FIITJEE

Solutions to JEE(Main)-2020

Test Date: 2nd September 2020 (Second Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours | **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of **3 hours** duration.
2. This **Test Paper** consists of **75** questions. The maximum marks are **300**.
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 25 questions in each part of equal weightage out of which 20 questions are MCQs and 5 questions are numerical value based. Each question is allotted **4 (four)** marks for correct response.
4. **(Q. No. 01 – 20, 26 – 45, 51 – 70)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
5. **(Q. No. 21 – 25, 46 – 50, 71 – 75)** contains 15 Numerical based questions with answer as numerical value. Each question carries **+4 marks** for correct answer. There is no negative marking.
6. Candidates will be awarded marks as stated above in **instruction No.3** for correct response of each question. One mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer box.
7. There is only one correct response for each question. Marked up more than one response in any question will be treated as wrong response and marked up for wrong response will be deducted accordingly as per **instruction 6** above.

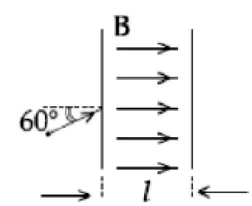
PART –A (PHYSICS)

1. A capillary tube made of glass of radius 0.15 mm is dipped vertically in a beaker filled with methylene iodide (surface tension = 0.05 Nm^{-1}, density = 667 kg m^{-3}) which rises to height h in the tube. It is observed that the two tangents drawn from liquid-glass interfaces
(from opposite sides of the capillary) make an angle of 60º with one another. Then h is close to (g = 10 ms^{-2})
(A) 0.172 m (B) 0.049 m
(C) 0.087 m (D) 0.137 m

2. When the temperature of metal wire is increased from 0ºC to 10ºC, its length increases by 0.02%.The percentage change in its mass density will be closed to:
(A) 0.008 (B) 0.8
(C) 0.06 (D) 2.3

3. A particle is moving 5 times as fast as an electron. The ratio of the de-Broglie wavelength of the particle to that of the electron is 1.878×10^{-4}. The mass of the particle is close to:
(A) 9.7×10^{-28} kg (B) 4.8×10^{-27} kg
(C) 1.2×10^{-28} kg (D) 9.1×10^{-31} kg

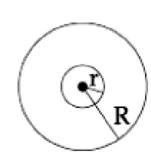
4. The figure shows a region of length 'ℓ' with a uniform magnetic field of 0.3 T in it and a proton entering the region with velocity 4×10^5 ms^{-1} making an angle 60º with the field. If the proton completes 10 revolution by the time it cross the region shown, 'ℓ' is close to (mass of proton = 1.67×10^{-27} kg, charge of the proton = 1.6×10^{-19} C)



(A) 0.44 m (B) 0.11 m (C) 0.88 m (D) 0.22 m

5. An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true?
(a) the mean free path of the molecules decreases
(b) the mean collision time between the molecules decreases.
(c) the mean free path remains unchanged.
(d) the mean collision time remains unchanged.
(A) (a) and (b) (B) (a) and (d)
(C) (c) and (d) (D) (b) and (c)

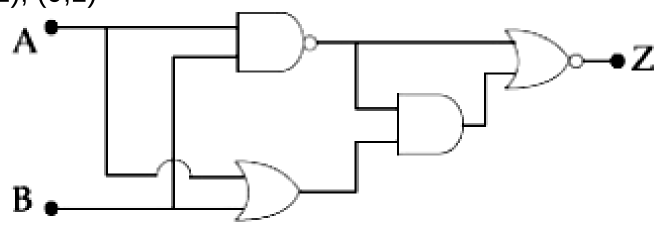
6. A charge Q is distributed over two concentric conducting thin spherical shells of radii r and R (R > r). If the surface charge densities on the two shells are equal, the electric potential at the common centre is:



(A) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)}{(R^2+r^2)}Q$ (B) $\frac{1}{4\pi\varepsilon_0}\frac{(R+2r)Q}{2(R^2+r^2)}$

(C) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)}{2(R^2+r^2)}Q$ (D) $\frac{1}{4\pi\varepsilon_0}\frac{(2R+r)}{(R^2+r^2)}Q$

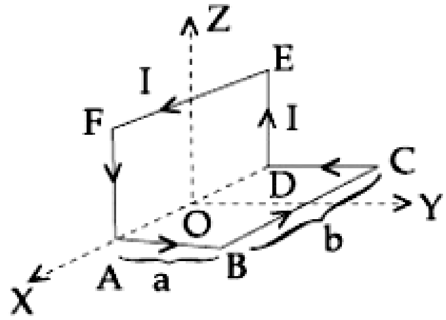
7. In the following, digital circuit, what will be the output at 'Z', when the input (A,B) are (1,0), (0,0), (1,1), (0,1)



(A) 0,1,0,0
(B) 1,0, 1, 1
(C) 1,1,0,1
(D) 0,0,1,0

8. An inductance coil has a reactance of 100Ω. When an AC signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. The self-inductance of the coil is:
(A) 1.1×10^{-2} H
(B) 1.1×10^{-1} H
(C) 6.7×10^{-7} H
(D) 5.5×10^{-5} H

9. A wire carrying current I is bent in the shape ABCDEFA as shown, where rectangles ABCDA and ADEFA are perpendicular to each other. If the sides of the rectangles are of lengths a and b, then the magnitude and direction of magnetic moment of the loop ABCDEFA is:



(A) $\sqrt{2}\,abI$, along $\left(\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}\right)$
(B) $\sqrt{2}\,abI$, along $\left(\frac{\hat{j}}{\sqrt{5}} + \frac{2\hat{k}}{\sqrt{5}}\right)$
(C) abI, along $\left(\frac{\hat{j}}{\sqrt{5}} + \frac{2\hat{k}}{\sqrt{5}}\right)$
(D) abI, along $\left(\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}\right)$

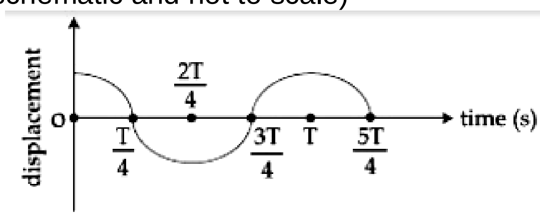
10. In a hydrogen atom, an electron makes a transition from $(n + 1)^{th}$ level to the n^{th} level. If n >> 1, the frequency of radiation emitted is proportional to:
(A) $\frac{1}{n}$
(B) $\frac{1}{n^2}$
(C) $\frac{1}{n^3}$
(D) $\frac{1}{n^4}$

11. The height ‘h’ at which the weight of a body will be the same as that at the same depth ‘h’ from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected)
(A) $\frac{\sqrt{5}R - R}{2}$
(B) $\frac{\sqrt{5}}{2}R - R$
(C) $\frac{R}{2}$
(D) $\frac{\sqrt{3}R - R}{2}$

12. Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are 0.1 kg-m^2 and 10 rad s^{-1} respectively while those for the second one are 0.2 kg-m^2 and 5 rad s^{-1} respectively. At some instant, they get stuck together and start rotating as a single system about their common axis with some angular speed. The kinetic energy of the combined system is:
(A) $\frac{20}{3}$ J (B) $\frac{2}{3}$ J (C) $\frac{5}{3}$ J (D) $\frac{10}{3}$ J

13. A 10 μF capacitor is fully charged to a potential difference of 50V. After removing the source voltage, it is connected to an uncharged capacitor in parallel. Now the potential difference across them becomes 20 V. The capacitance of the second capacitor is:
(A) 15 μF (B) 20 μF
(C) 10 μF (D) 30 μF

14. The displacement time graph of a particle executing SHM is given in figure: (sketch is schematic and not to scale)



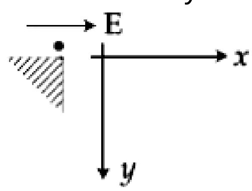
Which of the following statements is/are true for this motion?
(a) The force is zero at $t = \frac{3T}{4}$
(b) The acceleration is maximum at t = T
(c) The speed is maximum at $t = \frac{T}{4}$
(d) The P.E. is equal to K.E. of the oscillation at $t = \frac{T}{2}$
(A) (a), (b) and (d) (B) (a) and (d)
(C) (a), (b) and (c) (D) (b), (c) and (d)

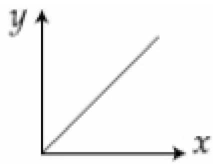
15. In a Young's double slit experiment, 16 fringes are observed in a certain segment of the screen when light of wavelength 700 nm is used. If the wavelength of light is changed to 400 nm, the number of fringes observed in the same segment of the screen would be:
(A) 30 (B) 24
(C) 18 (D) 28

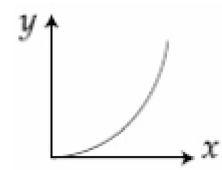
16. A heat engine is involved with exchange of heat of 1915 J, –40J, +125 J and –QJ, during one cycle achieving and efficiency of 50.0%. The value of Q is:
(A) 40 J (B) 640 J
(C) 400 J (D) 980 J

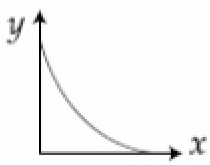
17. If momentum (P), area (A) and time (T) are taken to be the fundamental quantities, then the dimensional formula for energy is :
(A) $[P^2AT^{-2}]$ (B) $[PA^{-1}T^{-2}]$
(C) $[PA^{1/2}T^{-1}]$ (D) $[P^{1/2}AT^{-1}]$

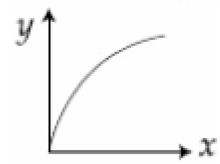
18. A small point mass carrying some positive charge on it, is released from the edge of a table. There is a uniform electric field in this region in the horizontal direction. Which of the following options then correctly describe the trajectory of the mass? (Curves are drawn schematically and are not to scale)



(A) 

(B) 

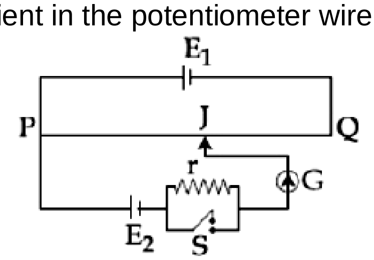
(C) 

(D) 

19. In a plane electromagnetic wave, the directions of electric field and magnetic field are represented by $\hat{k}$ and $2\hat{i}-2\hat{j}$, respectively. What is the unit vector along direction of propagation of the wave.

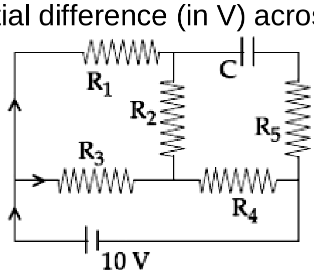
(A) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$
(B) $\frac{1}{\sqrt{5}}(\hat{i}+2\hat{j})$
(C) $\frac{1}{\sqrt{5}}(2\hat{i}+\hat{j})$
(D) $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$

20. A potentiometer wire PQ of 1m length is connected to a standard cell E_1. Another cell E_2 of emf 1.02 V is connected with a resistance 'r' and switch S (as shown in figure). With switch S open, the null position is obtained at a distance of 49 cm from Q. The potential gradient in the potentiometer wire is:

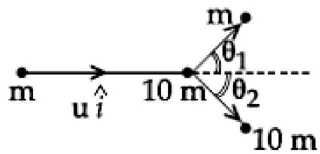


(A) 0.04 V/cm
(B) 0.01 V/cm
(C) 0.02 V/cm
(D) 0.03 V/cm

21. An ideal cell of emf 10V is connected in circuit shown in figure. Each resistance is 2Ω. The potential difference (in V) across the capacitor when it is fully charged is _________



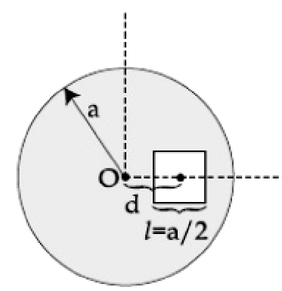
22. A particle of mass m is moving along the x-axis with initial velocity $u\hat{i}$. It collides elastically with a particle of mass 10m at rest and then moves with half its initial kinetic energy (see figure). If $\sin\theta_1 = \sqrt{n}\sin\theta_2$ then value of n is ________.



23. A light ray enters a solid glass sphere of refractive index $\mu = \sqrt{3}$ at an angle of incidence 60°. The ray is both reflected and refracted at the farther surface of the sphere. The angle (in degrees) between the reflected and refracted rays at this surface is _______.

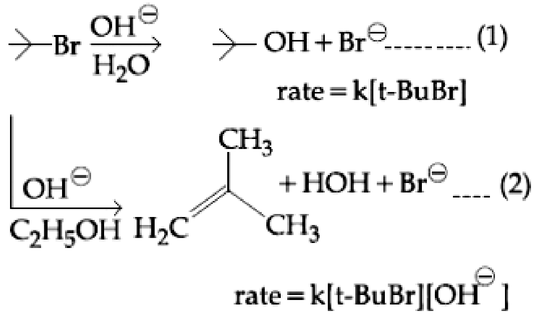
24. A wire of density 9×10^{-3} kg cm^{-3} is stretched between two clamps 1 m apart. The resulting strain in the wire is 4.9×10^{-4}. The lowest frequency of the transverse vibrations in the wire (Young's modulus of wire $Y = 9 \times 10^{10}$ Nm^{-2}), (to the nearest integer),_______

25. A square shaped hole of side $l = \frac{a}{2}$ is carved out at a distance $d = \frac{a}{2}$ from the centre O of a uniform circular disk of radius a. If the distance of the centre of mass of the remaining portion from O is $-\frac{a}{X}$, value of X (to the nearest integer) is :



PART –B (CHEMISTRY)

26. Consider the reaction sequence given below:



Which of the following statements is true:
(A) Doubling the concentration of base will double the rate of both the reactions
(B) Changing the concentration of base will have no effect on reaction (1)
(C) Changing the base from OH^- to ^-OR will have no effect on reaction(2)
(D) Changing the concentration of base will have no effect on reaction (2)

27. The number of subshells associated with n = 4 and m = –2 quantum numbers is:
(A) 2
(B) 16
(C) 8
(D) 4

28. Match the type of interaction in column A with the distance dependence of their interaction energy in column B :

	A		B
(I)	ion-ion	(a)	$\frac{1}{r}$
(II)	dipole-dipole	(b)	$\frac{1}{r^2}$
(III)	London dispersion	(c)	$\frac{1}{r^3}$
		(d)	$\frac{1}{r^6}$

(A) (I) – (a); (II) – (c); (III) – (d)
(B) (I) – (b); (II) – (d); (III) – (c)
(C) (I) – (a); (II) – (b); (III) – (c)
(D) (I) – (a); (II) – (b); (III) – (d)

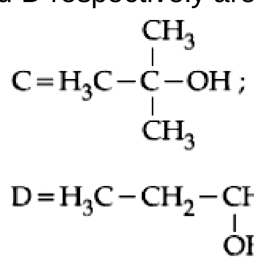
29. The shape/structure of $[XeF_5]^-$ and XeO_3F_2, respectively are :
(A) octahedral and square pyramidal
(B) trigonal bipyramidal and trigonal bipyramidal
(C) trigonal bipyramidal and pentagonal
(D) pentagonal planar and trigonal bipyramidal

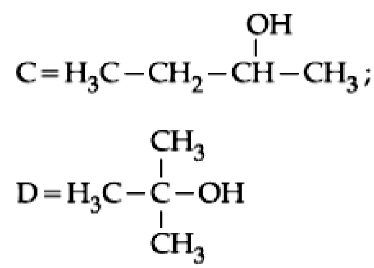
30. Two elements A and B have similar chemical properties. They don't form solid hydrogencarbonates, but react with nitrogen to form nitrides. A and B, respectively, are:
(A) Na and Ca
(B) Na and Rb
(C) Cs and Ba
(D) Li and Mg

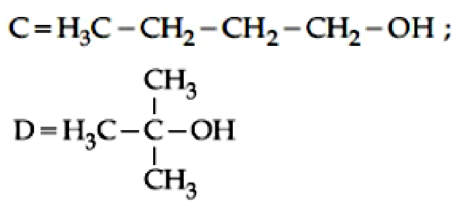
31. Two compounds A and B with same molecular formula (C_3H_6O) undergo Grignard reaction with methylmagnesium bromide to give products C and D. Products C and D show following chemical tests.

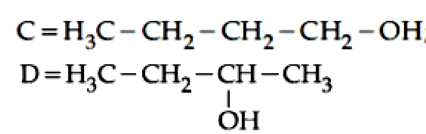
TEST	C	D
Ceric ammonium nitrate Test	Positive	Positive
Lucas Test	Tubridity obtained after five minutes	Turbidity obtained immediately
Iodoform Test	Positive	Negative

C and D respectively are:

(A) 

(B) 

(C) 

(D) 

32. Three elements X, Y and Z are in the 3rd period of the periodic table. The oxides of X, Y and Z, respectively, are basic, amphoteric and acidic. The correct order of the atomic numbers of X, Y and Z is:

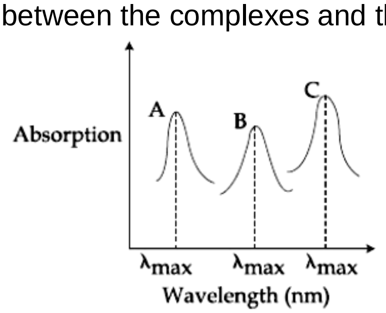
(A) X < Z < Y
(B) Z < Y < X
(C) X < Y < Z
(D) Y < X < Z

33. The correct observation in the following reaction is:

$$\text{Sucrose} \xrightarrow[\text{Cleavage (Hydrolysis)}]{\text{Gly cosidic bond}} A + B \xrightarrow[\text{Reagent}]{\text{Seliw anoff}} ?$$

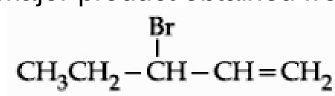
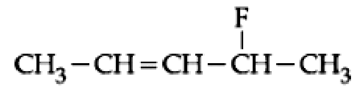
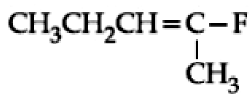
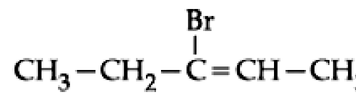
(A) Formation of blue colour
(B) Formation of violet colour
(C) Formation of red colour
(D) Gives no colour

34. Simplified absorption spectra of three complexes ((i) and (ii) and (iii)) of M^{n+} ion are provided below; their λ_{max} values are marked as A, B and C respectively. The correct match between the complexes and their λ_{max} values is:



(i) $[M(NCS)_6]^{(-6+n)}$ (ii) $[MF_6]^{(-6+n)}$
(iii) $[M(NH_3)_6]^{n+}$
(A) A-(i), (B)-(ii), C-(iii) (B) A-(ii), (B)-(iii), C-(i)
(C) A-(iii), (B)-(i), C-(ii) (D) A-(ii), (B)-(i), C-(iii)

35. The major product obtained from E_2-elimination of 3-bromo-2-fluoropentane is

(A) 
(B) 
(C) 
(D) 

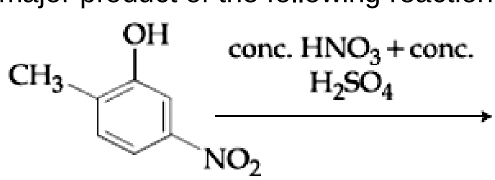
36. Cast iron is used for the manufacture of:
(A) pig iron, scrap iron and steel (B) wrought iron and steel
(C) wrought iron and pig iron (D) wrought iron, pig iron and steel

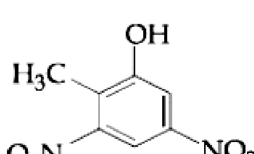
37. The results given in the below table were obtained during kinetic studies of the following reaction :
$2A + B \longrightarrow C + D$

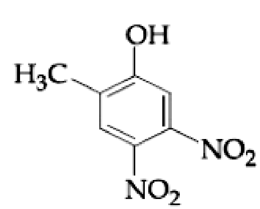
Experiment	[A]/ $molL^{-1}$	[B]/ $molL^{-1}$	Initial rate/ $molL^{-1}\ min^{-1}$
I	0.1	0.1	6.00×10^{-3}
II	0.1	0.2	2.40×10^{-2}
III	0.2	0.1	1.20×10^{-2}
IV	X	0.2	7.20×10^{-2}
V	0.3	Y	2.88×10^{-1}

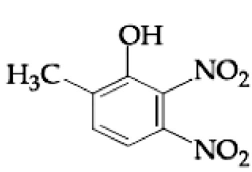
X and Y in the given table are respectively:
(A) 0.3, 0.4 (B) 0.4, 0.3
(C) 0.4, 0.4 (D) 0.3, 0.3

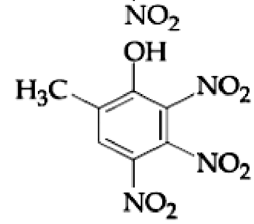
38. The major product of the following reaction is:



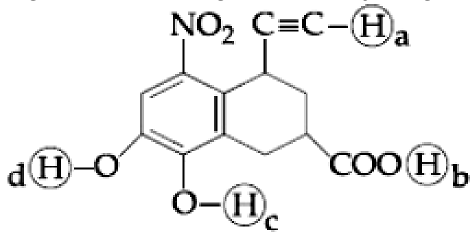
(A) 

(B) 

(C) 

(D) 

39. Arrange the following labelled hydrogens in decreasing order of acidity:

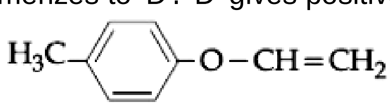


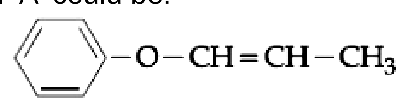
(A) c > b > a > d
(B) b > c > d > a
(C) b > a > c > d
(D) c > b > d > a

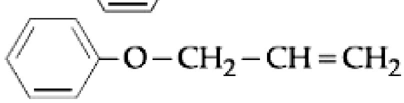
40. If you spill a chemical toilet cleaning liquid on your hand, your first aid would be:
(A) aqueous NH_3
(B) vinegar
(C) aqueous NaOH
(D) aqueous $NaHCO_3$

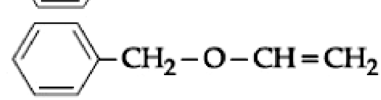
41. Amongst the following statements regarding adsorption, those that are valid are:
(a) ΔH becomes less negative as adsorption proceeds.
(b) On a given adsorbent, ammonia is adsorbed more than nitrogen gas.
(c) On adsorption, the residual force acting along the surface of the adsorbent increases.
(d) With increase in temperature, the equilibrium concentration of adsorate increases.
(A) (c) and (d)
(B) (a) and (b)
(C) (d) and (a)
(D) (b) and (c)

42. An organic compound 'A' ($C_9H_{10}O$) when treated with conc. HI undergoes cleavage to yield compound 'B' and 'C'. 'B' gives yellow precipitate with $AgNO_3$ where as 'C' tautomerizes to 'D'. 'D' gives positive iodoform test. 'A' could be:

(A) 

(B) 

(C) 

(D) 

43. The one that is not expected to show isomerism is:
(A) $[Ni(en)_3]^{2+}$
(B) $[Ni(NH_3)_4(H_2O)_2]^{2+}$
(C) $[Pt(NH_3)_2Cl_2]$
(D) $[Ni(NH_3)_2Cl_2]$

44. The size of a raw mango shrinks to a much smaller size when kept in a concentrated salt solution. Which one of the following process can explain this?
(A) Dialysis
(B) Osmosis
(C) Diffusion
(D) Reverse osmosis

45. The molecular geometry of SF_6 is octahedral. What is the geometry of SF_4 (including lone pair(s) of electrons, if (any)?
(A) Tetrahedral
(B) Trigonal bipyramidal
(C) Square planar
(D) Pyramidal

46. The ratio of the mass percentages of 'C & H' and 'C & O' of a saturated acyclic organic compound 'X' are 4 : 1 and 3 : 4 respectively. Then, the moles of oxygen gas required for complete combustion of two moles of organic compound 'X' is_________.

47. For the disproportionation reaction $2Cu^{+}(aq) \rightleftharpoons Cu(s) + Cu^{2+}$ at 298 K, ln K (where K is the equilibrium constant) is _______ $\times 10^{-1}$.
Given : $\left(E^{\circ}_{Cu^{2+}/Cu^{+}} = 0.16V\ E^{\circ}_{Cu^{+}/Cu} = 0.52V\ \frac{RT}{F} = 0.025\right)$

48. The heat of combustion of ethanol into carbon dioxides and water is –327 kcal at constant pressure. The heat evolved (in cal) at constant volume at 27°C (if all gases behave ideally) is (R = 2 cal mol^{-1} K^{-1})

49. The work function of sodium metal is 4.41×10^{-19} J. If photons of wavelength 300 nm are incident on the metal, the kinetics energy of the ejected electrons will be ($h = 6.63 \times 10^{-34}$ J s; $c = 3 \times 10^{8}$ m/s)______ $\times 10^{-21}$ J.

50. The oxidation states of transition metal atoms in $K_2Cr_2O_7$, $KMnO_4$ and K_2FeO_4, respectively, are x, y and z. The sum of x, y and z is_______.

PART–C (MATHEMATICS)

51. Let a, b, c $\in$ R be all non-zero and satisfy $a^3 + b^3 + c^3 = 2$. If the matrix $A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$ satisfies $A^T A = I$, then a value of abc can be

(A) $\frac{1}{3}$ (B) 3
(C) $-\frac{1}{3}$ (D) $\frac{2}{3}$

52. If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in the interval

(A) $\left(-\frac{5}{4}, -1\right)$ (B) $\left[-1, -\frac{1}{2}\right]$
(C) $\left[-\frac{3}{2}, -\frac{5}{4}\right]$ (D) $\left(-\frac{1}{2}, -\frac{1}{4}\right]$

53. Let f(x) be a quadratic polynomial such that f(-1) + f(2) = 0. If one of the roots of f(x) = 0 is 3, then its other root lies in

(A) (-3, -1) (B) (-1, 0)
(C) (0, 1) (D) (1, 3)

54. A plane passing through the point (3, 1, 1) contains two lines whose direction ratios are 1, -2, 2 and 2, 3, -1 respectively. If this plane also passes through the point (α, -3, 5), then α is equal to

(A) -5 (B) -10
(C) 10 (D) 5

55. If the sum of first 11 terms of an A.P. $a_1, a_2, a_3, \ldots$ is 0 ($a_1 \neq 0$), then the sum of the A.P. $a_1, a_3, a_5, \ldots, a_{23}$ is ka_1, where k is equal to

(A) $\frac{121}{10}$ (B) $-\frac{121}{10}$
(C) $\frac{72}{5}$ (D) $-\frac{72}{5}$

56. $\lim\limits_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$ is equal to

(A) 2 (B) e
(C) e^2 (D) 1

57. For some $\theta \in \left(0, \frac{\pi}{2}\right)$, if the eccentricity of the hyperbola $x^2 - y^2\sec^2\theta = 10$ is $\sqrt{5}$ times the eccentricity of the ellipse $x^2\sec^2\theta + y^2 = 5$, then the length of the latus rectum of the ellipse is

(A) $\frac{4\sqrt{5}}{3}$
(B) $2\sqrt{6}$
(C) $\sqrt{30}$
(D) $\frac{2\sqrt{5}}{3}$

58. Consider the region $R = \{(x, y) \in R^2 : x^2 \le y \le 2x\}$. If a line $y = \alpha$ divides the area of region R into two equal parts, then which of the following is true?

(A) $\alpha^3 - 6\alpha^{3/2} - 16 = 0$
(B) $3\alpha^2 - 8\alpha + 8 = 0$
(C) $\alpha^3 - 6\alpha^2 + 16 = 0$
(D) $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$

59. Let $n > 2$ be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is

(A) 201
(B) 101
(C) 199
(D) 200

60. The imaginary part of

$\left(3 + 2\sqrt{-54}\right)^{1/2} - \left(3 - 2\sqrt{-54}\right)^{1/2}$ can be

(A) $-2\sqrt{6}$
(B) 6
(C) $\sqrt{6}$
(D) $-\sqrt{6}$

61. Let $f : (-1, \infty) \to R$ be defined by $f(0) = 1$ and $f(x) = \frac{1}{x}\log_e(1 + x), x \ne 0$. Then the function f :

(A) increases in $(-1,0)$ and decreases in $(0, \infty)$
(B) increases in $(-1, \infty)$
(C) decreases in $(-1, \infty)$
(D) decreases in $(-1,0)$ and increases in $(0, \infty)$

62. If a curve $y = f(x)$, passing through the point(1, 2), is the solution of the differential equation $2x^2dy = (2xy + y^2)dx$, then $f(½)$ is equal to

(A) $\frac{-1}{1+\log_e 2}$
(B) $\frac{1}{1-\log_e 2}$
(C) $\frac{1}{1+\log_e 2}$
(D) $1 + \log_e 2$

63. If A = {X = $(x, y, z)^T$: PX = 0 and $x^2 + y^2 + z^2 = 1$}, where $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$,

then the set A

(A) is a singleton
(B) contains more than two elements
(C) is an empty set
(D) contains exactly two elements

64. The equation of the normal to the curve $y = (1 + x)^{2y} + \cos^2(\sin^{-1}x)$ at x = 0 is

(A) 2y + x = 4
(B) x + 4y = 8
(C) y = 4x + 2
(D) y + 4x = 2

65. Which of the following is tautology?

(A) $(\sim p) \wedge (p \vee q) \rightarrow q$
(B) $(p \rightarrow q) \wedge (q \rightarrow p)$
(C) $(\sim q) \vee (p \wedge q) \rightarrow q$
(D) $(q \rightarrow p) \vee \sim (p \rightarrow q)$

66. Let S be the sum of the first 9 terms of series $(x + ka) + (x^2 + (k + 2)a) + (x^3 + (k + 4)a) + (x^4 + (k + 6)a) + ...$, where $a \neq 0$ and $x \neq 1$. If $S = \frac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to

(A) 1
(B) 3
(C) – 5
(D) – 3

67. Let E^C denote the complement of an event E. Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$ and $P(E_1 \cap E_2 \cap E_3) = 0$. Then $P\left(E_2^C \cap E_3^C / E_1\right)$ is equal to

(A) $P(E_3)$ - $P(E_2^C)$
(B) $P(E_2^C) + P(E_3)$
(C) $P(E_3^C) - P(E_2)$
(D) $P(E_3^C) - P(E_2^C)$

68. The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola, is

(A) $192\sqrt{3}$
(B) $256\sqrt{3}$
(C) $64\sqrt{3}$
(D) $128\sqrt{3}$

69. Let f: R $\rightarrow$ R be a function which satisfies $f(x + y) = f(x) + f(y)$ $\forall$ x, y $\in$ R. If f(1) = 2 and $g(n) = \sum_{k=1}^{n-1} f(k)$, n $\in$ N, then the value of n for which g(n) = 20 is

(A) 9
(B) 20
(C) 5
(D) 4

70. The set of all possible values of θ in the interval $(0, \pi)$ for which the points (1,2) and $(\sin\theta, \cos\theta)$ lie on the same side of the line x + y = 1 is

(A) $\left(0, \frac{\pi}{4}\right)$
(B) $\left(0, \frac{3\pi}{4}\right)$
(C) $\left(0, \frac{\pi}{2}\right)$
(D) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

71. If y = $\sum_{k=1}^{6} k\cos^{-1}\left\{\frac{3}{5}\cos kx - \frac{4}{5}\sin kx\right\}$, then $\frac{dy}{dx}$ at x = 0 is _______

72. If the variance of the terms in an increasing A.P. b_1, b_2, b_3, …, b_{11} is 90, then the common difference of this A.P. is _______

73. Let [t] denote the greatest integer less than or equal to t. Then the value of $\int_1^2 |2x-[3x]|\,dx$ is _____

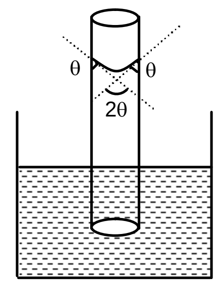
74. For a positive integer n, $\left(1+\frac{1}{x}\right)^n$ is expanded in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio 2 : 5 : 12, then n is equal to ____

75. Let the position vectors of points 'A' and 'B' be $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio λ : 1 (λ > 0). If O is the origin and $\overrightarrow{OB}.\overrightarrow{OP} - 3|\overrightarrow{OA}\times\overrightarrow{OP}|^2 = 6$, then λ is equal to ______

FIITJEE
Solutions to JEE (Main)-2020
PART –A (PHYSICS)

1. **C**

Sol. $2\theta = 60° \Rightarrow \theta = 30°$



$h = \frac{2T\cos\theta}{\rho rg}$

$= \frac{2(0.05)\cos 30^{o}}{(667)(0.15\times10^{-3})(10)}$

$= \frac{\sqrt{3}\times100}{667\times3} \simeq \frac{173.2}{2000}$ m. = 8.66 cm

2. **C**

Sol. $\Delta\ell = \ell \propto \Delta T \quad \Rightarrow \frac{\Delta\ell}{\ell} = \alpha\Delta T$ = 0.02%

$\Delta\rho = -\rho\gamma\Delta T$

$\Rightarrow \quad \left|\frac{\Delta\rho}{\rho}\right| = \gamma\Delta T = 3\propto\Delta T = 3(0.02\%)$ = 0.06%

3. **A**

Sol. $\lambda = \frac{h}{p} = \frac{h}{mv}$

$\Rightarrow \quad \frac{\lambda_p}{\lambda_e} = \frac{m_e v_e}{m_p v_p} \Rightarrow 1.878\times10^{-4} = \left(\frac{9.1\times10^{-31}}{m_p}\right)\left(\frac{1}{5}\right)$

$\Rightarrow \quad m_p = \frac{9.1\times10^{-31}}{5\times1.878\times10^{-4}}$ = 0.97 × 10^{-27} kg

4. **A**

Sol. ℓ = 10 × pitch

$= 10\times\frac{2\pi mv\cos\theta}{qB}$

$= \frac{20\times3.14\times1.67\times10^{-27}\times4\times10^{5}\times\frac{1}{2}}{1.6\times10^{-19}\times0.3}$

= 4.36 × 10^{-1} m. = 0.44 m.

5. **D**

Sol. On increasing the temperature, random velocity of molecules increases, therefore mean collision time between the molecules decreases. But the mean free path remains constant as it is product of velocity and time.

$\therefore$ B and C are correct option.

6. **A**

Sol. $\sigma 4\pi r^2 + \sigma 4\pi R^2 = Q$

$\Rightarrow \quad \sigma = \dfrac{Q}{4\pi(R^2 + r^2)}$

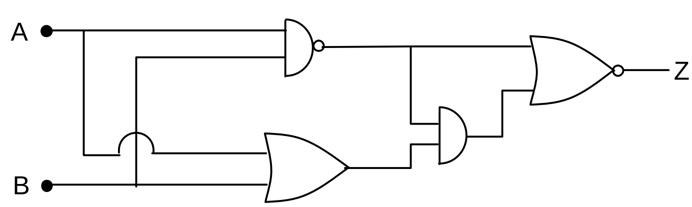
$\therefore \quad V_C = \dfrac{kq_1}{r} + \dfrac{kq_2}{R} = \dfrac{k(\sigma 4\pi r^2)}{r} + k\left(\dfrac{\sigma 4\pi R^2}{R}\right)$

$\Rightarrow \quad V_C = k\sigma 4\pi\ (r + R)$

$= K\left(\dfrac{Q}{R^2 + r^2}\right)(R + r)$

7. **D**

Sol.



A	B	Z
1	**0**	**0**
0	**0**	**0**
1	**1**	**1**
0	**1**	**0**

8. **A**

Sol. $R = 100\ \Omega$

$\tan\phi = \dfrac{X_C - X_L}{R} \Rightarrow \tan(-45^o) = \dfrac{-X_L}{R}$

$\Rightarrow \quad X_L = R = 100$

$\Rightarrow \quad Lw = 100$

$\Rightarrow \quad L = \dfrac{100}{2\pi f} = \dfrac{100}{2 \times 3.14 \times 1000}$

$\Rightarrow \quad L = 1.59 \times 10^{-2}$ H

Now of the option matches and the nearest option is (A).

9. **A**

Sol. $\vec{m} = Iab\,\hat{k} + Iab\,\hat{j}$

$\Rightarrow \quad |\vec{m}| = Iab\sqrt{2}$

Direction $\Rightarrow \dfrac{\hat{j}+\hat{k}}{\sqrt{2}}$

10. **C**

Sol. $\Delta E = h\nu = 13.6\ Z^2\left(\dfrac{1}{n_1^2}-\dfrac{1}{n_2^2}\right)eV$

$\Rightarrow \quad h\nu = 13.6(1)^2\left[\dfrac{1}{n^2}-\dfrac{1}{(n+1)^2}\right]eV$

$\Rightarrow \quad \nu = \left(\dfrac{13.6\ eV}{h}\right)\left[\dfrac{(n+1)^2-n^2}{n^2(n+1)^2}\right]$

$\Rightarrow \quad \nu = \left(\dfrac{13.6\ eV}{h}\right)\dfrac{2n+1}{n^2(n+1)^2}$

For n >> 1

$\Rightarrow \quad \nu = \left(\dfrac{13.6\ eV}{h}\right)\dfrac{2n}{(n^2)\,n^2}$

$\Rightarrow \quad \nu \propto \dfrac{1}{n^3}$

11. **A**

Sol. $g_{h=h} = g_{d=h}$

$\Rightarrow \quad \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2} = g\left(1-\dfrac{h}{R}\right)$

$\Rightarrow \quad 1 = \left(1+\dfrac{h}{R}\right)^2\left(1-\dfrac{h}{R}\right)$

$\Rightarrow \quad 1 = \left(1+\dfrac{2h}{R}+\dfrac{h^2}{R^2}\right)\left(1-\dfrac{h}{R}\right) \quad \Rightarrow \quad 1 = 1+\dfrac{h}{R}-\dfrac{h^2}{R^2}-\dfrac{h^3}{R^3}$

$\Rightarrow \quad \dfrac{h}{R}\left(\dfrac{h^2}{R^2}+\dfrac{h}{R}-1\right) = 0 \qquad \Rightarrow \quad \dfrac{h}{R} = \dfrac{-1\pm\sqrt{1+4}}{2}$

$\Rightarrow \quad h = \left(\dfrac{\sqrt{5}-1}{2}\right)R$

12. **A**

Sol. By conservation of angular momentum,

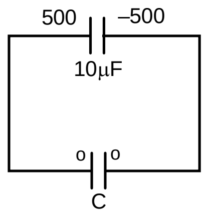
$I_1w_1 + I_2w_2 = (I_1 + I_2)\ w$

$\Rightarrow \quad w = \dfrac{I_1w_1+I_2w_2}{I_1+I_2} = \dfrac{(0.1)(10)+(0.2)(5)}{0.1+0.2} = \dfrac{2}{0.3} = \dfrac{20}{3}$

$K = \dfrac{1}{2}(I_1+I_2)w^2 = \dfrac{1}{2}(0.3)\left(\dfrac{20}{3}\right)\left(\dfrac{20}{3}\right) = \dfrac{20}{3}\ J$

13. **A**

Sol.



By conservation of charge,
(50) (10) + 0 = (20) (10) + (20) C
$\Rightarrow$ 500 = 20(10 + C)
$\Rightarrow$ 25 = 10 +C $\Rightarrow$ C = 15 μF

14. **C**

Sol. F = ma = m(–w^2 x) = 0 $\Rightarrow$ A
At t = T $\Rightarrow$ x $\rightarrow$ max $\Rightarrow$ a = max $\Rightarrow$ B
$v = \frac{dx}{dt}$ = slope of x – t curve $\Rightarrow$ C
$U = \frac{1}{2}mw^2x^2$ & $K = \frac{1}{2}mw^2(A^2 - x^2)$

15. **D**

Sol. $\ell = n_1\beta_1 = n_2\,\beta_2 \quad \Rightarrow \quad n_1\frac{\lambda_1 D}{d} = n_2\frac{\lambda_2 D}{d}$
$\Rightarrow$ $n_1\lambda = n_2\lambda_2$
$\Rightarrow$ (16) (700) = n_2(400) $\Rightarrow$ n_2 = 28

16. **D**

Sol. Efficiency (η) = $\frac{Q_{net}}{Q_+}$

$\Rightarrow \quad \frac{50}{100} = \frac{1915 - 40 + 125 - Q}{1915 + 125} \Rightarrow \frac{1}{2} = \frac{2000 - Q}{2040}$
$\Rightarrow$ 1020 = 2000 – Q
$\Rightarrow$ Q = 980 J

17. **C**

Sol. E = $P^a A^b T^c$
$\Rightarrow$ $ML^2T^{-2} = (MLT^{-1})^a\,(L^2)^b\,(T)^c$
$\Rightarrow$ a = 1
$\Rightarrow$ a + 2b = 2 $\Rightarrow$ $b = \frac{1}{2}$
–a + c = –2 $\Rightarrow$ c= –1
$\Rightarrow$ E = P $A^{1/2}$ T^{-1}

18. **A**

Sol. $X = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$; $y = \frac{1}{2}gt^2$

$\Rightarrow \quad \frac{x}{y} = \frac{qE}{mg} \quad \Rightarrow$ Straight line

19. **A**

Sol. In EM wave, wave velocity is in the direction perpendicular to both electric field & magnetic field.

$\Rightarrow \quad \hat{v} = \hat{E} \times \hat{B} = \hat{k} \times \left(\frac{2\hat{i} - 2\hat{j}}{2\sqrt{2}}\right) = \frac{\hat{j} + \hat{i}}{\sqrt{2}}$

20. **C**

Sol. $V_{PQ} = \varepsilon_2 = \left(\frac{dv}{dx}\right)\ell$

$\Rightarrow \quad 1.02 = \left(\frac{dv}{dx}\right)(51)$

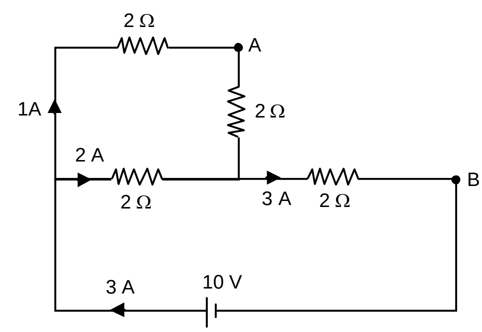
$\Rightarrow \quad \frac{dv}{dx} = \frac{1.02}{51} = 0.02$ V/cm

21. **8**

Sol. When capacitor is fully charged, no current flows through it. Therefore, we can remove capacitor branch.

$V_{AB} = 2(1) + 2(3)$

$= 2 + 6 = 8$ V



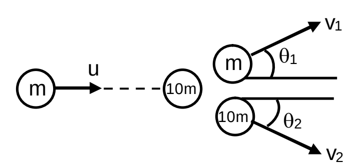
22. **10**

Sol. $\frac{1}{2}mv_1^2 = \frac{1}{2}\left(\frac{1}{2}mv^2\right) \Rightarrow v_1 = \frac{u}{\sqrt{2}}$

As the collision is perfectly elastic, energy remains conserved.

$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 + \frac{1}{2}(10\,m)v_2^2 \Rightarrow \frac{1}{4}mv^2 = 5mv_2^2$

$\Rightarrow \; v_2 = \frac{u}{2\sqrt{5}}$



By conservation of momentum along Y direction

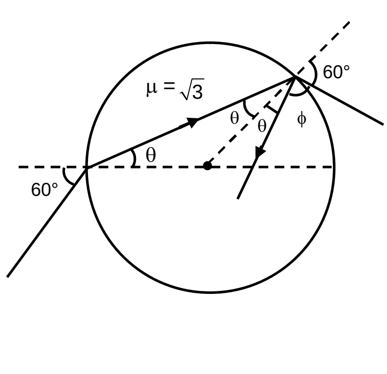
$0 + 0 = mv_1 \sin\theta, - 10\,m\; v_2 \sin\theta_2$

$\Rightarrow m\frac{u}{\sqrt{2}}\sin\theta_1 = 10\,m\left(\frac{u}{2\sqrt{5}}\right)\sin\theta_2$

$\Rightarrow \sin\theta_1 = \sqrt{10}\,\sin\theta$

23. **90**

Sol.



$1 \sin 60° = \sqrt{3}\sin\theta$

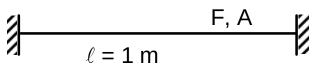
$\Rightarrow \quad \theta = 30°$

$\Rightarrow \quad \theta + \phi + 60° = 180$

$\Rightarrow \quad \phi = 120° - 30° = 90°$

24. **35**

Sol.



$$Y = \frac{\sigma}{\in} = \frac{F/A}{\in}$$

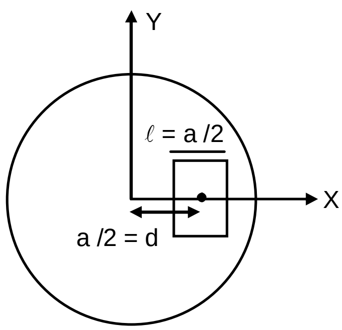
$$\Rightarrow \quad F = YA\in \quad \Rightarrow \quad v = \sqrt{\frac{F\ell}{m}} = \sqrt{\frac{(YA\in)\ell}{m}} = \sqrt{\frac{Y\in}{\rho}}$$

$$\Rightarrow \quad v = \sqrt{\frac{(9\times10^{10})(4.9\times10^{-4})}{9\times10^{3}}} = 70 \text{ m/s}$$

$$f = \frac{v}{2l} = \frac{70}{2\times1} = 35 \text{ Hz}$$

25. **23**

Sol.



Assuming the density of the disc is σ.

$$\Rightarrow \quad M_{disc} = \sigma\,\pi a^2$$

$$\Rightarrow \quad M_{hole} = \sigma\left(\frac{a}{2}\right)^2 = \frac{\sigma a^2}{4}$$

$$\Rightarrow \quad X_{cm} = \frac{m_D X_D - m_H X_H}{m_D - m_H}$$

$$\Rightarrow \quad X_{CM} = \frac{\sigma\pi a^2(0) - \sigma\left(\frac{a}{2}\right)^2\left(\frac{a}{2}\right)}{\sigma\pi a^2 - \sigma\left(\frac{a}{2}\right)^2}$$

$$\Rightarrow \quad X_{CM} = \frac{-\sigma\left(\frac{a}{4}\right)^2\left(\frac{a}{2}\right)}{\sigma a^2\left(\pi - \frac{1}{4}\right)} = -\frac{a}{2(4\pi-1)} = -\frac{a}{x} \qquad \Rightarrow \quad x = 23.12$$

PART –B (CHEMISTRY)

26. D

Sol. First reaction is S_N1 in which rate does not depend on conc. of nucleophile. Second reaction is E_2 reaction in which rate depends on conc. of base.

27. A

Sol. For n= 4 possible values of ℓ = 0, 1, 2, 3 only ℓ = 2 & ℓ = 3 can have m = –2. So possible subshells are 2

28. A

Sol. (i) ion-ion interaction energy is inversely proportional to the distance between ions $\left(\frac{1}{r}\right)$

(ii) dipole-dipole interaction energy is inversely proportional to the third power of r $\left(\frac{1}{r^3}\right)$

(iii) The interaction energy of London force is inversely proportional to sixth power of distance between two interaction particles $\left(\frac{1}{r^6}\right)$

29. D

Sol. (i) XeF_5^- St. No. = (5 + 2) = 7
so hybridisation is = sp^3d^3
and structure is pentagonal planar.

(ii) XeO_3F_2 St. No. = 5
so hybridisation is = sp^3d
and structure is trigonal bipyramidal.

30. D

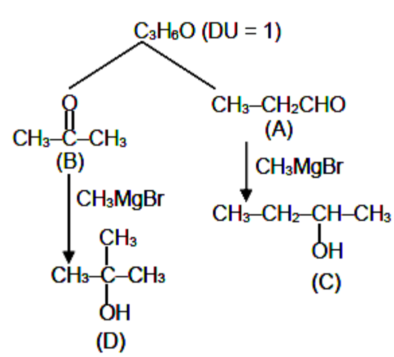
Sol. Li and Mg do not form solid bicarbonate. But react with N_2 to give nitrides.

$$6Li + N_2 \xrightarrow{\Delta} 2Li_3N$$

$$3Mg + N_2 \xrightarrow{\Delta} Mg_3N_2$$

31. B

Sol.



Iodoform Test	–ve	+ve
Lucas Test	Immediate	after 5-10 Mint.
Ceric Ammonium nitrate	+ve	+ve

32. C

Sol. On moving left to right in a period.
Acidic character of oxides is increase.
3rd period element oxides.

$$\underbrace{Na_2O \quad MgO}_{\text{Basic}} \quad \underbrace{Al_2O_3}_{\text{Amphoteric}} \qquad \underbrace{SiO_2 \quad P_2O_5 \quad Cl_2O_7}_{\text{Acidic}}$$

(i) Acidic character$\uparrow$
(ii) Atomic No$\uparrow$
So X have minimum Atomic No
& Z have maxima Atomic No
So correct order is X < Y < Z

33. C

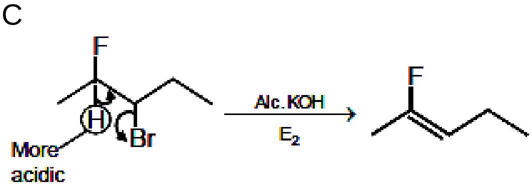
Sol. Seliwanoff reagent $\rightarrow$ [Resorcinol + Conc. HCl]
Use of Seliwanoff reagent is to distinguish aldoses and ketoses. Ketoses show red colour with Seliwanoff Reagent.

34. C

Sol. Stronger the ligand greater is splitting of d orbitals and smaller will be wave length of light absorbed.
The splitting power of ligands is $NH_3 > NC\bar{S} > F^-$
So order of wave length of light absorbed is $\lambda_{NH_3} < \lambda_{NC\bar{S}} < \lambda_{F^-}$

35. C

Sol.



36. B

Sol. Cast iron is made from pig iron which is used for production of wrought iron & steel.

37. A

Sol. Rate = $k[A]^a[B]^b$

from Exp (1) & (2) $\quad b = 2$

from Exp (1) & (3) $\quad a = 1$

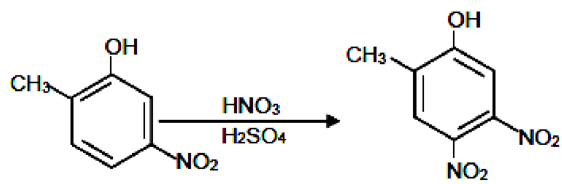
from Exp (2) & (4) $\Rightarrow 3 = \left(\frac{x}{0.1}\right)^1 \quad$ so $x = 0.3$

from Exp (1) & (5) $\Rightarrow 48 = (3)^1\left(\frac{y}{0.1}\right)^2$

$$(4)^2 = \left(\frac{y}{0.1}\right)^2 \quad \text{so } y = 0.4$$

38. B

Sol. This is electrophilic substitution reaction which is determine by electronic effect of $OH\backslash CH_3\backslash NO_2$.



39. B

Sol. Acidic strength $\propto$ Stability of conjugate base
General order of acidic strength
$R - COOH > Ph - OH > R - C \equiv CH$
'c' is more acidic due to –M effect of $-NO_2$.

40. D

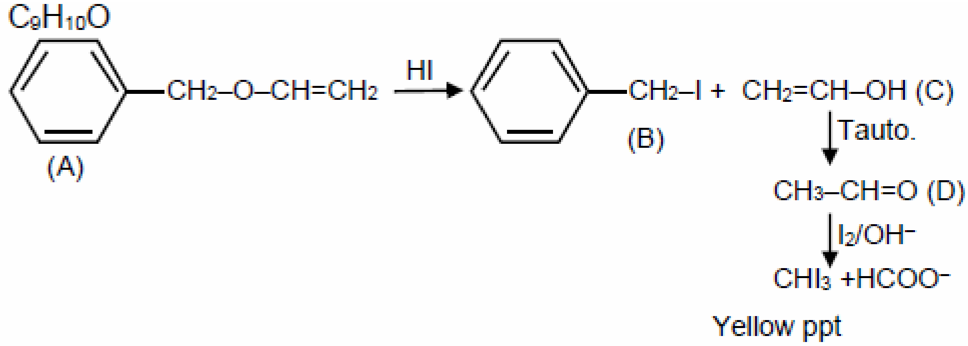
Sol. In toilet cleaning liquid the main constituent is HCl, which can cause skin burn so it should be treated with $NaHCO_3$ which can easily consume the acid.

41. B

Sol. (a) When gas is adsorbed on metal surface. ΔH become less negative with progress of reaction.
(b) Gas with greater value of critical temperature (TC) absorbed more. As $TC(NH_3) > TC(N_2)$
So NH_3 absorbed more than N_2.

42. D

Sol. $C_9H_{10}O$



43. D

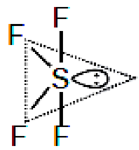
Sol. (1) $\overset{+2}{[Ni(NH_3)_2Cl_2]} \Rightarrow Ni^{2+} \Rightarrow 3d^8 4s^0$.
$\Rightarrow$ sp^3 hybridisition.
$\Rightarrow$ tetrahedral.
so $[Ni(NH_3)_2Cl_2]$ do not show isomerism.
(2) $[Ni(NH_3)_4(H_2O)_2]^{2+}$, show geometrical isomerism.
(3) $[Ni(en)_3]^{2+}$, show optical isomerism.
(4) $[Pt(NH_3)_2Cl_2]$, show geometrical isomerism.

44. B

Sol. When mango kept in concentrate salt solution then solvent (water) flow from mango to concentrate solution that's why mango shrinks this is called. "Osmosis"

45. B

Sol. $SF_4 \Rightarrow$ Steric No = 5 so hybridisation is sp^3d.



Geometry is trigonal bipyramidal but shape is "See Saw".

46. 5

Sol. Mass ratio of C : H is 4 : 1 $\Rightarrow$ 12 : 3

& C : O is 3 : 4 $\Rightarrow$ 12 : 16

		mass	mole	mole ratio
so	C	12	1	1
	H	3	3	3
	O	16	1	1

Empirical formula $\Rightarrow CH_3O$

as compound is saturated acyclic so molecular formula is $C_2H_6O_2$.

$$\underset{\text{mole}}{C_2H_6O_2} + \underset{\text{2 mole}}{\frac{5}{2}O_2(g)} \longrightarrow \underset{\text{5 mole}}{2CO_2(g)} + 3H_2O(g)$$

so required moles of O_2 is $\Rightarrow$ 5

47. 144

Sol. $E^\circ_{cell} = E^\circ_{Cu^+/Cu} - E^\circ_{Cu^{2+}/Cu^{+1}}$

$= 0.52 - 0.16$

$= 0.36$ V

$$\boxed{E^\circ_{cell} = \frac{RT}{nF}\ln K_{eq}}$$

$0.36 = \frac{0.025}{1}\ln k$

$\ln k = 14.4$

$= 144 \times 10^{-1}$

Ans. 144

48. **–326400**

Sol. $C_2H_5OH(\ell) + O_2(g) \longrightarrow 2O_2(g) + 3H_2O(\ell)$; $\Delta H_C = -327$ Kcal

$\Delta H_C = \Delta U_C + \Delta n_g RT$

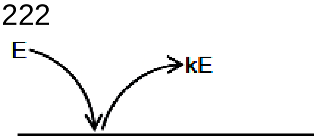
$-327 \times 10^3 = \Delta U_C + 1 \times 2 \times 300$

$\Delta U_C = -326400$ cal

So heat evolved as constant volume is –326400 cal

49. 222

Sol.



Metal (Work function = E_0)

$E = E_0 + (kE)_{max}$

$\frac{hC}{\lambda} = 4.41 \times 10^{-19} + kE$

$\frac{6.63 \times 10^{-34} \times 3 \times 108}{300 \times 10^{-9}} = 4.41 \times 10^{-19} + kE$

So, $(kE)_{max} = 6.63 \times 10^{-19} - 4.41 \times 10^{-19}$

$= 2.22 \times 10^{-19}$

$= 222 \times 10^{-21}$ J

50. 19

Sol.

	Compound	**Oxidation state of transition element.**
(i)	$K_2Cr_2O_7$	x = +6
(ii)	$KMnO_4$	y = +7
(iii)	K_2FeO_4	z = +6

so (x + y + z) = 19

PART–C (MATHEMATICS)

51. A

Sol. $A^TA = I$

$\Rightarrow a^2 + b^2 + c^2 = 1$ and $ab + bc + ca = 0$

Now, $(a+b+c)^2 = 1$

$\Rightarrow a+b+c = \pm 1$

So, $a^3 + b^3 + c^3 - 3abc$

$= (a+b+c)\left(a^2 + b^2 + c^2 - ab - bc - ca\right)$

$= (\pm 1)(1-0) = \pm 1$

$\Rightarrow 3abc = 2 \pm 1 = 3, 1$

$\Rightarrow abc = 1, \frac{1}{3}$

52. B

Sol. $\lambda = -\left(\sin^4\theta + \cos^4\theta\right)$

$\lambda = -\left(\left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta\right)$

$\lambda = \frac{\sin^2 2\theta}{2} - 1$

$\frac{\sin^2 2\theta}{2} \in \left[0, \frac{1}{2}\right]$

$\lambda \in \left[-1, -\frac{1}{2}\right]$

53. B

Sol. $f(x) = a(x-3)(x-\alpha)$

$f(2) = a(\alpha - 2)$

$f(-1) = 4a(1+\alpha)$

$f(-1) + f(2) = 0 \Rightarrow a(\alpha - 2 + 4 + 4\alpha) = 0$

$a \neq 0 \Rightarrow 5\alpha = -2$

$\alpha = -\frac{2}{5} = -0.4$

$\alpha \in (-1, 0)$

54. D

Sol. Here normal is $\perp^r$ to both the lines. So normal vector to the plane is

$\vec{n} = \left(\hat{i} - 2\hat{j} + 2\hat{k}\right) \times \left(2\hat{i} + 3\hat{j} - \hat{k}\right)$

$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = \hat{i}\left(2-6\right) - \hat{j}\left(-1-4\right) + \hat{k}\left(3+4\right)$

$\vec{n} = -4\hat{i} + 5\hat{j} + 7\hat{k}$

Now equation of plane passing through (3, 1, 1) is

$\Rightarrow -4\left(x-3\right) + 5\left(y-1\right) + 7\left(z-1\right) = 0$

$\Rightarrow -4x + 12 + 5y - 5 + 7z - 7 = 0$

$\Rightarrow -4x + 5y + 7z = 0$

The plane also passes through $\left(\alpha, -3, 5\right)$.

$-4\alpha + 5 \times \left(-3\right) + 7 \times \left(5\right) = 0$

$\Rightarrow -4\alpha - 15 + 35 = 0$

$\Rightarrow \alpha = 5$

55. D

Sol. $a_1 + a_2 + a_3 + ... + a_{11} = 0$

$\Rightarrow \left(a_1 + a_{11}\right) \times \dfrac{11}{2} = 0$

$\Rightarrow a_1 + a_{11} = 0$

$\Rightarrow a_1 + a_1 + 10d = 0$, where d is the common difference

$\Rightarrow a_1 = -5d$

$a_1 + a_3 + a_5 + ... + a_{23} = \left(a_1 + a_{23}\right) \times \dfrac{12}{2} = \left(a_1 + a_1 + 22d\right) \times 6$

$= \left(2a_1 + 22\left(-\dfrac{a_1}{5}\right)\right) \times 6 = -\dfrac{72}{5} a_1 \Rightarrow K = -\dfrac{72}{5}$

56. C

Sol. $\lim\limits_{x \to 0}\left(\tan\left(\dfrac{\pi}{4} + x\right)\right)^{1/x} = e^{\lim\limits_{x \to 0}\frac{1}{x}\left(\tan\left(\frac{\pi}{4}+x\right)-1\right)}$

$= e^{\lim\limits_{x \to 0}\left(\frac{1+\tan x - 1 + \tan x}{x(1-\tan x)}\right)} = e^{\lim\limits_{x \to 0}\frac{2\tan x}{x(1-\tan x)}} = e^2$

57. A

Sol. $x^2 - y^2 \sec^2\theta = 10$

$\Rightarrow \dfrac{x^2}{10} - \dfrac{y^2}{10\cos^2\theta} = 1$

Hence, eccentricity of hyperbola $\left(e_H\right)=\sqrt{1+\frac{10\cos^2\theta}{10}}$ ………….(1)

$\left\{e=\sqrt{1+\frac{b^2}{a^2}}\right\}$

Now, equation of ellipse is $x^2\sec^2\theta+y^2=5$.

$\Rightarrow \frac{x^2}{5\cos^2\theta}+\frac{y^2}{5}=1$

Hence, eccentricity of ellipse $\left(e_E\right)=\sqrt{1-\frac{5\cos^2\theta}{5}}$ $\left\{e=\sqrt{1-\frac{a^2}{b^2}}\right\}$

$=\sqrt{1-\cos^2\theta}=\left|\sin\theta\right|=\sin\theta$ …………..(2)

$\left\{\because \theta\in\left(0,\frac{\pi}{2}\right)\right\}$

Given $e_H=\sqrt{5}e_E$

Hence, $1+\cos^2\theta=5\sin^2\theta$

$1+\cos^2\theta=5\left(1-\cos^2\theta\right)$

$\cos^2\theta=\frac{2}{3}$ ………….(3)

Now, length of latus rectum of ellipse $=\frac{2a^2}{b}=\frac{10\cos^2\theta}{\sqrt{5}}=\frac{20}{3\sqrt{5}}=\frac{4\sqrt{5}}{3}$

58. D

Sol. $y\geq x^2 \Rightarrow$ upper region of $y=x^2$

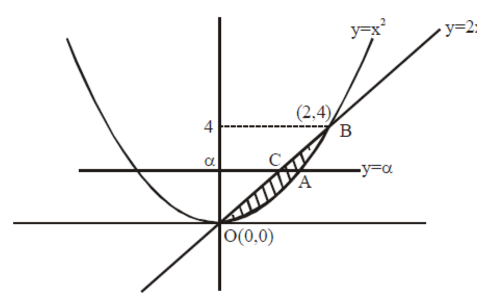
$y\leq 2x \Rightarrow$ lower region of $y=2x$

According to question, area of OABCO = 2 (area of OACO)

$\Rightarrow \int_0^4\left(\sqrt{y}-\frac{y}{2}\right)dy=2\int_0^{\alpha}\left(\sqrt{y}-\frac{y}{2}\right)dy$

$\Rightarrow \frac{4}{3}=2\left(\frac{2}{3}\alpha^{3/2}-\frac{1}{4}\alpha^2\right)$

$\Rightarrow 3\alpha^2-8\alpha^{3/2}+8=0$



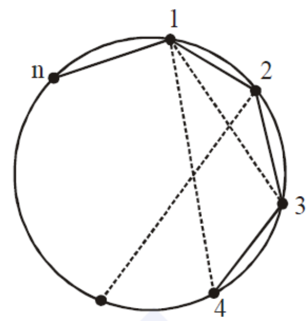
59. A

Sol. Number of blue lines = number of sides = n
Number of red lines = number of diagonals

$$= {}^nC_2 - n$$

$${}^nC_2 - n = 99n \Rightarrow \frac{n(n-1)}{2} = 100n$$

$$\Rightarrow n = 201$$



60. A

Sol. $3 + 2\sqrt{-54} = 3 + 6\sqrt{6}\ i = \left(3 + \sqrt{6}\ i\right)^2$

$$3 - 2\sqrt{54} = \left(3 - \sqrt{6}\ i\right)^2$$

$$\left(3 + 2\sqrt{-54}\right)^{1/2} - \left(3 - 2\sqrt{-54}\right)^{1/2} = \pm\left(3 + \sqrt{6}\ i\right) \pm \left(3 - \sqrt{6}\ i\right)$$

$$= 6, -6,\ 2\sqrt{6}\ i, -2\sqrt{6}\ i$$

61. C

Sol. $f'(x) = \dfrac{\dfrac{x}{1+x} - \ell n(1+x)}{x^2}$

$$= \frac{x - (1+x)\ell n(1+x)}{x^2(1+x)}$$

Suppose $h(x) = x - (1+x)\ell n(1+x)$

$$\Rightarrow h'(x) = 1 - \ell n(1+x) - 1 = -\ell n(1+x)$$

$$h'(x) > 0\ \forall x \in (-1, 0)$$

$$h'(x) < 0\ \forall x \in (0, \infty)$$

$$h(0) = 0 \Rightarrow h'(x) \le 0\ \forall x \in (-1, \infty)$$

$$\Rightarrow f'(x) \le 0\ \forall\ x \in (-1, \infty)$$

$\Rightarrow f(x)$ is a decreasing function for all $x \in (-1, \infty)$.

62. C

Sol. $2x^2dy = \left(2xy + y^2\right)dx$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy + y^2}{2x^2} \quad \text{(Homogeneous D.E.)}$$

$$\left\{\begin{array}{l} \text{Let } y = xt \\ \Rightarrow \dfrac{dy}{dx} = t + x\dfrac{dt}{dx} \end{array}\right\}$$

$$\Rightarrow t + x\frac{dt}{dx} = \frac{2x^2t + x^2t^2}{2x^2}$$

$$\Rightarrow t + x\frac{dt}{dx} = t + \frac{t^2}{2}$$

$$\Rightarrow 2\int\frac{dt}{t^2} = \int\frac{dx}{x}$$

$$\Rightarrow 2\left(-\frac{1}{t}\right) = \ell n x + C$$

$$\Rightarrow -\frac{2x}{y} = \ell n x + C$$

Put x = 1 and y = 2 to get C = –1.

$$\Rightarrow \frac{-2x}{y} = \ell n x - 1$$

$$\Rightarrow y = \frac{2x}{1 - \ell n x}$$

$$\Rightarrow f(x) = \frac{2x}{1 - \log_e x}$$

So, $f\left(\frac{1}{2}\right) = \frac{1}{1 + \log_e 2}$

63. D

Sol. Given $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$.

Here $|P| = 0$ and also given $PX = 0$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\Rightarrow \left.\begin{array}{c} x + 2y + z = 0 \\ -2x + 3y - 4z = 0 \\ x + 9y - z = 0 \end{array}\right\}$$

$D = 0$, so the system has infinitely many solutions.

By solving these equations, we get $x = -\frac{11\lambda}{2}$, $y = \lambda$, $z = \frac{7\lambda}{2}$.

Also, given $x^2 + y^2 + z^2 = 1$.

$$\Rightarrow \left(-\frac{11\lambda}{2}\right)^2 + (\lambda)^2 + \left(\frac{7\lambda}{2}\right)^2 = 1$$

$$\Rightarrow \lambda = \pm \frac{1}{\sqrt{\frac{121}{4}+1+\frac{49}{4}}}$$

There are 2 values of λ.
So, there are 2 solution sets for (x, y, z).

64. B

Sol. $y = (1+x)^{2y} + \cos^2\left(\sin^{-1} x\right)$

Put $x = 0$.

$y = (1+0)^{2y} + \cos^2\left(\sin^{-1} 0\right) = 2$

So, we have to find the normal at (0, 2).

Now, $y = e^{2y\ln(1+x)} + \cos^2\left(\cos^{-1}\sqrt{1-x^2}\right)$

$y = e^{2y\ln(1+x)} + \left(\sqrt{1-x^2}\right)^2$

$y = e^{2y\ln(1+x)} + \left(1-x^2\right)$ ....(1)

Now differentiate w.r.t. x.

$y' = e^{2y\ln(1+x)}\left[\frac{2y}{1+x} + \ln(1+x).2y'\right] - 2x$

Put $x = 0$ and $y = 2$.

$y' = e^{4\ln 1}\left[4 + \ln 1.2y'\right] - 0$

$y' = 4 =$ slope of tangent to the curve

Hence, equation of normal at (0, 2) is $y - 2 = -\frac{1}{4}(x-0)$

$\Rightarrow x + 4y = 8$

65. A

Sol. $\sim p \wedge (p \vee q) \rightarrow q$

$\equiv (\sim p \wedge p) \vee (\sim p \wedge q) \rightarrow q$

$\equiv C \vee (\sim p \wedge q) \rightarrow q$

$\equiv (\sim p \wedge q) \rightarrow q$

$\equiv \sim(\sim p \wedge q) \vee q$

$\equiv (p \vee \sim q) \vee q$

$\equiv (p \vee q) \vee (\sim q \vee q)$

$\equiv (p \vee q) \vee T$

So, $\sim p \wedge (p \vee q) \rightarrow q$ is a tautology.

66. D

Sol. $S = \left(x + x^2 + x^3 + x^4 + \ldots 9\,\text{terms}\right) + \left(ka + ka + ka + ka + \ldots 9\,\text{terms}\right)$

$+\left(0 + 2a + 4a + 6a + \ldots 9\ \text{terms}\right)$

$\Rightarrow S = x\left(\dfrac{x^9 - 1}{x - 1}\right) + 9ka + 72a$

$\Rightarrow S = \dfrac{\left(x^{10} - x\right) + (9k + 72)a(x - 1)}{x - 1}$

Comparing with the given sum, we get $9k + 72 = 45$.

$\Rightarrow k = -3$

67. C

Sol. Given E_1, E_2, E_3 are pairwise independent events.

So $P(E_1 \cap E_2) = P(E_1).P(E_2)$

and $P(E_2 \cap E_3) = P(E_2).P(E_3)$

and $P(E_3 \cap E_1) = P(E_3).P(E_1)$

and $P(E_1 \cap E_2 \cap E_3) = 0$.

Now $P\left(\dfrac{\bar{E}_2 \cap \bar{E}_3}{E_1}\right) = \dfrac{P\left(E_1 \cap \left(\bar{E}_2 \cap \bar{E}_3\right)\right)}{P(E_1)}$

$= \dfrac{P(E_1) - \left[P(E_1 \cap E_2) + P(E_1 \cap E_3) - P(E_1 \cap E_2 \cap E_3)\right]}{P(E_1)}$

$= \dfrac{P(E_1) - P(E_1).P(E_2) - P(E_1)P(E_3) + 0}{P(E_1)}$

$= 1 - P(E_2) - P(E_3)$

$= \left[1 - P(E_3)\right] - P(E_2)$

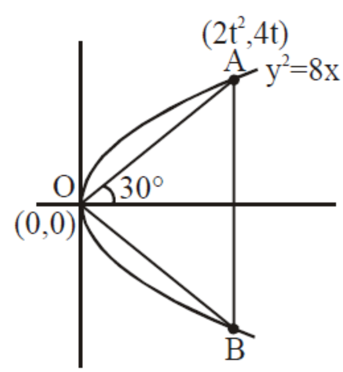
$= P\left(E_3^C\right) - P(E_2)$

68. A

Sol. $\tan 30^o = \dfrac{4t}{2t^2} = \dfrac{2}{t} \Rightarrow t = 2\sqrt{3}$

$AB = 8t = 16\sqrt{3}$

Area $= \dfrac{\sqrt{3}}{4}\left(16\sqrt{3}\right)^2 = 192\sqrt{3}$



69. C

Sol. $f(x+y)=f(x)+f(y)$

$\Rightarrow f(n)=nf(1)=2n$

$$g(n)=\sum_{k=1}^{n-1}(2k)=2\left(\frac{(n-1)n}{2}\right)=n(n-1)$$

$g(n)=20 \Rightarrow n(n-1)=20$

$n=5$

70. C

Sol. Given the points (1, 2) and $(\sin\theta, \cos\theta)$ lie on the same side of the line $x+y-1=0$.

$\Rightarrow (1+2-1)(\sin\theta+\cos\theta-1)>0$

$\Rightarrow \sin\theta+\cos\theta>1$

$$\Rightarrow \sin\left(\theta+\frac{\pi}{4}\right)>\frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\pi}{4}<\theta+\frac{\pi}{4}<\frac{3\pi}{4}$$

$$\Rightarrow 0<\theta<\frac{\pi}{2}$$

71. 91

Sol. Put $\cos\alpha=\frac{3}{5}, \sin\alpha=\frac{4}{5}, 0<\alpha<\frac{\pi}{2}$

Now $\frac{3}{5}\cos kx-\frac{4}{5}\sin kx=\cos\alpha.\cos kx-\sin\alpha.\sin kx=\cos(\alpha+kx)$

As we have to find derivate at x = 0, we have $\cos^{-1}(\cos(\alpha+kx))=\alpha+kx$

$$\Rightarrow y=\sum_{k=1}^{6}k(\alpha+kx)=\sum_{k=1}^{6}\left(k\alpha+k^2x\right)$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=0}=\frac{(6)(7)(13)}{6}=91$$

72. 3.00

Sol. Let a be the first term and d be the common difference of the given A.P., where d > 0.

$$\overline{X}=a+\frac{0+d+2d+...+10d}{11}=a+5d$$

$$\text{Variance}=\frac{\sum\left(\overline{X}-x_i\right)^2}{11}$$

$\Rightarrow 90\times 11=2\left(25d^2+16d^2+9d^2+4d^2+d^2\right)=110d^2$

$\Rightarrow d=\pm 3 \Rightarrow d=3$

73. 1.0

Sol. $3 < 3x < 6$

Take cases when $3 < 3x < 4,\ 4 < 3x < 5,\ 5 < 3x < 6$.

Now $\int_1^2 \left|2x - [3x]\right| dx$

$= \int_1^{4/3} (3-2x)dx + \int_{4/3}^{5/3} (4-2x)dx + \int_{5/3}^{2} (5-2x)dx$

$= \frac{2}{9} + \frac{3}{9} + \frac{4}{9} = 1$

74. 118

Sol. ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2:5:12$

Now $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{5}$

$\Rightarrow 7r = 2n + 2$ ……….(1)

$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{5}{12}$

$\Rightarrow 17r = 5n - 12$ ………….(2)

On solving (1) and (2), we get $n = 118$.

75. 0.8

Sol. Using section formula, we get

$\overline{OP} = \frac{2\lambda+1}{\lambda+1}\hat{i} + \frac{\lambda+1}{\lambda+1}\hat{j} + \frac{3\lambda+1}{\lambda+1}\hat{k}$

Now $\overline{OB}.\overline{OP} = \frac{4\lambda+2+\lambda+1+9\lambda+3}{\lambda+1}$

$= \frac{14\lambda+6}{\lambda+1}$

$$\overline{OA} \times \overline{OP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ \frac{2\lambda+1}{\lambda+1} & 1 & \frac{3\lambda+1}{\lambda+1} \end{vmatrix} = \frac{2\lambda}{\lambda+1}\hat{i} + \frac{-\lambda}{\lambda+1}\hat{j} + \frac{-\lambda}{\lambda+1}\hat{k}$$

$$\left|\overrightarrow{OA} \times \overrightarrow{OP}\right|^2 = \frac{4\lambda^2+\lambda^2+\lambda^2}{(\lambda+1)^2} = \frac{6\lambda^2}{(\lambda+1)^2}$$

$$\Rightarrow \frac{14\lambda+6}{\lambda+1} - 3 \times \frac{6\lambda^2}{(\lambda+1)^2} = 6$$

$$\Rightarrow 10\lambda^2 - 8\lambda = 0 \Rightarrow \lambda = 0, \frac{8}{10} \Rightarrow \lambda = 0.8$$