FIITJEE

Solutions to JEE(Main)-2020

Test Date: 5th September 2020 (First Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

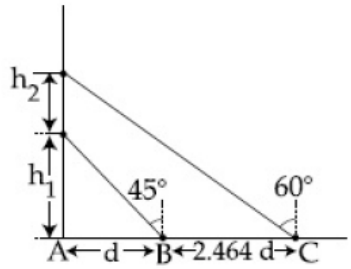
1. The test is of **3 hours** duration.
2. This **Test Paper** consists of **75** questions. The maximum marks are **300**.
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 25 questions in each part of equal weightage out of which 20 questions are MCQs and 5 questions are numerical value based. Each question is allotted **4 (four)** marks for correct response.
4. **(Q. No. 01 – 20, 26 – 45, 51 – 70)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
5. **(Q. No. 21 – 25, 46 – 50, 71 – 75)** contains 15 Numerical based questions with answer as numerical value. Each question carries **+4 marks** for correct answer. There is no negative marking.
6. Candidates will be awarded marks as stated above in **instruction No.3** for correct response of each question. One mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer box.
7. There is only one correct response for each question. Marked up more than one response in any question will be treated as wrong response and marked up for wrong response will be deducted accordingly as per **instruction 6** above.

PART –A (PHYSICS)

1. A hollow spherical shell at outer radius R floats just submerged under the water surface. The inner radius of the shell is r. If the specific gravity of the shell material is $\frac{27}{8}$ w.r.t water, the value of r is.

(A) $\frac{2}{3}R$ (B) $\frac{4}{9}R$ (C) $\frac{1}{3}R$ (D) $\frac{8}{9}R$

2. A balloon is moving up in air vertically above a point A on the ground. When it is at a height h_1, a girl standing at a distance d (point B) from A (see figure) sees it at the angle 45^0 with respect to the vertical. When the balloon climbs up a future height h_2, it is seen at the angle 60^0 with respect to the vertical if the girl moves further by a distance 2.464 d (point C). Then the height h_2 is (given tan 30^0 = 0.5774):

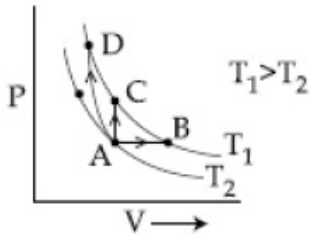


(A) 0.464 d (B) 0.732 d
(C) 1.464 d (D) d

3. A physical quantity z depends on four observables a, c, c and d, as z = $\frac{a^2b^{\frac{2}{3}}}{\sqrt{c}\,d^3}$. The percentages of error in the measurement of a, b, c and d are 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in z is:

(A) 13.5% (B) 16.5%
(C) 12.25% (D) 14.5%

4. Three different processes that can occur in an ideal monoatomic gas are shown in the P vs V diagram. The paths are labeled as A $\rightarrow$ B, A $\rightarrow$ C and A $\rightarrow$ D. The change in internal energies during these process are taken as E_{AB}, E_{AC} and E_{AD} and the work done by W_{AB}, W_{AC} and W_{AD}. The correct relation between these parameters are:



(A) $E_{AB} < E_{AC} < E_{AD}, W_{AB} > 0, W_{AC} > W_{AD}$

(B) $E_{AB} = E_{AC} = E_{AD}, W_{AB} > 0, W_{AC} = 0, W_{AD} > 0$

(C) $E_{AB} > E_{AC} > E_{AD}, W_{AB} < W_{AC} < W_{AD}$

(D) $E_{AB} = E_{AC} < E_{AD}, W_{AB} > 0, W_{AC} = 0, W_{AD} < 0$

5. A wheel is rotating freely with an angular speed ω on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of moment of inertia 3I initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is:

(A) $\frac{3}{4}$ (B) $\frac{5}{6}$ (C) $\frac{1}{4}$ (D) 0

6. In a resonance tube experiment when the tube is filled with water up to a height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m/s, the tuning fork frequency is:
(A) 1100 Hz (B) 2200 Hz
(C) 3300 Hz (D) 550 Hz

7. An electron is constrained to move along the y – axis with a speed of 0.1 c (c is the speed of light) in the presence of electromagnetic wave, whose electric field is $\vec{E} = 30\hat{j}\sin\left(1.5\times10^{7}t - 5\times10^{-2}x\right)V/m.$ The maximum magnetic force experienced by the electron will be: (given $c = 3\times10^{8} ms^{-1}$ and electron charge = $1.6\times10^{-19}C$)
(A) $3.2\times10^{-18}N$ (B) $1.5\times10^{-19}N$
(C) $2.4\times10^{-18}N$ (D) $4.8\times10^{-19}N$

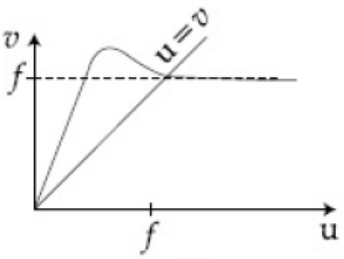
8. With increasing biasing voltage of a photodiode, the photocurrent magnitude:
(A) remains constant
(B) increases initially and after attaining
(C) increases linearly
(D) increases initially and saturates finally

9. A bullet of mass 5 g, traveling with a speed of 210 m/s, strikes a fixed wooden target. One half of its kinetic energy is converted into heat in the bullet while the other half is converted into heat in the wood. The rise of temperature of the bullet if the specific heat of its material is $0.030\ cal/\left(g-^{0}C\right)\left(1cal = 4.2\times10^{7}\ ergs\right)$ close to:
(A) 83.3^{0} C (B) 38.4^{0} C
(C) 87.5^{0} C (D) 119.2^{0} C

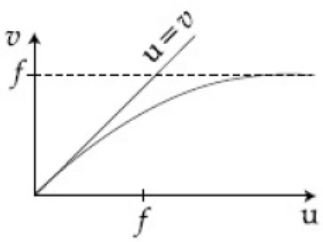
10. The value of the acceleration due to gravity is g_1 at a height $h = \frac{R}{2}$ (R= radius of the earth) from the surface of the earth. It is again equal to g_1 at a depth d below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$ equals:
(A) $\frac{7}{9}$ (B) $\frac{1}{3}$
(C) $\frac{5}{9}$ (D) $\frac{4}{9}$

11. For a concave lens of focal length f, the relation between object and image distances u and is υ, respectively, from its pole can best be represented by (u - υ is the reference line):

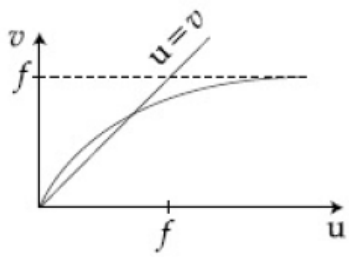
(A)



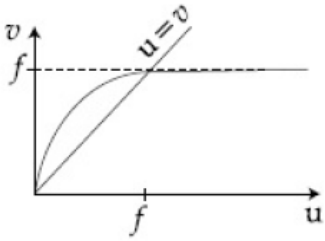
(B)



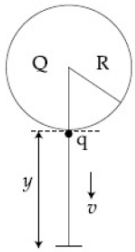
(C)



(D)



12. A solid sphere of radius R carries a charge Q + q distributed uniformly over its volume. A very small point like piece of it of mass m gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge q. If it acquires a speed υ when it has fallen through a vertical height h (see figure), then: (assume the remaining portion to be spherical)



(A) $v^2 = y\left[\frac{qQ}{4\pi \in_0 R^2 ym} + g\right]$

(B) $v^2 = 2y\left[\frac{qQ}{4\pi \in_0 R(R+y)m} + g\right]$

(C) $v^2 = y\left[\frac{qQ}{4\pi \in_0 R(R+y)m} + g\right]$

(D) $v^2 = 2y\left[\frac{QqR}{4\pi \in_0 (R+y)^3 m} + g\right]$

13. A galvanometer of resistance G is converted into a voltmeter of range 0 – IV by connecting a resistance R_1 in series with it. The additional resistance that should be connected in series with R_1 to increase the range of the voltmeter to 0 – 2V will be:

(A) $R_1 + G$

(B) R_1

(C) G

(D) $R_1 - G$

14. Number of molecules in a volume of 4 cm^3 of a perfect monoatomic gas at some temperature T and at a pressure of 2 cm of mercury is close to? (Given, mean kinetic energy of a molecule (at T) is 4 x 10^{-14} erg, g = 980 cm / s^2, density of mercury = 13.6 g / cm^3)

(A) 5.8×10^{18}

(B) 4.0×10^{16}

(C) 5.8×10^{16}

(D) 4.0×10^{18}

15. A square loop of side 2a, and carrying current I, is kept in XZ plane with its centre at origin. A long wire carrying the same current I is placed parallel to the z – axis and passing through the point (0, b, 0), (b > > a). The magnitude of the torque on the loop about z – axis is given by:

(A) $\frac{2\mu_0 I^2 a^3}{\pi b^2}$ (B) $\frac{\mu_0 I^2 a^2}{2\pi b}$ (C) $\frac{\mu_0 I^2 a^3}{2\pi b^2}$ (D) $\frac{2\mu_0 I^2 a^2}{\pi b}$

16. Assume that the displacement (s) of air is proportional to the pressure difference (Λp) created by a sound wave. Displacement (s) further depends on the speed of sound (v), density of air (ρ) and the frequency (f). If $\Lambda p \sim 10 Pa, \upsilon \sim 300$ m/ s, $\rho \sim \pi 1$ lg / m^3 and $f \sim 1000$ Hz, then s will be of the order of (take the multiplicative constant to be 1)

(A) $\frac{1}{10}$mm (B) 10 mm

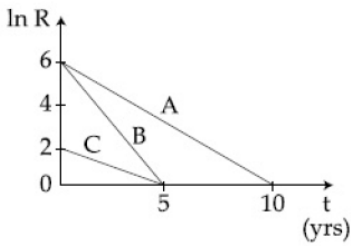
(C) $\frac{3}{100}$mm (D) 1 mm

17. A helicopter rises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is at a height h. The time taken by the packet to reach the ground is close to [g is the acceleration due to gravity]:

(A) $t = 1.8\sqrt{\frac{h}{g}}$ (B) $t = \sqrt{\frac{2h}{3g}}$

(C) $t = 3.4\sqrt{\left(\frac{h}{g}\right)}$ (D) $t = \frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$

18. Activities of three radioactive substances A, B and C are represented by the curves A, B and C in the figure. Then their half lives $T_{\frac{1}{2}}(A) : T_{\frac{1}{2}}(B) : T_{\frac{1}{2}}(C)$ are in the radio:



(A) 3:2:1 (B) 2:1:3
(C) 4:3:1 (D) 2:1:1

19. An electrical power line, having a total resistance of 2Ω, delivers 1 kW at 220 V. The efficiency of the transmission line is approximately:

(A) 96% (B) 72%
(C) 91% (D) 85%

20. Two capacitors of capacitances C and 2C are charged to potential differences V and 2V, respectively. These are then connected in parallel in such a manner that the positive terminal of one is connected to the negative terminal of the other. The final energy of this configuration is:

(A) $\frac{3}{2}CV^2$ (B) $\frac{9}{2}CV^2$

(C) $\frac{25}{6}CV^2$ (D) zero

21. A force $\vec{F} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ N acts at a point $\left(4\hat{i} + 3\hat{j} - \hat{k}\right)$m. Then the magnitude of torque about the point $\left(\hat{i} + 2\hat{j} + \hat{k}\right)$m will be $\sqrt{x}$ N-m. The value of x is _____.

22. A particle of mass 200 MeV/c^2 collides with a hydrogen atom at rest. Soon after the collision the particle comes to rest, and the atom recoils and goes to its first excited state. The initial kinetic energy of the particle (in eV) is $\frac{N}{4}$. The value of N is: (Given the mass of the hydrogen atom to be 1 GeV / c^2) _____.

23. Two concentric circular coils, C_1 and C_2 are placed in the XY plane. C_1 has 500 turns, and a radius of 1 cm. C_2 has 200 turns and radius of 20 cm. C_2 carries a time dependent current $I(t) = \left(5t^2 - 2t + 3\right)A$ where t is in s. The emf induced in C_1 (in mV), at the instant t = 1 s is $\frac{4}{x}$. The value of x is _____.

24. A beam of electrons of energy E scatters from a target having atomic spacing of $1\overset{0}{A}$. The first maximum intensity occurs at $\theta = 60^0$. Then E (in eV) is _____.
(Planck constant h = electron mass m = 9.1×10^{-31}kg)

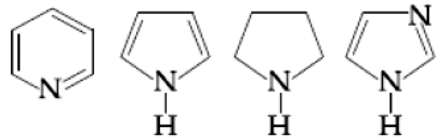
25. A compound microscope consists of an objective lens of focal length 1 cm and an eye piece of focal length 5 cm with a separation of 10 cm. The distance between an object and the objective lens, at which the strain on the eye is minimum is $\frac{n}{40}$ cm. The value of n is _____.

PART –B (CHEMISTRY)

26. The structure of PCl_5 in the solid state is:
(A) square planar $[PCl_4]^+$ and octahedral $[PCl_6]^-$
(B) trigonal biphyramidal
(C) tetrahedral $[PCl_4]^+$ and octahedral $[PCl_6]$
(D) square pyramidal

27. In the sixty period, the orbitals that are filled are:
(A) 6s, 5f, 6d, 6p
(B) 6s, 6p, 6d, 6f
(C) 6s, 4f, 5d, 6p
(D) 6s, 5d, 5f, 6p

28. The increasing order of basicity of the following compounds is:

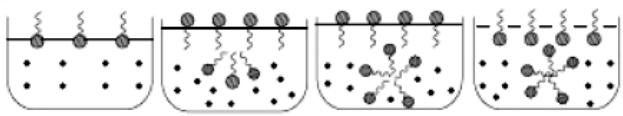


(A) (B) (C) (D)

(A) (A) < (B) < (C) < (D)
(B) (D) > (A) > (B) < (D)
(C) (B) < (A) < (C) < (D)
(D) (B) < (A) < (D) < (C)

29. Identify the correct molecular picture showing what happens at the critical micellar concentration (CMC) of an aqueous solution of a surfactant

(⊘ polar head; ∿ non-polar tail ; . water).



(A) (B) (C) (D)

(A) (B)
(B) (A)
(C) (C)
(D) (D)

30. Which of the following is not an essential amino acid?
(A) Valine
(B) Tyrosine
(C) Leucine
(D) Lysine

31. A flask contains a mixture of compounds A and B. Both compounds decompose by first – order kinetics. The half lives for A and B are 300 s and 180 s, respectively. If the concentrations of A and B are equal initially, the line required for the concentration of A to be four times that of B (in s) is : (Use ln 2 = 0.693)
(A) 120
(B) 300
(C) 900
(D) 180

32. The equation that represents the water – gas shift reactions is:

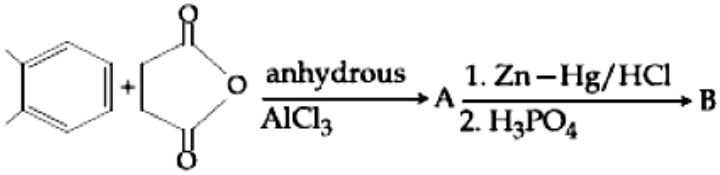
(A) $CO(g) + H_2O(g) \xrightarrow[\text{Catalyst}]{673\ K} CO_2(g) + H_2(g)$

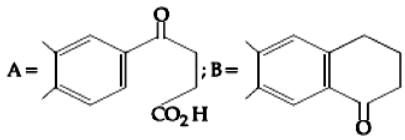
(B) $C(s) + H_2O(g) \xrightarrow{1270\ K} CO(g) + H_2(g)$

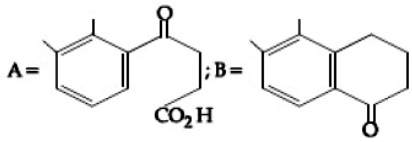
(C) $2C(s) + O_2(g) + 4N_2(g) \xrightarrow{1273\ K} 2CO(g) + 4N_2(g)$

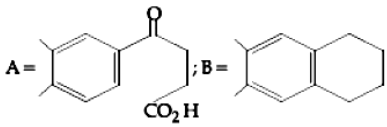
(D) $CH_4(g) + H_2O(g) \xrightarrow[Ni]{1270\ K} CO(g) + 3H_2(g)$

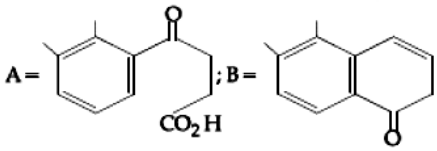
33. In the following reaction sequence the major products A and B are:



(A) 

(B) 

(C) 

(D) 

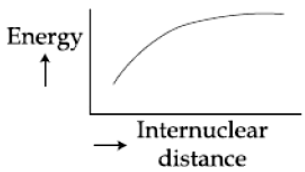
34. A diatomic molecule X_2 has a body centered cubic (bcc). The density of the molecule is 6.17 g cm^{-3}. The number of molecules present in 200 g of X_2 is:
(Avogadro constant (N_A) = 6 x 10^{23} mol^{-1})

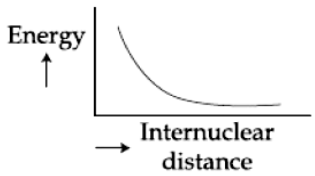
(A) 2 N_A (B) 8 N_A
(C) 14 N_A (D) 4 N_A

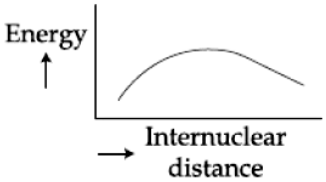
35. If a person is suffering from the deficiency of nor – adrenaline, what kind of drug can be suggested?

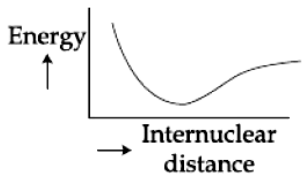
(A) Antidepressant (B) Antishistamine
(C) Analgesic (D) Anti – inflammatory

36. The potential energy curve for the H_2 molecule as a function of internuclear distance is :

(A) 

(B) 

(C) 

(D) 

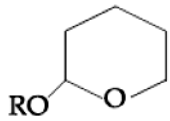
37. Consider the following reaction:

$N_2O_4 \rightleftharpoons 2NO_2(g)\ ;\ \Delta H^o = +58\ kJ$

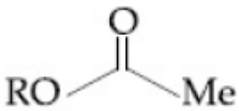
For each of the following cases (a, b) the direction in which the equilibrium shifts is:
(a) Temperature is decreased
(b) Pressure is increased by adding N_2 at constant T
(A) (a) towards reactant, (b) towards product
(B) (a) towards reactant, (b) no change
(C) (a) towards product, (b) no change
(D) (a) towards reactant

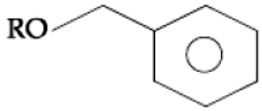
38. The most appropriate reagent for conversion of C_2H_5CN into $CH_3CH_2CH_2NH_2$ is:
(A) CaH_2 (B) $Na(CN)BH_3$
(C) $LiAlH_4$ (D) $NaBH_4$

39. Which of the following derivates of alcohol is unstable is an aqueous base?

(A) 

(B) RO – CMe_3

(C) 

(D) 

40. The condition that indicates a polluted environment is:
(A) 0.03% of CO_2 in the atmosphere (B) pH of rain water to be 5.6
(C) eutrophication (D) BOD value of 5 ppm

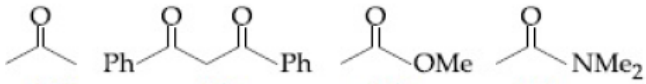
41. The values of the crystal field stabilization energies for a high spin d^6 metal ion in octahedral and tetrahedral fields, respectively, are:

(A) $-0.4\ \Delta_o$ and $-0.27\ \Delta_t$ (B) $-1.6\ \Delta_o$ and $-0.4\ \Delta_t$

(C) $-0.4\ \Delta_o$ and $-0.6\ \Delta_t$ (D) $-2.4\ \Delta_o$ and $-0.6\ \Delta_t$

42. The difference between the radii of 3rd and 4th orbits of Li^{2+} in ΔR_1. The difference between the radii of 3rd and 4th orbits of He^+ is ΔR_2. Ratio $\Delta R_1 : \Delta R_2$ is:
(A) 8:3 (B) 2:3
(C) 3:8 (D) 3:2

43. The increasing order of the acidity of the α-hydrogen of the following compounds is:



(A) (B) (C) (D)

(A) (C) < (A) < (B) < (D) (B) (B) < (C) < (A) < (D)
(C) (D) < (C) < (A) < (B) (D) (A) < (C) < (D) < (B)

44. The correct electronic configuration and spin – only magnetic moment (BM) of Gd^{3+} (Z = 64), respectively, are:

(A) $[Xe]4f^7$ and 7.9 (B) $[Xe]4f^7$ and 8.9

(C) $[Xe]5f^7$ and 8.9 (D) $[Xe]5f^7$ and 7.9

45. An Ellingham diagram provides information about:
1. the condition of pH and potential under which a species is thermodynamically stable.
2. the standard Gibbs energies of formation of some metal oxides
3. the kinetics of the reduction process
4. standard electrode potentials of reduction reactions involved in the extraction of metals

46. The total number of coordination sites in ethylenediaminetetraacetate ($EDTA^{4-}$) is ____.

47. The minimum number of moles of O_2 required for complete combustion of 1 mole of propane and 2 moles of butane is ___

48. An oxidation – reduction in which 3 electrons are transferred has a ΔG^0 of 17.37 kJ mol^{-1} at 25^0 C. The value of E^0_{cell} (in V is ____ $\times 10^{-2}$)
(1 F = 96,500 C mol^{-1})

49. The number of chiral carbon(s) present in peptide, Ile-Arg-Pro, is ___

50. A soft drink was bottled with a partial pressure of CO_2 of 3 bar over the liquid at room temperature. The partial pressure of CO_2 over the solution approaches a value of 30 bar when 44 g of CO_2 is dissolved in 1 kg of water at room temperature. The approximate pH of the soft drink is ___ x 10^{-1}/
(First dissociation constant of $H_2CO_3 = 4.0 \times 10^{-7}; \log 2 = 0.3;$ density of the soft drink = 1 g mL^{-1})

PART–C (MATHEMATICS)

51. If the four complex numbers, $z, \bar{z}, -\bar{z} - 2\mathrm{Re}(\bar{z})$ and z – 2Re(z) represent the vertices of a square of side 4 units in the Argand plane, then $|z|$ is equal to:

(A) $2\sqrt{2}$ (B) 2

(C) $4\sqrt{2}$ (D) 4

52. The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to

(A) $(x \wedge \sim y) \vee (\sim x \wedge y)$ (B) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$

(C) $(x \wedge y) \wedge (\sim x \vee \sim y)$ (D) $(x \wedge y) \vee (\sim x \wedge \sim y)$

53. If $2^{10} + 2^9.3^1 + 2^8.3^2 + ... + 2.3^9 + 3^{10} = S - 2^{11}$, then S is equal to

(A) $\frac{3^{11}}{2} + 2^{10}$ (B) $3^{11} - 2^{12}$

(C) 2.3^{11} (D) 3^{11}

54. Let $\lambda \varepsilon$ R. The system of linear equations.

$2x_1 - 4x_2 + \lambda x_3 = 1$

$x_1 - 6x_2 + x_3 = 2$

$\lambda x_1 - 10x_2 + 4x_3 = 3$

Is inconsistent for:

(A) exactly one positive value of λ (B) exactly one negative value of λ

(C) every value of λ (D) exactly two values of λ

55. If $\int \left(e^{2x} + 2e^x - e^{-x} - 1\right) e^{\left(e^x + e^{-x}\right)} dx = g(x) e^{\left(e^x + e^{-x}\right)} + c$ where c is a constant of integration, then g(0) is equal to:

(A) e (B) 2

(C) e^2 (D) 1

56. If $3^{2\sin 2\alpha - 1}$, 14 and $3^{4 - 2\sin 2\alpha}$ are the first three terms of an AP for some α, then the sixth term of this AP is:

(A) 65 (B) 78

(C) 66 (D) 81

57. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be:

(A) 63 (B) 54

(C) 38 (D) 36

58. If the common tangent to the parabolas $y^2 = 4x$ and $x^2 = 4y$ also touches the circle, $x^2 + y^2 = c^2$, then c is equal to

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

59. The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is:

(A) $\frac{5}{9}$ (B) $\frac{25}{9}$

(C) $\frac{5}{27}$ (D) $\frac{25}{81}$

60. The value of $\int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} dx$ is:

(A) $\frac{3\pi}{2}$ (B) $\frac{\pi}{2}$

(C) π (D) $\frac{\pi}{4}$

61. If S is the sum of the first 10 terms of the series $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \tan^{-1}\left(\frac{1}{21}\right) + ...,$ then tan(S) is equal to:

(A) $\frac{5}{6}$ (B) $\frac{10}{11}$

(C) $-\frac{6}{5}$ (D) $\frac{5}{11}$

62. If (a, b, c) is the image of the point (1, 2, –3) in the line, $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then a + b+ c is equal to

(A) 3 (B) 1

(C) 2 (D) – 1

63. If y = y (x) is the solution of the differential equation $\frac{5+e^x}{2+y}.\frac{dy}{dx} + e^x = 0$ satisfying y(0) = 1, then a value of $y\left(\log_e 13\right)$ is:

(A) 0 (B) 1

(C) – 1 (D) 2

64. If α is the positive root of the equation, $p(x) = x^2 - x - 2 = 0$, then $\lim\limits_{x\to\infty} \dfrac{\sqrt{1-\cos(p(x))}}{x+\alpha-4}$ is equal to:

(A) $\dfrac{1}{\sqrt{2}}$ (B) $\dfrac{1}{2}$

(C) $\dfrac{3}{\sqrt{2}}$ (D) $\dfrac{3}{2}$

65. If the co-ordinates of two points A and B are $(\sqrt{7},0)$ and $(-\sqrt{7},0)$ respectively and P is ny point on the conic $9x^2+16y^2=144$, then PA + PB is equal to:

(A) 8 (B) 16

(C) 9 (D) 6

66. If the function $f(x)\begin{cases} k_1(x-\pi)^2 - 1 & x \le \pi \\ k_2\cos x & x > \pi \end{cases}$ is twice differentiable, then the ordered pair (k_1,k_2) is equal to:

(A) (1, 0) (B) (1, 1)

(C) $\left(\dfrac{1}{2},-1\right)$ (D) $\left(\dfrac{1}{2},1\right)$

67. If the minimum and the maximum values of the function $f:\left[\dfrac{\pi}{2},\dfrac{\pi}{2}\right]\to R$, defined by $f(\theta)=\begin{vmatrix} -\sin^2\theta & -1-\sin^2\theta & 1 \\ -\cos^2\theta & -1-\cos^2\theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$ are m and M respectively, then the ordered pair (m, M) is equal to:

(A) (0, 4) (B) $(0,2\sqrt{2})$

(C) (–4, 4) (D) (–4, 0)

68. The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2, 4, 10, 12, 14, then the absolute difference of the remaining two observations is:

(A) 2 (B) 4

(C) 1 (D) 3

69. If the point P on the curve, $4x^2+5y^2=20$ is farthest from the point Q(0, –4) then PQ^2 is equal to:

(A) 21 (B) 48

(C) 36 (D) 29

70. If the volume of a parallelepiped, whose coterminus edges are given by the vectors $\vec{a} = \hat{i} + \hat{j} + n\hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - n\hat{k}$ and $\vec{c} = \hat{i} + n\hat{j} + 3\hat{k}$ $(n \geq 0)$, is 158 cu.units. then:

(A) $\vec{a}.\vec{c} = 14$ (B) n=7

(C) $\vec{b}.\vec{c} = 10$ (D) n=9

71. The natural number m, for which the coefficient of x in the binomial expansion of $\left(x^m + \frac{1}{x^2}\right)^{22}$ is 1540, is ____

72. Let $f(x) = x.\left[\frac{x}{2}\right]$, for – 10 < x < 10, where [t] denotes the greatest integer function. Then the number of points of discontinuity of f is equal to ___

73. If the line, $2x - y + 3 = 0$ is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines $4x - 2y + \alpha = 0$ and $6x - 3y + \beta = 0$, respectively, then the sum of all possible values of α and β is ____.

74. Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, a ___.

75. The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is ___.

FIITJEE

Solutions to JEE (Main)-2020

PART –A (PHYSICS)

1. **D**

Sol. As it just floats B = mg

$$\left(\frac{4\pi R^3}{3}\right)(\rho_\ell)\,(g) = \left(\frac{4\pi R^3}{3} - \frac{4\pi r^3}{3}\right)(\rho_s)\,(g)$$

$$R^3 = (R^3 - r^3)\left(\frac{27}{8}\right)$$

On solving we get,

$$r = \frac{8}{9}R \text{ (approx)}$$

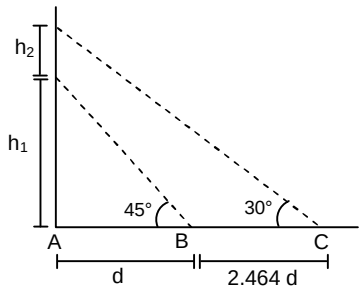
2. **D**

Sol. $\frac{h_1}{d} = 1$

$$\frac{h_1 + h_2}{3.464d} = \frac{1}{\sqrt{3}}$$

$$d + h_2 = \frac{3.464d}{\sqrt{3}}$$

$h_2 = 2d - d = d$



3. **D**

4. **No option.**

5. **A**

Sol. As $\text{Torque}_{net} = 0$

Hence, L = constant

$I\omega = (3I + I)\omega'$

$$\omega' = \frac{\omega}{4}$$

Loss in K.E. = $\frac{1}{2}I\omega^2 - \frac{1}{2}(I+3I)\frac{\omega^2}{16}$

$= \frac{1}{2}I\omega^2\left(1-\frac{1}{4}\right)$

Fractional loss = $\frac{\frac{1}{2}I\omega^2\frac{3}{4}}{\frac{1}{2}I\omega^2} = \frac{3}{4}$

6. **B**

Sol. $f = \frac{V(n)}{4\left(1-\frac{17}{100}\right)}$ (as closed from end)

$f = \frac{(n-2)(v)}{4\left(1-\frac{24.5}{100}\right)}$

$\frac{nV(100)}{4(83)} = \frac{(n-2)(v)(100)}{(4)(75.5)}$

$75.5\, n = 83\,(n-2)$

$75.5\, n = 83\,(M-2)$

$7.5\, n = 166$

$n = 22$ (approx)

$f = \frac{(330)(22)(100)}{4(83)} = 2200$ Hz

7. **D**

Sol. $\frac{E_0}{B_0} = C$

$B_0 = \frac{E_0}{C}$

Force, $F_{max} = QVB$

8. **D**

Sol. Photodiode operate in reverse bias. The photocurrent increases initially and saturates finally.

9. **C**

Sol. $\frac{1}{4}m(210)^2 = m(0.03)\times(4.2)\times1000\times\Delta T$; $Q = mS\Delta t$

$$\Delta T = \frac{(210)(210)}{(4)(4.2)(0.03)(1000)} = 87.5°C$$

10. **C**

Sol. $g' = g\left(\frac{R}{R+h}\right)^2$ above

$g' = g\left(\frac{2}{3}\right)^2$ $\quad h = \frac{R}{2}$

$g' = g\left(1-\frac{d}{R}\right)$ above

$I - \frac{d}{R} = \frac{4}{9}$ $\quad \frac{d}{R} = \frac{5}{9}$

11. **B**

Sol. V is always less than u.

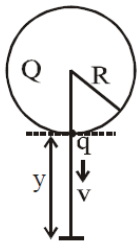
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$$

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{F}$$

$$V = \frac{uF}{u+F} = \frac{u}{\left(\frac{u}{F}+1\right)}$$

12. **B**

Sol.

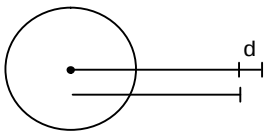


$dw_{ef} = Eqdx$

$$\int du_{et} = \int \frac{kQ}{x^2} dxq$$

$$= kQq\left(-\frac{1}{2}\right)_R^{R+y}$$

$$W_{ef} = \frac{kQq(y)}{(R)(R+y)}$$



$W_{all} = \Delta k$

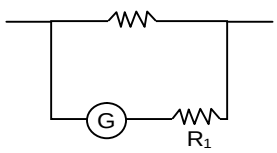
$W_{mg} + W_{ef} = \frac{1}{2}mv^2$

$$V^2 = \frac{2}{m}\left(\frac{kQqy}{(R)(R+y)} + mg\,y\right)$$

$$V^2 = 2y\left(\frac{kQq}{m(R)(R+y)} + g\right) \;;\; k = \frac{1}{4\pi\varepsilon_0}$$

13. **A**

Sol.



$V = I_G (G + R_1)$

$1 = I_G (G + R_1)$ ...(1)

$2 = I_G (G + R_1 + R_2)$

$2 = 1 + I_G R_2$

$I_G R_2 = 1$

$R_2 = G + R_1$

14. **D**

Sol. Use the equation to find 'N':

$$P = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{1}{2}mu^2\right)$$

15. **D**

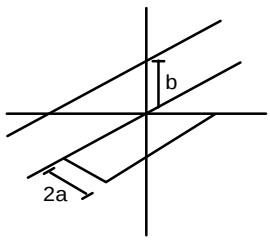
Sol. $\vec{B}_1 = \dfrac{\mu_o I}{2\pi(\ell)}$

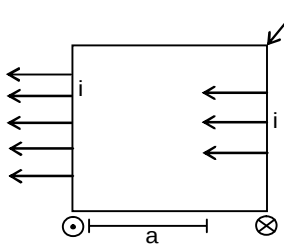
$|\vec{\tau}| = |\vec{r} \times \vec{F}|$

$\tau = 2aF$

$\tau = 2(a)(I)(2a)\left(\dfrac{\mu_o I}{2\pi b}\right)$

$\tau = \dfrac{2\mu_o I^2 a^2}{\pi b}$





B on the also equal to B_1 as

$$B = \frac{\mu_0 I}{2\pi\sqrt{b^2 + \frac{\mu_0^2}{4}}}$$

b >>> a

16. **C**

Sol. $\text{disp} = \left(\frac{\text{kg}}{\text{ms}^2}\right)\left(\frac{\text{s}}{\text{m}}\right)(\text{s})\left(\frac{\text{m}^3}{\text{kg}}\right)$

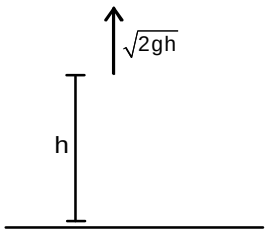
disp= (1) (16) $\left(\frac{1}{300}\right)\left(\frac{1}{1000}\right)$ (1)

disp in mm = $\frac{1000}{300\times100} = 0.03333$

$\Rightarrow \quad \frac{3}{100}$

17. **C**

Sol.



$V^2 = 2gh$

$V = \sqrt{2gh}$

$h = -\sqrt{2gh} + \frac{gt^2}{2}$

$gt^2 - 2\sqrt{2gh}\,t - 2h = 0$

$t = \frac{2\sqrt{2gh} \pm \sqrt{8gh + 8gh}}{2g}$

$t = \frac{2\sqrt{2gh} + 4\sqrt{gh}}{2g}$ as cannot be negative.

$\therefore \quad t = (2+\sqrt{2})\sqrt{\frac{h}{g}} \quad \Rightarrow \quad 3.4\sqrt{\frac{h}{g}}$

18. **B**

Sol. $\ln|R| = \ln|R_0| - \lambda t$

and $T_{1/2} \propto \frac{1}{\lambda}$

where 'N' is slope

$T_{1/2}(A) : T_{1/2}(B) : T_{1/2}(C) = \frac{10}{6} : \frac{5}{6} : \frac{5}{2}$

$= 2 : 1 : 3$

19. **A**

Sol. Power delivered = 1000 W

Voltage = 220 V

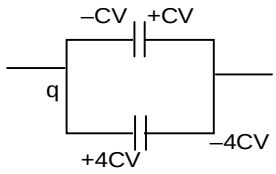
Transmission 'I' = $\frac{1000}{220}$

Power loss = I^2R

Efficiency = $\frac{1000\times100}{1000+I^2R}$

20. **A**

Sol. $\frac{4CV-q}{2\ell}=\frac{-CV+q}{c}$ (as same point)



$4cV - q = -2cV + 2q$

$q = 2cV$

Energy = $\frac{(2cv)^2}{4c}+\frac{(cV)^2}{2c} \Rightarrow \frac{3cV^2}{2}$

21. **195.00**

Sol. $\vec{F}=(\hat{i}+2\hat{j}+3\hat{k})N$

$\vec{r}=\left((4-1)\hat{i}+(3-2)\hat{j}+(-1-1)\hat{k}\right)$

$\vec{\tau}=\vec{r}\times\vec{F}$

$=\left(3\hat{i}+\hat{j}-2\hat{k}\right)\times\left(\hat{i}+2\hat{j}+3\hat{k}\right)$

$\hat{i}\ \ \hat{j}\ \ \hat{k}$

$3\ \ 1-2 \ \ \Rightarrow \ \ \hat{i}(3+4)-\hat{j}(11)+(6-1)\hat{k}$

$1\ \ 2\ \ 3 \qquad\qquad 7\hat{i}-11\hat{j}+5\hat{k}$

$|\vec{\tau}|=\sqrt{121+25+49}$

$|\vec{\tau}|=\sqrt{195}$; x = 195

22. **51.00**

Sol. M_o = 200 MeV/C^2, m = 1 Gev/C^2

Initial velocity of particle is 'V_o'.

Final velocity of hydrogen atom is 'V'

$M_oV_o = mV$

$V=\frac{M_oV_o}{m}$ …(1)

Also, $\frac{1}{2}M_0V_0^2=\frac{1}{2}mV^2+\frac{3}{4}\times13.6$

Put (1) and get answer

$\frac{1}{2}M_0V_0^2=\frac{51}{4}eV$

Hence, N = 51.

23. **5.00**

Sol. $\vec{B} = \frac{\mu_o I (200)}{2R_2}$ $\quad R_2 = 20$ cm ; $R_1 = 1$ cm

$$e = -\frac{d\phi}{dt} = \pi(R_1)^2 (500)\frac{dB}{dt}$$

$$= \pi(R_1^2)(500)\left(\frac{\mu_o I}{2R_2}\right)(200)(10t - 2)$$

$$= \pi\left(\frac{1}{10000}\right)\frac{(500)(200)(\mu_o)(8)(5)}{(2)}$$

$$= 16\pi^2 \times 10 \times 5 \times 10^{-7}$$

$$= 800\pi^2 \times 10^{-7}$$

$$e = 0.7887 \text{ mv}$$

$$\frac{4}{x} = 0.8 \quad ; \quad x = 5.$$

24. **50.47**

Sol. We know, $P = \sqrt{2\,Em}$ and $\lambda = \frac{h}{\sqrt{2\,Em}}$ …(1)

For 1st maxima, 2nd $\sin\theta = \lambda$

Put (1) and get the answer

E = 50.47 eV.

25. **50.00**

Sol. Image by objective must be at focus of eye piece.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_o}$$

$$\frac{1}{5} - \frac{1}{u} = \frac{1}{1} \quad ; \quad u = -\frac{5}{4}\text{cm}$$

Hence, N = 50.

PART –B (CHEMISTRY)

26. A

Sol. In solid state PCl_5 exists as an ionic solid with constituent ions PCl_4^+ (Tetrahedral) and PCl_6^- (Octahedral)

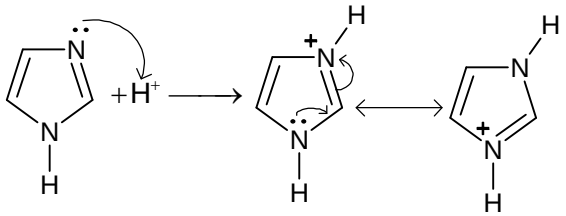
27. C

Sol. According to Aufbau's principal, the increasing energy of atomic orbitals for sixth period element follows the order
6s < 4f < 5d < 6p

28. C

Sol. B is least basic as lone pair of electron is present in resonance so as to make the system aromatic in nature
D is most basic as it results in formation of equivalent resonating structures upon attack of H^+
Among A and C, the former is less basic as sp^2 hybridistion of nitrogen decrease its basic strength.
Hence option 3 follows

Sol.



29. D

Sol. At CMC, the particles cluster together through lyophobic end to form associated colloid called micelle. Further all lyophilic ends(polar head) get projected towards water.

30. B

Sol. Tyrosine is a non essential amino acid.

31. C

Sol. For A

$$\frac{0.693}{300} = \frac{2.303}{t}\log\frac{A_0}{A_t}$$

For B

$$\frac{0.693}{180} = \frac{2.303}{t}\log\frac{B_0}{B_t}$$

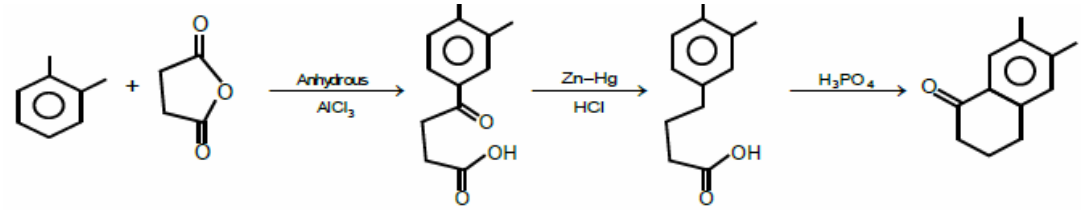
Given $A_0 = B_0$ & $A_t = 4B_t$
Substituting & solving we get t = 900 s

32. A

Sol. In water gas shift reaction carbon monoxide is oxidized into carbon dioxide by treating it with steam in presence of catalyst.

33. B

Sol.



34. D

Sol. $\rho = \frac{M}{V}$

$$\therefore 6.17 = \frac{2}{N_A} \times \frac{M_{X_2}}{\left(300 \times 10^{-10}\right)^3}$$

Solving M_{X_2} = 50 g

$\therefore$ No. of molecules in 200 g

$$= \frac{200}{50} \times N_A = 4N_A$$

35. A

Sol. Among its different uses noradrenaline is used as an anti-depressant.

36. D

Sol. With decrease in inter-nuclear distance, the potential energy of the system decreases, reaches a minimum value and then sharply increases due to rise in inter-electronic as well as inter-nuclear repulsions

37. B

Sol. In endothermic reaction formation of reactants is favoured upon decrease in temperature. Addition of inert gas at constant volume and temperature has no effect on equilibrium.

38. C

Sol. $LiAlH_4$ is a versatile reducing agent which can be used for reduction of cyanide into 1^o amine.

39. C

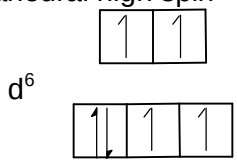
Sol. Esters are susceptible to reaction in basic medium.

40. C

Sol. Excess of nitrogen and phosphorus is primarily responsible for eutrophication and hence an indicator of polluted environment.

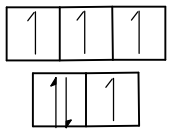
41. C

Sol. Octahedral high spin



$\therefore$ CFSE = $4(-0.4\Delta_0) + 2(0.6\Delta_0) = -0.44\Delta_0$

Tetrahedral

d^6 

$\therefore$ CFSE = $3(-0.6\Delta_t) + 3(0.4\Delta_t) = -0.6\Delta_0$

42. B

Sol. $\Delta R_1 = \frac{a_0}{3}(16-9)$

$\Delta R_2 = \frac{a_0}{2}(16-9)$

$\therefore \frac{\Delta R_1}{\Delta R_2} = \frac{2}{3}$

43. C

Sol. B is the most acidic as it is active methylene group. D is least acidic due to cross-conjugation in conjugate base.

$\therefore$ Option 3 follows

44. A

Sol. Gd : $[Xe]4f^7 5d^1 6s^2$

$\therefore$ Gd^{3+} is $[Xe]4f^7$

Also $\mu = \sqrt{n(n+2)}$ B.M

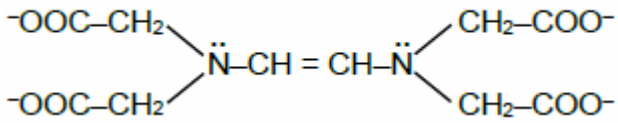
$= \sqrt{7\times 9}$ B.M $= 7.9$ B.M

45. B

Sol. Ellingham diagram provides information on Gibb's free energy for formation of oxides as a function of temperature.

46. 6

Sol. EDTA as shown is hexadentate



47. 18

Sol. $C_3H_8 + 5O_2 \longrightarrow 3CO_2 + 4H_2O$

$C_4H_{10} + \frac{13}{2}O_2 \longrightarrow 4CO_2 + 5H_2O$

$\because$ 2 mol C_4H_{10} is given

$\therefore$ 13 mol O_2 is required for combustion of Butane

$\therefore$ Total mol of O_2 = 5 + 13 = 18

48. -6

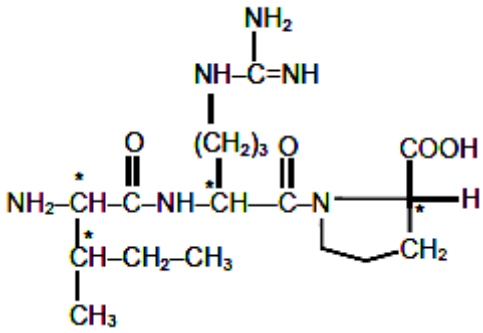
Sol. $\Delta G^0 = -nFE^o$

$\therefore 17.37 \times 10^3 = -3 \times 96500 \times E^o$

$\therefore E^o = -6 \times 10^{-2}$ V

49. 4

Sol. The structure of the given compound is as show below



50. 37

Sol. Mass of CO_2 dissolve at 3 bar pressure in 1 kg water(which is same as 1 litre since density is 1 gm/mL) = 4.4 gm = 0.1 mol.

For weak electrolytes of Ostwald's dilution law we have pH = ½(pK_a – log C)

$\therefore$ pH = ½ [6.4 + 1] = 37 = 37×10^{-1}

PART–C (MATHEMATICS)

51. A

Sol. $\therefore$ length of side = 4

then $\left|z-\bar{z}\right|=4$

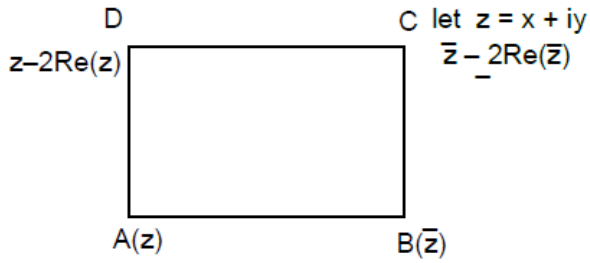
$\left|2iy\right|=4$

$\left|y\right|=2$

also $\left|z-\left(z-2\text{Re}\left(z\right)\right)\right|=4$

$\left|2x\right|=4\Rightarrow\left|x\right|=2$

$\left|z\right|=\sqrt{x^2+y^2}=2\sqrt{2}$



52. D

Sol. Negation of $x\leftrightarrow\sim y$

$\equiv\sim\left(x\leftrightarrow\sim y\right)$

$\equiv x\leftrightarrow\sim\left(\sim\left(y\right)\right)$

$\equiv x\leftrightarrow y$

$\equiv\left(x\wedge y\right)\vee\left(\sim x\wedge\sim y\right)$

53. D

Sol. $S'=2^{10}+2^9.3+2^8.3^2+.....+2.3^9+3^{10}$

G.P. $\rightarrow a=2^{10},r=\frac{3}{2},n=11$

$S'=2^{10}.\frac{\left(\left(\frac{3}{2}\right)^{11}-1\right)}{\frac{3}{2}-1}=2^{11}\left(\frac{3^{11}}{2^{11}}-1\right)$

$=3^{11}-2^{11}$

54. B

Sol. Here $D=\begin{vmatrix}2 & -4 & \lambda\\ 1 & -6 & 1\\ \lambda & -10 & 4\end{vmatrix}=\left(\lambda-3\right)\left(3\lambda+2\right)$

$D=0\quad\Rightarrow\quad\lambda=3,-\frac{2}{3}$

$$D_1 = \begin{vmatrix} 1 & -4 & \lambda \\ 2 & -6 & 1 \\ 3 & -10 & 4 \end{vmatrix} = 2(3-\lambda)$$

For $\lambda = -\frac{2}{3}$, $D_1 \neq 0$

55. B

Sol. $I = \int \left(e^{2x} + 2e^x - e^{-x} - 1\right) e^{\left(e^x + e^{-x}\right)} dx$

$I = \int \left(e^{2x} + e^x - 1\right) e^{\left(e^x + e^{-x}\right)} dx + \int \left(e^x - e^{-x}\right) e^{e^x + e^{-x}} dx$

$I = \int \left(e^x + 1 - e^{-x}\right) e^{e^x + e^{-x}} dx + e^{x^e + e^{-x}}$

$e^x + e^{-x} + x = du$

$\left(e^x - e^{-x} + 1\right) dx = du$

$I = e^{e^x + e^{-x}} + e^{e^x + e^{-x}} = e^{e^x + e^{-x}} \left(e^x + 1\right)$ then $g(x) = e^x + 1$

$g(0) = 2$

56. C

Sol. a, b, c are in AP then

$2b = a + c$

$28 = 3^{2\sin 2\theta - 1} + 3^{4 - 2\sin 2\theta}$

Put $3^{2\sin 2\theta} = x$

$28 = \frac{x}{3} + \frac{81}{x} \Rightarrow x^2 - 84x + 243 = 0$

$(x-3)(x-81) = 0$

$3^{2\sin 2\theta} = 3$ or 3^4

$2\sin 2\theta = 1$ or 4

$\sin 2\theta = \frac{1}{2}$

terms are 1, 14, 27,……then $T_6 = 1 + 5(13)$

57. D

Sol. $n(C) = 73, \quad n(T) = 65, \quad n(C \cap T) = x$

$n(C \cup T) \leq 100$

$\Rightarrow \quad n(C) + n(T) - n(C \cap T) \leq 100$

$\Rightarrow \quad x \geq 38$

$n(C \cap T) \leq \min(n(C), n(T)) \quad \Rightarrow \quad x \leq 65$

$\Rightarrow \quad 38 \leq x \leq 65$

58. B

Sol. $y^2 = 4x$ and $x^2 = 4y$

any tangent of $y^2 = 4x$ is $y = mx + \frac{1}{m}$

it also tangent for $x^2 = 4y$

$\therefore \quad \frac{1}{m} = -m^2 \Rightarrow \quad m = -1$

$\therefore \quad$ common tangent is $y = -x - 1$, it also touches $x^2 + y^2 = c^2$

$\therefore \quad 1 = c^2.(1+1) \Rightarrow c^2 = \frac{1}{2}$

59. D

Sol. $\therefore x^2 = |x|^2 = t$ let

$9t^2 - 18t + 5 = 0$

$(3t-1)(3t-5) = 0$

$|x| = \frac{1}{3}, \frac{5}{3}$

Product of roots $= \frac{1}{3}\left(-\frac{1}{3}\right)\left(\frac{5}{3}\right)\left(\frac{-5}{3}\right) = \frac{25}{81}$

60. B

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} dx . I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} dx$

$I = \int_{-\pi/2}^{\pi/2} \frac{e^{\sin x}}{e^{\sin x}} dx \begin{cases} \text{Replace} \\ x \to (a+b+x) \end{cases}$

$\int_a^b (f(x))dx = \int_c^b f(a+b+x)dx$

$2I = \int_{-\pi/2}^{\pi/2} 1dx \Rightarrow I = \frac{1}{2}\int_{-\pi/2}^{\pi/2} dx$

$I = \frac{1}{2}[x]_{-\pi/2}^{\pi/2} \Rightarrow I = \frac{\pi}{2}$

61. A

Sol. $S = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \ldots..$ upto 10 term

$S = \tan^{-1}\left(\frac{2-1}{1+1.2}\right) + \tan^{-1}\left(\frac{3-2}{1+2.3}\right) + \tan^{-1}\left(\frac{4-3}{1+3.4}\right) + \ldots.. + \tan^{-1}\left(\frac{11-10}{1+11.10}\right)$

$$S=\left(\tan^{-1}2-\tan^{-1}1\right)+\left(\tan^{-1}3-\tan^{-1}2\right)+.....+\left(\tan^{-1}11-\tan^{-1}10\right)$$

$$S=\tan^{-1}11-\tan^{-1}1$$

$$S=\tan^{-1}(11)-\frac{\pi}{4}$$

$$\tan(S)=\frac{5}{6}$$

62. C

Sol. $R(-1+2r,\ 3-2r,\ -r)$

dr's of PR are $(2-2r,\ -1+2r,\ -3+r)$

Then $2(2-2r)+2(1-2r)+1(3-r)=0$

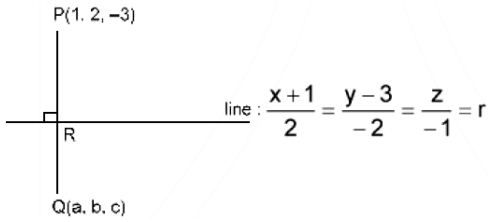
$9-9r=0 \quad \Rightarrow \quad r=1$

R (1, 1, –1)

then $a+1=2 \quad b+2=2 \quad c-3=-2$

$a=1 \quad b=0 \quad c=1$

$\therefore \quad a+b+c=2$



63. C

Sol. Given $\frac{dy}{2+y}=\frac{-e^x dx}{5+e^x}$

$$\ell n(2+y)=-\ell n\left(5+e^x\right)+\ell nC$$

$$y=\frac{C}{5+e^x}-2$$

$y(0)=1 \quad \therefore \quad C=18$

$$y=\frac{18}{5+e^x}-2$$

$\therefore \quad y=(\log_e 13)=-1$

64. C

Sol. $P(x)=0$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2,-1 \quad \therefore \quad \alpha=2$

Now $\lim\limits_{x\to 2^+}\frac{\sqrt{1-\cos\left(x^2-x-2\right)}}{x-2} \quad \Rightarrow \quad \lim\limits_{x\to 2^+}\frac{\sqrt{2\sin^2\left(\frac{x^2-x-2}{2}\right)}}{x-2}$

$$\Rightarrow \quad \lim_{x\to 2^+} \frac{\left|\sin\left(\frac{x^2-x-2}{2}\right)\right|}{x-2} \qquad \Rightarrow \quad \text{for } x\to 2^+,\ \frac{x^2-x-2}{2}\to 0^+$$

$$\lim_{x\to 2^+} \frac{\sqrt{2}\sin\frac{\left(x^2-x-2\right)}{2}}{\left(\frac{x^2-x-2}{2}\right)}.\frac{\frac{x^2-x-2}{2}}{(x-2)} \qquad \Rightarrow \quad \lim_{x\to 2^+}\frac{1}{\sqrt{2}}.\frac{(x-2)(x+1)}{(x-2)}=\frac{3}{\sqrt{2}}$$

65. A

Sol. For ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1,\ a=4,\ b=3,\ e=\sqrt{1-\frac{9}{16}}=\frac{\sqrt{7}}{4}$

A and B are foci then $PA+PB=2a=2(4)=8$

66. D

Sol. f(x) is differentiable then will also continuous then $f(\pi)=-1,\ f\left(\pi^{+1}\right)=-k_2$

$k_2=1$

Now $f'(x)=\begin{cases}2k_1(x-\pi) & x\le\pi\\ -k_2\sin x & x>\pi\end{cases}$ then $f'\left(\pi^-\right)=f'\left(\pi^+\right)=0$

$$f''(x)=\begin{cases}2k_1 & x\le\pi\\ -k_2\cos x & x>\pi\end{cases} \text{ then } 2k_1=k_2$$

$k_1=\frac{1}{2}$

67. D

Sol. $C_3\to C_2-C_1$

$$f(\theta)=\begin{vmatrix}-\sin^2\theta & -1 & 1\\ -\cos^2\theta & -1 & 1\\ 12 & -2 & -2\end{vmatrix}=4\left(\cos^2\theta-\sin^2\theta\right)=4(\cos 2\theta),\theta\in\left[\frac{\pi}{4},\frac{\pi}{2}\right]$$

$f(\theta)_{max}=M=0$

$f(\theta)_{min}=m=-4$

68. A

Sol. $\bar{x}=\frac{2+4+10+12+14+x+y}{7}=8$

$\Rightarrow \quad 42+x+y=56 \qquad \Rightarrow \quad x+y=14$

$$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

$$16 = \frac{4+16+100+144+196+x^2+y^2}{7} - (8)^2$$

$$\Rightarrow \quad 16+64 = \frac{460+x^2+y^2}{7}$$

$$\Rightarrow \quad 560 = 460 + x^2 + y^2 \qquad \Rightarrow \quad x^2+y^2 = 100 \qquad \ldots\ldots\ldots(2)$$

$$\Rightarrow \quad xy = 48$$

$$(x-y)^2 = (x+y)^2 - 4xy = 4$$

$$|x-y| = 2$$

69. C

Sol. Equation $\frac{x^2}{5} + \frac{y^2}{4} = 1$ then $P\left(\sqrt{5}\cos\theta, 2\sin\theta\right)$

$$(PQ)^2 = 5\cos^2\theta + 4(\sin\theta+2)^2 = \cos^2\theta + 16\sin\theta + 20$$

$$= -\sin^2\theta + 16\sin\theta + 21$$

$$= 85 - (\sin\theta - 8)^2$$

$$= (PQ)^2_{max} = 85 - 49 = 36,$$

$$\because \quad (\sin\theta - 8)^2 \in [49, 81]$$

70. C

Sol. Volume of parallelepiped $v = \left|\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]\right|$

$$v = \begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix} = \pm 158$$

$$1(12+n^2) - 1(6+n) + n(2n-4) = \pm 158$$

$3n^2 - 5n - 152 = 0$ or $3n^2 - 5n + 164 = 0$

$D < 0$ (no real roots)

$n = 8, -\frac{19}{3} \quad \Rightarrow \quad n = 8$

then $\vec{b}.\vec{c} = 2 + 4n - 3n = 10$

$\vec{a}.\vec{c} = 1 + n + 3n = 33$

71. 13.00

Sol. $T_{r+1} = {}^{22}C_r \left(x^m\right)^{22-r} x^{-2r}$

$T_{r+1} = {}^{22}C_r\, x^{m(22-r)-2r}$

$22m - mr - 2r = 1$

$22m - 1 = r(m+2)$

$r = \dfrac{22m-1}{m+2}$

$r = \dfrac{22m+44-45}{m+2}$

$r = 22 - \dfrac{3.3.5}{m+2}$

So possible value of m = 1, 3, 7, 13, 43

but ${}^{20}C_r = 1540$

only possible condition is m = 13

72. 08.00

Sol. $-5 < \dfrac{x}{2} < 5$

$I \Rightarrow \quad \left[\dfrac{x}{2}\right] = -5,-4,-3,-2,-1,0,1,2,3,4$

Hence, function is discontinues at = –4, –3, –2, –1, 1, 2, 3, 4

Number of values is 8.

73. 30.00

Sol. $2x - y + 3 = 0$ …….(i)

$4x - 2y + \alpha = 0 \quad \Rightarrow \quad 2x - y + \dfrac{\alpha}{2} = 0$ ……(ii)

$6x - 3y + \beta \quad \Rightarrow \quad 2x - y + \dfrac{\beta}{3} = 0$ ……(iii)

$d_1 = \dfrac{\left|\dfrac{\alpha}{2} - 3\right|}{\sqrt{2^2+1^2}} = \dfrac{1}{\sqrt{5}} \Rightarrow \left|\alpha - 6\right| = 2 \Rightarrow \alpha - 6 = 2,-2 \Rightarrow \alpha = 8, 4$

$d_2 = \dfrac{\left|\dfrac{\beta}{3} - 3\right|}{\sqrt{2^2+1^2}} = \dfrac{2}{\sqrt{5}} \Rightarrow \left|\beta - 9\right| = 6 \Rightarrow \beta - 9 = 6,-6 \Rightarrow \beta = 15,3$

Sum of all value of α and $\beta = 30$.

74. 11.00

Sol. P (at least 2 show 3 or 5) $= {}^4C_2 . \left(\dfrac{2}{6}\right)^2 \left(\dfrac{4}{6}\right)^2 + {}^4C_3 \left(\dfrac{2}{6}\right)^3 \left(\dfrac{4}{6}\right) + {}^4C_4 \left(\dfrac{2}{6}\right)^4$

$= \dfrac{384+128+16}{6^4} = \dfrac{11}{27}$

n = 27

$\therefore$ expectation of number of times = np

$= 27.\frac{11}{27} = 11$

75. 240.00

Sol. SYLLABUS

S – 2, L – 2, A, B, Y, U

Required $= {}^{2}C_{1}.{}^{5}C_{2}.\frac{4!}{2!} = 2.10.\frac{24}{2} = 240$