

FIITJEE

Solutions to JEE(Main)-2020

Test Date: 6th September 2020 (First Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours

Maximum Marks: 300

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

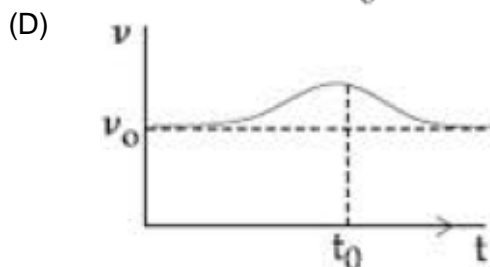
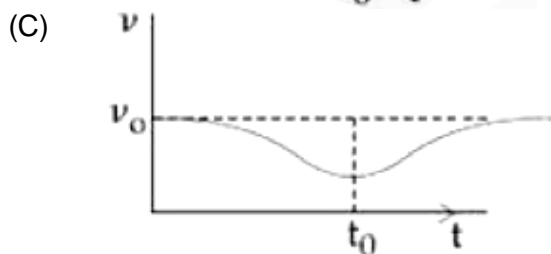
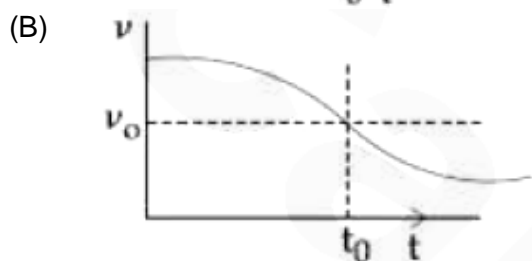
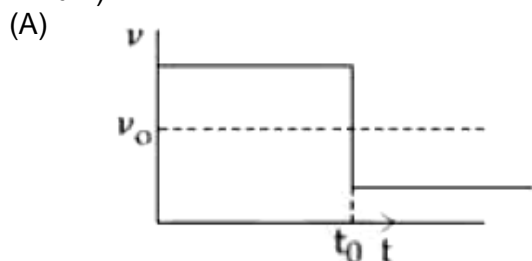
Important Instructions:

1. The test is of **3 hours** duration.
2. This **Test Paper** consists of **75** questions. The maximum marks are **300**.
3. There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 25 questions in each part of equal weightage out of which 20 questions are MCQs and 5 questions are numerical value based. Each question is allotted **4 (four)** marks for correct response.
4. **(Q. No. 01 – 20, 26 – 45, 51 – 70)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
5. **(Q. No. 21 – 25, 46 – 50, 71 – 75)** contains 15 Numerical based questions with answer as numerical value. Each question carries **+4 marks** for correct answer. There is no negative marking.
6. Candidates will be awarded marks as stated above in **instruction No.3** for correct response of each question. One mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer box.
7. There is only one correct response for each question. Marked up more than one response in any question will be treated as wrong response and marked up for wrong response will be deducted accordingly as per **instruction 6** above.

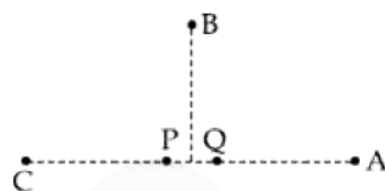
PART -A (PHYSICS)

1. A screw gauge has 50 divisions on its circular scale. The circular scale is 4 units ahead of the pitch scale marking, prior to use. Upon one complete rotation of the circular scale, a displacement of 0.5 mm is noticed on the pitch scale. The nature of zero error involved, and the least count of the screw gauge, are respectively:
 (A) Negative, $2\mu\text{m}$ (B) Positive, $10\mu\text{m}$
 (C) Positive, 0.1 mm (D) Positive, $0.1\mu\text{m}$

2. A sound source S is moving along a straight track with speed v , and is emitting sound of frequency ν_0 (see figure). An observer is standing at a finite distance, at the point O, from the track. The time variation of frequency heard by the observer is best represented by:
 (t_0 represents the instant when the distance between the source and observer is minimum)



3. In the figure below, P and Q are two equally intense coherent sources emitting radiation of wavelength 20 m. The separation between P and Q is 5 m and the phase of P is ahead of that of Q by 90° . A, B and C are three distinct points of observation, each equidistant from the midpoint of PQ. The intensities of radiation at A, B, C will be in the ratio:



- (A) 0 : 1 : 4
(B) 2 : 1 : 0
(C) 0 : 1 : 2
(D) 4 : 1 : 0

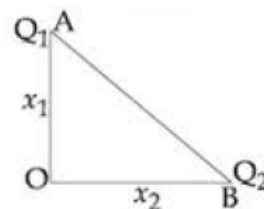
4. If the potential energy between two molecules is given by $U = \frac{A}{r^6} + \frac{B}{r^{12}}$, then at equilibrium, separation between molecules, and the potential energy are:

- (A) $\left(\frac{B}{2A}\right)^{1/6}, -\frac{A^2}{2B}$
(B) $\left(\frac{B}{A}\right)^{1/6}, 0$
(C) $\left(\frac{2B}{A}\right)^{1/6}, -\frac{A^2}{4B}$
(D) $\left(\frac{2B}{A}\right)^{1/6}, -\frac{A^2}{2B}$

5. An AC circuit has $R = 100\Omega$, $C = \mu F$ and $L = 80 \text{ mH}$, connected in series. The quality factor of the circuit is:

- (A) 2
(B) 0.5
(C) 20
(D) 400

6. Charge Q_1 and Q_2 are at point A and B of a right angle triangle OAB (see figure). The resultant electric field at point O is perpendicular to the hypotenuse, then $\frac{Q_1}{Q_2}$ is proportional to:



- (A) $\frac{x_1^3}{x_2^3}$
(B) $\frac{x_2}{x_1}$
(C) $\frac{x_1}{x_2}$
(D) $\frac{x_2^2}{x_1^2}$

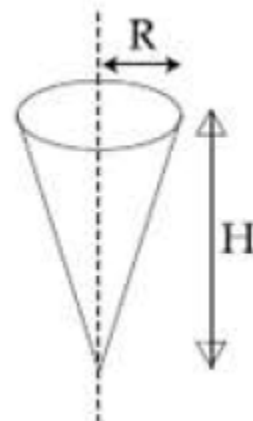
7. Shown in the figure is a hollow icecream cone (it is open at the top). If its mass is M , radius of its top, R and height, H , then its moment of inertia about its axis is:

(A) $\frac{MR^2}{2}$

(B) $\frac{M(R^2 + H^2)}{4}$

(C) $\frac{MH^2}{3}$

(D) $\frac{MR^2}{3}$



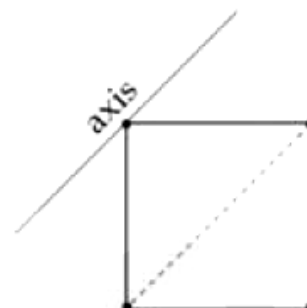
8. Four point masses, each of mass m , are fixed at the corners of a square of side l . The square is rotating with angular frequency ω , about an axis passing through one of the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is:

(A) $ml^2\omega$

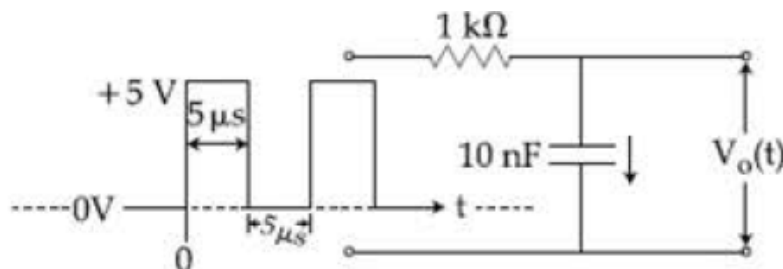
(B) $5ml^2\omega$

(C) $3ml^2\omega$

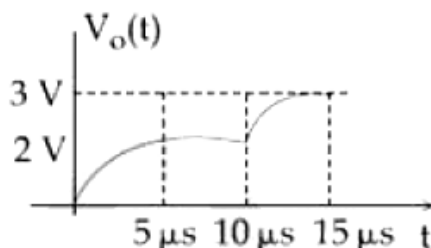
(D) $2ml^2\omega$

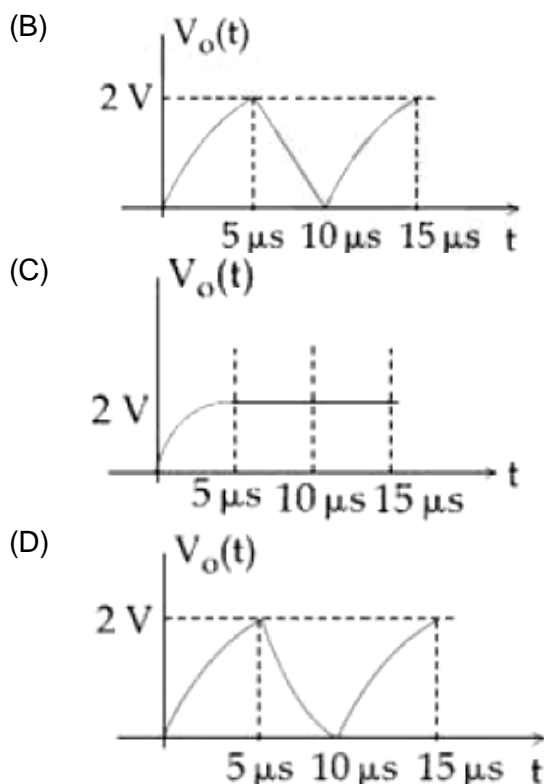


9. For the given input voltage waveform $V_{in}(t)$, the output voltage waveform $V_o(t)$, across the capacitor is correctly depicted by:



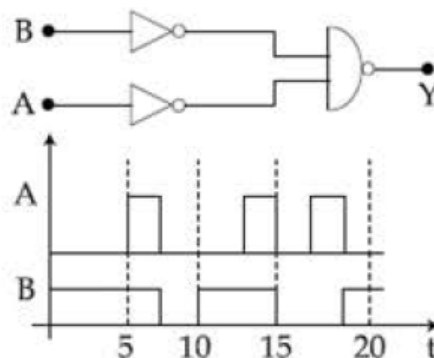
(A)





10. A particle of charge q and mass m is moving with a velocity $-v\hat{i}$ ($v \neq 0$) towards a large screen placed in the $Y-Z$ plane at a distance d . If there is a magnetic field $\vec{B} = B_0\hat{k}$, the minimum value of v for which the particle will not hit the screen is:
- (A) $\frac{qdB_0}{3m}$ (B) $\frac{2qdB_0}{m}$
 (C) $\frac{qdB_0}{m}$ (D) $\frac{qdB_0}{2m}$
11. An insect is at the bottom of a hemispherical ditch of radius 1 m. It crawls up the ditch but starts slipping after it is at height h from the bottom. If the coefficient of friction between the ground and the insect is 0.75, then h is: ($g = 10\text{ms}^{-2}$)
- (A) 0.20 m (B) 0.45 m
 (C) 0.60 m (D) 0.80 m
12. A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is:
- (A) 1 : 6 (B) 1 : 3
 (C) 1 : 2 (D) 3 : 4

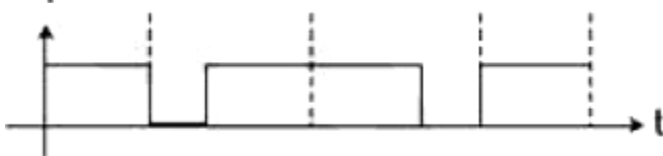
13. An electron, a doubly ionized helium ion (He^{++}) and a proton are having the same kinetic energy. The relation between their respective de – Broglie wavelength λ_e , $\lambda_{\text{He}^{++}}$ and λ_p is:
- (A) $\lambda_e > \lambda_{\text{He}^{++}} > \lambda_p$ (B) $\lambda_e < \lambda_{\text{He}^{++}} = \lambda_p$
 (C) $\lambda_e > \lambda_p > \lambda_{\text{He}^{++}}$ (D) $\lambda_e < \lambda_p < \lambda_{\text{He}^{++}}$
14. A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of:
- (A) 10^{-3} (B) 10^{-4}
 (C) 10^{-2} (D) 10^{-1}
15. You are given that Mass of ${}^7_2\text{Li} = 7.0160\text{u}$ Mass of ${}^4_2\text{He} = 4.0026\text{u}$ and ${}^1_1\text{H} = 1.0079\text{u}$. When 20 g of ${}^7_3\text{Li}$ is converted into ${}^4_2\text{He}$ by proton capture, the energy liberated, (in kWh), is [Mass of nucleon = $\frac{1\text{Gev}}{c^2}$]
- (A) 4.5×10^5 (B) 8×10^6
 (C) 6.82×10^5 (D) 1.33×10^6
16. A point like object is placed at a distance of 1 m in front of convex lens of focal length 0.5 m. A plane mirror is placed at a distance of 2m behind the lens. The position and nature of the final image formed by the system is:
- (A) 2.6 m from the mirror, real (B) 1 m from the mirror, virtual
 (C) 1 m from the mirror, real (D) 2.6 m from the mirror, virtual
17. Identify the correct output signal Y in the given combination of gates (as shown) for the given inputs A and B.

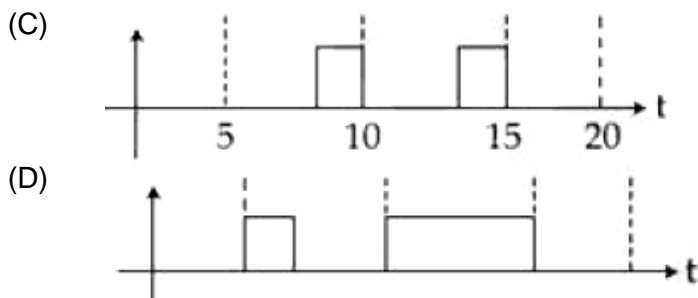


(A)



(B)





18. Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of T .

The total internal energy, U of a mole of this gas, and the value of $\gamma \left(= \frac{C_P}{C_V} \right)$ are given, respectively, by:

- (A) $U = \frac{5}{2}RT$ and $\gamma = \frac{6}{5}$ (B) $U = 5RT$ and $\gamma = \frac{7}{5}$
 (C) $U = \frac{5}{2}RT$ and $\gamma = \frac{7}{5}$ (D) $U = 5RT$ and $\gamma = \frac{6}{5}$

19. An object of mass m is suspended at the end of a massless wire of length L and area of cross – section, A . Young modulus of the material of the wire is Y . If the mass is pulled down slightly its frequency of oscillation along the vertical direction is:

- (A) $f = \frac{1}{2\pi} \sqrt{\frac{mL}{YA}}$ (B) $f = \frac{1}{2\pi} \sqrt{\frac{YA}{mL}}$
 (C) $f = \frac{1}{2\pi} \sqrt{\frac{mA}{YL}}$ (D) $f = \frac{1}{2\pi} \sqrt{\frac{YL}{mA}}$

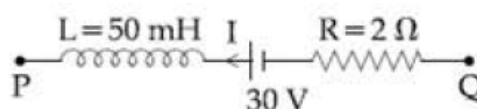
20. An electron is moving along $+x$ direction with a velocity of $6 \times 10^6 \text{ ms}^{-1}$. It enters a region of uniform electric field of 300 V/cm pointing along $+y$ direction. The magnitude and direction of the magnetic field set up in this region such that the electron keeps moving along the x direction will be:

- (A) $3 \times 10^{-4} \text{ T}$, along $+z$ direction (B) $5 \times 10^{-3} \text{ T}$, along $-z$ direction
 (C) $5 \times 10^{-3} \text{ T}$, along $+z$ direction (D) $3 \times 10^{-4} \text{ T}$, along $-z$ direction

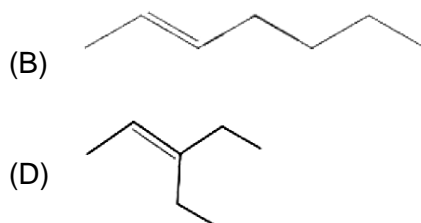
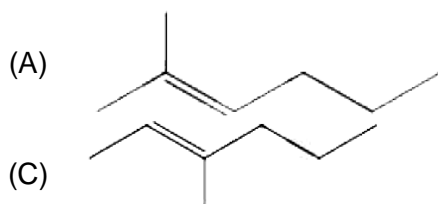
21. The density of a solid metal sphere is determined by measuring its mass and its diameter. The maximum error in the density of the sphere is $\left(\frac{x}{100} \right) \%$. If the relative errors in measuring the mass and the diameter are 6.0% and 1.5% respectively, the value of x is _____.

22. Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial speed. The angle between the initial velocities of the two bodies (in degree) is _____.

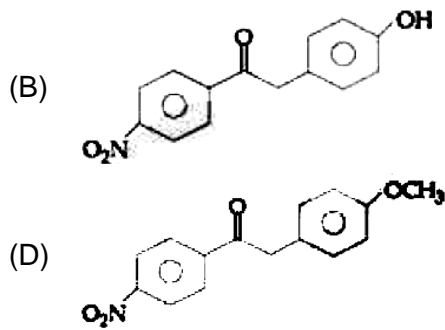
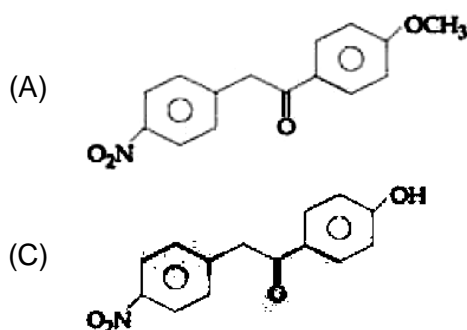
23. Suppose that intensity of a laser is $\left(\frac{315}{\pi}\right) \text{ W/m}^2$. The rms electric field, in units of V/m associated with this source is close to the nearest integer is _____.
 ($\epsilon_0 = 8.86 \times 10^{-12} \text{ C}^2\text{Nm}^{-2}$; $c = 3 \times 10^8 \text{ ms}^{-1}$)
24. Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure P_1 and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by P_2 . The ratio $\frac{P_2}{P_1}$ is _____.
25. A part of complete circuit is shown in the figure. At some instant, the value of current I is 1 A and it is decreasing at a rate of 10^2 A s^{-1} . The value of the potential difference $V_P - V_Q$, (in volts) at that instant, is _____.



27. Consider the following reactions:



- $$\text{O}_2\text{N}-\text{C}_6\text{H}_4-\text{C}\equiv\text{C}-\text{C}_6\text{H}_4-\text{OCH}_3 \xrightarrow[\text{H}_2\text{O}]{\text{Hg}^{2+}/\text{H}^+}$$



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$$(A) C_2 = \frac{1000x_2}{M_1 + x_2(M_2 - M_1)}$$

$$(B) C_2 = \frac{dx_2}{M_2 + x_2(M_2 - M_1)}$$

$$(C) C_2 = \frac{1000 dx_2}{M_1 + x_2(M_2 - M_1)}$$

$$(D) C_2 = \frac{dx_1}{M_2 + x_2(M_2 - M_1)}$$

30. The INCORRECT statement is:

- (A) bronze is an alloy of copper and tin
 (B) cast iron is used to manufacture wrought iron
 (C) german silver is an alloy of zinc, copper and nickel
 (D) brass is an alloy of copper and nickel.

31. Consider the Assertion and Reason given below.

Assertion (A): Ethene polymerized in the presence of Ziegler Natta Catalyst at high temperature and pressure is used to make buckets and dustbins.

Reason (R): High density polymers are closely packed and are chemically inert.

Choose the correct answer from the following:

- (A) (A) is correct but (R) is wrong
 (B) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
 (C) Both (A) and (R) are correct and (R) is the correct explanation of (A)
 (D) (A) and (R) both are wrong.

32. Arrange the following solutions in the decreasing order of pOH

(1) 0.01 M HCl

(2) 0.01 M NaOH

(3) 0.01 M CH₃COONa

(4) 0.01 M NaCl

(A) (1) > (3) > (4) > (2)

(B) (1) > (4) > (3) > (2)

(C) (2) > (3) > (4) > (1)

(D) (2) > (4) > (3) > (1)

33. Among the sulphates of alkaline earth metals, the solubilities of BeSO₄ and MgSO₄ in water, respectively, are:

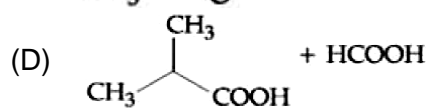
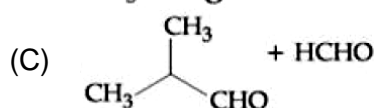
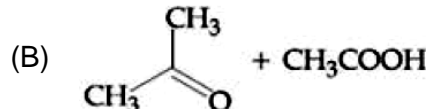
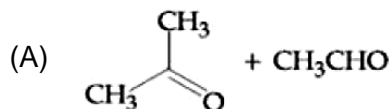
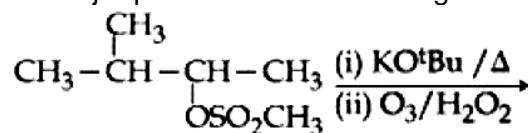
(A) poor and poor

(B) high and poor

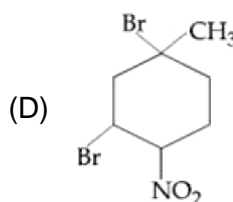
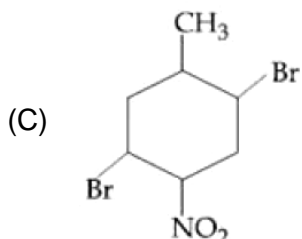
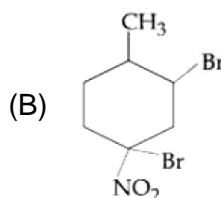
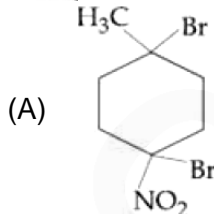
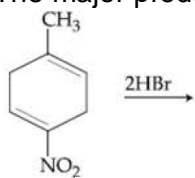
(C) high and high

(D) poor and high

34. The major products of the following reaction are:

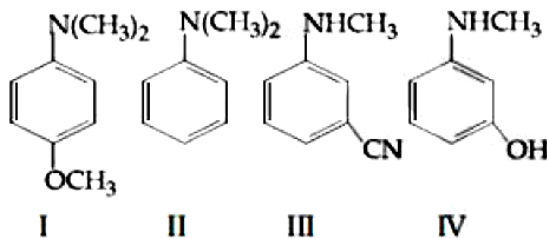


35. The major product of the following reaction is:



36. The present of soluble fluoride ion upto 1 ppm concentration in drinking water, is:
 (A) harmful for teeth (B) harmful to skin
 (C) harmful to bones (D) safe for teeth

37. The increasing order of pK_b values of the following compounds is:



- (A) II < IV < III < I
 (C) II < I < III < IV

- (B) I < II < IV < III
 (D) I < II < III < IV

38. Which of the following compounds shows geometrical isomerism?
 (A) 2-methylpent-2-ene (B) 4-methylpent-2-ene
 (C) 4-methylpent-1-ene (D) 2-methylpent-1-ene

39. The set that contains atomic numbers of only transition elements, is
 (A) 37, 42, 50, 64 (B) 21, 25, 42, 72
 (C) 9, 17, 34, 38 (D) 21, 32, 53, 64

40. The variation of equilibrium constant with temperature is given below:
 Temperature Equilibrium Constant
 $T_1 = 25^\circ\text{C}$ $K_1 = 10$
 $T_2 = 100^\circ\text{C}$ $K_2 = 100$

The value of $\Delta H^\circ, \Delta G^\circ$ at T_1 and ΔG° at T_2

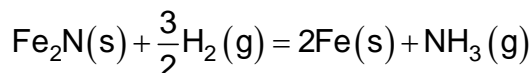
(in kJ mol^{-1}) respectively, are close to

[Use $R = 8.314 \text{ JK}^{-1} \text{ mol}^{-1}$]

- (A) 28.7, - 7.14 and - 5.71
(B) 0.64 - 7.14 and - 5.71
(C) 28.4, - 5.71 and - 14.29
(D) 0.64, - 5.71 and -14.29

41. Kraft temperature is the temperature:
(A) below which the aqueous solution of detergents starts freezing
(B) below which the formation of micelles takes place
(C) above which the aqueous solution of detergents starts boiling
(D) above which the formation of micelles takes place.

42. For the reaction



- (A) $K_c = K_p (RT)$
(B) $K_c = K_p (RT)^{-1/2}$
(C) $K_c = K_p (RT)^{1/2}$
(D) $K_c = K_p (RT)^{3/2}$

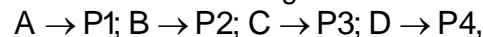
43. The species that has a spin – only magnetic moment of 5.9 BM, is:
(T_d = tetrahedral)

- (A) $[\text{Ni}(\text{CN})_4]^{2-}$ – (square planar)
(B) $[\text{NiCl}_4]^{2-}$ – (T_d)
(C) $\text{Ni}(\text{CO})_4$ (T_d)
(D) $[\text{MnBr}_4]^{2-}$ – (T_d)

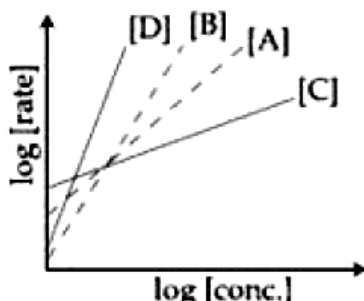
44. The lanthanoid that does NOT show +4 oxidation state is:

- (A) Dy
(B) Ce
(C) Eu
(D) Tb

45. Consider the following reactions



The order of the above reactions are a, b, c and d, respectively. The following graph is obtained when $\log[\text{rate}]$ vs. $\log[\text{conc.}]$ are plotted:



Among the following, the correct sequence for the order of the reactions is:

- (A) $d > a > b > c$
(B) $a > b > c > d$
(C) $c > a > b > d$
(D) $d > b > a > c$

46. In an estimation of bromine by Carius method, 1.6 g of an organic compound gave 1.88 g of AgBr. The mass percentage of bromine in the compound is _____
(Atomic mass, Ag = 108, Br = 80 g mol^{-1})

47. Potassium chlorate is prepared by the electrolysis of KCl in basic solution
 $6\text{HO}^- + \text{Cl}^- \rightarrow \text{ClO}_3^- + 3\text{H}_2\text{O} + 6\text{e}^-$
If only 60% of the current is utilized in the reaction, the time (rounded to the nearest hour) required to produce 10 g of KClO_3 using a current of 2A is _____.
(Given : $F = 96,500 \text{ C mol}^{-1}$; molar mass of $\text{KClO}_3 = 122 \text{ g mol}^{-1}$)
48. The number of Cl = O bonds in perchloric acid is, " _____".
49. The elevation of boiling point of 0.10 m aqueous $\text{CrCl}_3 \cdot x\text{NH}_3$ solution is two times that of 0.05 m aqueous CaCl_2 solution. The value of x is _____.
[Assume 100% ionisation of the complex and CaCl_2 , coordination number of Cr as 6, and that all NH_3 molecules are present inside the coordination sphere]
50. A spherical balloon of radius 3 cm containing helium gas has a pressure of 48×10^{-3} bar. At the same temperature, the pressure, of a spherical balloon of radius 12 cm containing the same amount of gas will be _____ $\times 10^{-6}$ bar.

PART-C (MATHEMATICS)

51. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of

$$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8} \text{ is:}$$

(A) 2
(C) 1

(B) 3
(D) 4

52. The area (in sq. units) of the region $A = \{(x, y) : |x| + |y| \leq 1, 2y^2 \geq |x|\}$ is:

(A) $\frac{1}{3}$
(C) $\frac{1}{6}$

(B) $\frac{7}{6}$
(D) $\frac{5}{6}$

53. The general solution of the differential equation $\sqrt{1+x^2+y^2+x^2y^2} + xy \frac{dy}{dx} = 0$ is:
(where C is a constant of integration)

(A) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2} \log_e \left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1} \right) + C$

(B) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2} \log_e \left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1} \right) + C$

(C) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2} \log_e \left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1} \right) + C$

(D) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2} \log_e \left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1} \right) + C$

54. Let L_1 be a tangent to the parabola $y^2 = 4(x+1)$ and L_2 be a tangent to the parabola $y^2 = 8(x+2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line:

(A) $x+3=0$
(C) $x+2=0$

(B) $2x+1=0$
(D) $x+2y=0$

55. If $f(x+y) = f(x)f(y)$ and $\sum_{x=1}^{\infty} f(x) = 2, x, y \in \mathbb{N}$ where \mathbb{N} is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is:
- (A) $\frac{2}{3}$ (B) $\frac{1}{9}$
 (C) $\frac{1}{3}$ (D) $\frac{4}{9}$
56. If $I_1 = \int_0^1 (1-x^{50})^{100} dx$ and $I_2 = \int_0^1 (1-x^{50})^{101} dx$ such that $I_2 = \alpha I_1$ then α equals to:
- (A) $\frac{5049}{5050}$ (B) $\frac{5050}{5049}$
 (C) $\frac{5050}{5051}$ (D) $\frac{5051}{5050}$
57. Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is:
- (A) $\frac{15}{101}$ (B) $\frac{5}{101}$
 (C) $\frac{5}{33}$ (D) $\frac{10}{99}$
58. A ray of light coming from the point $(2, 2\sqrt{3})$ is incident at an angle 30° on the line $x = 1$ at the point A. The ray gets reflected on the line $x = 1$ and meets x -axis at the point B. Then, the line AB passes through the point:
- (A) $\left(3, -\frac{1}{\sqrt{3}}\right)$ (B) $\left(4, -\frac{\sqrt{3}}{2}\right)$
 (C) $\left(3, -\sqrt{3}\right)$ (D) $\left(4, -\sqrt{3}\right)$
59. Which of the following points lies on the locus of the foot of perpendicular drawn upon any tangent to the ellipse, $\frac{x^2}{4} + \frac{y^2}{2} = 1$ from any of its foci?
- (A) $(-2, \sqrt{3})$ (B) $(-1, \sqrt{2})$
 (C) $(-1, \sqrt{3})$ (D) $(1, 2)$

60. The region represented by $\{z = x + iy \in \mathbb{C} : |z| - \operatorname{Re}(z) \leq 1\}$ is also given by the inequality:
- (A) $y^2 \geq 2(x+1)$ (B) $y^2 \leq 2\left(x + \frac{1}{2}\right)$
 (C) $y^2 \leq x + \frac{1}{2}$ (D) $y^2 \geq x+1$
61. The position of a moving car at time t is given by $f(t) = at^2 + bt + c$, $t > 0$, where a , b and c are real numbers greater than 1. Then the average speed of the car over the time interval $[t_1, t_2]$ is attained at the point:
- (A) $\frac{(t_2 - t_1)}{2}$ (B) $a(t_2 - t_1) + b$
 (C) $\frac{(t_1 + t_2)}{2}$ (D) $2a(t_1 + t_2) + b$
62. $\lim_{x \rightarrow 1} \left(\frac{\int_0^{(x-1)^2} t \cos(t^2) dt}{(x-1)\sin(x-1)} \right)$
- (A) is equal to $\frac{1}{2}$ (B) is equal to 1
 (C) is equal to $-\frac{1}{2}$ (D) does not exist
63. If $\sum_{i=1}^n (x_i - a) = n$ and $\sum_{i=1}^n (x_i - a)^2 = na$, ($n, a > 1$) then the standard deviation of n observations x_1, x_2, \dots, x_n is:
- (A) $a - 1$ (B) $n\sqrt{a-1}$
 (C) $\sqrt{n(a-1)}$ (D) $\sqrt{a-1}$
64. If $\{p\}$ denotes the fractional part of the number p , then $\left\{ \frac{3^{200}}{8} \right\}$, is equal to
- (A) $\frac{5}{8}$ (B) $\frac{7}{8}$
 (C) $\frac{3}{8}$ (D) $\frac{1}{8}$

65. The shortest distance between the lines $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$ and $x + y + z + 1 = 0$, $2x - y + z + 3 = 0$ is:
- (A) 1 (B) $\frac{1}{\sqrt{3}}$
(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$
66. The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to:
- (A) $p \wedge \sim q$ (B) $\sim p \wedge \sim q$
(C) $\sim p \vee \sim q$ (D) $\sim p \vee q$
67. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated?
- (A) $2!3!4!$ (B) $(3!)^3 \cdot (4!)$
(C) $(3!)^2 \cdot (4!)$ (D) $3!(4!)^3$
68. Let m and M be respectively the minimum and maximum values of $\begin{vmatrix} \cos^2 x & 1 + \sin^2 x & \sin 2x \\ 1 + \cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1 + \sin 2x \end{vmatrix}$. Then the ordered pair (m, M) is equal to:
- (A) $(-3, 3)$ (B) $(-3, -1)$
(C) $(-4, -1)$ (D) $(1, 3)$
69. Let a, b, c, d and p be any non zero distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$. Then:
- (A) a, c, p are in A.P. (B) a, c, p are in G.P.
(C) a, b, c, d are in G.P. (D) a, b, c, d are in A.P.
70. The value of λ and μ for which the system of linear equations
- $$\begin{aligned} x + y + z &= 2 \\ x + 2y + 3z &= 5 \\ x + 3y + \lambda z &= \mu \end{aligned}$$
- has infinitely many solutions are, respectively:
- (A) 6 and 8 (B) 5 and 7
(C) 5 and 8 (D) 4 and 9
71. Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of $m.n$ is _____.

72. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2, & x < 0 \\ 0, & x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2, & x > 0 \end{cases}$

The value of λ for which $f''(0)$ exists, is _____

73. If \vec{a} and \vec{b} are unit vectors, then the greatest value of $\sqrt{3}|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|$ is _____

74. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If AD = 8 m, BC = 11 m and AB = 10 m; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is _____.

75. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45° . After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75° . Then the height of the hill (in metres) is _____.

FIITJEE

Solutions to JEE (Main)-2020

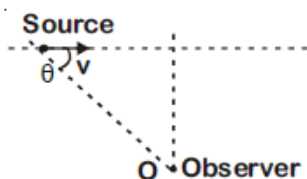
PART -A (PHYSICS)

1. **B**

Sol.
$$\begin{aligned} \text{L.C.} &= \frac{0.5 \text{ mm}}{50} \\ &= 10^{-2} \text{ mm} \\ &= 10^{-5} \text{ m} \\ &= 10 \text{ } \mu\text{m} \end{aligned}$$

2. **B**

Sol.



While approaching

$$v = v_0 \left(\frac{c}{c - v \cos \theta} \right)$$

While receding

$$v = v_0 \left(\frac{c}{c + v \cos \theta} \right)$$

3. **B**

Sol.
$$\phi_A = \frac{\pi}{2} - \frac{2\pi}{\lambda} \times \frac{5}{20} = 0$$

$$\phi_B = \frac{\pi}{2}$$

$$\phi_C = \frac{\pi}{2} + \frac{2\pi}{\lambda} \times \frac{5}{20} = \pi$$

$$I_A = 4I_0 \quad ; \quad I_B = 2I_0 \quad ; \quad I_C = 0$$

4. **C**

Sol.
$$F = -\frac{dU}{dr} = -\left[\frac{6A}{r^7} - \frac{12B}{r^{13}} \right]$$

$$F = 0$$

$$\Rightarrow r = \left(\frac{2B}{A} \right)^{1/6}$$

$$U \left(\text{at } r = \left(\frac{2B}{A} \right)^{1/6} \right) = -\frac{A^2}{4B}$$

5. **A**

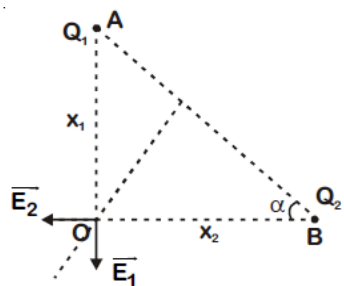
Sol. $Q = \frac{1}{R} \sqrt{\frac{L}{C}}$

$$= \frac{1}{100} \sqrt{\frac{80 \times 10^{-3}}{2 \times 10^{-6}}}$$

$$= 2$$

6. **C**

Sol.



Net field along AB at O must be zero.

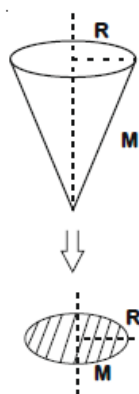
$$E_2 \cos \alpha = E_1 \sin \alpha$$

$$\frac{kQ_2}{x_2^2} \cdot \frac{x_2}{AB} = \frac{kQ_1}{x_1^2} \cdot \frac{x_1}{AB}$$

$$\frac{Q_1}{Q_2} = \frac{x_1}{x_2}$$

7. **A**

Sol.

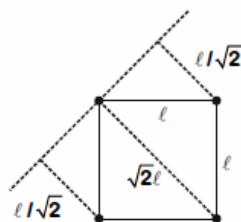


$$I = \frac{MR^2}{2}$$

Moment of inertia of this cone will same as circular disk of mass (M) and radius R.

8. **C**

Sol.



$$I = m \left(\frac{\ell^2}{2} \right) \times 2 + m \times (\sqrt{2} \ell)^2 = 3m\ell^2$$

$$\therefore L = I\omega = 3m\ell^2\omega$$

9. **A**Sol. $\tau = RC = 10 \mu s$

For $0 < t < 5 \mu s$, it will get charged. For $5 < t < 10 \mu s$ potential is constant and again gets charged after that.

10. **C**Sol. $r = \frac{mv}{qB_0}$

To not collide, $r < d$

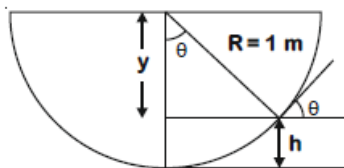
$$\Rightarrow \frac{mv}{qB_0} < d$$

$$\therefore v_{\max} = \frac{qB_0 d}{m}$$

Note: It should be maximum instead of minimum.

11. **A**

Sol.



$$\mu = \tan \theta$$

$$\Rightarrow \frac{3}{4} = \tan \theta$$

$$\Rightarrow \theta = 37^\circ$$

$$\therefore h = R - R \cos \theta = 1 - 1 \times \frac{4}{5} = 0.2 \text{ m}$$

12. **A**Sol. $\frac{V_{\max}}{V_{\min}} = \frac{(1+e)}{(1-e)}$

$$\frac{r_{\max}}{r_{\min}} = \frac{(1+e)}{1-e} = 6$$

13. **C**

Sol. $\lambda = \frac{h}{p}$
 $p = \sqrt{2mk}$
 $\lambda \propto \frac{1}{\sqrt{m}}$

14. **A**

Sol. $a = \omega^2 \times \ell$
 $= \left(\frac{2\pi}{T} \right)^2 \times 0.1$
 $= \left(\frac{2\pi}{60} \right)^2 \times 0.1$
 $= 1.1 \times 10^{-3} \text{ m/s}^2$

15. **D**

Sol. ${}^7_3\text{Li} + {}^1_1\text{H} \longrightarrow {}^4_2\text{He}$
 $\Delta m = (m_{\text{Li}} + m_{\text{H}} - 2m_{\text{He}})$
 $= .0187 \text{ u}$
 $Q \text{ value} = \Delta mc^2$
 $= 17.42 \text{ MeV}$

Energy liberated = $\frac{20}{7} \times 6.023 \times 10^{23} \times (Q\text{-value})$

$\approx 300 \times 10^{29} \text{ eV}$

$\approx 480 \times 10^{10} \text{ J}$

$= 1.33 \times 10^6 \text{ kWh}$

16. **A**

Sol.

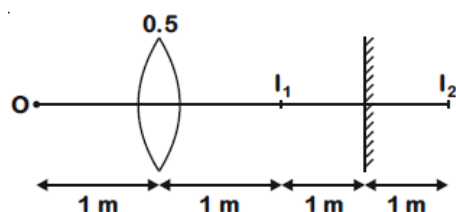


Image formed by one will be object for other.

$$\frac{1}{v_1} + \frac{1}{1} = \frac{1}{0.5} \Rightarrow v_1 = 1 \text{ m}$$

I_2 will be formed in behind the mirror.

$$\frac{1}{v_3} + \frac{1}{3} = \frac{1}{0.5} \Rightarrow v_3 = 0.6 \text{ m}$$

So, final image will be formed at 2.6 m from the mirror, real.

17. **None**

Sol. $Y = \overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$

Truth table

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

18. **C**

Sol. $f = 5$

$$\therefore U = \frac{5}{2}RT$$

$$\text{And } \gamma = 1 + \frac{2}{f} = 1 + \frac{2}{5} = \frac{7}{5}$$

19. **B**

Sol. $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

$$k = \frac{YA}{L}$$

$$f = \left(\frac{1}{2\pi} \right) \sqrt{\frac{YA}{mL}}$$

20. **C**

Sol. $F = q(\vec{E} + \vec{V} \times \vec{B})$

$$\vec{E} + \vec{V} \times \vec{B} = 0$$

21. **1050.00**

Sol. $\rho = \frac{m}{\frac{4}{3}\pi \left(\frac{d}{2}\right)^3}$

$$\therefore \% \frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + 3 \cdot \left(\frac{\Delta d}{d} \right)$$

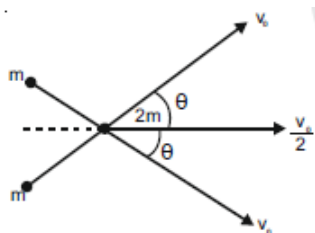
$$= 6 + 3 \times 1.5$$

$$= 10.5\%$$

$$= \left(\frac{1050}{100} \right) \%$$

22. **120.00**

Sol.



$$mv_0 \times \cos \theta \times 2 = 2m \times \left(\frac{v_0}{2} \right)$$

$$\Rightarrow \cos \theta = \frac{1}{2}$$

$$\Rightarrow \theta = 60^\circ$$

$$\therefore 2\theta = 120^\circ$$

23. **194**

Sol. $\therefore I = \frac{1}{2} \epsilon_0 E_0^2 c$

$$\Rightarrow E_0 = \sqrt{\frac{2I}{\epsilon_0 c}}$$

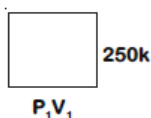
$$\therefore E_{\text{rms}} = \frac{E_0}{\sqrt{2}} = \sqrt{\frac{1}{\epsilon_0 c}}$$

$$= \sqrt{\frac{315}{\pi} \times \frac{1}{8.86 \times 8.86 \times 10^{-12} \times 3 \times 10^8}}$$

$$= 194$$

24. **05.00**

Sol. $n_0 = \frac{P_1 V_1}{R \times 250}$



$$n' = 0.75 n_0 + 0.5 n_0$$

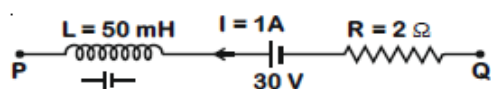
$$= 1.25 n_0 \text{ moles}$$

$$P_2 \times 2V_1 = (1.25) \frac{P_1 V_1}{R \times 250} \times R \times 2000$$

$$\Rightarrow \frac{P_2}{P_1} = 5$$

25. **33.00**

Sol.



$$V_Q - 2 \times 1 + 30 + (50 \times 10^{-3}) \times 10^2 = V_P$$

$$\Rightarrow V_P - V_Q = 33.0$$

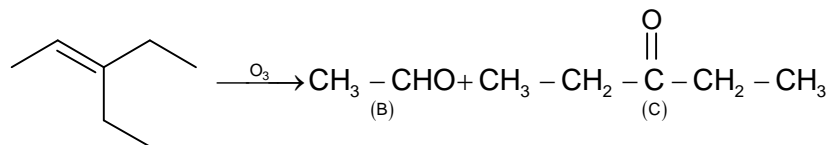
PART -B (CHEMISTRY)

26. D

Sol. Due to triple bond N_2 is inert.

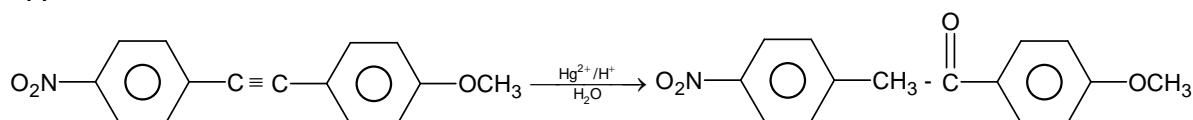
27. D

Sol.



28. A

Sol.



29. C

Sol. Let total mole = 1

$$n_2 = x_2 \quad n_1 = (1 - x_2)$$

$$m_2 = x_2 M_2 \quad m_1 = (1 - x_2) M_1$$

$$\therefore m_{\text{soln}} = [x_2 M_2 + (1 - x_2) M_1]$$

$$= [M_1 + x_2 (M_2 - M_1)]$$

$$V_{\text{soln}} = \frac{M_{\text{soln}}}{d} = \frac{M_1 + x_2 (M_2 - M_1)}{d} \text{ mole}$$

$$\therefore C_2 = \frac{x_2 \times 1000}{V_{\text{soln}}} = \frac{dx_2 \times 1000}{M_1 + x_2 (M_2 - M_1)}$$

30. D

Sol. Fact based

31. C

Sol. Fact based

32. B

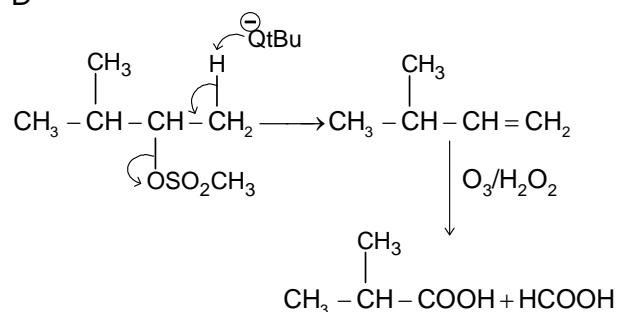
Sol. Most acidic has highest pOH

33. C

Sol. Solubility of sulphate decreases down.

34. D

Sol.



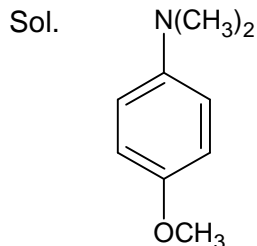
35. D

Sol. $-\text{NO}_2$ will repel \oplus
 $-\text{CH}_3$ will attract \oplus

36. D


Sol. Fact based

37. B



Due to +R of $-\text{OCH}_3$
 This is most basic hence pK_b is less

38. B

Sol. 
 $\text{CH}_3 - \text{CH} = \text{CH} - \text{CH}(\text{CH}_3) - \text{CH}_3$
 It will show geometrical isomerism

39. B

Sol. Fact based

40. C

Sol. $\log \frac{100}{10} = \frac{\Delta H}{2.303R} \left[\frac{1}{298} - \frac{1}{373} \right]$
 $\frac{2.303 \times 6.31 \times 298 \times 373}{75} = \Delta H$
 $\Delta H = 28.36$
 $\Delta G_1 = -2.303 \times 8.31 \times 298 \log 10 = -5.7$
 $\Delta G_2 = -2.303 \times 8.31 \times 373 \log 10 = -14.276$

41. D

Sol. Fact based

42. C

Sol. $\Delta n_g = -\frac{1}{2}$
 $K_P = K_c(RT)^{-1/2}$
 $K_c = K_P(RT)^{+1/2}$

43. D

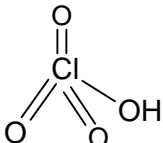
Sol. $\mu = 5.9 \text{ B.M}$
 i.e. it has 5 unpaired electrons
 hence $[\text{MnBr}_4]^{2-}$

44. C
Sol. Fact based

45. D
Sol. $\text{rate} = K[\text{Conc}]^n$
 $\text{Log}[\text{rate}] = \text{log}K + n\text{log}[\text{conc}]$
 \therefore "n" is slope
Higher the slope, higher order

46. 50
Sol. $\text{AgBr} = 1.88 \text{ g}$
 $\therefore \text{Br} = 0.8 \text{ gm}$
 $\% \text{Br} = \frac{0.8}{1.6 \times 100} = 50\%$

47. 11
Sol. $n_{\text{KClO}_3} = \left[\frac{10}{122} \right]$
 $\text{No. of F} = \left[\frac{10}{122} \times 6 \right]$
 $\text{No. of F supplied} = \left[\frac{10 \times 6}{122} \times \frac{100}{60} \right] = \frac{100}{122}$
 $\therefore \frac{100}{122} = \frac{2 \times t}{96500}$
 $T = 11 \text{ hrs}$

48. 3
Sol. 

49. 5
Sol. $i \text{ of } \text{CaCl}_2 = 3$
 $\text{Molality effective} = 3 \times 0.09 = 0.15$
 $\text{Molality effective of complex} = 0.15 \times 2 = 0.3$
 $\therefore [\text{Cr}(\text{NH}_3)_5\text{Cl}]\text{Cl}_2$

50. 750
Sol. $P_1V_1 = P_2V_2$
 $48 \times 10^{-3} \times 3^3 = P_2 \times 12^3$
 $P_2 = \frac{48 \times 10^{-3}}{4 \times 4 \times 4} = 0.75 \times 10^{-3} = 750 \times 10^{-6}$

PART-C (MATHEMATICS)

51. A

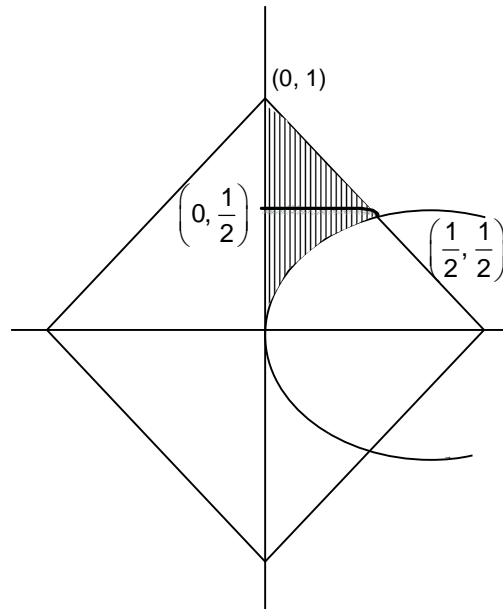
Sol. $\alpha + \beta = 64$; $\alpha\beta = 256 = 2^8$

$$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8} \\ = \frac{\alpha + \beta}{(\alpha\beta)^{5/8}} = \frac{64}{32} = 2$$

52. D

Sol. Required area

$$= 4 \left[\int_0^{\frac{1}{2}} 2y^2 dy + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right] = \frac{5}{6}$$



53. A

Sol.

$$\sqrt{1+x^2+y^2+x^2y^2} + xy \frac{dy}{dx} = 0$$

$$\sqrt{(1+x^2)(1+y^2)} + xy \frac{dy}{dx} = 0$$

$$\Rightarrow \int \frac{\sqrt{1+x^2}}{x} dx + \int \frac{y}{\sqrt{1+y^2}} dy = 0$$

$$\sqrt{1+x^2} + \frac{1}{2} \ln \left| \frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1} \right| + \sqrt{1+y^2} = C$$

$$\Rightarrow \sqrt{1+x^2} + \sqrt{1+y^2} = \frac{1}{2} \ln \left| \frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1} \right| + C$$

54. A

Sol. Equation of tangent to $y^2 = 4(x+1)$ is $y = m(x+1) + \frac{1}{m}$

Equation of tangent to $y^2 = 8(x+2)$ is $y = -\frac{1}{m}(x+2) - 2m$

Solving for point of intersection: $m(x+1) + \frac{1}{m} = -\frac{1}{m}(x+2) - 2m + 2m$

$$\Rightarrow \left(m + \frac{1}{m}\right)(x+3) = 0 \Rightarrow x+3 = 0$$

55. D

Sol. $f(x+y) = f(x)f(y)$

$$f(2) = (f(1))^2$$

$$f(3) = f(2+1) = f(2)f(1) = f(1)^3$$

:

:

and so on

$$f(n) = (f(1))^n$$

$$\Rightarrow \sum_{x=1}^{\infty} f(x) = 2 \Rightarrow \sum_{x=1}^{\infty} (f(1))^x = 2$$

$$\Rightarrow \frac{f(1)}{1-f(1)} = 2 \Rightarrow f(1) = \frac{2}{3}$$

$$\Rightarrow f(2) = \frac{4}{9} \Rightarrow f(4) = \frac{16}{81} \Rightarrow \frac{f(4)}{f(2)} = \frac{4}{9}$$

56. C

Sol. $I_2 = \int_0^1 (1-x^{50})^{101} dx$

$$= x(1-x^{50})^{101} \Big|_0^1 - \int_0^1 x \cdot 101(1-x^{50})^{100} (-50x^{49}) dx$$

$$= 0 + 5050 \int_0^1 \left\{ 1 - (1 - x^{50}) \right\} (1 - x^{50})^{100} dx$$

$$I_2 = 5050 I_1 - 5050 I_2$$

$$\Rightarrow 5051(I_2) = 5050(I_1)$$

$$I_2 = \frac{5050}{5051} I_1 \Rightarrow \alpha = \frac{5050}{5051}$$

57. C

Sol. If we select any two even numbers then their A.M. will be a natural number. Similarly, if we select two odd numbers their A.M. will be natural number. So, we need to select either two even numbers or two odd numbers to create an A.P. (Middle number is automatically fixed)

$$\text{Number of ways to do so} = {}^5C_2 + {}^6C_2 = 10 + 15 = 25$$

$$\text{Total number of possible ways to select three numbers} = {}^{11}C_3 = 165$$

$$\Rightarrow \text{Required probability} = \frac{25}{165} = \frac{5}{33}$$

58. C

Sol. $m_{AP} = \tan 60^\circ = \sqrt{3}$

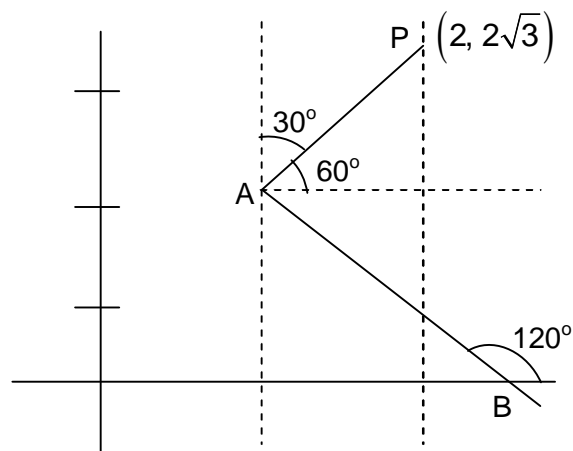
Equation of AP

$$y - 2\sqrt{3} = \sqrt{3}(x - 2)$$

Solving with $x = 1$

$$\Rightarrow A \equiv (1, \sqrt{3})$$

$$m_{AB} = \tan 120^\circ = -\cot 30^\circ = -\sqrt{3}$$



$$\text{Equation of AB: } y - \sqrt{3} = -\sqrt{3}(x - 1)$$

$$\Rightarrow \sqrt{3}x + y = 2\sqrt{3}$$

Clearly $(3, -\sqrt{3})$ satisfies the above equation.

59. C

Sol. Such feet of perpendicular lie on the auxiliary circle

$$\text{Equation of auxiliary circle: } x^2 + y^2 = 4$$

Clearly, $(-1, \sqrt{3})$ satisfies above equation.

60. B

Sol. $|z| - \operatorname{Re}(z) \leq 1$

$$\sqrt{x^2 + y^2} - x \leq 1$$

$$\sqrt{x^2 + y^2} \leq 1 + x$$

Squaring, $x^2 + y^2 \leq 1 + x^2 + 2x$

$$\Rightarrow y^2 \leq 2\left(x + \frac{1}{2}\right)$$

61. C

Sol. Average speed = $\frac{f(t_2) - f(t_1)}{t_2 - t_1} = \frac{(at_2^2 + bt_2 + c) - (at_1^2 + bt_1 + c)}{t_2 - t_1} = a(t_1 + t_2) + b$

Instantaneous speed = $2at + b$

$$\Rightarrow 2at + b = a(t_2 + t_1) + b \Rightarrow t = \frac{t_1 + t_2}{2}$$

62. **BONUS**

Sol. $\lim_{x \rightarrow 1} \frac{\int_0^{(x-1)^2} t \cos t^2 dt}{(x-1) \sin(x-1)}$

$$= \lim_{x \rightarrow 1} \frac{(x-1)^2 (\cos(x-1)^4) \cdot 2(x-1)}{\sin(x-1) + (x-1) \cos(x-1)}$$

$$= \lim_{y \rightarrow 0} \frac{y^2 \cos(y^4) \cdot 2y}{\sin y + y \cos y}$$

$$= \lim_{y \rightarrow 0} \frac{2y^2 \cos(y^4)}{\frac{\sin y}{y} + \cos y} = \frac{2(0)^2 \cos(0)}{1+1} = 0$$

63. D

Sol. S.D. = $\sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2} = \sqrt{\frac{\sum (x_i - a)^2}{n} - \left(\frac{\sum (x_i - a)}{n}\right)^2}$

$$= \sqrt{n \cdot \frac{a}{n} - \left(\frac{n}{n}\right)^2} = \sqrt{\frac{\sum (x_i - a)^2}{n} - \left(\frac{\sum (x_i - a)}{n}\right)^2}$$

$$= \sqrt{n \cdot \frac{a}{n} - \left(\frac{n}{n}\right)^2} = \sqrt{a - 1}$$

64. D

Sol. $\frac{3^{200}}{8} = \frac{81^{50}}{8} = \frac{(80+1)^{50}}{8} = \frac{\left({}^{50}C_0 80^{50} + {}^{50}C_1 80^{49} + {}^{50}C_2 80^{48} + \dots + {}^{50}C_{49} 80 + {}^{50}C_{50}\right)}{8}$

$$= \frac{80k+1}{8} = 10k + \frac{1}{8}; \quad (k \in \text{integers})$$

65. B

Sol. Family of planes containing line of intersection

 $x + y + z + 1 = 0 = 2x - y + z + 3$ is given by

$$x + y + z + 1 + \lambda(2x - y + z + 3) = 0$$

If this plane is parallel to line $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z-0}{1}$

then, $(2\lambda + 1)(0) + (1 - \lambda)(-1) + (1 + \lambda)(1) = 0 \Rightarrow \lambda = 0$

\Rightarrow Plane parallel to given line is $x + y + z + 1 = 0$

\Rightarrow Distance of $(1, -1, 0)$ from $x + y + z + 1 = 0$ is $\frac{|1 - 1 + 0 + 1|}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}$

66. B

Sol. $p \vee (\sim p \wedge q) = (p \vee \sim p) \wedge (p \vee q) = p \vee q$

$\sim(p \vee (\sim p \wedge q)) = \sim(p \vee \sim q) \wedge (p \vee q) = p \vee q$

$\sim(p \vee (\sim p \wedge q)) = \sim(p \vee q) = \sim p \wedge \sim q$

67. B

Sol. Let A_i, B_i, C_i be the family members of families A, B, C respectively

$$(A_1 A_2 A_3)(B_1 B_2 B_3)(C_1 C_2 C_3 C_4)$$

Required arrangement = $3! \times 3! \times 3! \times 4! = (3!)^3 (4!)$

68. B

$$\begin{aligned} \text{Sol. } & \begin{vmatrix} \cos^2 x & 1 + \sin^2 x & \sin 2x \\ 1 + \cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1 + \sin 2x \end{vmatrix} \\ &= \begin{vmatrix} \cos^2 x & 1 + \sin^2 x & \sin 2x \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{vmatrix} \quad (R_2 \rightarrow R_2 - R_1; R_3 \rightarrow R_3 - R_1) \\ &= -2 - \sin 2x \\ &m = -3; M = -1 \Rightarrow (m, M) \equiv (-3, -1) \end{aligned}$$

69. B

$$\begin{aligned} \text{Sol. } & (a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0 \\ & (a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bp + c^2) + (c^2p^2 - 2cdp + d^2) = 0 \\ & \Rightarrow (ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0 \\ & \Rightarrow p = \frac{b}{a} = \frac{c}{b} = \frac{d}{c} \\ & \Rightarrow a, b, c, d \text{ are in G.P.} \end{aligned}$$

70. C

$$\begin{aligned} \text{Sol. } & \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = \lambda - 5; \Delta_x = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 2 & 3 \\ \mu & 3 & \lambda \end{vmatrix} = -\lambda + \mu - 3 \\ & \Delta_y = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 5 & 3 \\ 1 & \mu & \lambda \end{vmatrix} = 3\lambda - 2\mu + 1; \Delta_z = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 1 & 3 & \mu \end{vmatrix} = \mu - 8 \\ & \text{For } \lambda = 5; \mu = 8 \\ & \Delta_x = \Delta_y = \Delta_z = \Delta = 0 \end{aligned}$$

71. 28

$$\begin{aligned} \text{Sol. } & 2^m - 2^n = 112 \\ & m = 7; n = 4 \text{ (By Hit and Trial)} \\ & m \cdot n = 28 \end{aligned}$$

72. 05

$$\text{Sol. } f'(x) = \begin{cases} 20x^3 \sin\left(\frac{1}{x}\right) - 5x^2 \cos\left(\frac{1}{x}\right) - 3x^2 \cos\left(\frac{1}{x}\right) - x \sin\left(\frac{1}{x}\right) + 10 & x < 0 \\ 0 & x = 0 \\ 20x^3 \cos\left(\frac{1}{x}\right) + 5x^2 \sin\left(\frac{1}{x}\right) + 3x^2 \sin\left(\frac{1}{x}\right) - x \cos\left(\frac{1}{x}\right) + 2\lambda & x > 0 \end{cases}$$

$$f''_-(0) = f''_+(0)$$

$$\Rightarrow 10 = 2\lambda$$

$$\Rightarrow \lambda = 5$$

73. 04

$$\begin{aligned} \text{Sol. } & \sqrt{3}|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}| \\ &= \sqrt{3}\sqrt{1+1+2\vec{a}\cdot\vec{b}} + \sqrt{1+1-2\vec{a}\cdot\vec{b}} \\ &= \sqrt{3}\sqrt{2+2\cos\theta} + \sqrt{2-2\cos\theta} \\ &= \sqrt{3}\cdot 2\cos\frac{\theta}{2} + 2\sin\frac{\theta}{2} \\ &= 2\left(\sqrt{3}\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right) \\ &\leq 2\sqrt{(\sqrt{3})^2 + 1} \\ &= 2\sqrt{3+1} \\ &= 4 \end{aligned}$$

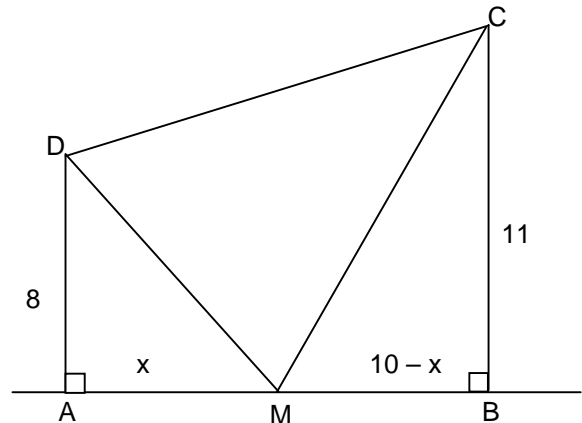
74. 05

Sol.

$$= 2x^2 - 20x + 285$$

$$= 2(x-5)^2 + 235$$

$$MD^2 + MC^2 \text{ is minimum when } x = 5$$



75. 80

Sol. $x + z = h$

$$x = 80 \cos 30^\circ = 40\sqrt{3}$$

$$y = 80 \sin 30^\circ = 40$$

$$\Rightarrow 40\sqrt{3} + z = h$$

$$\tan 75^\circ = \frac{h - y}{z}$$

$$2 + \sqrt{3} = \frac{h - 40}{h - 40\sqrt{3}}$$

$$h - 40\sqrt{3} = (h - 40)(2 - \sqrt{3})$$

$$h - 40\sqrt{3} = 2h - \sqrt{3}h - 80 + 40\sqrt{3}$$

$$(\sqrt{3} - 1)h = 80(\sqrt{3} - 1)$$

$$h = 80$$

