FIITJEE

Solutions to JEE(Main)-2020

Test Date: 7th January 2020 (Second Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of **3 hours** duration.
2. This **Test Paper** consists of **75** questions. The maximum marks are **300**.
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 25 questions in each part of equal weightage out of which 20 questions are MCQs and 5 questions are numerical value based. Each question is allotted **4 (four)** marks for correct response.
4. **(Q. No. 01 – 20, 26 – 45, 51 – 70)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
5. **(Q. No. 21 – 25, 46 – 50, 71 – 75)** contains 15 Numerical based questions with answer as numerical value. Each question carries **+4 marks** for correct answer. There is no negative marking.
6. Candidates will be awarded marks as stated above in **instruction No.3** for correct response of each question. One mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer box.
7. There is only one correct response for each question. Marked up more than one response in any question will be treated as wrong response and marked up for wrong response will be deducted accordingly as per **instruction 6** above.

PART – A (PHYSICS)

1. An emf of 20 V is applied at time t = 0 to a circuit containing in series 10 mH inductor and
5 Ω resistor. The ratio of the currents at time t = ∞ and at t = 40 s is close to: (take $e^2 = 7.389$)
(A) 1.06 (B) 0.84
(C) 1.46 (D) 1.15

2. A particle of mass m and charge q has an initial velocity $\vec{v} = v_0\hat{j}$. If an electric field $\vec{E} = E_0\hat{i}$ and magnetic field $\vec{B} = B_0\hat{i}$ act on the particle, its speed will double after a time:
(A) $\frac{2mv_0}{qE_0}$ (B) $\frac{\sqrt{2}mv_0}{qE_0}$ (C) $\frac{\sqrt{3}mv_0}{qE_0}$ (D) $\frac{3mv_0}{qE_0}$

3. In a building there are 15 bulbs of 45 W, 15 bulbs of 100 W, 15 small fans of 10 W and 2 heaters of 1 kW. The voltage of electric main is 220 V. The minimum fuse capacity (rated value) of the building will be
(A) 10 A (B) 20 A (C) 25 A (D) 15 A

4. An electron (of mass m) and a photon have the same energy E in the range of a few eV. The ratio of the de-Broglie wavelength associated with the electron and the wavelength of the photon is (c = speed of light in vacuum)
(A) $c\,(2mE)^{1/2}$ (B) $\frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$ (C) $\frac{1}{c}\left(\frac{2E}{m}\right)^{1/2}$ (D) $\left(\frac{E}{2m}\right)^{1/2}$

5. A thin lens made of glass (refractive index = 1.5) of focal length f = 16 cm is immersed in a liquid of refractive index 1.42. If its focal length in liquid is f_ℓ , then the ratio f_ℓ /f is closest to the integer:
(A) 17 (B) 1 (C) 9 (D) 5

6. A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much les than the speed of sound). The observers hears 2 beats/sec. The oscillation frequency of each tuning fork is
ν_0 = 1400 Hz and the velocity of sound in air is 350 m/s. The speed of each tuning fork is close to:
(A) $\frac{1}{4}$ m/s (B) 1 m/s (C) $\frac{1}{2}$ m/s (D) $\frac{1}{8}$ m/s

7. Mass per unit are of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre is:
(A) $2\pi a^4\left(\frac{A}{4}+\frac{B}{5}\right)$ (B) $2\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$
(C) $2\pi a^4\left(\frac{aA}{4}+\frac{B}{5}\right)$ (D) $\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$

8. A planar loop of wire rotates in a uniform magnetic field. Initially, at t = 0, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of 10 s about an axis in its plane then the magnitude of induced emf will be maximum and minimum, respectively at:
(A) 2.5 s and 5.0 s
(B) 5.0 s and 10.0 s
(C) 2.5 s and 7.5 s
(D) 5.0 and 7.5 s

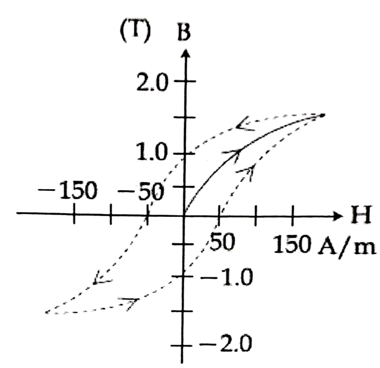
9. Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperatures, T_1 and T_2. The temperature of the hot reservoir of the first engine is T_1 and the temperature of the cold reservoir of the second engine is T_2. T is temperature of the sink of first engine which is also the source for the second engine. How is T related to T_1 and T_2, if both the engines perform equal amount of work?
(A) $T = \sqrt{T_1T_2}$
(B) $T = \frac{T_1 + T_2}{2}$
(C) $T = \frac{2T_1T_2}{T_1 + T_2}$
(D) T = 0

10. A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take g = 10 ms^{-2} at the north pole and the radius of the earth = 6400 km)
(A) 194.66 N
(B) 194.32 N
(C) 195.32 N
(D) 195.66 N

11. The dimension of $\frac{B^2}{2\mu_0}$, where B is magnetic field and μ_0 is the magnetic permeability of vacuum, is
(A) ML^2T^{-1}
(B) ML^2T^{-2}
(C) $ML^{-1}T^{-2}$
(D) MLT^{-2}

12. In a Young's double slit experiment, the separation between the slits is 0.15 mm. In the experiment, a source of light of wavelength 589 nm is used and the interference pattern is observed on a screen kept 1.5 m away. The separation between the successive bright fringes on the screen is:
(A) 4.9 mm
(B) 6.9 mm
(C) 3.9 mm
(D) 5.9 mm

13. The figure gives experimentally measured B vs H variation in a ferromagnetic material. The retentivity, co-ercivity and saturation, respectively, of the material are
(A) 1.0 T, 50 A / m and 1.5 T
(B) 1.5 T, 50 A / m and 1.0 T
(C) 150 A / m, 1.0 T and 1.5 T
(D) 1.5 T, 50 A / m and 1.0 T



14. The activity of a radioactive sample falls from 700 s^{-1} to 500 s^{-1} in 30 minutes. Its half life is close to
(A) 66 min (B) 52 min (C) 62 min (D) 72 min

15. An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities of fluid in this pipe is
(A) $\frac{3}{4}$ (B) $\frac{81}{256}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{9}{16}$

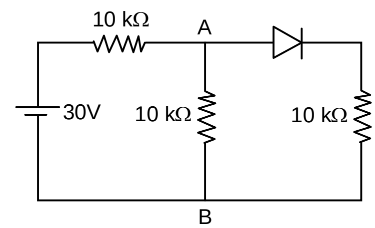
16. A mass of 10 kg is suspended by a rope of length 4 m, from the ceiling. A force F is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of 45° with the vertical. Then F equals: (Take g = 10 ms^{-2} and the rope to be massless)
(A) 100 N (B) 75 N (C) 70 N (D) 90 N

17. The electric field of a plane electromagnetic wave is given by
$$\vec{E} = E_0 \frac{\hat{i}+\hat{j}}{\sqrt{2}} \cos(kz + \omega t)$$
At t = 0, a positively charged particle is at the point (x, y, z) = $\left(0, 0, \frac{\pi}{k}\right)$. If its instantaneous velocity at (t = 0) is $v_0\hat{k}$, the force acting on it due to the wave is:
(A) zero
(B) parallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$
(C) parallel to $\hat{k}$
(D) antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

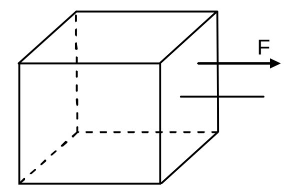
18. Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently the mean collision time between the gas molecule changes from τ_1 to τ_2. If $\frac{C_p}{C_v} = \gamma$ for the gas then a good estimate for $\frac{\tau_2}{\tau_1}$ is given by
(A) 2 (B) $\left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$ (C) $\left(\frac{1}{2}\right)^{\gamma}$ (D) $\frac{1}{2}$

19. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed of 3 m/s. the frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator (g = 10 m/s^2) must be at least:
(A) 56300 W (B) 66000 W (C) 62360 W (D) 48000 W

20. In the figure, potential difference between A and B is:
(A) 5 V
(B) 10 V
(C) zero
(D) 15 V



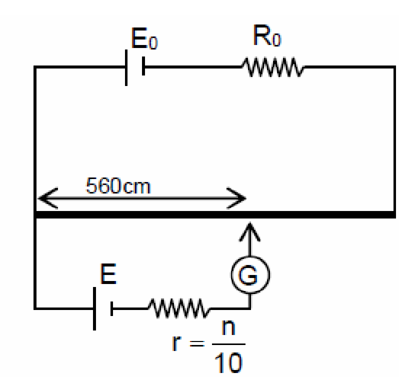
21. Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100 \times \frac{b}{a}$ for box not to topple before moving is _________.

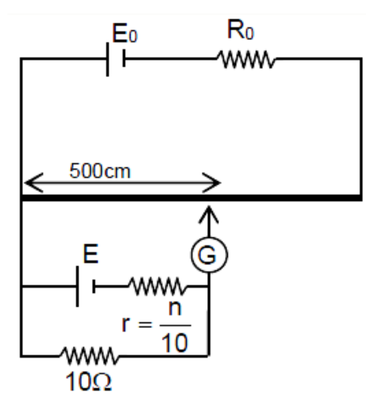


22. M grams of steam at 100°C is mixed with 200 g of ice at its melting point in a thermally insulated container. If it produces liquid water at 40°C [heat of vaporization of water is 540 cal/g and heat of fusion of ice is 80 cal/g], the value of M is _________.

23. A 60 pF capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged 60 pF capacitor in parallel. The electrostatic energy that is lost in this process by the time the charge is redistributed between them is (in nJ) _________.

24. The balancing length for a cell is 560 cm in a potentiometer experiment. When an external resistance of 10 Ω is connected in parallel to the cell, the balancing length changes by 60 cm. If the internal resistance of the cell is $\frac{N}{10}\Omega$, where N is an integer then value of N is _________.





25. The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultant of $2\vec{P}$ and $\vec{Q}$ will make with $\vec{Q}$ is, _________.

PART –B (CHEMISTRY)

26. In the following reactions, products(A) and (B) respectively are

$$\underset{\text{(Hot and Conc)}}{NaOH + Cl_2} \longrightarrow (A) + \text{side products}$$

$$\underset{\text{(Dry)}}{Ca(OH)_2}\, Cl_2 \longrightarrow (B) + \text{side products}$$

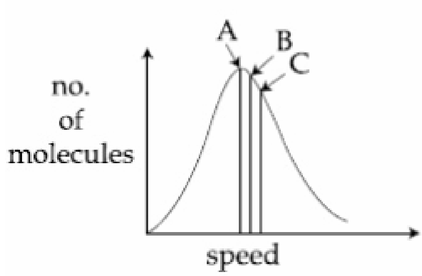
(A) $NaClO_3$ and $Ca(OCl)_2$
(B) $NaOCl$ and $Ca(OCl)_2$
(C) $NaOCl$ and $Ca(ClO_3)_2$
(D) $NaClO_3$ and $Ca(ClO_3)_2$

27. The refining method used when the metal and the impurities have low and high melting temperatures, respectively is
(A) zone refining
(B) vapour phase refining
(C) liquation
(D) distillation

28. The redox reaction among the following is
(A) reaction of $[Co(H_2O)_6]Cl_3$ with $AgNO_3$
(B) combination of dinitrogen with dioxygen at 2000 K
(C) reaction of H_2SO_4 with NaOH
(D) formation of ozone from atmospheric oxygen in the presence of sunlight

29. Within each pair of elements F & Cl, S & Se and Li & Na, respectively, the elements that release more energy upon an electron gain are
(A) F, Se and Na
(B) Cl, Se and Na
(C) F, S and Li
(D) Cl, S and Li

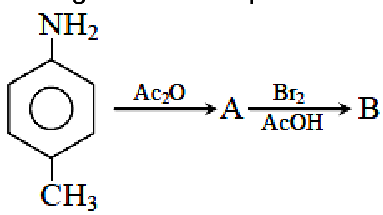
30. Identify the correct labels of A, B and C in the following graph from the options given below



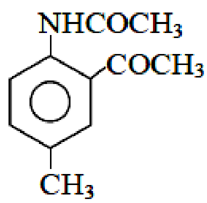
Root mean square speed(V_{rms}); most probable speed(V_{mp}); Average speed(V_{av})
(A) A-V_{av}, B-V_{rms}, C-V_{mp}
(B) A-V_{rms}, B-V_{mp}, C-V_{av}
(C) A-V_{mp}, B-V_{rms}, C-V_{av}
(D) A-V_{mp}, B-V_{av}, C-V_{rms}

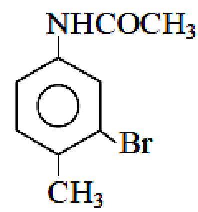
31. For the reaction
$2H_2(g) + 2NO(g) \longrightarrow N_2(g) + 2H_2O$, the observed rate expression is, rate = $k_f[NO]^2[H_2]$. The rate expression for the reverse reaction is:
(A) $k_b[N_2][H_2O]$
(B) $k_b[N_2][H_2O]^2/[NO]$
(C) $k_b[N_2][H_2O]^2/[H_2]$
(D) $k_b[N_2][H_2O]^2$

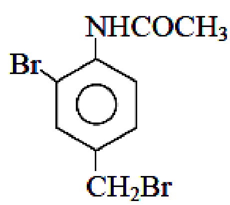
32. In the following reaction sequence

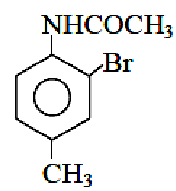


The major product B is

(A) 

(B) 

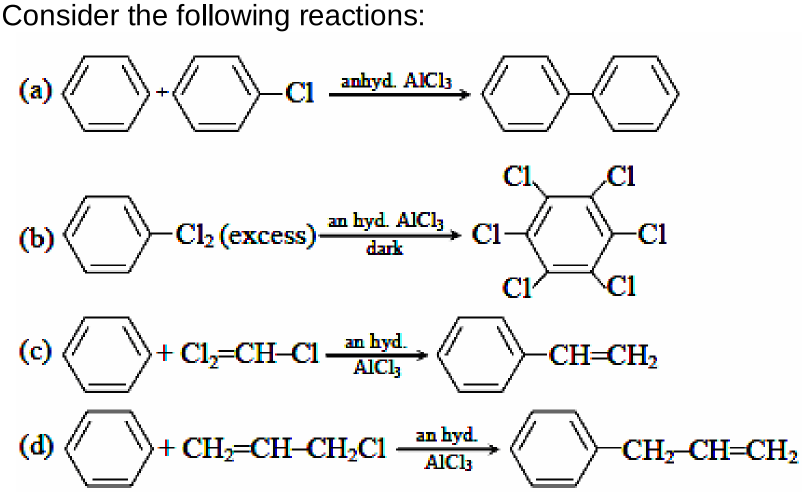
(C) 

(D) 

33. Which of the following statements is correct?
(A) Gluconic acid is a dicarboxylic acid
(B) Gluconic acid can form cyclic(acetal/hemiacetal) structure
(C) Gluconic acid is a partial oxidation product of glucose
(D) Gluconic acid is obtained by oxidation of glucose with HNO_3

34. The equation that is incorrect is
(A) $\left(\Lambda_m^0\right)_{NaBr} - \left(\Lambda_m^0\right)_{NaI} = \left(\Lambda_m^0\right)_{KBr} - \left(\Lambda_m^0\right)_{NaBr}$
(B) $\left(\Lambda_m^0\right)_{NaBr} - \left(\Lambda_m^0\right)_{NaCl} = \left(\Lambda_m^0\right)_{KBr} - \left(\Lambda_m^0\right)_{KCl}$
(C) $\left(\Lambda_m^0\right)_{H_2O} = \left(\Lambda_m^0\right)_{HCl} + \left(\Lambda_m^0\right)_{NaOH} - \left(\Lambda_m^0\right)_{NaCl}$
(D) $\left(\Lambda_m^0\right)_{KCl} - \left(\Lambda_m^0\right)_{NaCl} = \left(\Lambda_m^0\right)_{KBr} - \left(\Lambda_m^0\right)_{NaBr}$

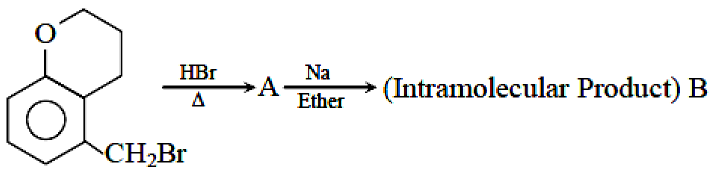
35. Consider the following reactions:

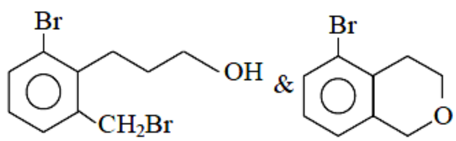


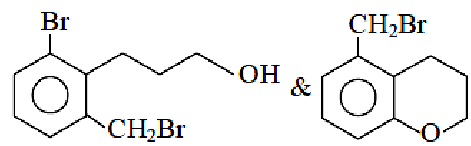
Which of these reactions are possible?
(A) (b) and (d)
(B) (a) and (d)
(C) (a) and (b)
(D) (b), (c) and (d)

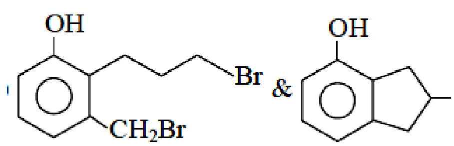
36. The number of possible optical isomers for the complexes MA_2B_2 with sp^3 and dsp^2 hybridized metal atom, respectively is:
[Note: A and B are unidentate neutral and unidentate monoanionnic ligands, respectively]
(A) 2 and 2
(B) 0 and 2
(C) 0 and 1
(D) 0 and 0

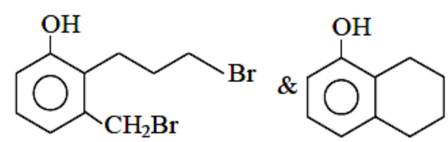
37. In the following reaction sequence, structures of A and B respectively will be



(A) 

(B) 

(C) 

(D) 

38. The bond order and the magnetic characteristics of CN^- are
(A) 3, paramagnetic
(B) 2½, diamagnetic
(C) 3, diamagnetic
(D) 2½, paramagnetic

39. For the following reactions:

$$CH_3CH_2CH_2Br + Z^{\ominus} \xrightarrow[\text{substitution}]{k_s} CH_3CH_2CH_2Z + Br^{\ominus}$$

$$CH_3CH_2CH_2Br + Z^{\ominus} \xrightarrow[\text{elimination}]{k_e} CH_3CH{=}CH_2 + HZ + Br^{\ominus}$$

where,

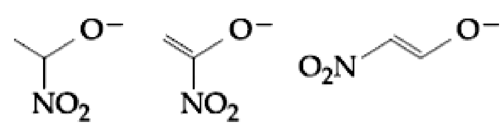
$$Z^{\ominus} = CH_3CH_2O^{\ominus}\ \text{(A)}\ \text{or}\ H_3C-\underset{CH_3}{\overset{CH_3}{\underset{|}{\overset{|}{C}}}}-O^{\ominus}\ \text{(B)},$$

k_s and k_e are respectively, the rate constant for substitution and elimination, and $\mu = \dfrac{k_s}{k_e}$, the correct option is______________
(A) $\mu_B > \mu_A$ and $k_e(A) > k_e(B)$
(B) $\mu_A > \mu_B$ and $k_e(A) > k_e(B)$
(C) $\mu_A > \mu_B$ and $k_e(B) > k_e(A)$
(D) $\mu_B > \mu_A$ and $k_e(B) > k_e(A)$

40. Among the statements (a - d), the incorrect ones are:
(a) Octahedral Co(III) complexes with strong field ligands have very high magnetic moments
(b) When $\Delta_0 < P$, the d-electron configuration of Co(III) in an octahedral complex is $t_{eg}^4 e_g^2$
(c) Wavelength of light absorbed by $[Co(en)_3]^{3+}$ is lower than that of $[CoF_6]^{3-}$
(d) If the Δ_0 for an octahedral complex of Co(III) is 18,000 cm^{-1}, the Δ_t for its tetrahedral complex with the same ligand will be 16,000 cm^{-1}
(A) (b) and (c) only (B) (a) and (b) only
(C) (c) and (d) only (D) (a) and (d) only

41. A Chromatography column, packed with silica gel as stationary phase, was used to separate a mixture of compounds consisting of (a) benzanilide, (b) aniline and (c) acetophenone. When the column is eluted with a mixture of solvents, hexane; ethyl acetate(20:80), the sequence of obtained compounds is
(A) (b), (c) and (a) (B) (a), (b) and (c)
(C) (c), (a) and (b) (D) (b), (a) and (c)

42. The correct order of stability for the following alkoxides is



(A) (B) (C)

(A) (B) > (C) > A (B) (C) > (A) > (B)
(C) (C) > (B) > (A) (D) (B) > (A) > (C)

43. Among statements (a) – (d), the correct ones are
(a) decomposition of hydrogen peroxide gives dioxygen.
(b) like hydrogen peroxide, compounds such as $KClO_3$, $Pb(NO_3)_2$ and $NaNO_3$ when heated liberated dioxygen.
(c) 2-Ethylanthraquinone is useful for the industrial preparation of hydrogen peroxide
(d) Hydrogen peroxide is used for the manufacture of sodium perborate
(A) (a), (b), (c) and (d) (B) (a) and (c) only
(C) (a), (b) and (c) only (D) (a), (c) and (d) only

44. The ammonia(NH_3) released on quantitative reaction of 0.6 g urea (NH_2CONH_2) with sodium hydroxide(NaOH) can be neutralized by
(A) 100 mL of 0.1 N HCl (B) 100 mL of 0.2 N HCl
(C) 200 mL of 0.2 N HCl (D) 200 mL of 0.4 N HCl

45. Two open beakers one containing a solvent and the other containing a mixture of that solvent with a non-volatile solute are together sealed in a container. Over time
(A) the volume of the solution increases and the volume of the solvent decreases
(B) the volume of the solution and the solvent does not change
(C) the volume of the solution does not change and the volume of the solvent decreases
(D) the volume of the solution decreases and the volume of the solvent increases

46. Consider the following reactions:

$$NaCl + K_2Cr_2O_7 + \underset{(Conc.)}{H_2SO_4} \longrightarrow (A) + \text{side products}$$

$$(A) + NaOH \longrightarrow (B) + \text{Side products}$$

$$(B) + \underset{(dilute)}{H_2SO_4} + H_2O_2 \longrightarrow (C) + \text{Side products}$$

The sum of the total number of atoms in one molecule each of (A), (B) and (C) is _____

47. The standard heat of formation $\left(\Delta_f H^0_{298}\right)$ of ethane (in kJ/mol), if the heat of combustion of ethane, hydrogen and graphite are -1560, -393.5 and -286 kJ/mol respectively is _____

48. The flocculation value of HCl for arsenic sulphide sol is 30 m mol L^{-1}. If H_2SO_4 is used for the flocculation of arsenic sulphide, the amount, in grams of H_2SO_4 in 250 mL required for the above purpose is __________
(Molecular mass of H_2SO_4 = 98 g/mol)

49. The number of sp^2-hybridized carbons present in "Aspartame" is_________

50. 3 g of acetic acid is added to 250 mL of 0.1 M HCl and the solution made up to 500 mL. To 20 mL of this solution ½ mL of 5 M NaOH is added. The pH of the solution is_________
[Given: pK_a of acetic acid = 4.75, molar mass of acetic acid = 60 g/mol, log 3 = 0.4771]
Neglect any changes in volume.

PART–C (MATHEMATICS)

51. The number of ordered pairs (r, k) for which $6 \cdot {}^{35}C_r = \left(k^2 - 3\right) . {}^{36}C_{r+1}$, where k is an integer, is:
(A) 3 (B) 6
(C) 4 (D) 2

52. If $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ for some $a \in R$, then the distance between the foci of the ellipse is:
(A) $2\sqrt{2}$ (B) $2\sqrt{7}$
(C) 4 (D) $2\sqrt{5}$

53. If the sum of the first 40 terms of the series, $3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + ...$ is (102)m, then m is equal to:
(A) 10 (B) 5
(C) 20 (D) 25

54. Let $A = \left[a_{ij}\right]$ and $B = \left[b_{ij}\right]$ be two 3×3 real matrices such that $b_{ij} = \left(3\right)^{(i+j-2)} a_{ji}$, where $i, j = 1, 2, 3$. If the determinant of B is 81, then the determinant of A is:
(A) $\frac{1}{3}$ (B) $\frac{1}{9}$
(C) $\frac{1}{81}$ (D) 3

55. The value of c in the Lagrange's mean value theorem for the function $f\left(x\right) = x^3 - 4x^2 + 8x + 11$, when $x \in \left[0,1\right]$ is:
(A) $\frac{\sqrt{7} - 2}{3}$ (B) $\frac{4 - \sqrt{5}}{3}$
(C) $\frac{4 - \sqrt{7}}{3}$ (D) $\frac{2}{3}$

56. The value of α for which $4\alpha \int_{-1}^{2} e^{-\alpha|x|} dx = 5$, is
(A) $\log_e\left(\frac{3}{2}\right)$ (B) $\log_e\left(\frac{4}{3}\right)$
(C) $\log_e \sqrt{2}$ (D) $\log_e 2$

57. Let $y = y(x)$ be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and $y\left(\frac{1}{2}\right) = -\frac{1}{4}$. Then $\frac{dy}{dx}$ at $x = \frac{1}{2}$, is equal to:

(A) $-\frac{\sqrt{5}}{4}$
(B) $-\frac{\sqrt{5}}{2}$
(C) $\frac{2}{\sqrt{5}}$
(D) $\frac{\sqrt{5}}{2}$

58. Let $y = y(x)$ be the solution curve of the differential equation, $\left(y^2 - x\right)\frac{dy}{dx} = 1,$ satisfying $y(0) = 1$. This curve intersects the x – axis at a point whose abscissa is:

(A) $2+e$
(B) –e
(C) 2
(D) 2 – e

59. Let $f(x)$ be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim\limits_{x\to 0}\left(2 + \frac{f(x)}{x^3}\right) = 4$, then which one of the following is not true?

(A) x = 1 is a point of minima and x = –1 is a point of maxims of f.
(B) x = 1 is a point of maxima and x = –1 is a point of minimum of f
(C) f is an odd function
(D) $f(1) - 4f(-1) = 4$

60. The area (in sq. units of the region $\left\{(x,y) \in R^2 \,\middle|\, 4x^2 \le y \le 8x + 12\right\}$ is:

(A) $\frac{124}{3}$
(B) $\frac{125}{3}$
(C) $\frac{128}{3}$
(D) $\frac{127}{3}$

61. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. If $\lambda = \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ and $\vec{d} = \vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}$, then the ordered pair, $(\lambda, \vec{d})$ is equal to:

(A) $\left(\frac{3}{2}, 3\vec{b}\times\vec{c}\right)$
(B) $\left(\frac{3}{2}, 3\vec{a}\times\vec{c}\right)$
(C) $\left(-\frac{3}{2}, 3\left(\vec{a}\times\vec{b}\right)\right)$
(D) $\left(-\frac{3}{2}, 3\vec{a}\times\vec{b}\right)$

62. The locus of the mid – points of the perpendiculars drawn from points on the line, $x = 2y$ to the line $x = y$ is:

(A) $7x - 5y = 0$
(B) $3x - 2y = 0$
(C) $2x - 3y = 0$
(D) $5x - 7y = 0$

63. Let $a_1, a_2, a_3, ...$ be a G.P. such that $a_1 < 0, a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to:

(A) –171
(B) –513
(C) 171
(D) $\frac{511}{3}$

64. Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $P_k = (\alpha)^k + (\beta)^k, k \geq 1$, then which one of the following statements is not true?

(A) $p_5 = 11$
(B) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$
(C) $p_3 = p_5 - p_4$
(D) $p_5 = p_2 . p_3$

65. If $\frac{3 + i\sin\theta}{4 - i\cos\theta}, \theta \in [0, 2\pi]$, is a real number, then an argument of $\sin\theta + i\cos\theta$ is:

(A) $-\tan^{-1}\left(\frac{3}{4}\right)$
(B) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$
(C) $\pi - \tan^{-1}\left(\frac{3}{4}\right)$
(D) $\tan^{-1}\left(\frac{4}{3}\right)$

66. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to:

(A) 4
(B) $\frac{17}{8}$
(C) $\frac{17}{2}$
(D) $\frac{17}{4}$

67. Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. Then $(AB)^2$ is equal to:

(A) $\frac{56}{5}$
(B) $\frac{32}{5}$
(C) $\frac{64}{5}$
(D) $\frac{52}{5}$

68. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0,2\pi)-\{\pi\}$ which satisfy the equation, $2\cot^2\theta-\frac{5}{\sin\theta}+4=0$, then $\int_{\theta_1}^{\theta_2}\cos^2 3\theta\,d\theta$ is equal to:

(A) $\frac{\pi}{3}+\frac{1}{6}$
(B) $\frac{\pi}{3}$
(C) $\frac{\pi}{9}$
(D) $\frac{2\pi}{3}$

69. The coefficient of x^7 in the expression $(1+x)^{10}+x(1+x)^9+x^2(1-x)^8+......+x^{10}$ is:

(A) 120
(B) 420
(C) 330
(D) 210

70. Let A, B, C and D be four non – empty sets. The contrapositive statement of "If $A\subseteq B$ and $B\subseteq D$, then $A\subseteq C$" is:

(A) If $A\subseteq C$, then $B\subset A$ or $D\subset B$
(B) If $A\not\subseteq C$, then $A\not\subseteq B$ or $B\subseteq D$
(C) If $A\not\subseteq C$, then $A\not\subseteq B$ or $B\not\subseteq D$
(D) If $A\not\subseteq C$, then $A\subseteq B$ or $B\subseteq D$

71. If the function f defined on $\left(-\frac{1}{3},\frac{1}{3}\right)$ by $f(x)=\begin{cases}\frac{1}{x}\log_e\left(\frac{1+3x}{1-2x}\right), & \text{when } x\neq 0\\ k & \text{,when } x=0\end{cases}$ is continuous, then k is equal to ___________

72. If the system of linear equations,

$x+y+z=6$

$x+2y+3z=10$

$3x+2y+\lambda z=\mu$

has more than two solutions, then $\mu-\lambda^2$ is equal to ____________

73. If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then xy is equal to ___________

74. If the foot of the perpendicular drawn from the point (1, 0, 3) on a line passing through $(\alpha,7,1)$ is $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$, then α is equal to ___________

75. Let $X=\{n\in N:1\le n\le 50\}$. If $A=\{n\in X:n \text{ is a multiple of } 2\}$ and $B=\{n\in X, n \text{ is a multiple of } 7\}$, then the number of elements in the smallest subset of X containing both A and B is ________

JEE (Main) – 2020
ANSWERS

PART A – PHYSICS

1.	**B**	2.	**C**	3.	**B**	4.	**B**
5.	**C**	6.	**A**	7.	**B**	8.	**A**
9.	**B**	10.	**C**	11.	**C**	12.	**D**
13.	**A**	14.	**C**	15.	**D**	16.	**A**
17.	**D**	18.	**Bonus**	19.	**B**	20.	**B**
21.	**50.00**	22.	**40**	23.	**6**	24.	**12**
25.	**90°**						

PART B – CHEMISTRY

26.	**A**	27.	**C**	28.	**B**	29.	**D**
30.	**D**	31.	**C**	32.	**D**	33.	**C**
34.	**A**	35.	**A**	36.	**D**	37.	**D**
38.	**C**	39.	**C**	40.	**D**	41.	**C**
42.	**C**	43.	**A**	44.	**B**	45.	**A**
46.	**18**	47.	**192.5**	48.	**0.3675**	49.	**9**
50.	**5.2271**						

PART C – MATHEMATICS

51.	**C**	52.	**B**	53.	**C**	54.	**B**
55.	**C**	56.	**D**	57.	**B**	58.	**D**
59.	**B**	60.	**C**	61.	**C**	62.	**D**
63.	**A**	64.	**D**	65.	**B**	66.	**B**
67.	**C**	68.	**B**	69.	**C**	70.	**C**
71.	**5**	72.	**13**	73.	**54**	74.	**4**
75.	**29**						

HINTS AND SOLUTIONS

PART A – PHYSICS

1. **B**

Sol. $i = \frac{V}{r}\left(1 - e^{-Rt/L}\right)$

$= 4(1 - e^{-500\,t})$

At $t = \infty$

$i_1 = 4A$

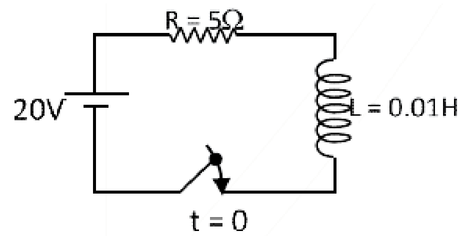
at $t = 40s$

$i_2 = 4(1 - e^{-20000})$

$= 4\left[1 - \frac{1}{(e^2)^{10000}}\right]$

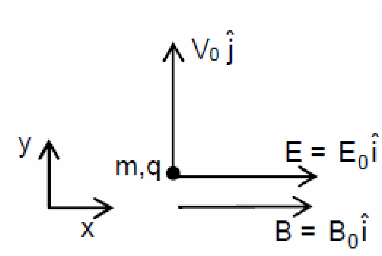
$= 4\left[1 - \frac{1}{(7.389)^{10000}}\right]$

$\frac{i_1}{i_2}$ is slightly greater than 1.



2. **C**

Sol. Since magnetic force cannot change the speed. So only electric field which is along x-direction will change the speed along x-direction only.



$v_x = \frac{E_o q}{m} t$ but $v_y = v_0$

$2v_0 = \sqrt{v_x^2 + v_y^2}$

$4v_0^2 = \frac{E_o^2 q^2 t^2}{m^2} + V_0^2$

$t = \frac{\sqrt{3}\, mv_o}{qE_o}$

3. **B**

Sol. Total power is (15 × 45) + (15 × 100) + (15 × 10) + (2 × 1000) = 4325 W.

So, current is $\frac{4325}{220} = 19.66$ A

Answer is 20 Amp.

4. **B**

Sol. $\lambda_{electron} = \frac{h}{\sqrt{2mE}}$; $\frac{\lambda_{electron}}{\lambda_{photon}} = \frac{1}{c}\sqrt{\frac{E}{2m}}$

$\lambda_{photon} = \frac{hc}{E}$

5. **C**

Sol. $\frac{1}{f} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ …(i)

$\frac{1}{f_\ell} = \left(\frac{\mu_g}{\mu_\ell} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ …(ii)

$\frac{(i)}{(ii)}$ $\frac{f_\ell}{f} = \frac{\mu_g - 1}{\frac{\mu_g}{\mu_\ell} - 1} = \frac{1.5 - 1}{\frac{1.5}{1.41} - 1}$ = 8.875 ≈ 9.

6. **A**

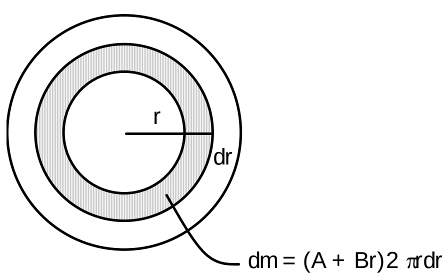
Sol. $f_1 = f_o\left(\frac{v}{v - v_s}\right)$ $f_1 - f_2 = 2$

$f_2 = f_o\left(\frac{v}{v + v_s}\right)$ $f_0 v\left[\frac{v + v_s - (v - v_s)}{v^2 - v_s^2}\right] = 2$; $\frac{2f_0 v\, v_s}{v^2 - v_s^2} = 2$

$\because$ $v_s << v$ $\therefore$ $v_s = \frac{1}{4}\, m/s$

7. **B**

Sol.



Moment of inertia of ring

$dI = dmr^2$

$I = \int_0^a (A + Br)2\pi r dr \pi r^2$

$= 2\pi A\int_0^a r^3 dr + 2\pi B\int_0^a r^4 dr$

$= 2\pi\left[A\frac{a^4}{4} + B\frac{a^5}{5}\right]$

$= 2\pi\, a^4\left[\frac{A}{4} + \frac{Ba}{5}\right]$

8. **A**

Sol. At any time t

$\phi = BA\cos\frac{\pi t}{5}$ $\left(\because \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{10} = \frac{\pi}{5}\right)$

$\frac{d\phi}{dt} = -\frac{\pi}{5}BA\sin\left(\frac{\pi t}{5}\right)$

$e = \frac{\pi}{5}BA\sin\left(\frac{\pi t}{5}\right)$

e will be maximum when $\frac{\pi t}{5}$ is $\frac{\pi}{2}$

t = 2.5 sec.

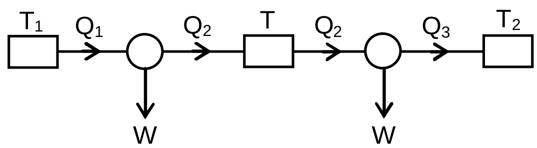
e will be minimum when $\frac{\pi t}{5}$ is $\pi, 0$

t = 0, 5 sec

So, answer will be (1).

9. **B**

Sol.



$W = Q_1 - Q_2 = Q_2 - Q_3$

$$Q_2 = \frac{Q_1 + Q_3}{2}$$

$$2 = \frac{Q_1}{Q_2} + \frac{Q_3}{Q_2}$$

$$2 = \frac{T_1}{T} + \frac{T_2}{T}$$

$$T = \frac{T_1 + T_2}{2}$$

10. **C**

Sol. $W_{equator} = W_{pole} - m\omega^2 R$

$$= 196 - 19.6 \times \left(\frac{2\pi}{24 \times 60 \times 60}\right)^2 \times 6400 \times 10^3$$

$$= 195.33 \text{ N}$$

11. **C**

Sol. Magnetic energy per unit volume = $\frac{B^2}{2\mu_0}$

$$\frac{\text{Energy}}{\text{Volume}} = \frac{ML^2T^{-2}}{L^3} = (ML^{-1}T^{-2})$$

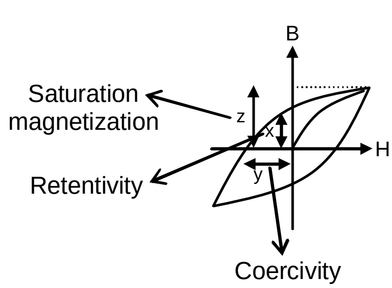
12. **D**

Sol. The distance between two successive bright fringes is fringe width (β).

$$\beta = \frac{\lambda D}{d} = \frac{589 \times 10^{-9} \times 1.5}{0.15 \times 10^{-3}} = 5.9 \text{ mm}$$

13. **A**

Sol.



x = retentivity
y = coercivity
z = saturation magnetization

14. **C**

Sol. $A = A_o e^{-\lambda t}$

$500 = 700 e^{-\lambda t}$

$\lambda t = \ell n \frac{7}{5}$

$\frac{\ell n 2}{t_{1/2}} \times 30 = \ell n \frac{7}{5}$

$\because \quad t_{1/2} = \frac{\ell n 2 \times 30}{\ell n \frac{7}{5}}$

$t_{1/2} = 61.8$ min ≈ 62 min.

15. **D**

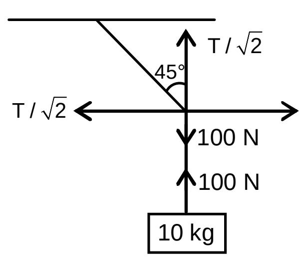
Sol. Using equation of continuity

$A_1V_1 = A_2V_2$

$\frac{V_1}{V_2} = \frac{A_2}{A_1} = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{4.8}{6.4}\right)^2 = \frac{9}{16}$

16. **A**

Sol.



$\frac{T}{\sqrt{2}} = 100 \quad ; \quad \frac{T}{\sqrt{2}} = F$

F = 100 N

17. **D**

Sol. Electric field at t = 0 & (x, y, z) = $\left(0, 0, \frac{\pi}{k}\right)$ is

$\vec{E} = -\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right) E_0$ and $\vec{F} = \vec{E}q$

$\therefore \quad \vec{F} = -\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right) q$

Which is antiparallel to $\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right)$

18. **Bonus**

Sol. Relaxation time $(\tau) \propto \frac{V}{\sqrt{T}}$

and $\quad T \propto \frac{1}{V^{\gamma-1}}$

$\tau \propto V^{1+\frac{\gamma-1}{2}}$; $\tau \propto V^{\frac{1+\gamma}{2}}$

$\frac{\tau_f}{\tau_i} = \left(\frac{2V}{V}\right)^{\frac{1+\gamma}{2}}$; $\frac{\tau_f}{\tau_i} = (2)^{\frac{1+\gamma}{2}}$

19. **B**

Sol. Net force on motor will be

F_{min} = [920 + 68(10)]g + 6000

= 22000 N

So, required power for motor

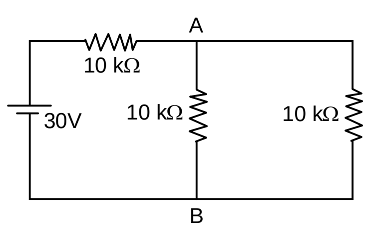
$P_{min} = \vec{F}_{min} \cdot \vec{v}$

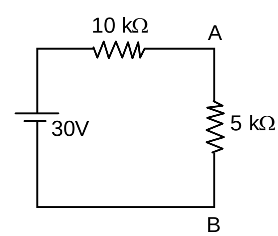
= 22000 × 3

= 66000 watt

20. **B**

Sol. In forward bias diode act as a short circuit wire. Hence, the equivalent circuit is now.





So, $V_{ab} = \frac{30}{5+10} \times 5 = 10$ V.

21. **50.00**

Sol. For no toppling

$F\left(\frac{a}{2}+b\right) \le mg\frac{a}{2}$

$\mu\frac{a}{2} + \mu b \le \frac{a}{2}$

0.2 a + 0.4 b ≤ 0.5a

0.4b ≤ 0.3a

$b \le \frac{3a}{4}$

b ≤ 0.75a (in limiting case)

But is not possible as maximum value of b can be equal to 0.5a only.

$\therefore \quad \left(100\dfrac{b}{a}\right)_{max} = 50.00$

22. **40**

Sol. $M_{ice}\, L_f + m_{ice}\,(40 - 0)\, C_w = m_{steam}\, L_v + m_{steam}\,(100 - 40)\, C_w$

$\Rightarrow \; 200[80 + 40(1)] = M[540 + 60(1)]$

$\Rightarrow \; 200(120) = M(600)$

M = 40 gm.

23. **6**

Sol. Common potential after connection.

$V_{common} = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2} = \dfrac{60\times 20 + 0}{120} = 10 \text{ Volt}$

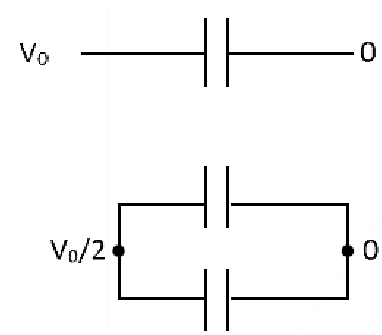
Loss of energy = $\dfrac{1}{2}CV^2 - \dfrac{1}{2}(2C)\times V_{Common}^2$

$= \dfrac{1}{2}\times 60\times 10^{-12}\times (20)^2 - 60\times 10^{-12}\times (10)^2$

$= 60 \times 10^{-12}\,(200 - 100)$

$= 6000 \times 10^{-12}$

$= 6$ nJ

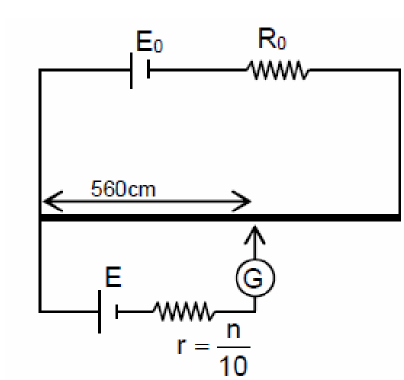


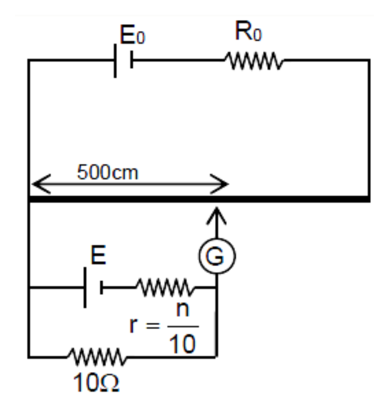
24. **12**

Sol. $r_{in} = \left(\dfrac{\ell_1 - \ell_2}{\ell_2}\right)R_{ext} = \dfrac{60}{500}\times 10$

$r = \dfrac{6}{5} = 1.2\ \Omega$

n = 12





25. **90°**

Sol. $\left|\vec{P}+\vec{Q}\right| = \left|\vec{P}\right|$

$P^2 + Q^2 + 2PQ\cos\theta = P^2$

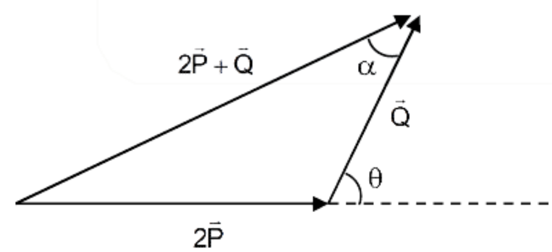
$\Rightarrow \quad Q + 2P\cos\theta = 0$

$\Rightarrow \quad \cos\theta = -\dfrac{Q}{2P}$

$\tan\alpha = \dfrac{2P\sin\theta}{2P\cos\theta + Q} = \infty$

$\because \quad [2P\cos\theta + Q = 0]$

$\alpha = 90°$



PART B – CHEMISTRY

26. A

Sol. $6NaOH + 3Cl_2 \longrightarrow 5NaCl + NaClO_3 + 3H_2O$

$2Ca(OH)_2 + Cl_2 \longrightarrow Ca(OCl)_2 + CaCl_2 + H_2O$

27. C

Sol. Concept based.

28. B

Sol. $N_2 + O_2 \longrightarrow 2NO$

Oxidation state changes.

29. D

Sol. Cl > F (exception)
S > Se (size effect)
Li > Na (size effect)

30. D

Sol. $V_{rms} > V_{av} > V_{mp}$

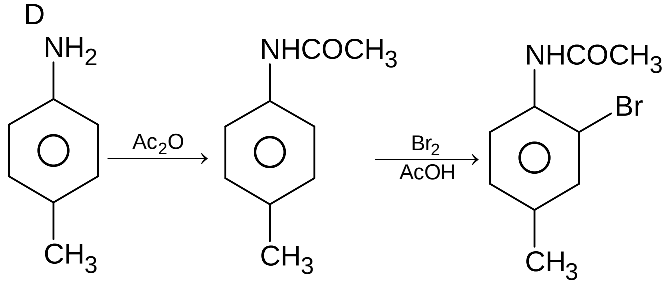
31. C

Sol. Al equilibrium

$$K_f[H_2][NO]^2 = \frac{K_b[N_2][H_2O]^+}{[H_2]}$$

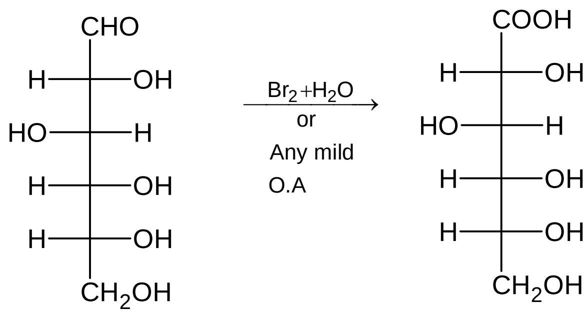
32. D

Sol.



33. C

Sol.



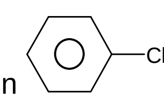
34. A

Sol. Each ion makes a definite contribution irrespective of the other ion

$\left(\Lambda_m^0\right)_{NaBr} - \left(\Lambda_m^0\right)_{NaI} = \Lambda^0_{m\,Br^-} - \Lambda^0_{m\,I^-}$

$\left(\Lambda_m^0\right)_{KBr} - \left(\Lambda_m^0\right)_{NaBr} = \Lambda^0_{m\,K^+} - \Lambda^0_{m\,Na^+}$

35. A

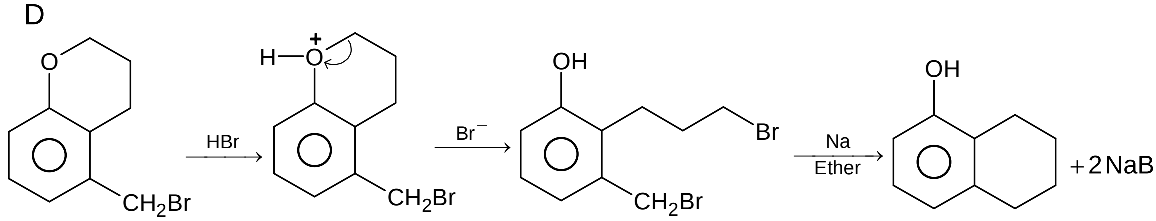
Sol. Carbon-chlorine bond cleavage is not possible in  and $CH_3 = CH - Cl$

36. D

Sol. MA_2B_2 shows geometrical and not optical isomerism.

37. D

Sol.

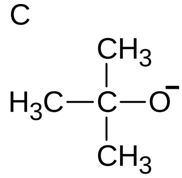


38. C

Sol. $CN^- \Rightarrow$ 14 electrons, so B.O = 3 and diamagnetic.

39. C

Sol.



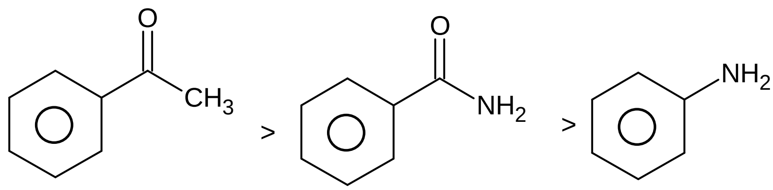
Is bulky base, so elimination is dominating.

40. D

Sol. In strong ligand field Co^{3+} will have $t_{2g}^6 e_g^0$ of configuration and $\Delta t = \frac{4}{9}\Delta_0$

41. C

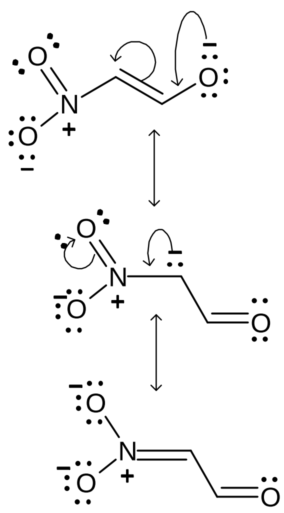
Sol.



Aniline has higher viscosity due to intermolecular H-bonding.

42. C

Sol. C has maximum resonating structure than in B



43. A

Sol. Conceptual.

44. B

Sol. Moles of HCl = moles of NH_3

= 2 × moles of urea $= 2 \times \frac{0.6}{60} = 0.02$

$\Rightarrow$ N.V = 0.02

45. A

Sol. Vapour pressure over solvent is greater than that over solution.

46. 18

Sol. $NaCl + K_2Cr_2O_7 \xrightarrow{H^+} \underset{(A)}{CrO_2Cl_2} \xrightarrow{NaOH} \underset{(B)}{Na_2CrO_4} \xrightarrow[H^+]{H_2O_2} \underset{(C)}{CrO_5}$

$CrO_2Cl_2 \rightarrow 5$

$Na_2CrO_4 \rightarrow 7$

$CrO_3 \rightarrow 6$

47. 192.5

Sol. $2C(gr) + 3H_2(g) \longrightarrow C_2H_6(g)$

$\Delta H_f = 2 \times \Delta H_{comb}(C) + 3 \times \Delta\sigma H_{comb}(H_2) - \Delta H_{Comb}(C_2H_6)$

$\Rightarrow \sigma H_f = 2(-286) + 3(-393.5) - (-1560) = 192.5$

48. 0.3675

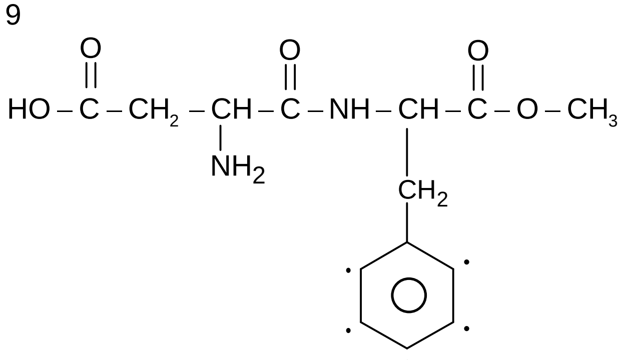
Sol. 30 m.mol L^{-1} of HCl

Then in 250 mL HCl will be $\frac{30}{4}$ m.mol

H_2SO_4 will be $\frac{1}{2}\times\frac{30}{4}\text{m.mol} = 98\times\frac{1}{2}\times\frac{30}{4}\times 16^{-3}\text{g} = 0.3675\text{ g}$

49. 9

Sol.



The dotted ones are sp^2 carbons.

50. 5.2271

Sol. 500 mL has HCl = 25×10^{-3} mol = 25 m.mol

120 mL has = 1 m.mol

and 500 mL has CH_3COOH = $\frac{1}{20}\times 10^3\text{m.mol}$

so 20 mL has $\frac{10^3}{20}\times\frac{20}{500} = 2\text{m.mol}$

NaOH added in 20 mL is $5\times\frac{1}{2} = 2.5\text{m.mol}$

So, $\underset{(\text{left})}{NaOH} + CH_3COOH \longrightarrow CH_3COOH + H_2O$

$\quad 1.5 \qquad\qquad 2 \qquad\qquad\qquad 1.5$

$\Rightarrow$ pH = $p^{K_a} + \log\frac{\text{salt}}{\text{acid}} = 4.75 + \log 0.4771 = 5.2271$

PART C – MATHEMATICS

51. C

Sol. $6.\,^{35}C_r = \left(k^2-3\right).\frac{36}{r+1}\,^{35}C_r$

$\Rightarrow k^2 - 3 = \frac{r+1}{6},\ k^2 - 3 > 0$

(i) $k = \pm 2$ gives r = 5

(ii) $k = \pm 3$ gives r = 35

4 ordered pairs

52. B

Sol. $y=-\frac{3x}{4}+3\sqrt{2}$ line is tangent to ellipse

$\therefore\ c^2=a^2m^2+b^2$

$18=\frac{9a^2}{16}+9$

$a^2=16$

$\therefore e^2=1-\frac{b^2}{a^2}$

$e=\frac{\sqrt{7}}{4}$

Distance between focii = 2ae

$=2\sqrt{7}$

53. C

Sol. $3+4+8+9+13+14+......$ upto 40 terms

$\Rightarrow 7+17+27+........20$ terms

$S=\frac{20}{2}\left[2\times 7+19\times 10\right]$

$=102\times 20=102\ m$

$\therefore\ m=20$

54. B

Sol. $|B|=\begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix}=\begin{vmatrix} 3^0a_{11} & 3^1a_{21} & 3^2a_{31} \\ 3^1a_{12} & 3^2a_{22} & 3^3a_{32} \\ 3^2a_{13} & 3^3a_{23} & 3^4a_{33} \end{vmatrix}$

$81=3^3.3^3.3^2|A|$

$\Rightarrow |A|=\frac{1}{9}$

55. C

Sol. Using LMVT

$f'(c)=\frac{f(1)-f(0)}{1-0}$

$3c^2-8c+8=\frac{16-11}{1-0}$

$3c^2-8c+3=0$

$c=\frac{4-\sqrt{7}}{3}\in(0,1)$

56. D

Sol. $4\alpha\int_{-1}^{0} e^{\alpha x}dx + \int_{0}^{2} e^{-\alpha x}dx = 5$

$\Rightarrow 4\alpha\left(\frac{1-e^{-\alpha}}{\alpha} - \frac{e^{-2\alpha}-1}{-\alpha}\right) = 5$

Let $e^{-\alpha} = t$

$\therefore 4t^2 + 4t - 3 = 0$

$\Rightarrow t = \frac{1}{2}$

$\alpha = \ell n 2$

57. B

Sol. Differentiating

$y.\frac{-2x}{2\sqrt{1-x^2}} + \sqrt{1-x^2}.y' = \frac{x.2yy'}{2\sqrt{1-y^2}} - \sqrt{1-y^2}$

Put $x = \frac{1}{2}, y = -\frac{1}{4}$ and $x, y = -\frac{1}{8}$

$y' = \frac{-\sqrt{5}}{2}$

58. D

Sol. $\frac{dx}{dy} + x = y^2$

I.F. $= e^{\int 1dy} = e^y$

$\Rightarrow x.e^y = \int y^2.e^y\,dy$

$xe^y = y^2e^y - 2ye^y + 2e^y + C$

$\because y(0) = 1$

$\Rightarrow C = -e$

$\therefore xe^y = y^2e^y - 2ye^y + 2e^y - e$

Put $y = 0$

$\therefore x = 0 - 0 + 2 - e$

$\Rightarrow x = 2 - e$

59. B

Sol. $f'(x) = a(x+1)(x-1)x^2$

$f'(x) = ax^4 - x^2 f$

$f(x) = \frac{ax^5}{5} - \frac{ax^3}{5} + C$

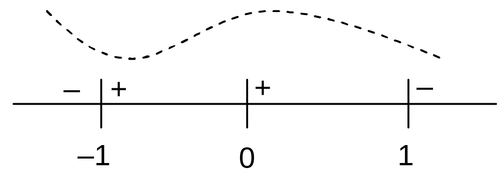
$\because f(0) = 0 \Rightarrow c = 0$

$\lim\limits_{x\to 0} \dfrac{f(x)}{x^3} = 2$

$\Rightarrow a = -6$

$\therefore f'(x) = -6(x^2-1)(x^2)$

Minima at x = –1
Maxima at x = 1



60. C

Sol. $4x^2 = 8x + 12$

$x = -1, 3$

Area $= \int_{-1}^{3} \left[(8x+12) - 4x^2\right] dx$

$= \left[\dfrac{8x^2}{2} + 12x - \dfrac{4x^3}{3}\right]_{-1}^{3}$

$= \dfrac{128}{3}$

61. C

Sol. $3 + 2\left(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}\right) = 0$

$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -\dfrac{3}{2}$

$\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$

$\vec{d} = 3\left(\vec{a} \times \vec{b}\right)$

62. D

Sol. Slope $AB = \dfrac{k-\alpha}{h-2\alpha} = -1$

$\Rightarrow \alpha = \dfrac{k+k}{3}$ …….(1)

also $\dfrac{\beta+2\alpha}{2} = h$, $\dfrac{\beta+\alpha}{2} = k$

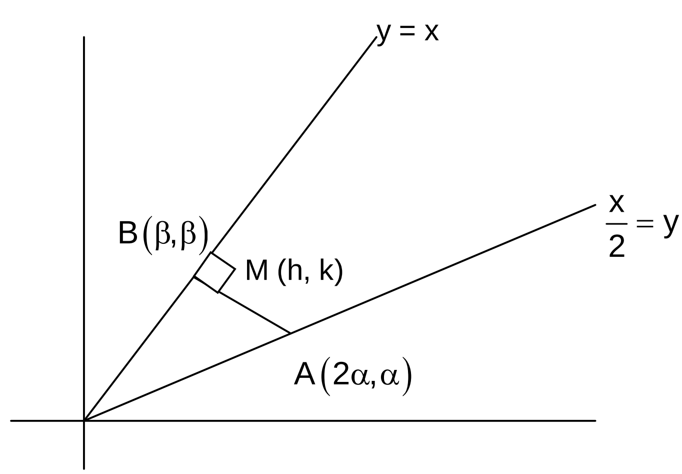
$\alpha = 2h - 2k$ …….(2)

From (1) and (2)

$\dfrac{h+k}{3} = 2h - 2k$

$\Rightarrow 5h = 7k$

$\Rightarrow 5x = 7y$



63. A

Sol. $a_1 + a_2 = 4 \Rightarrow a_1 + a_1 r = 4$ ....(i)

$a_3 + a_4 = 16 \Rightarrow a_1 r^2 + a_1 r^3 = 16$ ....(ii)

$\Rightarrow r = \pm 2$

$r = 2 \Rightarrow a_i = \frac{4}{3}$

$r = -2 \Rightarrow a_1 = -4$

$\sum_{i=1}^{9} a_i = \frac{a\left(r^9 - 1\right)}{\left(r - 1\right)} = \left(-4\right)\frac{\left(\left(-2\right)^9 - 1\right)}{\left(-2-1\right)}$

$= \frac{4}{3}\left(-513\right) = 4\lambda$

$\Rightarrow \lambda = -171$

64. D

Sol. $p_5 = \alpha^5 + \beta^5$

$= \left(\alpha + 1\right)^2 . \alpha + \left(\beta - 1\right)^2 . \beta$

$= 5\alpha + 5\beta + 6$

$= 5\left(1\right) + 6 = 11$

$p_2 = \alpha^2 + \beta^2 = \alpha + \beta + 2 = 3$

$p_3 = \alpha^3 + \beta^3 = \left(\alpha + 1\right).\alpha + \left(\beta + 1\right).\beta$

$= 1 + 3 = 4$

Hence $p_5 \neq p_2 . p_3$

65. B

Sol. Let $z = \frac{3 + i\sin\theta}{4 - i\cos\theta} \times \frac{\left(4 + i\cos\theta\right)}{\left(4 + i\cos\theta\right)}$

$= \frac{12 - \sin\theta\cos\theta + i\left(4\sin\theta + 3\cos\theta\right)}{16 + \cos^2\theta}$

z is real

$\therefore 4\sin\theta + 3\cos\theta = 0$

$\Rightarrow \tan\theta = \frac{-3}{4}\left[\because \theta \text{lies is 2nd quadrant}\right]$

$\arg\left(\sin\theta + i\cos\theta\right) = \pi + \tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$

$= \pi - \tan^{-1}\left(\frac{4}{3}\right)$

66. B

Sol. Probability (at most two machines will be out of service) $=\left(\frac{3}{4}\right)^3.k$

$$\Rightarrow {}^5C_0\left(\frac{3}{4}\right)^5+{}^5C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4+{}^5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3=\left(\frac{3}{4}\right)^3.k$$

$$\Rightarrow \frac{17}{8}.\left(\frac{3}{4}\right)^3=\left(\frac{3}{4}\right)^3.k$$

$$\Rightarrow k=\frac{17}{8}$$

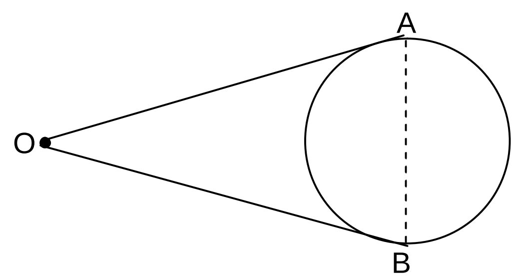
67. C

Sol. $OS=\sqrt{S_1}$

Radius = R = 2

Length of AB $=\frac{2RL}{\sqrt{L^2+R^2}}=\frac{16}{\sqrt{20}}$

$AB^2=\frac{64}{5}$



68. B

Sol. $2\frac{\cos^2\theta}{\sin^2\theta}-\frac{5}{\sin\theta}+4=0$

$(2\sin\theta-1)(\sin\theta-2)=0$

$\sin\theta=\frac{1}{2}$ only

$$\therefore \theta=\underset{\theta_1}{\underset{\downarrow}{\frac{\pi}{6}}},\underset{\theta_2}{\underset{\downarrow}{\frac{5\pi}{6}}}$$

$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\cos^2 3\theta\, d\theta=\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\left(\frac{1+\cos 6\theta}{2}\right)d\theta=\frac{\pi}{3}$$

69. C

Sol. $(1+x)^{10}+x(1+x)^9+x^2(1+x)^8+........+x^{10}$

$$=(1-x)^{10}\frac{\left[1-\left(\frac{x}{1+x}\right)^{11}\right]}{\left(1-\frac{x}{1+x}\right)}$$

$\Rightarrow (1+x)^{11}-x^{11}$

Coefficient of x^7 is ${}^{11}C_7=330$

70. C

Sol. If $A\subseteq B$ and $B\subseteq D$ then $A\subseteq C$
Contrapositive is
If $A\not\subseteq C$, then $A\not\subseteq B$ or $B\not\subseteq D$

71. 5

Sol. $\lim\limits_{x\to 0} f(x)=\lim\limits_{x\to 0}\left(\frac{1}{x}\ell n(1+3x)-\frac{1}{x}\ell n(1-2x)\right)$

$=\lim\limits_{x\to 0}\left(\frac{3\ell n(1+3x)}{3x}-\frac{2\ell n(1-2x)}{-2x}\right)$

$=3+2=5$

f is continuous

$\therefore \lim\limits_{x\to 0} f(x)=f(0)$

$\therefore f(0)=5=k$

72. 13

Sol. $\Delta=\begin{vmatrix}1&1&1\\1&2&3\\3&2&\lambda\end{vmatrix}=0$

$\Rightarrow 1(2\lambda-6)-1(\lambda-9)+1(-4)=0$

$\Rightarrow \lambda=1$

$\Delta_x=\begin{vmatrix}6&1&1\\10&2&3\\\mu&2&\lambda\end{vmatrix}=0$

$\Rightarrow 2\lambda+\mu=16$

$\Rightarrow \mu=14$

$\mu-\lambda^2=14-1=13$

73. 54

Sol. Mean $=10=\dfrac{3+7+9+12+13+20+x+y}{8}$

$16 = x + y$ ………(1)

Variance $\sigma^2 = 25 = \dfrac{\sum x_i^2}{8} - (\text{mean})^2$

$$25 = \frac{3^2 + 7^2 + 9^2 + 12^2 + 13^2 + 20^2 + x^2 + y^2}{8} = 100$$

$x^2 + y^2 = 148$ ……….(2)

$(x+y)^2 = x^2 + y^2 + 2xy$

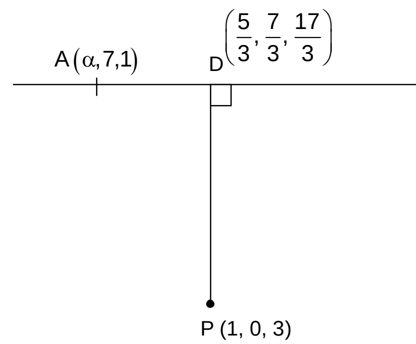
$256 = 148 + 2xy$

$x.y = 54$

74. 4

Sol. $D\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$

$\overrightarrow{AD}.\overrightarrow{PD} = 0$



$$\left(\left(\frac{5}{3} - \alpha\right)\hat{i} + \left(\frac{7}{3} - 7\right)\hat{j} + \left(\frac{17}{3} - 1\right)\hat{k}\right)\left(\frac{2}{3}\hat{i} + \frac{7}{3}\hat{j} + \frac{8}{3}\hat{k}\right) = 0$$

$$\left(\frac{5}{2} - \alpha\right)\frac{2}{3} + \frac{7}{3} \times \left(\frac{14}{3}\right) + \frac{14}{3} \times \frac{8}{3} = 0$$

$\Rightarrow 3\alpha = 12$

$\alpha = 4$

75. 29

Sol. $A = \{2, 4, 6, 8, ...50\} \Rightarrow$ 25 element

$A = \{7, 14, 21, ...49\} \Rightarrow$ 7 elements

$A \cap B = \{14, 28, 42\} = 3$ elements

Required number of elements $= 25 + 7 - 3 = 29$