ m^{-3}$
(C) $10^{10}\ kg\ m^{-3}$
(D) $10^{24}\ kg\ m^{-3}$

17. The mass density of a planet of radius R varies with the distance r from its centre as $\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right)$. Then the gravitational field is maximum at:
(A) $r = \frac{1}{\sqrt{3}}R$
(B) $r = R$
(C) $r = \sqrt{\frac{5}{9}}R$
(D) $r = \sqrt{\frac{3}{4}}R$

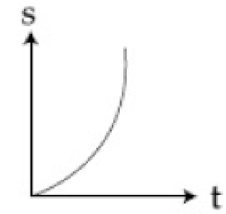
18. A uniform magnetic field B exists in a direction perpendicular to the plane of a square loop made of a metal wire. The wire has a diameter of 4 mm and a total length of 30 cm. The magnetic field changes with time at a steady rate $\frac{dB}{dt}=0.032\,Ts^{-1}$. The induced current in the loop is close to (Resistivity of the metal wire is $1.23\times10^{-8}\,\Omega m$)

(A) $0.61A$ (B) $0.43\,A$
(C) $0.53\,A$ (D) $0.34\,A$

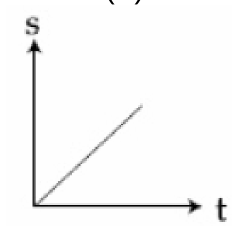
19. A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement(s) – time (t) graph that describes the motion of the particle is (graph are drawn schematically and are not to scale):



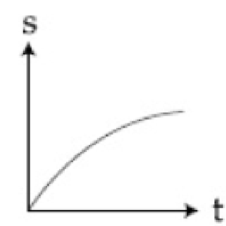
(A)



(B)



(C)



(D)

20. Two resistors 400Ω and 800Ω are connected in series across a 6 V battery. The potential difference measured by a voltmeter of $10k\Omega$ across 400Ω resistor is close to:

(A) 2 V (B) 2.05V
(C) 1.95 V (D) 1.8 V

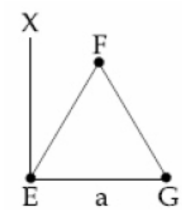
21. A block starts moving up an inclined plane of inclination 30^o with an initial velocity of ν_0.

It comes back to its initial position with velocity $\frac{\nu_0}{2}$. The value of the coefficient f kinetic friction between the block and the inclined plane is close to $\frac{I}{1000}$, The nearest integer to I is

22. A galvanometer coil has 500 turns and each turn has an average are of $3\times10^{-4}m^2$. If a torque of 1.5 Nm is required to keep this coil parallel to a magnetic field when a current of 0.5 A is flowing through it, the strength of the field (in T) is _________.

23. If minimum possible work is done by a refrigerator in converting 100 grams of water at $0^{o}C$ to ice, how much heat (in calories) is released to the surroundings at temperature $27^{o}C$ (Latent heat of ice = 80 Cal/gram) to the nearest integer?

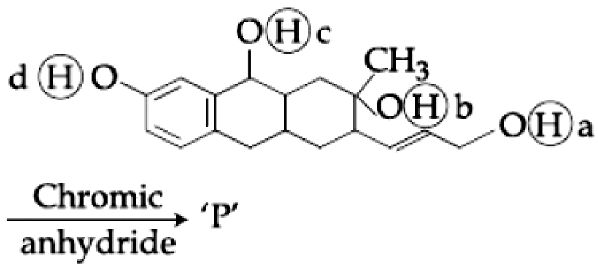
24. A massless equilateral triangle EFG of side 'a' (As shown in figure) has three particles of mass m situated at its vertices. The moment of inertia of the system about the line EX perpendicular to EG in the plane of EFG is $\frac{N}{20}ma^2$ where N is an integer. The value of N is _________.



25. When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm from the mirror. If the object is moved with a speed of 9 cm s^{-1}, the speed (in cm s^{-1}) with which image moves at that instant is _______________.

PART – B (CHEMISTRY)

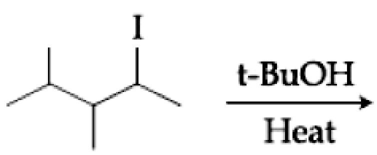
26. Consider the following reaction:

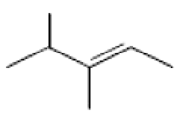


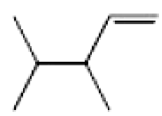
The product 'P' gives positive ceric ammonium nitrate test. This is because of the presence of which of these – OH group(s)?

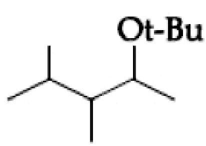
(A) (d) only
(B) (b) and (d)
(C) (b) only
(D) (c) and (d)

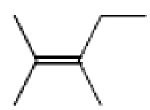
27. The major product in the following reaction is:



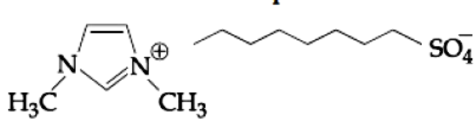
(A) 

(B) 

(C) 

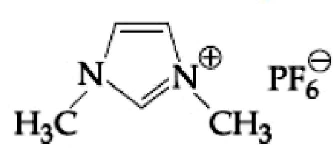
(D) 

28. An ionic micelles is formed on the addition of:

(A) excess water to liquid 

(B) sodium stearate to pure toluene

(C) liquid diethyl ether to aqueous NaCl solution

(D) excess water to liquid 

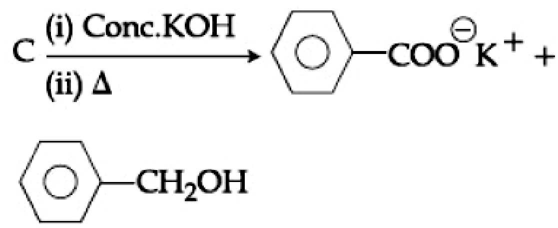
29. The five successive ionization enthalpies of an element are 800, 2427, 3658, 25024 and 32824 kJ mol^{-1}. The number of valence electrons in the element is:

(A) 4
(B) 2
(C) 5
(D) 3

30. The compound A in the following reactions is:

$$A \xrightarrow[\text{(ii) Conc. } H_2SO_4/\Delta]{\text{(i) } CH_3MgBr/H_2O}$$

$$B \xrightarrow[\text{(ii) } Zn/H_2O]{\text{(i) } O_3} C + D$$



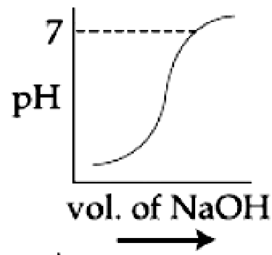
$$D \xrightarrow[\Delta]{Ba(OH)_2} H_3C-\overset{\overset{\large CH_3}{|}}{C}=CH-\overset{\overset{\large O}{||}}{C}-CH_3$$

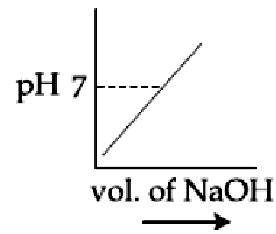
(A) $C_6H_5-\overset{\overset{O}{||}}{C}-CH_2CH_3$
(B) $C_6H_5-CH_2-\overset{\overset{O}{||}}{C}-CH_3$
(C) $C_6H_5-\overset{\overset{O}{||}}{C}-CH_3$
(D) $C_6H_5-\overset{\overset{O}{||}}{C}-CH\begin{matrix} CH_3 \\ CH_3 \end{matrix}$

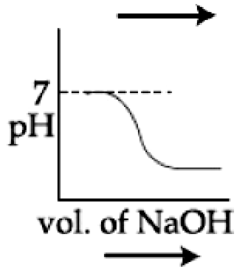
31. Consider the hypothetical situation where the azimuthal quantum number, l, takes values 0, 1, 2,…….n + 1, where n is the principal quantum number. Then, the element with atomic number:
(A) 9 is the first alkali metal
(B) 8 is the first noble gas
(C) 13 has a half – filled valence subshell
(D) 6 has a 2p – valence subshell

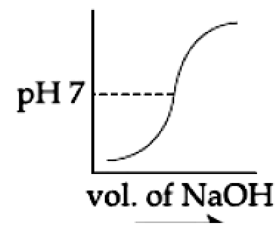
32. The strengths of 5.6 volume hydrogen peroxide (of density 1 g/mL) in terms of mass percentage and molarity (M), respectively, are:
(Take molar mass of hydrogen peroxide as 34 g/mol)
(A) 0.85 and 0.25
(B) 0.85 and 0.5
(C) 1.7 and 0.5
(D) 1.7 and 0.25

33. 100 mL of 0.1 M HCl is taken in a beaker and to it 100 mL of 0.1 M NaOH is added in steps of 2 mL and the pH is continuously measured. Which of the following graphs correctly depicts the change in pH?

(A) 

(B) 

(C) 

(D) 

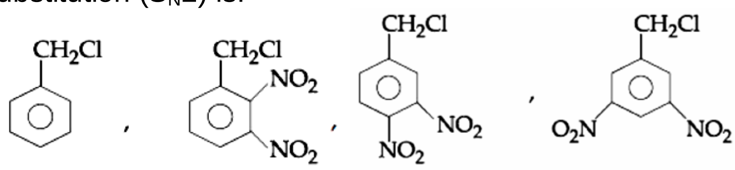
34. The incorrect statement(s) among (a) – (d) regarding acid rain is (are):
(a) It can corrode water pipes.
(b) It can damage structures made up of stone.
(c) It cannot cause respiratory ailments in animals
(d) It is not harmful for trees
(A) (a), (b) and (d)
(B) (c) and (d)
(C) (a), (c) and (d)
(D) (c) only

35. The increasing order of the reactivity of the following compounds in nucleophilic addition reaction is:
Propanal, Benzaldehyde, Propanone, Butanone
(A) Benzaldehyde < Propanal < Propanone < Butanone
(B) Butanone < Propanone < Benzaldehyde < Propanal
(C) Propanal < Propanone < Butanone < Benzaldehyde
(D) Benzaldehyde < Butanone < Propanone < Propanal

36. Complex A has a composition of $H_{12}O_6Cl_3Cr$. If the complex on treatment with conc. H_2SO_4 loses 13.5% of its original mass, the correct molecular formula of A is:
[Given : atomic mass of Cr = 52 amu and Cl = 35 amu]
(A) $\left[Cr\left(H_2O\right)_4Cl_2\right]Cl.2H_2O$
(B) $\left[Cr\left(H_2O\right)_6\right]Cl_3$
(C) $\left[Cr\left(H_2O\right)_5Cl\right]Cl_2.H_2O$
(D) $\left[Cr\left(H_2O\right)_3Cl_3\right].3H_2O$

37. The incorrect statement is
(A) In manganate and permanganateions, the π - bounding takes place by overlap of p –orbitals of oxygen and d – orbitals of manganese
(B) Manganate ion is green in colour and permanganateion is purple in colour
(C) Manganate and permanganate ions are paramagnetic
(D) Manganate and permanganate ions are tetrahedral

38. The decreasing order or reactivity of the following compounds towards nucleophilic substitution (S_N2) is:



(I) (II) (III) (IV)

(A) (II) > (III) > (I) > (IV)
(B) (III) > (II) > (IV) > (I)
(C) (II) > (III) > (IV) > (I)
(D) (IV) > (II) > (III) > (I)

39. A mixture of one mole each of H_2, He and O_2 each are enclosed in a cylinder of volume V at temperature T. If the partial pressure of H_2 is 2 atm, the total pressure of the gases in the cylinder is:
(A) 14 atm
(B) 6 atm
(C) 22 atm
(D) 38 atm

40. Among the statements (I – IV), the correct ones are:
(I) Be has smaller atomic radius compared to Mg.
(II) Be has higher ionization enthalpy than Al.
(III) Charge/radius ratio of Be is greater than that of Al.
(IV) Both Be and Al form mainly covalent compounds.
(A) (I), (III) and (IV)
(B) (II), (III) and (IV)
(C) (I), (II) and (III)
(D) (I), (II) and (IV)

41. Three isomers A, B and C (mol. formula $C_8H_{11}N$) give the following results:

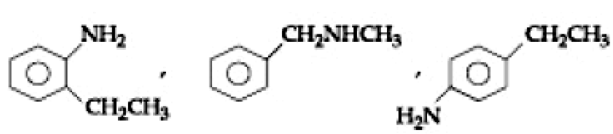
$$\text{A and C} \xrightarrow{\text{Diazotization}} P + Q \xrightarrow[\text{(ii) oxidation } (KMnO_4 + H^+)]{\text{(i) Hydrolysis}} \begin{matrix} \text{R (product of A)} \\ + \\ \text{S (product of C)} \end{matrix}$$

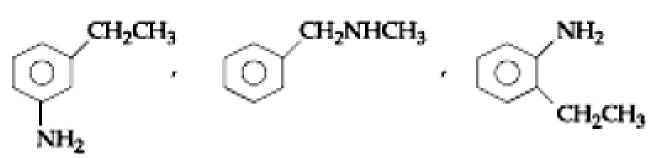
R has lower boiling point than S

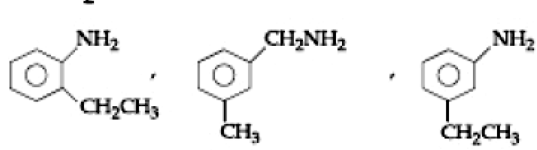
$$B \xrightarrow{C_6H_5SO_2Cl} \text{alkali – insoluble product}$$

A, B and C, respectively are:

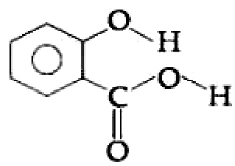
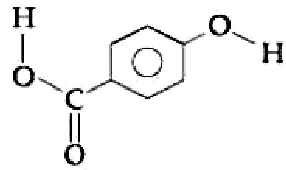
(A) 

(B) 

(C) 

(D) 

42. Consider the following molecules and statements related to them:

(A)  (B) 

(a) (B) is more likely to be crystalline
(b) (B) has higher boiling point than (A)
(c) (B) dissolves more readily than (A) in water

Identify the correct option from below:

(A) (a) and (c) are true
(B) (b) and (c) are true
(C) (a) and (b) are true
(D) only (a) is true

43. For the reaction $2A + 3B + \frac{3}{2}C \rightarrow 3P$, which statement is correct?

(A) $\frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt} = \frac{3}{4}\frac{dn_C}{dt}$
(B) $\frac{dn_A}{dt} = \frac{dn_B}{dt} = \frac{dn_C}{dt}$
(C) $\frac{dn_A}{dt} = \frac{3}{2}\frac{dn_B}{dt} = \frac{3}{4}\frac{dn_C}{dt}$
(D) $\frac{dn_A}{dt} = \frac{2}{3}\frac{dn_B}{dt} = \frac{4}{3}\frac{dn_C}{dt}$

44. The d – electron configuration of $\left[Ru(en)_3\right]Cl_2$ and $\left[Fe(H_2O)_6\right]Cl_2$, respectively are:

(A) $t_{2g}^6 e_g^0$ and $t_{2g}^4 e_g^2$
(B) $t_{2g}^4 e_g^2$ and $t_{2g}^4 e_g^2$
(C) $t_{2g}^6 e_e^0$ and $t_{2g}^6 e_g^0$
(D) $t_{2g}^4 e_g^2$ and $t_{2g}^6 e_g^0$

45. Match the following drugs with their therapeutic actions:

Column-I		Column-II	
(i)	Ranitidine	(a)	Antidepressant
(ii)	Nardil (Phenelzine)	(b)	Antibiolic
(iii)	Chloramphenicol	(c)	Antihistamine
(iv)	Dimetane (Brompheniramine)	(d)	Antacid
		(e)	Analgesic

(A) (i) – (e); (ii) – (a); (iii) – (c); (iv) – (d)
(B) (i) – (a); (ii) – (c); (iii) – (b); (iv) – (e)
(C) (i) – (d); (ii) – (c); (iii) – (a); (iv) – (e)
(D) (i) – (d); (ii) – (a); (iii) – (b); (iv) – (c)

46. The volume (in mL) of 0.1 N NaOH required to neutralise 10 mL of 0.1 N phosphinic acid is _______.

47. The number of $\rangle C=O$ groups present in a tripeptide Asp – Glu – Lys is _____________.

48. If 250 cm^3 of an aqueous solution containing 0.73 g of a protein A is isotonic with one litre of another aqueous solution containing 1.65 g of a protein B, at 298 K, the ratio of the molecular masses of A and B is _________________ $\times 10^{-2}$ (to the nearest integer).

49. An acidic solution of dichromate is electrolyzed for 8 minutes using 2A current. As per the following equation $Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$. The amount of Cr^{3+} obtained was 0.104 g. The efficiency of the process (in %) is (Take: F = 96000 C, At. Mass of chromium = 52) ______________.

50. 6.023×10^{22} molecules are present in 10 g of a substance 'x'. The molarity of a solution containing 5 g of substance 'x' in 2 L solution is ___________ $\times 10^{-3}$.

PART–C (MATHEMATICS)

51. If the sum of the series $20+19\frac{3}{5}+19\frac{1}{5}+18\frac{4}{5}+....$ upto n^{th} term is 488 and the n^{th} term is negative, then:

(A) n^{th} term is $-4\frac{2}{5}$
(B) $n = 41$
(C) n^{th} term is -4
(D) $n = 60$

52. If z_1, z_2 are complex numbers such that $\text{Re}(z_1)=|z_1-1|, \text{Re}(z_2)=|z_2-1|$ and $\arg(z_1-z_2)=\frac{\pi}{6}$, then $\text{Im}(z_1+z_2)$ is equal to:

(A) $2\sqrt{3}$
(B) $\frac{\sqrt{3}}{2}$
(C) $\frac{1}{\sqrt{3}}$
(D) $\frac{2}{\sqrt{3}}$

53. Let $a, b, c \in R$ be such that $a^2+b^2+c^2=1$. If $a\cos\theta = b\cos\left(\theta+\frac{2\pi}{3}\right)=c\cos\left(\theta+\frac{4\pi}{3}\right)$, where $\theta=\frac{\pi}{9}$, then the angle between the vectors $a\hat{i}+b\hat{j}+c\hat{k}$ and $b\hat{i}+c\hat{j}+a\hat{k}$ is:

(A) $\frac{\pi}{9}$
(B) $\frac{2\pi}{3}$
(C) 0
(D) $\frac{\pi}{2}$

54. Let the latus rectum of the parabola $y^2=4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is:

(A) $4\sqrt{5}$
(B) $8\sqrt{5}$
(C) 8
(D) 12

55. Let R_1 and R_2 be two relations defined as follows:
$R_1 = \{(a,b) \in R^2 : a^2 + b^2 \in Q\}$ and
$R_2 = \{(a,b) \in R^2 : a^2 + b^2 \notin Q\}$, where Q is the set of all rational numbers. Then:

(A) Neither R_1 nor R_2 is transitive.
(B) R_1 is transitive but R_2 is not transitive.
(C) R_1 and R_2 are both transitive
(D) R_2 is transitive but R_1 is not transitive.

56. If the value of the integral $\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}} dx$ is $\frac{k}{6}$, then k is equal to:

(A) $2\sqrt{3} + \pi$
(B) $3\sqrt{2} - \pi$
(C) $2\sqrt{3} - \pi$
(D) $3\sqrt{2} + \pi$

57. If $x^3 dy + xy\,dx = x^2 dy + 2y\,dx; y(2) = e$ and $x > 1$, then y(4) is equal to:

(A) $\frac{\sqrt{e}}{2}$
(B) $\frac{1}{2} + \sqrt{e}$
(C) $\frac{3}{2}\sqrt{e}$
(D) $\frac{3}{2} + \sqrt{e}$

58. The probability that a randomly chosen 5 – digit number is made from exactly two digits is:

(A) $\frac{134}{10^4}$
(B) $\frac{121}{10^4}$
(C) $\frac{150}{10^4}$
(D) $\frac{135}{10^4}$

59. If the surface area of a cube is increasing at a rate of 3.6 cm^2/sec, retaining its shape; then the rate of change of its volume (in cm^3/sec), when the length of a side of the cube is 10 cm, is:

(A) 9
(B) 18
(C) 10
(D) 20

60. The set of all real values of λ for which the quadratic equations,
$(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0, 1) is:

(A) (0, 2)
(B) $(2, 4]$
(C) $(-3, -1)$
(D) $(1, 3]$

61. If a ΔABC have vertices A (–1, 7), B (–7, 1) and C (5, –5), then its orthocenter has coordinates:
(A) $\left(\frac{3}{5}, -\frac{3}{5}\right)$ (B) (–3, 3)
(C) (3, –3) (D) $\left(-\frac{3}{5}, \frac{3}{5}\right)$

62. Let p, q, r be three statements such that the truth value of $(p \wedge q) \rightarrow (\sim q \vee r)$ is F. Then the truth values of p, q,r are respectively:
(A) T, F, T (B) T, T, F
(C) F, T, F (D) T, T, T

63. The plane which bisects the line joining the points (4, –2, 3) and (2, 4, –1) at right angles also passes through the point:
(A) (0, –1, 1) (B) (4, 0, 1)
(C) (4, 0, –1) (D) (0, 1, –1)

64. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25} + \frac{y^2}{b^2} = 1(b < 5)$ and the hyperbola, $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$ respectively satisfying $e_1 e_2 = 1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to :
(A) (8, 10) (B) $\left(\frac{24}{5}, 10\right)$
(C) $\left(\frac{20}{3}, 12\right)$ (D) (8, 12)

65. Let A be a 3×3 matrix such that $\text{adj}\,A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$ and $B = \text{adj}(\text{adj}\,A)$.
If $|A| = \lambda$ and $\left|\left(B^{-1}\right)^T\right| = \mu$, then the ordered pair, $(|\lambda|.\mu)$ is equal to:
(A) $\left(9, \frac{1}{9}\right)$ (B) (3, 81)
(C) $\left(9, \frac{1}{81}\right)$ (D) $\left(3, \frac{1}{81}\right)$

66. If $\int \sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right)dx = A(x)\tan^{-1}\left(\sqrt{x}\right)+B(x)+C$, where C is a constant of integration, then the ordered pair $\left(A(x), B(x)\right)$ can be:

(A) $\left(x-1, -\sqrt{x}\right)$
(B) $\left(x-1, \sqrt{x}\right)$
(C) $\left(x+1, -\sqrt{x}\right)$
(D) $\left(x+1, \sqrt{x}\right)$

67. Let $x_i\,(1 \le i \le 10)$ be ten observations of a random variable X. If $\sum_{i=1}^{10}(x_i - p) = 3$ and $\sum_{i=1}^{10}(x_i - p)^2 = 9$ where $0 \ne p \in R$, then the standard deviation of these observations is:

(A) $\sqrt{\frac{3}{5}}$
(B) $\frac{4}{5}$
(C) $\frac{7}{10}$
(D) $\frac{9}{10}$

68. If the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is k, then 18k is equal to:

(A) 9
(B) 5
(C) 7
(D) 11

69. Suppose $f(x)$ is a polynomial of degree four, having critical points at –1, 0, 1. If $T = \left\{x \in R \,\middle|\, f(x) = f(0)\right\}$, then the sum of squares of all the elements of T is:

(A) 8
(B) 6
(C) 2
(D) 4

70. $\lim\limits_{x \to a} \frac{(a+2x)^{\frac{1}{3}} - (3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}} - (4x)^{\frac{1}{3}}}\ (a \ne 0)$ is equal to:

(A) $\left(\frac{2}{3}\right)^{\frac{4}{3}}$
(B) $\left(\frac{2}{9}\right)^{\frac{4}{3}}$
(C) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$
(D) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$

71. If the tangent to the curve, $y = e^x$ at a point $\left(c, e^c\right)$ and the normal to the parabola, $y^2 = 4x$ at the point (1, 2) intersect at the same point on the x – axis, then the value of c is ________.

72. The total number of 3 – digit numbers, whose sum of digits is 10, is ___________.

73. Let S be the set of all integer solutions, (x, y, z), of the system of equation
$x - 2y + 5z = 0$
$-2x + 4y + z = 0$
$-7x + 14y + 9z = 0$
such that $15 \le x^2 + y^2 + z^2 \le 150$. Then, the number of elements in the set S is equal to ______.

74. Let a plane P contain two lines $\vec{r} = \hat{i} + \lambda\left(\hat{i} + \hat{j}\right), \lambda \in R$ and $\vec{r} = -\hat{j} + \mu\left(\hat{j} - \hat{k}\right), \mu \in R$. If $Q(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn from the point M (1, 0, 1) to P, then $3(\alpha + \beta + \gamma)$ equals ___________ .

75. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4th A.M. is equal to 2nd G.M., then m is equal to ______________.

FIITJEE
Solutions to JEE (Main)-2020
PART –A (PHYSICS)

1. **B**

Sol. $\vec{B}$ is $\perp$ to $\vec{E}$ and direction of propagation of wave. Also, $vB_0 = E_0$

$$B_0 = \frac{E_0}{v} = E_0\sqrt{\mu_0 \in_0}$$

2. **A**

Sol. $\vec{P}_i = \vec{P}_f \Rightarrow 0.1 \times 20 = 2 \times v_x$

$\Rightarrow \quad v_x = 1$ m/s

$v_y = \sqrt{2gh} = \sqrt{2\times10\times1}$, KE = $\frac{1}{2}m\,(V_x^2 + V_y^2)$

$KE = \frac{1}{2}\times 2\times(1+20) = 21\,J$

3. **D**

Sol. $qV = \frac{1}{2}mv^2$

$\Rightarrow \quad v = \sqrt{\frac{2qV}{m}} \quad \Rightarrow \quad v \propto \sqrt{\frac{q}{m}}$

$\frac{v_H}{v_{He}} = \sqrt{\frac{1}{1}\times\frac{4}{1}} = \frac{2}{1}$

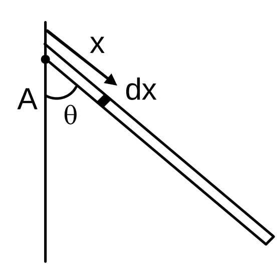
4. **C**

Sol. Torque of centrifugal force about A

$= \int d = \int [dm\,\omega^2(x\sin\theta)]\,(x\cos\theta)$

$= \int_0^{\ell} \frac{m}{\ell}dx\,\omega^2 x^2 \sin\theta\cos\theta$

$= \frac{m}{\ell}\omega^2\sin\theta\cos\theta\left[\frac{x^3}{3}\right]_0^{\ell} = \frac{m\omega^2\ell^2\sin\theta\cos\theta}{3}$



$\tau_{mg} = \tau_{centrifugal}$ (about A)

$mg\frac{\ell}{2}\sin\theta = \frac{m\omega^2\ell^2\sin\theta\cos\theta}{3}$

$\cos\theta = \frac{3g}{2\ell\omega^2}$

5. **C**

Sol. Solar constant = $\dfrac{\text{Energy}}{\text{Area} \times \text{Time}} = \dfrac{ML^2T^{-2}}{L^2 \times T} = ML^oT^{-3}$

6. **C**

Sol. Heat lost by steam = Heat gained by water and calorimeter.
m × 540 + m × 1 × (100 – 31) = 200
540 m + 69 m = 1200
$m = \dfrac{1200}{609} \approx 2$

7. **C**

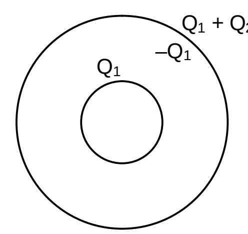
Sol. Multimeter will show deflection when current will flow to charge the capacitor.

8. **A**

Sol. Power = Number of photons emitted per sec × Energy of 1 photon
$P = n \times \dfrac{hc}{\lambda} \quad \Rightarrow \quad n \propto \lambda$

9. **C**

Sol. $\sigma_1 = \sigma_2$



$\dfrac{Q_1}{4\pi R^2} = \dfrac{Q_2 + Q_1}{4\pi(16R^2)}$
$\Rightarrow \quad Q_1 + Q_2 = 16\,Q_1$
$\Rightarrow \quad 15Q_1 = Q_2$

$V(R) - V(4R) = \left(\dfrac{KQ_1}{R} + \dfrac{KQ_2}{4R}\right) - \left(\dfrac{KQ_1}{4R} + \dfrac{KQ_2}{4R}\right)$
$= \dfrac{3KQ_1}{4R} = \dfrac{3Q_1}{16\pi \in R}$

10. **B**

Sol. $V_{max} = A'\,\omega' = A\omega$
$A'\sqrt{\dfrac{k}{m/2}} = A\sqrt{\dfrac{k}{m}}$
$A' = \dfrac{A}{\sqrt{2}}$

11. **C**

Sol. $\phi = \dfrac{2\pi}{\lambda} \times \Delta x = \dfrac{2\pi}{\lambda}\left[n_1L_1 - n_2L_2\right]$

12. **C**

Sol. $\dfrac{50-40}{300} = -k\left[\dfrac{50+40}{2} - 20\right]$

$$\frac{40-T}{300}-k\left[\frac{T+40}{2}-20\right]$$
$\Rightarrow \quad \frac{10}{40-T}=\frac{25}{T}\times 2$
$\Rightarrow \quad T = 200 - 5T$
$\Rightarrow \quad T = \frac{100}{3} \approx 33°C$

13. **A**

Sol. Band gap energy = $\frac{hc}{\lambda}=\frac{1240\ \text{eV nm}}{400\,\text{nm}}$ = 3.1 eV

14. **D**

Sol. $\Delta Q = nC_p\Delta T \;\Rightarrow\; 160 = nC_p\,50$
$\Delta Q = nC_v\Delta T \;\Rightarrow\; 240 = nC_v\,100$
$\Rightarrow \quad \frac{C_p}{C_v}=\frac{16}{5}\times\frac{10}{24}=\frac{4}{3}$
$\Rightarrow \quad 1+\frac{2}{f}=\frac{4}{3} \quad \Rightarrow f = 6$

15. **A**

Sol. Due to perfect diamagnetic sphere, effect of external field cannot be felt inside it.

16. **B**

Sol. $\text{Density}=\frac{M}{V}=\frac{1.67\times10^{-27}\times A}{\frac{4}{3}\pi\,(1.3)^3\times10^{-45}\times A} \approx 10^{17}$

17. **C**

Sol. $E=\frac{G\,m_{enc}}{r^2}=\frac{G}{r^2}\int_0^r \rho_0\left(1-\frac{r^2}{R^2}\right)4\pi r^2 dr$

$E=\frac{G}{r^2}4\pi\,\rho_0\left[\frac{r^3}{3}-\frac{r^5}{5R^2}\right]_0^r$

$= 4\pi G\rho_0\left[\frac{r}{3}-\frac{r^3}{5R^2}\right]$

For E to be max, $\frac{dE}{dr}=0$

$\Rightarrow \quad \frac{1}{3}-\frac{3r^2}{5R^2}=0$

$r=\sqrt{\frac{5}{9}}\,R$

18. **A**

Sol. $i = \frac{E}{R} = \frac{1}{R}\frac{d\phi}{dt} = \frac{a^2}{R}\frac{dB}{dt}$

$i = \frac{(7.5)^2 \times 10^{-4} \times \pi \times 4 \times 10^{-6}}{1.23 \times 10^{-8} \times 0.3} \times 0.032$

$= 0.61$ A

19. **B**

Sol. P = constant then $S = \sqrt{\frac{8P}{9m}}\, t^{3/2}$

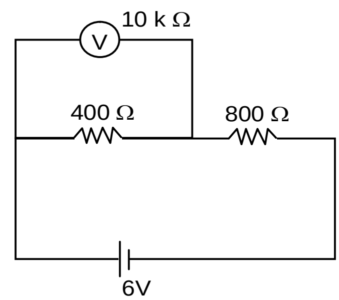
$S \propto t^{3/2}$

20. **C**

Sol. Parallel of 10 kΩ and 400 Ω = 384.61 Ω

$V_{400\,\Omega} = \frac{384.61}{384.61 + 800} \times 6$

$= 1.95$ V



21. **346.00**

Sol. Upward journey

$0 - v_0^2 = -2\left(\frac{g}{2} + \frac{mg\sqrt{3}}{2}\right)S$

$S = \frac{v_0^2}{g(1 + \mu\sqrt{3})}$

Downward journey

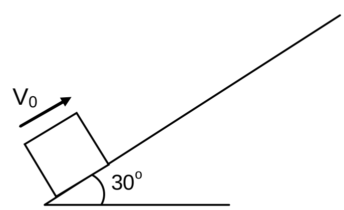
$\frac{v_0^2}{4} - 0 = 2\left(\frac{g}{2} - \frac{4g\sqrt{3}}{2}\right)\frac{v_0^2}{g(1 + \mu\sqrt{3})}$

$\frac{1}{4} = \frac{1 - \sqrt{3}\,\mu}{1 + \sqrt{3}\,\mu}$

$\frac{5}{3} = \frac{2}{2\sqrt{3}\,\mu} \quad \Rightarrow \quad \mu = \frac{\sqrt{3}}{5} = \frac{1732}{5}$

$\Rightarrow \quad \mu = 0.346$

$\Rightarrow$ I = 346.00



22. **20.00**

Sol. τ = B I N A sin θ

$\Rightarrow \quad 1.5 = B \times 0.5 \times 500 \times 3 \times 10^{-4} \times 1$

$\Rightarrow \quad B = 20.00$

23. **8791**

Sol. $Q_1 = mL = 100\text{ gm} \times \dfrac{80\text{ cal}}{\text{gm}} = 8000\text{ cal}$

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \Rightarrow Q_2 = \frac{8000 \times 300}{273} = 8791\text{ cal}$$

24. **25.00**

Sol. $I = 0 + ma^2 + m\left(\dfrac{a}{2}\right)^2$

$$I = ma^2 + \frac{ma^2}{4} = \frac{5ma^2}{4}$$

$\Rightarrow$ N = 25.00

25. **1.00**

Sol. $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$$-\frac{1}{v^2}\frac{dV}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$$

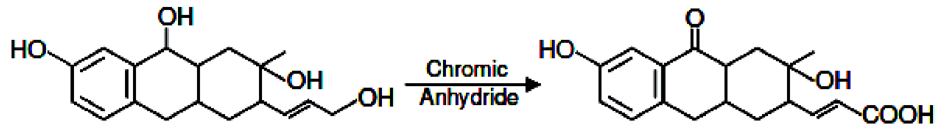
$$\frac{dv}{dt} = \frac{-v^2}{u^2}\frac{du}{dt}$$

$$V_I = -\left(\frac{10}{30}\right)^2 \times 9 = 1\text{ cm/s}$$

PART – B (CHEMISTRY)

26. C

Sol.

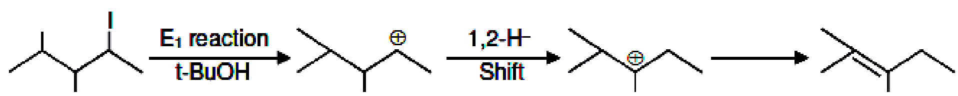


Product 'P'

3° Alcohol gives Red colour with ceric ammonium nitrate

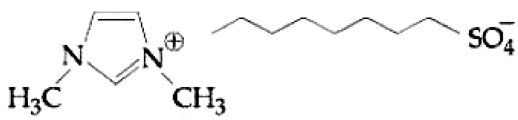
27. B

Sol.



28. A

Sol. excess water to liquid



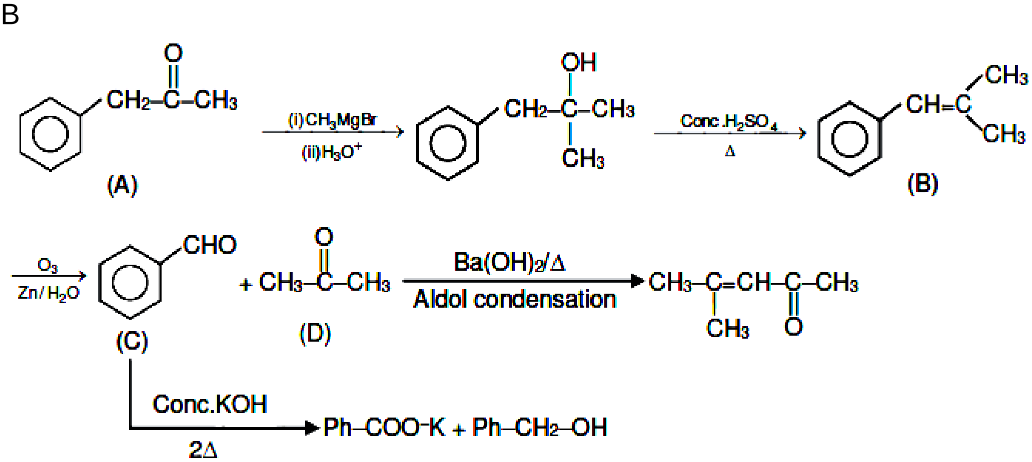
Due to presence of hydrophobic chain it forms micelle

29. D

Sol. As difference in 3rd and 4th ionisation energies is high so atom contains 3 valence electrons.

30. B

Sol.



31. B

Sol. For n = 1 value of ℓ = 0, 1, 2

For n = 2 value of ℓ = 0, 1, 2, 3

So, according to n + l rule the filling order of subshells will be:
1s 1p 2s 1d 2p 3s 2d 3p 4s ……..
(1) 1st noble gas will have configuration $1s^2\ 1p^6$ so atomic number will be 8.
(2) 1st alkali metal will have electronic configuration $\Rightarrow 1s^1 \Rightarrow$ (Z = 1)
(3) Electronic configuration of C (Z = 6) $\Rightarrow 1s^2\ 1p^4$
(4) Z = 13, Electronic configuration = $1s^2\ 1p^6\ 2s^2\ 1d^3$
So it will not have half-filled electronic configuration

32. C
Sol. For H_2O_2

$$\text{Molarity} = \frac{\text{Volume strength}}{11.2} = \frac{5.6}{11.2} = 0.5\ \text{M}$$

$$\text{Molarity} = \frac{\%(w/w) \times 10 \times d}{\text{GMM}}$$

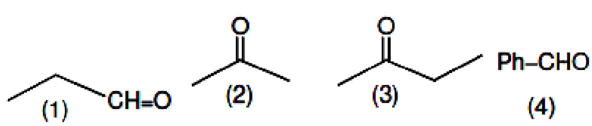
$$0.5 = \frac{\%(w/w) \times 10 \times d}{34}$$

$$\%(w/w) = \frac{0.5 \times 34}{10} = 1.7$$

33. D
Sol. At equivalence point pH is 7 and pH increases with addition of NaOH so correct graph is (1).

34. B
Sol. (b) It is harmful for trees and plants
(c) It causes breathing problem in human being and animals

35. B
Sol. Rate of NAR α – I – M on substate



1 > 4 > 2 > 3

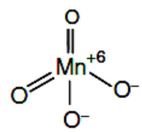
36. A
Sol. Conc. H_2SO_4 acts as dehydrating agent.
Molar mass of given complex = 266.5 g/mol.
On treating with conc. H_2SO_4 the mass
lost by the complex = $= \frac{13.5}{100}(266.5) \approx 36\text{g}$ = 2 moles of H_2O
Formula of the complex = $[Cr(H_2O)_4Cl_2]Cl.\ 2H_2O$

37. C

Sol.

Manganate

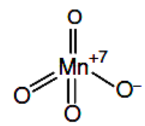
MnO_4^{2-}



Paramagnetic, green in colour,
Tetrahedral & contains $p\pi$-$d\pi$ bond

Permanganate

$MnO_4^{\ominus}$



Diamagnetic, purple in colour,
Tetrahedral & contains $p\pi$-$d\pi$ bond

38. B

Sol. S_N2 reaction depend upon –I, –M effect on substrate. On increase –I, –M, effect rate of S_N2 reaction increase.

39. B

Sol.

$$\boxed{P_{gas} = \frac{n_{gas}RT}{V}}$$

as n, T & V constant So

$P_{H_2} = P_{O_2} = P_{He} = 2\,atm$

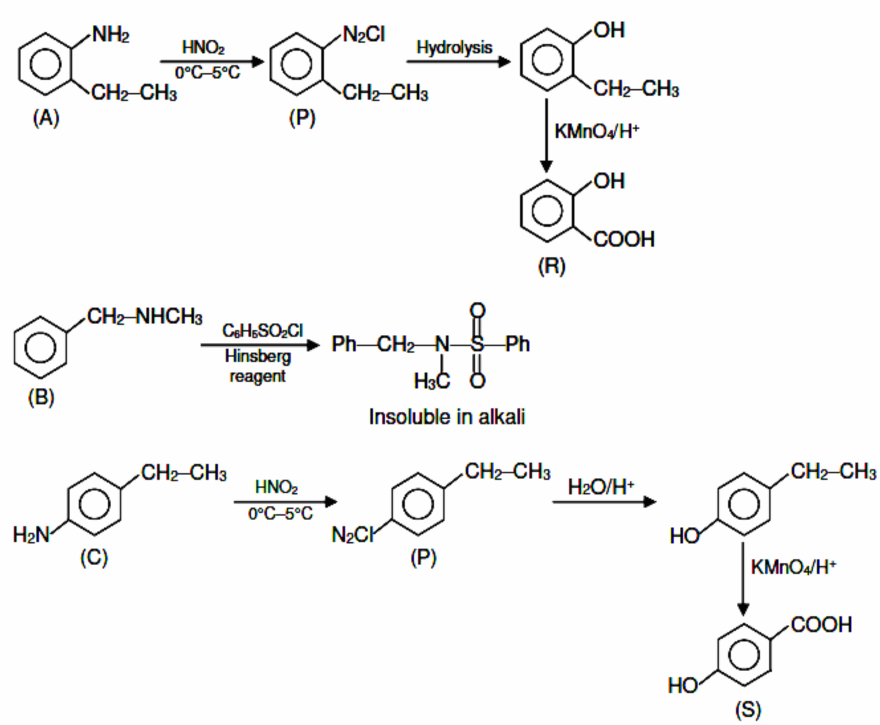
So, $P_{Total} = P_{H_2} + P_{O_2} + P_{He} = 6\ atm$

40. D

Sol. Charge / radius ratio of Be and Al is same because of diagonal relationship. Remaining statements are correct.

41. B

Sol.



42. C

Sol. Due to inter molecular H-Bonding in B, than A, B is more soluble and having more B.P point than A.

43. D

Sol. For a given reaction, $\text{rate} = -\frac{1}{2}\frac{dn_A}{dt} = -\frac{1}{3}\frac{dn_B}{dt} = -\frac{2}{3}\frac{dn_C}{dt}$

$$\text{rate} = \frac{dn_A}{dt} = \frac{2}{3}\frac{d\,n_B}{dt} = \frac{4}{3}\frac{dn_C}{dt}$$

44. A

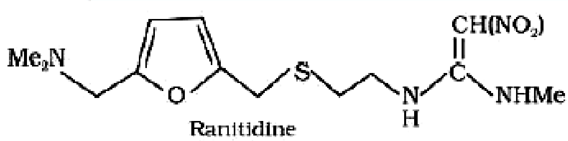
Sol. $\left[Ru(en)_3\right]Cl_2 \Rightarrow Ru^{2+} = 4d^6 = t_{2g}^6,\ e_g^0$

$[Fe(H_2O)_6]^{2+} \Rightarrow Fe^{2+} = 3d^6 = t_{2g}^4,\ e_g^2$

So, correct answer is (A).

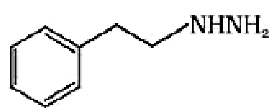
45. D

Sol.



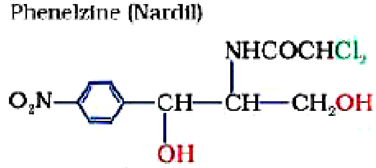
Ranitidine

$\rightarrow$ Antacid



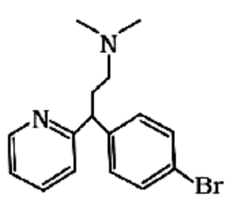
$\rightarrow$ Antidepressant drugs

Phenelzine (Nardil)



$\rightarrow$ Antibiotics

Chloramphenicol



$\rightarrow$ Antihistamine

Brompheniramine
(Dimetapp, Dimetane)

46. 10

Sol. Phosphinic acid is hypo phosphorous acid (H_3PO_2).

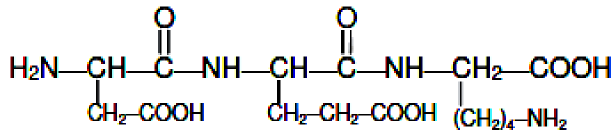
$NaOH + H_3PO_2 \longrightarrow NaH_2PO_2 + H_2O$

For neutrization

$(N_1V_1)_{acid} = (N_2V_2)_{base}$

$0.1 \times 10 = 0.1 \times (V_{mL})_{NaOH}$

47. 5
Sol. Asp – Glu – Lys pripeptide is:



No. of CO group = 5

48. 177
Sol. For isotonic solution
$i_1C_1 = i_2C_2$ {for protein i = 1}
$C_1 = C_2$

$$\frac{0.73\times1000}{M_A\times250}=\frac{1.65}{M_B\times1}$$

$$\frac{M_A}{M_B}=\frac{0.73\times4}{1.65}=1.77=177\times10^{-2}$$

49. 60
Sol. According to Faraday law
$W = ZIt \times \eta$ Where η = efficiency

or $W=\frac{E}{96500}\times I\times t\times\eta$

Putting values

$$.104=\frac{\left(52/3\right)\times2\times8\times60\times\eta}{96500}\Rightarrow\eta=0.6$$

So percentage efficiency = 60%

50. 25
Sol.

Number of mole of X $=\frac{6.022\times10^{22}}{6.022\times10^{23}}=\frac{10}{\text{Molar mass of X}}$

So molar mass of X = 100g

Molarity $=\frac{5}{100\times2}=0.025M$

Ans. = 0.025 M

$M = 25\times10^{-3}$

So P = 25

PART–C (MATHEMATICS)

51. C

Sol. $488 = \frac{n}{2}\left[2\left(\frac{100}{5}\right)+(n-1)\left(\frac{2}{5}\right)\right]$

$488 = \frac{n}{2}(101-n)$

$\Rightarrow \quad n^2 - 101n + 2440 = 0$

$\Rightarrow \quad n = 61 \quad \text{or} \quad 40$

For $n = 40 \quad \Rightarrow \quad T_n > 0$

For $n = 61 \quad \Rightarrow \quad T_n < 0$

$T_n = \frac{100}{5} + (61-1)\left(-\frac{2}{5}\right) = -4$

52. A

Sol. $|z_1 - 1| = \text{Re}(z_1)$ Let $z_1 x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$(x_1 - 1)^2 + y_1^2 = x_1^2$

$y_1^2 - 2x_1 + 1 = 0$ ....(1)

$|z_2 - 1| = \text{Re}(z_2)$

$(x_2 - 1)^2 + y_2^2 = x_2^2$

$y_2^2 - 2x_2 + 1 = 0$ ....(2)

$y_1^2 - y_2^2 - 2(x_1 - x_2) = 0$

$(y_1 - y_2)(y_1 + y_2) = 2(x_1 - x_2)$

$y_1 + y_2 = 2\left(\frac{x_1 - x_2}{y_1 - y_2}\right)$ ....(3)

$\arg(z_1 - z_2) = \frac{\pi}{6}$

$\tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{6}$

$\frac{y_1 - y_2}{x_1 - x_2} = \frac{1}{\sqrt{3}}$ ....(4)

$\therefore \quad y_1 + y_2 = 2\sqrt{3}$

$\Rightarrow \quad \text{Im}(z_1 + z_2) = 2\sqrt{3}$

53. D

Sol. $a\cos\theta = b\cos\left(\theta+\frac{2\pi}{3}\right) = c\cos\left(\theta+\frac{4\pi}{3}\right) = k$

$$a = \frac{k}{\cos\theta},\ b = \frac{k}{\cos\left(\theta+\frac{2\pi}{3}\right)},\ c = \frac{k}{\cos\left(\theta+\frac{4\pi}{3}\right)}$$

$$ab+bc+ca = k^2\frac{\left[\cos\left(\theta+\frac{4\pi}{3}\right)+\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)\right]}{\cos\left(\theta+\frac{4\pi}{3}\right)\cos\theta\cos\left(\theta+\frac{2\pi}{3}\right)}$$

$$= k^2\left[\frac{\cos\theta+2\cos(\theta+\pi).\cos\left(\frac{\pi}{3}\right)}{\cos\theta.\cos\left(\theta+\frac{2\pi}{3}\right).\cos\left(\theta+\frac{4\pi}{3}\right)}\right]$$

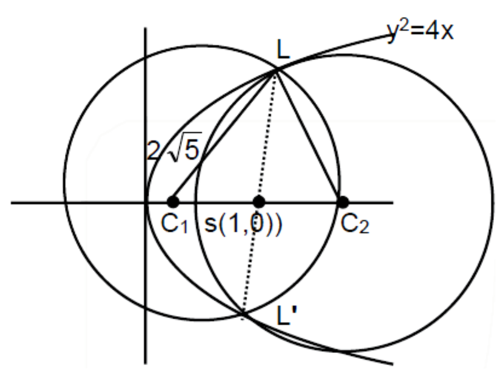
$$= k^2\left[\frac{\cos\theta-2\cos\theta.\frac{1}{2}}{\cos\theta.\cos\left(\theta+\frac{2\pi}{3}\right).\cos\left(\theta+\frac{4\pi}{3}\right)}\right] = 0$$

$$\cos\phi = \frac{\left(a\hat{i}+b\hat{j}+c\hat{k}\right).\left(b\hat{i}+c\hat{j}+a\hat{k}\right)}{\sqrt{a^2+b^2+c^2}.\sqrt{b^2+c^2+a^2}} = ab+bc+ca = 0$$

$$\phi = \frac{\pi}{2}$$

54. C

Sol. $C_1C_2 = 2C_1S = 2\sqrt{20-4} = 8$



55. A

Sol. For R_1 let $a = 1+\sqrt{2},\ b = 1-\sqrt{2},\ c = 8^{1/4}$

$aR_1b \quad \Rightarrow \quad a^2+b^2 = \left(1+\sqrt{2}\right)^2+\left(1-\sqrt{2}\right)^2 = 6 \in Q$

$aR_1c \quad \Rightarrow \quad b^2+c^2 = \left(1-\sqrt{2}\right)^2+\left(8^{1/4}\right)^2 = 3 \in Q$

$aR_1c \quad \Rightarrow \quad a^2+c^2=\left(1+\sqrt{2}\right)^2+\left(8^{1/4}\right)^2=3+4\sqrt{2}\notin Q$

$\therefore$ R_1 is not transitive.

For R_2 let $a=1+\sqrt{2},\ b=\sqrt{2},\ c=1-\sqrt{2}$

$aR_2B \quad \Rightarrow \quad a^2+b^2=\left(1+\sqrt{2}\right)^2+\left(\sqrt{2}\right)^2=5+2\sqrt{2}\notin Q$

$bR_2b \quad \Rightarrow \quad b^2+c^2=\left(\sqrt{2}\right)^2+\left(1-\sqrt{2}\right)^2=5-2\sqrt{2}\notin Q$

$aR_2c \quad \Rightarrow \quad a^2+c^2=\left(1+\sqrt{2}\right)^2+\left(1-\sqrt{2}\right)^2=6\in Q$

$\therefore$ R^2 is not transitive.

56. C

Sol. $\frac{k}{6}=\int_0^{\frac{1}{2}}\frac{x^2}{\left(1-x^2\right)^{3/2}}dx \qquad x=\sin\theta; dx=\cos\theta\,d\theta$

$$\Rightarrow \frac{k}{6}=\int_0^{\frac{\pi}{6}}\frac{\sin^2\theta}{\left(1-\sin^2\theta\right)^{3/2}}.\cos\theta\,d\theta$$

$$\Rightarrow \frac{k}{6}=\int_0^{\frac{\pi}{6}}\frac{\sin^2\theta}{\cos^3\theta}.\cos\theta\,d\theta$$

$$\Rightarrow \frac{k}{6}=\int_0^{\frac{\pi}{6}}\tan^2\theta\,d\theta=\int_0^{\frac{\pi}{6}}\left(\sec^2\theta-1\right)d\theta$$

$$\Rightarrow \frac{k}{6}=\left(\tan\theta-\theta\right)_0^{\pi/6}=\left(\frac{1}{\sqrt{3}}-\frac{\pi}{6}\right)=\frac{2\sqrt{3}-\pi}{6}$$

$$\Rightarrow k=2\sqrt{3}-\pi$$

57. C

Sol. $x^3dy+xydx=2ydx+x^2dy$

$$\Rightarrow \left(x^3-x^2\right)dy=\left(2-x\right)y\,dx$$

$$\Rightarrow \frac{dy}{y}=\frac{2-x}{x^2\left(x-1\right)}dx$$

$$\Rightarrow \int\frac{dy}{y}=\int\frac{2-x}{x^2\left(x-1\right)}dx \qquad \ldots\ldots\ldots\ldots\ldots..(i)$$

Let $\frac{2-x}{x^2\left(x-1\right)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1}$

$\Rightarrow\ 2-x=A(x-1)+B(x-1)+Cx^2$

$\Rightarrow\ C=1, B=-2$ and $A=-1$

$\Rightarrow\ \int\frac{dy}{y}=\int\left\{\frac{-1}{x}-\frac{2}{x^2}+\frac{1}{x-1}\right\}dx$

$\Rightarrow \ell n y=-\ell n x+\frac{2}{x}+\ell n|x-1|+C$

$\therefore\ y(2)=e$

$\Rightarrow\ 1=-\ell n 2+1+0+C$

$\Rightarrow\ C=\ell n 2$

$\Rightarrow\ \ell n y=-\ell n x+\frac{2}{x}+\ell n|x-1|+\ell n 2$

at $x=4$

$\Rightarrow\ \ell n y(4)=-\ell n 4+\frac{1}{2}+\ell n 3+\ell n 2$

$\Rightarrow\ \ell n y(4)=\ell n\left(\frac{3}{2}\right)+\frac{1}{2}=\ell n\left(\frac{3}{2}e^{1/2}\right)$

$\Rightarrow\ y(4)=\frac{3}{2}e^{1/2}$

58. D

Sol. Total $=9\left(10^4\right)$

Fav. Way $={}^9C_2\left(2^5-2\right)+{}^9C_1\left(2^4-1\right)=36(30)+9(15)=1080+135$

Probability $=\frac{36\times30+9\times15}{9\times10^4}=\frac{4\times30+15}{10^4}=\frac{135}{10^4}$

59. A

Sol. $S=6a^2\Rightarrow\frac{ds}{dt}=12a.\frac{da}{dt}=3.6$

$\Rightarrow 12(10)\frac{da}{dt}=3.6$

$\Rightarrow\frac{da}{dt}=0.03$

$V=a^3\Rightarrow\frac{dv}{dt}=3a^2.\frac{da}{dt}$

$=3(10)^2.\left(\frac{3}{100}\right)=9$

60. D

Sol. $f(0)f(1)\le 0$

$\Rightarrow \quad 2\left(\lambda^2+1-4\lambda+2\right)\le 0 \quad \Rightarrow \quad 2\left(\lambda^2-4\lambda+3\right)\le 0$

$(\lambda-1)(\lambda-3)\le 0$

$\Rightarrow \quad \lambda[1,3]$

But at $\lambda = 1$, both roots are 1 so $\lambda \ne 1$

61. B

Sol. $m_{BC} = \dfrac{6}{-12} = -\dfrac{1}{2}$

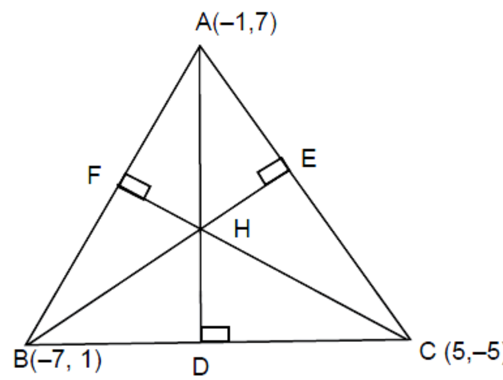
$\therefore$ Equation of AD is $y-7 = 2(x+1)$

$y = 2x+9$ ……………..(1)

$m_{AC} = \dfrac{12}{-6} = -2$

$\therefore$ Equation of BE is $y-1 = \dfrac{1}{2}(x+7)y$

$y = \dfrac{x}{2}+\dfrac{9}{2}$ ………………(2)



by (1) and (2)

$2x+9 = \dfrac{x+9}{2}$

$\Rightarrow 4x+18 = x+9$

$\Rightarrow 3x = 9 \Rightarrow x = -3$

$\therefore y = 3$

62. B

Sol. $(p \wedge q)$ should be TRUE and $(\sim q \vee r)$ should be FALSE.

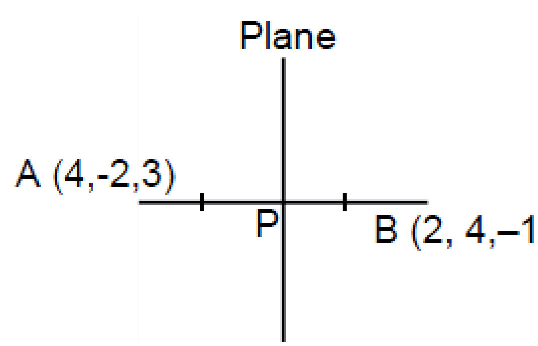
63. C

Sol. Mid point $P \equiv (3,1,1)$

Normal of plane is along the line AB.
D.R,'s of normal = 4 –2, –2 –4, 3 –1 (–1) = 2, –6, 4,
= 1, –3, 2

Plane $\rightarrow 1(x-3)-3(y-1)+2(z-1)=0$

$\Rightarrow \quad x-3y+2z-2=0$



64. A

Sol. $e_1 = \sqrt{1-\dfrac{b^2}{25}}; \ e_2 = \sqrt{1+\dfrac{b^2}{16}}$

$e_1 e_2 = 1$

$$\Rightarrow (e_1e_2)^2 = 1$$

$$\Rightarrow \left(1-\frac{b^2}{25}\right)\left(1+\frac{b^2}{16}\right) = 1$$

$$\Rightarrow 1+\frac{b^2}{16}-\frac{b^2}{25}-\frac{b^4}{25\times16} = 1$$

$$\Rightarrow \frac{9}{16.25}b^2 - \frac{b^4}{25.16} = 0$$

$$\Rightarrow \quad b^2 = 9$$

$$e_1 = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$$

$$e_2 = \sqrt{1+\frac{9}{16}} = \frac{5}{4}$$

$$\alpha = 2(5)(e_1) = 8$$

$$\beta = 2(4)(e_2) = 10$$

$$(\alpha,\beta) = (8,10)$$

65. D

Sol. $|\text{adj}\, A| = |A|^2 = 9$

$$\Rightarrow |A| = \pm 3 = \lambda \qquad \Rightarrow \qquad |\lambda| = 3$$

$$\Rightarrow |B| = |\text{adj}\, A|^2 = 81$$

$$\Rightarrow \left|\left(B^{-1}\right)^T\right| = \left|B^{-1}\right| = |B|^{-1} = \frac{1}{|B|} = \frac{1}{81} = \mu$$

66. C

Sol. $I = \int \sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right)dx$

$$\int \underset{\text{I}}{\tan^{-1}\left(\sqrt{x}\right)}\underset{\text{II}}{dx} = x\tan^{-1}\sqrt{x} - \int \frac{1}{1+x}.\frac{1}{2\sqrt{x}}.x\,dx + C = x\tan^{-1}\sqrt{x} - \frac{1}{2}\int \frac{t.2t.dt}{1+t^2} + C\left(x = t^2\right)$$

$$= x\tan^{-1}\sqrt{x} - \int \frac{t^2}{1+t^2}dt + C = x\tan^{-1}\sqrt{x} - t + \tan^{-1}t + C = x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C$$

$$= (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C \qquad \Rightarrow \qquad (Ax) = x+1 \Rightarrow B(x) = -\sqrt{x}$$

67. D

Sol. $$\text{S.D.} = \sqrt{\frac{\sum_{i=1}^{10}(x_i - p)^2}{10} - \left(\frac{\sum_{i=1}^{10}(x_i - p)}{10}\right)^2}$$

$$\sqrt{\frac{9}{10} - \left(\frac{3}{10}\right)^2} = \frac{9}{10}$$

68. C

Sol. $$T_{r+1} = {}^9C_r \left(\frac{3x^2}{2}\right)^{9-r} \left(-\frac{1}{3x}\right)^r$$

$$= {}^9C_r \left(\frac{3}{5}\right)^{9-r} \left(-\frac{1}{3}\right)^r x^{18-3r}$$ for the term independent of x put r = 6

$$\Rightarrow T_7 = {}^9C_6 \left(\frac{3}{2}\right)^3 \left(-\frac{1}{3}\right)^6$$

$$= {}^9C_3 \left(\frac{1}{6}\right)^3 = \frac{9\times8\times7}{3\times2\times1}\left(\frac{1}{6}\right)^3 = \left(\frac{7}{18}\right)$$

69. D

Sol. $$f'(x) = k.x(x+1)(x-1) = k\left(x^3 - x\right)$$

$$\Rightarrow \quad f(x) = k\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + C$$

$$\Rightarrow \quad f(0) = C$$

$$\Rightarrow \quad f(x) = f(0)$$

$$\Rightarrow \quad k\frac{\left(x^4 - 2x^2\right)}{4} + C = C$$

$$\Rightarrow \quad x^2\left(x^2 - 2\right) = 0$$

$$\Rightarrow \quad x = \left\{0, \sqrt{2}, -\sqrt{2}\right\}$$

70. D

Sol. $$\lim_{x\to a} \frac{\frac{1}{3}(a+2x)^{-2/3}.2 - \frac{1}{3}.(3x)^{-2/3}.3}{\frac{1}{3}(3a+x)^{-2/3}.-\frac{1}{3}(4x)^{-2/3}.4}$$

$$=\frac{\frac{1}{3}(3a)^{-2/3}.(2-3)}{\frac{1}{3}(4a)^{-2/3}.(1-4)}=\frac{3^{-2/3}}{4^{-2/3}}.\frac{1}{3}$$

$$=\frac{2^{4/3}}{9^{1/3}}.\frac{1}{3}=\frac{2}{3}.\left(\frac{2}{9}\right)^{1/3}$$

71. 04.00

Sol. For (1, 2) of $y^2=4x \Rightarrow t=1, a=1$

normal $\Rightarrow tx+y=2at+at^3$

$\Rightarrow x+y=3$ intersect x – axis at (3, 0)

$y=e^x \quad \Rightarrow \frac{dy}{dx}=e^x$

tangent $\Rightarrow y-e^c=e^c(x-c)$

at $(3,0) \Rightarrow 0-e^c=e^c(3-c) \Rightarrow c=4$

72. 54.00

Sol. Let xyz be the three digit number

$x+y+z=10,\ x\le 1,\ y\ge 0,\ z\ge 0$

$x-1=t \quad \Rightarrow \quad x=1+t \quad x-1\ge 0$

$t\ge 0$

$t+y+z=10-1$

$t+y+z=9, \quad 0\le t, z,\ z\le 9$

total number of non negative integral solution $={}^{9+3-1}C_{3-1}={}^{11}C_2=\frac{11.10}{2}=55$

But for t = 9, x = 10, so required number or integers = 55 – 1 = 54

73. 08.00

Sol. $x-2y+5z=0$ ……………………(i)

$-2x+4y+z=0$ ……………………(ii)

$-7x+14y+9z=0$ ……………………(iii)

$2\times(i)+(ii) \Rightarrow z=0$

$\Rightarrow \quad x=2y$

$\Rightarrow \quad 15\le x^2+y^2+z^2\le 150$

$\Rightarrow \quad 15\le 4y^2+y^2\le 150$

$\Rightarrow \quad 3\le y^2\le 30$

$\Rightarrow \quad y=\pm 2, \pm 3, \pm 4, \pm 5$

$\Rightarrow$ 8 solutions.

74. 05.00

Sol. Normal of plane $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$

$\vec{n} = -\hat{i} + \hat{j} + \hat{k}$

D.R.'s = –1, 1, 1

Plane $\Rightarrow -1(x-1)+1(y-0)+1(z-0)=0$

$\Rightarrow x-y-z-1=0$

If (x, y, z) is foot of perpendicular of M (1, 0, 1) on the plane then

$$\Rightarrow \quad \frac{x-1}{1} = \frac{y-0}{-1} = \frac{z-1}{-1} = \frac{-(1-0-1-1)}{3}$$

$$x = \frac{4}{3},\ y = -\frac{1}{3},\ z = \frac{2}{3}$$

$$\alpha+\beta+\gamma = \frac{4}{3}-\frac{1}{3}+\frac{2}{3} = \frac{5}{3}$$

75. 39.00

Sol. $3, A_1, A_2, A_3, ...A_m, 243$

$$d = \frac{243-3}{m+1} = \frac{240}{m+1}$$

$3, G_1, G_2, G_3, 243$

$$r = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = (81)^{1/4} = 3$$

$G_2 = A_4$

$$\Rightarrow \quad 3(3)^2 = 3 + 4\left(\frac{240}{m+1}\right)$$

$$\Rightarrow \quad 27 = 3 + \frac{960}{m+1}$$

$\Rightarrow \quad m+1 = 40$

$\Rightarrow \quad m = 39$