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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2021

COMPUTER BASED TEST (CBT)
Memory Based Questions & Solutions

Date: 20 July, 2021 (SHIFT-2) | TIME : (3.00 p.m. to 6.00 p.m)

Duration: 3 Hours | Max. Marks: 300

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PART : PHYSICS

Ans. (2)

Sol. K.E. $\Rightarrow K = \frac{P^2}{2m}$

$P \propto \sqrt{K}$

$\frac{P_2}{} \quad \sqrt{\frac{K_2}{}} = \frac{P_2}{} \quad \sqrt{\frac{4K}{}}$

$$\frac{P_2}{P_1} = \sqrt{\frac{K_2}{K_1}} \Rightarrow \frac{P_2}{P_1} = \sqrt{\frac{4K}{K}}$$

$\Rightarrow \quad \frac{P_2}{P_1} = 2$

$\Rightarrow \quad \frac{P_2 - P_1}{P_1}\% = \left(\frac{P_2}{P_1} - 1\right) \times 100 = (2 - 1) \times 100 = 100$

$\Rightarrow \quad \frac{\Delta P}{P_1}\% = 100\%$

2. A RLC circuit is in its resonance condition. Its circuit components have value

$R = 5\Omega$

$L = 2H$

Then find power in circuit ?

(1) 6kW (2) 10kW (3) 12kW (4) 12.5kW

Ans. (4)

Sol. As circuit is in resonance. Thus

$X_L = X_C$

$\therefore \quad Z = R$ so $i_{rms} = V/Z = V/R$

$P = i_{rms}^2 R$

$P = \frac{V^2}{R} = \frac{250 \times 250}{5} = 12500$ J/s = 12.5 kW

3. A wheel rotating with an angular speed of 600 rpm is given an constant angular acceleration of 1800 rpm^2 for 10 sec. Number of revolutions revolved by wheel is :

(1) 125 (2) 100 (3) 75 (4) 50

Ans. (1)

Sol. $\omega_0 = 600$ rpm

$\alpha = 1800$ rpm^2

t = 10 sec = 1/6 minute

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

60 2 36

= 100 + 25 = 125 revolution.

4. $|\vec{P}| = |\vec{Q}|$, $|\vec{P} + \vec{Q}| = |\vec{P} - \vec{Q}|$. Find angle between $\vec{P}$ & $\vec{Q}$

(1) 45° (2) 90° (3) 135° (4) 150°

Ans. (2)

Sol. $|\vec{P} + \vec{Q}| = |\vec{P} - \vec{Q}|$

$|\vec{P}|^2 + |\vec{Q}|^2 + 2|\vec{P}||\vec{Q}|\cos\theta = |\vec{P}|^2 + |\vec{Q}|^2 - 2|\vec{P}||\vec{Q}|\cos\theta$

$|\vec{P}||\vec{Q}|\cos\theta = 0°$

Thus, $\theta = 90°$

5. A body is moved from rest along straight line by a machine delivering a constant power. Time taken by body to travel a distance "S" is proportional to

(1) $S^{1/3}$ (2) $S^{2/3}$ (3) $S^{1/2}$ (4) $S^{1/4}$

Ans. (2)

Sol. Energy supply = Pt

$V \propto \sqrt{t}$

$\frac{dS}{dt} = C\sqrt{t}$

$\int_0^S dS = C\int_0^t t^{1/2}dt$

$S = \frac{2Ct^{3/2}}{3}$

$t^{3/2} = \frac{3S}{2C}$

$t = S^{2/3}\left(\frac{3}{2C}\right)^{2/3}$

$T \propto S^{2/3}$

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6. A uniform rod of young's modulus Y is stretched by two tension T_1 and T_2 such that rods get expanded to length L_1 and L_2 respectively. Find initial length of rod ?

(1) $\frac{L_1T_1 - L_2T_2}{T_1 - T_2}$ (2) $\frac{L_2T_1 - L_1T_2}{T_2 - T_1}$ (3) $\frac{L_1T_2 - L_2T_1}{T_2 - T_1}$ (4) $\frac{L_1}{T_1} \times \frac{T_2}{L_2}$

Ans. (3)

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As $\frac{T}{A} = Y\frac{\Delta\ell}{\ell}$

So, $\frac{T_1}{A} = \frac{Y(L_1 - L_0)}{L_0}$

$\frac{T_2}{A} = \frac{Y(L_2 - L_0)}{L_0}$

Dividing

$\frac{T_1}{T_2} = \frac{L_1 - L_0}{L_2 - L_0}$; $T_1L_2 - T_1L_0 = T_2L_1 - T_2L_0$; $L_0 = \frac{L_1T_2 - L_2T_1}{T_2 - T_1}$

7. Time (T), velocity (C) and angular momentum (h) are choosen as fundamental quantities instead of mass, length and time. In term of these, dimension of mass would be :

(1) $[M] = [T^{-1}C^{-2}h]$ (2) $[M] = [T^{-1}C^{2}h]$

(3) $[M] = [T^{-1}C^{-2}h^{-1}]$ (4) $[M] = [T^{-1}C^{-2}h]$

Ans. (1)

Sol. $M \propto T^xC^yh^z$

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$M^1L^0T^0 = T^{x-y-z}\,L^{y+2z}\,M^z$

On comparing powers

$z = 1$ ...(1)

$x - y - z = 0$ ...(2)

$y + 2z = 0$ ...(3)

$y + 2 \times 1 = 0$

$y = -2$

$x - (-2) - 1 = 0$

$x = -1$

$M \propto T^{-1}C^{-2}h^{1}$

$[M] \propto [T^{-1}C^{-2}h]$

8. Find relation between γ (adiabatic constant) and degree of freedom (f)

(1) $f = \frac{2}{\gamma - 1}$ (2) $f = \frac{\gamma}{\gamma - 1}$ (3) $f = \frac{\gamma - 1}{2}$ (4) $f = \frac{\gamma - 1}{\gamma}$

Ans. (1)

Sol. $C_V = \frac{fR}{2}$

$C_P = \left(\frac{f}{2} + 1\right)R$

$\Rightarrow \gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$

$\gamma = 1 + \frac{2}{f}$

$f = \frac{2}{\gamma - 1}$

9. Two identical drops of Hg coalesce to form a bigger drop. Find ratio of surface energy of bigger drop to smaller drop.

(1) $2^{3/2}$ (2) $3^{2/5}$ (3) $2^{2/3}$ (4) $5^{2/3}$

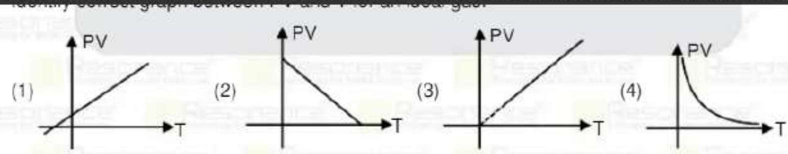
Ans. (3)

Sol. $2 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$\frac{R}{r} = 2^{1/3}$ ...(1)

Now $\frac{U_{bigger}}{U_{smaller}} = \frac{S \times 4\pi R^2}{S \times 4\pi r^2} = \left(\frac{R}{r}\right)^2 = 2^{2/3}$

10. Identify correct graph between PV and T for an ideal gas.



Ans. (3)

Sol. $PV = nRT$

$\Rightarrow PV = CT$

Therefore, PV v/s T graph is straight line.

11. For a body in pure rolling, its rotational kinetic energy is 1/2 times of its translation kinetic energy. They body should be ?

(1) solid cylinder (2) Ring (3) solid sphere (4) Hollow sphere

Ans. (1)

Sol. Given

$\frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{1}{2} mv^2$

as $v = R\omega$ (pure rolling

$I = \frac{1}{2}mR^2$

Thus, solid cylinder.

12. Magnetic susceptibility of material is 499 & $\mu_0 = 4\pi \times 10^{-7}$. SI unit then find μ_r

(1) 500 (2) 400 (3) 300 (4) 200

Ans. (1)

Sol. $\mu_r = 1 + \chi$

$= 1 + 499 = 500$

13. A plane electromagnetic wave travels in free space. Electric field is $\vec{E} = E_0\hat{i}$ and magnetic field is represented by $\vec{B} = B_0\hat{k}$. What is the unit vector along the direction of propagation of electromagnetic wave?

(1) $\hat{j}$ (2) $-\hat{k}$ (3) $-\hat{j}$ (4) $\hat{k}$

Ans. (3)

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Unit vector in direction $\vec{E} \times \vec{B} \Rightarrow \frac{\vec{E} \times \vec{B}}{|\vec{E} \times \vec{B}|}$

$\Rightarrow \frac{E_0\hat{i} \times B_0\hat{k}}{E_0B_0 \sin 90}$

$\Rightarrow \hat{i} \times \hat{k}$

$\Rightarrow -\hat{j}$

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14. Two satellites of mass M_A and M_B are revolving around a planet of mass M in radius R_A and R_B respectively. Then ?

(1) $T_A > T_B$ if $R_A > R_B$ (2) $T_A > T_B$ if $M_A > M_B$

(3) $T_A = T_B$ if $M_A > M_B$ (4) $T_A > T_B$ if $R_A < R_B$

Ans. (1)

Sol. $T \propto r^{3/2}$

$\frac{T_A}{T_B} \propto \left(\frac{R_A}{R_B}\right)^{3/2}$

15. If N_0 active nuclei becomes $\frac{N_0}{16}$ in 80 days. Find half life of nuclei ?

Sol. $N_0 \xrightarrow{t_{1/2}} \frac{N_0}{2} \xrightarrow{t_{1/2}} \frac{N}{4} \xrightarrow{t_{1/2}} \frac{N}{8} \xrightarrow{t_{1/2}} \frac{N}{16}$

$4 \times t_{1/2} = 80$ days

$t_{1/2} = 20$ days

16. A satellite is revolving around a planet in an orbit of radius R. Suddenly radius of orbit becomes 1.02 R then what will be percentage change in its time period of revolution ?

Ans. 3

Sol. As $T \propto R^{3/2}$

$T_1 = kR^{3/2}$

$\frac{\Delta T}{T} = \frac{3}{2} \times \frac{\Delta R}{R} = 3\%$

17. A person walks up a stationary escalator in the time t_1. If he remains stationary on the escalator, then it can take him up in time t_2. Determine the time it would take to walk up on the moving escalator ?

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Ans. (1)

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Sol. Suppose length of escalator = L

Speed of man wrt escalator = $\frac{L}{t_1}$

Speed of escalator = $\frac{L}{t_2}$

Speed of man wrt ground when escalator is moving = $\frac{L}{t_1} + \frac{L}{t_2}$

Time taken by the man to walk on the moving escalator = $\frac{L}{\frac{L}{t_1} + \frac{L}{t_2}} = \frac{t_1 t_2}{t_1 + t_2}$

18. For given graph between decay rate & time. Find half life (where R = decay rate)

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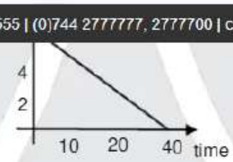


(1) $\frac{10}{3} \ln 2$ (2) $\frac{20}{3} \ln 2$ (3) $\frac{3}{20} \ln 2$ (4) $\frac{20}{3} \ln 2$

Ans. (2)

Sol. $R = R_0e^{-\lambda t}$

$\ln R = \ln R_0 - \lambda t$

$\text{slope} = -\lambda = \frac{-6}{40}$

$\lambda = \frac{3}{20}$

$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{3} \times 20 = \frac{20}{3}\ln 2$

time period of oscillation ?

(1) $2\pi\sqrt{\frac{x_2^2 + x_1^2}{v_1^2 - v_2^2}}$ (2) $2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 + v_2^2}}$ (3) $2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$ (4) $2\pi\sqrt{\frac{x_2^2 + x_1^2}{v_1^2 + v_2^2}}$

Ans. (3)

Sol. $v = \omega\sqrt{A^2 - x^2}$

$v_1 = \omega\sqrt{A^2 - x_1^2}$

$v_2 = \omega\sqrt{A^2 - x_2^2}$

$\left(\frac{v_1}{\omega}\right)^2 - \left(\frac{v_2}{\omega}\right)^2 = x_2^2 - x_1^2$

$\omega^2 = \frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}$

$\omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$

$T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$

(1) $3\lambda_0$ (2) $2\lambda_0$ (3) $4\lambda_0$ (4) $8\lambda_0$

Ans. (3)

Sol. $KE = h\nu - W$

$eV = \frac{hc}{\lambda} - W$

For first case

$e(3V_0) = \frac{hc}{\lambda_0} - W$ ...(i)

For second case

$eV_0 = \frac{hc}{2\lambda_0} - W$ ...(ii)

From equation (i) and (ii)

$W = \frac{hc}{4\lambda_0}$

For λ_{th}

$$\Rightarrow \quad \frac{hc}{4\lambda_0} = \frac{hc}{\lambda_{th}} \quad \Rightarrow \quad \lambda_{th} = 4\lambda_0$$

(1) $\tan^{-1}\left(\frac{1}{\sqrt{6}}\right)$ (2) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (3) $\tan^{-1}\left(\frac{1}{\sqrt{4}}\right)$ (4) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Ans. (1)

Sol. Let vertical and horizontal component of earth's magnetic field at meridian will be V and H.

Angle of dip, $\tan\theta = \frac{V}{H}$ ....(i)

at angle of 45° from magnetic meridian, angle of dip = 30°

$$\tan 30^\circ = \frac{V}{H\cos 45^\circ} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{V}{H\cos 45^\circ}$$

$$\frac{V}{H} = \frac{1}{\sqrt{6}}$$

$$\tan\theta = \frac{V}{H} \Rightarrow \frac{1}{\sqrt{6}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{6}}\right)$$

22. A sodium lamp in space was emitting waves of wavelength 2880Å. When observed from a planet, its wavelength was recorded 2886 Å. Find the speed of planet ?

(1) 4.25×10^5 m/s (2) 6.25×10^5 m/s (3) 2.75×10^5 m/s (4) 3.75×10^5 m/s

Ans. (2)

Sol. $\frac{V_{rel}}{C} = \frac{\Delta\lambda}{\lambda}$

$$V_{rel} = \frac{6}{2880} \times 3 \times 10^8$$

$$= 6.25 \times 10^5 \text{ m/s}$$

23. An electron having debroglie wavelength is falls on an X-ray tube. The cut off wave length of emitted X-Ray is

(1) $\frac{2mc\lambda^2}{h}$ (2) $\frac{2h}{mc}$ (3) $\frac{h}{mc}$ (4) $\frac{2}{3}\frac{mc\lambda^2}{h}$

Ans. (1)

Sol. De-broglie wavelength

$\lambda_B = \frac{h}{P}$

$\Rightarrow P = \frac{h}{\lambda_B}$

$\therefore$ Kinetic energy of electron $\Rightarrow E = \frac{P^2}{2m_e} = \frac{h^2}{2m_e\lambda_B^2}$

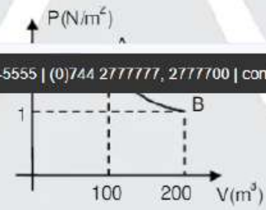
For cut-off wavelength of emitted X-Ray

$E = \frac{hc}{\lambda}$

$\Rightarrow \frac{h^2}{2m_e\lambda_B^2} = \frac{hc}{\lambda}$

$\Rightarrow \lambda = \frac{2m_e c\lambda_B^2}{h} = \frac{2mc\lambda^2}{h}$ where $\lambda_B = \lambda$ & $m_e = m$.

24. A gas is undergoing change in state by an isothermal process AB as follows. Work done by gas in process AB is



(1) 100 ln2 Joule (2) – 100 ln2 Joule (3) 200 ln2 Joule (4) – 200 ln2 Joule

Ans. (3)

Sol. $W_{isothermal} = P_1V_1 \ln\frac{V_2}{V_1}$

$V_1 = 100m^3$

$V_2 = 200m^3$

$P_1 = 2\ N/m^2$

$W = 2 \times 100 \ln\frac{200}{100}$

$= 200 \ln 2$ Joule

25. A block is projected upto a rough plane of inclination 30°. If time of ascending is half the time for descending and the coefficient of friction is $\mu = \frac{3}{5\sqrt{n}}$. Then n =......

Ans. 3

Sol. $S=\frac{1}{2}a_A t_A^2$ .....(1)

$S=\frac{1}{2}a_D t_D^2$ .....(2)

From Equation (1) & (2)

$\frac{t_A^2}{t_D^2}=\frac{a_D}{a_A}$

$\Rightarrow \quad \frac{t_A^2}{t_D^2}=\frac{g\sin\theta-\mu g\cos\theta}{g\sin\theta+\mu g\cos\theta}$

$\Rightarrow \quad \frac{t_A}{t_D}=\sqrt{\frac{g\sin\theta-\mu g\cos\theta}{g\sin\theta+\mu g\cos\theta}}$

$\Rightarrow \quad \frac{1}{2}=\sqrt{\frac{1-\sqrt{3}\mu}{1+\sqrt{3}\mu}}$

$\Rightarrow \quad 1+\sqrt{3}\mu=4-4\sqrt{3}\mu$

$\Rightarrow \quad 5\sqrt{3}\mu=3$

$\Rightarrow \quad \mu=\frac{3}{5\sqrt{3}}$

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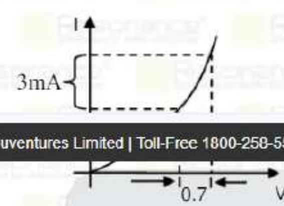
26. I – V characteristic curve of a diode in forward bias is given in fig. find out dynamic resistance -



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(1) 212.3Ω (2) 205.3Ω (3) 245.3Ω (4) 233.3Ω

Ans. (4)

Sol. Dynamic resistance = $\frac{\Delta V}{\Delta I}$

$=\frac{0.7}{3mA}=233.3\Omega$

27. An electron is accelerated through a voltage of 40 kV. What will be its wavelength ?

(1) 0.061Å (2) 0.011Å (3) 0.021Å (4) 0.161Å

Ans. (1)

Sol. $\lambda_B=\frac{h}{P}$

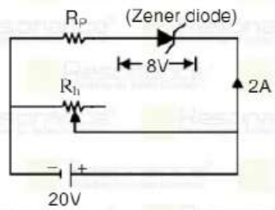
h

$= \frac{}{\sqrt{2meV}}$

$\sqrt{V}$

$= \frac{12.27}{\sqrt{40 \times 10^3}} Å = 0.061 Å$

28. Find value of R_P in given ckt ? (V_Z = 8V)



(1) 4Ω (2) 6Ω (3) 3Ω (4) 5Ω

Ans. (2)

Sol. Applying KVL

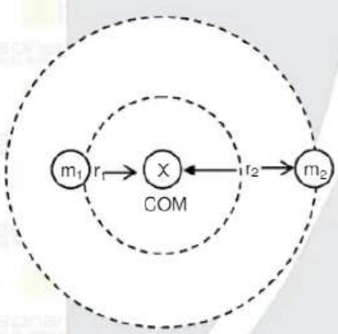
$20 - 8 - 2R_P = 0$

$R_P = 6\Omega$

29. Two stars of masses m_1 and m_2 are in mutual interaction and revolving in orbits of radii r_1 and r_2 respectively. Time period of revolution for this system will be ?

Ans. (2)

Sol.



Let angular velocity will be ω

For mass m_1

$\frac{Gm_1m_2}{} = m_1 r_1 \omega^2 = m_1 \times \frac{m_2(r_1+r_2)}{} \omega^2$

$\omega = \frac{\sqrt{G(m_1+m_2)}}{(r_1+r_2)^{3/2}}$

$T = \frac{2\pi}{\omega}$

$= 2\pi\sqrt{\frac{(r_1+r_2)^3}{G(m_1+m_2)}}$

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SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1 If element of matrix A is defined as $A = [a_{ij}]_{3\times3}$ where $A = \begin{cases} (-1)^{j-i} & i<j \\ 2 & i=j \\ (-1)^{i+j} & i>j \end{cases}$, then the value of $|3\text{Adj}(2A^{-1})|$ is:

(1) 72 (2) 36 (3) 108 (4) 48

Ans. (3)

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Sol. $A = \begin{bmatrix} -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

So, $|A| = \begin{vmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix}$

$= 2(4-1) + 1(-2+1) + 1(1-2)$

$= 2(3) + 1(-1) + 1(-1)$

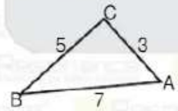
$= 4$

$|3\text{Adj}(2A^{-1})| = 3^3\,|\text{Adj}(2A^{-1})| = 3^3 \times |2A^{-1}|^2$

$= 3^3 \times 2^6 \times |A^{-1}|^2 = 3^3 \times 2^6 \times \dfrac{1}{|A|^2} = 108$

2. In a ΔABC, if $|\overrightarrow{AB}| = 7$, $|\overrightarrow{BC}| = 5$, and $|\overrightarrow{CA}| = 3$. If projection of $\overrightarrow{BC}$ on $\overrightarrow{CA}$ is $\left(\frac{k}{2}\right)$, then the value of k

Sol. $|\overrightarrow{AB}| = 7, \quad |\overrightarrow{BC}| = 5, \quad |\overrightarrow{CA}| = 3,$



Projection of $\overrightarrow{BC}$ on $\overrightarrow{CA}$ is $= |\overrightarrow{BC}| \cos\angle BCA$

$$5\left|\left(\frac{3^2+5^2-7^2}{2.3.5}\right)\right| = = 5\left|\frac{-15}{30}\right| = \frac{5}{2}$$

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3. The value of $\tan(2\tan^{-1}(3/5) + \sin^{-1}(5/13))$ is :

(1) $\frac{220}{21}$ (2) $\frac{110}{21}$ (3) $\frac{55}{21}$ (4) $\frac{20}{11}$

Ans. (1)

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Sol. $\tan\left(\tan^{-1}\frac{\frac{5}{3}}{1-\frac{9}{25}} + \tan^{-1}\frac{5}{12}\right)$

$$\tan\left(\tan^{-1}\left(\frac{15}{8}\right)+\tan^{-1}\left(\frac{5}{12}\right)\right) = \frac{\frac{15}{8}+\frac{5}{12}}{1-\frac{15}{8}.\frac{5}{12}} = \frac{220}{21}$$

4. Mean of 6 observations is 10 and their variance is $\frac{20}{3}$. If observations are 15, 11, 10, 7, a, b then $|a - b|$ is equal to :

(1) 2 (2) 1 (3) 3 (4) 4

Ans. (2)

Sol. Mean = 10

$$\frac{7+10+11+15+a+b}{6} = 10$$

$a + b = 17$ ....(1)

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$$\frac{49+100+121+225+a^2+b^2}{6} - 100 = \frac{20}{3}$$

$a^2 + b^2 = 145$ ....(2)

$(a + b)^2 = 289$

$ab = 72$

$(a - b)^2 = (a + b)^2 - 4ab$

$(a - b)^2 = 289 - 288 = 1$

$|a - b| = 1$

5. If $f(x) = x + 1$, then find $\lim\limits_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+f\left(\frac{10}{n}\right)+..........+f\left(\frac{5(n-1)}{n}\right)\right]$

(1) $\frac{7}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{1}{2}$

Ans. (1)

Sol. $= \lim \frac{1}{n}\sum^{n-1} f\left(\frac{5r}{n}\right) = \int_0^1 f(5x)dx = \int_0^1 (5x+1)dx$

$= \left(\frac{5x^2}{2} + x\right)_0^1 = \frac{5}{2} + 1 = \frac{7}{2}$ Ans.

x is :

(1) 7 (2) 243 (3) 9 (4) 81

Ans. (3)

Sol. $2 \log_9 x + 3 \log_9 x + 4 \log_9 x$ ......21 terms

$= (2 + 3 + 4 + 5 + \ldots\ldots + 22) \log_9 x = \frac{21}{2}(2+22)\log_9 x$

$= 21 \times 12 \log_9 x$

$= 252 \log_9 x$

Given sum $= 252 \Rightarrow \log_9 x = 1$

$\Rightarrow x = 9$

7. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ([x] - [\sin x])\,dx = ?$ (Where $[\cdot]$ represents G.I.F.)

Ans. (3)

Sol. $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ([x] - [\sin x])dx$

using property $\int_{-a}^{a} f(x)dx = \int_{0}^{a} f(x)dx + \int_{0}^{a} f(-x)dx$

$I = \int_{0}^{\frac{\pi}{2}} ([x] + [-x])dx - \int_{0}^{\frac{\pi}{2}} ([\sin x] + [-\sin x])dx = 0$

8. If $\lim_{x \to 0} \frac{\alpha x e^x - \beta \ell n(1+x) + \gamma x^2 e^{-x}}{x^3} = 10$, then the value of $\alpha + \beta + \gamma$ is :

Ans. (3)

Sol. $\lim_{x \to 0} \frac{\ldots}{x^3} = 10$

$\Rightarrow \alpha - \beta = 0, \Rightarrow \alpha = \beta$

$\Rightarrow \alpha + \frac{\beta}{2} + \gamma = 0 \Rightarrow \gamma = -\frac{3\beta}{2}$

$\Rightarrow \frac{\alpha}{2} - \frac{\beta}{3} - \gamma = 10$

$\Rightarrow \frac{\beta}{2} - \frac{\beta}{3} + \frac{3\beta}{2} = 10 \Rightarrow \frac{3\beta - 2\beta + 9\beta}{6} = 10$

$\because \beta = 6, \alpha = 6, \gamma = -9$

So, the value of $\alpha + \beta + \gamma = 3$

9. The value of x satisfying the equation $\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x+1)^2 = 4$ is :

(1) -2 (2) 2 (3) -4 (4) 4

Ans. (2)

Sol. $\log_{(x+1)}((2x+5)(x+1)) + \log_{(2x+5)}(x+1)^2 = 4$

$1 + \log_{(x+1)}(2x+5) + 2\log_{(2x+5)}(x+1) = 4$

$t^2 + t + 2 = 4t \Rightarrow t^2 - 3t + 2 = 0$

$t = 1, t = 2$

For t = 1	For t = 2
$2x + 5 = x + 1$	$2x+5 = (x+1)^2$
$\Rightarrow x = -4$ (rejected)	$x = 2, x = -2$ (rejected)

10. If (α, β) is the point on $y^2 = 6x$, that is closest to $\left(3, \frac{3}{2}\right)$ then find $2(\alpha + \beta)$

(1) 6 (2) 9 (3) 7 (4) 5

Ans. (2)

Sol.

$y^2 = 6x$

$2yy' = 6$

$\frac{dy}{dx} = \frac{3}{\beta}$

$-\frac{\beta}{3} = \frac{\beta - 3/2}{\alpha - 3}$

$-\frac{\beta}{3} = \frac{2\beta - 3}{2\alpha - 6}$

$-\beta(2\alpha - 6) = 6\beta - 9$

$6\beta - 2\alpha\beta = 6\beta - 9$

$\alpha\beta = \frac{9}{2} \Rightarrow \beta = \frac{9}{2\alpha}$

$4\alpha^2$

$\alpha^3 = \frac{27}{8} \quad \alpha = \frac{3}{2}$, $\beta^2 = 9 \Rightarrow \beta = \pm 3$

$\alpha = \frac{3}{2}$, $\beta = 3$

$2(\alpha + \beta) = 9$

11. Two circles pass through (–1,4) and their centres lie on $x^2 + y^2 + 2x + 4y = 4$ r. If r_1 and r_2 are maximum 4 minimum radii and $\frac{r_1}{r_2} = a + b\sqrt{2}$ then the value of a + b is

Ans. 3

Sol. Given circle

$(x + 1)^2 + (y + 2)^2 = (3)^2$

any point on this circle is $(3\cos\theta - 1, 3\sin\theta 2)$ equation of circle having centre $(3\cos\theta - 1, 3\sin\theta - 2)$

$r = \sqrt{(3\cos\theta - 1 + 1)^2 + (3\sin\theta - 2 - 4)^2}$

$= \sqrt{9\cos^2\theta + 9\sin^2\theta + 36 - 36\sin\theta}$

$\sqrt{45 - 36\sin\theta}$

$\Rightarrow r_{max} = 9 = r_1$ and $r_{min} = 3 = r_2$

$\Rightarrow \frac{r_1}{r_2} = \frac{9}{3} = 3 \; 3 + 0\sqrt{2}$

$\Rightarrow a + b = 3$

12. If ΔABC is right angled triangle with sides a,b & c and smallest angle θ. If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are also the sides of right angled triangle then find $\sin\theta$

(1) $\sqrt{\frac{3-\sqrt{5}}{2}}$ (2) $\frac{3-\sqrt{5}}{2}$ (3) $\sqrt{\frac{3+\sqrt{5}}{2}}$ (4) $\frac{3+\sqrt{5}}{2}$

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Ans. (1)

Sol. Let a > b > c

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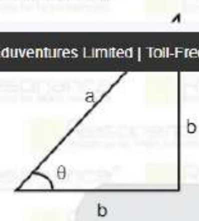

$\sin\theta = \frac{c}{a}$

$\frac{1}{a} < \frac{1}{b} < \frac{1}{c}$

$\frac{1}{c^2} = \frac{1}{a^2} + \frac{1}{b^2}$

$1 = \frac{c^2}{a^2} + \frac{c^2}{b^2}$

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$1 = \sin^2\theta + \frac{1}{\frac{a^2}{c^2} - 1} = \sin^2\theta + \frac{1}{\operatorname{cosec}^2\theta - 1}$

$1 = \frac{1-\sin^2\theta+1}{\text{cosec}^2\theta-1} \Rightarrow \sin^2 q + \text{cosec}^2\theta = 3$

13. If Re $[(1+\cos\theta + 2i\sin\theta)^{-1}] = 4$ then value of θ is :

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $-\frac{\pi}{2}$ (4) π

Ans. (4)

Sol. $\frac{1}{1+\cos^2\theta+2i\sin\theta} \times \frac{1+\cos\theta-2i\sin\theta}{1+\cos\theta-2i\sin\theta}$

$= \frac{1+\cos\theta-2i\sin\theta}{(1+\cos\theta)^2+4\sin^2\theta}$

$1+\cos^2\theta+2\cos\theta+4\sin^2\theta$

$\Rightarrow \frac{1+\cos\theta}{1+\cos^2\theta+2\cos\theta+4-4\cos^2\theta} = 4$

$\Rightarrow \frac{1+\cos\theta}{5+2\cos\theta-3\cos^2\theta} = 4$

$\Rightarrow 1 + \cos\theta = 20 + 8\cos\theta - 12\cos^2\theta$

$\Rightarrow 12\cos^2\theta - 7\cos\theta - 19 = 0$

$\Rightarrow 12\cos^2\theta - 19\cos\theta + 12\cos\theta - 19 = 0$

$\Rightarrow \cos\theta(12\cos\theta - 19) + 1(12\cos\theta - 19) = 0$

$\Rightarrow \cos\theta = -1$ or $\cos\theta = \frac{19}{12}$ (rejected)

$\Rightarrow \theta = \pi$

14. If $x = ay - 1 = z - 2$, and $x = 3y - 2 = bz - 2$ lie in same plane then the value of a, b, is

(1) a = 2, b = 3 (2) a = 1, b = 1 (3) b = 1, $a \in R-\{0\}$ (4) a = 3, b = 2

Ans. (3)

$1 \quad \frac{1}{a} \quad 1 \quad \frac{1}{3} \quad \frac{1}{b}$

$(\vec{a}_1 - \vec{a}_2)\cdot(\vec{b}_1 \times \vec{b}_2) = 0$

$$\begin{vmatrix} 0 & \frac{1}{a}-\frac{2}{3} & 2-\frac{2}{b} \\ 1 & \frac{1}{a} & 1 \\ 1 & \frac{1}{3} & \frac{1}{b} \end{vmatrix} = 0$$

$\Rightarrow \frac{1}{ab} - \frac{1}{a} = 0$

$b = 1$, $a \in R - \{0\}$

15. If $P(\bar{A} \cap B) + P(A \cap \bar{B}) = 1 - K$

$P(\bar{A} \cap C) + P(A \cap \bar{C}) = 1 - 2K$

$P(A \cap B \cap C) = K^2, K \in (0,1)$

Then the value of P(at least one of A,B,C) is:

(1) $> \frac{1}{\ }$ (2) $\left[\frac{1}{\ }, \frac{1}{\ }\right]$ (3) $< \frac{1}{\ }$ (4) $\frac{1}{\ }$

2 [8 4] 4 4

Ans. (1)

Sol. $P(A) + P(B) - 2P(A\cap B) = 1 - K$

$P(A) + P(C) - 2P(A\cap C) = 1 - 2K$

$P(B) + P(C) - 2P(B\cap C) = 1 - K$

$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$

$= \frac{3-4k}{2} + k^2 = \frac{2k^2-4k+3}{2}$

$\because$ The value of $2k^2 - 4k + 3$ is greater than 1

$\therefore$ $P(A\cup B\cup C) > \frac{1}{2}$

(1) –5 (2) 5 (3) 6 (4) –6

Ans. (1)

Sol. $f(f(x)) = \frac{5f(x)+3}{6f(x)+a} = x \Rightarrow 5f(x) + 3 = 6x\, f(x) + ax$

$\Rightarrow \frac{25x+15}{6x+a} + 3 = 6x\left(\frac{5x+3}{6x+a}\right) + ax$

$\Rightarrow 25x + 15 + 18x + 3a = 30x^2 + 18x + 6ax^2 + a^2x$

$\Rightarrow (30+6a)x^2 + (a^2 - 25)x - (3a+15) = 0$

$\Rightarrow 6(a+5)x^2 + (a-5)(a+5)x - 3(a+5) = 0, \quad \forall x$

$\Rightarrow a + 5 = 0 \Rightarrow a = -5$

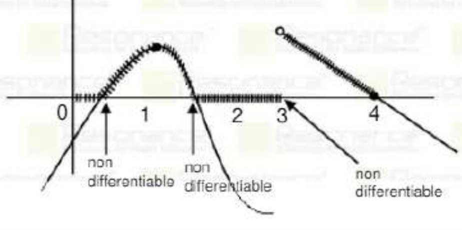
17. If $g(t) = \begin{cases} \max(t^3 - 6t^2 + 9t - 3, 0) & , t \in [0,3] \\ 4 - t & , t \in (3,4] \end{cases}$ then the number of points at which g(t) is non differentiable is :

(1) 1 (2) 2 (3) 3 (4) 4

Sol. $y = t^3 - 6t^2 + 9t - 3$

$y' = 3t^2 - 12t + 9$

$= 3(t^2 - 4t + 3)$



18. A : if 2 + 4 = 7, then 3+4 = 8

B : if 3 + 5 = 8, then earth is flat

C : if A and B are true, then 5+4 = 11

(1) A is true, B and C are false (2) B is true, A and C are false

(3) C is true, A and B are false (4) B is false, A and C are true

Ans. (4)

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

A is true, B is false, C is true .

19. If $A=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, $B\sum_{r=1}^{2021}A^r$ then value of $|B|$ is

(1) 2021 (2) $(2021)^2$ (3) -2021 (4) 0

Ans. (2)

Sol. A=I, B = I + I + ... 2021 times

$B = 2021\begin{bmatrix}1 & 0\\ \end{bmatrix}$

$|B| = (2021)^2$

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Memory Based Questions & Solutions

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Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1 If element of matrix A is defined as $A = [a_{ij}]_{3\times3}$ where $A = \begin{cases} (-1)^{j-i} & i<j \\ 2 & i=j \\ (-1)^{i+j} & i>j \end{cases}$, then the value of $|3\text{Adj}(2A^{-1})|$ is:

(1) 72 (2) 36 (3) 108 (4) 48

Ans. (3)

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Sol. $A = \begin{bmatrix} -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

So, $|A| = \begin{vmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix}$

$= 2(4-1) + 1(-2+1) + 1(1-2)$

$= 2(3) + 1(-1) + 1(-1)$

$= 4$

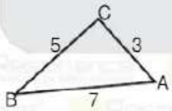
$|3\text{Adj}(2A^{-1})| = 3^3\,|\text{Adj}(2A^{-1})| = 3^3 \times |2A^{-1}|^2$

$= 3^3 \times 2^6 \times |A^{-1}|^2 = 3^3 \times 2^6 \times \dfrac{1}{|A|^2} = 108$

2 In a ΔABC, if $|\overrightarrow{AB}| = 7$, $|\overrightarrow{BC}| = 5$ and $|\overrightarrow{CA}| = 3$. If projection of $\overrightarrow{BC}$ on $\overrightarrow{CA}$ is $\left(\frac{n}{\ }\right)$, then the value of n

2. In a ΔABC, if $|\overrightarrow{AB}| = 7$, $|\overrightarrow{BC}| = 5$, and $|\overrightarrow{CA}| = 3$. If projection of $\overrightarrow{BC}$ on $\overrightarrow{CA}$ is $\left|\frac{5}{2}\right|$, then the value of ...

Sol. $|\overrightarrow{AB}| = 7$, $|\overrightarrow{BC}| = 5$, $|\overrightarrow{CA}| = 3$,



Projection of $\overrightarrow{BC}$ on $\overrightarrow{CA}$ is $= |\overrightarrow{BC}| \cos\angle BCA$

$$5\left|\left(\frac{3^2+5^2-7^2}{2.3.5}\right)\right| = 5\left|\frac{-15}{30}\right| = \frac{5}{2}$$

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(1) $\frac{220}{21}$ (2) $\frac{110}{21}$ (3) $\frac{55}{21}$ (4) $\frac{20}{11}$

Ans. (1)

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Sol. $\tan\left(\tan^{-1}\frac{\frac{6}{5}}{1-\frac{9}{25}} + \tan^{-1}\frac{5}{12}\right)$

$$\tan\left(\tan^{-1}\left(\frac{15}{8}\right)+\tan^{-1}\left(\frac{5}{12}\right)\right) = \frac{\frac{15}{8}+\frac{5}{12}}{1-\frac{15}{8}.\frac{5}{12}} = \frac{220}{21}$$

4. Mean of 6 observations is 10 and their variance is $\frac{20}{3}$. If observations are 15, 11, 10, 7, a, b then $|a-b|$ is equal to :

(1) 2 (2) 1 (3) 3 (4) 4

Ans. (2)

Sol. Mean = 10

$$\frac{7+10+11+15+a+b}{6} = 10$$

$a + b = 17$ ....(1)

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$$\frac{49+100+121+225+a^2+b^2}{6} - 100 = \frac{20}{3}$$

$a^2 + b^2 = 145$ ....(2)

$(a + b)^2 = 289$

$ab = 72$

$(a - b)^2 = (a + b)^2 - 4ab$

$(a - b)^2 = 289 - 288 = 1$

$|a - b| = 1$

5. If $f(x) = x + 1$, then find $\lim\limits_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+f\left(\frac{10}{n}\right)+..........+f\left(\frac{5(n-1)}{n}\right)\right]$

(1) $\frac{7}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{1}{2}$

Ans. (1)

Sol. $= \lim \frac{1}{n}\sum_{r=0}^{n-1} f\left(\frac{5r}{n}\right) = \int_0^1 f(5x)dx = \int_0^1 (5x+1)dx$

$= \left(\frac{5x^2}{2} + x\right)_0^1 = \frac{5}{2} + 1 = \frac{7}{2}$ Ans.

x is :

(1) 7 (2) 243 (3) 9 (4) 81

Ans. (3)

Sol. $2 \log_9 x + 3 \log_9 x + 4 \log_9 x$ ......21 terms

$= (2 + 3 + 4 + 5 + \ldots\ldots + 22) \log_9 x = \frac{21}{2}(2+22)\log_9 x$

$= 21 \times 12 \log_9 x$

$= 252 \log_9 x$

Given sum $= 252 \Rightarrow \log_9 x = 1$

$\Rightarrow x = 9$

7. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ([x] - [\sin x])\, dx = ?$ (Where [·] represents G.I.F.)

Ans. (3)

Sol. $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ([x] - [\sin x])dx$

using property $\int_{-a}^{a} f(x)dx = \int_{0}^{a} f(x)dx + \int_{0}^{a} f(-x)dx$

$I = \int_{0}^{\frac{\pi}{2}} ([x] + [-x])dx - \int_{0}^{\frac{\pi}{2}} ([\sin x] + [-\sin x])dx = 0$

8. If $\lim_{x \to 0} \frac{\alpha x e^x - \beta \ell n(1+x) + \gamma x^2 e^{-x}}{x^3} = 10$, then the value of $\alpha + \beta + \gamma$ is :

Ans. (3)

Sol. $\lim_{x \to 0} \frac{\ldots}{x^3} = 10$

$\Rightarrow \alpha - \beta = 0, \Rightarrow \alpha = \beta$

$\Rightarrow \alpha + \frac{\beta}{2} + \gamma = 0 \Rightarrow \gamma = -\frac{3\beta}{2}$

$\Rightarrow \frac{\alpha}{2} - \frac{\beta}{3} - \gamma = 10$

$\Rightarrow \frac{\beta}{2} - \frac{\beta}{3} + \frac{3\beta}{2} = 10 \Rightarrow \frac{3\beta - 2\beta + 9\beta}{6} = 10$

$\because \beta = 6, \alpha = 6, \gamma = -9$

So, the value of $\alpha + \beta + \gamma = 3$

9. The value of x satisfying the equation $\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x+1)^2 = 4$ is :

(1) -2 (2) 2 (3) -4 (4) 4

Ans. (2)

Sol. $\log_{(x+1)}((2x+5)(x+1)) + \log_{(2x+5)}(x+1)^2 = 4$

$1 + \log_{(x+1)}(2x+5) + 2\log_{(2x+5)}(x+1) = 4$

$t^2 + t + 2 = 4t \Rightarrow t^2 - 3t + 2 = 0$

$t = 1, t = 2$

For $t = 1$ | For $t = 2$

$2x + 5 = x + 1$ | $2x + 5 = (x+1)^2$

$\Rightarrow x = -4$ (rejected) | $x = 2, x = -2$ (rejected)

10. If (α, β) is the point on $y^2 = 6x$, that is closest to $\left(3, \frac{3}{2}\right)$ then find $2(\alpha + \beta)$

(1) 6 (2) 9 (3) 7 (4) 5

Ans. (2)

Sol.

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$y^2 = 6x$

$2yy' = 6$

$\frac{dy}{dx} = \frac{3}{\beta}$

$-\frac{\beta}{3} = \frac{\beta - 3/2}{\alpha - 3}$

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Resonance | JEE MAIN-2021 | DATE : 20-07-2021 (SHIFT-2) | PAPER-1 | MEMORY BASED | MATHEMATICS

$-\frac{\beta}{3} = \frac{2\beta - 3}{2\alpha - 6}$

$-\beta(2\alpha - 6) = 6\beta - 9$

$6\beta - 2\alpha\beta = 6\beta - 9$

$\alpha\beta = \frac{9}{2} \Rightarrow \beta = \frac{9}{2\alpha}$

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$\alpha^3 = \frac{27}{8}\ \ \alpha = \frac{3}{2}$, $\beta^2 = 9 \Rightarrow \beta = \pm 3$

$\alpha = \frac{3}{2}, \beta = 3$

$2(\alpha + \beta) = 9$

11. Two circles pass through (–1,4) and their centres lie on $x^2 + y^2 + 2x + 4y = 4$ r. If r_1 and r_2 are maximum 4 minimum radii and $\frac{r_1}{r_2} = a + b\sqrt{2}$ then the value of a + b is

Ans. 3

Sol. Given circle

$(x + 1)^2 + (y + 2)^2 = (3)^2$

any point on this circle is $(3\cos\theta - 1, 3\sin\theta 2)$ equation of circle having centre $(3\cos\theta - 1, 3\sin\theta - 2)$

$r = \sqrt{(3\cos\theta - 1 + 1)^2 + (3\sin\theta - 2 - 4)^2}$

$= \sqrt{9\cos^2\theta + 9\sin^2\theta + 36 - 36\sin\theta}$

$\sqrt{45 - 36\sin\theta}$

$\Rightarrow r_{max} = 9 = r_1$ and $r_{min} = 3 = r_2$

$\Rightarrow \frac{r_1}{r_2} = \frac{9}{3} = 3\ 3 + 0\sqrt{2}$

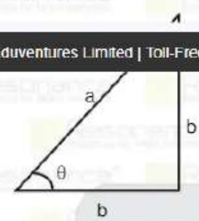
$\Rightarrow a + b = 3$

12. If ΔABC is right angled triangle with sides a,b & c and smallest angle θ. If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are also the sides of right angled triangle then find $\sin\theta$

(1) $\sqrt{\frac{3-\sqrt{5}}{2}}$ (2) $\frac{3-\sqrt{5}}{2}$ (3) $\sqrt{\frac{3+\sqrt{5}}{2}}$ (4) $\frac{3+\sqrt{5}}{2}$

Ans. (1)

Sol. Let a > b > c



$\sin\theta = \frac{c}{a}$

$\frac{1}{a} < \frac{1}{b} < \frac{1}{c}$

$\frac{1}{c^2} = \frac{1}{a^2} + \frac{1}{b^2}$

$1 = \frac{c^2}{a^2} + \frac{c^2}{b^2}$

$1 = \sin^2\theta + \frac{1}{\frac{a^2}{c^2} - 1} = \sin^2\theta + \frac{1}{\mathrm{cosec}^2\theta - 1}$

$1 = \dfrac{1-\sin^2\theta+1}{\text{cosec}^2\theta-1} \Rightarrow \sin^2 q + \text{cosec}^2\theta = 3$

13. If Re $[(1+\cos\theta + 2i\sin\theta)^{-1}] = 4$ then value of θ is :

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $-\frac{\pi}{2}$ (4) π

Ans. (4)

Sol. $\dfrac{1}{1+\cos^2\theta+2i\sin\theta} \times \dfrac{1+\cos\theta-2i\sin\theta}{1+\cos\theta-2i\sin\theta}$

$= \dfrac{1+\cos\theta-2i\sin\theta}{(1+\cos\theta)^2+4\sin^2\theta}$

$1+\cos^2\theta+2\cos\theta+4\sin^2\theta$

$\Rightarrow \dfrac{1+\cos\theta}{1+\cos^2\theta+2\cos\theta+4-4\cos^2\theta} = 4$

$\Rightarrow \dfrac{1+\cos\theta}{5+2\cos\theta-3\cos^2\theta} = 4$

$\Rightarrow 1 + \cos\theta = 20 + 8\cos\theta - 12\cos^2\theta$

$\Rightarrow 12\cos^2\theta - 7\cos\theta - 19 = 0$

$\Rightarrow 12\cos^2\theta - 19\cos\theta + 12\cos\theta - 19 = 0$

$\Rightarrow \cos\theta(12\cos\theta - 19) + 1\,(12\cos\theta - 19) = 0$

$\Rightarrow \cos\theta = -1$ or $\cos\theta = \dfrac{19}{12}$ (rejected)

$\Rightarrow \theta = \pi$

14. If $x = ay - 1 = z - 2$, and $x = 3y - 2 = bz - 2$ lie in same plane then the value of a, b, is

(1) $a = 2, b = 3$ (2) $a = 1, b = 1$ (3) $b = 1, a \in R - \{0\}$ (4) $a = 3, b = 2$

Ans. (3)

$1 \quad \frac{1}{a} \quad 1 \qquad \frac{1}{3} \quad \frac{1}{b}$

$(\vec{a}_1 - \vec{a}_2)\cdot(\vec{b}_1 \times \vec{b}_2) = 0$

$$\begin{vmatrix} 0 & \frac{1}{a}-\frac{2}{3} & 2-\frac{2}{b} \\ 1 & \frac{1}{a} & 1 \\ 1 & \frac{1}{3} & \frac{1}{b} \end{vmatrix} = 0$$

$\Rightarrow \dfrac{1}{ab} - \dfrac{1}{a} = 0$

$b = 1,\ a \in R - \{0\}$

15. If $P(\bar{A} \cap B) + P(A \cap \bar{B}) = 1 - K$

$P(\bar{A} \cap C) + P(A \cap \bar{C}) = 1 - 2K$

$P(A \cap B \cap C) = K^2, K \in (0,1)$

Then the value of P(at least one of A,B,C) is:

(1) $> \frac{1}{\ }$ (2) $\left[\frac{1}{\ }, \frac{1}{\ }\right]$ (3) $< \frac{1}{\ }$ (4) $\frac{1}{\ }$

2 [8 4] 4 4

Ans. (1)

Sol. $P(A) + P(B) - 2P(A\cap B) = 1 - K$

$P(A) + P(C) - 2P(A\cap C) = 1 - 2K$

$P(B) + P(C) - 2P(B\cap C) = 1 - K$

$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$

$= \frac{3-4k}{2} + k^2 = \frac{2k^2 - 4k + 3}{2}$

$\because$ The value of $2k^2 - 4k + 3$ is greater than 1

$\therefore\ P(A\cup B\cup C) > \frac{1}{2}$

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(1) –5 (2) 5 (3) 6 (4) –6

Ans. (1)

Sol. $f(f(x)) = \frac{5f(x)+3}{6f(x)+a} = x \Rightarrow 5f(x) + 3 = 6x\,f(x) + ax$

$\Rightarrow \frac{25x+15}{6x+a} + 3 = 6x\left(\frac{5x+3}{6x+a}\right) + ax$

$\Rightarrow 25x + 15 + 18x + 3a = 30x^2 + 18x + 6ax^2 + a^2x$

$\Rightarrow (30+6a)x^2 + (a^2 - 25)x - (3a+15) = 0$

$\Rightarrow 6(a+5)x^2 + (a-5)(a+5)x - 3(a+5) = 0, \quad \forall x$

$\Rightarrow a+5 = 0 \Rightarrow a = -5$

17. If $g(t) = \begin{cases} \max(t^3 - 6t^2 + 9t - 3, 0), & t \in [0,3] \\ 4 - t, & t \in (3,4] \end{cases}$ then the number of points at which g(t) is non differentiable is :

(1) 1 (2) 2 (3) 3 (4) 4

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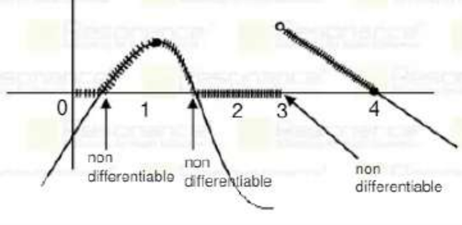
Sol. $y = t^3 - 6t^2 + 9t - 3$

$y' = 3t^2 - 12t + 9$

$= 3(t^2 - 4t + 3)$



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18. A : if $2+4=7$, then $3+4=8$

B : if $3+5=8$, then earth is flat

C : if A and B are true, then $5+4=11$

(1) A is true, B and C are false (2) B is true, A and C are false

(3) C is true, A and B are false (4) B is false, A and C are true

Ans. (4)

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

A is true, B is false, C is true .

19. If A= $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, B $\sum_{r=1}^{2021} A^r$ then value of |B| is

(1) 2021 (2) $(2021)^2$ (3) -2021 (4) 0

Ans. (2)

Sol. A=I, B = I + I + ... 2021 times

$B = 2021\begin{bmatrix} 1 & 0 \\ \end{bmatrix}$

$|B| = (2021)^2$