PART –A (PHYSICS)

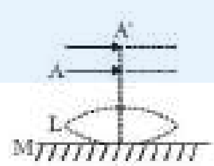
1. Two coils P and Q are separated by some distance. When a current of 3 A flows through coil P a magnetic flux of 10^{-3} Wb passes through Q. No current is passed through Q. When no current passes through P and a current of 2 A passes through Q, the flux through P is:
   (A) 6.67×10^{-3} Wb (B) 6.67×10^{-4} Wb
   (C) 3.67×10^{-4} Wb (D) 3.67×10^{-3} Wb

2. A metal wire of resistance 3Ω is elongated to make a uniform wire of double its previous length. The new wire is now bent and the ends joined to make a circle. If two points on this circle make an angle 60^0 at the centre, the equivalent resistance between these two points will be:
   (A) $\frac{12}{5}\Omega$ (B) $\frac{5}{3}\Omega$
   (C) $\frac{5}{2}\Omega$ (D) $\frac{7}{2}\Omega$

3. The resistance of a galvanometer is 50 ohm and the maximum current which can be passed through it is 0.002 A. What resistance must be connected to it in order to convert it into an ammeter of range 0 – 0.5 A?
   (A) 0.2 ohm (B) 0.002 ohm
   (C) 0.02 ohm (D) 0.5 ohm

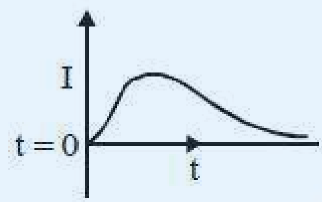
4. The position of a particle as a function of time t, is given by $x(t)=at+bt^2-ct^3$ where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be:
   (A) $a+\frac{b^2}{4c}$ (B) $a+\frac{b^2}{c}$
   (C) $a+\frac{b^2}{2c}$ (D) $a+\frac{b^2}{3c}$

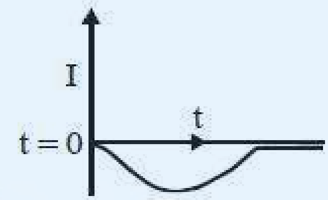
5. A thin convex lens L (refractive index = 15) is placed on a plane mirror M. When a pin is placed at A, such that OA = 18 cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_1 is put between the lens and the mirror, the pin has to be moved to A'. such that OA' = 27cm, to get its inverted real image at A′ itself. The value of μ_1 will be:

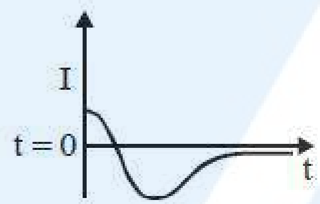


   (A) $\sqrt{2}$ (B) $\frac{4}{3}$
   (C) $\sqrt{3}$ (D) $\frac{3}{2}$

6. A moving coil galvanometer has a coil with 175 turns and area 1 cm^2 It uses a torsion band of torsion constant 10^{-6} N – m/rad. The coil is placed in a magnetic field B parallel to its plane. The coil deflects by 1^0 for a current of 1 mA. The value of B (in Tesla) is approximately:-
(A) 10^{-3}
(B) 10^{-1}
(C) 10^{-4}
(D) 10^{-2}

7. A very long solenoid of radius R is carrying current $I(t) = kte^{-\alpha t}(k > 0)$, as a function of time ($t \geq 0$). counter clockwise current is taken to be positive. A circular conducting coil of radius 2R is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by:-
(A)


(B)


(C)


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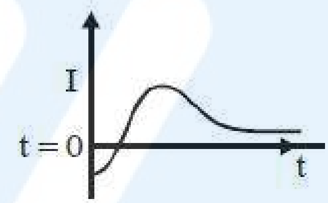


8. A massless spring (k = 800 N/m), attached with a mass (500 g) is completely immersed in 1 kg of water. The spring is stretched by 2 cm and released so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely? (Assume that the water container and spring receive negligible heat and specific heat of mass = 400 J/kg K, specific heat of water = 4184 J/kg K)
(A) 10^{-3} K
(B) 10^{-4}
(C) 10^{-1} K
(D) 10^{-5} K

9. A particle P is formed due to a completely inelastic collision of particles x and y having de – Broglie wavelengths λ_x and λ_y respectively. If x and y were moving in opposite directions, then the de – Broglie wavelength of P is:
(A) $\lambda_x + \lambda_y$
(B) $\frac{\lambda_x\lambda_y}{\lambda_x + \lambda_y}$
(C) $\frac{\lambda_x\lambda_y}{|\lambda_x - \lambda_y|}$
(D) $\lambda_x - \lambda_y$

10. A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distances x_1 and x_2 ($x_1 > x_2$) from the lens. The ratio of x_1 and x_2 is:
(A) 5 : 3
(B) 2 : 1
(C) 4 : 3
(D) 3 : 1

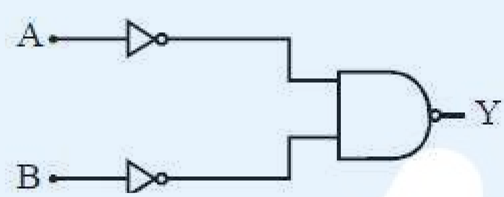
11. Diameter of the objective lens of a telescope is 250 cm. For light of wavelength 600 nm. Coming from a distant object, the limit of resolution of the telescope is close to:-

(A) 1.5×10^{-7} rad
(B) 2.0×10^{-7} rad
(C) 3.0×10^{-7} rad
(D) 4.5×10^{-7} rad

12. Moment of inertia of a body about a given axis is 1.5 kg m^2 Initially the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular acceleration of 20 rad/s^2 must be applied about the axis of rotation for a duration of:

(A) 2 s
(B) 5 s
(C) 2.5 s
(D) 3 s

13. The logic gate equivalent to the given logic circuit is:



(A) OR
(B) AND
(C) NOR
(D) NAND

14. 50 W/m^2 energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on 1 m^2 surface area will be close to $\left(c = 3 \times 10^8 \text{ m/s}\right)$

(A) 15×10^{-8}N
(B) 35×10^{-8}N
(C) 10×10^{-8}N
(D) 20×10^{-8}N

15. The area of a square is 5.29 cm^2 The area of 7 such squares taking into account the significant figures is:

(A) 37 cm^2
(B) 37.0 cm^2
(C) 37.03 cm^2
(D) 37.030 cm^2

16. The physical sizes of the transmitter and receiver antenna in a communication system are:

(A) Proportional of carrier frequency
(B) Inversely proportional to modulation frequency
(C) Inversely proportional to carrier frequency
(D) Independent of both carrier and modulation frequency

17. Four point charges –q, +q, +q and –q are placed on y axis at y = -2d, y = -d, y = +d and y = +2d, respectively. The magnitude of the electric field E at a point on the x – axis at x = D, with D > > d, will vary as:

(A) $E \propto \frac{1}{D}$
(B) $E \propto \frac{1}{D^3}$
(C) $E \propto \frac{1}{D^2}$
(D) $E \propto \frac{1}{D^4}$

18. The specific heats, C_P and C_V of a gas of diatomic molecules, A, are given (in units of J mol^{-1} K^{-1}) by 29 and 22, respectively. Another gas of diatomic molecules B, has the corresponding values 30 and 21. If they are treated as ideal gases, then:-
(A) A has one vibrational mode and B has two
(B) Both A and B have a vibrational mode each
(C) A is rigid but B has a vibrational mode
(D) A has a vibrational mode but B has none

19. The position vector of a particle changes with time according to the relation $\vec{r}(t) = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$. What is the magnitude of the acceleration at t = 1?
(A) 40 (B) 100
(C) 25 (D) 50

20. A test particle is moving in a circular orbit in the gravitational field produced by a mass density $\rho(r) = \frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T:
(A) T/R^2 is a constant (B) TR is constant
(C) T^2/R^3 is a constant (D) T/R is a constant

21. A particle of mass m is moving with speed 2 v collides with a mass 2m moving with speed v in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass m, which move at angle 45^0 with respect to the original direction. The speed of each of the moving particle will be:
(A) $v/(2\sqrt{2})$ (B) $2\sqrt{2}v$
(C) $\sqrt{2}v$ (D) $v/\sqrt{2}$

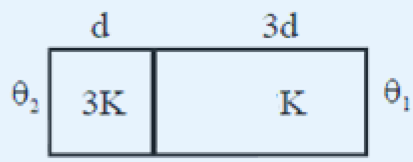
22. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is:-
(A) 0.5 (B) 0.7
(C) 0.6 (D) 0.8

23. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of 20 ms^{-1} with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source of car B? (speed of sound in air = 340 ms^{-1})
(A) 2250 Hz (B) 2060 Hz
(C) 2150 Hz (D) 2300 Hz

24. A wedge of mass M = 4m lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by:
(A) $\frac{2v^2}{7g}$ (B) $\frac{v^2}{g}$
(C) $\frac{2v^2}{5g}$ (D) $\frac{v^2}{2g}$

25. A He+ ion is in its first excited state. Its ionization energy is:
(A) 6.04 eV (B) 13.60 eV
(C) 54.40 eV (D) 48.36 eV

26. Two materials having coefficients of thermal conductivity 3K and K and thickness d and 3d, respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are θ_2 and θ_1 respectively $(\theta_2 > \theta_1)$. The temperature at the interface is:



(A) $\frac{\theta_2 + \theta_1}{2}$ (B) $\frac{\theta_1}{10} + \frac{9\theta_2}{10}$
(C) $\frac{\theta_1}{3} + \frac{2\theta_2}{3}$ (D) $\frac{\theta_1}{6} + \frac{5\theta_2}{6}$

27. A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its centre. Two beads of mass m and negligible size are at the centre of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod will be:-
(A) $\frac{M\omega_0}{M+3m}$ (B) $\frac{M\omega_0}{M+m}$
(C) $\frac{M\omega_0}{M+2m}$ (D) $\frac{M\omega_0}{M+6m}$

28. The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitor are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is:
(A) $\frac{V}{K+n}$ (B) V
(C) $\frac{(n+1)V}{(K+n)}$ (D) $\frac{nV}{K+n}$

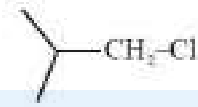
29. In a conductor, if the number of conduction electrons per unit volume is 8.5 x 10^{28} m^{-3} and mean free time is 25fs (femto second), its approximate resistivity is: $(m_e = 9.1\times10^{-31}kg)$
(A) 10^{-5} Ωm (B) 10^{-6} Ωm
(C) 10^{-7} Ωm (D) 10^{-8} Ωm

30. A string 2.0 m long and fixed at its end is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is:
(A) 320 m/s, 120 Hz (B) 180 m/s, 80 Hz
(C) 180m/s, 120 Hz (D) 320 m/s, 80 Hz

PART –B (CHEMISTRY)

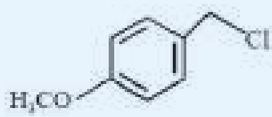
31. Increasing order of reactivity of the following compounds for S_N1 substitution is:

(a)

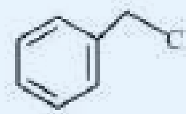


(b)

(c)



(d)



(A) (b) < (c) < (d) < (a)
(B) (a) < (b) < (d) < (c)
(C) (b) < (a) < (d) < (c)
(D) (b) < (c) < (a) < (d)

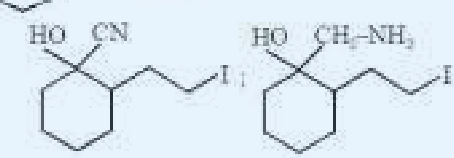
32. The one that is not a carbonate is:
(A) bauxite
(B) siderite
(C) calamine
(D) malachite

33. During compression of a spring the work done is 10kJ and 2kJ escaped to the surroundings as heat. The change in internal energy, ΔU (in kJ) is:
(A) 8
(B) 12
(C) -12
(D) -8

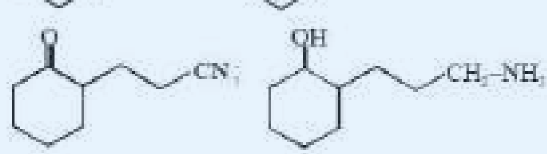
34. The amorphous form of silica is:
(A) quartz
(B) kieselguhr
(C) cristobalite
(D) tridymite

35. The major products A and B for the following reactions are, respectively

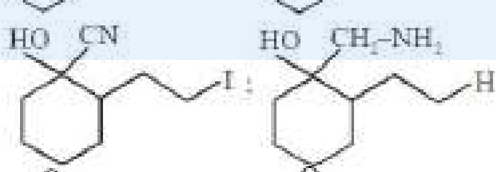
(A)



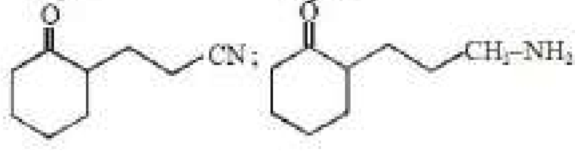
(B)



(C)

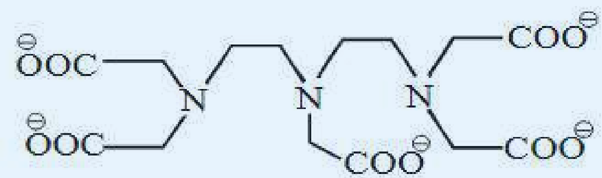


(D)



36. Which one of the following about an electron occupying the 1s orbital in a hydrogen atom is incorrect? (The Bohr radius is represented by a_0)
(A) The electron can be found at a distance $2a_0$ from the nucleus
(B) The probability density of finding the electron is maximum at the nucleus
(C) The magnitude of potential energy is double that of its kinetic energy on an average
(D) The total energy of the electron is maximum when it is at a distance a_0 from the nucleus

37. The maximum possible denticities of a ligand given below towards a common transition and inner – transition metal ion, respectively, are:



(A) 6 and 8
(B) 8 and 6
(C) 8 and 8
(D) 6 and 6

38. The correct statements among I to III regarding group 13 element oxides are,
I. Boron trioxide is acidic
II. Oxides of aluminium and gallium are amphoteric
III. Oxides of indium and thallium are basic
(A) I, II and III
(B) II and III only
(C) I and III only
(D) I and II only

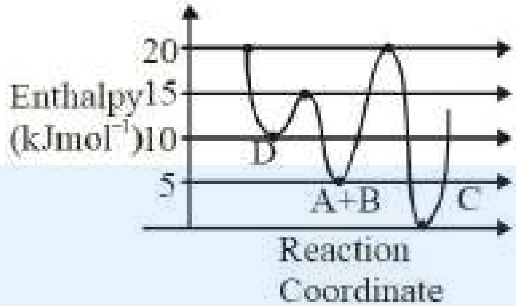
39. Among the following species, the diamagnetic molecule is
(A) O_2
(B) NO
(C) B_2
(D) CO

40. The peptide that gives positive ceric ammonium nitrate and carbylamine tests is:
(A) Lys - Asp
(B) Ser – Lys
(C)Gln - Asp
(D) Asp – Gln

41. Assertion : For the extraction of iron, haematite ore is used
Reason : Haematite is a carbonate ore of iron
(A) Only the reason is correct.
(B) Both the assertion and reason are correct and the reason is the correct explanation for the assertion.
(C) Only the assertion is correct
(D) Both the assertion and reason are correct, but the reason is not the correct explanation for the assertion

42. 10 mL of 1mM surfactant solution forms a monolayer covering 0.24 cm^2 on a polar substrate. If the polar head is approximated as cube, what is its edge length?
(A) 2.0 pm
(B) 2.0 nm
(C)1.0 pm
(D) 0.1 nm

43. Consider the given plot of enthalpy of the following reaction between A and B.
$A + B \rightarrow C + D$
Identify the incorrect statement



(A) C is the thermodynamically stable product
(B) Formation of A and B from C has highest enthalpy of activation
(C) D is kinetically stable product
(D) Activation enthalpy to form C is 5kJ mol^{-1} less than that to form D

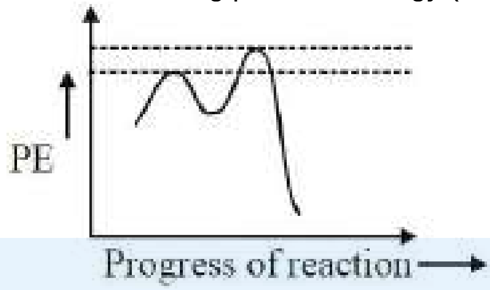
44. At a given temperature T, gases Ne, Ar, Xe and Kr are found to deviate from ideal gas behaviour. Their equation of state is given as $P = \frac{RT}{V-b}$ at T. Here, b is the van der Waals constant. Which gas will exhibit steeper increase in the plot of Z (compression factor) versus P?
(A) Ne (B) Ar
(C) Xe (D) Kr

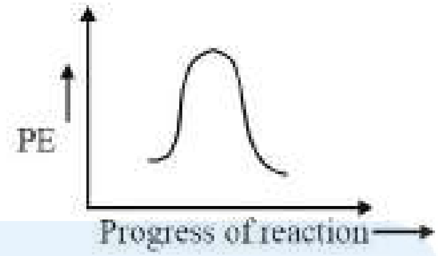
45. A solution of $Ni(NO_3)_2$ is electrolysed between platinum electrodes using 0.1 Faraday electricity. How many mole of Ni will be deposited at the cathode?
(A) 0.20 (B) 0.05
(C). 0.10 (D) 0.15

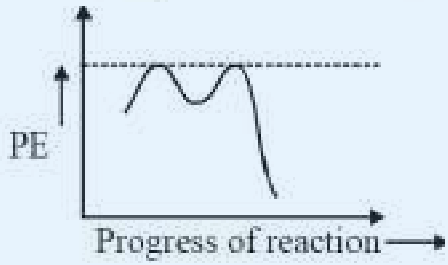
46. In the following reaction
Carbonyl compound + MeOH $\xrightleftharpoons{HCl}$ acetal
Rate of the reaction is the highest for:
(A) Acetone as substrate and methanol in stoichiometric amount
(B) Proponal as substrate and methanol in stoichiometric amount
(C) Acetone as substrate and methanol in excess
(D) Propanal as substrate and methanol in excess

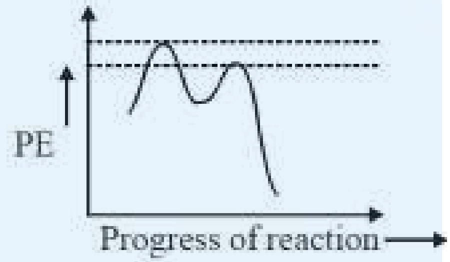
47. The structures of beryllium chloride in the solid state and vapour, phase, respectively, are:
(A) chain and dimeric (B) chain and chain
(C)dimeric and dimeric (D) dimeric and chain

48. Which of the following potential energy (PE) diagrams represents the S_N1 reaction?

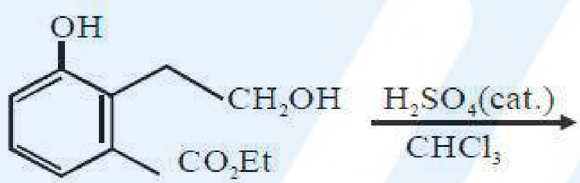
(A) 

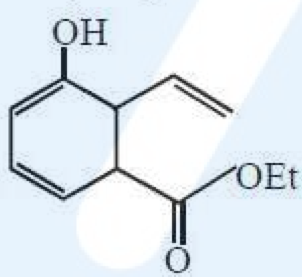
(B) 

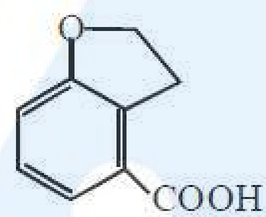
(C) 

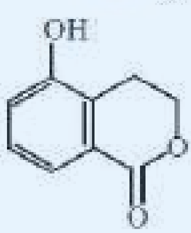
(D) 

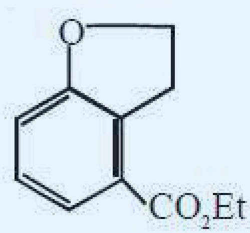
49. The major product of the following reaction is:



(A) 

(B) 

(C) 

(D) 

50. The maximum number of possible oxidation states of actinoids are shown by

(A) berkelium (Bk) and californium (Cf)
(B) nobelium (No) and lawrencium (Lr)
(C) actinium (Ac) and thorium (Th)
(D) neptunium (Np) and plutonium (Pu)

51. Molal depression constant for a solvent is 4.0 K Kg mol^{-1}. The depression in the freezing point of the solvent for 0.03 mol kg^{-1} solution of K_2SO_4 is: (Assume complete dissociation of the electrolyte)

(A) 0.12 K
(B) 0.36 K
(C) 0.18 K
(D) 0.24 K

52. Noradrenaline is a/an
(A) Neurotransmitter
(B) Antidepresessant
(C) Antihistamine
(D) Antacid

53. Hinsberg's reagent is:
(A) $C_6H_5SO_2Cl$
(B) C_6H_5COCl
(C) $SOCl_2$
(D) $(COCl)_2$

54. The layer of atmosphere between 10 km to 50 km above the sea level is called as:
(A) troposphere
(B) mesosphere
(C)stratosphere
(D) thermosphere

55. HF has highest boiling point among hydrogen halides, because it has:
(A) lowest dissociation enthalpy
(B) strongest van der Waals' interactions
(C) strongest hydrogen bonding
(D) lowest ionic character

56. What would be the molality of 20% (mass/mass) aqueous solution of KI?
(molar mass of KI = 166 g mol^{-1})
(A) 1.08
(B) 1.48
(C)1.51
(D) 1.35

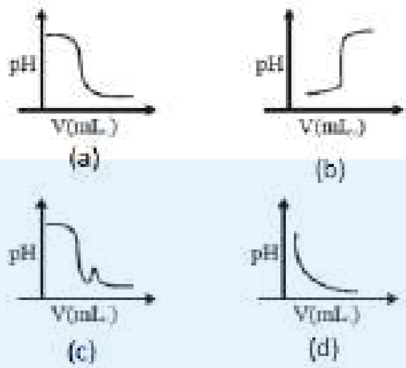
57. Which of the following compounds is a constituent of the polymer?

$$\left[HN - \overset{\overset{\displaystyle O}{\|}}{C} - NH - CH_2 \right]_n \ ?$$

(A) Formaldehyde
(B) Ammonia
(C) Methylamine
(D) N – methyl urea

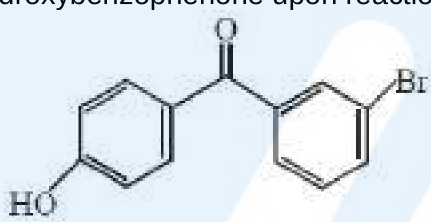
58. The correct statements among I to III are:
I. Valence bond theory cannot explain the color exhibited by transition metal complexes
II. Valence bond theory can predict quantitatively the magnetic properties of transition metal complexes
III. Valence bond theory cannot distinguish ligands as weak and strong field ones
(A) I and II only
(B) I, II and III
(C) I and III only
(D) II and III only

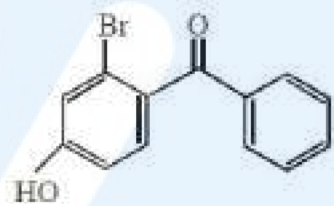
59. In an acid – base titration, 0.1 M HCI solution was added to the NaOH solution of unknown strength. Which of the following correctly shows the change of pH of the titration mixture in this experiment?

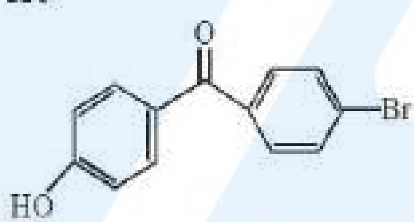


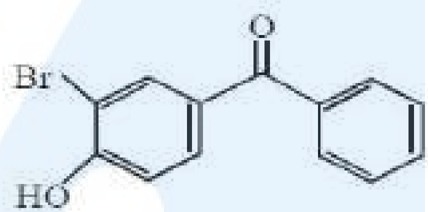
(A) (a)
(B) (c)
(C) (d)
(D) (b)

60. p-Hydroxybenzophenone upon reaction with bromine in carbon tetrachloride gives:

(A) 

(B) 

(C) 

(D) 

PART–C (MATHEMATICS)

61. If the tangent to the parabola $y^2 = x$ at a point (α, β), ($\beta > 0$) is also a tangent to the ellipse, $x^2 + 2y^2 = 1$, then a is equal to

(A) $2\sqrt{2}+1$
(B) $\sqrt{2}-1$
(C) $\sqrt{2}+1$
(D) $2\sqrt{2}-1$

62. Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is

(A) 190
(B) 262
(C) 225
(D) 157

63. If $f: R \to R$ is a differentiable function and $f(2) = 6$, then $\lim_{x\to 2} \int_6^{f(x)} \frac{2\,t\,dt}{(x-2)}$ is:

(A) 0
(B) $2f'(2)$
(C) 12f' (2)
(D) 24f' (2)

64. If the system of equations $2x + 3y - z = 0$, $x + ky - 2z = 0$ and $2x - y + z = 0$ has a non – trivial solution (x, y, z), then $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k$ is equal to

(A) $\frac{3}{4}$
(B) –4
(C) $\frac{1}{2}$
(D) $-\frac{1}{4}$

65. The common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point

(A) (–4, 6)
(B) (6. –2)
(C) (–6, 4)
(D) (4, –2)

66. If the sum and product of the first three term in an A.P. are 33 and 1155, respectively, then a value of its 11^{th} tern is

(A) –25
(B) 25
(C) –36
(D) –35

67. The value of the integral $\int_0^1 x\cot^{-1}(1 - x^2 + x^4)\,dx$ is

(A) $\frac{\pi}{4} - \frac{1}{2}\log_e 2$
(B) $\frac{\pi}{2} - \log_e 2$
(C) $\frac{\pi}{2} - \frac{1}{2}\log_e 2$
(D) $\frac{\pi}{4} - \log_e 2$

68. The value of $\sin 10^{o} \sin 30^{o} \sin 50^{o} \sin 70^{o}$ is

(A) $\frac{1}{36}$ (B) $\frac{1}{32}$

(C) $\frac{1}{18}$ (D) $\frac{1}{16}$

69. Let $z \in C$ be such that $|z|<1$. If $w=\frac{5+3z}{5(1-z)}$, then

(A) $5\operatorname{Im}(w)<1$ (B) $4\operatorname{Im}(w)>5$

(C) $5\operatorname{Re}(w)>1$ (D) $5\operatorname{Re}(w)>4$

70. If some three consecutive in the binomial expansion of $(x+1)^{n}$ in powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficient is:

(A) 964 (B) 625

(C) 227 (D) 232

71. If $\cos x \frac{dy}{dx}-y \sin x=6x, \left(0<x<\frac{\pi}{2}\right)$ and $y\left(\frac{\pi}{3}\right)=0,$ then $y\left(\frac{\pi}{6}\right)$ is equal to:

(A) $-\frac{\pi^{2}}{4\sqrt{3}}$ (B) $-\frac{\pi^{2}}{2}$

(C) $\frac{\pi^{2}}{2\sqrt{3}}$ (D) $-\frac{\pi^{2}}{2\sqrt{3}}$

72. If the two lines $x+(a-1)y=1$ and $2x+a^{2}y=1\left(a \in R-\{0,1\}\right)$ are perpendicular, then the distance of their point of intersection from the origin is:

(A) $\frac{2}{5}$ (B) $\frac{\sqrt{2}}{5}$

(C) $\frac{2}{\sqrt{5}}$ (D) $\sqrt{\frac{2}{5}}$

73. A water tank has the shape of an inverted right circular cone, whose semi vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$.Water is poured in at a constant rage of 5 cubic meter per minute. Then the rate (in m/min) at which the level of water is rising at the instant when the depth of water in the tank is 10 m is:

(A) $\frac{2}{\pi}$ (B) $\frac{1}{5\pi}$

(C) $\frac{1}{10\pi}$ (D) $\frac{1}{15\pi}$

74. Two poles standing on a horizontal ground are of heights 5m and 10 m respectively. The line joining their tops makes an angle of 15^{o} with ground. Then the distance (in m) between the poles, is

(A) $\frac{5}{2}\left(2+\sqrt{3}\right)$
(B) $5\left(\sqrt{3}+1\right)$
(C) $5\left(2+\sqrt{3}\right)$
(D) $10\left(\sqrt{3}-1\right)$

75. The vertices B and C of a ΔABC lie on the line, $\frac{x+2}{3}=\frac{y-1}{0}=\frac{z}{4}$ such that BC = 5 units. Then the area (in sq. units) of this triangle, given that the point A (1, –1, 2) is:

(A) $2\sqrt{34}$
(B) $\sqrt{34}$
(C) 6
(D) $5\sqrt{17}$

76. The total number of matrices $A=\begin{bmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1 \end{bmatrix}$; $\left(x, y \in R,\ x \neq y\right)$ for which $A^{T}A=3I_{3}$

(A) 6
(B) 2
(C) 3
(D) 4

77. The area in sq. units) of the smaller of the two circles that touch the parabola, $y^{2}=4x$ at the points (1, 2) and the axis is

(A) $4\pi\left(2-\sqrt{2}\right)$
(B) $8\pi\left(3-2\sqrt{2}\right)$
(C) $4\pi\left(3+\sqrt{2}\right)$
(D) $8\pi\left(2-\sqrt{2}\right)$

78. If the function $f\left(x\right)=\begin{cases} a\left|\pi-x\right|+1, & x\le 5 \\ b\left|\pi-x\right|+3, & x>5 \end{cases}$ is continuous at x = 5, then the value of a – b is

(A) $\frac{2}{5-\pi}$
(B) $\frac{2}{\pi-5}$
(C) $\frac{2}{\pi+5}$
(D) $\frac{-2}{\pi+5}$

79. If $f\left(x\right)=\left[x\right]-\left[\frac{x}{4}\right], x \in R$, where [x] denotes the greatest integer function, then:

(A) Both $\lim\limits_{x\to 4-} f\left(x\right)$ and $\lim\limits_{x\to 4+} f\left(x\right)$ exist but are not equal
(B) $\lim\limits_{x\to 4-} f\left(x\right)$ exists but $\lim\limits_{x\to 4+} f\left(x\right)$ does not exist
(C) $\lim\limits_{x\to 4+} f\left(x\right)$ exists but $\lim\limits_{x\to 4-} f\left(x\right)$ does not exist
(D) f is continuous at x = 4

80. If $\int e^{\sec x}\left(\sec x+\tan x f(x)+\left(\sec x \tan x+\sec^{2} x\right)\right) dx=e^{\sec x} f(x)+C$, then a possible choice of $f(x)$ is

(A) $\sec x-\tan x-\frac{1}{2}$
(B) $x \sec x+\tan x+\frac{1}{2}$
(C) $\sec x+x \tan x-\frac{1}{2}$
(D) $\sec x+\tan x+\frac{1}{2}$

81. If m is chosen in the quadratic equation $\left(m^{2}+1\right) x^{2}-3 x+\left(m^{2}+1\right)^{2}=0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is:

(A) $8\sqrt{3}$
(B) $4\sqrt{3}$
(C) $10\sqrt{5}$
(D) $8\sqrt{5}$

82. Two newspaper A and B are published in a city. It is known that 25% of the city populations reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisement is:

(A) 12.8
(B) 13.5
(C) 13.9
(D) 13

83. Let P be the plane, which contains the line of intersection of the planes, $x+y+z-6=0$ and $2x+3y+z+5=0$ and it is perpendicular to the xy – plane. Then the distance of the point (0, 0, 256) from P is equal to:

(A) $63\sqrt{5}$
(B) $205\sqrt{5}$
(C) $\frac{17}{\sqrt{5}}$
(D) $\frac{11}{\sqrt{5}}$

84. If $P \Rightarrow (q \vee r)$ is false, then the truth values of p, q, r are respectively

(A) F, T, T
(B) T, F, F
(C) T, T, F
(D) F, F, F

85. The domain of the definition of the function $f(x)=\frac{1}{4-x^{2}}+\log\left(x^{3}-x\right)$ is

(A) $(1,2) \cup (2,\infty)$
(B) $(-1,0) \cup (1,2) \cup (3,\infty)$
(C) $(-1,0) \cup (1,2) \cup (2,\infty)$
(D) $(-2,-1) \cup (-1,0) \cup (2,\infty)$

86. The sum of the series $1+2\times 3+3\times 5+4\times 7+....$ upto 11^{th} term is

(A) 915
(B) 946
(C) 945
(D) 916

87. The mean and the median of the following ten numbers in increasing order 10, 22, 26, 29, 34, x, 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to

(A) $\frac{7}{3}$
(B) $\frac{9}{4}$
(C) $\frac{7}{2}$
(D) $\frac{8}{3}$

88. The area (in sq. units) of the region $A = \left\{(x, y) : \frac{y^2}{2} \le x \le y + 4\right\}$ is:

(A) $\frac{53}{3}$
(B) 18
(C) 30
(D) 16

89. If a unit vector $\vec{r}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and $\theta \in (0, \pi)$ with $\hat{k}$, then a value of θ is

(A) $\frac{5\pi}{12}$
(B) $\frac{5\pi}{6}$
(C) $\frac{2\pi}{3}$
(D) $\frac{\pi}{4}$

90. A rectangle is inscribed in a circle with a diameter lying along the line $3y = x + 7$. If the two adjacent vertices of the rectangle are (–8, 5) and (6, 5) then the area of the rectangle (in sq. units) is

(A) 72
(B) 84
(C) 98
(D) 56

HINTS AND SOLUTIONS

PART A – PHYSICS

1. Mutual induction

$\phi_q = MI_p$

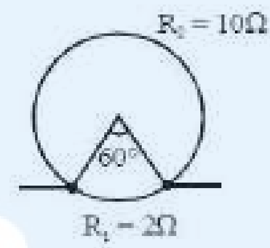
$10^{-3} = M(3)$

$\Rightarrow \quad M = \frac{1}{3} \times 10^{-3}$

$\phi_p = MI_q$

$\Rightarrow \quad \phi_p = 6.67 \times 10^{-4}$ Wb

2. $R = \frac{\rho\ell}{A} = \frac{\rho\ell}{(V/\ell)} = \frac{\rho\ell^2}{V}$ (V $\rightarrow$ Volume of wire)

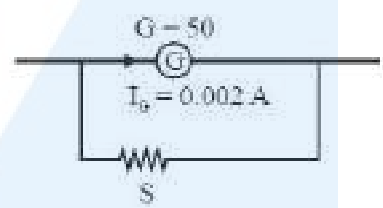


$\Rightarrow$ Final resistance = $3 \times (B)^2 = 12\ \Omega$

$R_{eq} = 2\ \Omega \parallel 10\ \Omega$

$= \frac{5}{3}\Omega$

3. Since shunt resistance is connected in parallel with galvanometer, both will have same voltage drop.



$(0.002)(R_G) = (0.5 - 0.002)(r_S)$

$\Rightarrow \quad r_s \approx 0.2\ \Omega$

4. $x = at + bt^2 - ct^3$

$V = \frac{dx}{dt} = a + 2bt - 3ct^2$

$a = \frac{dv}{dt} = 2b - 6ct$

Put acceleration = 0

$\Rightarrow \quad t = \frac{b}{3c}$

Find V at $t = \frac{b}{3c}$

$V = a + \frac{b^2}{3c}$

5. For image to form at object itself, says must retrace their path back to object. Hence must incident on mirror normally.

Case 1: Object will be at focus of lens

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{1}{-18}$$

$\Rightarrow \quad R = 18$ cm

Case 2: Retraction at 1st surface:

$$\frac{1}{-27}-\frac{1.5}{V_1}=\frac{1-1.5}{R} \qquad \ldots(i)$$

2^{nd} retraction:

$$\frac{1.5}{V_1}-\frac{\mu}{\infty}=\frac{1.5-u}{-R} \qquad \ldots(ii)$$

From (i) and (ii)

$\mu = \frac{4}{3}$.

6. $\tau = \vec{M}\times\vec{B}$

$C\theta$ = i N A B

$10^{-6}\times\frac{\pi}{180}=10^{-3}\times10^{-4}\times175\times B$

B = 10^{-3} Tesla

7. $\varepsilon=-\frac{d\phi}{dt} = \frac{-d}{dt}(\mu_o nI)(\pi R^2)$

$= -(\mu_o n\,\pi R^2)\frac{dI}{dt}$

$= -\mu_o n\,\pi R^2\,\frac{d(kt\,e^{-\alpha t})}{dt}$

$= -C\,(t(e^{-\alpha t})(-\alpha) + e^{-\alpha t})$ (C $\rightarrow$ constant)

$= -C\,(e^{-\alpha t})(1-\alpha t)$

Clearly, at t = 0

Induced current $\neq 0$

Also, apply Lenz law to find correct option.

8. By law of conservation of energy

$\frac{1}{2}kx^2 = (m_1s_1 + m_2s_2)\Delta T$

$\Delta T=\frac{16\times10^{-2}}{4384} = 3.65 \times 10^{-5}$

9. Conservation of momentum

$\vec{p}_x+\vec{p}_y=\vec{p}_{final}$

$m_xv_x - m_yv_y = (m_x + m_y)\,V$

$\frac{h}{\lambda_x}-\frac{h}{\lambda_y}=\frac{h}{\lambda}$

$\Rightarrow \lambda=\frac{\lambda_x\lambda_y}{|\lambda_x-\lambda_y|}$

10. Magnification is 2

If image is real, $x_1 = \frac{3f}{2}$

If image is virtual, $x_2 = \frac{f}{2}$

$\frac{x_1}{x_2} = 3 : 1$

11. Limit of resolution = $\frac{1.22\,\lambda}{d}$

$= \frac{1.22 \times 600 \times 10^{-9}}{250 \times 10^{-2}}$

$= 2.9 \times 10^{-7}$ rad.

12. KE = $\frac{1}{2} I\omega^2 = 1200$ (given)

$\Rightarrow$ $\omega = 40$ rad/s

$\omega = \omega_0 + \alpha t$

$40 = 0 + (20)\ t$

$\Rightarrow$ t = 2 sec.

13. Truth table can be formed as

A	B	Equivalent
0	0	0
0	1	1
1	0	1
1	1	1

Hence the equivalent is “OR” gate.

14. Radiation pressure for 100% reflection = $\frac{2I}{C}$

Radiation pressure for 0% reflection = $\frac{I}{C}$

Hence, in given case, radiation pressure = $(0.25)\left(\frac{2I}{C}\right) + (0.75)\left(\frac{I}{C}\right)$

$= (1.25)\left(\frac{I}{C}\right)$

Force = P × (Area)

$= 20.83 \times 10^{-8}$ N

15. Total Area = $A_1 + A_2 + \ldots\ldots\ldots A_7$

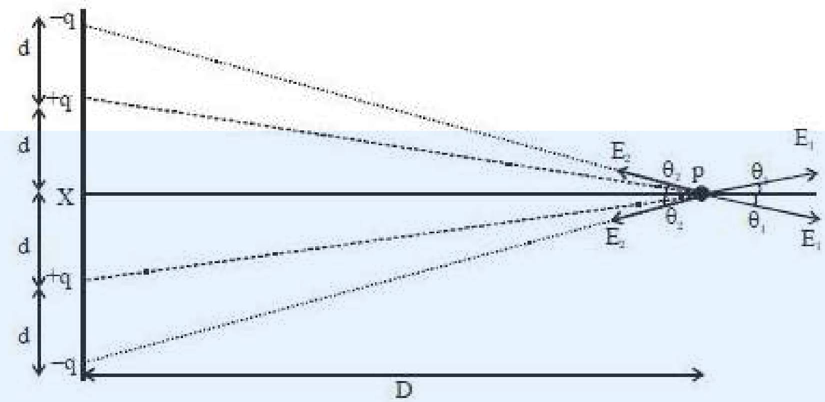
$= A + A + \ldots\ldots\ldots\ldots$ 7 times

$= 37.03\ m^2$

Addition of 7 terms all having 2 terms beyond decimal, so final answer must have 2 terms beyond decimal (as per rules of significant digits)

16. The physical size of antenna of receiver and transmitter both are inversely proportional to carrier frequency.

17.



Electric field at p = $2E_1\cos\theta_1 - 2E_2\cos\theta_2$

$$= \frac{2Kq}{(d^2+D^2)} \times \frac{D}{(d^2+D^2)^{1/2}} - \frac{2Kq}{[(2d)^2+D^2]} \times \frac{D}{[(2d)^2+D^2]^{1/2}}$$

$$= 2KqD\left[(d^2+D^2)^{-3/2} - (4d^2+D^2)^{-3/2}\right]$$

$$= \frac{2KqD}{D^3}\left[\left(1+\frac{d^2}{D^2}\right)^{-3/2} - \left(1+\frac{4d^2}{D^2}\right)^{-3/2}\right]$$

Applying binomial approximation $\because$ d << D

$$= \frac{2KqD}{D^3}\left[1-\frac{3}{2}\frac{d^2}{D^2} - \left(1-\frac{3\times 4d^2}{2D^2}\right)\right]$$

$$= \frac{2KqD}{D^3}\left[\frac{12}{2}\frac{d^2}{D^2} - \frac{3}{2}\frac{d^2}{D^2}\right]$$

$$= \frac{9kqd^2}{D^4}$$

18. For A:

$$\frac{C_p}{C_v} = \gamma = 1+\frac{2}{f} = \frac{29}{22}$$

It gives f = 6.3 ≈ 6 (3 translational, 2 rotational and 1 vibrational)

For B:

$$\frac{C_p}{C_v} = \gamma = 1+\frac{2}{f} = \frac{30}{21}$$

$\Rightarrow$ f = 4.67

$\Rightarrow$ ≈ 5 (3 translational, 2 rotational, no vibrational)

19. $\vec{r} = (15t^2)\hat{i} + (4 - 20t^2)\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = (30t)\hat{i} - (40t)\hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt} = (30)\hat{i} - (40)\hat{j}$

$|\vec{a}| = 50$

20. For circular motion of particle:

$\frac{mV^2}{r} = mE$

$= m\left(\frac{GM}{r^2}\right)$

Where M = $\int_0^r (4\pi x^2 dx)\left(\frac{k}{x^2}\right)$

$= 4\pi kr$

$\Rightarrow \quad \frac{mV^2}{r} = m\left(\frac{G(4\pi k)}{r}\right)$

$\Rightarrow$ V = constant

T = $\frac{2\pi R}{V}$

$\Rightarrow \quad \frac{T}{R} = \text{Constant}$

21.

Linear momentum conservation

$m\,2v + 2m\,v = m \times 0 + m\frac{v'}{\sqrt{2}} \times 2$

$v' = 2\sqrt{2}\,v.$

22. In 1st situation

$V_b\rho_b g = V_s\rho_w g$

$\frac{V_s}{V_b} = \frac{\rho_b}{\rho_w} = \frac{4}{5}$ ...(i)

Here V_b is volume of block

V_s is submerged volume of block

ρ_b is density of block

ρ_w is density of water & Let ρ_o is density of oil

Finally in equilibrium condition

$V_b\rho_b g = \frac{V_b}{2}\rho_o g + \frac{V_b}{2}\rho_w g$

$2\rho_b = \rho_0 + \rho_w$

$\Rightarrow \dfrac{\rho_o}{\rho_w} = \dfrac{3}{5} = 0.6$

23. Doppler effect:

$f = \left(\dfrac{v + u_o}{v - u_s}\right)(f_o)$

$2000 = \left(\dfrac{340 + (-20)}{340 - (-20)}\right)(f_o)$

$f_o = 2250$ Hz

24. Let mass attains height 'h' on wedge and at that time, both attain velocity v_f.
Conservation of momentum:
$mv = (m + 4m)V_f$ ...(A)

COE:

$\dfrac{1}{2}mv^2 = \dfrac{1}{2}(m + 4m)V_f^2 + mgh$ ...(B)

From (A) and (B)

$h = \dfrac{2V^2}{5g}$

25. T.E. = $-(13.6)\left(\dfrac{z^2}{n^2}\right)$eV

$z = n = 2$

$\Rightarrow$ Ionisation energy = –T.E.
= 13.6 eV

26. At steady state:

$\left(\dfrac{\Delta q}{\Delta t}\right)_1 = \left(\dfrac{\Delta q}{\Delta t}\right)_2$

$\dfrac{3kA(\theta_2 - \theta)}{d} = \dfrac{kA(\theta - \theta_1)}{3d}$

$\Rightarrow \theta = \dfrac{\theta_1 + 9\theta_2}{10}$

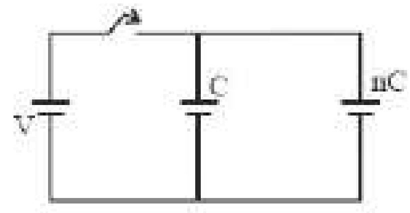
27. Conservation of angular momentum about rotation axes:

$L_i = L_f$

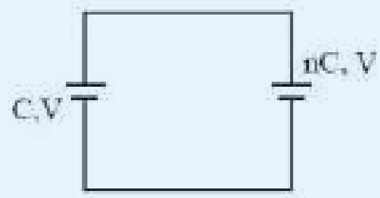
$\left(\dfrac{M\ell^2}{12}\right)\omega_o = \left[\dfrac{M\ell^2}{12} + 2\left(m\left(\dfrac{\ell}{2}\right)^2\right)\right]\omega_f$

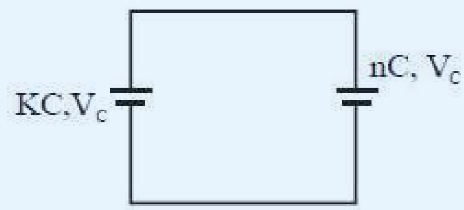
$\Rightarrow \omega_f = \left(\dfrac{M}{M + 6m}\right)\omega_o$

28.



After fully charging, battery is disconnected.





Total chare of the system = CV + nCV

= (n + 1)CV

After the insertion of dielectric of constant K

New potential (common)

$$V_c = \frac{\text{Total charge}}{\text{Total capacitance}}$$

$$= \frac{(n+1)CV}{KC+nC} = \frac{(n+1)V}{K+n}$$

29. $\rho = \frac{2m}{ne^2\tau}$

$= 3.34 \times 10^{-8}\ \Omega$ m

30. We have:

$f = \frac{nv}{2\ell}$

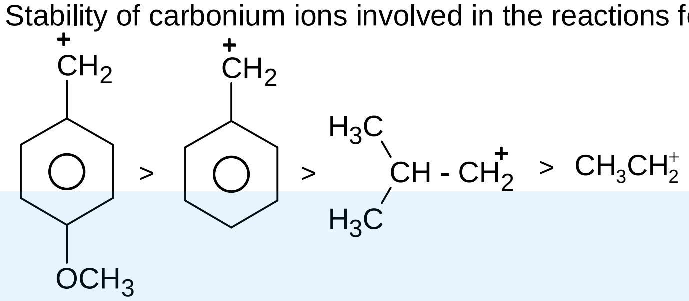
$240 = \frac{3 \times v}{2 \times 2}$

$\Rightarrow$ v = 320 m/s

Fundamental frequency = $\frac{v}{2\ell}$ = 80 Hz.

PART B – CHEMISTRY

31. Stability of carbonium ions involved in the reactions follow the order:



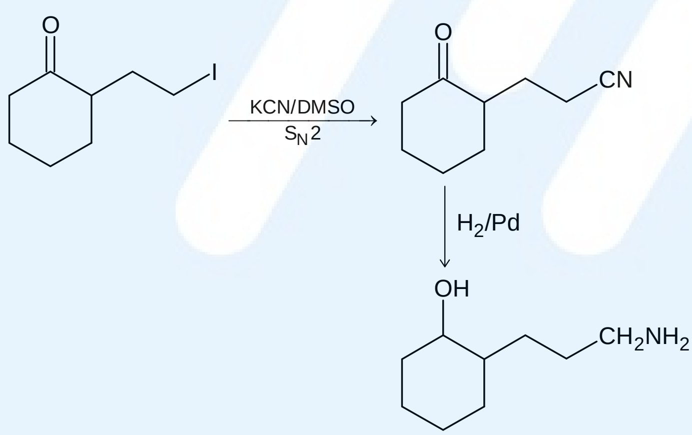
$\therefore$ Reactivity α Stability of carbonium ions

32. Malachite = $CuCO_3.Cu(OH)_2$
Bauxite = $Al_2O_3.2H_2O$
Calamine = $ZnCO_3$
Siderite = $FeCO_3$

33. Work done on system = +10 kJ
Heat escaped = -2kJ
$\Delta U = q + w$
$= 10 - 2 = 8$ kJ

34. Kieselguhr is amorphous form of silica.

35.



36. $T.E = -K.E = \frac{PE}{2}$
The energy of electrons increases as they are away from the nucleus. The probability of finding the 1s electron may be higher at a_0 but the energy is not. The probability density of finding the electron is not zero at any place in the atom. So, the energy may be higher when it is far from a_0.

37. General coordination number of CN^- in transition element is 6 and in inner transition element is 8-12. because inner transition metal ions can make available more number of vacant orbitals of nearly same energy than transition metal ions. The high effective nuclear charge of inner-transition metal ions make them form complex with high coordination number.

38. B_2O_3 is acidic in nature
Al_2O_3 and Ga_2O_3 are amphoteric
Oxides of In and Tl are basic in nature. Because the metallic character of the elements increases on moving down the group.

39. CO is diamagnetic in nature due to absence of any unpaired electron.

40. Due to –OH group of serine it give cerric ammonium nitrate test whereas due to $–NH_2$ group lysine give +ve carbylamine test.

41. Extraction of Fe is done form haematite ore this is true but reason is wrong as haematite is Fe_2O_3.

42. $\text{Moles} = \frac{\text{MVml}}{1000} = \frac{10^{-3} \times 10}{1000} = 10^{-5}\text{mole}$
10^{-5} N_A molecules covering area = 0.24 cm^2
1 -------------------------------------- = $\frac{0.24}{10^{-5}N_A}cm^2$
$\frac{0.24}{10^{-5} \times 6 \times 10^{23}} = a^2$
$a^2 = 4 \times 10^{-20}$ cm^2
$a = 2 \times 10^{-10}$ cm
$a = 2 \times 10^{-12}$ m
a = 2 pm

43. $E_a = (D \rightarrow C)$
= 15 – 0 = 15 kJ mol^{-1}
$E_a = (A + B) \rightarrow C = 15$ kJ mol^{-1}
$E_a = (A + B) \rightarrow D = 10$ kJ mol^{-1}
$E_a = C \rightarrow (A + B) = 20$ kJ mol^{-1} (high activation enthalpy in the reaction)
$\therefore$ Activation enthalpy to form C is 5 kJ mol^{-1} more than that of form D.

44. $P = \frac{RT}{(V - b)}$
P(V – b) = RT
$\left(P + \frac{a}{V^2}\right)(V - b) = RT$
At high pressure.
P(V - b) = RT
PV – Pb = RT
$\frac{PV}{RT} - \frac{Pb}{RT} = 1$
$Z = 1 + \frac{Pb}{RT}$
$Z > 1$, $Z \alpha b$
The value of ‘b’ for the gases follows the order
Ne < Ar < Kr < Xe

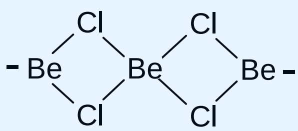
45. 1F deposits 1 g equivalent of Ni or ½ mole of Ni

$\therefore$ 0.1 F will deposit $\frac{1}{20} = 0.05$ moles of Ni

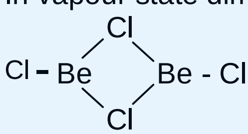
46. Ketones < Aldehyde $\longrightarrow$ Rate of Nucleophilic addition reaction

Only aldehydes are responsible for formation of acetals. Excess of MeOH is used to drive the reaction towards forward direction.

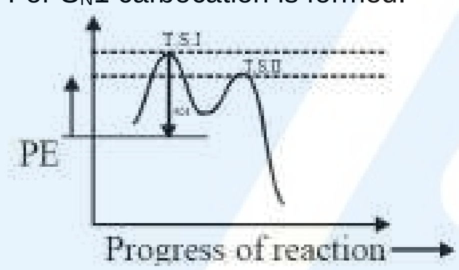
47. Solid state – chain



In vapour state dimeric

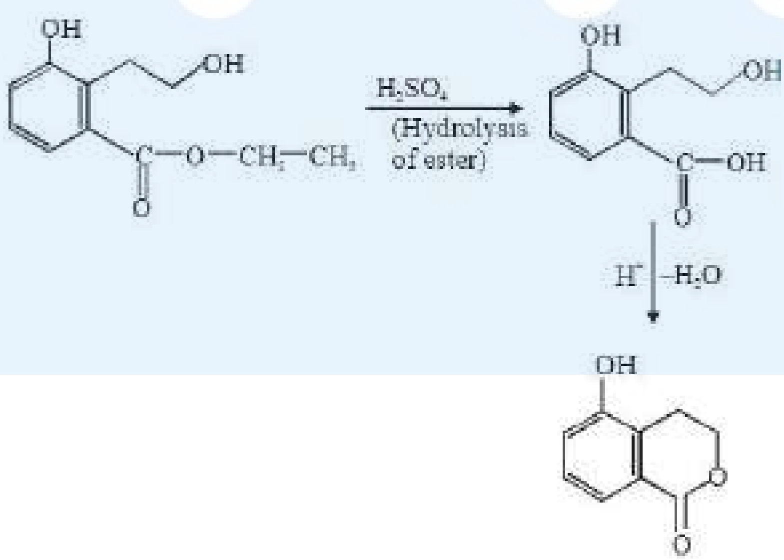


48. For S_N1 carbocation is formed.



The first transition state should have higher energy than the second transition state. Reaction intermediate is also formed.

49.



Chloroform is used as a solvent in this reaction

50.

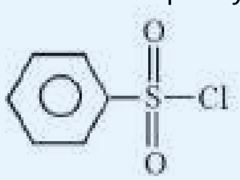
Ac	Th	Pu	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr
+3		+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+3	+2	+3
	+4	+4	+4	+4	+4	+4	+4	+4						
		+5	+5	+5	+5	+5								
				+6	+6	+6								
			+6	+7	+7									

$\therefore$ Np and Pu has maximum no. of possible oxidation states

51. $K_f = 4$ K Kg mol^{-1}
i = 3
Molality = 0.03
$\Delta T_f = i K_f m$
= 3(4) (0.03)
$\Delta T_f = 0.36$ K

52. It is an organic chemical in the catecholamine family that functions in the brain and body as hormone and neurotransmitter.

53. Benzene sulphonyl chloride $C_6H_5SO_2Cl$



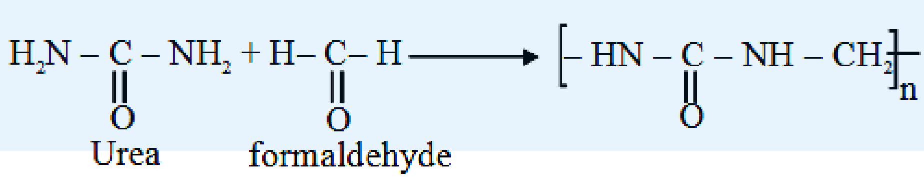
54. Fact based.

55. HF has strong hydrogen bond. $\therefore$ It has highest boiling point among hydrogen halides. The strong hydrogen bond is due to more difference in electronegativity between F and H atoms.

56. 20% w/w KI
Mass of solute(KI) = 20 g
Mass of solvent = 100 – 20 = 80 g
Molar mass of KI = 38 + 128 = 166

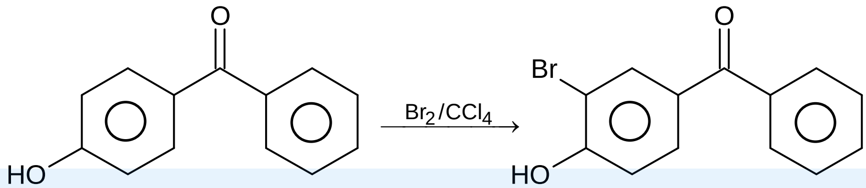
$$\text{Molality} = \frac{\text{gm(solute)}}{\text{mw} \times \text{Kg(solvent)}} = \frac{20 \times 1000}{166 \times 80} = 1.506 = 1.51$$

57.



58. (i) VBT does not explain colour exhibited by complex of transition metal because splitting of d-orbitals in explained by CFT.
(ii) VBT does not distinguish between strong and weak field complex.
(iii) VBT does not give quantitative interpretation of magnetic properties.
$\therefore$ According to question statement I and III are correct.

59. The nature of the curve is such that the pH change of the reaction becomes larger for very small change in volume of titrant.

60. 

$-\overset{\overset{\displaystyle O}{||}}{C}-$ $\longrightarrow$ – R effect, deactivate the ring for EAS. OH $\rightarrow$ +R effect activate the ring towards EAS

PART C – MATHEMATICS

61. Equation of tangent to the parabola $y^2 = x$

At (α, β) is $T = 0$

$$y\beta = \frac{x+\alpha}{2}$$

$$\Rightarrow y\beta = \frac{x+\beta^2}{2} \left(\because \beta^2 = \alpha\right)$$

$$\Rightarrow y = \frac{1}{2\beta}x + \frac{\beta}{2}$$

$$\left(m = \frac{1}{2\beta}, c = \frac{\beta}{2}\right)$$

This is also a tangent to ellipse $x^2 + 2y^2 = 1$

$$\therefore C = \pm\sqrt{a^2m^2 + b^2}$$

$$\Rightarrow \frac{\beta}{2} = \pm\sqrt{\frac{1}{4\beta^2} + \frac{1}{2}}$$

$$\Rightarrow \frac{\beta^2}{4} = \frac{1}{4\beta^2} + \frac{1}{2}$$

$$\Rightarrow \beta^4 - 2\beta^2 - 1 = 0$$

$$\Rightarrow \left(\beta^2 - 1\right)^2 = 2$$

$$\Rightarrow \beta^2 - 1 = \sqrt{2}$$

$$\Rightarrow \beta^2 = \sqrt{2} + 1$$

$$\alpha = \beta^2 = \sqrt{2} + 1$$

62. $\frac{n(n+1)}{2} + 99 = (n-2)^2$

$$\Rightarrow n^2 + n + 198 = 2n^2 - 8n + 8$$

$$\Rightarrow n^2 - 9n - 190 = 0$$

$\Rightarrow (n-19)(n+10)=0$

$\Rightarrow n=19$

$\therefore$ Number of balls is $\dfrac{19\times 20}{2}=190$

63. $\lim\limits_{x\to 2}\int_6^{f(x)} \dfrac{2t\,dt}{(x-2)}dx$ {given that $f(2)=6$}

$\dfrac{0}{0}$ from, so we use L – Hospital Rule

$=\lim\limits_{x\to 2}\dfrac{f'(x).2f(x)}{1}$

$=f'(2).2f(2)$

$=12f'(2)$

64. system of equations has non trival solution

$\therefore D=0=\begin{vmatrix} 2 & 3 & -1 \\ 1 & k & -2 \\ 2 & -1 & 1 \end{vmatrix}=0$

$\Rightarrow k=\dfrac{9}{2}$

So equation are $2x+3y-z=0$ ……….(1)

$x+\dfrac{9}{2}y-2z=0$ ……….(2)

$2x-y+z=0$ ……….(3)

(1) – (3) $\Rightarrow 4y-2z=0$

$\Rightarrow 2y=z$ ………….(4)

$\Rightarrow \dfrac{y}{z}=\dfrac{1}{2}$

From equation (1) and (4)

$2x+3y-2y=0$

$\Rightarrow 2x+y=0$

$\Rightarrow \dfrac{x}{y}=\dfrac{-1}{2}$ or $\dfrac{z}{x}=-4$

$\dfrac{x}{y}+\dfrac{y}{z}+\dfrac{z}{x}+k=\dfrac{1}{2}$

65. Circle $x^2+y^2=4$

$\Rightarrow c_1(0,0);\ r_1=2$

and circle $x^2+y^2+6x+8y-24=0$

$\Rightarrow c_2(-3,\ 4);\ r_2=7$

$\Rightarrow d=c_1c_2=5$

also $d=|r_1-r_2|$
circles touch internally
Equation of common tangent $S_1-S_2=0$
$\Rightarrow 6x+8y-20=0$
$\Rightarrow 3x+4y-10=0$
Point (6, –2) satisfy it.

66. Let the three numbers in A.P. are a – d, a, a + d
Given that $a-d+a+a+d=33$
$\Rightarrow a=11$ and $(a-d)(a)(a+d)=1155$
$\Rightarrow a\left(a^2d^2\right)=1155$
$\Rightarrow 11\left(121-d^2\right)=1155$
$\Rightarrow d^2=16$
$\Rightarrow d=\pm 4$
If d = 4 then first term a – d = 7
If d = –4 then first term a – d = 15
$T_{11}=7+10(4)=47$, $T_{11}=15+10(-4)=-25$

67. $I=\int_0^1 x\cot^{-1}\left(1-x^2+x^4\right)dx$
$I=\int_0^1 x\tan^{-1}\left(\frac{1}{1-x^2+x^4}\right)dx$
$I=\int_0^1 x\ \tan^{-1}\left\{\frac{x^2-\left(x^2-1\right)}{1+x^2\left(x^2-1\right)}\right\}dx$
$I=\int_0^1 x\left\{\tan^{-1}x^2-\tan^{-1}\left(x^2-1\right)\right\}dx$
It $x^2=t\Rightarrow 2x\,dx=dt$
$I=\frac{1}{2}\int_0^1\left\{\tan^{-1}t-\tan^{-1}(t-1)\right\}dt$
$=\frac{1}{2}\int_0^1\tan^{-1}t\ dt-\frac{1}{2}\int_0^1\tan^{-1}\left(t^{-1}\right)dt$
$=\frac{1}{2}\int_0^1\tan^{-1}t\,dt-\frac{1}{2}\int_0^1\tan^{-1}(-t)\,dt$
$=\frac{1}{2}\int_0^1\tan^{-1}t\,dt+\frac{1}{2}\int_1^1\tan^{-1}(t)\,dt$
$=\int_0^1\tan^{-1}(t)\,dt$
$=\left(t.\tan^{-1}t\right)_0^1-\int_0^1\frac{t}{1+t^2}dt$
$=\left(\frac{\pi}{4}\right)-\frac{1}{2}\left[\log\left(1+t^2\right)\right]_0^1$
$=\frac{\pi}{4}-\frac{1}{2}\log_e{}^2$

68. $\sin 10^{o} \sin 30^{o} \sin 50^{o} \sin 70^{o}$

$= \sin 10^{o} \sin 30^{o} \sin 50^{o} \sin 70^{o}$

$= \sin 30^{o} \left\{ \sin 10^{o} \sin\left(60^{o} - 10^{o}\right) \sin\left(60^{o} + 10^{o}\right) \right\}$

$= \sin 30^{o} \left\{ \frac{1}{4} \sin 3\left(10^{o}\right) \right\}$

$= \frac{1}{2}\left(\frac{1}{4} \times \frac{1}{2}\right)$

$= \frac{1}{16}$

69. $W = \frac{5+3z}{5(1-z)}$

$\Rightarrow 5w - 5wz = 5 + 3z$

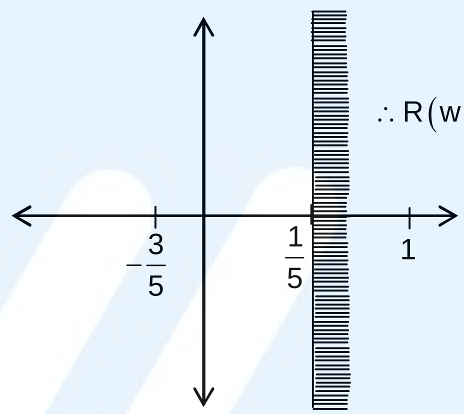
$\Rightarrow z = \frac{5w-5}{3+5w}$

given $|z| < 1$

$\Rightarrow \left|\frac{5w-5}{3+5w}\right| < 1$

$\Rightarrow |5w-5| < |3+5w|$

$\Rightarrow |w-1| < \left|\frac{3}{5} + w\right|$



$\therefore R(w) > \frac{1}{5}$

70. Given: $\frac{{}^{n}C_{r-1}}{{}^{n}C_{r}} = \frac{2}{15} \Rightarrow \frac{r}{n-r+1} = \frac{2}{15}$

$\Rightarrow 15r = 2n - 2r + 2$

$\Rightarrow 17r = 2n + 2$ ………..(1)

also given $\frac{{}^{n}C_{r}}{{}^{n}C_{r+1}} = \frac{15}{70} \Rightarrow \frac{r+1}{n-r} = \frac{3}{14}$

$\Rightarrow 3n - 3r = 14r + 14$

$\Rightarrow 17r = 3n - 14$ ………..(2)

Solving (1) and (2)

n = 16, r = 2

Average of coefficient $= \frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3}$

$= \frac{16+120+560}{3}$

= 232

71. $\cos x \frac{dy}{dx} - y \sin x = 6x$

$\Rightarrow \frac{dy}{dx} - y \tan x = 6x \sec x$

$= e^{-\int \tan x\,dx} = e^{-\log_e \sec x} = \frac{1}{\sec x}$

: solution of equation

$\Rightarrow y.\frac{1}{\sec x} = \int 6x \sec x.\frac{1}{\sec x}dx$

$\Rightarrow \frac{y}{\sec x} = 3x^2 + c$ …….(1)

given $y\left(\frac{\pi}{3}\right) = 0$

So, $0 = 3\frac{\pi^2}{9} + C$

$\Rightarrow C = -\frac{\pi^2}{3}$

Now from (1)

$\Rightarrow \frac{y}{\sec x} = 3x^2 - \frac{\pi^2}{3}$

At $x = \frac{\pi}{6}$

$\Rightarrow \frac{\sqrt{3}y}{2} = \frac{3\pi^2}{36} - \frac{\pi^2}{3}$

$\Rightarrow y = -\frac{\pi^2}{2\sqrt{3}}$

72. Two lines are perpendicular

$\therefore \quad m_1m_2 = -1$

$\Rightarrow \left(\frac{-1}{a-1}\right)\left(\frac{-2}{a^2}\right) = -1$

$\Rightarrow a^3 - a^2 + 2 = 0$

$\Rightarrow (a+1)(a^2 - 2a + 2) = 0$

$\therefore a = -1$

So lines are $\left.\begin{array}{l} L_1 : x - 2y + 1 = 0 \\ L_2 : 2x + y - 1 = 0 \end{array}\right\}$

Solving these equation we get point of intersection

$P\left(\frac{1}{5}, \frac{3}{6}\right)$

Now distance of P from origin

$OP = \sqrt{\frac{1}{25} + \frac{9}{25}} = \sqrt{\frac{2}{5}}$

73. Given $\theta = \tan^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \tan\theta = \frac{1}{2} = \frac{r}{h}$

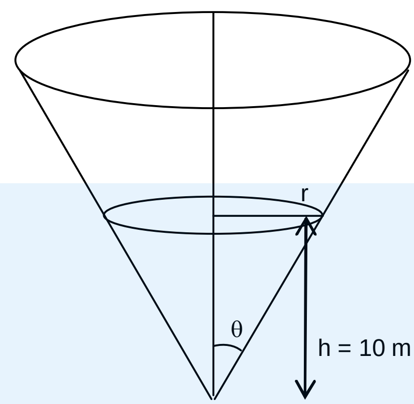
$\Rightarrow r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \frac{h^3}{4}$

$\frac{dv}{dt} = \frac{\pi}{12}\left(3h^2\right)\frac{dh}{dt}$

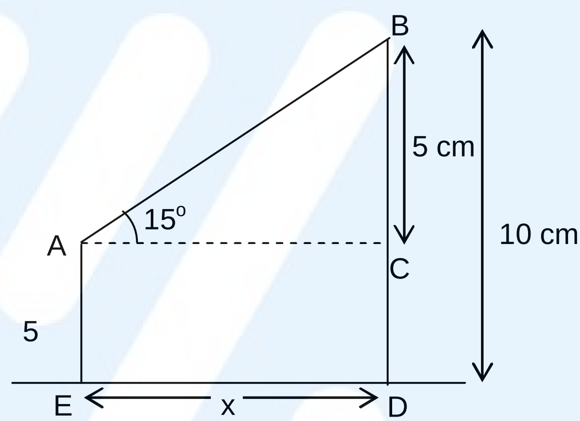
$5 = \frac{\pi}{4}(100)\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{5\pi}$



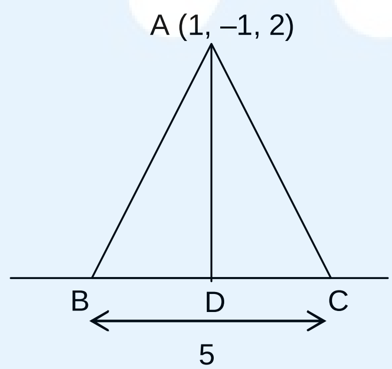
74. In $\Delta ABC \Rightarrow \tan 15^o = \frac{5}{x}$

$\Rightarrow 2-\sqrt{3} = \frac{5}{x}$

$\Rightarrow x = 5\left(2+\sqrt{3}\right)$



75. Let any point on given line is
$D(3\lambda-2, 1, 4\lambda)$
Now $AD \perp BC$
D.R. of BC
$\Rightarrow a_1 = 3,\ b_1 = 0,\ c_1 = y$
D.R. of AD
$\Rightarrow a_2 = 3\lambda-3,\ b_2 = 2,\ c_2 = 4\lambda-2$



$\Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow 3(3\lambda-3) + 0 + 4(4\lambda-2) = 0$

$25\lambda = 17$

$\Rightarrow \lambda = \frac{17}{25}$

Co – ordinate of point D

$\left(\frac{1}{25}, 1, \frac{68}{25}\right)$

$AD = \sqrt{\frac{576}{625} + 4 + \frac{324}{25}} = \frac{2}{5}\sqrt{34}$

Area of $\Delta ABC = \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 5 \times \frac{2}{6}\sqrt{34}$

$= \sqrt{34}$

76. $A^T A = 3I_3$

$\Rightarrow \begin{bmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\Rightarrow 8x^2 = 3 \Rightarrow x = \pm\sqrt{\frac{3}{8}}$

$\Rightarrow 6y^2 = 3 \Rightarrow y = \pm\sqrt{\frac{1}{2}}$

4 matrices are possible

77. Equation of tangent to the parabola $y^2 = 4x$ at

(1, 2) is $2y = 4\left(\frac{x+1}{2}\right)$

$\Rightarrow y = x + 1$

Equation of normal $y = -x + 3$

Let centre be C $(3 - r, r)$

Now $PC^2 = r^2$

$\Rightarrow (3 - r - 1)^2 + (r - 2)^2 = r^2$

$\Rightarrow 2(2 - r)^2 = r^2$

$\Rightarrow r^2 - 8r + 8 = 0$

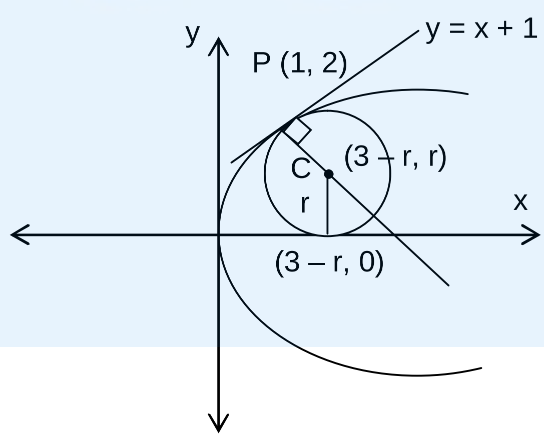
$\Rightarrow r = 4 \pm 2\sqrt{2}$

For $r = 4 + 2\sqrt{2}$ $(3 - r < 0)$ Not possible

So $r = 4 - 2\sqrt{2}$

Area $= \pi r^2 = \pi\left(16 + 8 - 16\sqrt{2}\right)$

$= 8\pi\left(3 - 2\sqrt{2}\right)$



78. $f(x)=\begin{cases} a|\pi-x|+1, & x\le 5 \\ b|\pi-x|+3, & x>5 \end{cases}$

Contributes at x = 5

$\therefore$ L.H.L. = R.H.L = f (5)

$\Rightarrow b|\pi-5|+3=a|\pi-5|+1$

$\Rightarrow -b(\pi-5)+3=-a(5-\pi)+1$

$\Rightarrow (a-b)(\pi-5)=-2$

$\Rightarrow a-b=\dfrac{-2}{\pi-5}=\dfrac{2}{5-\pi}$

79. $f(x)=[x]-\left[\dfrac{x}{4}\right]$

$\lim\limits_{x\to 4^+} f(x)=\lim\limits_{x\to 4^+}\left([x]-\left[\dfrac{x}{4}\right]\right)=4-1=3$

$\lim\limits_{x\to 4^-} f(x)=\lim\limits_{x\to 4^-}\left([x]-\left[\dfrac{x}{4}\right]\right)=3-0=3$

$f(4)=3$

$\therefore$ Continuous at x = 4

80. $\int e^{\sec x}\left(\sec x+\tan x f(x)+\left(\sec x\tan x+\sec^2 x\right)\right)dx=e^{\sec x}f(x)+C$

Diff. both side w.r.t.x

$=e^{\sec x}\left(\sec x+\tan x+f(x)+\left(\sec x\tan x+\sec^2 x\right)\right)$

$=e^{\sec x}.\sec x\tan x f(x)+e^{\sec x}f(n)$

$\Rightarrow f'(x)=\sec^2 x+\tan x\sec x$

$\Rightarrow f(x)=\tan x+\sec x+C$

81. $(m^2+1)x^2-3x+(m+1)^2=0$

$\Rightarrow \alpha+\beta=\dfrac{3}{m^2+1}$

$\alpha\beta=\dfrac{(m+1)^2}{m^2+1}$

$\because \alpha+\beta$ is maximum

$\therefore m^2+1$ is minimum

$\Rightarrow m=0$

$\therefore \alpha+\beta=3$ and $\alpha\beta=1$

$|\alpha^3-\beta^3|=\left|(\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)\right|$

$\left|\sqrt{(\alpha+\beta)^2-4\alpha\beta}\left\{(\alpha+\beta)^2-\alpha\beta\right\}\right|$

$=\left|\sqrt{9-4}(9-1)\right|$

$=8\sqrt{5}$

82. Let population = 100

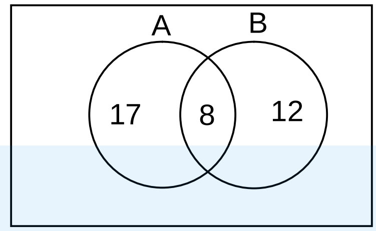
$n(A) = 25$

$n(B) = 20$

$n(A \cap B) = 8$

$n(A \cap \overline{B}) = 17$

$n(\overline{A} \cap B) = 12$



Now % of the population who look advertisement

$= \frac{30}{100} \times 17 + \frac{40}{100} \times 12 + \frac{50}{100} \times 8$

$= 13.9$

83. Equation of plane $P_1 + \lambda P_2 = 0$

$(x + y + z - 6) + \lambda(2x + 3y + z + 5) = 0$

$\Rightarrow x(1 + 2\lambda) + y(1 + 3\lambda) + 2(1 + \lambda) - 6 + 5\lambda = 0$

This plane is $\perp$ to xy – plane

$\therefore 1 + \lambda = 0$

So, equation of plane

$-x - 2y - 11 = 0$

$\Rightarrow x + 2y + 11 = 0$

distance of the point (0, 0, 256) from this plane.

$= \left| \frac{0 + 0 + 11}{\sqrt{1 + 4}} \right| = \frac{11}{\sqrt{5}}$

84. $p \Rightarrow (q \vee r)$ is false

$(\because T \Rightarrow F = F)$

So, $p = T$, $q = F$ and $r = F$

85. $f(x) = \frac{1}{4 - x^2} + \log_{10}(x^3 - x)$

Let $f_1 = \frac{1}{4 - x^2}$ and $f_2 = \log_{10}(x^3 - x)$

$\Rightarrow 4 - x^2 \neq 0$ $\quad$ $x^3 - x > 0$

$\Rightarrow x \neq \pm 2$ $\quad$ $\Rightarrow x(x + 1)(x + 1) > 0$

$x \in (-1, 0) \cup (1, \infty) - \{2\}$

$x \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

86. $S = 1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + ... +$ upto 11 terms

n^{th} term of the series is $T_n = n(2n - 1)$

$\Rightarrow S = \sum_{n=1}^{11} T_n = \sum_{n=1}^{11} (2n^2 - n)$

$\Rightarrow S_n = \frac{2n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$

Put $n = 1$

$\Rightarrow S_{11} = \frac{2(11)(12)(23)}{6} - \frac{11(12)}{2}$

$\Rightarrow S_{11} = 946$

87. mean = 42

$\Rightarrow \frac{10+22+26+29+34+x+42+67+70+y}{10} = 45$

$\Rightarrow x + y = 120$ ....(i)

and median = 35

$\Rightarrow \frac{34+x}{2} = 35 \Rightarrow x = 36$

from (i) y = 84

$\frac{y}{x} = \frac{84}{36} = \frac{7}{3}$

88. $y^2 = 2x$ .....(i)

and $x - y - 4 = 0$ ....(ii)

solving (1) and (2)

$(x-y)^2 = 2x$

$\Rightarrow x^2 - 10x + 16 = 0$

$\Rightarrow x = 8, 2$ and $y = 4, -2$

$A = \int_{-2}^{4} \left(y + 4 - \frac{y^2}{2} \right) dy$

$A = \left(\frac{y^2}{4} + 4y - \frac{y^3}{6} \right)_{-2}^{4}$

$A = \left(4 + 16 - \frac{64}{6} \right) - \left(1 - 8 + \frac{8}{6} \right) = 18$

89. $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

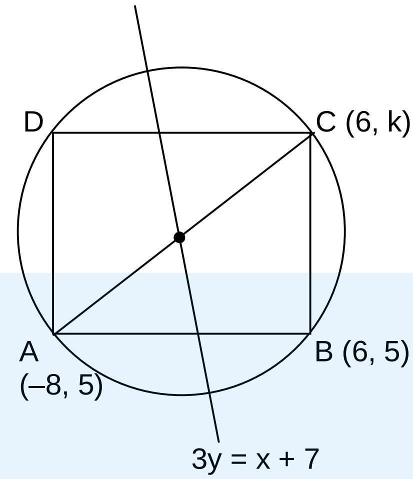
$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2 \gamma = 1$

$\Rightarrow \cos^2 \gamma = 1 - \frac{3}{4} = \frac{1}{4}$

$\Rightarrow \cos \gamma = \pm \frac{1}{2}$

$\Rightarrow \gamma = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

90. Let vertex C is $(6, k)$ then centre of circle $\left(-1, \frac{5+k}{2}\right)$



It lies on diameter $3y = x + 7$

$\Rightarrow 3\left(\frac{5+k}{2}\right) = -1 + 7$

$\Rightarrow k = -1$

So, AB = 14 and BC = 6

Area = $14 \times 6 = 84$