FIITJEE

Solutions to JEE(Main)-2020

Test Date: 2nd September 2020 (First Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of **3 hours** duration.
2. This **Test Paper** consists of **75** questions. The maximum marks are **300**.
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 25 questions in each part of equal weightage out of which 20 questions are MCQs and 5 questions are numerical value based. Each question is allotted **4 (four)** marks for correct response.
4. **(Q. No. 01 – 20, 26 – 45, 51 – 70)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
5. **(Q. No. 21 – 25, 46 – 50, 71 – 75)** contains 15 Numerical based questions with answer as numerical value. Each question carries **+4 marks** for correct answer. There is no negative marking.
6. Candidates will be awarded marks as stated above in **instruction No.3** for correct response of each question. One mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer box.
7. There is only one correct response for each question. Marked up more than one response in any question will be treated as wrong response and marked up for wrong response will be deducted accordingly as per **instruction 6** above.

PART –A (PHYSICS)

1. A plane electromagnetic wave, has frequency of 2.0×10^{10} Hz and its energy density is 1.02×10^{-8} J/m^3 in vacuum. The amplitude of the magnetic field of the wave is close to ($\frac{1}{4\pi \in_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$ and speed of light = 3×108 ms^{-1}) :

(A) 150 nT
(B) 180 nT
(C) 190 nT
(D) 160 nT

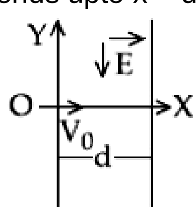
2. Magnetic materials used for making permanent magnets (P) and magnets in a transformer (T) have different properties of the following, which property best matches for the type of magnet required ?

(A) T : Large retentivity, large coercivity
(B) P : Small retentivity, large coercivity
(C) P : Large retentivity, large coercivity
(D) T : Large retentivity, small coercivity

3. Two identical strings X and Z made of same material have tension T_x and T_Z in then If their fundamental frequencies are 450 Hz and 300 Hz, respectively, then the ratio T_x/T_Z is :

(A) 1.25
(B) 2.25
(C) 1.5
(D) 0.44

4. A charged particle (mass m and charge q) moves along X axis with velocity V_0. When it passes through the origin it enters a region having uniform electric field $\vec{E} = -E\hat{j}$ which extends upto x = d. Equation of path of electron in the region x > d is:

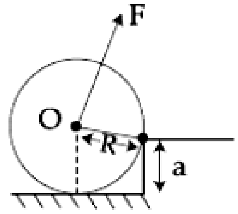


(A) $y = \frac{qEd}{mV_0^2}\left(\frac{d}{2} - x\right)$
(B) $y = \frac{qEd}{mV_0^2}x$
(C) $y = \frac{qEd^2}{mV_0^2}x$
(D) $y = \frac{qEd}{mV_0^2}\left(x - d\right)$

5. The least count of the main scale of a vernier callipers is 1 mm. Its vernier scale is divided into 10 divisions and coincide with 9 divisions of the main scale. When jaws are touching each other, the 7th division of vernier scale coincides with a division of main scale and the zero of vernier scale is lying right side of the zero of main scale. When this vernier is used to measure length of cylinder the zero of the vernier scale between 3.1 cm and 3.2 cm and 4th VSD coincides with a main scale division. The length of the cylinder is (VSD is vernier scale division)

(A) 3.21 cm
(B) 3.07 cm
(C) 3.2 cm
(D) 2.99 cm

6. A uniform cylinder of mass M and radius R is to be pulled over a step of height a (a < R) by applying a force F at its centre 'O' perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is:



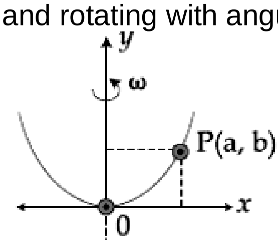
(A) $Mg\sqrt{1-\frac{a^2}{R^2}}$
(B) $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$
(C) $Mg\sqrt{\left(\frac{R}{R-a}\right)^2-1}$
(D) $Mg\frac{a}{R}$

7. The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large distance 'r' from its center. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as:
(A) $T \propto R$
(B) $T^2 \propto R^3$
(C) $T^2 \propto \frac{1}{R^3}$
(D) $T^2 \propto R$

8. Interference fringes are observed on a screen by illuminating two thin slits 1 mm apart with a light source (λ = 632.8 nm). The distance between the screen and the slits is 100 cm. If a bright fringe is observed on a screen at distance of 1.27 mm from the central bright fringe, then the path difference between the waves, which are reaching this point from the slits is close to:
(A) 2 nm
(B) 2.05 μm
(C) 2.87 nm
(D) 1.27 μm

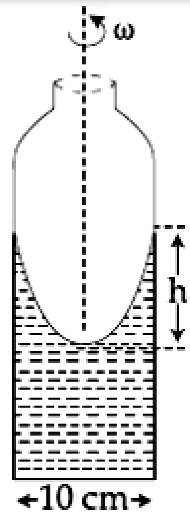
9. Consider four conducting materials copper, tungsten, mercury and aluminum with resistivity ρ_C, ρ_T, ρ_M and ρ_A respectively. Then
(A) $\rho_A > \rho_T > \rho_C$
(B) $\rho_M > \rho_A > \rho_C$
(C) $\rho_C > \rho_A > \rho_T$
(D) $\rho_A > \rho_M > \rho_C$

10. A bead of mass m stays at point P (a, b) on a wire bent in the shape of a parabola $y = Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction)



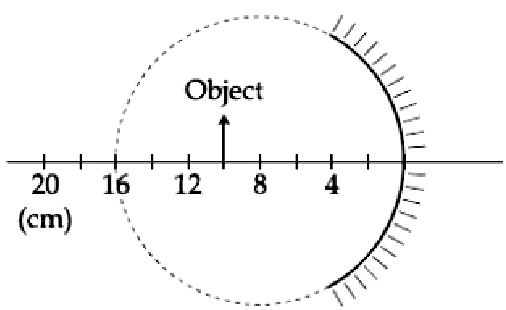
(A) $\sqrt{\frac{2gC}{ab}}$
(B) $\sqrt{\frac{2g}{C}}$
(C) $2\sqrt{2gC}$
(D) $2\sqrt{gC}$

11. A cylindrical vessel containing a liquid is rotated about its axis so that the liquid rises at its sides as shown in the figure. The radius of vessel is 5 cm and the angular speed of rotation is ω rad s^{-1}. The difference in the height, h (in cm) of liquid at the centre of vessel and at the will be :



(A) $\frac{25\omega^2}{2g}$ (B) $\frac{5\omega^2}{2g}$

(C) $\frac{2\omega^2}{25g}$ (D) $\frac{2\omega^2}{5g}$

12. 

A spherical mirror is obtained as shown in the figure from a hollow glass sphere. if an object is positioned infront of the mirror, what will be the nature and magnification of the image of the object? (Figure drawn as schematic and not to scale)

(A) Inverted, real and magnified (B) Erect, virtual and unmagnified

(C) Inverted, real and unmagnified (D) Erect, virtual and magnified

13. A particle of mass m with an initial velocity $u\hat{i}$ collides perfectly elastically with a mass 3 m at rest. It moves with a velocity $\upsilon\hat{j}$ after collision, then υ is given by

(A) $\upsilon = \frac{u}{\sqrt{2}}$ (B) $\upsilon = \frac{1}{\sqrt{6}}u$

(C) $\upsilon = \frac{u}{\sqrt{3}}$ (D) $\upsilon = \sqrt{\frac{2}{3}}u$

14. If speed V, area A and force F are chosen as fundamental units, then the dimension of Young's modulus will be
(A) FA^2V^{-3}
(B) FA^2V^{-2}
(C) FA^2V^{-1}
(D) $FA^{-1}V^0$

15. An amplitude modulated wave is represented by the expression υ_m = 5(1 + 0.6 cos 6280t) sin (211 × 10^4t) volts. The minimum and maximum amplitudes modulated wave are respectively
(A) 3V, 5V
(B) $\frac{3}{2}$V , 5V
(C) $\frac{5}{2}$V , 8V
(D) 5V, 8V

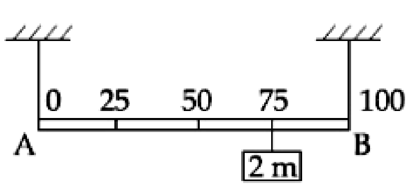
16. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) of the mixture is:
(A) 15
(B) 20
(C) 13
(D) 11

17. A beam of protons with speed 4 × 105 ms^{-1} enters a uniform magnetic field of 0.3T at an angle of 60º to the magnetic field. The pitch of the resulting helical path of protons is close to : (Mass of the proton = 1.67 × 10^{-27} kg, charge of the proton = 1.69 × 10^{-19} C)
(A) 2 cm
(B) 4 cm
(C) 12 cm
(D) 5 cm

18. In a reactor, 2 kg of $_{92}U^{235}$ fuel is fully used up in 30 days. The energy released per fission is 200 MeV. Given that the Avogadro number, N = 6.023 × 1026 per kilo mole and 1 eV = 1.6 × 10^{-19} J. The power output of the reactor is close to :
(A) 60 MW
(B) 35 MW
(C) 125 MW
(D) 54 MW

19. 72 km/hour, respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km/hour. Speed (in ms^{-1}) of this person as observed from train B will be close to : (take the distance between the tracks as negligible)
(A) 28.5 ms^{-1}
(B) 31.5 ms^{-1}
(C) 30.5 ms^{-1}
(D) 29.5 ms^{-1}

20. Shown in the figure is rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass 'm' and has another weight of mass 2m hung at a distance of 75 cm from A. The tension in the string at A is :



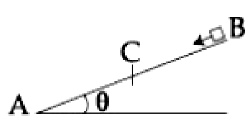
(A) 0.75 mg
(B) 1 mg
(C) 2 mg
(D) 0.5 mg

21. A circular coil of radius 10 cm is placed in a uniform magnetic field of 3.0×10^{-5} T with its plane perpendicular to the field initially. It is rotated at constant angular speed about an axis along the diameter of coil and perpendicular to magnetic field so that it undergoes half of rotation in 0.2 s. The maximum value of EMF induced (in μV) in the coil will be close to the integer…

22. An engine takes in 5 moles of air at 20°C and 1 atm, and compresses it adiabatically to $1/10^{th}$ of the original volume. Assuming air to be a diatomic ideal gas made up of rigid molecules, the change in its internal energy during this process comes out to be X kJ. The value of X to the nearest integer is:

23. When radiation of wavelength A is used to illuminate a metallic surface, the stopping potential is V. When the same surface is illuminated with radiation of wavelength 3λ, the stopping potential is $\frac{V}{4}$V . If the threshold wavelength for the metallic surface is $n\lambda$ then value of n will be:

24. A 5μF capacitor is charged fully by a 220 V supply. It is then disconnected from the supply and is connected in series to another uncharged 2.5 μF capacitor. If the energy change during the charge redistribution is $\frac{X}{100}$ J then value of X to the nearest integer is:

25. 

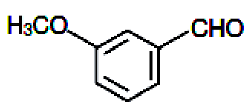
A small block starts slipping down from a point B on an inclined plane AB, which is making an angle θ with the horizontal section BC is smooth and the remaining section CA is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If BC = 2AC, the coefficient of friction is given by $\mu = k\tan\theta$. The value of k is ……….

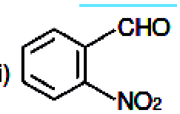
PART –B (CHEMISTRY)

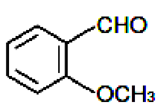
26. If AB_4 molecule is a polar molecule, a possible geometry of AB_4 is
(A) rectangular planar
(B) square pyramidal
(C) tetrahedral
(D) square planar

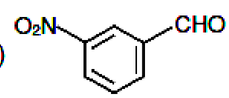
27. For the following assertion and reason, the correct option is
Assertion(A): When Cu(II) and sulphide ions are mixed they react together extremely quickly to give a solid
Reason(R): The equilibrium constant of $Cu^{2+}(aq) + S^{2-}(aq) \rightleftharpoons CuS(s)$ is high because the solubility product is low.
(A) (A) is false and (R) is true.
(B) Both (A) and (R) are false.
(C) Both (A) and (R) are true but (R) is not the explanation for (A).
(D) Both (A) and (R) are true and (R) is the explanation for (A).

28. The increasing order of the following compounds towards HCN addition is :

(i) 

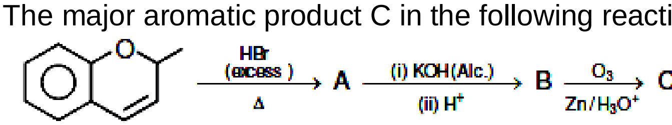
(ii) 

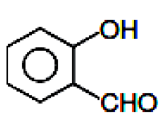
(iii) 

(iv) 

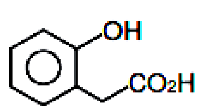
(A) (iii) < (iv) < (i) < (ii)
(B) (i) < (iii) < (iv) < (ii)
(C) (iii) < (i) < (iv) < (ii)
(D) (iii) < (iv) < (ii) < (i)

29. The major aromatic product C in the following reaction sequence will be :

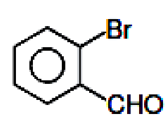




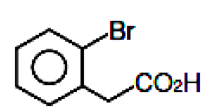
(A)



(B)



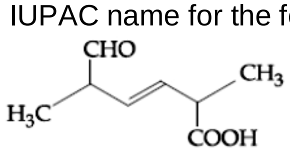
(C)



(D)

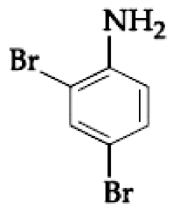
30. On heating compound (A) gives a gas (B) which is a constituent of air. This gas when treated with H_2 in the presence of a catalyst gives another gas (C) which is basic in nature. (A) should not be :
(A) NH_4NO_2
(B) $(NH_4)_2Cr_2O_7$
(C) $Pb(NO_3)_2$
(D) NaN_3

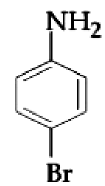
31. The IUPAC name for the following compound is:



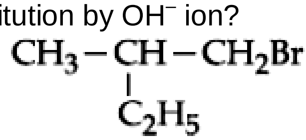
(A) 2, 5-dimethyl-6-carboxy-hex-3-enal
(B) 6-formyl-2-methyl-hex-3-enoic
(C) 2, 5-dimethyl-6-oxo-hex-3-enoic acid
(D) 2, 5-dimethyl-5-carboxy-hex-3-enal

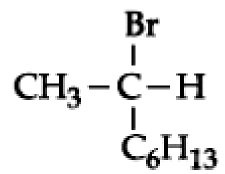
32. In Carius method of estimation of halogen, 0.172 g of an organic compound showed presence of 0.08 g of bromine. Which of these is the correct structure of the compound?

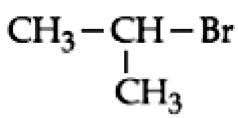
(A) $H_3C - CH_2 - Br$
(B) $H_3C - Be$
(C)


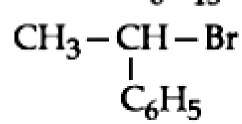
(D)


33. Which of the following compounds will show retention in configuration on nucleophile substitution by OH^- ion?

(A)


(B)


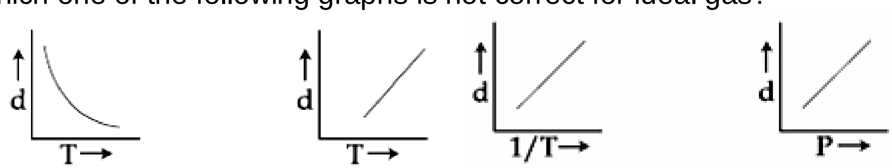
(C)


(D)


34. The statement that is not true about ozone is:
(A) in the atmosphere, it is depleted by CFCs.
(B) in the stratosphere, it forms a protective shield against UV radiation.
(C) in the stratosphere, CFCs release chlorine free radicals (Cl) which reacts with O_3 to give chlorine dioxide radicals.
(D) it is a toxic gas and its reaction with NO gives NO_2

35. While titration dilute HCl solution with aqueous NaOH, which of the following will not be required?
(A) Burette and porcelain tile
(B) Clamp and phenolphthalein
(C) Pipette and distilled water
(D) Bunsen burner and measuring cylinder

36. Which one of the following graphs is not correct for ideal gas?



I II III IV

d = Density, P = Pressure, T = Temperature

(A) I
(B) IV
(C) II
(D) III

37. Consider that d^6 metal ion (M^{2+}) forms a complex with aqua ligands, and the spin only magnetic moment of the complex is 4.90 BM. The geometry and the crystal field stabilization energy of the complex is :
(A) tetrahedral and $-1.6\Delta_t + 1P$
(B) tetrahedral and $-0.6\Delta_t$
(C) octahedral and $-2.4\Delta_0 + 2P$
(D) octahedral and $-1.6\Delta_0$

38. Which of the following is used for the preparation of colloids?
(A) Mond Process
(B) Van Arkel Method
(C) Bredig's Arc Method
(D) Ostwald process

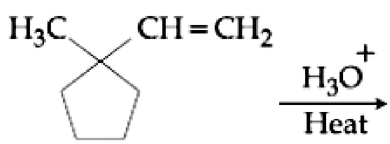
39. An open beaker of water in equilibrium with water vapour is in a sealed container. When a few grams of glucose are added to the beaker of water, the rate at which water molecules :
(A) leaves the vapour increases
(B) leaves the solution decreases
(C) leaves the solution increases
(D) leaves the vapour decreases

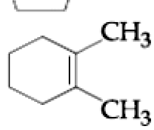
40. In general, the property (magnitudes only) that shows an opposite trend in comparison to other properties across a period is :
(A) Ionization enthalpy
(B) Electron gain enthalpy
(C) Atomic radius
(D) Electronegativity

41. For octahedral Mn(II) and tetrahedral Ni(II) complexes, consider the following statements:
(I) both the complexes can be high spin.
(II) Ni(II) complex can very rarely be of low spin.
(III) with strong field ligands, Mn(II) complexes can be low spin.
(IV) aqueous solution of Mn(II) ions is yellow in color.

The correct statements are :
(A) (II), (III) and (IV) only
(B) (I), (II) and (III) only
(C) (I), (III) and (IV) only
(D) (I) and (II) only

42. The major product in the following reaction is :



(A)


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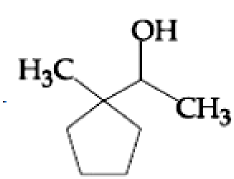
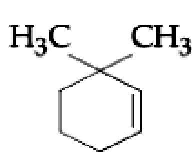
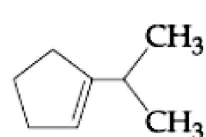


(C)


(D)


43. Consider the following reactions :

(i) Glucose+ROH $\xrightarrow{\text{dry HCl}}$ Acetal $\xrightarrow[(CH_3CO)_2O]{x \text{ eq. of}}$ acetyl derivative

(ii) Glucose $\xrightarrow{Ni/H_2}$ A $\xrightarrow[(CH_3CO)_2O]{y \text{ eq. of}}$ acetyl derivative

(iii) Glucose $\xrightarrow[(CH_3CO)_2O]{z \text{ eq. of}}$ acetyl derivative

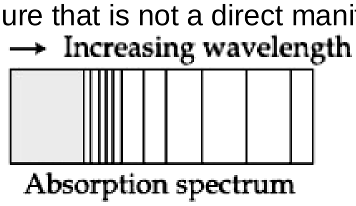
'x', 'y' and 'z' in these reactions are respectively.

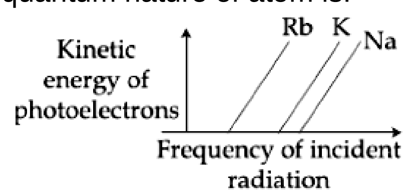
(A) 4, 6 & 5
(B) 5, 6 & 5
(C) 5, 4 & 5
(D) 4, 5 & 5

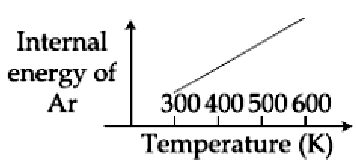
44. The metal mainly used in devising photoelectric cells is:

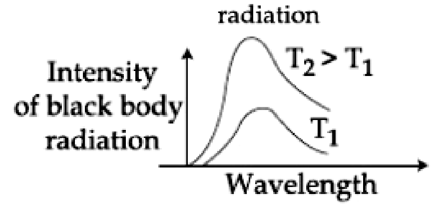
(A) Rb
(B) Cs
(C) Li
(D) Na

45. The figure that is not a direct manifestation of the quantum nature of atom is:

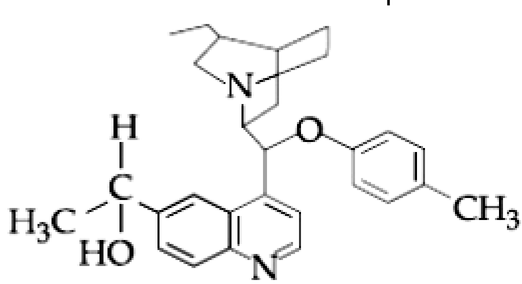
(A)


(B)


(C)


(D)


46. The number of chiral carbons present in the molecule given below is……………



47. The Gibbs energy change(in J) for the given reaction at $[Cu^{2+}] = [Sn^{2+}] = 1$ M and 298 K is

$$Cu(s) + Sn^{2+}(aq) \longrightarrow Cu^{2+}(aq) + Sn(s); \left(E^0_{Sn^{2+}|Sn} = -0.16\,V,\ E^0_{Cu^{2+}|Cu} = -0.34\,V\right)$$

Take F = 96500 C mol^{-1}

48. The oxidation states of iron atoms in compounds (A), (B) and (C), respectively, are x, y and z. The sum of x, y and z is ……………

$$\underset{(A)}{Na_4\left[Fe(CN)_5(NOS)\right]} \quad \underset{(B)}{Na_4\left[FeO_4\right]} \quad \underset{(C)}{\left[Fe_2(CO_9)\right]}$$

49. The internal energy change (in J) when 90 g of water undergoes complete evaporation at 100ºC is……………
(Given : ΔH_{vap} for water at 373 K = 41 kJ/mol, R = 8.314 JK^{-1} mol^{-1})

50. The mass of gas adsorbed, x, per unit mass of adsorbate, m, was measured at various pressures, p. A graph between $\log\frac{x}{m}$ and log p gives a straight line with slope equal to 2 and the intercept equal to 0.4771. The value of $\frac{x}{m}$ at a pressure of 4 atm is :
(Given log 3 = 0.4771)

PART–C (MATHEMATICS)

51. If $|x| < 1$, $|y| < 1$ and $x \neq y$, then the sum to infinity of the following series $(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots$ Is

(A) $\frac{x+y-xy}{(1-x)(1-y)}$ (B) $\frac{x+y+xy}{(1-x)(1-y)}$

(C) $\frac{x+y-xy}{(1+x)(1+y)}$ (D) $\frac{x+y+xy}{(1+x)(1+y)}$

52. Let $\alpha > 0$, $\beta > 0$ be such that $\alpha^3 + \beta^2 = 4$. If the maximum value of the term independent of x in the binomial expansion of $\left(\alpha x^{\frac{1}{9}} + \beta x^{-\frac{1}{6}}\right)^{10}$ is 10 k, then k is equal to

(A) 176 (B) 352

(C) 84 (D) 336

53. Let P(h, k) be a point on the curve $y = x^2 + 7x + 2$, nearest to the line, $y = 3x - 3$. Then the equation of the normal to the curve at P is

(A) $x + 3y - 62 = 0$ (B) $x - 3y - 11 = 0$

(C) $x + 3y + 26 = 0$ (D) $x - 3y + 22 = 0$

54. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is

(A) $-\frac{1}{2}\left(1-i\sqrt{3}\right)$ (B) $-\frac{1}{2}\left(\sqrt{3}-i\right)$

(C) $\frac{1}{2}\left(\sqrt{3}-i\right)$ (D) $\frac{1}{2}\left(1-i\sqrt{3}\right)$

55. The plane passing through the points (1, 2, 1), (2, 1, 2) and parallel to the line, $2x = 3y$, $z = 1$ also passes through the point :

(A) (2, 0, –1) (B) (–2, 0, 1)

(C) (0, –6, 2) (D) (0, 6, -2)

56. Let A be a 2×2 real matrix with entries from {0, 1} and $|A| \neq 0$. Consider the following two statements;

(P) If $A \neq I_2$, then $|A| = -1$

(Q) If $|A| = 1$, then tr(A) = 2

where I_2 denotes 2×2 identity matrix and tr(A) denotes the sum of the diagonal entries of A. Then :

(A) Both (P) and (Q) are true (B) Both (P) and (Q) are false

(C) (P) is true and (Q) are false (D) (P) is false and (Q) is true

57. Area(in sq. units) of the region outside $\frac{|x|}{2}+\frac{|y|}{3}=1$ and inside the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is
(A) $3(4-\pi)$
(B) $3(\pi-2)$
(C) $6(\pi-2)$
(D) $6(4-\pi)$

58. Let X = {x ∈ N : 1 ≤ x ≤ 17} and Y{ax + b : x ∈ X and a, b ∈ R, a > 0}. If mean and variance of elements of Y are 17 and 216 respectively then a + b is equal to
(A) – 27
(B) 9
(C) 7
(D) – 7

59. If the tangent to the curve y = x + siny at a point (a, b) is parallel to the line joining $\left(0,\frac{3}{2}\right)$ and $\left(\frac{1}{2},2\right)$, then:
(A) b = $\frac{\pi}{2}+a$
(B) |a + b| = 1
(C) |b – a| = 1
(D) b = a

60. Let y = y(x) be the solution of the differential equation,
$\frac{2+\sin x}{y+1}.\frac{dy}{dx}=-\cos x, y>0, y(0)=1$.
If $y(\pi)$ = a and $\frac{dy}{dx}$ at x = π is b, then the ordered pair (a, b) is equal to
(A) (2, 1)
(B) (1, –1)
(C) $\left(2,\frac{3}{2}\right)$
(D) (1, 1)

61. The contrapositive of the statement "If I reach the station in time, then I will catch the train" is:
(A) If I do not reach the station in time, then I will catch the train.
(B) If I do not reach the station in time, then I will not catch the train.
(C) If I will catch the train, then I reach the station in time.
(D) If I will not catch the train, then I do not reach the station in time.

62. The domain of the function $f(x)=\sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is
$(-\infty,-a]\cup[a,\infty)$
(A) $\frac{\sqrt{17}-1}{2}$
(B) $\frac{\sqrt{17}}{2}$
(C) $\frac{\sqrt{17}}{2}+1$
(D) $\frac{1+\sqrt{17}}{2}$

63. Let α and β be the roots of the equation, $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, n = 1, 2, 3, …., then
(A) $6S_6 + 5S_5 = 2S_4$
(B) $6S_6 + 5S_5 + 2S_4 = 0$
(C) $5S_6 + 6S_5 = 2S_4$
(D) $5S_6 + 6S_5 + 2S_4 = 0$

64. If R = {(x, y) : x, y $\in$ Z, $x^2 + 3y^2 \leq 8$} is a relation on the set of integers Z, then the domain R^{-1} is:
(A) {–2, –1, 0, 1, 2}
(B) {–2, –1, 1, 2}
(C) {–1, 0, 1}
(D) {0, 1}

65. If p(x) be a polynomial of degree three that has a local maximum value 8 at x = 1 and a local minimum value 4 at x = 2; then p(0) is equal to
(A) 6
(B) –24
(C) 12
(D) –12

66. If a function f(x) defined by

$$f(x) = \begin{cases} ae^x + be^{-x}, & -1 \leq x < 1 \\ cx^2 \quad , & 1 \leq x \leq 3 \\ ax^2 + 2cx\ , & 3 < x \leq 4 \end{cases}$$

be continuous for some a, b, c $\in$ R and $f'(0) + f'(2) = e$, then the value of a is
(A) $\frac{1}{e^2 - 3e + 13}$
(B) $\frac{e}{e^2 - 3e + 13}$
(C) $\frac{e}{e^2 + 3e + 13}$
(D) $\frac{e}{e^2 - 3e - 13}$

67. The sum of the first three terms of a G.P is S and their product is 27. Then all such S lie in
(A) $(-\infty, 9]$
(B) $(-\infty, -3] \cup [9, \infty)$
(C) $(-\infty, -9] \cup [3, \infty)$
(D) $[-3, \infty)$

68. Let S be the set of all $\lambda \in R$ for which the system of linear equations
$$2x - y + 2z = 2$$
$$x - 2y + \lambda z = -4$$
$$x + \lambda y + z = 4$$
has no solution. Then the set S
(A) contains more than two elements
(B) is a singleton
(C) is an empty set
(D) contains exactly two elements

69. A line parallel toe the straight line 2x – y = 0 is tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point(x_1, y_1). Then $x_1^2 + 5y_1^2$ is equal to
(A) 6
(B) 5
(C) 5
(D) 10

70. Box I contains 30 cards numbered I to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is:
(A) $\frac{8}{17}$
(B) $\frac{2}{3}$
(C) $\frac{2}{5}$
(D) $\frac{4}{17}$

71. If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is ……

72. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $|\vec{a}-\vec{b}|^2 + |\vec{a}-\vec{c}|^2 = 8$. Then $|\vec{a}+2\vec{b}|^2 + |\vec{a}+2\vec{c}|^2$ is equal to _______

73. If $\lim\limits_{x\to 1} \dfrac{x + x^2 + x^3 + ... + x^n - n}{x-1} = 820, \left(n \in N\right)$ then the value of n is equal to _____

74. The number of integral values of k for which the line, 3x + 4y= k intersects the circle, $x^2 + y^2 - 2x - 4y + 4 = 0$ at two distinct points is ….

75. The integral $\int_0^2 \left| |x-1| - x \right| dx$ is equal to ___________

FIITJEE
Solutions to JEE (Main)-2020
PART –A (PHYSICS)

1. **D**

Sol. Energy Density = $\frac{1}{2}\frac{B^2}{\mu_0}$

$B = \sqrt{2\times\mu_0\times\text{Energy density}}$

$B = \sqrt{2\times 4\pi\times 10^{-7}\times 1.02\times 10^{-8}}$ = 160 × 10^9 = 160 nT

2. **B**

Sol. Based on theory.

3. **B**

Sol. $f_x = \frac{1}{2\ell}\sqrt{\frac{T_x}{\mu}}$

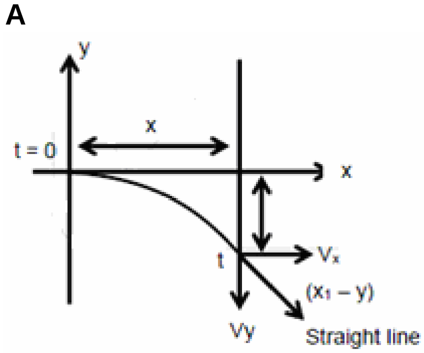
$f_y = \frac{1}{2\ell}\sqrt{\frac{T_y}{\mu}}$

$\frac{f_x}{f_y} = \frac{450}{300} = \sqrt{\frac{T_x}{T_y}}$

$\Rightarrow$ T_x/T_y = 9/4 = 2.25

4. **A**

Sol.



x > d path is straight line

$$\frac{-y = \frac{1}{2}at^2}{x-d} = \frac{at}{V_0}$$

$$\frac{-y - \frac{1}{2}a^2}{at} = \frac{x-d}{V_0}$$

$$\frac{-y}{at} - \frac{1}{2}\frac{d}{V_0} = \frac{x}{V_0} - \frac{d}{V_0}$$

$$-\frac{myV_0}{qEd} = \frac{x}{V_0} - \frac{d}{2V_0}$$

$$y = \frac{-qEd}{mV_0}\left(\frac{x}{V_0} - \frac{d}{2V_0}\right) \;;\; y = \frac{qEd}{mV_0^2}\left(\frac{d}{2} - x\right)$$

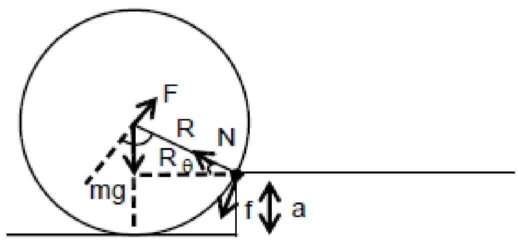
5. **B**

Sol. Zero error = 0 + 7 × 0.1 = 0.070

Vernier reading = (3.1 + 4 × 0.01) –0.07 = 3.07

6. **B**

Sol.



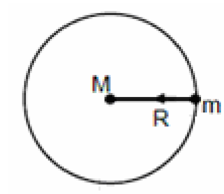
FR > mg cosθ R

F > mg cos θ

$$F > mg\frac{\sqrt{R^2-(R-a)^2}}{R} \Rightarrow Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$$

7. **D**

Sol.



$$M = \int \rho dV$$

$$M = \int_0^{r=R_0} \frac{k}{r} 4\pi r^2 dr$$

$$M = \frac{4\pi k R_0^2}{2} = 2\pi k R^2$$

$$F_G = \frac{GMm}{R_0^2} = 2\omega_0^2 R$$

$$\Rightarrow \quad \frac{G\frac{4\pi kR^2}{2}}{R^2} = \omega_0^2 R \quad \Rightarrow \quad \omega_0 = \sqrt{\frac{2\pi KG}{R}}$$

$$\because \quad T = \frac{2\pi}{\omega_0} = \frac{2\pi\sqrt{R}}{\sqrt{2\pi KG}} = \sqrt{\frac{2\pi R}{KG}}$$

$$\Rightarrow \quad T^2 \propto R$$

8. **D**

Sol. $\Delta P = d\sin\theta$

$= d\theta$

$= \frac{dy}{D} = \frac{10^{-3} \times 1.270\,mm}{1m} \quad 1.27\ \mu m$

9. **B**

Sol. $\rho_m = 98 \times 10^{-8}$

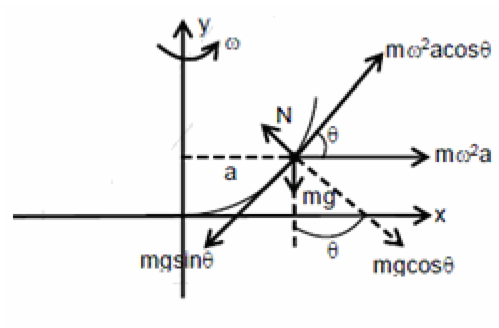
$\rho_A = 2.65 \times 10^{-8}$

$\rho_C = 1.724 \times 10^{-8}$

$\rho_T = 5.65 \times 10^{-8}$

10. **C**

Sol.



$m\omega^2 a\cos\theta = mg\sin\theta$

$\omega = \sqrt{\dfrac{g\tan\theta}{a}}$

$y = 4cx^2$

$\tan\theta = \dfrac{dy}{dx} = 8xC$

$(\tan\theta)_{a,\,b} = 8aC$

$\omega = \sqrt{\dfrac{g\times 8ac}{a}} = 2\sqrt{2gc}$

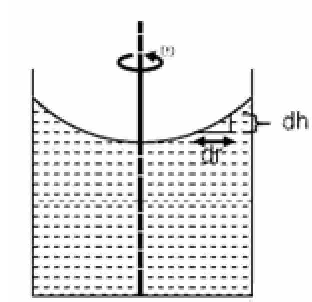
11. **A**

Sol. $\rho dr\omega^2 r = \rho gdh$

$\omega^2\int_0^R rdr = g\int_0^h dh$

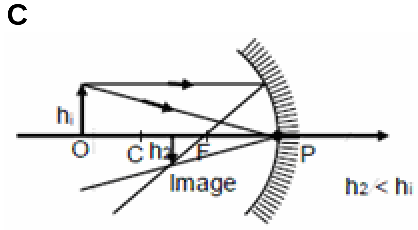
$\dfrac{\omega^2R^2}{2} = gh$

$h = \dfrac{\omega^2R^2}{2g} = \dfrac{25\omega^2}{2g}$



12. **C**

Sol.



13. **A**

Sol. From momentum conservation

$mu\hat{i} + 0 = mv\hat{j} + 3m\vec{v}'$

$\vec{v}' = \dfrac{u}{3}\hat{i} - \dfrac{v}{3}\hat{j}$

From kinetic energy conservation $\dfrac{1}{2}mu^2 = \dfrac{1}{2}mv^2 + \dfrac{1}{2}(3\,m)\left(\left(\dfrac{u}{3}\right)^2 + \left(\dfrac{v}{3}\right)^2\right)$

Solving, $v = \dfrac{u}{\sqrt{2}}$

14. **D**

Sol. $Y \propto F^aV^bA^c$ $\qquad Y = \left(\dfrac{F}{A}\right)$

$\dfrac{MLT^{-2}}{L^2} \propto (M^1L^1T^{-2})^a(L^1T^{-1})^b(L^2)^c$

$M^1L^{-1}T^{-2} \propto M^aL^{a+b+2c}\,T^{-2a-b}$

$a + b + 2c = -1$
$-2a + b = -2$
$a = 1, b = 0, c = -1$
$Y = F^1 v^0 A^{-1}$

15. **B**
Sol. From Given equation
$\mu = 0.6$
$A_m = \mu A c$
$\frac{A_{max} - A_{min}}{2} = A_c = 5$ …(1)
$\frac{A_{max} - A_{min}}{2} = 3$ …(2)
From equation (1) + (2)
$A_{max} = 8$
From equation (1) – (2)
$A_{min} = 2$

16. **A**
Sol. $\frac{f_1 n_1 RT_1}{2} + \frac{r_2 n_2 RT_2}{2} = 3 \times \frac{5}{2} RT + \frac{5}{2} \times 3RT = 15$

17. **B**
Sol. Pitch = $(V \cos\theta)T$
$= (V\cos\theta)\frac{2\pi m}{eB}$
$= (4\times 10^5 \cos 60^o)\,\frac{2\pi}{0.3\times 10}\left(\frac{1.67\times 10^{-27}}{1.69\times 10^{19}}\right)$
= 4 cm

18. **A**
Sol. $P = \frac{E}{t}$
$= \frac{2}{235}\times\frac{6.023\times 10^{26}\times 200\times 1.6\times 10^{-19}}{30\times 24\times 60\times 60}$ = 60 W

19. **D**
Sol. V_A = 36 km/hr = 10 m/s
V_B = –72 km/hr = –20 m/s
V_{MA} = –1.8 km/hr = –0.5 m/s
$V_{man, B} = V_{man, A} + V_{A, B}$
$= V_{man, A} + V_A - V_B$
= –0.5 + 10 – (–20)
= –0.5 + 30 = 29.5 m/s

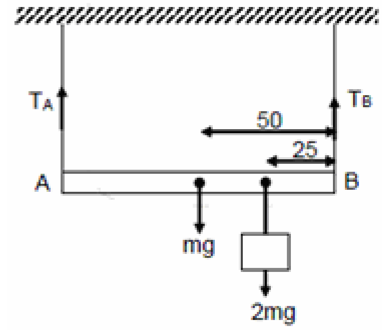
20. **B**

Sol. τ_{net} about B is zero at equilibrium

$T_A\,100 - mg \times 50 - 2\,mg \times 25 = 0$

$T_A\,100 = 100\,mg$

$T_A = 1\,mg$



21. **15**

Sol. Flux as a function of time $\phi = \vec{B} \cdot \vec{A} = AB\cos(\omega t)$

emf induced,

$$e = \frac{-d\phi}{dt} = AB\omega\sin(\omega t)$$

Maximum value of emf = $AB\omega$

= $\pi R^2 B\omega$

$$= 3.14 \times 0.1 \times 0.1 \times 3 \times 10^{-5} \times \frac{\text{o}}{0.2} = 15$$

22. **46**

Sol. $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= T_1 (10)^{\frac{7}{5}-1}$$

$T_2 = T_1\,(10)^{2/5}$

$$\Delta V = \frac{5}{2} nR \ ; \ \frac{5}{2} \times 5 \times 3[10^{2/5} - 1]\,(293)$$

$$\frac{625}{6} \times 1.5 \times 293 = 461440 \approx 46\,\text{ks}$$

23. **9**

Sol.
$$\frac{hc}{\lambda} = \phi + eV \qquad \ldots(1)$$

$$\frac{hc}{3\lambda} = \phi + \frac{eV}{4} \qquad \ldots(2)$$

from (1) and (2)

$$\frac{hc}{\lambda}\left(1 - \frac{1}{3}\right) = \frac{3}{4} eV \quad ; \quad \frac{hc}{\lambda}\frac{2}{3} = \frac{3}{4} eV$$

$$eV = \frac{8}{9}\frac{hc}{\lambda} \ ; \ \frac{hc}{\lambda} = \phi + \frac{8}{9}\frac{hc}{\lambda}$$

$\phi = \frac{hc}{9\lambda} = \frac{hc}{\lambda_{th}}$

$\lambda_{th} = 9\lambda$; $\therefore$ k = 9

24. **4**

Sol. $c_1 = 5\mu F$ $V_1 = 220$ Volt

$c_2 = 2.5\ \mu F$ $V_2 = 0$

Heat loss; $\Delta H = U_i - U_f = \frac{1}{2}\frac{c_1 c_2}{c_1 + c_2}(v_1 - v_2)^2$

$= \frac{1}{2} \times \frac{5 \times 2.5}{(5 + 2.5)}(220 - 0)^2 \mu J$

$= \frac{5}{2 \times 3} \times 22 \times 22 \times 100 \times 10^{-6} J$

$= \frac{5 \times 11 \times 22}{3} \times 10^{-4} J = \frac{55 \times 22}{3} \times 10^{-4} J$

$= \frac{1210}{3} \times 10^{-4} J = \frac{1210}{3} \times 10^{-3} J \times 4 \times 10^{-2}$

According to questions

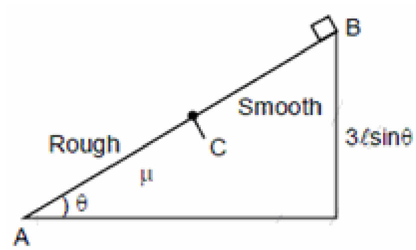
$\frac{x}{100} = 4 \times 10^{-2}$

So, x = 4

Note: But given answer by JEE Main x = 36

25. **3**

Sol. Let AC = ℓ $\therefore$ BC = 2ℓ $\therefore$ AB = 3ℓ



Apply work – Energy theorem

$W_f + W_{mg} = \Delta KE$

$Mg\,(3\ell)\sin\theta - \mu mg\cos\theta(\ell) = 0 + 0$

$\mu mg\cos\theta\ell = 3mg\ell\sin\theta$

$\mu = 3\tan\theta = k\tan\theta$

$\therefore$ k = 3

PART –B (CHEMISTRY)

26. B

Sol. For AB_4 compound possible geometry are

S. No.	Bond pair	Lone pair	Total	Hybridisation	Geometry	Polarity
1	4	0	4	SP^3	Tetrahedral	non polar
2	4	1	5	SP^3d	Sea-saw	Polar
3	4	2	6	sp^3d^2	Square Planar	non polar

Square pyramidal can be polar due to lone pair moment as the bond pair moments will get cancelled out.

27. C

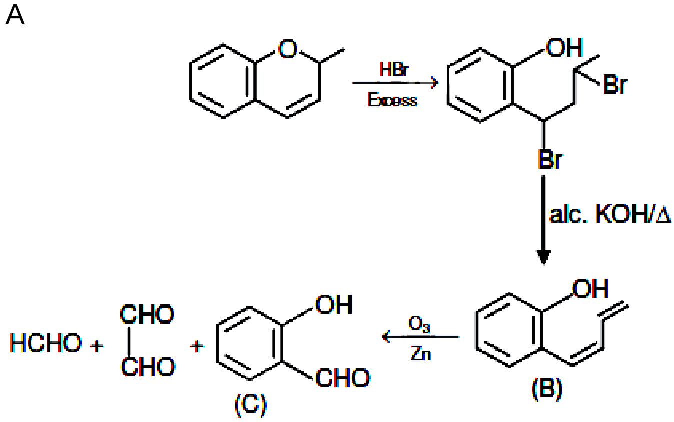
Sol. Rate of chemical reaction has nothing to do with value of equilibrium constant.

28. C

Sol. –I, – M effect of NO_2 increase reactivity towards nucleophilic addition reaction with HCN.

29. A

Sol.



30. C

Sol. (1) $Pb(NO_3)_2 \xrightarrow{\Delta} PbO + 2NO_2$

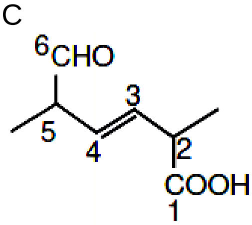
(2) $NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

(3) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + H_2O$

(4) $NaN_3 \xrightarrow{300°C} 3N_2 + 2Na$

31. C

Sol.



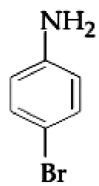
(2, 5-dimethyl-6-oxo-hex-3-enoic acid

32. D

Sol. Mole of Bromine = $\frac{0.08}{80} = 10^{-3}$mole

Molar mass of compound = $\frac{0.172}{M} = 10^{-3}$

$M = \frac{0.172}{10^{-3}} = 172$ g

Molar mass of  = 80 + 72 + 6 + 14 = 172 g

33. A

Sol. Reaction path is S_N2 because OH^- is strong nucleophile, but hydroxyl ion will not attack on chiral centres and so there is retension of configuration.

34. C

Sol. In presence of sunlight CFC's molecule divides & release chlorine free radical, which react with ozone give chlorine monoxide radical ($ClO^{\bullet}$) and oxygen.

$$CF_2Cl_2(g) \xrightarrow{UV} \dot{C}l(g) + \dot{C}F_2Cl(g)$$

$$Cl^{\bullet}(g) + O_3(g) \longrightarrow ClO^{\bullet}(g) + O_2(g)$$

$$ClO^{\bullet}(g) + O(g) \longrightarrow Cl^{\bullet}(g) + O_2(g)$$

35. D

Sol. In this acid base Titrating there is no use of Bunsen burner and measuring cylinder other laboratory equipments will be required for getting the end point of titration.

36. C

Sol. For ideal gas

PM = dRT

$d = \left[\frac{PM}{R}\right]\frac{1}{T}$

So graph between d Vs T is not straight line.

37. B

Sol. Since spin only magnetic moment is 4.90 BM so number of unpaired electrons must be 4, so If the complex is octahedral, then it has to be high spin complex with configuration $t_{2g}^{2,1,1}e_g^{1,1}$ in that case

CFSE = 4 × (-0.4Δ_0) + 2 × 0.6Δ_0 = -0.4Δ_0

If the complex is tetrahedral then its electronic configuration will be = $e_g^{2,1}t_{2g}^{1,1,1}$ and CFSE will be = 3 × (-0.6Δ_t) + 3 × (0.4Δ_t) = -0.6Δ_t

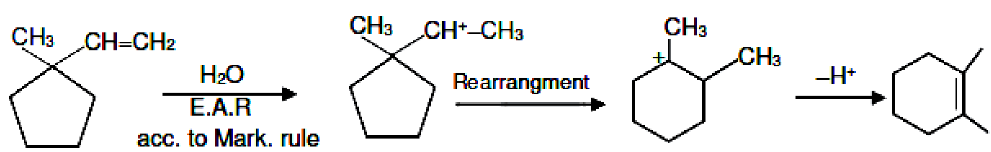
38. C

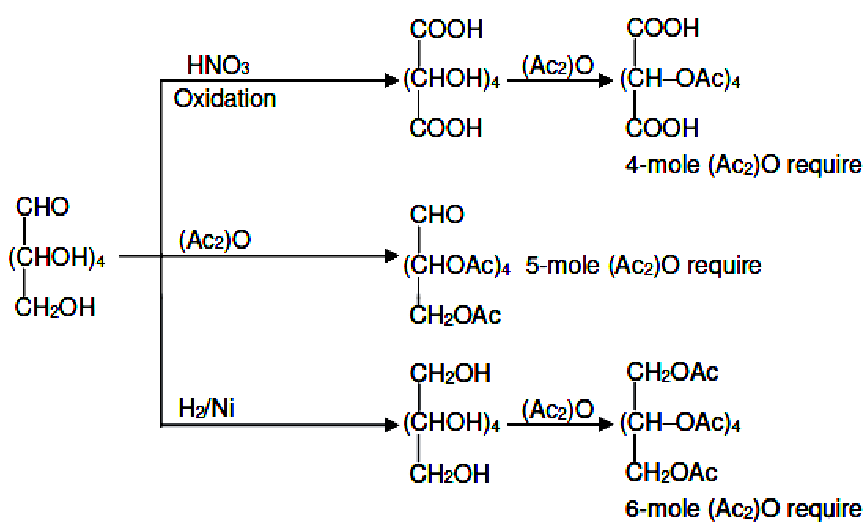
Sol. Bredig's Arc Method is used for preparation of colloidal sol's of less reactive metal like Au, Ag, Pt.

39. A & C
Sol. The vapour pressure of solution will be less than vapour pressure of pure solvent, so some vapour molecules will get condensed to maintain new equilibrium.

40. C
Sol. On moving Left to Right along a period.
Atomic Radius $\rightarrow$ decreases.
Electronegativity $\rightarrow$ Increases.
Electron gain enthalpy $\rightarrow$ Increases.
Ionisation Enthalpy $\rightarrow$ Increases.

41. B
Sol. With weak field ligands Mn(II) will be of high spin and with strong field ligands it will be of low spin. Ni(II) tetrahedral complexes will be generally of high spin due to sp^3 hybridisation. Mn(II) is of light pink color in aqueous solution.

42. A
Sol.


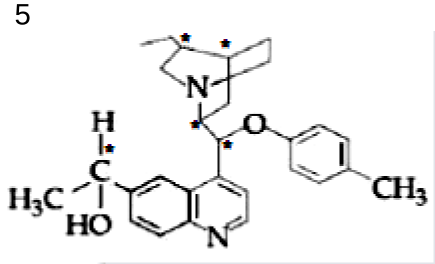
43. A
Sol.


44. B
Sol. Cesium has lowest ionisation enthalpy and hence it can show photoelectric effect to the maximum extent hence it is used in photo electric cell.

45. C
Sol. 1, 2 and 3 are according to quantum theory but (4) is statement of kinetic theory of gases

46. 5

Sol.



47. 96500

Sol. $E^0_{cell} = E^0_{Sn^{2+}|Sn} - E^0_{Cu^{2+}|Cu}$

= -0.16 – 0.34 = -0.50 V

$\Delta G^0 = -nFE^0_{cell}$

= -2 × 96500 × (-0.5) = 96500 J

= 96.5 kJ = 96500 J

48. 6

Sol. The oxidation states of iron in these compounds will be

A = +2

B = +4

C = 0

The sum of oxidation states will be = 6.

49. 189000 to 190000

Sol. $\Delta H = \Delta U + \Delta n_g RT$

$41000 \times 5 = \Delta U + 5 \times 8.314 \times 373$

$205000 = \Delta U + 15505.61$

$\Delta U = 189494.39$ J = 189494 J

50. 48

Sol.

$\left(\frac{x}{m}\right) = k(P)^{\frac{1}{n}}$

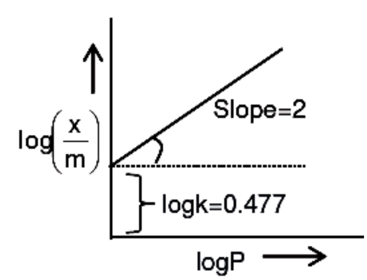
$\log\left(\frac{x}{m}\right) = \log k + \frac{1}{n}\log P$

Slope = $\frac{1}{n} = 2$ So $n = \frac{1}{2}$.

Intercept ⇒ logk = 0.477 So k = Antilog (0.477) = 3

So $\left(\frac{x}{m}\right) = k(P)^{\frac{1}{n}}$

$= 3[4]^2 = 48$



PART–C (MATHEMATICS)

51. A

Sol. $|x|<1, |y|<1,\ x \neq y$

$$(x+y)+\left(x^2+xy+y^2\right)+\left(x^3+x^2y+xy^2+y^3\right)+....$$

By multiplying and dividing x – y:

$$\frac{\left(x^2-y^2\right)+\left(x^3-y^3\right)+\left(x^4-y^4\right)+....}{x-y}$$

$$=\frac{\left(x^2+x^3+x^4+...\right)-\left(y^2+y^3+y^4+.....\right)}{x-y}$$

$$=\frac{\frac{x^2}{1-x}-\frac{y^2}{1-y}}{x-y}$$

$$=\frac{\left(x^2-y^2\right)-xy(x-y)}{(1-x)(1-y)(x-y)}$$

$$=\frac{x+y-xy}{(1-x)(1-y)}$$

52. D

Sol. Let t_{r+1} denotes

$r+1^{th}$ term of $\left(\alpha x^{\frac{1}{9}}+\beta x^{-\frac{1}{6}}\right)^{10}$

$$t_{r+1}={}^{10}C_r\alpha^{10-r}(x)^{\frac{10-r}{9}}.\beta^r x^{-\frac{r}{6}}$$

$$={}^{10}C_r\,\alpha^{10-r}\beta^r(x)^{\frac{10-r}{9}-\frac{r}{6}}$$

If t_{r+1} is independent of x

$$\frac{10-r}{9}-\frac{r}{6}=0 \qquad \Rightarrow \qquad r=4$$

maximum value of t_5 is 10 K (given)

$\Rightarrow \qquad {}^{10}C_4\ \alpha^6\beta^4$ is maximum

By AM $\geq$ GM (for positive numbers)

$$\frac{\frac{\alpha^3}{2}+\frac{\alpha^3}{2}+\frac{\beta^2}{2}+\frac{\beta^2}{2}}{4}\geq\left(\frac{\alpha^6\beta^4}{16}\right)^{\frac{1}{4}}$$

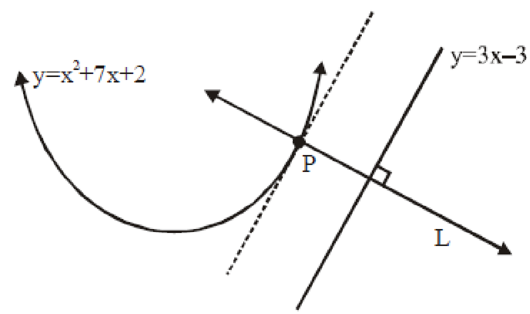
$$\Rightarrow \alpha^6\beta^4\leq 16$$

So, $10K = {}^{10}C_4 16$

$\Rightarrow \quad K = 336$

53. C

Sol. Let L be the common normal to parabola $y = x^2 + 7x + 2$ and line $y = 3x - 3$



$\Rightarrow$ slope of tangent of $y = x^2 + 7x + 2$ at P = 3

$$\Rightarrow \left.\frac{dy}{dx}\right]_{\text{For P}} = 3$$

$\Rightarrow 2x + 7 = 3 \Rightarrow x = -2 \Rightarrow y = -8$

So P (–2, –8)

Normal at P : x + 3y + C = 0

$\Rightarrow$ C = 26 (P satisfies the line)

Normal : x + 3y + 26 = 0

54. B

Sol. The value of $\left(\dfrac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{8}-i\cos\frac{2\pi}{9}}\right)^3$

$$= \left(\frac{1+\sin\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)+i\cos\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)}{1+\sin\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)-i\cos\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)}\right)^3$$

$$= \left(\frac{1+\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}}{1+\cos\frac{5\pi}{18}-i\sin\frac{5\pi}{18}}\right)^3$$

$$= \left(\frac{2\cos^2\frac{5\pi}{36}+2i\sin\frac{5\pi}{36}\cos\frac{5\pi}{36}}{2\cos^2\frac{5\pi}{36}-2i\sin\frac{5\pi}{36}.\cos\frac{5\pi}{36}}\right)^3$$

$$= \left(\frac{\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36}-i\sin\frac{5\pi}{36}}\right)^3$$

$$= \left(\frac{e^{i5\pi/36}}{e^{-i\,5\pi/36}}\right)^3 = \left(e^{i5\pi/18}\right)^3$$

$$= \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}$$
$$= -\frac{\sqrt{3}}{2} + \frac{i}{2}$$

55. B

Sol. Two points on the line (L say) $\frac{x}{3} = \frac{y}{2}, z = 1$ are (0, 0, 1) and (3, 2, 1)

So dr's of the line is < 3, 2, 0 >

Line passing through (1, 2, 1), parallel to L and coplanar with given plane is

$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + t\left(3\hat{i} + 2\hat{j}\right)$, $t \in R\left(-2, 0, 1\right)$ satisfies the line (for t = –1)

$\Rightarrow \left(-2, 0, 1\right)$ lies on given plane.

Answer of the question is (B)

We can check other options by finding equation of plane

Equation plane : $\begin{vmatrix} x-1 & y-2 & z-1 \\ 1+2 & 2-0 & 1-1 \\ 2+2 & 1-0 & 2-1 \end{vmatrix} = 0$

$\Rightarrow 2(x-1) - 3(y-2) - 5(z-1) = 0$

$\Rightarrow 2x - 3y - 5z + 9 = 0$

56. D

Sol. $|A| \neq 0$

For $(P): A \neq I_2$

So, $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

or $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$|A|$ can be –1 or 1

So (P) is false.

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow tr(A) = 2$

$\Rightarrow$ Q is true

57. C

Sol. $\frac{|x|}{2}+\frac{|y|}{3}=1$

$\frac{x^2}{4}+\frac{y^2}{9}=1$

Area of Ellipse $=\pi ab=6\pi$

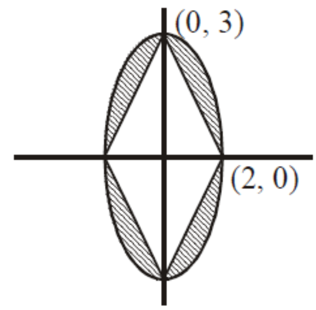
Required area,

$=\pi\times 2\times 3$ – (Area of quadrilateral)

$=6\pi-\frac{1}{2}6\times 4$

$=6\pi-12$

$=6(\pi-2)$



58. D

Sol. $\sigma^2 =$ variance

μ = mean

$\sigma^2=\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}$

$\mu=17$

$\Rightarrow \frac{\sum_{x=1}^{17}(ax+b)}{17}=17$

$\Rightarrow 9a+b=17$ ………..(1)

$\sigma^2=216$

$\Rightarrow \frac{\sum_{x=1}^{17}(ax+b-17)^2}{17}=216$

$\Rightarrow \frac{\sum_{x=1}^{17}a^2(x-9)^2}{17}=216$

$\Rightarrow\ a^2 81-18\times 9a^2+a^2 3\times(35)=216$

$\Rightarrow\ a^2=\frac{216}{24}=9 \quad \Rightarrow \quad a=3(a>0)$

$\Rightarrow$ From (1), b = – 10

So, $a+b=-7$

59. C

Sol. Slope of tangent to the curve $y=x+\sin y$

at (a, b) is $\dfrac{2-\dfrac{3}{2}}{\dfrac{1}{2}-0}=1$

$\Rightarrow \left.\dfrac{dy}{dx}\right]_{x=a}=1$

$\dfrac{dy}{dx}=1+\cos y.\dfrac{dy}{dx}$ (from equation of curve)

$\Rightarrow \left.\dfrac{dy}{dx}\right]_{x=a \text{ and } y=b}=1+\cos b\ =1$

$\Rightarrow \quad \cos b=0$

$\Rightarrow \quad \sin b=\pm 1$

Now, from curve $y=x+\sin y$

$b=a+\sin b$

$\Rightarrow \ |b-a|=|\sin b|=1$

60. D

Sol. $\dfrac{2+\sin x}{y+1}\dfrac{dy}{dx}=-\cos x, y>0$

$\Rightarrow \dfrac{dy}{y+1}=\dfrac{-\cos x}{2+\sin x}dx$

By integrating both sides:

$\ell n|y+1|=-\ell n|2+\sin x|+\ell n K$

$\Rightarrow y+1=\dfrac{K}{2+\sin x} \qquad (y+1>0)$

$\Rightarrow y(x)=\dfrac{K}{2+\sin x}-1$

Given $y(0)=1 \Rightarrow K=4$

So, $y(x)=\dfrac{4}{2+\sin x}-1$

$a=y(\pi)=1$

$b=\left.\dfrac{dy}{dx}\right]_{x=\pi}=\left.\dfrac{-\cos x}{2+\sin x}\left(y(x)+1\right)\right]_{x=\pi}=1$

So, $(a,b)=(1,1)$

61. D

Sol. Let p denotes statement

p : I reach the station in time.

q : I will catch the train.

Contrapositive of $p \to q$ is $\sim q \to \sim p$

$\sim q \to \sim p$: I will not catch the train, then I do not reach the station in time.

62. D

Sol. $f(x) = \sin\left(\frac{|x|+5}{x^2+1}\right)$

For domain:

$-1 \le \frac{|x|+5}{x^2+1} \le 1$

Since $|x|+5$ and x^2+1 is always positive

So $\frac{|x|+5}{x^2+1} \ge 0 \ \forall x \in R$

So for domain:

$\frac{|x|+5}{x^2+1} \le 1$

$\Rightarrow |x|+5 \le x^2+1$

$\Rightarrow 0 \le x^2 - |x| - 4$

$\Rightarrow 0 \le \left(|x| - \frac{1+\sqrt{17}}{2}\right)\left(|x| - \frac{1-\sqrt{17}}{2}\right)$

$\Rightarrow |x| \ge \frac{1+\sqrt{17}}{2}$ or $|x| \le \frac{1-\sqrt{17}}{2}$ (Rejected)

$\Rightarrow x \in \left(-\infty, -\frac{1+\sqrt{17}}{2}\right] \cup \left[\frac{1+\sqrt{17}}{2}, \infty\right)$

So, $a = \frac{1+\sqrt{17}}{2}$

63. C

Sol. α and β are roots of $5x^2 + 6x - 2 = 0$

$\Rightarrow 5\alpha^2 + 6\alpha - 2 = 0$

$\Rightarrow 5\alpha^{n+2} + 6\alpha^{n+1} - 2\alpha^n = 0$ ……….(1)

(By multiplying α^n)

Similarly $5\beta^{n+2} + 6\beta^{n+1} - 2\beta^n = 0$ ………..(2)

By adding (1) and (2)

$5S_{n+2} + 6S_{n+1} - 2S_n = 0$

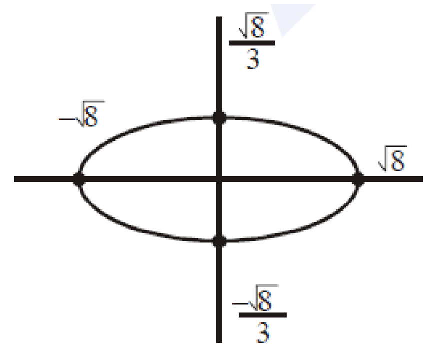
For n = 4

$5S_6 + 6S_5 = 2S_4$

64. C

Sol. $R\left\{(x,y): x, y \in z,\ x^2+3y^2 \le 8\right\}$

For domain of R^{-1}
Collection of all integral of y's
For $x=0,\ 3y^2 \le 8$
$\Rightarrow y \in \{-1, 0, 1\}$



65. D

Sol. Since $p(x)$ has relative extreme at x = 1 and 2

so $p'(x)=0$ at x = 1 and 2

$\Rightarrow p'(x) = A(x-1)(x-2)$

$\Rightarrow p(x) = \int A\left(x^2-3x+2\right)dx$

$p(x) = A\left(\frac{x^3}{3}-\frac{3x^2}{2}+2x\right)+C$ ………….(1)

$P(1)=8$

From (1)

$8 = A\left(\frac{1}{3}-\frac{3}{2}+2\right)+C$

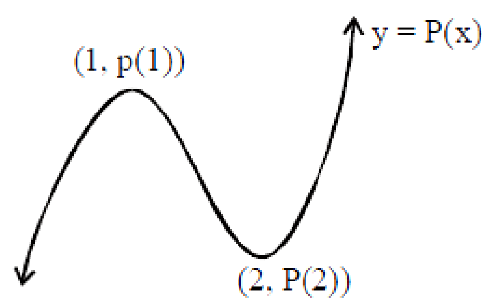
$\Rightarrow 8 = \frac{5A}{6}+C \Rightarrow 48 = 5A+6C$ ……….(3)

$P(2)=4$

$\Rightarrow 4 = A\left(\frac{8}{3}-6+4\right)+C$

$\Rightarrow 4 = \frac{2A}{3}+C \Rightarrow 12 = 2A+3C$ ………..(4)

From 3 and 4, C = –12
So P (0) = C = – 12



66. B

Sol. $f(x) = \begin{cases} ae^x + be^{-x}, & -1 \le x < 1 \\ cx^2\ , & 1 \le x \le 3 \\ ax^2+2cx\ , & 3 < x \le 4 \end{cases}$

For continuity at x = 1

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x)$

$\Rightarrow \quad ae + be^{-1} = c \quad \Rightarrow \quad b = ce - ae^2$ ……….(1)

For continuity at x = 3

$\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^+} f(x)$

$\Rightarrow 9c = 9a + 6c$

$\Rightarrow c = 3a$ ………(2)

$f'(0) + f'(2) = e$

$\left(ae^x - be^x\right)_{x=0} + (2cx)_{x=2} = e$

$\Rightarrow \ a - b + 4c = e$ ………..(3)

From (1), (2) and (3)

$a - 3ae + ae^2 + 12a = e$

$\Rightarrow \ a\left(e^2 + 13 - 3e\right) = e$

$$\Rightarrow a = \frac{e}{e^2 - 3e + 13}$$

67. B

Sol. Let three terms of G.P. are $\frac{a}{r}, a, ar$ product = 27

$\Rightarrow a^3 = 27 \Rightarrow a = 3$

$S = \frac{3}{r} + 3r + 3$

For $r > 0$

$\frac{\frac{3}{r} + 3r}{2} \geq \sqrt{3^2}$ (By AM $\geq$ GM)

$\Rightarrow \frac{3}{r} + 3r \geq 6$ ……….(1)

For $r < 0$, $\frac{3}{r} + 3r \leq -6$ …………(2)

From (1) and (2)

$S \in (-\infty, -3] \cup [9, \infty)$

68. D

Sol. $2x - y + 2z = 2$

$x - 2y + \lambda z = -4$

$x + \lambda y + z = 4$

For no solution:

$$D = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix} = 0$$

$$\Rightarrow 2\left(-2-\lambda^2\right)+1(1-\lambda)+2(\lambda+2)=0$$

$$\Rightarrow -2\lambda^2+\lambda+1=0$$

$$\Rightarrow \lambda = 1, -\frac{1}{2}$$

$$D_x = \begin{vmatrix} 2 & -1 & 2 \\ -4 & 2 & \lambda \\ 4 & \lambda & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & -1 & 2 \\ -2 & -2 & \lambda \\ \lambda & \lambda & 1 \end{vmatrix}$$

$=2(1+\lambda)$ which is not equal to zero for $\lambda = 1, -\frac{1}{2}$

69. A

Sol. Slope of tangent is 2, Tangent of hyperbola

$\frac{x^2}{4}-\frac{y^2}{2}=1$ at the point (x_1, y_1) is $\frac{xx_1}{4}-\frac{yy_1}{2}=1$ (T = 0)

Slope : $\frac{1}{2}\frac{x_1}{y_1}=2 \Rightarrow x_1 = 4y_1$ ....(1)

(x_1, y_1) lies on hyperbola

$\Rightarrow \frac{x_1^2}{4}-\frac{y_1^2}{2}=1$ ....(2)

From (1) and (2)

$$\frac{(4y_1)^2}{4}-\frac{y_1^2}{2}=1 \Rightarrow 4y_1^2-\frac{y_1^2}{2}=1$$

$$\Rightarrow 7y_1^2 = 2 \Rightarrow y_1^2 = \frac{2}{7}$$

Now $x_1^2+5y_1^2 = (4y_1)^2+5y_1^2$

$$=(21)y_1^2 = 21\times\frac{2}{7}=6$$

70. A

Sol. Let B_1 be the event where Box – I is selected and $B_2 \rightarrow$ where box – II selected

$$P(B_1) = P(B_2) = \frac{1}{2}$$

Let E be the event where selected card is non prime.

For B_1: Prime numbers:

$$\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$$

For B_2: Prime numbers:

$$\{31, 37, 41, 43, 47\}$$

$$P(E) = P(B_1) \times P\left(\frac{E}{B_1}\right) + P(B_2)P\left(\frac{E}{B_2}\right)$$

$$= \frac{1}{2} \times \frac{20}{30} + \frac{1}{2} \times \frac{15}{20}$$

Required probability:

$$P\left(\frac{B_1}{E}\right) = \frac{\frac{1}{2} \times \frac{20}{30}}{\frac{1}{2} \times \frac{20}{30} + \frac{1}{2} \times \frac{15}{20}} = \frac{\frac{2}{3}}{\frac{2}{3} + \frac{3}{4}} = \frac{8}{17}$$

71. 309.00

Sol. MOTHER

$1 \to E$
$2 \to H$
$3 \to M$
$4 \to O$
$5 \to R$
$6 \to T$

So position of word MOTHER in dictionary

$2 \times 5! + 2 \times 4! + 3 \times 3! + 2! + 1$
$= 240 + 48 + 18 + 2 + 1$
$= 309$

72. 2.00

Sol. $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$|\vec{a} - \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 8$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b} + |\vec{a}|^2 + |\vec{c}|^2 - 2\vec{a}.\vec{c} = 8$

$\Rightarrow 4 - 2(\vec{a}.\vec{b} + \vec{a}.\vec{c}) = 8$

$\Rightarrow \vec{a}.\vec{b} + \vec{a}.\vec{c} = -2$

$|\vec{a} + 2\vec{b}|^2 + |\vec{a} + 2\vec{c}|^2$

$= |\vec{a}|^2 + 4|\vec{b}|^2 + 4\vec{a}.\vec{b} + |\vec{a}|^2 + 4|\vec{c}|^2 + 4\vec{a}.\vec{c}$

$= 10 + 4(\vec{a}.\vec{b} + \vec{a}.\vec{c})$

$= 10 - 8$
$= 2$

73. 40.00

Sol. $\lim\limits_{x \to 1} \dfrac{x + x^2 + x^3 + ... + x^n - n}{x - 1} = 820$

$$\Rightarrow \lim_{x\to1}\left(\frac{x-1}{x-1}+\frac{x^2-1}{x-1}+......\frac{x^n-1}{x-1}\right)=820$$

$\Rightarrow 1+2+.....+n=820$

$\Rightarrow n(n+1)=2\times820$

$\Rightarrow n(n+1)=40\times41$

Since $n\in N$, so $n=40$

74. 9.00

Sol. Circle $x^2+y^2-2x-4y+4=0$

$\Rightarrow (x-1)^2+(y-2)^2=1$

Centre : (1, 2) radius = 1

line $3x+4y-k=0$ intersects the circle at two distinct points.

$\Rightarrow$ distance of centre from the line < radius

$$\Rightarrow \left|\frac{3\times1+4\times2-k}{\sqrt{3^2+4^2}}\right|<1$$

$\Rightarrow |11-k|<5$

$\Rightarrow 6<k<16$

$\Rightarrow k\in\{7,8,9,.......15\}$ since $k\in I$

Number of K is 9.

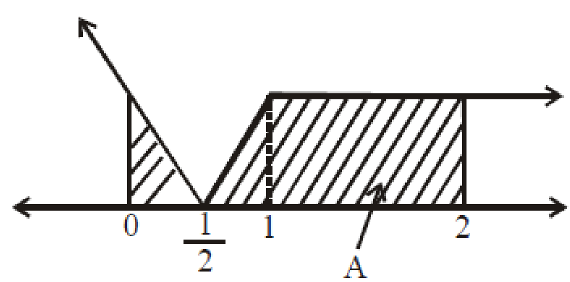
75. 1.50

Sol. $\int_0^2 \left||x-1|-x\right|dx$

Let $f(x)=\left||x-1|-x\right|$

$$=\begin{cases}1, & x\ge1\\ |1-2x|, & x\le1\end{cases}$$

$A=\frac{1}{2}+1=\frac{3}{2}$



Or

$$\int_0^{1/2}(1-2x)dx+\int_{1/2}^{1}(2x-1)+\int_0^2 1dx$$

$$=\left[x-x^2\right]_0^{\frac{1}{2}}+[x]_1^2$$

$$=\frac{3}{2}$$