FIITJEE

Solutions to JEE(Main)-2020

Test Date: 4th September 2020 (First Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

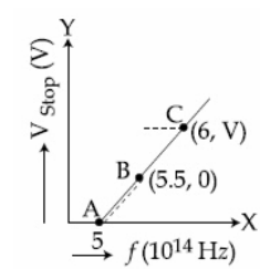
1. The test is of **3 hours** duration.
2. This **Test Paper** consists of **75** questions. The maximum marks are **300**.
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 25 questions in each part of equal weightage out of which 20 questions are MCQs and 5 questions are numerical value based. Each question is allotted **4 (four)** marks for correct response.
4. **(Q. No. 01 – 20, 26 – 45, 51 – 70)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
5. **(Q. No. 21 – 25, 46 – 50, 71 – 75)** contains 15 Numerical based questions with answer as numerical value. Each question carries **+4 marks** for correct answer. There is no negative marking.
6. Candidates will be awarded marks as stated above in **instruction No.3** for correct response of each question. One mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer box.
7. There is only one correct response for each question. Marked up more than one response in any question will be treated as wrong response and marked up for wrong response will be deducted accordingly as per **instruction 6** above.

PART –A (PHYSICS)

1. On the x-axis and at a distance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{\left(x^2+a^2\right)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x, taking its value to be zero at infinity is:

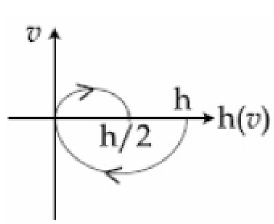
(A) $A\left(x^2+a^2\right)^{1/2}$
(B) $\frac{A}{\left(x^2+a^2\right)^{3/2}}$
(C) $A\left(x^2+a^2\right)^{3/2}$
(D) $\frac{A}{\left(x^2+a^2\right)^{1/2}}$

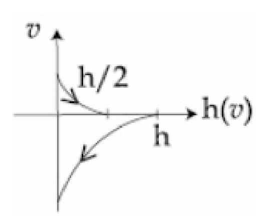
2. Given figure shows few data points in a photo electric effect experiment for a certain metal. The minimum energy for ejection of electron from its surface is: **(Plancks constant h = 6.62 × 10^{-34} J.s)**

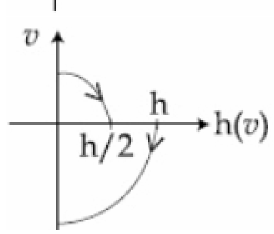


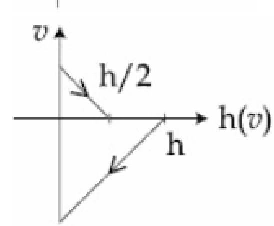
(A) 1.93 eV
(B) 2.10 eV
(C) 2.59 eV
(D) 2.27 eV

3. A Tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height $\frac{h}{2}$. The velocity versus height of the ball during its motion may be represented graphically by:
(graphs are drawn schematically and on not to scale)

(A)


(B)


(C)


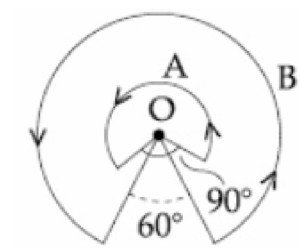
(D)


4. Dimensional formula for thermal conductivity is **(here K denotes the temperature):**

(A) $MLT^{-3}K^{-1}$
(B) $MLT^{-2}K^{-2}$
(C) $MLT^{-2}K$
(D) $MLT^{-3}K$

5. Particle A of mass $m_A = \frac{m}{2}$ moving along the x-axis with velocity v_0 collides elastically with another particle B at rest having mass $m_B = \frac{m}{3}$. If both particles move along the x-axis after the collision, the change $\Delta\lambda$ in de-Broglie wavelength of particle A, in terms of its de-Broglie wavelength (λ_0) before collision is:
(A) $\Delta\lambda = \frac{3}{2}\lambda_0$ (B) $\Delta\lambda = \frac{5}{2}\lambda_0$
(C) $\Delta\lambda = 2\lambda_0$ (D) $\Delta\lambda = 4\lambda_0$

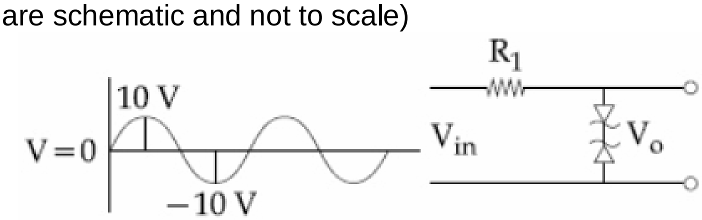
6. A wire A, bent in the shape of an arc of a circle, carrying a current of 2 A and having radius 2 cm and another wire B, also bent in the shape of arc of a circle, carrying a current of 3 A and having radius of 4 cm, are placed as shown in the figure. The ratio of the magnetic fields due to the wires A and B at the common centre O is:

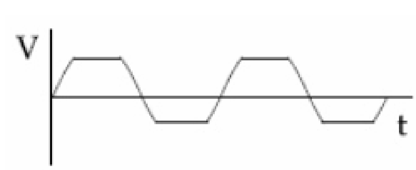


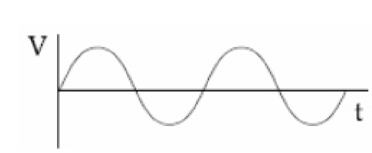
(A) 6 : 5 (B) 2 : 5
(C) 6 : 4 (D) 4 : 6

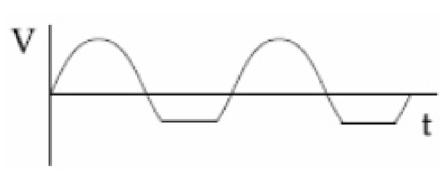
7. The specific heat of water = 4200 J $kg^{-1}K^{-1}$ and the latent heat of ice = 3.4×10^5 J kg^{-1}. 100 grams of ice at 0°C is placed in 200 g of water at 25°C. The amount of ice that will melt as the temperature of water reaches 0°C is close to **(in grams)**:
(A) 64.6 (B) 61.7
(C) 69.3 (D) 63.8

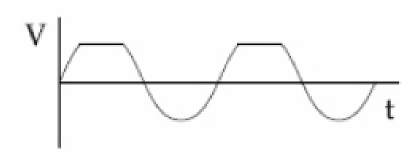
8. Take the breakdown voltage of the zener diode used in the given circuit as 6V. For the input voltage shown in figure below, the time variation of the output voltage is:
(Graphs drawn are schematic and not to scale)



(A)


(B)


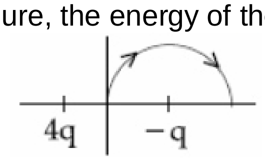
(C)


(D)


9. A beam of plane polarized light of large cross-sectional area and uniform intensity of 3.3 Wm^{-2} falls normally on a polarizer (cross sectional area 3×10^{-4} m^2) which rotates about its axis with an angular speed of 31.4 rad/s. The energy of light passing through the polariser per revolution, is close to:
(A) 1.0×10^{-5} J
(B) 1.5×10^{-4} J
(C) 5.0×10^{-4} J
(D) 1.0×10^{-4} J

10. A small bar magnet placed with its axis at 30° with an external field of 0.06 T experiences a torque of 0.018 Nm. The minimum work required to rotate it from its stable to unstable equilibrium position is:
(A) 11.7×10^{-3} J
(B) 9.2×10^{-3} J
(C) 7.2×10^{-2} J
(D) 6.4×10^{-2} J

11. A two point charges 4q and –q are fixed on the x-axis at $x = -\frac{d}{2}$ and $x = \frac{d}{2}$, respectively. If a third point charge 'q' is taken from the origin to x = d along the semicircle as shown in the figure, the energy of the charge will:



(A) decrease by $\frac{4q^2}{3\pi \epsilon_0 d}$
(B) increase by $\frac{2q^2}{3\pi \epsilon_0 d}$
(C) increase by $\frac{3q^2}{4\pi \epsilon_0 d}$
(D) decrease by $\frac{q^2}{4\pi \epsilon_0 d}$

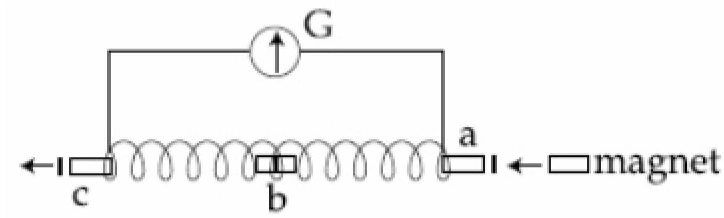
12. Choose the correct option relating wavelengths of different parts of electromagnetic wave spectrum:
(A) $\lambda_{x-rays} < \lambda_{micro\ waves} < \lambda_{radio\ waves} < \lambda_{visible}$
(B) $\lambda_{visible} < \lambda_{micro\ waves} < \lambda_{radio\ waves} < \lambda_{x-rays}$
(C) $\lambda_{visible} > \lambda_{x-rays} > \lambda_{radio\ waves} > \lambda_{micro\ waves}$
(D) $\lambda_{radio\ waves} > \lambda_{micro\ waves} > \lambda_{visible} > \lambda_{x-rays}$

13. Match the C_P/C_V ratio for ideal gases with different type of molecules:

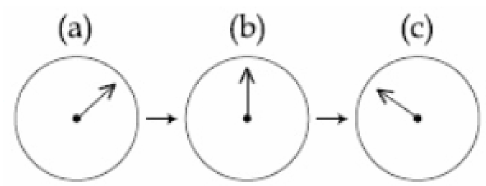
Molecule Type			**C_P/C_V**
(A)	Monatomic	(I)	$\frac{7}{5}$
(B)	Diatomic rigid molecules	(II)	$\frac{9}{7}$
(C)	Diatomic non-rigid molecules	(III)	$\frac{4}{3}$
(D)	Triatomic rigid molecules	(IV)	$\frac{5}{3}$

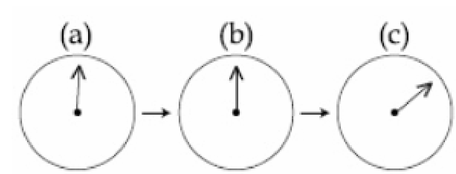
Match the correct option?
(A) A → III; B → IV; C → II; D → I
(B) A → IV; B → I; C → II; D → III
(C) A → II; B → III; C → I; D → IV
(D) A → IV; B → II; C → I; D → III

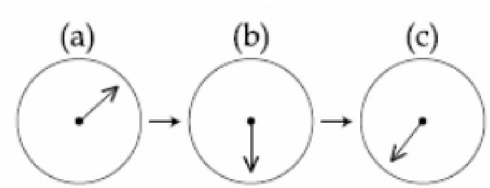
14. A small bar magnet is moved through a coil at constant speed from one end to the other. Which of the following series of observations will be seen on the galvanometer G attached across the coil?

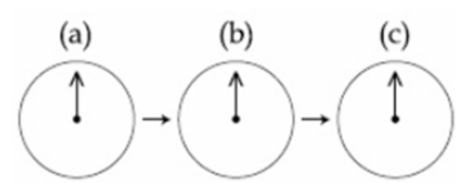


Three positions shown describe:
(a) the magnet's entry
(b) magnet is completely inside and
(c) magnet's exit

(A)


(B)


(C)


(D)


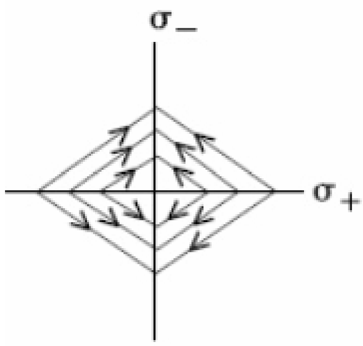
15. For a transverse wave traveling along a straight line, the distance between two peaks (crests) is 5 m, while the distance between one crest and one trough is 1.5 m. The possible wavelengths (in m) of the waves are:

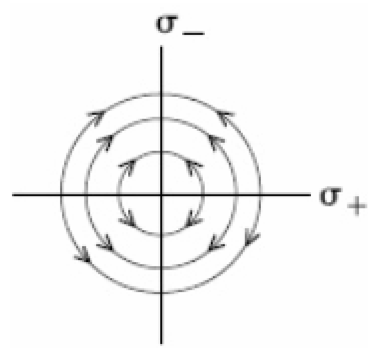
(A) 1, 3, 5, …..
(B) $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}, ...$
(C) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, ...$
(D) 1, 2, 3, ……

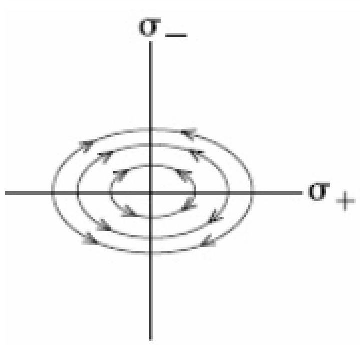
16. A air bubble of radius 1 cm in water has an upward acceleration 9.8 cm s^{-2}. The density of water is 1 gm cm^{-3} and water offers negligible drag force on the bubble. The mass of the bubble is:
($g = 980\ cm/s^2$).
(A) 4.51 gm
(B) 1.52 gm
(C) 3.15 gm
(D) 4.15 gm

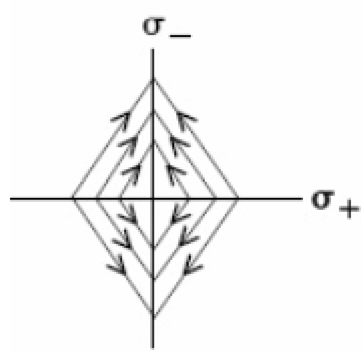
17. Starting from the origin at time t=0, with initial velocity $5\hat{j}\ ms^{-1}$, a particle moves in the x – y plane with a constant acceleration of $\left(10\hat{i} + 4\hat{j}\right) ms^{-2}$. At time t, its coordinates are (20 m, y_0 m). The values of t and y_0 are, respectively:
(A) 2 s and 24 m
(B) 4 s and 52 m
(C) 5 s and 25 m
(D) 2 s and 18 m

18. Two charged thin infinite plane sheets of uniform surface charge density σ_+ and σ_-, where $|\sigma_+| > |\sigma_-|$, intersect at right angle. Which of the following best represents the electric field lines for this system.

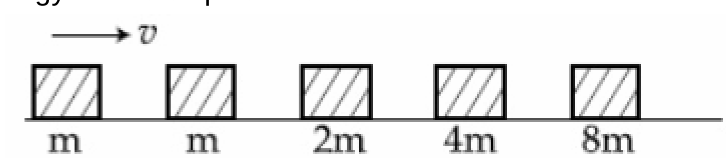
(A) 

(B) 

(C) 

(D) 

19. Blocks of masses m, 2m and 8m are arranged in a line on a frictionless floor. Another block of mass m, moving with speed ν along the same line (see figure) collides with mass m in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass 8m starts moving the total energy loss is p% of the original energy. Value of 'p' is close to:



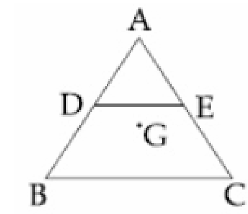
(A) 77
(B) 37
(C) 87
(D) 94

20. A battery of 3.0 V is connected to a resistor dissipating 0.5 W of power. If the terminal voltage of the battery is 2.5 V, the power dissipated within the internal resistance is:
(A) 0.50 W
(B) 0.10 W
(C) 0.125 W
(D) 0.072 W

21. A closed vessel contains 0.1 mole of a monatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be close to _______.

22. In a compound microscope, the magnified virtual image is formed at a distance of 25 cm from the eye-piece. The focal length of its objective lens is 1 cm. If the magnification is 100 and the tube length of the microscope is 20 cm, then the focal length of the eye-piece lens (in cm) is ________.

23. ABC is a plane lamina of the shape of an equilateral triangle. D, E are mid points of AB, AC and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is I_0. If part ADE is removed, the moment of inertia of the remaining part about the same axis is $\frac{NI_0}{16}$ where N is an integer. Value of N is _________.



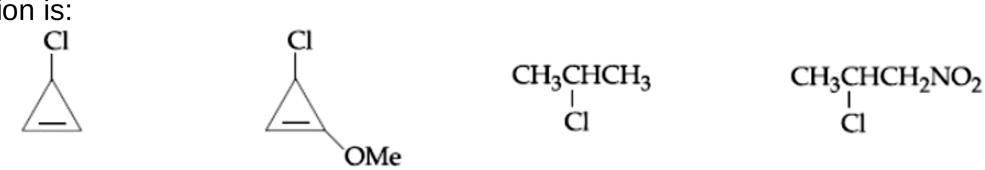
24. In the line spectra of hydrogen atom, difference between the largest and the shortest wavelengths of the Lyman series is $304\,\mathring{A}$. The corresponding difference for the Paschan series in Å is: _________.

25. A circular disc of mass M and radius R is rotating about its axis with angular speed ω_1. If another stationary disc having radius $\frac{R}{2}$ and same mass M is dropped co-axially on to the rotating disc. Gradually both discs attain constant angular speed ω_2. The energy lost in the process is p% of the initial energy. Value of p is ________.

PART –B (CHEMISTRY)

26. Among statements (a) – (d), the correct ones are:
(a) Lime stone is decomposed to CaO during the extraction of iron from its oxides.
(b) In the extraction of silver, silver is extracted as an anionic complex.
(c) Nickel is purified by Mond's process.
(d) Zr and Ti are purified by Van Arkel method.
(A) (a), (b), (c) and (d) (B) (c) and (d) only
(C) (a), (c) and (d) only (D) (b), (c) and (d) only

27. The decreasing order of reactivity of the following organic molecules towards $AgNO_3$ solution is:



(A) (B) (C) (D)

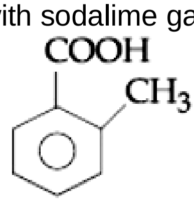
(A) (A) > (B) > (D) > (C) (B) (C) > (D) > (A) > (B)
(C) (A) > (B) > (C) > (D) (D) (B) > (A) > (C) > (D)

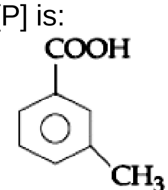
28. The region in the electromagnetic spectrum where the Balmer series lines appear is:
(A) Ultraviolet (B) Visible
(C) Infrared (D) Microwave

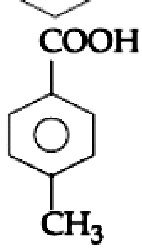
29. The pair in which both the species have the same magnetic moment (spin only) is:
(A) $\left[Co(OH)_4\right]^{2-}$ and $\left[Fe(NH_3)_6\right]^{2+}$ (B) $\left[Mn(H_2O)_6\right]^{2+}$ and $\left[Cr(H_2O)\right]^{2+}$
(C) $\left[Cr(H_2O)_6\right]^{2+}$ and $\left[Fe(H_2O)_6\right]^{2+}$ (D) $\left[Cr(H_2O)_6\right]^{2+}$ and $\left[CoCl_4\right]^{2-}$

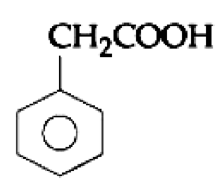
30. On heating, lead (II) nitrate gives a brown gas (A). The gas (A) on cooling changes to a colourless solid/liquid (B). (B) on heating with NO changes to a blue solid (C). The oxidation number of nitrogen in solid (C) is:
(A) +4 (B) +3
(C) +2 (D) +5

31. [P] on treatment with $Br_2/FeBr_3$ in CCl_4 produced a single isomer $C_8H_7O_2Br$ while heating [P] with sodalime gave toluene. The compound [P] is:

(A) 

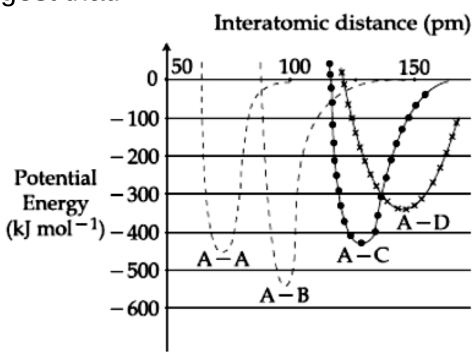
(B) 

(C) 

(D) 

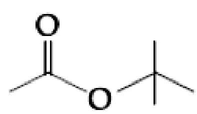
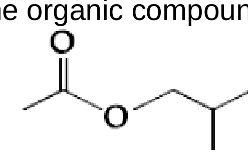
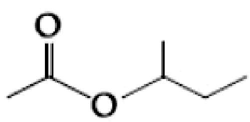
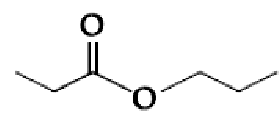
32. The ionic radii of O^{2-}, F^{-}, Na^{+} and Mg^{2+} are in the order:
(A) $O^{2-} > F^{-} > Mg^{2+} > Na^{+}$
(B) $Mg^{2+} > Na^{+} > F^{-} > O^{2-}$
(C) $F^{-} > O^{2-} > Na^{+} > Mg^{2+}$
(D) $O^{2-} > F^{-} > Na^{+} > Mg^{2+}$

33. The intermolecular potential energy for the molecules A, B, C and D given below suggest that:

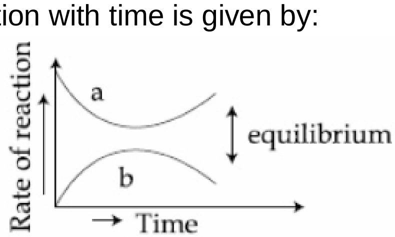


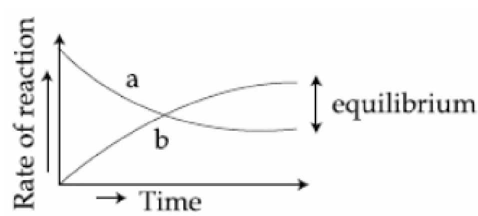
(A) A – A has the largest bond enthalpy
(B) A – D has the shortest bond length
(C) D is more electronegative than other atoms
(D) A – B has the stiffest bond.

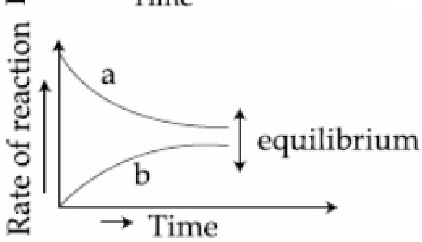
34. An organic compound (A) (molecular formula $C_6H_{12}O_2$) was hydrolysed with dil. H_2SO_4 to give a carboxylic acid (B) and an alcohol (C). 'C' gives white turbidity immediately when treated with anhydrous $ZnCl_2$ and conc. HCl. The organic compound (A) is:

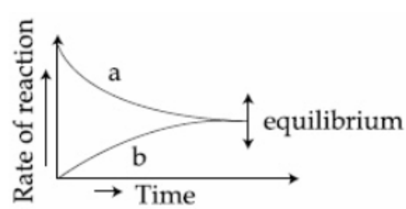
(A) 
(B) 
(C) 
(D) 

35. For the equilibrium $A \rightleftharpoons B$, the variation of the rate of the forward (a) and reverse (B) reaction with time is given by:

(A) 

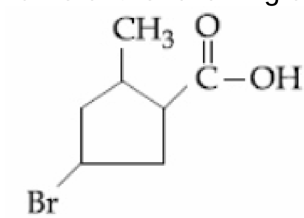
(B) 

(C) 

(D) 

36. The number of isomers possible for $\left[Pt(en)(NO_2)_2\right]$ is:
(A) 4 (B) 3
(C) 1 (D) 2

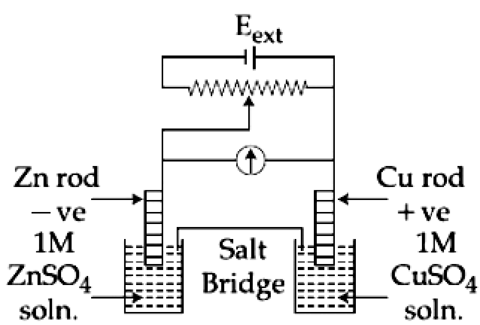
37. The IUPAC name of the following compound is:



(A) 4–Bromo–5–methylcyclopentanoic acid
(B) 4–Bromo–2–methylcyclopentane carboxylic acid
(C) 3–Bromo–5– methylcyclopentane carboxylic acid
(D) 5–Bromo–3–methylcyclopentanoic acid

38. What are the functional groups present in the structure of maltose?
(A) One acetal and one hemiacetal (B) One ketal and one hemiketal
(C) One acetal and one ketal (D) Two acetals

39.



$E^{\circ}_{Cu^{2+}|Cu} = +0.34\ V$

$E^{\circ}_{Zn^{2+}|Zn} = -0.76\ V$

Identify the incorrect statement from the options below for the above cell:
(A) If $E_{ext} < 1.1\,V$, Zn dissolves at anode and Cu deposits at cathode
(B) If $E_{ext} = 1.1\,V$, no flow of e^- or current occurs
(C) If $E_{ext} > 1.1\,V$, Zn dissolves at Zn electrode and Cu deposits at Cu electrode
(D) If $E_{ext} > 1.1\,V$, e^- flows from Cu to Zn

40. Which of the following will react with $CHCl_3$ + alc. KOH?
(A) Adenine and proline (B) Adenine and lysine
(C) Adenine and thymine (D) Thymine and proline

41. Match the following:

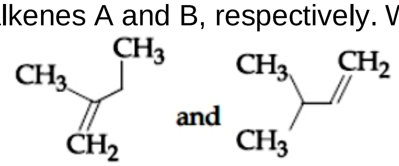
Column I			Column II
(i)	Foam	(a)	smoke
(ii)	Gel	(b)	cell fluid
(iii)	Aerosol	(c)	jellies
(iv)	Emulsion	(d)	rubber
		(e)	froth
		(f)	milk

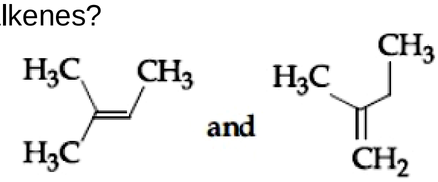
(A) (i)–(d), (ii)–(b), (iii)–(a), (iv)–(e)
(B) (i)–(d), (ii)–(b), (iii)–(e), (iv)–(f)
(C) (i)–(b), (ii)–(c), (iii)–(e), (iv)–(d)
(D) (i)–(e), (ii)–(c), (iii)–(a), (iv)–(f)

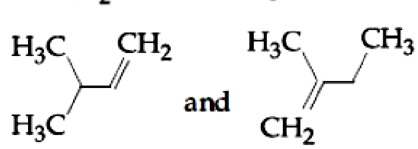
42. For one mole of an ideal gas, which of these statements must be true?
(a) U and H each depends only on temperature
(b) Compressibility factor z is not equal to 1
(c) $C_{P,m} - C_{V,m} = R$
(d) $dU = C_V dT$ for any process
(A) (a), (c) and (d)
(B) (b), (c) and (d)
(C) (a) and (c)
(D) (c) and (d)

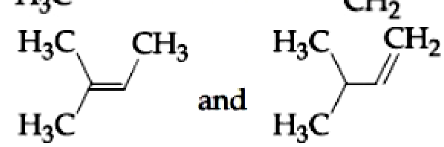
43. On combustion of Li, Na and K in excess of air, the major oxides formed, respectively, are:
(A) Li_2O, Na_2O_2 and K_2O
(B) Li_2O, Na_2O_2 and KO_2
(C) Li_2O, Na_2O and K_2O_2
(D) Li_2O, Na_2O_2 and K_2O_2

44. When neopentyl alcohol is heated with an acid, it slowly converted into an 85:15 mixture of alkenes A and B, respectively. What are these alkenes?

(A) 

(B) 

(C) 

(D) 

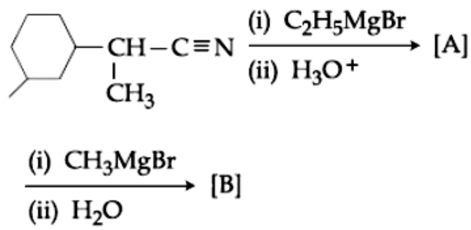
45. The elements with atomic numbers 101 and 104 belong to, respectively:
(A) Actinoids and Group 6
(B) Actinoids and Group 4
(C) Group 11 and Group 4
(D) Group 6 and Actinoids

46. At 300K, the vapour pressure of a solution containing 1 mole of n-hexane and 3 moles of n-heptane is 550 mm of Hg. At the same temperature, if one more mole of n-heptane is added to this solution, the vapour pressure of the solution increases by 10 mm of Hg. What is the vapour pressure in mm Hg of n-heptane in its pure state _______?

47. If 75% of a first order reaction was completed in 90 minutes, 60% of the same reaction would be completed in approximately (in minutes) _________.
(Take: log 2 = 0.30; log 2.5 = 0.40)

48. The mass of ammonia in grams produced when 2.8 kg of dinitrogen quantitatively reacts with 1 kg of dihydrogen is __________.

49. The number of chiral centres present in [B] is _________.



50. At 20.0 mL solution containing 0.2 g impure H_2O_2 reacts completely with 0.316 g of $KMnO_4$ in acid solution. The purity of H_2O_2 (in %) is ______________.
(mol. wt. of H_2O_2 = 34; mol. wt. of $KMnO_4$ = 158)

PART–C (MATHEMATICS)

51. Let α and β be roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$, form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is:
(A) 33 : 31
(B) 5 : 3
(C) 3 : 1
(D) 9 : 7

52. Let $f(x) = |x - 2|$ and $g(x) = f(f(x)), x \in [0, 4]$. Then $\int_0^3 (g(x) - f(x))\,dx$ is equal to:
(A) $\frac{1}{2}$
(B) $\frac{3}{2}$
(C) 1
(D) 0

53. Let *f* be a twice differentiable function on (1, 6). If f(2) = 8, $f'(2) = 5$, $f'(x) \geq 1$ and $f''(x) \geq 4$, for all $x \in (1, 6)$, then:
(A) $f(5) + f'(5) \leq 26$
(B) $f(5) \leq 10$
(C) $f(5) + f'(5) \geq 28$
(D) $f'(5) + f''(5) \leq 20$

54. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ (a > b)$ be given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t) = \frac{5}{12} + t - t^2$, then $a^2 + b^2$ is equal to:
(A) 126
(B) 135
(C) 116
(D) 145

55. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is:
(A) 3
(B) 5
(C) 7
(D) 9

56. Let x_0 be the point of local maxima of $f(x) = \vec{a} \cdot (\vec{b} \times \vec{c})$ where $\vec{a} = x\hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + x\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$. Then the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ at $x = x_0$ is
(A) – 4
(B) – 22
(C) – 30
(D) 14

57. If $\left(a + \sqrt{2}\,b\cos x\right)\left(a - \sqrt{2}\,b\cos y\right) = a^2 - b^2$, where a > b > 0, then $\frac{dx}{dy}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is:
(A) $\frac{a - 2b}{a + 2b}$
(B) $\frac{2a + b}{2a - 2}$
(C) $\frac{a + b}{a - b}$
(D) $\frac{a - b}{a + b}$

58. Two vertical poles AB = 15 m and CD = 10 m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is:

(A) $\frac{10}{3}$ (B) 6

(C) 5 (D) $\frac{20}{3}$

59. If 1 + (1 – $2^2\cdot1$) + (1 – $4^2\cdot3$) + (1 – $6^2\cdot5$) + …….. + (1 – $20^2\cdot19$) = α – 220β, then an ordered pair (α, β) is equal to:

(A) (11, 97) (B) (10, 97)

(C) (10, 103) (D) (11, 103)

60. Given the following two statements:

$(S_1):(q \vee p) \to (p \leftrightarrow \sim q)$ is a tautology.

$(S_2):\sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy. Then:

(A) both (S_1) and (S_2) are correct (B) only (S_2) is correct

(C) both (S_1) and (S_2) are incorrect (D) only (S_1) is correct

61. Let $u = \frac{2z+i}{z-ki}, z = x+iy$ and k > 0. If the curve represented by Re(u) + Im(u) = 1 intersects the y-axis at the points P and Q where PQ = 5, then the value of k is:

(A) 2 (B) $\frac{1}{2}$

(C) $\frac{3}{2}$ (D) 4

62. A triangle ABC lying in the first quadrant has two vertices as A(1, 2) and B(3, 1). If $\angle$BAC = 90°, and $ar(\Delta ABC) = 5\sqrt{5}$ sq. units, then the abscissa of the vertex C is:

(A) $1+2\sqrt{5}$ (B) $1+\sqrt{5}$

(C) $2\sqrt{5}-1$ (D) $2+\sqrt{5}$

63. Let P (3, 3) be a point on the hyperbola, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal to it at P intersects the x-axis at (9, 0) and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to:

(A) $\left(\frac{3}{2}, 2\right)$ (B) $\left(\frac{9}{2}, 2\right)$

(C) (9, 3) (D) $\left(\frac{9}{2}, 3\right)$

64. The integral $\int\left(\frac{x}{x\sin x+\cos x}\right)^2 dx$ is equal to (where C is constant of integration):

(A) $\tan x+\frac{x\sec x}{x\sin x+\cos x}+C$ (B) $\tan x-\frac{x\sec x}{x\sin x+\cos x}+C$

(C) $\sec x+\frac{x\tan x}{x\sin x+\cos x}+C$ (D) $\sec x-\frac{x\tan x}{x\sin x+\cos x}+C$

65. If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$, $\left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $i = \sqrt{-1}$, then, which one of the following is not true?

(A) $a^2 - c^2 = 1$
(B) $a^2 - b^2 = \frac{1}{2}$
(C) $0 \le a^2 + b^2 \le 1$
(D) $a^2 - d^2 = 0$

66. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to:

(A) ${}^{50}C_6 - {}^{30}C_6$
(B) ${}^{51}C_7 + {}^{30}C_7$
(C) ${}^{51}C_7 - {}^{30}C_7$
(D) ${}^{50}C_7 - {}^{30}C_7$

67. Let [t] denote the greatest integer $\le$t. Then the equation in x, $[x]^2 + 2[x + 2] - 7 = 0$ has:

(A) infinitely many solutions
(B) exactly two solutions
(C) no integral solution
(D) exactly four integral solutions

68. Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx \; (x \ge 0)$. Then f(3) – f(1) is equal to:

(A) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$
(B) $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$
(C) $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$
(D) $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$

69. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. if x% of the people read both the newspapers, then a possible value of x can be:

(A) 29
(B) 55
(C) 37
(D) 65

70. Let y = y(x) be the solution of the differential equation, $xy' - y = x^2(x\cos x + \sin x)$, $x > 0$. If $y(\pi) = \pi$, then $y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ is equal to:

(A) $1 + \frac{\pi}{2} + \frac{\pi^2}{4}$
(B) $2 + \frac{\pi}{2} + \frac{\pi^2}{4}$
(C) $2 + \frac{\pi}{2}$
(D) $1 + \frac{\pi}{2}$

71. Let $\left(2x^2 + 3x + 4\right)^{10} = \sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to ___________.

72. If the equation of a plane P, passing through the intersection of the planes, x + 4y – z + 7 = 0 and 3x + y + 5z = 8 is ax + by + 6z = 15 for some a, b $\in$ R, then the distance of the point (3, 2, –1) from the plane P is _________.

73. If the system of equations
$x - 2y + 3z = 9$
$2x + y + z = b$
$x - 7y + az = 24,$
has infinitely many solutions, then a – b is equal to _________.

74. Suppose a differentiable function f(x) satisfies the identity $f(x+y) = f(x) + f(y) + xy^2 + x^2y$, for all real x and y. If $\lim_{x \to 0} \frac{f(x)}{x} = 1$ then $f'(3)$ is equal to ________.

75. The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is ________.

FIITJEE

Solutions to JEE (Main)-2020

PART –A (PHYSICS)

1. **D**

Sol. $g = \frac{Ax}{(x^2 + a^2)^{3/2}}$

$\Rightarrow \quad \int_v^0 dV = -\int_x^\infty g\,dx$

$\Rightarrow \quad O - V = -\left[\int \frac{Ax}{(a^2 + x^2)^{3/2}}\right]$

Let, $a^2 + x^2 = t^2$

$\Rightarrow \quad 2xdx = 2t\,dt$

$\Rightarrow \quad xdx = tdt$

$\Rightarrow \quad V = \int \frac{At\,dt}{t^3} \quad \Rightarrow \quad -\frac{A}{t} \quad \Rightarrow \quad -\frac{A}{\sqrt{a^2 + x^2}}\Big|_x^\infty$

$\Rightarrow \quad V = \frac{A}{\sqrt{a^2 + x^2}}$

2. **D**

Sol. $eVs = h\nu - \phi$

At $V_s = 0 \quad \Rightarrow \quad h\nu = \phi$

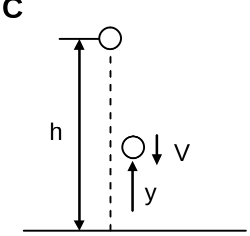
$\Rightarrow \quad \phi = [6.62 \times 10^{-34}]\,[10^{14}]\,[5.5]$

$\Rightarrow \quad \phi = \frac{[6.62 \times 10^{-34}]\,[10^{14}]\,[5.5]\text{ eV}}{\left(1.6 \times 10^{-19}\right]}$

= 2.27

3. **C**

Sol.



$\Rightarrow \quad V^2 = U^2 + 2gS$

$\Rightarrow \quad V^2 = O + 2g\,(h - y)$

$\Rightarrow \quad V^2 = 2gh - 2gy$

$\Rightarrow \quad V = \sqrt{2gh - 2gy}$

4. **A**

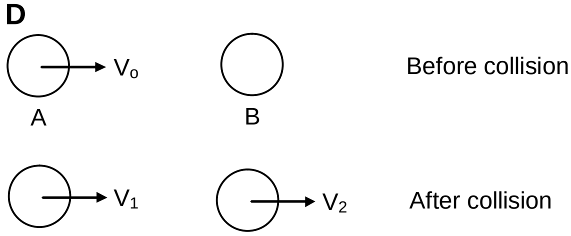
Sol. $K = \frac{\left(\frac{Q}{t}\right)\Delta x}{A\ \Delta T}$

$\Rightarrow \quad \dfrac{(ML^2T^{-2})(L)}{(L^2)(\theta)(T)}$

$\Rightarrow \quad M^1 L^{1-} T^{-3} \theta^{-1}$

5. **D**

Sol.



$\Rightarrow \quad \vec{V}_1 = 2\overrightarrow{U_{cm}} - \overrightarrow{U_1}$

$$\Rightarrow \quad 2\left[\frac{m/2\ V_o}{\frac{m}{2}+\frac{m}{3}}\right] - V_o$$

$$\Rightarrow \quad \frac{6}{5}V_o - V_o \Rightarrow \frac{V_o}{5}$$

$$\Rightarrow \quad \lambda_o = \frac{hc}{\left(\frac{m}{2}V_o\right)} \qquad \lambda_f = \frac{hc}{\left(\frac{M}{2}\frac{V_o}{5}\right)}$$

$$\Rightarrow \quad \Delta\lambda = \frac{8hc}{mV_o}$$

6. **A**

Sol. $B_A = \dfrac{\mu_o I\theta}{4\pi R}$

$$\Rightarrow \quad \frac{B_A}{B_B} = \frac{I_A\theta_A R_B}{I_B\theta_B R_A}$$

$$\Rightarrow \quad \frac{2\left(\frac{3\pi}{2}\right)(4)}{3\left[\frac{5\pi}{3}\right][2]}$$

$$\Rightarrow \quad \frac{6}{5}.$$

7. **B**

Sol. $m(L) = m_1S_1(\Delta T)$

$\Rightarrow \quad m(3.4 \times 10^5) = (200)(4200)(25)$

$\Rightarrow \quad m = 61.7$

8. **A**

9. **D**

Sol. $E = (I)(t)(A) <\cos^2\theta>$

$\Rightarrow \quad (3.3)\left[\frac{2\pi}{31.4}\right][3\times10^{-4}]\times\frac{1}{2}$

$\Rightarrow \quad 0.99 \times 10^{-4}$

10. **D**

Sol. $\vec{\tau} = \vec{\mu}\times\vec{B}$

$\Rightarrow \quad 0.018 = \mu(0.06)\ (\sin 30°)$

$\Rightarrow \quad \mu = 0.6$

$\Rightarrow \quad \text{Work} = U_f - U_i$

$\Rightarrow \quad 2\mu B$

$\Rightarrow \quad 7.2 \times 10^{-2}$ J.

11. **A**

Sol. $U_{initial} = \frac{k(4q)(q)}{(d/2)} + \frac{k(q)(-q)}{(d/2)}$

$\Rightarrow \quad \frac{6kq^2}{d}$

$\Rightarrow \quad U_{final} = \frac{4(4q)(q)}{\left(\frac{3d}{2}\right)} + \frac{k(q)(-q)}{(d/2)}$

$\Rightarrow \quad \frac{2}{3}\frac{kq^2}{d}$

$\Rightarrow \quad \Delta U = \left(\frac{2}{3}-6\right)\frac{kq^2}{d} \quad \Rightarrow \quad \frac{-16}{3}\ \frac{kq^2}{d}$

12. **D**

13. **B**

Sol. Mono atomic $\longrightarrow C_V = \frac{3R}{2} \quad C_p = \frac{5R}{2}$

Di- atomic $\longrightarrow C_V = \frac{5R}{2} \quad C_p = \frac{7R}{2}$
(Rigid)

Di-atomic $\longrightarrow C_V = \frac{7R}{2} \quad C_p = \frac{9R}{2}$
(Non-Rigid)

Tri–atomic $\longrightarrow C_V = 3R \qquad C_P = 4R$
(Rigid)

14. **A**

15. **B**

Sol. (n) $\lambda = 5$
(n, m): Integers

$$\left(\frac{2m+1}{2}\right)\lambda = \frac{3}{2}$$

$\Rightarrow \quad \frac{3/2}{5} = \frac{2m+1}{2n}$

$\Rightarrow \quad 3n = 10\ m + 5$

N, m are integers.

So, $m = 1, \quad n = 5, \quad \lambda = 1$

$m = 4 \quad n = 15 \quad \lambda = \frac{1}{3}$

$m = 7 \quad n = 25 \quad \lambda = \frac{1}{5}$

16. **D**

Sol. $\rho vg - mg = ma$

$\Rightarrow \quad \frac{\rho vg}{m} = g + a$

$\Rightarrow \quad m = \frac{\rho vg}{g+a}$

$\Rightarrow \quad \frac{10^3\left(\frac{4}{3}\pi \times 10^{-6}\right)(9.8)}{9.898}$

$\Rightarrow \quad 4.15\ gm$

17. **D**

Sol. $\vec{U} = 5\hat{j}$

$\vec{a} = 10\hat{i} + 4\hat{j}$

$\Rightarrow \quad \vec{S} = \vec{U}t + \frac{1}{2}(\vec{a})t^2$

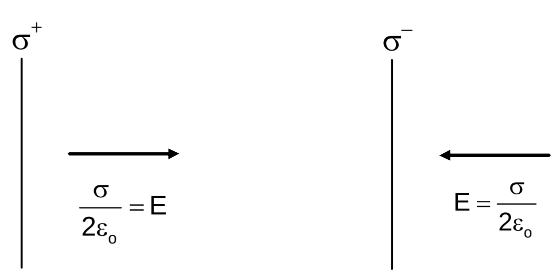
$\Rightarrow \quad 20\hat{i} + y_o\hat{j} = (5t^2)\hat{i} + (5t + 2t^2)\hat{j}$

$20 = 5t^2 \quad ; \quad y_o = 5t + 2t^2$

$t = 2 \quad \Rightarrow \quad 18\ m.$

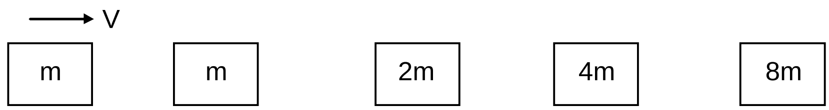
18. **D**

Sol. $\Rightarrow$ Electric field due to infinite sheet is uniform.



19. **D**

Sol.



$\Rightarrow$ mv = 16 mv$_1$

$\Rightarrow$ $V_1 = \frac{V}{16}$

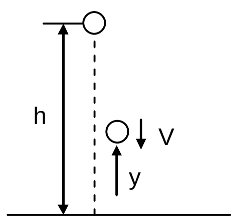
$\Rightarrow$ Δk loss = $\frac{1}{2}mv^2 - \frac{1}{2}(16m)\left(\frac{V}{16}\right)^2$

$\Rightarrow$ $\frac{1}{2}mv^2\left(\frac{15}{16}\right)$

% loss = $\frac{15}{16}\times 100$ = 93.75%

20. **B**

Sol.



(1) $\varepsilon = 3$

(2) ε – Ir = 2.5 V

$\Rightarrow$ Ir = 0.5

Now, IR = 2.5

$\Rightarrow$ $\frac{R}{r} = 5.$

$\Rightarrow$ $\frac{P_R}{R_r} = \frac{I^2R}{I^2r} = \frac{R}{r} = 5$

$\Rightarrow$ $P_r = \frac{0.5}{5} = 0.1$

21. **266.67**

Sol. $\Rightarrow$ $n_1T_1 + n_2T_2 = nT$

$\Rightarrow$ (0.1) (200) + (0.05) (400) = (0.15) T

$\Rightarrow$ T = 266.67

22. **6.25**

Sol. $M = \frac{1}{f_o}\left[1 + \frac{D}{Fe}\right]$

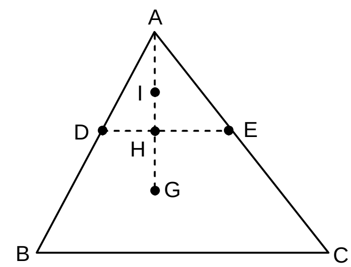
$\Rightarrow$ $100 = \frac{20}{1}\left[1 + \frac{25}{Fe}\right]$

$\Rightarrow$ $1 + \frac{25}{Fe} = 5$

$\Rightarrow$ $Fe = \frac{25}{4}$ = 6.25 cm

23. **11**

Sol.



$AH \to \frac{H}{2}$ $\qquad H = \frac{\sqrt{3}\,a}{2}$

$AG \to \frac{H}{\sqrt{3}}$ $\qquad AI = \frac{H}{2\sqrt{3}}$

$M_{ABC} = M$ $\qquad M_{ADE} = \frac{M}{4}$

$$\Rightarrow \quad I_G = \frac{Ma^2}{12} - \left[\frac{M}{4}\frac{\left[\frac{a}{2}\right]^2}{12} + \frac{M}{4}\left[\frac{a}{2\sqrt{3}}\right]^2\right] = \left(\frac{11}{16}\right)\frac{Ma^2}{12}$$

24. **10553.33**

Sol. $\frac{1}{\lambda_{min}} = R\left[\frac{1}{1} - \frac{1}{\infty}\right] = R$ $\qquad [n = \infty \to n = 1]$

$\frac{1}{\lambda_{max}} = R\left[\frac{1}{1} - \frac{1}{4}\right] = \frac{3R}{4}$ $\qquad [n = 2 \to n = 1]$

$\Rightarrow \quad \Delta\lambda \quad \Rightarrow \quad \frac{4}{3R} - \frac{1}{R} \quad \Rightarrow \quad \frac{1}{3R} = 340 \quad \ldots(1)$

For Paschan $\Rightarrow \frac{1}{\lambda_{min}} = R\left[\frac{1}{9}\right]$ $\qquad [n = \infty \to h = 3]$

$\Rightarrow \quad \frac{1}{\lambda_{max}} = R\left[\frac{1}{9} - \frac{1}{16}\right] = \frac{7R}{144}$

$\Rightarrow \quad \Delta\lambda = \frac{81}{7R} \quad \ldots(2)$

25. **20**

Sol. Angular momentum conservation:

$\Rightarrow \quad I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f$

$\Rightarrow \quad \frac{MR^2}{2}\omega_o = \left(\frac{MR^2}{2} + \frac{MR^2}{8}\right)\omega_f$

$\Rightarrow \quad \omega_f = \frac{4}{5}\omega_o$

$\Rightarrow \quad KE_{final} = \frac{1}{2}(I_1 + I_2)\omega_f^{\,2} = \frac{MR^2\omega^2}{5}$

$\Rightarrow \quad KE_{initial} = \frac{1}{2}I_1\,\omega_0^2 = \frac{MR^2\omega^2}{4}$

$\Rightarrow \quad$ % loss $\Rightarrow$ 20%.

PART –B (CHEMISTRY)

26. A
Sol. (a) In extraction of iron lime stone is added on a flux

$$CaCO_3 \longrightarrow CaO + CO_2$$

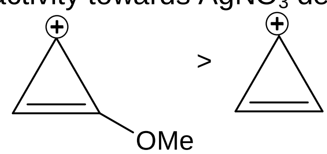
$$CO_2 + SiO_2 \longrightarrow CaSiO_3$$

(b) Extraction of silver

$$Ag + NaCN \longrightarrow \underset{\text{anionic complex}}{\left[Ag(CN)_2\right]^-}$$

(c) Nickel is purified by Mond's process
(d) Zr and Ti are purified by Van Arkel method by converting to volatile iodide.

27. D
Sol. Reactivity towards $AgNO_3$ depend on stability of carbocation formed



$$CH_3\overset{\oplus}{C}HCH_3 > CH_3\overset{\oplus}{C}H\underset{NO_2}{C}H_2$$

28. B
Sol. Balmer series lies in the visible region.

29. C
Sol. Cr^{2+} and Fe^{2+} both have 4 unpaired electrons.

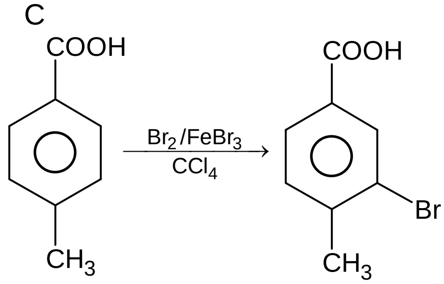
30. B
Sol. $2Pb(NO_3)_2 \xrightarrow{\Delta} 2PbO + 2NO_2 + O_2$

$2NO_2 \longrightarrow N_2O_4$ colourless

$N_2O_4 + 2NO \longrightarrow \underset{\text{Blue solid}}{2N_2O_3}$

Oxidation state in N_2O_3 is +3

31. C
Sol.

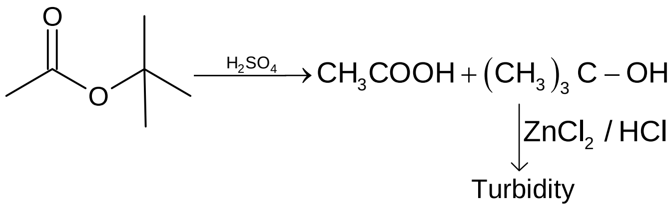


32. D
Sol. $O^{2-} > F^- > Na^+ > Mg^{2+}$. Greater the Zeff, smaller is size.

33. D
Sol. A – B has lowest potential energy and it means it has stronger bond.

34. A

Sol.

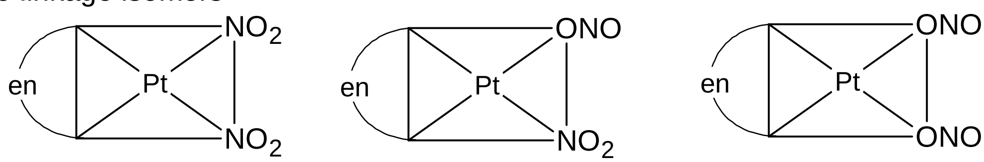


35. D

Sol. At equilibrium rate of forward and backward reaction becomes equal.

36. B

Sol. Two linkage isomers

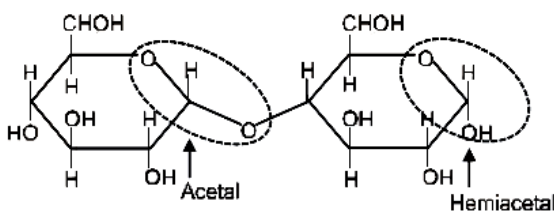


37. B

Sol. Correct IUPAC name is 4–Bromo–2–methylcyclopentane carboxylic acid

38. A

Sol.



Maltose

39. C

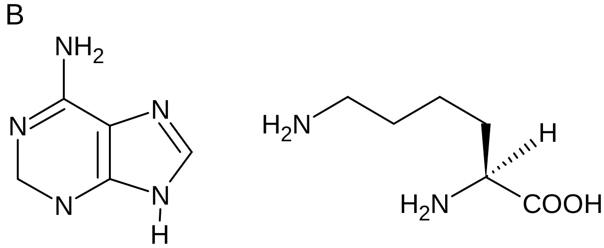
Sol. $E_{ext} > 1.1$ V, cell reaction is reversed

i.e. $Cu \longrightarrow Cu^{2+}$ anode

$Zn^{2+} + 2e^{-} \longrightarrow Zn$ cathode

40. B

Sol.



Adenine Lysine

Both contains $-NH_2$ group

41. D

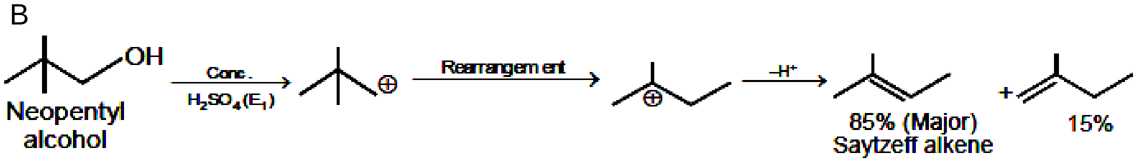
Sol. (i) Foam – Froth (e)

(ii) Gel – Jellies (c)

(iii) Aerosol – smoke(a)
(iv) Emulsion – milk(f)

42. A
Sol. U and H are temperature dependent
$C_{P,m} - C_{V,m} = R$ (for 1 mole of ideal gas)
$dU = C_V dt$

43. A
Sol. $4Li + O_2 \longrightarrow 2Li_2O$ oxide
$2Na + O_2 \longrightarrow Na_2O_2$ peroxide
$K + O_2 \longrightarrow KO_2$ superoxide

44. B
Sol.


45. B
Sol. Z = 101 belong to actinoids
104 belong to group 4

46. 600
Sol. $P_T = X_A\left(P_A^0 - P_B^0\right) + P_B^0$
ATQ
$550 = \frac{1}{4}\left(P_A^0 - P_B^0\right) + P_B^0$
$2200 = P_A^0 - P_B^0 + 4P_B^0$ …… (1)
$560 = \frac{1}{5}\left(P_A^0 - P_B^0\right) + P_B^0$
$2200 = P_A^0 - P_B^0 + 5P_B^0$ …… (2)
$P_A^0 + 3P_B^0 = 2200$
$P_A^0 \pm 4P_B^0 = 2800$
$P_B^0 = 600$ $\quad P_A^0 = 400$ mm Hg

47. 59.51
Sol. first order reaction
$K = \frac{2.303}{t}\log\frac{a_0}{a_0 - x}$
$K = \frac{2.303}{90}\log\frac{a_0}{0.25a_0}$ ....(1)
= 0.0154

$$t = 60\% = \frac{2.303}{K}\log\frac{a_0}{a_0}....(2)$$
$$= \frac{2.303}{0.0154}\times(1-0.602) = 59.51\text{ mins} \approx 60$$

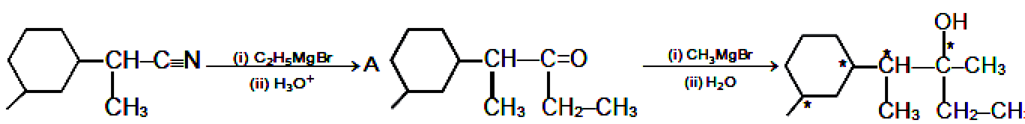
48. 3400

Sol. $N_2 + 3H_2 \longrightarrow 2NH_3$

$$\frac{2.8\times10^3}{28} \qquad \frac{1\times10^3}{2}$$
$$\frac{0.1\times10^3\text{mol}}{LR} \qquad 0.5\times10^3\text{mol}$$

Mass of NH_3 produced = $0.2 \times 10^3 \times 17 = 3.4$ kg = 3400 g

49. 4

Sol. 

50. 85

Sol. eq of H_2O_2 = eq of $KMnO_4$

$$= \frac{0.316}{158}\times5 = 0.01$$
$$= \frac{0.01\times17}{0.2}\times100 = 85\%$$

PART–C (MATHEMATICS)

51. D

Sol. $x^2 - 3x + p = 0$

$\alpha, \beta, \gamma, \delta$ in G.P.

$\alpha + \alpha r = 3$ ……………..(1)

$x^2 - 6x + q = 0$

$\alpha r^2 + \alpha r^3 = 6$ …………..(2)

$(2) \div (1) \Rightarrow r^2 = 2$

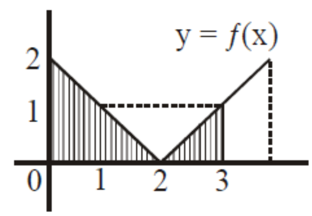
So, $\dfrac{2q+p}{2q-p} = \dfrac{2r^5+r}{2r^5-r} = \dfrac{2r^4+1}{2r^4-1} = \dfrac{9}{7}$

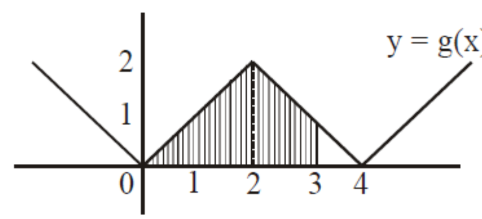
52. C

Sol. $\int_0^3 g(x) - f(x) = \int_0^3 \left\| x-2 \right| - 2 \right| dx - \int_0^3 |x-2| dx$

$= \left(\frac{1}{2}\times 2\times 2 + 1 + \frac{1}{2}\times 1\times 1\right) - \left(\frac{1}{2}\times 2\times 2 + \frac{1}{2}\times 1\times 1\right)$

$= \left(2 + 1 + \frac{1}{2}\right) - \left(2 + \frac{1}{2}\right) = 1$





53. C

Sol. $f(2) = 8, f'(2) = 5, f'(x) \ge 1, f''(x) \ge 4, \forall x \in (1, 6)$

Using LMVT

$f''(x) = \dfrac{f'(5) - f'(2)}{5-2} \ge 4 \Rightarrow f'(5) \ge 17$ ………..(1)

$f'(x) = \dfrac{f(5) - f(2)}{5-2} \ge 1 \Rightarrow f(5) \ge 11$ ……….(2)

Therefore $f'(5) + f(5) \ge 28$

54. A

Sol. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 (a > b); \dfrac{2b^2}{a} = 10 \Rightarrow b^2 = 5a$ ……….(i)

Now, $\phi(t) = \dfrac{5}{12} + t - t^2 = \dfrac{8}{12} - \left(t - \dfrac{1}{2}\right)^2$

$\phi(t)_{max} = \frac{8}{12} = \frac{2}{3} = e \Rightarrow e^2 = 1 - \frac{b^2}{a^2} = \frac{4}{9}$ ……….(ii)

$\Rightarrow a^2 = 81$ (From (i) and (ii)

So, $a^2 + b^2 = 81 + 45 = 126$

55. C

Sol. $\bar{x} = 10$

$\Rightarrow \bar{x} = \frac{63 + a + b}{8} = 10$

$\Rightarrow a + b = 17$ …………(1)

Since, variance is independent of origin.
So, we subtract 10 from each observation.

So, $\sigma^2 = 13.5 = \frac{79 + (a-10)^2 + (b-10)^2}{8}$

$\Rightarrow a^2 + b^2 - 20(a+b) = -171$

$\Rightarrow a^2 + b^2 = 169$ ………..(2)

From (1) and (2) ; a = 12 and b = 5

56. B

Sol. $f(x) = \vec{a}.(\vec{b} \times \vec{c}) = \begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix}$

$= x^3 - 27x + 26$

$f'(x) = 3x^2 - 27 = 0 \Rightarrow x = \pm 3$ and $f''(-3) < 0$

$\Rightarrow$ local maxima at $x = x_0 = -3$

Thus, $\vec{a} = -3\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} - \hat{k}$, and $\vec{c} = 7\hat{i} - 2\hat{j} - 3\hat{k}$

$\Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 9 - 5 - 26 = -22$

57. C

Sol. $\left(a + \sqrt{2}b\cos x\right)\left(a - \sqrt{2}b\cos y\right) = a^2 - b^2$

$\Rightarrow a^2 - \sqrt{2}\,ab\cos y + \sqrt{2}\,ab\cos x - 2b^2\cos x\cos y = a^2 - b^2$

Differentiating both sides:

$0 - \sqrt{2}ab\left(-\sin y\frac{dy}{dx}\right) + \sqrt{2}ab(-\sin x) - 2b^2\left[\cos x\left(-\sin y\frac{dy}{dx}\right) + \cos y(-\sin x)\right] = 0$

At $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$:

$$ab\frac{dy}{dx}-ab-2b^2\left(-\frac{1}{2}\frac{dy}{dx}-\frac{1}{2}\right)=0$$

$$\Rightarrow \frac{dx}{dy}=\frac{ab+b^2}{ab-b^2}=\frac{a+b}{a-b};\ a,b>0$$

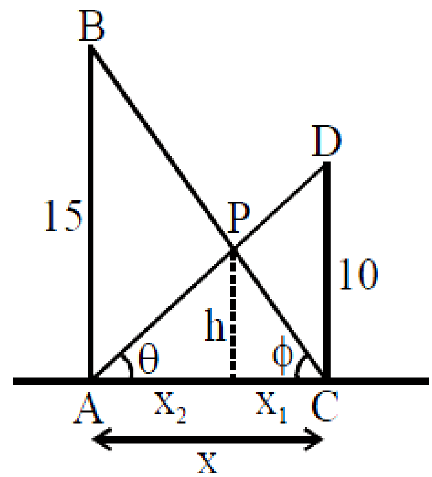
58. B

Sol. $\tan\theta=\frac{10}{x}=\frac{h}{x_2}\Rightarrow x_2=\frac{hx}{10}$

$\tan\phi=\frac{15}{x}=\frac{h}{x}\Rightarrow x_1=\frac{hx}{15}$

Now, $x_1+x_2=x=\frac{hx}{15}+\frac{hx}{10}$

$\Rightarrow 1=\frac{h}{10}+\frac{h}{15}\Rightarrow h=6$



59. D

Sol. $1+\left(1-2^2.1\right)+\left(1-4^2.3\right)+.......+\left(1-20^2.19\right)$

$=\alpha-220\beta$

$=11-\left(2^2.1+4^2.3+.....+20^2.19\right)$

$=11-2^2.\sum_{r=1}^{10}r^2\left(2r-1\right)=11-4\left(\frac{110^2}{2}-35\times11\right)$

$=11-220\left(103\right)$

$\Rightarrow \alpha=11,\ \beta=103$

60. C

Sol. Let TV (r) denotes truth value of a statement r.
Now, if TV (p) = TV (q) = T

$\Rightarrow TV\left(S_1\right)=F$

Also, if TV (p) = T and TV (q) = F

$\Rightarrow TV\left(S_2\right)=T$

61. A

Sol. $u=\frac{2z+i}{z-ki}$

$$=\frac{2x^2+\left(2y+1\right)\left(y-k\right)}{x^2+\left(y-k\right)^2}+i\frac{\left(x\left(2y+1\right)-2x\left(y-k\right)\right)}{x^2+\left(y-k\right)^2}$$

Since Re (u) + Im (u) = 1

$\Rightarrow 2x^2+(2y+1)(y-k)+x(2y+1)-2x(y-k)=x^2+(y-k)^2$

$\left.\begin{matrix} P(0,y_1) \\ Q(0,y_2) \end{matrix}\right\rangle \Rightarrow y^2+y-k-k^2=0 \left\langle \begin{matrix} y_1+y_2=-1 \\ y_1y_2=-k-k^2 \end{matrix}\right.$

$\because PQ=5$

$\Rightarrow |y_1-y_2|=5 \Rightarrow k^2+k-6=0$

$\Rightarrow k=-3, 2$

So, $k=2(k>0)$

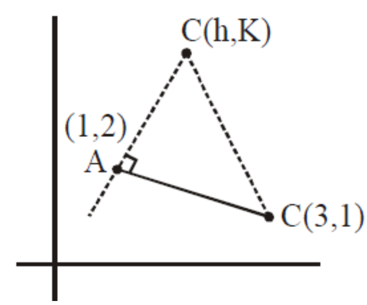
62. A

Sol. $\left(\frac{K-2}{h-1}\right)\left(\frac{1-2}{3-1}\right)=-1 \Rightarrow K=2h$ …….(1)

$\because [\Delta ABC]=5\sqrt{5}$

$\Rightarrow \frac{1}{2}(\sqrt{5})\sqrt{(h-1)^2+(K-2)^2}=5\sqrt{5}$ ……….(2)

$\Rightarrow h=2\sqrt{5}+1(h>0)$



63. D

Sol. Since (3, 3) lies on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\frac{9}{a^2}-\frac{9}{b^2}=1$ …………..(1)

Now, normal at (3, 3) is $y-3=-\frac{a^2}{b^2}(x-3)$,

which passes through $(9, 0) \Rightarrow b^2=2a^2$ ……….(2)

So, $e^2=1+\frac{b^2}{a^2}=3$

Also, $a^2=\frac{9}{2}$ (From (i) and (ii))

Thus, $(a^2, e^2)=\left(\frac{9}{2}, 3\right)$

64. B

Sol. $\int\left(\frac{x}{x\sin x+\cos x}\right)^2 dx=\int\left(\frac{x}{\cos x}\right).\frac{x\cos x\,dx}{(x\sin x+\cos x)^2}$

$$=\frac{x}{\cos x}\left(-\frac{1}{x\sin x+\cos x}\right)+\int\left(\frac{\cos x+x\sin x}{\cos^2 x}\right)\left(\frac{1}{x\sin x+\cos x}\right)dx$$
$$=-\frac{x\sec x}{x\sin x+\cos x}+\int\sec^2 x\,dx$$
$$=-\frac{x\sec x}{x\sin x+\cos x}+\tan x+C$$

65. B

Sol. $A^2=\begin{pmatrix}\cos 2\theta & i\sin 2\theta\\ i\sin 2\theta & \cos 2\theta\end{pmatrix}$

Similarly, $A^5=\begin{pmatrix}\cos 5\theta & i\sin 5\theta\\ i\sin 5\theta & \cos 5\theta\end{pmatrix}=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$

(1) $a^2+b^2=\cos^2 5\theta-\sin^2 5\theta=\cos 10\theta=\cos 75^o$

(2) $a^2-d^2=\cos^2 5\theta-\cos^2 5\theta=0$

(3) $a^2-b^2=\cos^2 5\theta+\sin^2 5\theta=1$

(4) $a^2-c^2=\cos^2 5\theta+\sin^2 5\theta=1$

66. C

Sol. $\sum_{r=0}^{20}{}^{50-r}C_6={}^{50}C_6+{}^{49}C_6+{}^{48}C_6+....+{}^{30}C_6$

$={}^{50}C_6+{}^{49}C_6+{}^{48}C_6+....+\left({}^{30}C_6+{}^{30}C_7\right)-{}^{30}C_7$

$={}^{50}C_6+{}^{49}C_6+{}^{48}C_6+....+\left({}^{31}C_6+{}^{31}C_7\right)-{}^{30}C_7$

$={}^{50}C_6+{}^{50}C_7-{}^{30}C_7$

$={}^{51}C_7-{}^{30}C_7$

67. A

Sol. $[x]^2+2[x+2]-7=0$

$\Rightarrow [x]^2+2[x]+4-7=0$

$\Rightarrow [x]=1,-3$

$\Rightarrow x\in[1,2)\cup[-3,-2)$

68. A

Sol. $f(x)=\int_1^3\frac{\sqrt{x}\,dx}{(1+x)^2}=\int_1^{\sqrt{3}}\frac{t.2t\,dt}{\left(1+t^2\right)^2}$ (put $\sqrt{x}=t$)

$=\left(-\frac{t}{1+t^2}\right)_1^{\sqrt{3}}+\left(\tan^{-1}t\right)_1^{\sqrt{3}}$ [Applying by parts]

$$=-\left(\frac{\sqrt{3}}{4}-\frac{1}{2}\right)+\frac{\pi}{3}-\frac{\pi}{4}$$
$$=\frac{1}{2}-\frac{\sqrt{3}}{4}+\frac{\pi}{12}$$

69. B

Sol. $\max\{n(A),\ n(B)\}\le n(A\cup B)\le n(U)$
$\Rightarrow 76\le 76+63-x\le 100$
$\Rightarrow -63\le -x\le -39$
$\Rightarrow 63\ge x\ge 39$

70. C

Sol. $x\frac{dy}{dx}-y=x^2(x\cos x+\sin x),\ x>0$
$\frac{dy}{dx}-\frac{y}{x}=x(x\cos x+\sin x)\Rightarrow \frac{dy}{dx}+Py=Q$
So, I.F. $=e^{\int -\frac{1}{x}dx}\ \frac{1}{|x|}=\frac{1}{x}(x>0)$
Thus, $\frac{y}{x}=\int\frac{1}{x}(x(x\cos x+\sin x))dx$
$\Rightarrow \frac{y}{x}=x\sin x+C$
$\because y(\pi)=\pi\Rightarrow C=1$
So, $y=x^2\sin x+x\Rightarrow (y)_{\pi/2}=\frac{\pi^2}{4}+\frac{\pi}{2}$
Also, $\frac{dy}{dx}=x^2\cos x+2x\sin x+1$
$\Rightarrow \frac{d^2y}{dx^2}=-x^2\sin x+4x\cos x+2\sin x$
$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{\frac{\pi}{2}}=-\frac{\pi^2}{4}+2$

71. 8

Sol. Given $\left(2x^2+3x+4\right)^{10}=\sum_{r=0}^{20}a_rx^r$ …………(1)
replace x by $\frac{2}{x}$ in above identity :-

$$\frac{2^{10}\left(2x^2+3x+4\right)^{10}}{x^{20}}=\sum_{r=0}^{20}\frac{a_r 2^r}{x^r}$$

$$\Rightarrow 2^{10}\sum_{r=0}^{20}a_r x^r=\sum_{r=0}^{20}a_r 2^r x^{(20-r)} \text{ (from(i))}$$

now, comparing coefficient of x^7 from both sides
(take r = 7 in L.H.S. and r = 13 in R.H.S.)

$$2^{10}a_7=a_{13}2^{13}\Rightarrow\frac{a_7}{a_{13}}=2^3=8$$

72. 3

Sol. $$D_1=\begin{vmatrix}-7 & 4 & -1\\ 8 & 1 & 5\\ 15 & b & 6\end{vmatrix}=0\Rightarrow b=-3$$

$$D=\begin{vmatrix}1 & 4 & -1\\ 3 & 1 & 5\\ a & b & 6\end{vmatrix}=0\Rightarrow 21a-8b-66=0 \qquad \ldots\ldots\ldots\ldots(1)$$

$$P: 2x-3y+6z=15$$

so required distance $=\frac{21}{7}=3$

73. 5

Sol. $$D=\begin{vmatrix}1 & -2 & 3\\ 2 & 1 & 1\\ 1 & -7 & a\end{vmatrix}=0\Rightarrow a=8$$

also, $$D_1=\begin{vmatrix}9 & -2 & 3\\ b & 1 & 1\\ 24 & -7 & 8\end{vmatrix}=0\Rightarrow b=3$$

hence, $a-b=8-3=5$

74. 10

Sol. Since, $\lim\limits_{x\to 0}\frac{f(x)}{x}$ exist $\Rightarrow f(0)=0$

Now, $f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim\limits_{h\to 0}\frac{f(h)+xh^2+x^2h}{h}$ (take y = h)

$$=\lim_{h\to 0}\frac{f(h)}{h}+\lim_{h\to .0}(xh)+x^2$$

$$\Rightarrow f'(x)=1+0+x^2$$

$$\Rightarrow f'(3)=10$$

75. 3

Sol. We have, 1 – (probability of all shots result in failure) > $\frac{1}{4}$

$$\Rightarrow 1-\left(\frac{9}{10}\right)^n > \frac{1}{4}$$

$$\Rightarrow \frac{3}{4} > \left(\frac{9}{10}\right)^n \Rightarrow n \geq 3$$