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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2021

COMPUTER BASED TEST (CBT)

Memory Based Questions & Solutions

Date: 22 July, 2021 (SHIFT-2) | TIME : (3.00 p.m. to 6.00 p.m)

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SUBJECT: PHYSICS

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PART : PHYSICS

1. If length of simple pendulum is made $\frac{1}{16}$ times, then time period of simple pendulum changes by .....

(1) $\frac{1}{3}$ times (2) $\frac{1}{4}$ times (3) $\frac{1}{5}$ times (4) $\frac{1}{6}$ times

Ans. **(2)**

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$T' = 2\pi\sqrt{\frac{\ell}{16g}} = \frac{T}{4}$

2. Ring, solid sphere and solid cylinder rolls an inclined plane without slipping then order of velocity at lowest point of inclined will be :

(1) $V_{Ring} > V_{Solid\ sphere} > V_{Solid\ cylinder}$ (2) $V_{Solid\ sphere} > V_{Solid\ cylinder} > V_{Ring}$

Ans. (2)

Sol. $mgh = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mv^2$

$V = \sqrt{\frac{2gh}{1+\frac{I_{cm}}{mR^2}}}$

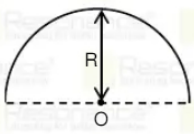
I↑ V↓

3. In given ac circuit correct phase diagram will be :

Ans. (2)

Sol. In pure capacitive circuit current leads with voltage.

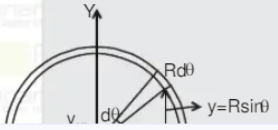
4. Distance of center of mass from point O is given by $\frac{\lambda R}{\pi}$ for uniform semi-circular ring final value of λ.



(1) 1 (2) 2 (3) 3 (4) $\frac{1}{2}$

Ans. (2)

Sol.



To find y_{cm} we use $y_{cm} = \frac{1}{M}\int dmy$ .....(i)

Here for dm we consider an elemental arc of the ring at an angle θ from the x-direction of angular width dθ. If radius of the ring is R then its y coordinate will be R sinθ, here dm is given as

$dm = \frac{M}{\pi R} \times R\, d\theta$

So from equation ....(i), we have

$$y_{cm} = \frac{1}{M}\int_0^{\pi}\frac{M}{\pi R}Rd\theta\ (R\sin\theta) = \frac{R}{\pi}\int_0^{\pi}\sin\theta d\theta$$

$$y_{cm} = \frac{2R}{\pi} \qquad .....(ii)$$

$\therefore\ \lambda = 2$

5. Find the ratio of De-Broglie wavelength of an electron and a proton when accelerated through same potential difference ?

(1) $\sqrt{1803}$ (2) $\sqrt{1621}$ (3) $\sqrt{1417}$ (4) $\sqrt{1230}$

Sol. $\lambda = \frac{}{p} = \frac{}{\sqrt{2mqV}}$

$$\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p \times 1}{m_e \times e}} = \sqrt{1803}$$

6. For an iron rod temperature is increased by 10°C. Given $\alpha = 10^{-5}$ per °C, Y = 10^{11} N/m^2, area cross section A = 10^{-2} m^2. Find energy stored per unit length

(1) 5 J/m (2) 10 J/m (3) 15 J/m (4) 20 J/m

Ans. **(1)**

Sol. $U = \frac{1}{2} \times$ stress $\times$ strain $\times$ volume

$U = \frac{1}{2} \times$ stress $\times$ strain $\times$ volume $\times A\ell$

$\frac{U}{\ell} = \frac{1}{2} \times$ stress $\times$ strain $\times$ volume $\times$ A

$= \frac{1}{2} \times Y \times (\text{strain})^2 \times A$

$= \frac{1}{2} \times Y\left(\frac{\Delta\ell}{\ell}\right)^2 \times A$

$= \frac{1}{2} Y\alpha^2 \Delta t^2 A$

$= \frac{1}{2} \times 10^{11} \times 10^{-10} \times 10 \times 10 \times 10^{-2}$

= 5 Joule/m

7. A gun of mass 4 kg fire a bullet of mass 4g with muzzle velocity equal to 50 m/s. Find the velocity of bullet.

(1) 48.59 m/s (2) 49.95 m/s (3) 45.59 m/s (4) 40.59 m/s

Ans. **(2)**

Sol.

Initial momentum = final momentum

$0 = m_G V_G + M_B V_b$

$\Rightarrow \quad V_G = -\frac{V_b}{1000} \qquad ....(1)$

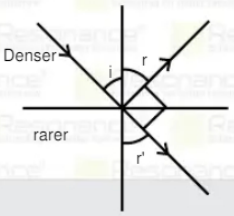
$V_{bG} = V_b - V_G$

$\Rightarrow \quad 50 = V_B - V_G \qquad ....(2)$

$\Rightarrow \quad 50 = \frac{+v_b}{1000} + V_b$

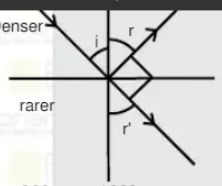
$\Rightarrow \quad V_b = \frac{50 \times 1000}{1001} \quad \Rightarrow \quad V_b = 49.95 \text{ m/s}$

8. A ray incident from denser medium to rarer medium. If angle between reflected and refracted ray is 90^0 then angle of reflection r and angle of refraction r' will be respectively:



(1) i, $\sin^{-1}$ (sin i) (2) $\sin^{-1}$ (cos i), i (3) i, $\sin^{-1}$ (cos i) (4) i, $\sin^{-1}$ (tan i)

Ans. **(3)**



r' + 90° + r = 180°
r' + 90° + I = 180
r' = 90 – I = $\sin^{-1}$(cos i)

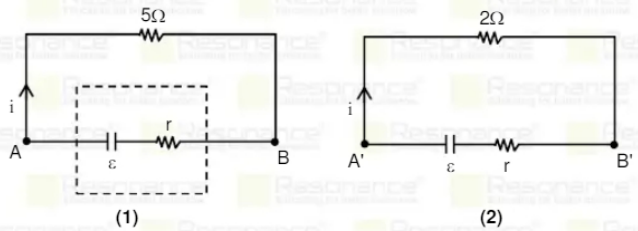
9. The average kinetic energy of a molecule of a monoatomic gas is :

(1) 5/2 kT (2) 3/2 kT (3) 7/2 kT (4) 1/2 kT

Ans. **(2)**

10. A cell first connected across 5 Ω resistance develops a potential difference of 1.25 V across it. Same cell again connected across 2Ω resistance develops 1V potential difference across it find the emf of cell :

Sol.



(1) $I = \frac{1.25}{5} = 0.25\ A$

$V_{AB} = 5V = \varepsilon - ir$

$1.25 = \varepsilon - 0.25\ r$ ...(1)

(2) $I = \frac{1}{2} = 0.5\ A$

$V_{A'B'} = 1 = \varepsilon - 0.5r$ ...(2)

Solving (1) & (2)

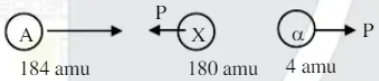
$\varepsilon = 15\ v$

then find the Kinetic energy of α -particle. Assume that there is no γ emission

(1) 5.0 MeV (2) 5.38 MeV (3) 3.60 Mev (4) 2.10 MeV

Ans. **(2)**

Sol.



$k_\alpha = \left(\frac{A-4}{A}\right)Q = 5.38\ MeV$

12. If $\vec{A} = \hat{i} + \hat{j} + \hat{k}$

$\vec{B} = \hat{i} + \hat{j}$

Find the projection of $\vec{A}$ on $\vec{B}$?

Ans. **(1)**

Sol. Projection of $\vec{A}$ on $\vec{B} = \vec{A}.\vec{B}$

$= (\hat{i} + \hat{j} + \hat{k}).\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right) = \frac{1+1}{\sqrt{2}} = \sqrt{2}$

13. In a room of temperature 25°C, a body in initial temperature of 75°C cools down to 65°C in 5 min. After 5 more minutes, the final temperature of body will be?

(1) 60°C (2) 58°C (3) 57°C (4) 55°C

Ans. (3)

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Sol. $\frac{\Delta T}{t} = K\left(\frac{T_1 + T_2}{2} - T_0\right)$

$\frac{75-65}{5} = K\left(\frac{75+65}{2} - 25\right)$ ...(1)

$\frac{65-T}{5} = K\left(\frac{T+65}{2} - 25\right)$ ...(2)

Eq(2)/Eq(1)

$\frac{T+65}{\quad} - 25$

$$\frac{65-T}{10}=\frac{T+15}{90}$$

$90 \times 65 - 90\,T = 10\,T + 10 \times 15$

$100\,T = 90 \times 65 - 15 \times 10$

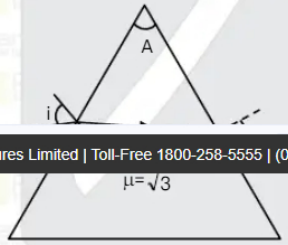
$T = 57°C$

14. A ray is incident on prism of refractive index $\mu = \sqrt{3}$. If angle of incident is twice of angle of refraction when deviation of ray is minimum. Then find the prism angle.

(1) 30 ° (2) 60° (3) 45° (4) 90°

Ans. **(2)**

Sol.



$$A = \frac{i}{2}+\frac{i}{2}=i$$

$$1\sin i = \mu \sin\frac{i}{2}$$

$$2\sin\frac{i}{2}\cos\frac{i}{2}=\sqrt{3}\sin\frac{i}{2}$$

$$\cos\frac{i}{2}=\frac{\sqrt{3}}{2}$$

$$\frac{i}{2}=30°$$

$i = 60°$

$\therefore \quad A = i = 60°$

15. Find height of antenna if coverage of signals from Antenna is 150 km and radius of earth is 6400 km. Also find the total population covered by antenna signal, if population density is 200 people/km^2

(1) $\frac{625}{124}\times10^3$ m ; 6.057×10^6 people (2) $\frac{225}{128}\times10^3$ m ; 12.057×10^8 people

(3) $\frac{125}{84}\times10^3$ m ; 8.057×10^9 people (4) $\frac{725}{72}\times10^3$ m ; 10.057×10^7 people

Ans. **(2)**

Sol.

$d = 150$ km

height of Antena = ?

$d = \sqrt{2Rh}$

$$h = \frac{d^2}{2R}=\frac{150\times150\times10^6}{2\times6400\times10^3}=\frac{225}{128}\times10^{8-5}=\frac{225}{128}\times10^3\,m$$

Population covered $\Rightarrow 2\pi Rh \times$ density

$= 2\pi \times 6400 \times 150 \times 200 = 12.057 \times 10^8$

16. A magnetic needle is placed vertically but in a wrong plane, which is at an angle θ with the magnetic meridian. If apparent dip in this wrong plane is δ', then find the real dip angle.

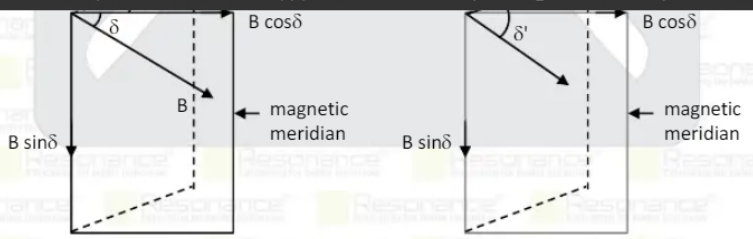
(1) $\tan^{-1}(\tan\delta' \sec\theta)$ (2) $\tan^{-1}(\tan\delta' \cos\theta)$ (3) $\tan^{-1}(\tan\delta' \sin\theta)$ (4) $\tan^{-1}(\tan\delta' \operatorname{cosec}\theta)$

Ans. **(2)**

Sol.

$(B\cos\delta)\cos\theta$

wrong plane wrong plane



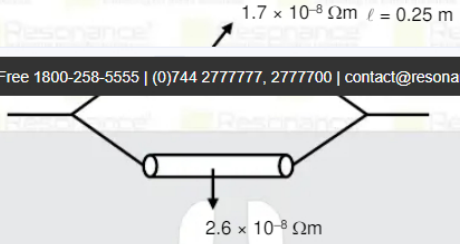
component in magnetic meridian component in wrong plane

$$\tan\delta' = \frac{B\sin\delta}{(B\cos\delta)\cos\theta}$$

$$\Rightarrow \quad \tan\delta = \tan\delta'\cos\theta$$

$$\delta = \tan^{-1}(\tan\delta'\cos\theta)$$

17. Two rods of length 0.25 m and area 3 mm^2 are connected as shown in figure & their resistivities are 1.7×10^{-8} Ωm & 2.6×10^{-8} Ωm. Find the equivalent resistance ?



(1) 0.85 mΩ (2) 0.95 mΩ (3) 0.80 mΩ (4) 0.75 mΩ

Ans. (1)

Sol. $$R_{eq} = \frac{R_1R_2}{R_1+R_2} = \frac{\rho_1\frac{\ell}{A}\rho_2\left(\frac{\ell}{A}\right)}{\rho_1\left(\frac{\ell}{A}\right)+\rho_2\left(\frac{\ell}{A}\right)}$$

$$= \frac{\ell}{A}\left(\frac{\rho_1\rho_2}{\rho_1+\rho_2}\right) = \frac{0.25}{3\times10^{-6}}\left(\frac{1.7\times10^{-8}\times2.6\times10^{-8}}{1.7\times10^{-8}+2.6\times10^{-8}}\right)$$

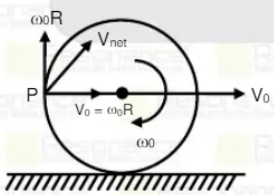
$$= 0.085 \times 10^{-2} = 0.85\ m\Omega$$

which is in the same horizontal level as the centre of mass.

(1) V_0 (2) $2V_0$ (3) $\sqrt{2}V_0$ (4) zero

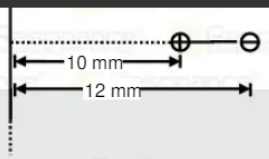
Ans. (3)

Sol.



$$(V_p)_{net} = \sqrt{V_0^2 + (\omega_0 R)^2} = \sqrt{V_0^2 + V_0^2} = \sqrt{2}V_0$$

19. A dipole is kept near the infinite linear charge of density 3×10^{-6} C/m along its perpendicular direction as shown in figure. The dipole is experiencing a force of 4 N then find charge of dipole.



(1) 2.14 μC (2) 2.24 μC (3) 4.44 μC (4) 3.14 μC

Ans. (3)

Sol. Let charge be Q

$$\text{Net force} = \frac{2k\lambda Q}{r_1} + \frac{2k\lambda(-Q)}{r_2}$$

$$4N = 2k\lambda Q\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$4N = 2 \times 9 \times 10^9 \times 3 \times 10^{-6} Q\left[\frac{1}{10} - \frac{1}{12}\right] \times 10^3$$

$$Q = 4.44 \times 10^{-6} \text{ C} = 4.44\ \mu\text{C}$$

20. A photo diode activeness when photon of wavelength 612 nm incident on it. Then depletion layer voltage of photodiode will be : (Given $hc = 1224\ \text{ev} - \text{nm}$)

(1) 2 volt (2) 1 volt (3) 4 volt (4) 3 volt

Ans. (2)

Sol. $E = \frac{hc}{\lambda} = eV$

$$V = \frac{hc}{\lambda e}$$

$$= \frac{1224\ \text{ev} - \text{nm}}{e \times 612\text{nm}}$$

$= 2$ volt

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21. $\vec{p}$ is vector perpendicular to both $\vec{a} = \hat{i} + \hat{j}$ & $\vec{b} = \hat{j} + \hat{k}$ vector. q is a vector perpendicular to both $\vec{b} = \hat{j} + \hat{k}$

(1) $\pi - \cos^{-1}\left(\frac{1}{3}\right)$ (2) $\pi - \cos^{-1}\left(\frac{2}{3}\right)$ (3) $\pi - \cos^{-1}\left(\frac{3}{4}\right)$ (4) $\pi - \cos^{-1}\left(\frac{1}{2}\right)$

Ans. **(1)**

Sol. $\vec{p} = \vec{a} \times \vec{b}$

$$\begin{bmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \hat{i} - \hat{j} + \hat{k}$$

$\vec{q} = \vec{b} \times \vec{c}$

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$= -\hat{i} + \hat{j} + \hat{k}$$

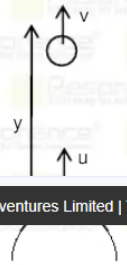
$$\theta = \pi - \cos^{-1}\left(\frac{1}{3}\right)$$

22. An object is projected from earth surface to reach infinity. Find expression for time required to reach y height

(1) $\frac{\frac{2}{3}\left[(R-y)^{\frac{3}{2}} + R^{\frac{3}{2}}\right]}{\sqrt{Gm}}$ (2) $\frac{\frac{2}{3}\left[(R+y)^{\frac{3}{2}} - R^{\frac{3}{2}}\right]}{\sqrt{2Gm}}$ (3) $\frac{\frac{2}{3}\left[(R+y)^{\frac{3}{2}} + R^{\frac{3}{2}}\right]}{\sqrt{2Gm}}$ (4) $\frac{\frac{2}{3}\left[(R-y)^{\frac{3}{2}} + R^{\frac{3}{2}}\right]}{\sqrt{2Gm}}$

Ans. (2)

Sol.



$$\Rightarrow v = \sqrt{\frac{2GM}{R+y}}$$

$$\Rightarrow \frac{dy}{dt} = \sqrt{\frac{2Gm}{R+y}}$$

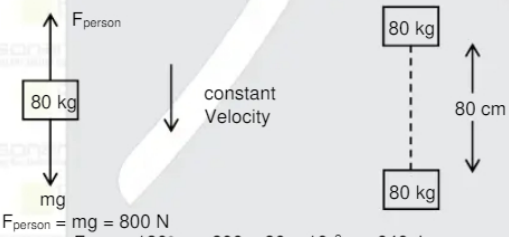
$$\Rightarrow \int_0^y \sqrt{R+y}\,dy = \sqrt{2Gm}\int_0^t dt \qquad \Rightarrow \left[\frac{2(R+y)^{\frac{3}{2}}}{3}\right]^y = \sqrt{2Gm}\ t$$

$$\Rightarrow \frac{2}{3}\left[\left((R+y)^{\frac{3}{2}}-R^{\frac{3}{2}}\right)\right]=\sqrt{2Gm}\,.t \qquad \Rightarrow t=\frac{\frac{2}{3}\left[(R+y)^{\frac{3}{2}}-R^{\frac{3}{2}}\right]}{\sqrt{2Gm}}$$

23. A man is standing on horizontal platform carrying a heavy box of mass 80 kg. Suddenly he lowered heavy box with constant velocity downward by 80 cm find the work done by person?

Ans. **(1)**

Sol.



$F_{person} = mg = 800$ N

$w_{person} = F_p s \cos 180° = -800 \times 80 \times 10^{-2} = -640$ J

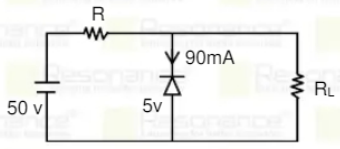
24. If Intensity of sunlight at a point is 92 W/m^2, then find Amplitude of magnetic field at this point? (Given $\mu_0 = 4\pi \times 10^{-7}$)

(1) 100 T (2) 200 T (3) 352 T (4) 500 T

Ans. (3)

$$B_0 = \sqrt{\frac{2\mu_0 I}{C}} = \sqrt{\frac{2\times 4\pi\times 10^{-7}\times 92}{3\times 10^8}} = 351.5 \approx 352$$

25. If current through diode is 90 mA Find the maximum value of R?



(1) 200 Ω (2) 500 Ω (3) 300 Ω (4) 400 Ω

Ans. (2)

Sol. $\frac{45}{R} \geq 90\text{mA}$

$R \leq \frac{45}{90} \times 10^3$

$R \leq 500\ \Omega$

if at t = 0, x = 0

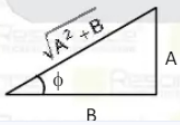
displacement $x(t) = C\cos(\omega t - \phi)$ then find value of C and ϕ.

(1) $\sqrt{A^2+B^2}, \frac{\pi}{2}$ (2) $\sqrt{A^2+B^2}, 0$ (3) $A^2+B^2, \frac{\pi}{2}$ (4) $A^2 + B^2, 0$

Ans. **(1)**

Sol. $x(t) = A\sin\omega t + B\cos\omega t$

$= \sqrt{A^2+B^2}\cos(\omega t - \phi)$



$0 = \cos(\phi)$

$\therefore \quad \phi = \frac{\pi}{2}$

$C = \sqrt{A^2+B^2}$ and $\phi = \frac{\pi}{2}$

27. **Statement-1:** On increase in temperature ferromagnetic material converts into paramagnetic material.

Statement-2: At high temperature, random ness of domains ferromagnetic material increases

(1) Statement 1 & 2 both are true

(2) Statement-1 & 2 both are true statement -2 is correct explant of statement-1

(3) Statement-1 is false Statement -2 is true

(4) Statement-2 is true Statement-1 is false.

Ans. (1)

28. Match the column

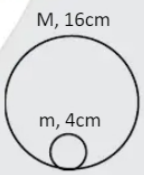
(i)	$\omega C > \frac{1}{\omega L}$	(a)	Current lag behind EMF
(ii)	$\omega C = \frac{1}{\omega L}$	(b)	EMF lag behind current
(iii)	$\omega C < \frac{1}{\omega L}$	(c)	Same phase

(1) (i) – a, (ii)– c, (iii) – b, (iv) – c (2) (i) – c, (ii)– a, (iii) – b, (iv) – c

(3) (i) – a, (ii)– c, (iii) – c, (iv) – b (4) (i) – c, (ii)– c, (iii) – b, (iv) – a

Ans. (1)

29. Find the time period of oscillation of ring of mass m while ring of mass M always remains at rest ,if r = 4 cm and R = 16 cm.



(1) 0.7 second (2) 0.8 second (3) 1 second (4) 0.9 second

Ans. **(3)**

Sol.

$mg\sin\theta - f = ma$

$f \times r = mr^2 \times \frac{a}{r}$ $\left[\sin\theta = \theta = \frac{x}{R-r}\right]$

$f = ma$

$\Rightarrow \quad mg\sin\theta = 2ma$

$\Rightarrow$ $mg\sin\theta = 2ma$

$\Rightarrow \quad a = \dfrac{gx}{2(R-r)} \quad \Rightarrow \quad T = 2\pi\sqrt{\dfrac{2(R-r)}{g}} = 2\pi\sqrt{\dfrac{0.12}{5}} \approx 1 \text{ second}$

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JEE (Main)

2021

COMPUTER BASED TEST (CBT)

~~Memory Based Questions & Solutions~~

Date: 22 July, 2021 (SHIFT-2) | TIME : (3.00 p.m. to 6.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: CHEMISTRY

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1. Find total number of electrons in p-orbitals of vanadium (Z = 23)

Ans. (12)

Sol. $_{23}V = 1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^3\,4s^2$

2. Identify the correct sequence of hybridization of following species

$NH_4^{\oplus}$, $NO_2^{\oplus}$, SF_4, IF_5

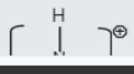
(1) sp^3, sp, sp^3d, sp^3d^2

(2) sp, sp^2, sp^3, sp^3d

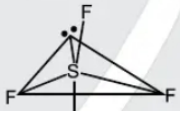
(3) sp^3, sp, sp^3d, sp^3d

(4) sp^3, sp^2, sp^3d, sp^3d^2

Ans. (1)



$NO_2^{\oplus}$ $O=\overset{\oplus}{N}=O$ sp



SF_4 sp^3d

IF_5 sp^3d^2

3. Identify the incorrect statement regarding Mendeleev.

(2) Mendeleev proposed the periodic table when structure of atoms were unknown

(3) Atomic number 101 element named after Mendeleev

(4) Mendeleev developed accurate barometer.

Ans. (2)

4. Identify the correct set which is paramagnetic and coloured.

(1) Cu^{2+}, Sc^{+}, Cr^{3+} (2) Mn^{7+}, Cr^{3+}, Hg^{2+}

(3) Cu^{+} , Sc^{3+} , Co^{+} (4) Mn^{7+}, Cu^{+}, Cr^{3+}

Ans. **(1)**

Sol.

Ion	**No. of unpaired e^-**
Cu^{2+}	1

This set is "paramagnetic & coloured"

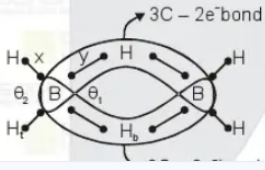
5. Identify the correct statement regarding diborane (B_2H_6)

(1) Hybridisation of boron is sp^2 (2) It contain one 3 center- 2 electron bond.

(3) It is planer molecule (4) $NaBH_4$ on oxidation with I_2 give B_2H_6

Ans. **(4)**

Sol. $2NaBH_4 + I_2 \xrightarrow{\text{ether}} B_2H_6 + 2NaI + H_2\uparrow$



6. K_P for the reaction $N_2O_4(g) \rightleftharpoons 2NO_2(g)$ at 288 k is 47·9, then value of K_C is -

[Report your answer to nearest integer]

[Given R = 0.083 bar lit / mole K]

Ans. **(2)**

Sol. $K_P = K_C(RT)^{\Delta n_g}$

$47·9 = K_C\,(0·083 \times 288)^1$

$K_C = 2$

7. How many total number of unpaired electrons are present in $[Co(NH_3)_6]Cl_2$ and $[Co(NH_3)_6]Cl_3$

Ans. (1)

Sol. Complex

(i) $[Co(NH_3)_6]Cl_2 \Rightarrow Co^{2+} = 3d^7$ unpaired electron = 1

Total unpaired electrons = 1

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8. Isotopes of hydrogen which emits low energy β^- particle with half life greater than 12 year is :

(1) Tritium and Deuterim (2) Deuterium

(3*) Tritium (4) Protium

Sol. Only tritium is radioactive and emits low energy β particles ($t_{½}$, 12.33 years)

9. The concentration of glucose ($C_6H_{12}O_6$) in blood is 0.72 gram/lit, the molarity of glucose is $[x] \times 10^{-3}$ M, then value of x is

Ans. (4)

Sol. $$M = \frac{W_{solute}}{M_{solute} \times V_{soln}(inlit.)}$$

$$= \frac{0.72}{180}$$

$$= 0.004 = 4 \times 10^{-3}$$

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(1) K_2SO_4 (2) KI (3) $C_6H_{12}O_6$ (4) $Al_2(SO_4)_3$

Ans. **(4)**

Sol. $\Delta T_f = i K_f m$

Greater the i value lower will be freezing point

11. N_2O_5 dissociate according to 1st order kinetic as $(2N_2O_{5(g)} \rightarrow 4NO_{2(g)} + O_{2(g)}$. kinetics initial concentration of N_2O_5 is $2{\cdot}4 \times 10^{-2}$ M and concentration of N_2O_5 after 1 hour is $1{\cdot}6 \times 10^{-2}$ M, then the rate constant k in min^{-1} for this dissociation is $[x] \times 10^{-6}$ min^{-1}, then x is.

Ans. 346

Sol. $2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$

Initial $2{\cdot}4 \times 10^{-2}$ M

After 1 hour $1{\cdot}6 \times 10^{-2}$ M

$$t = \frac{1}{\quad} \ln\left(\frac{a}{\quad}\right)$$

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$$k = \frac{2{\cdot}303}{2 \times 60} \log\left(\frac{2{\cdot}4 \times 10^{-2}}{1{\cdot}6 \times 10^{-2}}\right)$$

$$k = \frac{2{\cdot}303}{2 \times 60} \log\left(\frac{3}{2}\right)$$

$$= \frac{2{\cdot}303}{2 \times 60} \times [0{\cdot}48 - 0{\cdot}30]$$

$$= 0{\cdot}00346$$

$$= 346 \times 10^{-6} \text{ min}^{-1}$$

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(ii) $\underset{0.45M}{Ag^+}(aq) + e^- \longrightarrow \underset{0.5M}{Ag}$ $\quad E^\circ = 0.80\ V$

[Given log 2.5 = 0.4] $\dfrac{2.303RT}{F} = 0.06$

Report your answer as $[E_{cell}] \times 10^{-3}$.

Ans. (448)

Sol. $E^\circ_{cell} = (E^\circ_{RP})_C - (E^\circ_{RP})_A$

$= 0.80 - 0.34 = 0.46V]$

$$E_{cell} = E^\circ_{cell} - \frac{0.06}{2}\log\frac{[Cu^{2+}]}{[Ag^+]^2} = 0.46 - \frac{0.06}{2}\log\left\{\frac{0.5}{(0.45)^2}\right\}$$

$$= 0.46 - \frac{0.06}{2}\log 2.5 = 0.46 - \frac{0.06}{2}\times 2.5$$

14. Match column-I with Column-II

Column-I	**Column-II**
(a) Li	(i) soluble in organinc compound
(b) Na	(ii) outer electronic configuration is $6s^2$
(c) Ca	(iii) oxalate is not soluble in aqueous solution
(d) Ba	(iv) form strong monobasic compound

(1*) a–i, b–iv, c-iii, d–ii (2) a–i, b–ii, c-iii, d–iv

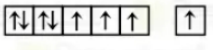
(3) a–iii, b–iv, c-i, d–ii (4) a–i, b–iv, c-iii, d–ii

Ans. (1)

15. Find the sum of magnetic moment (spin only) of following ion Co^+, Zn^{2+}, V^{5+}

[Report your answer to nearest integer]

Sol. $_{27}Co^+ = [_{18}Ar]3d^7\ 4s^1$



Unpaired electron = 4

$_{30}Zn^{2+} = [Ar]3d^{10}$ unpaired electron = 0

$_{23}V^{5+} = [_{18}Ar]3d^0$ unpaired electron = 0

So $\mu = \sqrt{n(n+2)}\ BM$

$= \sqrt{24}\ BM$

$= 4.89\ BM$

16. Which of the following have strong reducing power

(1) PH_3 (2) BiH_3 (3) AsH_3 (4) SbH_3

Sol. NH_3

PH_3

AsH_3

SbH_3

BiH_3

As we move down the group reducing power is increase.

17. When $AgNO_3$ solution is added to KI, the sol produced is

(1) KI/NO_3^- (2*) AgI/Ag^+ (3) AgI/I^- (4) $AgNO_3/NO_3^-$

Sol. $AgNO_3 + KI \rightarrow AgI \mid Ag^+$

[Report your answer to nearest integer]

Ans. 2

Sol. $C(graphite) + O_2 \rightarrow CO_2(g)\ \Delta H = -2.48$ kJ mole $\frac{10}{12}$ mole

Total heat released $= 2.48 \times \frac{10}{12} = 1.90$ kJ

19. 10 gram Benzene (GMM = 78) on methylation give 9.2 gram of Toluene (GMM = 92), then percentage yield of process is :

Ans. 78.00

Sol. $\underset{\frac{10}{78}\text{mole}}{C_6H_6} \xrightarrow{\text{Methylation}} \underset{\frac{10}{78}\text{mole}}{C_6H_5CH_3}$

(Theoretical) $\frac{10}{78}$

$$\% \text{ yield} = \frac{W_{actual}}{W_{theoritical}} \times 100$$

$$= \left[\frac{9.2}{10 \times 92} \times 78\right] \times 100 = 78\%$$

20. The total number of isomers of square planar complex $[MFCl(SCN)(NO_2)]$ is

(1) 12 (2) 6 (3) 8 (4) 4

Ans. (1)

Sol. SCN/NO_2 – 3 arrangements

NCS/NO_2 – 3 arrangements

SCN/ONO – 3 arrangements

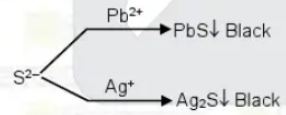
NCS/ONO – 3 arrangements

21. Consider sulphide ion [S^{2-} ion] as a soft base. Which of the following ion will form sulphide **[36 T]**

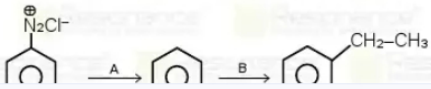
(1) Pb^{2+}, Ag^{+} (2) Ag^{+}, Mg^{2+} (3) Al^{3+} , Ag^{+} (4) Al^{3+}, Mg^{2+}

Ans. (3)

Sol.



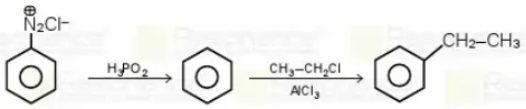
22. In the following seuqnce of reactions identify A & B respectively : **[OC, Aromatic, XII, M]**



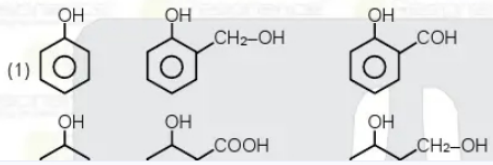
(3) $CH_3–CH_2–OH$ H_3PO_2 (4) $CH_3–CH_2–Cl$ H_3PO_2

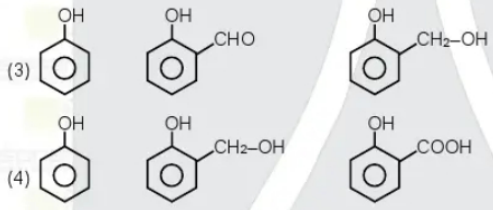
Ans. (1)

Sol.



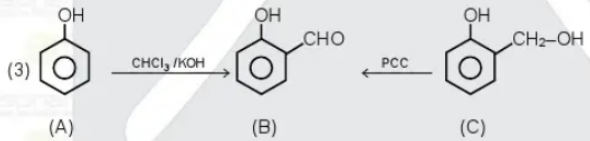
23. A(C_6H_6O) gives dark green colouration with $FeCl_3$. A on reaction with $CHCl_3$, KOH gives B. B can also be prepared from C by PCC. The correct option for A, B and C is :





Ans. (3)

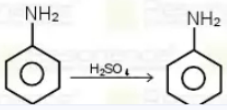
Sol.

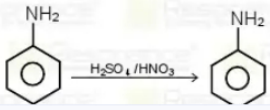


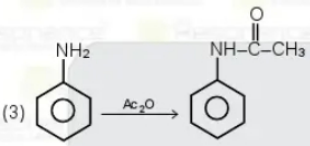
Ans. (3)

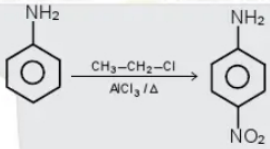
Sol.  has no conjugation between π-bond and lone-pair hence there will be no resonance in this compound.

25. Which of the following reacton is not possible :

(1) 

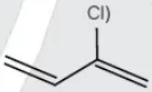
(2) 

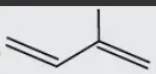
(3) 

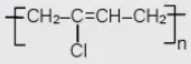
(4) 

Sol. Friedel-craft alkylation is not passible in aniline due to formation of highly deactivated cationic salt.

26. Match the following :

(a) Chloroprene (p) 

(c) Neoprene (r) 

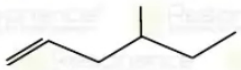
(d) Acrylonitrile (s) 

Ans. (1*) (a) → (p), (b) → (r), (c) → (s) , (d) → (q) (2) (a) → (r), (b) → (s), (c) → (p) , (d) → (q)
(3) (a) → (r), (b) → (p), (c) → (q) , (d) → (s) (4) (a) → (q), (b) → (r), (c) → (p) , (d) → (s)

Sol. NCERT

27. Which of the following does not show stereoisomerism

(1) 

(2) 

(3) 

(4) 

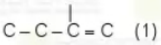
Ans. (4)

28. Total acyclic number of structures including geometrical of pentene is

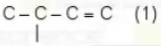
Ans. 6

Sol. C–C–C–C=C (1)

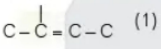
C–C–C=C–C (2)



(1)



(1)



(1)

29. Thiamin & pyridoxine vitamin are respectively :

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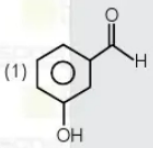
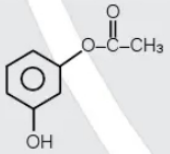
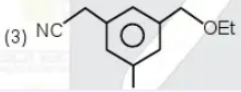
Ans. (1)

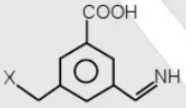
Sol. NCERT

30. Which of the following give tertiary alcohol with excess Grignard reagent (CH_3MgBr)

(1) 

(2) 

(3) 

(4) 

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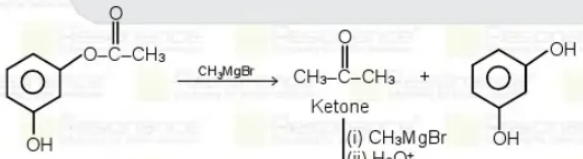
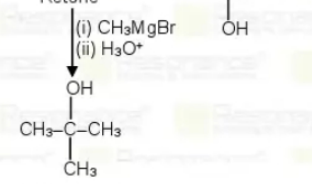
Ans. (2)

Sol. 



(tertiory Alcohol)

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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2021

COMPUTER BASED TEST (CBT)

Memory Based Questions & Solutions

Date: 22 July, 2021 (SHIFT-2) | TIME : (3.00 p.m. to 6.00 p.m)

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1 If $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and B be a 3×3 matrix whose entries are {1,2,3,4,5}, then the number of possible matrices B such that AB = BA are :

(1) 240 (2) 320 (3) 120 (4) 100

Ans. **(3)**

Sol. $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Let $B = \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$

$\Rightarrow \begin{bmatrix} p & q & r \\ a & b & c \\ x & y & z \end{bmatrix} = \begin{bmatrix} b & a & c \\ q & p & r \\ y & x & z \end{bmatrix}$

$\Rightarrow p = b$, $a = q$, $r = c$, $x = y$, & $z = z$

Hence number of such matrices are 5! = 120

2 The number of all possible numbers less than 10000 that can be formed using the digits 0, 2, 4, 6, 8

(without repetition) are :

Ans. **165**

Sol. 1 digit numbers = 5

2 digit numbers = $\underline{4}.\underline{4} = 16$

3 digit numbers = $\underline{4}.\underline{4}\,\underline{3} = 48$

4 digit numbers = $\underline{4}.\underline{4}\,\underline{3}\,\underline{2} = 96$

Total = 5 + 16 + 48 + 96 =165

intersection of lines $x - 2y = 4$ and $2x - y = 5$ also lies inside the circle, then the value of c lies in :

(1) $81 < c < 156$ (2) $100 < c < 156$ (3) $81 < c < 150$ (4) $100 < c < 150$

Ans. **(2)**

Sol. Intersection point of $2x - y = 5$ and $x - 2y = 4$ is $(2, -1)$

So, $(2, -1)$ lies indie the circle $\Rightarrow S_1 < 0$

$36(2)^2 + 36(-1)^2 - 108(2) + 120(-1) + c < 0$

$c < 156$ .....(i)

$\because$ circle $36x^2 + 36y^2 - 108x + 120y + c = 0$ neither touches nor cuts the co-ordinate axis so

$g^2 - c < 0 \Rightarrow \left(\frac{-3}{2}\right)^2 - \frac{c}{36} < 0 \Rightarrow c > 81$.....(ii)

and $f^2 - c < 0 \Rightarrow \left(\frac{5}{3}\right)^2 - \frac{c}{36} < 0 \Rightarrow c > 100$ ....(iii)

From (i), (ii) and (iii)

$100 < c < 156$

4. If line $2x + y = k$, $(k < 0)$ is a tangent to both the curves $x^2 - y^2 = 3$ and $y^2 = \alpha x$, then the value of α is

Ans. **24.00**

Sol. Given slope of line (m) = – 2

slope form of tangent to the curve $x^2 - y^2 = 3$ is $y = mx + \sqrt{a^2m^2 - b^2}$

On comparing, with the equation $2x + y = k$, $(k < 0) \Rightarrow k = -3$

Now, slope form of tangent to the parabola $y^2 = \alpha x$ is $y = mx + \frac{\alpha}{4m}$

But m = – 2 so

$y = -2x + \frac{\alpha}{4(-2)} \Rightarrow -3 = \frac{\alpha}{4 \times (-2)}$

$\alpha = 24$

5. The number of all possible values of $n \in \{1,2,3,\_\_\_\_ 100\}$ which satisfy the condition $11^n > 10^n + 9^n$ is:

Ans. (96)

Sol. Let $11^n > 10^n + 9^n$ $\quad n \in \{1,2,3,\_\_\_\_ 100\}$

$\Rightarrow 11^n - 9^n > 10^n$

$\Rightarrow (10+1)^n - (10-1)^n > 10^n$

$\Rightarrow \frac{1}{5}[{}^nC_1 10^n + {}^nC_3 10^{n-2} + {}^nC_5 10^{n-4} + ...] > 10^n$

$\Rightarrow \frac{1}{5}[{}^nC_1 + {}^nC_3 10^{-2} + {}^nC_5 10^{-4} + ...] > 1$

Clearly the above inequality is true for $n > 5$

For n = 4 we have $\frac{1}{5}\left[4+\frac{4}{10^2}\right]=\frac{4}{5}\left(\frac{101}{100}\right)<1$, Rejected

Hence, number of such $n\in\{1,2,3,\_\_\_\_100\}$ is equal to 96

6. If an AP, $S_{10} = 530$ & $S_5 = 140$ (where S_n denotes the sum of first n terms of an AP), then the value of $S_{20} - S_6$ is

(1) 1562 (2) 1862 (3) 1762 (4) 1662

Ans. **(2)**

Sol. $S_{10} = 530$

$\frac{10}{2}[2a+9d]=530$

$2a + 9d = 106$ ... (1)

$S_5 = 140$

$\frac{5}{2}[2a+4d]=140$

$5d = 50$

$d = 10$

$a = 8$

Now,

$S_{20} - S_6 =$

$10[2a + 19d] - 3[2a + 5d]$

$14a + 175d$

$14 \times 8 + (175)10 = 1862$

7. The value of r for which the term independent of x in the expansion of $\left(2x^r+\frac{1}{x^2}\right)^{10}$ is 180 is:

(1) 7 (2) 8 (3) 9 (4) 10

Ans. (2)

Sol. $T_{k+1} = {}^{10}C_k\,(2x^r)^{10-k}\,(x)^{-2k} \Rightarrow {}^{10}C_k\,2^{(10-k)}.\,x^{10r-rk-2k}$

Now, $10r - rk - 2k = 0 \Rightarrow r = \frac{2k}{10-k}$

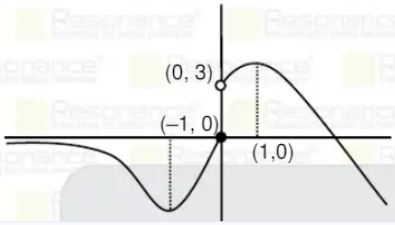
$r = \frac{2\times 8}{10-8} = 8$

8. If $f(x)=\begin{cases}-\frac{4}{3}x^3+2x^2+3 ; & x>0\\ 3xe^x ; & x\le 0\end{cases}$ then interval in which f(x) is increasing, is :

(1) $\left(-1,\frac{3}{2}\right)$ (2) (0, 1) (3) $\left(\frac{-3}{2},1\right)$ (4) $\left(\frac{1}{2},2\right)$

Ans. (B)

Sol. $f'(x)=\begin{cases}-4x^2+4x & , \ x>0\\ 3(x.e^x+e^x) & , \ x\le 0\end{cases}$



9. If $z^2+3\bar{z}=0$ has n solutions, then the value of $\sum_{k=0}^{\infty}\frac{1}{n^k}$ is :

(1) $\frac{3}{4}$ (2) $\frac{4}{3}$ (3) $\frac{5}{5}$ (4) $\frac{1}{2}$

Ans. (2)

Sol. Let z = x + iy

$(x + iy)^2 + 3(x – iy) = 0$

$x^2 – y^2 + 2ixy + 3x – 3iy = 0$

$x^2 – y^2 + 3x = 0$ & $2xy – 3y = 0$

Case-1: y = 0

$x^2 – y^2 + 3x = 0$

$\Rightarrow$ x = 0 or x = –3

Solutions are z=0 and z=–3

Case-2: $x = \frac{3}{2}$

$x^2 – y^2 + 3x = 0$

$\Rightarrow y=\frac{3\sqrt{3}}{2}$ or $y=\frac{-3\sqrt{3}}{2}$

Solutions are $z=\frac{3}{2}+i\frac{3\sqrt{3}}{2}$ and $z=\frac{3}{2}-i\frac{3\sqrt{3}}{2}$

Total number of solutions = n = 4

So $\sum_{k=0}^{\infty}\frac{1}{4^k}=\frac{1}{1-\frac{1}{4}}=\frac{4}{3}$ Ans.

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10 The number of solutions of the equation $\sin^7 x + \cos^7 x = 1$ in the interval $[0, 4\pi]$ is :

(1) 7 (2) 9 (3) 14 (4) 5

Ans. (4)

Sol. $\sin^2 x + \cos^2 x = 1$, $\sin^2 x \le 1$ and $\cos^2 x \le 1$

$\sin^7 x \le \sin^2 x$

$\cos^7 x \le \cos^2 x$

so, $\sin^7 x + \cos^7 x \le 1$

$\sin^7 x + \cos^7 x = 1$ when $\sin^7 x = \sin^2 x$ & $\cos^7 x = \cos^2 x$

Case-2 : $\sin x = 1, \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{5\pi}{2}$

Total number of solutions = 5

11. The sum of all natural numbers belonging to the set {1, 2, 3 ..... 100}, whose HCF with 2304 is 1, is

(1) 2449 (2) 1633 (3) 1449 (4) 2633

Ans. (2)

Sol. $2304 = 2^8 \cdot 3^2$

Hence n can not be multiple of 2 or 3

Then sum is

$\Rightarrow n(1) - (n(2) + n(3) - n(6))$

(where n(a) means the sum of all numbers belonging to the set {1, 2, 3 .....100},which are divisible by a)

$$\Rightarrow \frac{100\times101}{2} - \frac{2\times50\times51}{2} - 3\times\frac{33\times34}{2} + 6\times\frac{17\times16}{2}$$

12. If f(x) is a continuous function defined as $f(x) = \begin{cases} \frac{x^3}{(1-\cos 2x)^2} \ln\left(\frac{1+\alpha x e^x}{(1+xe^x)^2}\right) & ; \ x<0 \\ \alpha & ; \ x\geq 0 \end{cases}$,then the value of α is

(1) $\frac{-2}{3}$ (2) $\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $\frac{-1}{3}$

Ans. (1)

Sol. $= \lim\limits_{x\to0^-} \frac{x^3}{(1-\cos2x)^2} \ln\left(\frac{1+\alpha xe^x}{(1+xe^x)^2}\right)$

$$= \lim_{x\to0^-} \frac{x^3\times x}{4\sin^4 x} \frac{\ln(1+\alpha xe^x) - 2\ln(1+xe^x)}{x}$$

$$= \lim_{x\to0^-} \frac{1}{4} \frac{\ln(1+\alpha xe^x) - 2\ln(1+xe^x)}{x}$$

$$= \lim_{x\to0^-} \frac{1}{4}\left\{\frac{\ell n(1+\alpha xe^x)\alpha e^x}{\alpha e^x} - \frac{2\ell n(1+xe^x)}{\alpha e^x}e^x\right\}$$

$$= \frac{1}{4}(\alpha - 2)$$

Now, $\lim\limits_{x\to0^+} \alpha = \alpha$

$\because$ f(x) is continuous function so

$$\frac{\alpha-2}{4} = \alpha \Rightarrow \alpha = \frac{-2}{3}$$

13 If the domain of $f(x) = \frac{\cos^{-1}\sqrt{x^2-x+1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is $(\alpha + \beta]$, then the value of $\alpha + \beta$ is :

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) 1 (4) 2

Ans. (2)

Sol. $0 < x^2 - x + 1 < 1$ and $0 < \frac{2x-1}{x} < 1$

$\Rightarrow$ $x(x-1) \le 0$ & $-1 < 2x \le 3$

$\Rightarrow$ $x \in [0,1] \cap x \in \left(\frac{1}{2}, \frac{3}{2}\right]$

$\Rightarrow$ $x \in \left(\frac{1}{2}, 1\right]$

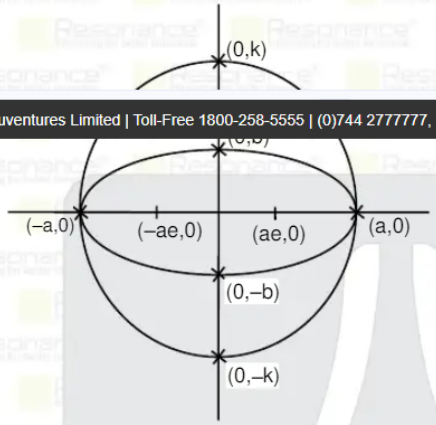
Hence $\alpha + \beta = \frac{1}{2} + 1 = \frac{3}{2}$

14. Let E_1 and E_2 be two Ellipses with same eccentricity. Where $E_1 = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, (a > b) and E_2 is such that it passes through the end points of major axis of E_1 also end points of minor axis of E_1 are foci of E_2. Find the eccentricity of E_1

(1) $\sqrt{\frac{\sqrt{5}-3}{2}}$ (2) $\sqrt{\frac{\sqrt{5}+3}{2}}$ (3) $\frac{\sqrt{5}+1}{2}$ (4) $\frac{\sqrt{5}-1}{2}$

Ans. (4)

Sol.



Eccentricity of E_1 is e $\Rightarrow$ $e^2 = 1 - \frac{b^2}{a^2}$

Eccentricity of E_2 is e $\Rightarrow$ $e^2 = 1 - \frac{a^2}{k^2}$

So, $e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{a^2}{k^2} \Rightarrow k = \frac{a^2}{b}$ .....(i)

From equation (i) and (ii) $e = \frac{b^2}{a^2}$

Since $e^2 = 1 - \frac{b^2}{a^2} \Rightarrow e^2 = 1 - e$

$\Rightarrow e^2 + e - 1 = 0$

$\Rightarrow e = \frac{\sqrt{5}-1}{2}$

15. Value of x for which $[e^x]^2 + [e^x + 1] - 3 = 0$ is

(1) $[e, e^2]$ (2) $[0, e]$ (3) $[e, \ell n 2]$ (4) $[0, \ell n 2]$

Ans. (4)

Sol. $[e^x]^2 + [e^x+1] - 3 = 0$

$[e^x]^2 + [e^x] - 2 = 0$

Let $[e^x] = t$

$t^2 + t - 2 = 0$

$(t+2)(t-1) = 0$

$t = 1, -2$

$x \in [0, \ell n 2]$

16. Evaluates $\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}dx$

(1) $e^x\sqrt{\frac{1+x}{1-2x}} + C$ (2) $e^x\sqrt{\frac{1+x}{1-x}} + C$ (3) $e^x\sqrt{\frac{1-x}{1+x}} + C$ (4) $e^x\left(\frac{1+x}{1-x}\right) + C$

Ans. **(2)**

Sol. $I = \int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}dx$

$= \int e^x \left\{ \frac{1}{(1-x)^{3/2}(1+x)^{1/2}} + \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}} \right\} dx$

$\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$ $\frac{1}{}$

$\Rightarrow I = \int e^x (f'(x) + f(x))dx$

$= e^x f(x) + C = e^x\sqrt{\frac{1+x}{1-x}} + C$

17. Four dice are rolled and the outcomes are put in 2 × 2 matrices. Find the probability that such a matrix will be non singular and all its entries are different.

(1) $\frac{71}{81}$ (2) $\frac{80}{81}$ (3) $\frac{30}{71}$ (4) $\frac{25}{71}$

Ans. **(2)**

Sol. $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$|x| = ad - bc = 0$

$\left.\begin{matrix} (1,6) & (3,2) \\ (3,4) & (6,2) \end{matrix}\right\} 8 + 8$ possibilites

18. Find the values of λ & μ for which the system of equations

$x + y + z = 6$

$x + 2y + \lambda z = \mu$ has no solution

(1) $\lambda = 2, \mu \neq 10$ (2) $\lambda \neq 2, \mu = 10$ (3) $\lambda \neq 3, \mu = 10$ (4) $\lambda \neq 2, \mu \neq 10$

Ans. **(1)**

Sol. For no solution $\Delta = 0$

$\Delta = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 3 & 5 & 5 \\ 1 & 2 & \lambda \end{vmatrix} = 0$$

$\Rightarrow 1(5\lambda - 10) - 1(3\lambda - 5) + 1(6 - 5) = 0$

$\Rightarrow 2\lambda - 4 = 0$

$\Rightarrow \lambda = 2$

$$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 26 & 5 & 5 \\ \mu & 2 & 2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & \mu & 2 \end{vmatrix}$$

$= 52 - 5\mu - 6 + 3\mu - 26$

$\Delta_2 = 20 - 2\mu$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 3 & 5 & 26 \\ 1 & 2 & \mu \end{vmatrix} = 1(5\mu - 52) - 1(3\mu - 26) + 6(6 - 5)$$

$\Delta_3 = 2\mu - 20$

Case-I

$\lambda = 2, \mu = 10 \Rightarrow \Delta = 0, \Delta_1 = 0, \Delta_2 = 0, \Delta_3 = 0$

system of equations are

$x + y + z = 6$

$3x + 5y + 5z = 26$

$x + 2y + 2z = 10$ has infinite many solutions

Case - II

system has not solution

19. The number of elements in the set $\{x \in R : (|x| - 3)|x - 4| = 6\}$ is equal to

Ans. **(3)**

Sol. $(|x| - 3)|x - 4| = 6$

Case - 1

$x \geq 4$

$(x - 3)(x - 4) = 6$

$x^2 - 7x + 6 = 0$

$(x - 1)(x - 6) = 0$

$x = 1, 6 \Rightarrow x = 6$

Case – 2

$0 < x < 4$

$(x - 3)(4 - x) = 6$

$x^2 - 7x + 18 = 0$

$D < 0$, No solution

Case -3

$(x + 3)(x - 4) = 6$

$x^2 - x - 18 = 0$

$x = \frac{1 \pm \sqrt{73}}{2}$

$x = \frac{1 - \sqrt{73}}{2}$

20. If $\int_0^{100\pi} \frac{\sin^2 x}{e^{\left(\frac{x}{\pi} - \left[\frac{x}{\pi}\right]\right)}} dx = \frac{\alpha\pi^3}{1 + 4\pi^2}$, $\alpha \in R$ where [x] is greatest integer function, then α is

(1) $50(e - 1)$ (2) $150(e^{-1} - 1)$ (3) $200(1 - e^{-1})$ (4) $100(1 - e)$

Ans. **(3)**

Sol. $\int_0^{100\pi} \frac{\sin^2 x}{e^{\left\{\frac{x}{\pi}\right\}}} dx$

$\Rightarrow 50\int_0^{\pi} e^{-x/\pi}[1 - \cos 2x]dx$

$\Rightarrow 50\left[e^{-x/\pi} \times (-\pi)\right]_0^{\pi} - 50\int_0^{\pi} e^{-x/\pi} \cos 2x dx$

$\Rightarrow 50 \times (-\pi)(e^{-1} - 1) - \frac{50 \times \left[e^{-x/\pi}\left(\frac{-1}{\pi} \times \cos 2x + 2\sin 2x\right)\right]_0^{\pi}}{\left(\frac{1}{\pi^2} + 4\right)}$

$\Rightarrow -50\pi(e^{-1} - 1) - \frac{50\pi^2}{(1 + 4\pi^2)}\left[e^{-1}\left(\frac{-1}{\pi}\right) + \frac{1}{\pi}\right]$

$\Rightarrow \frac{200\pi^3(1 - e^{-1})}{1 + 4\pi^2}$ So $\alpha = 200(1 - e^{-1})$