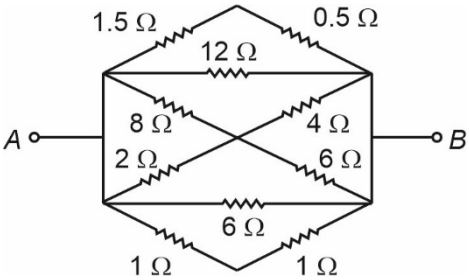
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. In the given circuit the resistance between terminals *A* and *B* is equal to

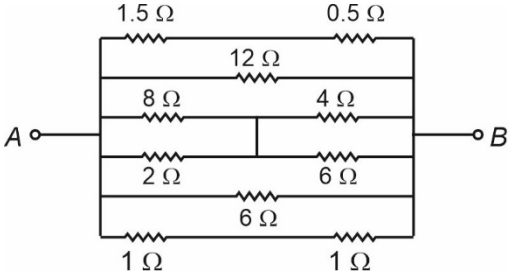


(1) 2 Ω (2) $\frac{3}{2}$ Ω

(3) $\frac{2}{3}$ Ω (4) 6 Ω

Answer (3)

Sol. The circuit can be redrawn as



So the net resistance across *A* and *B* is

$$\frac{1}{R_{net}} = \frac{1}{2}+\frac{1}{12}+\frac{1}{4}+\frac{1}{6}+\frac{1}{2}$$

$$\frac{1}{R_{net}} = \frac{6+1+3+2+6}{12}$$

$$R_{net} = \left(\frac{2}{3}\right)\Omega$$

2. A car travels 4 km distance with a speed of 3 km/h and next 4 km with a speed of 5 km/h. Find average speed of car.

(1) $\frac{15}{2}$ km/h (2) $\frac{15}{4}$ km/h

(3) 15 km/h (4) 10 km/h

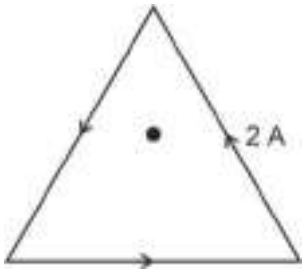
Answer (2)

Sol. $v_{avg} = \frac{\text{Distance}}{\text{Time}}$

$$= \frac{4+4}{\frac{4}{3}+\frac{4}{5}} \text{ km/h}$$

$$= \frac{15}{4} \text{ km/h}$$

3. A current 2 A is flowing through the sides of an equilateral triangular loop of side $4\sqrt{3}$ m as shown. Find the magnetic field induction at the centroid of the triangle.

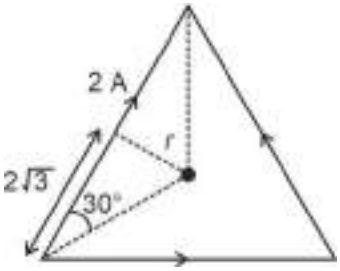


(1) $3\sqrt{3}\times10^{-7}$ T (2) $\sqrt{3}\times10^{-7}$ T

(3) $2\sqrt{3}\times10^{-7}$ T (4) $5\sqrt{3}\times10^{-7}$ T

Answer (1)

Sol.



$$\frac{r}{2\sqrt{3}} = \tan 30°$$

r = 2 m

Magnetic field at centroid

$$= 3\times\frac{\mu_0 I}{4\pi r}(\sin 60° + \sin 60°)$$

$$= 3\times\frac{\mu_0}{4\pi}\times\frac{2}{2}\left[\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right]$$

$$= 3\sqrt{3}\times\frac{\mu_0}{4\pi} \text{ T}$$

$$= 3\sqrt{3}\times10^{-7} \text{ T}$$

4. A particle is released at a height equal to radius of the earth above the surface of the earth. Its velocity when it hits the surface of earth is equal to

(M_e : mass of earth, R_e : Radius of earth)

(1) $v = \sqrt{\frac{2GM_e}{R_e}}$ (2) $v = \sqrt{\frac{GM_e}{2R_e}}$

(3) $v = \sqrt{\frac{GM_e}{R_e}}$ (4) $v = \sqrt{\frac{2GM_e}{3R_e}}$

Answer (3)

Sol. Using energy conservation.

$$-\frac{GMm}{2R_e} = -\frac{GMm}{R_e} + \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{GM_e}{R_e}}$$

5. A faulty scale reads 5°C at melting point and 95°C at steam point.

Find original temperature if this faulty scale reads 41°C.

(1) 40°C (2) 41°C

(3) 36°C (4) 45°C

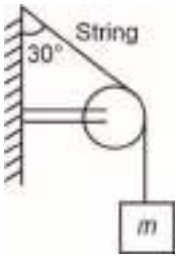
Answer (1)

Sol. $\frac{41-5}{95-5} = \frac{x-0}{100-0}$

$\Rightarrow$ $9x = 360$

$\Rightarrow$ $x = 40$

6. A block stays in equilibrium as shown:



Find the tension in the string if $m = \sqrt{3}$ kg

(1) $\sqrt{3}g$ N (2) $3g$ N

(3) $\frac{g}{2}$ N (4) $\frac{g}{\sqrt{3}}$ N

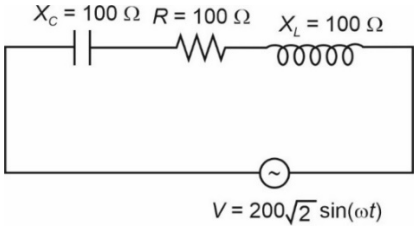
Answer (1)

Sol. Since block in equilibrium

$\Rightarrow$ $T = mg$

$\Rightarrow$ $T = \sqrt{3}g$

7. In the AC circuit shown in the figure the value of I_{rms} is equal to



(1) 2A (2) $2\sqrt{2}$A

(3) 4A (4) $\sqrt{2}$A

Answer (1)

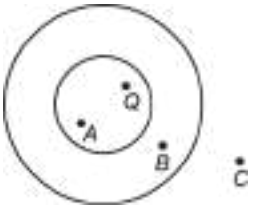
Sol. $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{100^2 + (100-100)^2} = 100\ \Omega$

So, $i_0 = \frac{200\sqrt{2}}{100} = 2\sqrt{2}$

So, $i_{rms} = \frac{i_0}{\sqrt{2}} = 2$A

8. A point charge Q is placed inside the cavity made in uniform conducting solid sphere as shown. E_A, E_B and E_C are electric field magnitudes at points A, B and C respectively, Then



(1) $E_A = 0$, $E_B = 0$ and $E_C \neq 0$

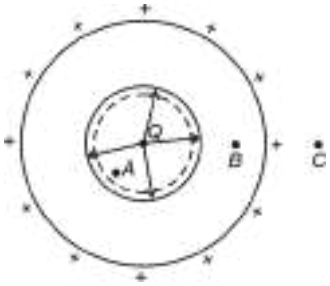
(2) $E_A \neq 0$, $E_B = 0$ and $E_C \neq 0$

(3) $E_A \neq 0$, $E_B = 0$ and $E_C = 0$

(4) $E_A \neq 0$, $E_B \neq 0$ and $E_C \neq 0$

Answer (2)

Sol. Taking Q as positive

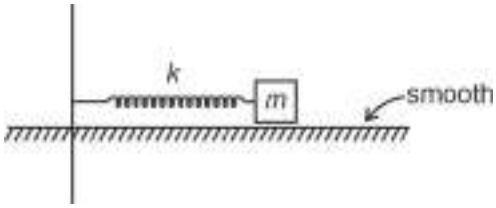


$E_A \neq 0$ (electric field due to both Q and induced charge on the inner surface of cavity)

$E_B = 0$ (No field line inside conductor)

$E_C \neq 0$ (electric field due to charge induced on outer surface of conductor).

9. In the shown mass-spring system when it is set into oscillations along the spring, it has angular frequency ω_1, when m = 1 kg and ω_2 if m = 2 kg. Then value of $\frac{\omega_1}{\omega_2}$ is equal to



(1) 1 (2) $\sqrt{2}$

(3) $\frac{1}{\sqrt{2}}$ (4) 2

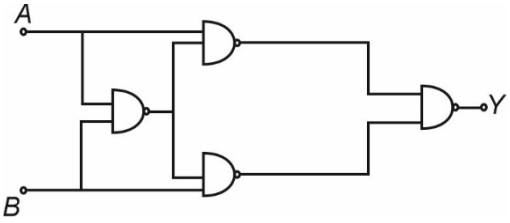
Answer (2)

Sol. $\omega_1 = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{1}}$

$\omega_2 = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{2}}$

So $\frac{\omega_1}{\omega_2} = \sqrt{\frac{k}{k/2}} = \sqrt{2}$

10. For the given logic circuit which of the following truth table is correct?



(1)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(2)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(3)

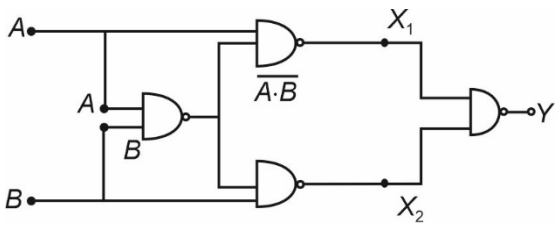
A	B	Y
0	0	1
0	1	1
1	0	1
1	0	0

(4)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Answer (1)

Sol.



$X_1 = \overline{\overline{A \cdot (\overline{A \cdot B})} \cdot \overline{B \cdot (\overline{A \cdot B})}}$

$= A \cdot (\overline{AB}) + B.(\overline{AB})$

$= A \cdot (\overline{A} + \overline{B}) + B \cdot (\overline{A} + \overline{B})$

$= A\overline{B} + B\overline{A}$

= XOR gate

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

11. A particle of mass m is moving under a force whose delivered power P is constant. Initial velocity of particle is zero. Find position of particle at t = 4s.

(1) $x = \frac{16}{3}\sqrt{\frac{2P}{m}}$ (2) $x = \frac{4}{3}\sqrt{\frac{2P}{m}}$

(3) $x = \frac{2}{3}\sqrt{\frac{P}{m}}$ (4) $x = \frac{3}{10}\sqrt{\frac{P}{m}}$

Answer (1)

Sol. $P = \frac{W}{t}$

$\Rightarrow \frac{1}{2}mv^2 = P \cdot t$

$\Rightarrow v = \sqrt{\frac{2Pt}{m}} = \frac{dx}{dt}$

$\Rightarrow x = \frac{16}{3}\sqrt{\frac{2P}{m}}$

12. Column-I list few physical quantities and column-II lists their dimensions. Choose the correct option matching the two lists correctly

	Column-I		Column-II
(P)	Pressure gradient	(A)	$[M^1L^2T^{-2}]$
(Q)	Energy density	(B)	$[M^1L^1T^{-1}]$
(R)	Torque	(C)	$[M^1L^{-2}T^{-2}]$
(S)	Impulse	(D)	$[M^1L^{-1}T^{-2}]$

(1) P-C, Q-A, R-B, S-D (2) P-C, Q-D, R-A, S-B

(3) P-A, Q-D, R-B, S-C (4) P-A, Q-C, R-B, S-D

Answer (2)

Sol. [Pressure gradient] $\Rightarrow \left[\frac{dp}{dz}\right] = \left[\frac{ML^{-1}T^{-2}}{L}\right]$

$= [ML^{-2}T^2]$

[Energy density] $\Rightarrow \left[\frac{dU}{dV}\right] = \left[\frac{ML^2T^{-2}}{L^3}\right] = [ML^{-1}T^{-2}]$

[Torque] $\Rightarrow [F] \times [r] = [MLT^{-2}] \times [L] = [ML^2T^{-2}]$

[Impulse] $\Rightarrow [F]\,[t] = [MLT^{-2}]\,[T] = [MLT^{-1}]$

So, P $\rightarrow$ C, Q $\rightarrow$ D, R $\rightarrow$ A, S $\rightarrow$ B

13. Consider the following assertion & reason:

Assertion (A): At sink temperature of –273°C, the efficiency of a Carnot engine will be 1.

Reason (R): Efficiency of a Carnot engine is given by $\eta = 1 - \frac{T_{sink}}{T_{Source}}$.

(1) (A) is correct, (R) is correct and correctly explains A

(2) (A) is not correct, (R) is correct

(3) Both (A) & (R) are incorrect

(4) Both (A) & (R) are correct, (R) does not explain (A)

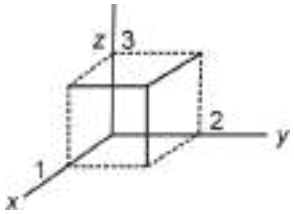
Answer (1)

Sol. $\eta = 1 - \frac{T_{sink}}{T_{Source}}$

If $T_{sink} = 0$ K $\Rightarrow \eta = 1$

14. Electric field in a region is

$\vec{E} = 2x^2\hat{i} - 4y\hat{j} + 6z\hat{k}$



Find the charge inside the cuboid shown:

(1) $-8\varepsilon_0$ (2) $36\varepsilon_0$

(3) $12\varepsilon_0$ (4) $24\varepsilon_0$

Answer (4)

Sol. $\phi_{total} = 2(1)^2[2 \times 3] - 4(2)[1 \times 3] + 6(3)[1 \times 2]$

$= 12 - 24 + 36$

$= 24$

$\Rightarrow \frac{q}{\varepsilon_0} = 24$

$\Rightarrow q = 24\varepsilon_0$

15. Find the ratio of de Broglie wavelength of proton, when it is accelerated across v and $3v$ potential difference.

(1) 3 : 1 (2) $1:\sqrt{3}$

(3) 1 : 3 (4) $\sqrt{3}:1$]

Answer (4)

Sol. When proton is accelerated by potential difference V, the linear momentum of proton

$\frac{P^2}{2m} = eV$

$P = \sqrt{2meV} \Rightarrow \lambda_1 = \frac{h}{\sqrt{2meV}}$

When accelerated by potential difference of 3V, then linear momentum of proton is

$\frac{P^2}{2m} = 3eV$

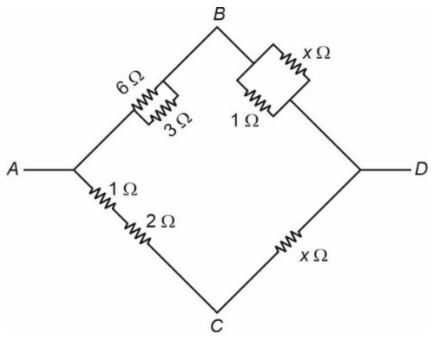
$P = \sqrt{6meV} \Rightarrow \lambda_2 = \frac{h}{\sqrt{6meV}}$

$\frac{\lambda_1}{\lambda_2} = \sqrt{3}$

SECTION - B

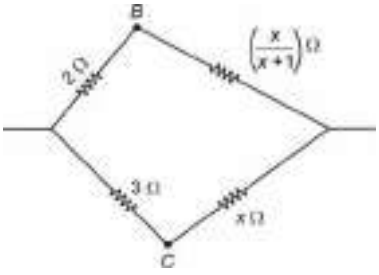
Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. For the given electrical circuit, the potential difference between points B and C is zero. The value of x is _____.



Answer (00.50)

Sol.



$V_B = V_C$

then $\frac{2}{3} = \frac{\left(\frac{x}{x+1}\right)}{x}$

$\Rightarrow \frac{2}{3} = \frac{1}{x+1}$

$x + 1 = \frac{3}{2}$

$\Rightarrow x = \frac{1}{2}\Omega$

22. Two waves of same intensity from sources in phase are made to superimpose at a point. If path difference between these two coherent waves is zero then resultant intensity is I_0. If this path difference is $\frac{\lambda}{2}$ where λ is wavelength of these waves, then resultant intensity is I, and if the path difference is $\frac{\lambda}{4}$ then resultant intensity is I_2. Value of $\frac{I_1 + I_2}{I_0}$ is equal to

Answer (00.50)

Sol. Let individual intensity from source is I thus

$I_0 = I + I + 2\sqrt{I \times I}\cos\left(0 \times \frac{2\pi}{\lambda}\right)$

$\Rightarrow I_0 = 4I$

$I_1 = I + I + 2\sqrt{I \times I}\cos\left(\frac{\lambda}{2} \times \frac{2\pi}{\lambda}\right)$

$\Rightarrow I_1 = 0$

$I_2 = I + I + 2\sqrt{I \times I}\cos\left(\frac{\lambda}{4} \times \frac{2\pi}{\lambda}\right)$

$\Rightarrow I_2 = 2I$

So, $\frac{I_1 + I_2}{I_0} = \frac{1}{2}$ or 0.5

23. A bullet (mass 10 grams) is fired from a gun (mass 10 kg without the bullet) with a speed of 100 m/s. The recoil speed of gun is $\frac{x}{10}$ m/s. Find x.

Answer (1)

Sol. Conserving momentum

$10 \times V = \frac{10}{1000} \times 100$

$\Rightarrow \quad V = \frac{1}{10}$ m/s

24. The ratio of temperature (in K) of hydrogen and oxygen is 2 : 1. The ratio of their average kinetic energy per molecule is

Answer (02.00)

Sol. Average kinetic energy = $\frac{f}{2}K_BT$

$\frac{(\text{Average kinetic energy})_{H_2}}{(\text{Average kinetic energy})_{O_2}} = \frac{T_{H_2}}{T_{O_2}} = \left(\frac{2}{1}\right)$

25. The relation between velocity (v) and position (x) of a particle moving along x-axis is given by $4v^2 = 50 - x^2$. The time period of the oscillatory motion of the particle is $\frac{88}{n}$ seconds.

Find n $\left[\text{use } \pi = \frac{22}{7}\right]$

Answer (07.00)

Sol. $4v^2 = 50 - x^2$

$v^2 = \frac{1}{4}(50 - x^2)$

$v = \frac{1}{2}\sqrt{50 - x^2}$

Comparing equation of S.H.M.

$v = \omega\sqrt{A^2 - x^2}$

$A^2 = 50$

$A = \sqrt{50} = 5\sqrt{2}$

$w = \frac{1}{2} = 0.5$ rad/sec

$T = \frac{2\pi}{w} = \frac{2\pi}{0.5} = 4\pi$ second

$\pi = \left(\frac{22}{7}\right)$

$T = \frac{88}{7} = \frac{88}{n}$

So, $n = 7$

26. Prism A has angle of prism equal to 6° and its material has refractive index 1.5. It is used in combination with prism B of refractive index 1.8 to produce dispersion without deviation. Prism angle of prism B is equal to _____ degrees.

Answer (03.75°)

Sol. For dispersion without deviation

$A_A(\mu_A - 1) + A_B(\mu_B - 1) = 0$

$6(1.5 - 1) + A(1.8 - 1) = 0$

$A = -\frac{3}{0.8} = -3.75°$

CHEMISTRY

SECTION – A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Maximum no. of e^- in n = 4 shell
   (1) 72
   (2) 50
   (3) 16
   (4) 32

Answer (4)

Sol. Maximum number of $e^- = 2n^2$

$= 2(4)^2$

$= 32$

2. BOD value of a water sample is 3 ppm.
   Select the correct option about the given sample of water.
   (1) It is highly polluted water
   (2) It is clean water
   (3) Concentration of oxygen in the given sample is very less
   (4) None of these

Answer (2)

Sol. The given sample of water is clean water as BOD value of clean water ranges between 3 to 5.

3. Which of the following chloride is more soluble in organic solvent?
   (1) Be
   (2) K
   (3) Ca
   (4) Mg

Answer (1)

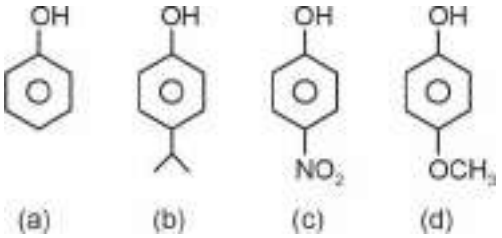
Sol. Out of the given elements, the chlorides of K and Ca are largely ionic. So, they will be more soluble in water and less soluble in organic solvents. $BeCl_2$ has higher covalent character than $MgCl_2$. Therefore, $BeCl_2$ is more soluble in organic solvents than $MgCl_2$.

4. The correct order of bond strength
   H_2O, H_2S, H_2Se, H_2Te
   (1) $H_2O > H_2S > H_2Se > H_2Te$
   (2) $H_2S > H_2O > H_2Se > H_2Te$
   (3) $H_2Te > H_2Se > H_2S > H_2O$
   (4) $H_2Te > H_2S > H_2O > H_2Se$

Answer (1)

Sol. The correct order of bond strength is
$H_2O > H_2S > H_2Se > H_2Te$

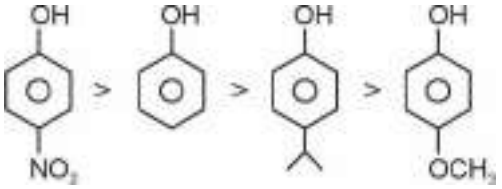
5. The correct order of acidic strength of the following compounds is



   (1) a > b > c > d
   (2) c > a > b > d
   (3) d > c > b > a
   (4) c > b > a > d

Answer (2)

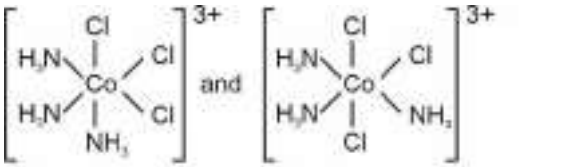
Sol. The correct acidic order is



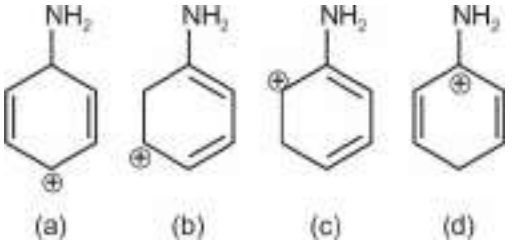
6. What is Cl – Co – Cl bond angle in $[Co(NH_3)_3Cl_3]$?
   (1) 120° and 90°
   (2) 90° and 180°
   (3) 90°
   (4) 180°

Answer (2)

Sol.



7. The correct decreasing order of stability of the following compounds is



(1) a > b > c > d

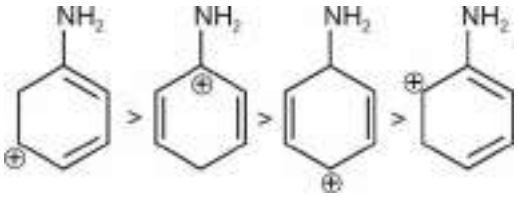
(2) d > b > c > a

(3) b > d > a > c

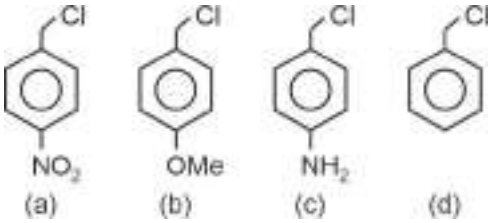
(4) b > a > d > c

Answer (3)

Sol. The correct stability order is



8. Which of the following is correct order of S_N1 reaction?

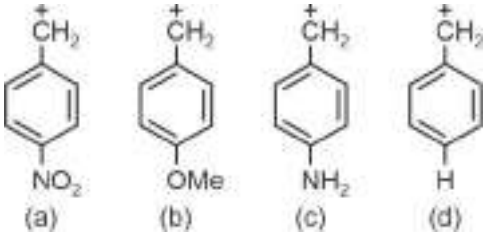


(1) a > b > c > d (2) c > b > d > a

(3) c > a > b > d (4) d > a > b > c

Answer (2)

Sol. The reactivity order of the given aralkyl halides towards S_N1 reaction will be decided by the stability of their corresponding carbocations.



The benzyl carbocation is stabilised by resonance. The presence of $-NH_2$ group at the p-position promotes the resonance stabilisation due to +R effect. The –OMe group also promotes but to a lesser extent due to higher electronegativity of O-atom than N-atom. The $-NO_2$ group opposes the resonance stabilisation due to its –R effect.

$\therefore$ The correct order is c > b > d > a.

9. Lead storage battery have 38% (w/w) H_2SO_4. Find the temperature at which the liquid of battery will freeze

(i = 2.67); k_f of water = $1.86\ \frac{K\cdot kg}{mole}$

(1) –3.1°C

(2) –31°C

(3) –0.31°C

(4) –0.031°C

Answer (2)

Sol. $\Delta T_f = ik_f \cdot m$

$= (2.67)(1.86)(m)$

$m = \frac{38(1000)}{(98)(62)} = 6.25$

$\Delta T_f = (2.67)(1.86)(6.25)$

$= 31.06°C$

Freezing point = –31.06°C

10. $KMnO_4$ oxidises I^- in acidic & neutral medium in which form – respectively.

(1) IO_3^-, IO^-

(2) IO_3^-, IO_3^-

(3) IO_3^-, I_3^-

(4) I_2, IO_3^-

Answer (4)

Sol. : $I^{\ominus}$ converts to I_2 in acidic medium and converts to $IO_3^{\ominus}$ in neutral medium.

11. Which of the following equation is correct?

(1) $LiNO_3 \rightarrow Li + NO_2 + O_2$

(2) $LiNO_3 \rightarrow LiNO_2 + O_2$

(3) $LiNO_3 \rightarrow Li_2O + NO_2 + O_2$

(4) $LiNO_3 \rightarrow Li_2O + N_2O_4 + O_2$

Answer (3)

Sol. $2LiNO_3 \xrightarrow{\Delta} Li_2O + 2NO_2 + \frac{1}{2}O_2$

12. The option containing correct match is

(List-I)		(List-II)
A.	$Ni(CO)_4$	(i) sp^3
B.	$[Ni(CN)_4]^{2-}$	(ii) sp^3d^2
C.	$[Cu(H_2O)_6]^{+2}$	(iii) d^2sp^3
D.	$[Fe(CN)_6]^{4-}$	(iv) dsp^2

(1) A(i), B(iv), C(ii), D(iii)

(2) A(iii), B(ii), C(iv), D(i)

(3) A(ii), B(iii), C(iv), D(i)

(4) A(iv), B(ii), C(i), D(iii)

Answer (1)

Sol. $Ni(CO)_4 \rightarrow sp^3$

$[Ni(CN)_4]^{2-} \rightarrow dsp^2$

$\left[Cu\left(H_2O\right)_6\right]^{+2} \rightarrow sp^3d^2$

$\left[Fe\left(CN\right)_6\right]^{4-} \rightarrow d^2sp^3$

13. Statement 1:– Antihistamine prevents the secretion of acid in stomach

Statement 2: – Antiallergic and antacid work on same receptors

(1) 1 is correct, 2 is incorrect

(2) Both are correct

(3) 1 is incorrect, 2 is correct

(4) Both are incorrect

Answer (4)

Sol. Antihistamines do not affect the secretion of acid in stomach. Antiallergic and antacid drugs work on different receptors. Therefore, both the statements are incorrect.

14. **Statement-1:** During hall-heroult process mixing of CaF_2 and Na_3AlF_6 decreases the M.P. of Al_2O_3.

Statement-2: During electrolytic refining Anode is pure and cathode is impure.

(1) Both are correct

(2) Statement-1 is correct, statement-2 is incorrect

(3) Both are incorrect

(4) Statement-1 is incorrect, statement-2 is correct

Answer (2)

Sol. Mixture of CaF_2 and Na_3AlF_6 decreases the melting point of Al_2O_3.

15. Nessler's reagent is

(1) $K_2[HgI_4]$

(2) $K_3[HgI_4]$

(3) Hg_2I_2

(4) HgI_2

Answer (1)

Sol. Nessler's reagent is $K_2[HgI_4]$

16. Boric acid is present in solid state while BF_3 is a gas at room temperature because

(1) Hydrogen bonding is present in boric acid

(2) Boric acid has more molar mass as compared to BF_3

(3) BF_3 is polymeric in nature

(4) Both (2) and (3)

Answer (1)

Sol. Due to H-bonding, boric acid is solid at room temperature.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. For given E_{cell},

$X | X^{2+}(0.001M) || Y^{2+}(0.01M) | Y$ at 298 K

$E^{\circ}_{X^{2+}/X} = -0.76$

$E^{\circ}_{Y^{2+}/Y} = +0.34$

$\frac{2.303\,RT}{F} = 0.06$

If E_{cell} = t, find 5t (closest integer).

Answer (6)

Sol. $E_{cell} = E^{\circ}_{cell} - \frac{0.06}{2}\log\frac{10^{-3}}{10^{-2}}$

$= 1.10 - 0.03\,(-1)$

$= 1.10 + 0.03$

$t = 1.13$ V

$5t = 5.65$ V

Nearest integer = 6

22. Find the number of formula units of FeO per unit cell (Round off to the nearest integer)

Given that density = 4.0 gm/cm^3

a = 5Å

$N_A = 6.0 \times 10^{23}$

Answer (04)

Sol. Density = $\frac{ZM}{N_A \times a^3} \Rightarrow Z = \frac{\text{density} \times N_A \times a^3}{M}$

$= \frac{4 \times 6.0 \times 10^{23} \times (5 \times 10^{-8})^3}{(56+16)}$

$= \frac{4 \times 6 \times 125 \times 10^{-1}}{72} = 4.16$

23. For 1st order reaction, 540 s is required for 60% completion, then the time for 90% completion is 1.35×10^x. Find x.

($\log 4 = 0.6$)

Answer (3)

Sol. $\frac{t_{90}}{t_{60}} = \frac{\log\frac{100}{100-90}}{\log\left(\frac{100}{100-60}\right)} = \frac{1}{\log\frac{10}{4}} = \frac{1}{1-0.6} = \frac{1}{0.4}$

$t_{90} = \frac{540}{0.4} = 1350$ sec

$1350 = 1.35 \times 10^x$

x = 3

24. 1 mole of a gas undergoes adiabatic process given that $C_V = 20\ JK^{-1}\ mol^{-1}$, w = 3 kJ, $T_1 = 27°C$, $T_2 = ?$ (°C)

Answer (177)

Sol. $w = +nC_v(T_2 - T_1)$

$3000 = 1 \times 20 \times (T_2 - 300)$

$150 = T_2 - 300$

$T_2 = 450$ K

$\Rightarrow T_2 = 177°C$

25. Volume strength of H_2O_2 solution is 60 'V', strength of solution is_____ g/L.

(Round off to the nearest integer)

Answer (182)

Sol. Volume strength of H_2O_2 = 60 volume

Molarity of H_2O_2 solution = $\frac{60}{11.2}$ M

Strength of H_2O_2 solution = $\frac{60 \times 34}{11.2}$

= 182.14 g/L

$\simeq$ 182 g/L

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Common tangent is drawn to $y^2 = 16x$ and $x^2 + y^2 = 8$. Find square of distance between point of contact of common tangent on both the curves.

(1) 78 (2) 72

(3) 42 (4) 76

Answer (2)

Sol. $y = mx + 4/m$

Perpendicular from (0,0)

$$\left|\frac{\frac{4}{m}}{\sqrt{m^2+1}}\right| = \sqrt{8}$$

$\Rightarrow \quad m = \pm 1$

Point of contact on parabola $= \left(\frac{a}{m^2}, \frac{2a}{m}\right)$

$= (4, \pm 8)$

Point of contact in circle = (–2, 2)

OR (2, –2)

Distance between (4, 8) & (–2, 2)

$= \sqrt{72}$

2. Let $f(x) = \begin{cases} \frac{x}{|x|} & , \quad x \neq 0 \\ 1 & x = 0 \end{cases}$

$$g(x) = \begin{cases} \frac{\sin(x+1)}{x+1} & x \neq -1 \\ 1 & x = -1 \end{cases}$$

$h(x) = 2[x] + f(x)$

([·] denotes greatest integer function)

then $\lim\limits_{x \to 1} g\left(h(x-1)\right)$ is

(1) $\frac{\sin 1}{1}$ (2) $\frac{\sin 2}{2}$

(3) –1 (4) 2

Answer (2)

Sol. $h(x-1) = 2[x-1] + f(x-1)$

$$\lim_{x \to 1^+} h(x-1) = 2.0 + f(0^+)$$

$= 1$

$$\lim_{x \to 1^-} h(x-1) = 2(-1) + f(0^-)$$

$= -3$

RHL

$$\lim_{x \to 1^+} g\left(h(x-1)\right) = \frac{\sin 2}{2}$$

LHL

$$\lim_{x \to 1^-} g\left(h(x-1)\right) = \frac{\sin 2}{2}$$

LHL = RHL

$$\therefore \quad \lim_{x \to 1} g\left(h(x-1)\right) = \frac{\sin 2}{2}$$

3. If $|\vec{a}| = 1, |\vec{b}| = 2, \vec{a} \cdot \vec{b} = 4, \quad \vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$ then $\vec{b} \cdot \vec{c}$ equals

(1) – 48 (2) – 12

(3) 12 (4) 48

Answer (2)

Sol. $\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$

$\vec{b} \cdot \vec{c} = -3\vec{b} \cdot \vec{b}$

$= -3|\vec{b}|^2$

$= -12$

4. $\lim\limits_{n \to \infty} \frac{3}{n}\left[4 + \left(2 + \frac{1}{n}\right)^2 + \left(2 + \frac{2}{n}\right)^2 + \ldots + \left(3 - \frac{1}{n}\right)^2\right]$ is

(1) 19 (2) 21

(3) –19 (4) 0

Answer (1)

Sol. $\lim\limits_{n \to \infty} \frac{3}{n} \sum\limits_{r=0}^{n-1} \left(2 + \frac{r}{n}\right)^2$

$= 3\int_0^1 (2+x)^2 dx$

$= 3\frac{(2+x)^3}{3}\Big|_0^1$

$= 27 - 8$

$= 19$

5. Let $f(x)=\sqrt{3-x}+\sqrt{x+2}$. The range of $f(x)$ is

(1) $[2\sqrt{2},\ \sqrt{10}]$ (2) $[\sqrt{5},\ \sqrt{10}]$

(3) $[\sqrt{2},\ \sqrt{7}]$ (4) $[\sqrt{7},\ \sqrt{10}]$

Answer (2)

Sol. $y=\sqrt{3-x}+\sqrt{x+2}$

$$y'=\frac{1}{2\sqrt{3-x}}(-1)+\frac{1}{2\sqrt{x+2}}=0$$

$$\Rightarrow \quad \sqrt{x+2}=\sqrt{3-x}$$

$$\Rightarrow \quad x=\frac{1}{2}$$

$$\Rightarrow \quad y\left(\frac{1}{2}\right)=\sqrt{\frac{5}{2}}+\sqrt{\frac{5}{2}}$$

$y_{max}=\sqrt{10}$

y_{min} at $x=-2$ or $x=3$ is $\sqrt{5}$

$$\therefore \quad y\in\left[\sqrt{5},\ \sqrt{10}\right]$$

6. The value of $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{1}{1+a_2a_3}\right)+....+\tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$

if $a_1=1$ and a_i are consecutive natural numbers

(1) $\frac{\pi}{4}-\cot^{-1}(2021)$

(2) $\frac{\pi}{4}-\cot^{-1}(2022)$

(3) $\frac{\pi}{4}-\tan^{-1}(2021)$

(4) $\frac{\pi}{4}-\tan^{-1}(2022)$

Answer (2)

Sol. $\tan^{-1}\left(\frac{a_2-a_1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{a_3-a_2}{1+a_2a_3}\right)+....+\tan^{-1}\left(\frac{a_{2022}-a_{2021}}{1+a_{2021}a_{2022}}\right)$

$$=\left(\tan^{-1}a_2-\tan^{-1}a_1\right)+\left(\tan^{-1}a_3-\tan^{-1}a_2\right)+...+\left(\tan^{-1}a_{2022}-\tan^{-1}a_{2021}\right)$$

$$=\tan^{-1}a_{2022}-\tan^{-1}a_1$$

$$\therefore \quad a_1=1,\ a_2=2....a_{2022}=2022$$

$$=\tan^{-1}2022-\tan^{-1}1$$

$$=\tan^{-1}2022-\frac{\pi}{4}$$

$$=\frac{\pi}{2}-\cot^{-1}2022-\frac{\pi}{4}$$

$$=\frac{\pi}{4}-\cot^{-1}2022$$

7. Let $P=\left(8\sqrt{3}+13\right)^{13}, Q=\left(6\sqrt{2}+9\right)^{9}$ then (where [ ] represents greatest integer function)

(1) [P] = Odd, [Q] = Even

(2) [P] = Even, [Q] = Odd

(3) [P] = Odd, [Q] = Odd

(4) [P] + [Q] = Even

Answer (4)

Sol. Let $P=I_1+f_1, f_1'=\left(8\sqrt{3}-13\right)^{13}$

$$I_1+f_1-f_1'=\left(8\sqrt{3}+13\right)^{13}-\left(8\sqrt{3}-13\right)^{13}$$

$$=2\left({}^{13}C_1\left(8\sqrt{3}\right)^{12}(13)^1+{}^{13}C_3\left(8\sqrt{3}\right)^{10}(13)^3+{}^{13}C_5\left(8\sqrt{3}\right)^{8}(13)^5+...+{}^{13}C_{13}\left(8\sqrt{3}\right)^{0}(13)^{13}\right)$$

$f_1-f'=0$

So, I_1 is even

Let $Q=I_2+f_2, f_2'=\left(9-6\sqrt{2}\right)^9$

$$I_2+f_2-f_2'=\left(9+6\sqrt{2}\right)^9-\left(9-6\sqrt{2}\right)^9$$

$$=2\left[{}^9C_0 9^9+{}^9C_2 9^7\left(6\sqrt{2}\right)^2+...\right]$$

Again $f_2-f_2'=0$

I_2 = even

8. Let p : I am well.

q : I will not take rest

r : I will not sleep properly, then

"If I am not well then I will not take rest and I will not sleep properly" is logically equivalent to

(1) $(\sim p\rightarrow q)\vee r$ (2) $\sim p\rightarrow(q\wedge r)$

(3) $(\sim p\wedge q)\rightarrow r$ (4) $(\sim p\vee q)\rightarrow r$

Answer (2)

Sol. ~ p : I am not well

q : I will not take rest

r : I will not sleep properly

I will not take rest and I will not sleep properly $\equiv q \wedge r$

If I am not well then I will not take rest and I will not sleep properly $\equiv \sim p \rightarrow (q \wedge r)$

9. q is maximum value of p lying in interval [0, 10], roots of $x^2 - px + \frac{5p}{d} = 0$ are having rational roots.

Find area of region

$S : \{0 \le y \le (x-q)^2\}$

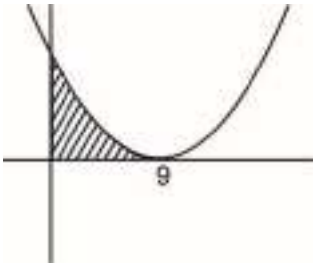
(1) 243 (2) 723

(3) 81 (4) 3

Answer (1)

Sol. $D = p^2 - 5p$ must be a perfect square i.e. possible when $p = 9$

Region for $0 \le y \le (x-9)^2$, in 1st quadrant



$$A = \int_0^9 (x-9)^2 dx$$

$$= \frac{(x-9)^3}{3}\Big|_0^9 = 0 + \frac{9^3}{3}$$

= 243 sq. unit

10. If $\frac{dy}{dx} = -\frac{3x^2+y^2}{3y^2+x^2}$, $y(1) = 0$, then $f(x)$ is

(1) $\log(x+y) + \frac{2xy}{(x+y)^2} = 0$

(2) $\log(x+y) - \frac{2xy}{(x+y)^2} = 0$

(3) $3 = (3y^2 - 2xy + 3x^2)(x+y)^2$

(4) $3 = (3y^2 - 2xy + 3x^2)(x+y)$

Answer (3)

Sol. $\frac{dy}{dx} = -\frac{3x^2+y^2}{3y^2+x^2} = -\frac{3+\left(\frac{y}{x}\right)^2}{3\left(\frac{y}{x}\right)^2+1}$

Let, $\frac{y}{x} = u$

$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

$$u + x\frac{du}{dx} = \frac{-(3+u^2)}{3u^2+1}$$

$$x\frac{du}{dx} = \frac{-(3+u^2) - u(3u^2+1)}{3u^2+1}$$

$$x\frac{du}{dx} = \frac{-\left[3u^3+u^2+u+3\right]}{(3u^2+1)}$$

$$x\frac{du}{dx} = \frac{-(u+1)(3u^2-2u+3)}{3u^2+1}$$

$$\int \frac{3u^2+1}{(u+1)(3u^2-2u+3)} du = -\int \frac{dx}{x}$$

$$\int \frac{\frac{1}{2}}{u+1} + \frac{\frac{1}{4}(6u-2)}{3u^2-2u+3} du = -\int \frac{dx}{x}$$

$$\frac{1}{2}\ln|(u+1)| + \frac{1}{4}\ln|3u^2-2u+3| = -\ln x + C$$

$$\frac{1}{2}\ln(x+y) - \frac{1}{2}\ln x + \frac{1}{4}\ln(3y^2-2xy+3x^2)$$

$$-\frac{1}{4}\times 2\ln x = -\ln x + C$$

$$\ln(x+y)^2 + \ln(3y^2-2xy+3x^2) = C$$

$$(x+y)^2(3x^2-2xy+3y^2) = C$$

$y(1) = 0$

$\Rightarrow$ $C = 3$

$$\boxed{(x+y)^2(3x^2-2xy+3y^2) = 3}$$

11. A bag contains 3 same balls and 3 different balls of three different colours. Two balls are drawn randomly with replacement. The probability they have same colour is m. Again four balls are drawn one by one with replacement, then probability of getting three same balls is n. The value of $m.n$ is

(1) $\frac{3}{49}$ (2) $\frac{6}{49}$

(3) $\frac{43}{147}$ (4) $\frac{8}{81}$

Answer (4)

Sol. For m

both balls is one of different colours $= \left(\frac{1}{6}\times\frac{1}{6}\right)\cdot 3$

both balls is from the same balls $= \frac{1}{2}\times\frac{1}{2}$

$\therefore \quad m = \frac{1}{4}+\frac{1}{12}=\frac{1}{3}$

For n

Same ball is from the different coloured balls

$$= 3\left(4\left(\frac{1}{6}\right)^3\cdot\frac{5}{6}\right)$$

Or same ball is from the 3 same balls

$$= = \left(4\left(\frac{1}{2}\right)^3\cdot\frac{1}{2}\right)$$

$\therefore \quad n = \frac{10}{6^3}+\frac{1}{4}=\frac{8}{27}$

$\therefore \quad m\cdot n = \frac{8}{81}$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Two A.P.'s are given as under

3, 7, 11, ……

1, 6, 11, 16,….

Find 8^{th} common term that is appearing in both the series

Answer (151)

Sol. First common term is 11 and common terms will appear in an A.P. having common difference as LCM of (4, 5) = 20

$T_8 = 11 + (8 - 1)\,20$

$= 151$

22. Using 1, 2, 2, 2, 3, 3, 5 find number of 7-digit odd numbers that can be formed

Answer (240)

Sol. - - - - - - $1 \rightarrow \frac{6!}{2!3!} = 60$

- - - - - - $3 \rightarrow \frac{6!}{3!} = 120$

- - - - - - $5 \rightarrow \frac{6!}{3!2!} = 60$

Total = 240

23. 50^{th} root of x is 12

50^{th} root of y is 18

Remainder when $x + y$ is divided by 25.

Answer (23)

Sol. $12^{50} + 18^{50} = 144^{25} + 324^{25}$

$= (25K_1 - 6)^{25} + (25K_2 - 1)^{25}$

$= 25\lambda - 6^{25} - 1$

$6^{25} + 1 = (6^5)^5 + 1$

$= (7776)^5 + 1$

$= (25\lambda_1 + 1)^5 + 1 = 25p + 2$

$\Rightarrow \quad 12^{50} + 18^{50} = 25\lambda - (25p + 2)$

$\Rightarrow$ Remainder = 23

24. Let $a = \{1, 3, 5, \ldots 99\}$

and $b = \{2, 4, 6, \ldots 100\}$

The number of ordered pair (a, b) such that $a + b$ when divided by 23 leaves remainder 2 is

Answer (108)

Sol. $a + b = 23\lambda + 2$

$\lambda = 0, 1, 2, \ldots.$

But λ can't be even

$\therefore$ if $\lambda = 1$ $(a, b) \rightarrow 12$ pairs

$\lambda = 3$ $(a, b) \rightarrow 35$ pairs

$\lambda = 5$ $(a, b) \rightarrow 42$ pairs

$\lambda = 7$ $(a, b) \rightarrow 19$ pairs

$\lambda = 9$ $(a, b) \rightarrow 0$ pairs

$\vdots$

Total = 12 + 35 + 42 + 19 = 108 ordered paris

25. Let a line parallel to $x + 3y - 2z - 2 = 0 = x - y + 2z$ and passes through (2, 3, 1). If distance of point (5, 3, 8) from the line is α, then $3\alpha^2$ is

Answer (158)

Sol. Let $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$

$\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$

Line will be parallel to $\vec{a} \times \vec{b}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ 1 & -1 & 2 \end{vmatrix} = \hat{i}(4) - \hat{j}(4) + \hat{k}(-4)$$

$\Rightarrow \quad \vec{n} = \hat{i} - \hat{j} - \hat{k}$

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{n}|}{|\vec{n}|}$$

where $\vec{a}_2 = 5\hat{i} + 3\hat{j} + 8\hat{k}$, $\vec{a}_1 = 2\hat{i} + 3\hat{j} + \hat{k}$

$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 7\hat{k}$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 7 \\ 1 & -1 & -1 \end{vmatrix}$$

$$= \hat{i}(7) - \hat{j}(-10) + \hat{k}(-3)$$

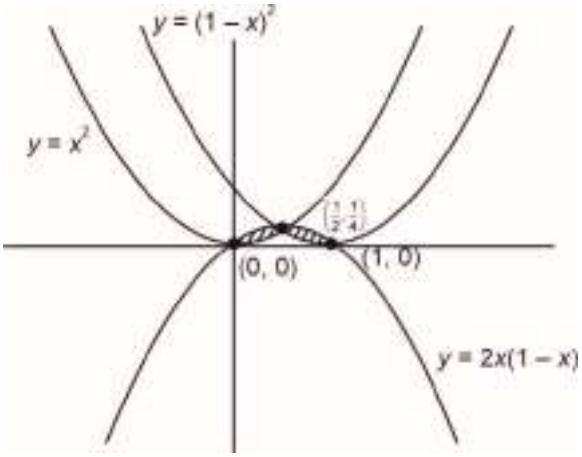
$$d = \frac{\sqrt{100 + 49 + 9}}{\sqrt{3}} = \frac{\sqrt{158}}{\sqrt{3}} = \alpha$$

$3\alpha^2 = 158$

26. If area of the region bounded by the curves $y = x^2$, $y = (1 - x)^2$ and $y = 2x(1 - x)$ is A, then find the value of $540A$,

Answer (135)

Sol. $$A = \int_0^{1} 2x(1-x)dx - \int_0^{1/2} x^2 dx - \int_{1/2}^{1} (1-x)^2 dx$$



$$= x^2 - \frac{2x^3}{3}\Big]_0^1 - \frac{x^3}{3}\Big]_0^1 + \frac{(1-x)^3}{3}\Big]_{1/2}^1$$

$$= \frac{1}{4}$$

$540A = 135$

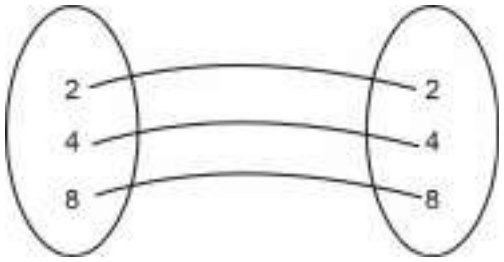
27. $A = \{2, 4, 6, 8, 10\}$

Find total no. of functions defined on A such that $f(m \cdot n) = f(m) \cdot f(n)$, $m, n \in A$

Answer (25)

Sol. $f(4) = (f(2)^2 = 4$

$f(8) = (f(2)^3 = 8$



For 6 and 10 we have 5 options

Total functions = 5 × 5 = 25