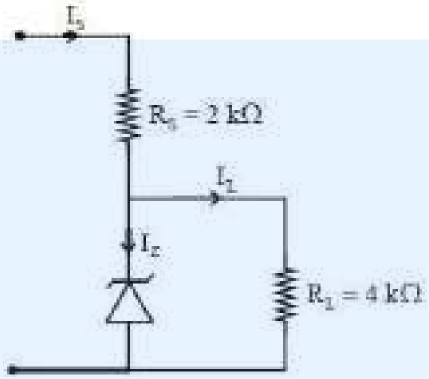
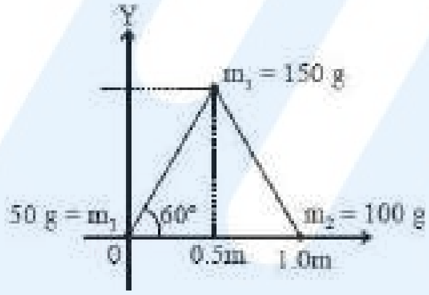
PART – A (PHYSICS)

1. Figure shown a DC voltage regulator circuit, with a Zener diode of breakdown voltage = 6V. If the unregulated input voltage varies between 10 V to 16 V, then what is the maximum Zener current?



(A) 2.5 mA
(B) 3.5 mA
(C) 7.5 mA
(D) 1.5 mA

2. Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be :

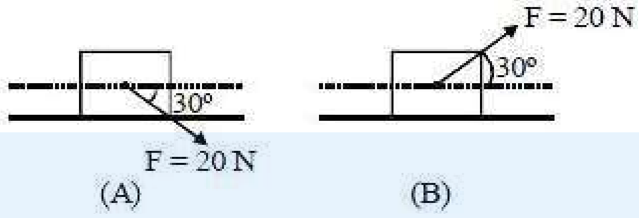


(A) $\left(\frac{\sqrt{3}}{7}m, \frac{7}{12}m\right)$
(B) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{8}m\right)$
(C) $\left(\frac{\sqrt{3}}{4}m, \frac{5}{12}m\right)$
(D) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{4}m\right)$

3. The ratio of the weights of a body on the Earth's surface to that on the surface of a planet is 9 : 4. The mass of the planet is $\frac{1}{9}$th of that of the Earth. If 'R' is the radius of the Earth, what is the radius of the planet ? (Take the planets to have the same mass density)

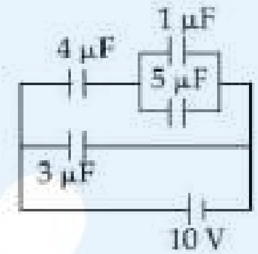
(A) $\frac{R}{3}$
(B) $\frac{R}{4}$
(C) $\frac{R}{9}$
(D) $\frac{R}{2}$

4. A block of mass 5 kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force F = 20 N, making an angle of 30º with the horizontal, as shown in the figures. The coefficient of friction between the block and floor is m = 0.2. The difference between the accelerations of the block, in case (B) and case (A) will be : (g = 10 ms^{-2})



(A) 0.4 ms^{-2} (B) 3.2 ms^{-2}
(C) 0 ms^{-2} (D) 0.8 ms^{-2}

5. In the given circuit, the charge on 4 μF capacitor will be :



(A) 13.4 μC
(B) 24 μC
(C) 9.6 μC
(D) 5.4 μC

6. A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process?
(A) 40 J (B) 30 J
(C) 35 J (D) 25 J

7. A Carnot engine has an efficiency of 1/6. When the temperature of the sink is reduced by 62ºC, its efficiency is doubled. The temperatures of the source and the sink are, respectively
(A) 62ºC, 124ºC (B) 99ºC, 37ºC
(C) 37ºC, 99ºC (D) 124ºC, 62ºC

8. Consider an electron in a hydrogen atom, revolving in its second excited state (having radius 4.65 Å). The de-Broglie wavelength of this electron is :
(A) 12.9 Å (B) 9.7 Å
(C) 6.6 Å (D) 3.5 Å

9. A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity sound? [Given reference intensity of sound as 10^{-12}W/m^2]
(A) 30 cm (B) 10 cm
(C) 40 cm (D) 20 cm

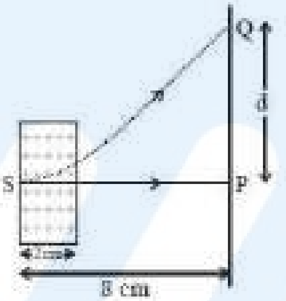
10. A system of three polarizers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at 60º to the pass axis of P_3. When a beam of unpolarized light of intensity I0 is incident on P_1, the intensity of light transmitted by the three polarizers is I. The ratio (I_0/I) equals (nearly) :
(A) 10.67 (B) 1.80
(C) 5.33 (D) 16.00

11. Let a total charge 2Q be distributed in a sphere of radius R, with the charge density given by r(r) = kr, where r is the distance from the centre. Two charges A and B, of –Q each, are placed on diametrically opposite points, at equal distance, a, from the centre. If A and B do not experience any force, then :

(A) $a = \frac{3R}{2^{1/4}}$ (B) $a = 2^{-1/4}R$

(C) $a = 8^{-1/4}R$ (D) $a = R/\sqrt{3}$

12. An electron, moving along the x-axis with an initial energy of 100 eV, enters a region of magnetic field $\vec{B} = \left(1.5 \times 10^{-3}\,T\right)\hat{k}$ at S (See figure). The field extends between x = 0 and x = 2 cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q (on the screen) is :
(electron's charge = 1.6×10^{-19} C, mass of electron = 9.1×10^{-31} kg)



(A) 12.87 cm (B) 2.25 cm
(C) 1.22 cm (D) 11.65 cm

13. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct?
(A) $R^2 = 4\,h_1h_2$ (B) $R^2 = 2\,h_1h_2$
(C) $R^2 = 16\,h_1h_2$ (D) $R^2 = h_1h_2$

14. A particle is moving with speed $\nu = v\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time t = t(assume that the particle is at origin at t = 0).

(A) $b^2\tau$ (B) $\frac{b^2\tau}{2}$

(C) $\frac{b^2\tau}{\sqrt{2}}$ (D) $\frac{b^2\tau}{4}$

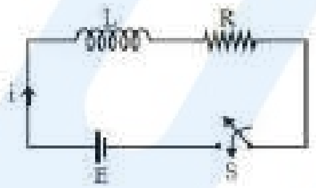
15. One kg of water, at 20ºC, is heated in an electric kettle whose heating element has a mean (temperature averaged) resistance of 20 Ω. The rms voltage in the mains is 200 V. Ignoring heat loss from the kettle, time taken for water to evaporate fully, is close to :
[Specific heat of water = 4200 J/kg ºC),
Latent heat of water = 2260 kJ/kg]
(A) 3 minutes (B) 10 minutes
(C) 22 minutes (D) 16 minutes

16. Half lives of two radioactive nuclei A and B are 10 minutes and 20 minutes, respectively. If, initially a sample has equal number of nuclei, then after 60 minutes, the ratio of decayed numbers of nuclei A and B will be :
(A) 9 : 8 (B) 1 : 8
(C) 8 : 1 (D) 3 : 8

17. In an amplitude modulator circuit, the carrier wave is given by, $C(t) = 4 \sin (20000\, \pi t)$ while modulating signal is given by, $m(t) = 2 \sin (2000\, \pi t)$. The values of modulation index and lower side band frequency are:
(A) 0.5 and 9 kHz (B) 0.3 and 9 kHz
(C) 0.5 and 10 kHz (D) 0.4 and 10 kHz

18. A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of Elasticity equals Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion, of the material of the rod, is (nearly) equals to :
(A) $9F/(\pi r^2 YT)$ (B) $F/(3\pi r^2 YT)$
(C) $3F/(\pi r^2 YT)$ (D) $6F/(\pi r^2 YT)$

19. Consider the LR circuit shown in the figure. If the switch S is closed at t = 0 then the amount of charge that passes through the battery between t = 0 and $t = \frac{L}{R}$ is :

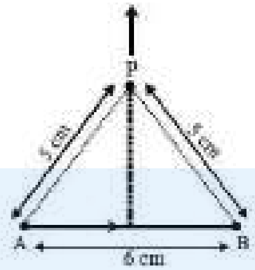


(A) $\frac{EL}{7.3R^2}$ (B) $\frac{EL}{2.7R^2}$
(C) $\frac{7.3\,EL}{R^2}$ (D) $\frac{2.7\,EL}{R^2}$

20. Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then, u equals:
(A) 15.0 m/s (B) 10.0 m/s
(C) 5.5 m/s (D) 2.5 m/s

21. A plane electromagnetic wave having a frequency n = 23.9 GHz propagates along the positive z-direction in free space. The peak value of the electric field is 60 V/m. Which among the following is the acceptable magnetic field component in the electromagnetic wave?
(A) $\vec{B} = 2 \times 10^{-7} \sin\left(1.5 \times 10^{2} x + 0.5 \times 10^{11} t\right)\hat{j}$ (B) $\vec{B} = 60 \sin\left(0.5 \times 10^{3} x + 1.5 \times 10^{11} t\right)\hat{k}$
(C) $\vec{B} = 2 \times 10^{-7} \sin\left(0.5 \times 10^{2} z - 1.5 \times 10^{11} t\right)\hat{i}$ (D) $\vec{B} = 2 \times 10^{7} \sin\left(0.5 \times 10^{3} z + 1.5 \times 10^{11} t\right)\hat{i}$

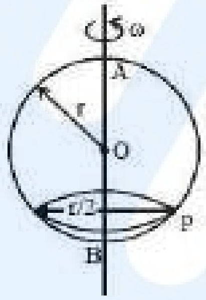
22. Find the magnetic field at point P due to a straight line segment AB of length 6 cm carrying a current of 5 A. (See figure)
($\mu 0$ = 4p × 10–7 N-A–2)



(A) 2.0×10^{-5} T
(B) 3.0×10^{-5} T
(C) 2.5×10^{-5} T
(D) 1.5×10^{-5} T

23. The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths, λ_1/λ_2, of the photons emitted in this process is :
(A) 20/7
(B) 7/5
(C) 9/7
(D) 27/5

24. A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed w about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to:

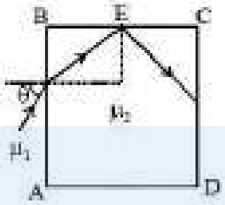


(A) $\frac{\sqrt{3}g}{2r}$
(B) $\left(g\sqrt{3}\right)/r$
(C) $2g/r$
(D) $2g/\left(r\sqrt{3}\right)$

25. A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2, when falling through the same fluid, the ratio (v_1/v_2) equals :
(A) 27
(B) 1/27
(C) 9
(D) 1/9

26. The number density of molecules of a gas depends on their distance r from the origin as, $n(r) = n_0 e^{-\alpha r^4}$. Then the total number of molecules is proportional to :
(A) $n_0\alpha^{-3/4}$
(B) $n_0\alpha^{-3}$
(C) $n_0\alpha^{1/4}$
(D) $\sqrt{n_0}\,\alpha^{1/2}$

27. A transparent cube of side d, made of a material of refractive index μ_2, is immersed in a liquid of refractive index $\mu_1(\mu_1 < \mu_2)$. A ray is incident on the face AB at an angle q(shown in the figure). Total internal reflection takes place at point E on the face BC.



The q must satisfy :

(A) $\theta > \sin^{-1}\frac{\mu_1}{\mu_2}$

(B) $\theta > \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$

(C) $\theta < \sin^{-1}\frac{\mu_1}{\mu_2}$

(D) $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$

28. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (ν) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column, $\ell_1 = 30$ cm and $\ell_2 = 70$ cm. Then n is equal to :

(A) 332 ms^{-1}
(B) 338 ms^{-1}
(C) 384 ms^{-1}
(D) 379 ms^{-1}

29. A spring whose unstretched length is ℓ has a force constant k. The spring is cut into two pieces of unstretched lengths ℓ_1 and ℓ_2 where, $\ell_1 = n\ell_2$ and n is an integer. The ratio k_1/k_2 of the corresponding force constants, k_1 and k_2 will be:

(A) n

(B) $\frac{1}{n^2}$

(C) n^2

(D) $\frac{1}{n}$

30. A moving coil galvanometer, having a resistance G, produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into (i) an ammeter of range 0 to I0 ($I_0 > I_g$) by connecting a shunt resistance RA to it and (ii) into a voltmeter of range 0 to V($V = GI_0$) by connecting a series resistance RV to it. Then,

(A) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \frac{I_g}{(I_0 - I_g)}$

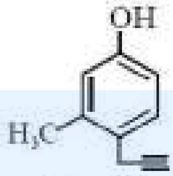
(B) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

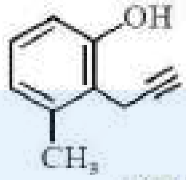
(C) $R_A R_V = G^2\left(\frac{I_g}{I_0 - I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_0 - I_g}{I_g}\right)^2$

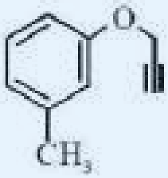
(D) $R_A - R_V = G^2\left(\frac{I_0 - I_g}{I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{(I_0 - I_g)}\right)^2$

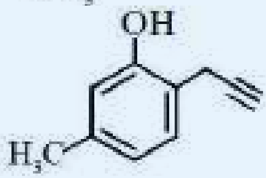
PART –B (CHEMISTRY)

31. What will be the major product when m-cresol is reacted with propargyl bromide ($HC \equiv C - CH_2Br$) in presence of K_2CO_3 in acetone

(A) 

(B) 

(C) 

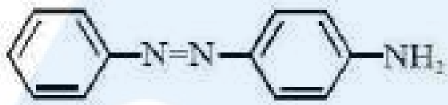
(D) 

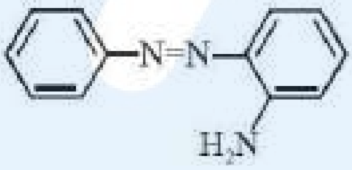
32. Which one of the following is likely to give a precipitate with $AgNO_3$ solution?
(A) $CH_2 = CH - Cl$
(B) $CHCl_3$
(C) $(CH_3)_3CCl$
(D) CCl_4

33. The INCORRECT match in the following is:
(A) $\Delta G^\circ < 0, K > 1$
(B) $\Delta G^\circ < 0, K < 1$
(C) $\Delta G^\circ = 0, K = 1$
(D) $\Delta G^\circ > 0, K < 1$

34. Benzene diazonium chloride on reaction with aniline in the presence of dilute hydrochloric acid gives :

(A) 

(B) 

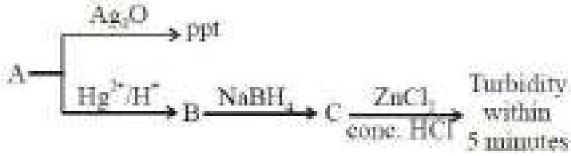
(C) 

(D) 

35. The INCORRECT statement is:
(A) $LiNO_3$ decomposes on heating to give $LiNO_2$ and O_2.
(B) Lithium is least reactive with water among the alkali metals.
(C) LiCl crystallises from aqueous solution as $LiCl.2H_2O$.
(D) Lithium is the strongest reducing agent among the alkali metals.

36. The C–C bond length is maximum in
(A) graphite
(B) C_{70}
(C) diamond
(D) C_{60}

37. Consider the following reactions :



'A' is:

(A) $CH_2 = CH_2$ (B) $CH_3 - C \equiv CH$

(C) $CH \equiv CH$ (D) $CH_3 - C \equiv C - CH_3$

38. In which one of the following equilibria, $K_p \neq K_c$?

(A) $2NO(g) \rightleftharpoons N_2(g) + O_2(g)$ (B) $2C(s) + O_2(g) \rightleftharpoons 2CO(g)$

(C) $NO_2(g) + SO_2(g) \rightleftharpoons NO(g) + SO_3(g)$ (D) $2HI(g) \rightleftharpoons H_2(g) + I_2(g)$

39. The coordination numbers of Co and Al in $[Co(Cl)(en)_2]Cl$ and $K_3[Al(C_2O_4)_3]$, respectively, are

(en = ethane-1,2-diamine)

(A) 6 and 6 (B) 5 and 3

(C) 3 and 3 (D) 5 and 6

40. Which of the given statements is INCORRECT about glycogen?

(A) It is present in some yeast and fungi

(B) It is present in animal cells

(C) Only α-linkages are present in the molecule

(D) It is a straight chain polymer similar to amylase

41. The primary pollutant that leads to photochemical smog is:

(A) acrolein (B) nitrogen oxides

(C) ozone (D) sulphur dioxide

42. The ratio of number of atoms present in a simple cubic, body centered cubic and face centered cubic structure are, respectively :

(A) 1 : 2 : 4 (B) 4 : 2 : 3

(C) 4 : 2 : 1 (D) 8 : 1 : 6

43. Thermal decomposition of a Mn compound (X) at 513 K results in compound Y, MnO_2 and a gaseous product. MnO_2 reacts with NaCl and concentrated H_2SO_4 to give a pungent gas Z. X, Y and Z, respectively.

(A) K_2MnO_4, $KMnO_4$ and SO_2 (B) K_3MnO_4, K_2MnO_4 and Cl_2

(C) K_2MnO_4, $KMnO_4$ and Cl_2 (D) $KMnO_4$, K_2MnO_4 and Cl_2

44. An ' Assertion' and a 'Reason' are given below. Choose the correct answer from the following options.

Assertion (A): Vinyl halides do not undergo nucleophilic substitution easily.

Reason (R): Even though the intermediate carbocation is stabilized by loosely held p-electrons, the cleavage is difficult because of strong bonding.

(A) Both (A) and (R) are correct statements but (R) is not the correct explanation of (A)

(B) Both (A) and (R) are wrong statements

(C) Both (A) and (R) are correct statements and (R) is the correct explanation of (A)

(D) (A) is a correct statement but (R) is a wrong statement.

45. The molar solubility of $Cd(OH)_2$ is 1.84×10^{-5} M in water. The expected solubility of $Cd(OH)_2$ in a buffer solution of pH = 12 is :
(A) 6.23×10^{-11} M
(B) 1.84×10^{-9} M
(C) $\frac{2.49}{1.84} \times 10^{-9}$ M
(D) 2.49×10^{-10} M

46. The compound used in the treatment of lead poisoning is:
(A) EDTA
(B) Cis-platin
(C) D-penicillamine
(D) desferrioxime B

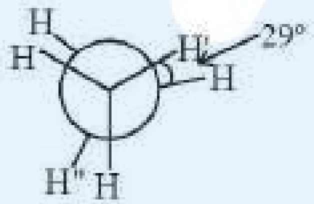
47. The pair that has similar atomic radii is:
(A) Ti and Hf
(B) Mn and Re
(C) Sc and Ni
(D) Mo and W

48. 25 g of an unknown hydrocarbon upon burning produces 88 g of CO_2 and 9 g of H_2O. This unknown hydrocarbon contains.
(A) 24g of carbon and 1 g of hydrogen
(B) 22g of carbon and 3 g of hydrogen
(C) 18g of carbon and 7 g of hydrogen
(D) 20g of carbon and 5 g of hydrogen

49. The decreasing order of electrical conductivity of the following aqueous solutions is:
0.1 M Formic acid (a)
0.1 M Acetic acid (b)
0.1 M Benzoic acid (c)
(A) a > c > b
(B) c > a > b
(C) c > b > a
(D) a > b > c

50. Among the following, the energy of 2s orbital is lowest in:
(A) K
(B) Na
(C) H
(D) Li

51. In the following skew conformation of ethane, H'–C–C–H" dihedral angle is :



(A) 58°
(B) 120°
(C) 149°
(D) 151°

52. The correct statement is:
(A) leaching of bauxite using concentrated NaOH solution gives sodium aluminate and sodium silicate
(B) the Hall-Heroult process is used for the production of aluminium and iron
(C) the blistered appearance of copper during the metallurgical process is due to the evolution of CO_2
(D) pig iron is obtained from cast iron

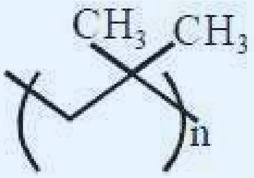
53. NO_2 required for a reaction is produced by the decomposition of N_2O_5 in CCl_4 as per the equation

$2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$

The initial concentration of N_2O_5 is 3.00 mol L^{-1} and it is 2.75 mol L^{-1} after 30 minutes. The rate of formation of NO_2 is :

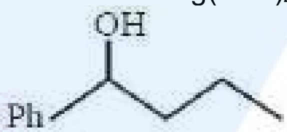
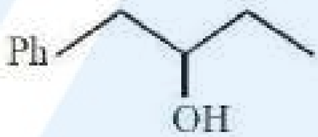
(A) 1.667×10^{-2} mol L^{-1} min^{-1}
(B) 4.167×10^{-3} mol L^{-1} min^{-1}
(C) 8.333×10^{-3} mol L^{-1} min^{-1}
(D) 2.083×10^{-3} mol L^{-1} min^{-1}

54. The correct name of the following polymer is:

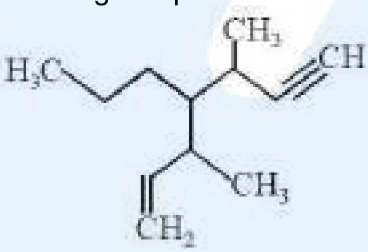


(A) Polyisoprene
(B) Polytert-butylene
(C) Polyisobutane
(D) Polyisobutylene

55. Heating of 2-chloro-1-phenylbutane with EtOK/EtOH gives X as the major product. Reaction of X with $Hg(OAc)_2/H_2O$ followed by $NaBH_4$ gives Y as the major product. Y is:

(A) 
(B) 
(C) 
(D) 

56. The IUPAC name of the following compound is :



(A) 3, 5-dimethyl-4-propylhept-1-en-6-yne
(B) 3-methyl-4-(1-methylprop-2-ynyl)-1-heptene
(C) 3-methyl-4-(3-methylprop-1-enyl)-1-heptyne
(D) 3, 5-dimethyl-4-propylhept-6-en-1-yne

57. Among the following, the INCORRECT statement about colloids is :
(A) The range of diameters of colloidal particles is between 1 and 1000 nm
(B) The osmotic pressure of a colloidal solution is of higher order than the true solution at the same concentration
(C) They can scatter light
(D) They are larger than small molecules and have high molar mass

58. In comparison to boron, berylium has:
(A) greater nuclear charge and greater first ionisation enthalpy
(B) lesser nuclear charge and lesser first ionisation enthalpy
(C) greater nuclear charge and lesser first ionisation enthalpy
(D) lesser nuclear charge and greater first ionisation enthalpy

59. A solution is prepared by dissolving 0.6 g of urea (molar mass = 60 g mol^{-1}) and 1.8 g of glucose (molar mass = 180 g mol^{-1}) in 100 mL of water at 27ºC. The osmotic pressure of the solution is :
(R = 0.08206 L atm K^{-1} mol^{-1})
(A) 8.2 atm
(B) 1.64 atm
(C) 4.92 atm
(D) 2.46 atm

60. The temporary hardness of a water sample is due to compound X. Boiling this sample converts X to compound Y. X and Y, respectively, are :
(A) $Mg(HCO_3)_2$ and $MgCO_3$
(B) $Ca(HCO_3)_2$ and CaO
(C) $Mg(HCO_3)_2$ and $Mg(OH)_2$
(D) $Ca(HCO_3)_2$ and $Ca(OH)_2$

PART–C (MATHEMATICS)

61. The general solution of the differential equation $(y^2 - x^3)\,dx - xy\,dy = 0$ $(x \neq 0)$ is : (where c is a constant of integration)
(A) $y^2 + 2x^3 + cx^2 = 0$
(B) $y^2 - 2x^3 + cx^2 = 0$
(C) $y^2 + 2x^2 + cx^3 = 0$
(D) $y^2 - 2x^2 + cx^3 = 0$

62. Let $\alpha \in R$ and the three vectors $\vec{a} = \alpha\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$ and $\vec{c} = \alpha\hat{i} - 2\hat{j} + 3\hat{k}$. Then the set $S = \left(\alpha : \vec{a}, \vec{b} \text{ and } \vec{c} \text{ are coplanar}\right)$
(A) Contains exactly two numbers only one of which is positive
(B) is empty
(C) Contains exactly two positive numbers
(D) is singleton

63. Let $\alpha \in (0, \pi/2)$ be fixed. If the integral $\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha}dx =$ $A(x)\cos 2\alpha + B(x)\sin 2\alpha + C$, where C is a constant of integration, then the functions A(x) and B(x) are respectively:
(A) $x + \alpha$ and $\log_e\left|\sin(x - \alpha)\right|$
(B) $x - \alpha$ and $\log_e\left|\cos(x - \alpha)\right|$
(C) $x - \alpha$ and $\log_e\left|\sin(x - \alpha)\right|$
(D) $x + \alpha$ and $\log_e\left|\sin(x + \alpha)\right|$

64. Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to :
(A) [3, 7]
(B) R
(C) [2, 6]
(D) [1, 4]

65. Let $f(x) = 5 - |x - 2|$ and $g(x) = |x + 1|, x \in R$. If f(x) attains maximum value at α and g(x) attains minimum value at β, then $\lim\limits_{x \to -\alpha\beta} \frac{(x-1)(x^2 - 5x + 6)}{x^2 - 6x + 8}$ is equal to :
(A) $\frac{3}{2}$
(B) $\frac{-3}{2}$
(C) $\frac{1}{2}$
(D) $\frac{-1}{2}$

66. Let $z \in C$ with Im(z) = 10 and it satisfies $\frac{2z - n}{2z + n} = 2i - 1$ for some natural number n. Then :
(A) n = 40 and Re(z) = 10
(B) n = 20 and Re(z) = 10
(C) n = 40 and Re(z) = – 10
(D) n = 20 and Re(z) = – 10

67. The term independent of x in the expansion of $\left(\frac{1}{60} - \frac{x^8}{81}\right)\cdot\left(2x^2 - \frac{3}{x^2}\right)^6$ is equal to:
(A) 36
(B) – 36
(C) – 108
(D) – 72

68. If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is $\frac{1}{9}$, then λ is equal to :

(A) 48
(B) $4\sqrt{3}$
(C) $2\sqrt{6}$
(D) 24

69. If [x] denotes the greatest integer $\le$ x, then the system of linear equations
$[\sin\theta]x + [-\cos\theta]y = 0$
$[\cot\theta]x + y = 0$

(A) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

(B) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(C) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

(D) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

70. For an initial screening of an admission test, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is $\frac{4}{5}$, then the probability that he is unable to solve less than two problems is :

(A) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$
(B) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$
(C) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$
(D) $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$

71. If $a_1, a_2, a_3, \ldots\ldots$ are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is:

(A) 200
(B) 280
(C) 150
(D) 120

72. An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points?

(A) $\left(2, \sqrt{2}\right)$
(B) $\left(2, 2\sqrt{2}\right)$
(C) $\left(1, 2\sqrt{2}\right)$
(D) $\left(\sqrt{2}, 2\right)$

73. The length of the perpendicular drawn from the point (2, 1, 4) to the plane containing the lines $\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right)$ and $\vec{r} = \left(\hat{i} + \hat{j}\right) + \mu\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$ is:

(A) $\frac{1}{3}$
(B) $\sqrt{3}$
(C) $\frac{1}{\sqrt{3}}$
(D) 3

74. Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true?
(A) If $(A - C) \subseteq B$, then $A \subseteq B$
(B) If $(A - B) \subseteq C$, then $A \subseteq C$
(C) $(C \cup A) \cap (C \cup B) = C$
(D) $B \cap C \neq \phi$

75. If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to :
(A) $\alpha\gamma$
(B) 0
(C) $\alpha\beta$
(D) $\beta\gamma$

76. The tangents to the curve $y = (x - 2)^2 - 1$ at its points of intersection with the line $x - y = 3$, intersect at the point :
(A) $\left(\frac{5}{3}, 1\right)$
(B) $\left(-\frac{5}{2}, -1\right)$
(C) $\left(-\frac{5}{2}, 1\right)$
(D) $\left(\frac{5}{2}, -1\right)$

77. The angle of elevation of the top of vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is:
(A) $15\left(1+\sqrt{3}\right)$
(B) $15\left(3-\sqrt{3}\right)$
(C) $15\left(3+\sqrt{3}\right)$
(D) $15\left(5-\sqrt{3}\right)$

78. A triangle has a vertex at (1, 2) and the mid points of the two sides through it are (–1, 1) and (2,3). Then the centroid of this triangle is :
(A) $\left(1, \frac{7}{3}\right)$
(B) $\left(\frac{1}{3}, 1\right)$
(C) $\left(\frac{1}{3}, 2\right)$
(D) $\left(\frac{1}{3}, \frac{5}{3}\right)$

79. A person throws two fair dice. He wins Rs. 15 for throwing a doublet (same numbers on the two dice), wins Rs.12 when the throw results in the sum of 9, and loses Rs. 6 for any other outcome on the throw. Then the expected gain/loss (in Rs.) of the person is :
(A) $\frac{1}{4}$ loss
(B) 2 gain
(C) $\frac{1}{2}$ gain
(D) $\frac{1}{2}$ loss

80. If ${}^{20}C_1 + \left(2^2\right)\ {}^{20}C_3 + \left(3^2\right)\ {}^{20}C_3 + \left(2^2\right) + ... + \left(20^2\right)\ {}^{20}C_{20} = A\left(2^{\beta}\right)$, then the ordered pair (A, β) is equal to:
(A) (420, 18)
(B) (380, 18)
(C) (420, 19)
(D) (380, 19)

81. The derivative of $\tan^{-1}\left(\frac{\sin x-\cos x}{\sin x+\cos x}\right)$, with respect to $\frac{x}{2}$, where $\left(x \in\left(0, \frac{\pi}{2}\right)\right)$ is:

(A) 2 (B) $\frac{1}{2}$

(C) 1 (D) $\frac{2}{3}$

82. A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point :

(A) (3, 5) (B) (1, 5)

(C) (3, 10) (D) (2, 3)

83. The equation of a common tangent to the curves, $y^2 = 16x$ and $xy = -4$ is :

(A) $x - 2y + 16 = 0$ (B) $2x - y + 2 = 0$

(C) $x + y + 4 = 0$ (D) $x - y + 4 = 0$

84. A plane which bisects the angle between the two given planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point:

(A) (1, 4, –1) (B) (2, –4, 1)

(C) (2, 4, 1) (D) (1, –4, 1)

85. A value of α such that $\int_{\alpha}^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \log_e\left(\frac{9}{8}\right)$ is:

(A) $-\frac{1}{2}$ (B) –2

(C) $\frac{1}{2}$ (D) 2

86. A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to:

(A) 24 (B) 28

(C) 27 (D) 25

87. A value of $\theta \in (0, \pi/3)$, for which $\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$, is:

(A) $\frac{\pi}{18}$ (B) $\frac{\pi}{9}$

(C) $\frac{7\pi}{36}$ (D) $\frac{7\pi}{24}$

88. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60° with the line x + y = 0. Then an equation of the line L is :

(A) $\left(\sqrt{3}-1\right)x+\left(\sqrt{3}+1\right)y = 8\sqrt{2}$ (B) $\sqrt{3}x + y = 8$

(C) $x + \sqrt{3}y = 8$ (D) $\left(\sqrt{3}+1\right)x+\left(\sqrt{3}-1\right)y = 8\sqrt{2}$

89. $\lim\limits_{x \to 0} \frac{x + 2\sin x}{\sqrt{x^2 + 2\sin x + 1} - \sqrt{\sin^2 x - x + 1}}$ is:

(A) 2
(B) 6
(C) 3
(D) 1

90. The Boolean expression $\sim(p \Rightarrow (\sim q))$ is equivalent to:

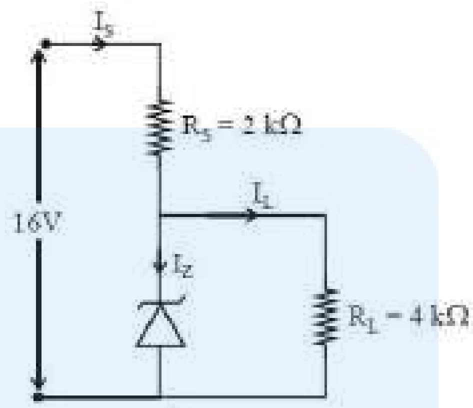
(A) $(\sim p) \Rightarrow q$
(B) $p \vee q$
(C) $p \wedge q$
(D) $q \Rightarrow \sim p$

HINTS AND SOLUTIONS

PART A – PHYSICS

1. Maximum current will flow from zener if input voltage is maximum.

When zener diode works in breakdown state, voltage across the zener will remain same.



$\therefore$ $V_{across\ 4k\Omega} = 6V$

$\therefore$ Current through 4KΩ = $\frac{6}{4000}A = \frac{6}{4}mA$

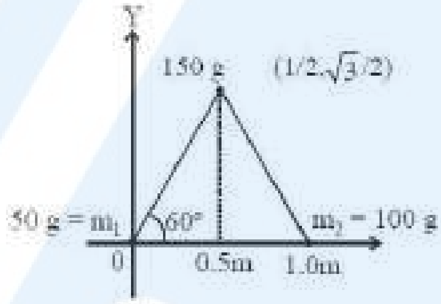
Since input voltage = 16 V

$\therefore$ Potential difference across 2KΩ = 10V

$\therefore$ Current through 2kΩ = $\frac{10}{2000} = 5mA$

$\therefore$ Current through zener diode
$= (I_s - I_L) = 3.5$ mA

2. The co-ordinates of the centre of mass



$$\vec{r}_{cm} = \frac{0 + 150 \times \left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) + 100 \times \hat{i}}{300}$$

$$\vec{r}_{cm} = \frac{7}{12}\hat{i} + \frac{\sqrt{3}}{4}\hat{j}$$

$\therefore$ Co-ordinate $\left(\frac{7}{12}, \frac{\sqrt{3}}{4}\right)$m

3. Since mass of the object remains same

$\therefore$ Weight of object will be proportional to 'g' (acceleration due to gravity)

Given:

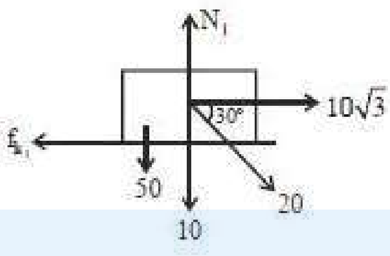
$$\frac{W_{earth}}{W_{planet}} = \frac{9}{4} = \frac{g_{earth}}{g_{planet}}$$

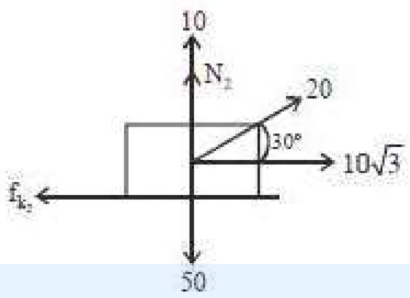
Also, $g_{surface} = \frac{GM}{R^2}$ (M is mass planet, G is universal gravitational constant, R is radius of planet)

$$\therefore \quad \frac{9}{4} = \frac{GM_{earth}\,R^2_{planet}}{GM_{planet}\,R^2_{earth}} = \frac{M_{earth}}{M_{planet}} \times \frac{R^2_{planet}}{R^2_{earth}} = 9\frac{R^2_{planet}}{R^2_{earth}}$$

$$\therefore \quad R_{planet} = \frac{R_{earth}}{2} = \frac{R}{2}$$

4.



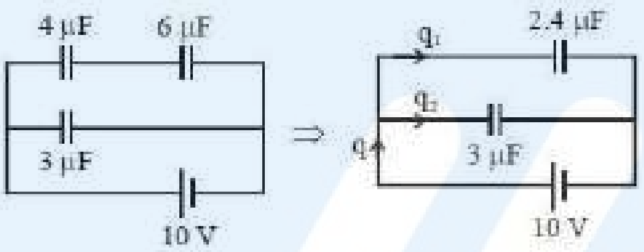


$N_1 = 60$ $\qquad$ $N_2 = 40$

$a_1 = \dfrac{10\sqrt{3} - 0.2 \times 60}{5}$ $\qquad$ $a_2 = \dfrac{10\sqrt{3} - 0.2 \times 40}{5}$

$a_1 - a_2 = 0.8$

5.



So total charge flow = q = 5.4 μF × 10V

= 54 μF

The charge will be distributed in the ratio of capacitance

$\Rightarrow \quad \dfrac{q_1}{q_2} = \dfrac{2.4}{3} = \dfrac{4}{5}$

$\therefore \quad 9X = 54\ \mu C$

$\therefore \quad X = 6\ \mu C$

$\therefore$ Charge on 4 μF capacitor will be = 4X = 4 × 6 μC

= 24 μC

6. For a diatomic gas, $C_p = \dfrac{7}{2}R$

Since gas undergoes isobaric process

$\Rightarrow \ \Delta Q = n\dfrac{7}{2}R\Delta T = \dfrac{7}{2}(nR\Delta T) = 35\ J$

7. Efficiency of Carnot engine = $1 - \dfrac{T_{sink}}{T_{source}}$

Given,

$\dfrac{1}{6} = 1 - \dfrac{T_{sink}}{T_{source}} \Rightarrow \dfrac{T_{sink}}{T_{source}} = \dfrac{5}{6}$ $\qquad$ …(i)

Also,

$$\frac{2}{6} = 1 - \frac{T_{sink} - 62}{T_{source}} \Rightarrow \frac{62}{T_{source}} = \frac{1}{6} \quad \ldots(ii)$$

$\therefore$ T_{source} = 372 K = 99°C

Also, $T_{sink} = \frac{5}{6} \times 372$ = 310 K = 37°C

(**Note:** Temperature of source is more than temperature of sink)

8. $2\pi r_n = n\lambda_n$

$$\lambda_3 = \frac{2\pi(4.65 \times 10^{-10})}{3}$$

λ_3 = 9.7 Å

9. Loudness of sound is given by

$dB = 10 \log \frac{I}{I_0}$ (I is intensity of sound, I_0 is reference intensity of sound)

$\therefore$ $120 = 10 \log\left(\frac{I}{I_0}\right)$

$\Rightarrow$ I = 1 W/m^2

Also, $I = \frac{P}{4\pi r^2} = \frac{2}{4\pi r^2}$

$\therefore$ $r = \sqrt{\frac{2}{4\pi}} = \sqrt{\frac{I}{2\pi}}$ m = 0.399 m

$\approx$ 40 cm

10. Since unpolarised light falls on P_1

$\Rightarrow$ Intensity of light transmitted from $P_1 = \frac{I_0}{2}$.

Pass axis of P_2 will be at an angle of 30° with P_1

$\therefore$ Intensity of light transmitted from

$$P_2 = \frac{I_0}{2}\cos^2 30° = \frac{3I_0}{8}$$

Pass axis of P_3 is at an angle of 60° with P_2

$\therefore$ Intensity of light transmitted from

$$P_3 = \frac{3I_0}{8}\cos^2 60° = \frac{3I_0}{32}$$

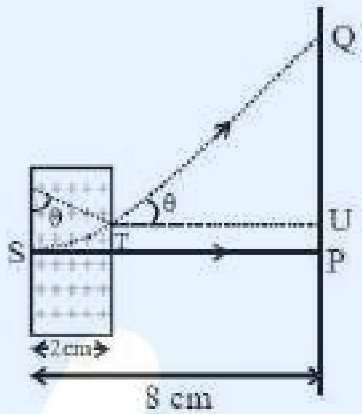
$$\therefore \left(\frac{I_0}{I}\right) = \frac{32}{3} = 10.67$$

11. $E\, 4\pi a^2 = \frac{\int_0^{\theta} kr\, 4\pi r^2 dr}{e_0}$

$$E = \frac{k\, 4\pi a^4}{4 \times 4\pi\varepsilon_0}$$

$$2Q = \int_0^R kr\, 4\pi r^2\, dr$$
$$k = \frac{2Q}{\pi R^4}$$
$$QE = \frac{1}{4\pi\varepsilon_0}\frac{QQ}{(2a)^2}$$
$R = a8^{1/4}$

12. $R = \frac{mv}{qB}$



$$= \frac{\sqrt{2m\,(K.E.)}}{qB}$$
$$R = \frac{\sqrt{2\times 9.1\times 10^{-31}\times(100\times 1.6\times 10^{-19})}}{1.6\times 10^{-19}\times 1.5\times 10^{-3}}$$
R = 2.248 cm
$$\sin\theta = \frac{2}{2.248} \quad ; \quad \tan\theta = \frac{QU}{TU}$$
$$\frac{2}{1.026} = \frac{QU}{6}$$
QU = 11.69
PU = R(1 – cos θ)
= 1.22
d = QU + PU

13. For same range angle of projection will be θ will be θ & 90 – θ

$$R = \frac{u^2 2\sin\theta\cos\theta}{g}$$
$$h_1 = \frac{u^2\sin^2\theta}{g}$$
$$h_2 = \frac{u^2\sin^2(90-\theta)}{g}$$
$$\frac{R^2}{h_1h_2} = 16$$

14. $v = b\sqrt{x}$
$$\frac{dv}{dt} = \frac{b}{2\sqrt{x}}\frac{dx}{dt} \quad ; \quad a = \frac{bv}{2\sqrt{x}}$$
$$a = \frac{b\left(b\sqrt{x}\right)}{2\sqrt{x}} \quad ; \quad \frac{dv}{dt} = a = \frac{b^2}{2} \quad ; \quad v = \frac{b^2}{2}\tau$$

15. $Q = P \times t$

$Q = mc\Delta T + mL$

$P = \frac{V_{rms}^2}{R}$

$4200 \times 80 + 2260 \times 10^3 = \frac{(200)^2}{20} \times t$

$t = 1298$ sec

$t \simeq 22$ min

16. $N_A = N_{OA} e^{-\lambda t} = \frac{N_{OA}}{2^{t/t_{1/2}}} = \frac{N_{OA}}{2^6}$

$\therefore$ Number of nuclei decayed

$= N_{OA} - \frac{N_{OA}}{2^6} = \frac{63 N_{OA}}{64}$

$N_B = N_{OBe^{-\lambda t}} = \frac{N_{OB}}{2^{t/t_{1/2}}} = \frac{N_{OB}}{2^3}$

$\therefore$ Number of nuclei decayed

$= N_{OB} - \frac{N_{OB}}{2^3} = \frac{7N_{OB}}{8}$

Since, $N_{OA} = N_{OB}$

$\therefore$ Ratio of decayed numbers of nuclei

A & B = $\frac{63\ N_{OA} \times 8}{64 \times 7N_{OB}} = \frac{9}{8}$

17. Modulation index is given by

$m = \frac{A_m}{A_c} = \frac{2}{4} = 0.5$

& (a) carrier wave frequency is given by

$= 2\pi f_c = 2 \times 10^4\ \pi$

$f_c = 1$ kHz

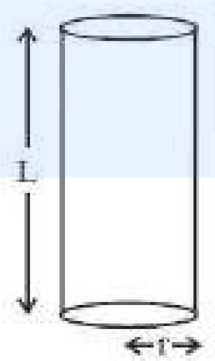
lower side band frequency

$\Rightarrow$ $f_c - f_m$

$\Rightarrow$ 10 kHz – 1 kHz = 9 kHz

18. $\because$ Length of cylinder remains unchanged

so $\left(\frac{F}{A}\right)_{Compressive} = \left(\frac{F}{A}\right)_{Thermal}$



$\frac{F}{\pi r^2} = Y\alpha T$ (α is linear coefficient of expansion)

$\therefore$ $\alpha = \frac{F}{YT\pi r^2}$

$\therefore$ The coefficient of volume expansion $\gamma = 3\alpha$

$\therefore$ $\gamma = 3\frac{F}{YT\pi r^2}$

19. $q = \int I dt$

$$q = \int_0^{L/R} \frac{E}{R}\left[1 - e^{\frac{-Rt}{L}}\right]dt$$

$$q = \frac{EL}{R^2}\frac{1}{e} \quad ; \quad q = \frac{EL}{2.7R^2}$$

20. f = 660 Hz, v = 330 m/s

$$f_1 = f\left(\frac{v-u}{v}\right) \quad ; \quad f_2 = f\left(\frac{v+u}{v}\right)$$

$$f_2 - f_1 = \frac{f}{v}[v + u - (v - u)]$$

$$10 = f_2 - f_1 = \frac{f}{v}[2u]$$

u = 2.5 m/s

21. Magnetic field when electromagnetic wave propagates in +z direction.
$B = B_0 \sin (kz - \omega t)$
where

$$B_0 - \frac{60}{3 \times 10^8} = 2 \times 10^{-7}$$

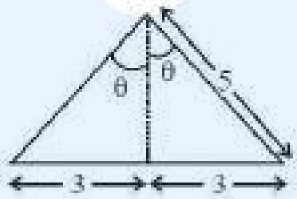
$$k = \frac{2\pi}{\lambda} = 0.5 \times 10^3$$

$$\omega = 2\pi f = 1.5 \times 10^{11}$$

22. $B = \frac{\mu_0 I}{4\pi d} 2 \sin\theta$

d = 4 cm

$\sin\theta = \frac{3}{5}$



23. $$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \quad ; \quad \frac{1}{\lambda_1} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$\frac{1}{\lambda_1} = R\left(\frac{7}{9 \times 16}\right) \quad ; \quad \frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$= R\left(\frac{5}{4 \times 9}\right)$$

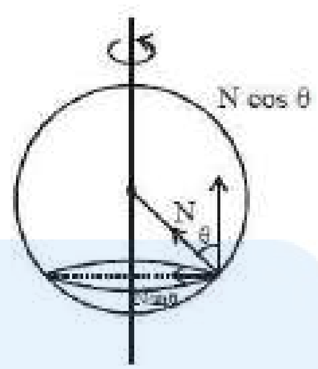
$$\frac{\lambda_1}{\lambda_2} = \frac{\frac{5}{36}}{\frac{7}{9 \times 16}} = \frac{20}{7}$$

24. $N\sin\theta = m\frac{r}{2}\omega^2$ ...(i)

$N\cos\theta = mg$ ...(ii)

$\tan\theta = \frac{r\omega^2}{2g}$

$\frac{r}{2\frac{\sqrt{3}\,r}{2}} = \frac{r\omega^2}{2g}$; $\omega^2 = \frac{2g}{\sqrt{3}\,r}$



25. We have

$$V_T = \frac{2}{9}\frac{r^2}{\eta}(\rho_0 - \rho_\ell)g$$

$\Rightarrow V_T \propto r^2$

Since mass of the sphere will be same

$\therefore \rho\frac{4}{3}\pi R^3 = 27\cdot\frac{4}{3}\pi r^3\rho$

$\Rightarrow r = \frac{R}{3}$

$\therefore \frac{v_1}{v_2} = \frac{R^2}{r^2} = 9$

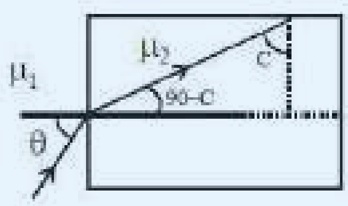
26. Given number density of molecules of gas as a function of r is

$n(r) = n_0 e^{-\alpha r^4}$

$\therefore$ Total number of molecule = $\int_0^\infty n(r)dV = \int_0^\infty n_0 e^{-\alpha r^4}\, 4\pi r^2\, dr$

$\therefore$ Number of molecules is proportional to $n_0\alpha^{-3/4}$

27. $\sin c = \frac{\mu_1}{\mu_2}$



$\mu_1\sin\theta = \mu_2\sin(90° - C)$

$$\sin\theta = \frac{\mu_2\sqrt{1-\frac{\mu_1^2}{\mu_2^2}}}{\mu_1}$$

$$\theta = \sin^{-1}\sqrt{\frac{\mu_1^2-\mu_2^2}{\mu_1^2}}$$

For TIR

$$\theta < \sin^{-1}\sqrt{\frac{u_2^2}{\mu_1^2}-1}$$

28. $v = 2f(l_2 - l_1)$

$v = 2 \times 480 \times (70 - 30) \times 10^{-2}$

$v = 960 \times 40 \times 10^{-2}$

$v = 38400 \times 10^{-2}$ m/s

$v = 384$ m/s

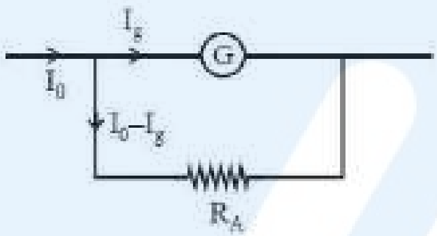
29. $k_1 = \frac{C}{\ell_1}$

$k_2 = \frac{C}{\ell_2}$

$\frac{k_1}{k_2} = \frac{C\ell_2}{\ell_1 C} = \frac{\ell_2}{n\ell_2} = \frac{1}{n}$

30. When galvanometer is used an ammeter shunt is used in parallel with galvanometer.

$\therefore$



$\therefore \quad I_g G = (I_0 - I_g)R_A$

$\therefore \quad R_A = \left(\frac{I_g}{I_0 - I_g}\right)G$

When galvanometer is used as a voltmeter, resistance is used in series with galvanometer.

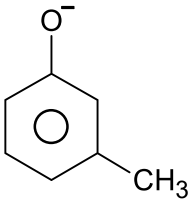
$I_g(G + R_v) = V = GI_0$ (given $V = GI_0$)

$\therefore \quad R_V = \frac{(I_0 - I_g)G}{I_g}$

$\therefore \quad R_A R_V = G^2 \;\; \& \;\; \frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

PART B – CHEMISTRY

31.

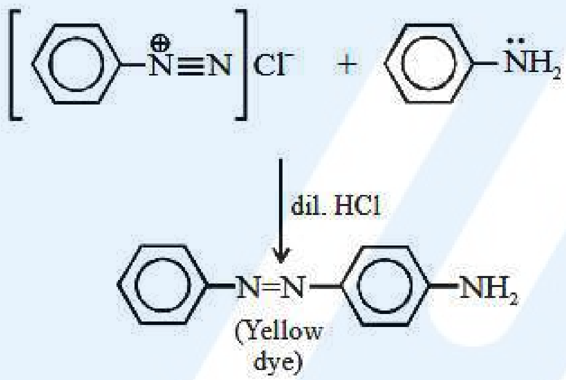


Above anion act as nucleophile for S_N2 attack on $CH \equiv C - CH_2 - Br$ and acetone acting as polar aprotic solvent.

32. $(CH_3)_3CCl$ will give Cl^- and most stable carbocation. Hence $(CH_3)_3CCl$ likely to give a precipitate with $AgNO_3$ solution.

33. $\Delta G° = -2.303\ RT \log K_{eq}$

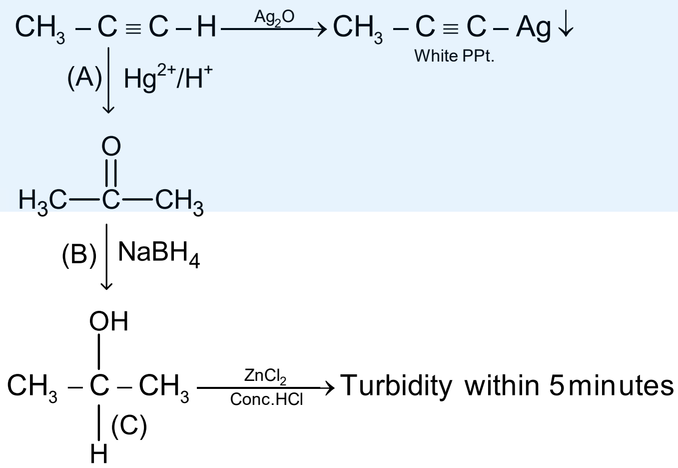
34. According to NCERT C – N coupling take place, when diazonium ion is treated with aniline.



35. $2LiNO_3 \xrightarrow{\Delta} Li_2O + 2NO_2 \uparrow + \frac{1}{2}O_2 \uparrow$

36. Since carbon in diamond is sp^3 hybridized and its C – C bond order is 1. In graphite and fullerene there is both C – C and C = C in conjugation, hence there is partial double bond character between carbon atoms.

37.



38. If $\Delta n_g = 0$
$K_P = K_C$
If $\Delta n_g \neq 0$, $K_P \neq K_C$
Hence (B) is correct answer.

39. en and $C_2O_4^{2-}$ are a bidentate ligand. So coordination number of $[Co(Cl)(en)_2]Cl$ is 5 and $K_3[Al(C_2O_4)_3]$ is 6

40. Glycogen is a multibranched polysaccharide.

41. NO_2 and Hydrocarbons are primary precursors of photochemical smog.

42. No. of atoms in simple cubic = 1, bcc = 2 & fcc = 4

43. $$\underset{(X)}{2KMnO_4} \xrightarrow[\Delta]{513K} \underset{(Y)}{K_2MnO_4} + MnO_2 + O_{2(g)}$$
$$MnO_2 + 4NaCl + 4H_2SO_4 \rightarrow MnCl_2 + 4NaHSO_4 + 2H_2O + \underset{\substack{(Z)\\ \text{pungent} \\ \text{gas}}}{Cl_{2(g)}}$$

44. Due to resonance there is partial double bond character between carbon and chlorine, hence it do not undergoes nucleophilic substitution reaction.

45. K_{sp} of $Cd(OH)_2 = 4s^3 = 4 \times (1.84 \times 10^{-5})^3$
If pH = 12
pOH = 2
$[OH^-] = 10^{-2}$ M
$K_{sp} = [Cd^{2+}]\,[OH^-]^2$
$4 \times (1.84 \times 10^{-5})^3 = [Cd^{2+}]\,[OH^-]^2$
$[Cd^{2+}] = \dfrac{4 \times (1.84)^3 \times 10^{-15}}{10^{-4}}$
$Cd^{2+} = 4 \times 6.22 \times 10^{-11} = 2.49 \times 10^{-10}$ M

46. (A) EDTA (ethylene diamine tetra acetate) is used for lead poisoning
(B) Cis platin is used as a anti cancer drug
(C) D-penicillamine is used for copper poisoning
(D) desferrioxime B is used for iron poisoning

47. Due to lanthanoide contraction Mo & W have similar atomic radii.

48.

$$C_xH_y + \left(x+\frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$$

$$\left(\frac{25}{M}\right) \qquad x\times\frac{25}{M} = 2 \qquad \frac{y}{2}\times\frac{25}{M} = 0.5$$

C $\qquad x\times\frac{25}{M} = 2$

H $\qquad y\times\frac{25}{M} = 1$

$C_{2y}H_y$ = 24y gm C + y gm H

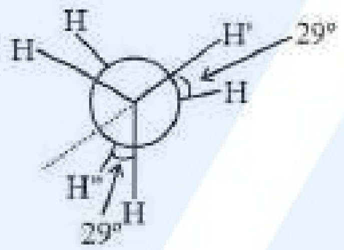
or

24 : 1 ratio by mass

49. Stronger the acidic strength greater will be its electrical conductivity.
K_a value of formic acid > benzoic acid > acetic acid.

50. Greater the nuclear charge, stronger will be the attraction, hence lower will be energy of 2s

51. Dihedral angle is then angle between bond pairs present on adjacent atoms.



H'–C–C–H"
Hence angle between
H'—C–C–H" is
(120° + 29°) = 149°

52. Since NaOH is a strong base hence it reacts with Al_2O_3 and SiO_2 to form salts.

53.

$$2N_2O_5(g) \longrightarrow 4NO_2(g) + O_2(g)$$

t=0 $\quad$ 3.0M

t=30 $\quad$ 2.75 M

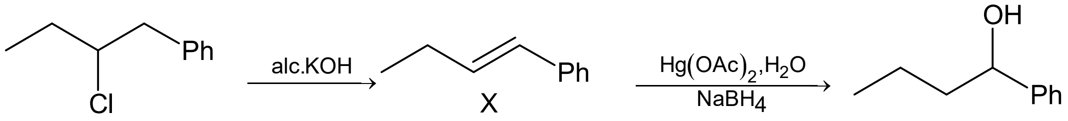
$$\frac{-\Delta[N_2O_5]}{\Delta t} = \frac{0.25}{30}$$

$$\frac{1}{2}\times\frac{-\Delta[N_2O_5]}{\Delta t} = \frac{1}{4}\times\frac{\Delta[NO_2]}{\Delta t}$$

$$\frac{\Delta[NO_2]}{\Delta t} = \frac{0.25}{30}\times 2 = 1.66\times 10^{-2}\ M/min$$

54. Isobutylene on polymerization will form given polymer.

55.



OMDM follow Markonikovs addition rule.

56. According to IUPAC rules, select the largest chain including functional group, if alkene and alkyne are present at equivalent position then priority is given to alkene.

57. Definitions and property of colloidal will explain & solve above question..

58. Since boron has higher nuclear charge because it has greater atomic number and lower 1^{st} I.E. then beryllium due to fully filled s-orbital.

59. $\pi = CRT = \left(\frac{6}{60} + \frac{18}{180}\right) \times .0821 \times 300$

$= 0.2 \times .082 \times 300 = 4.926$ atm.

60. $Mg(HCO_3)_2(aq) \longrightarrow Mg(OH)_2 \downarrow + 2CO_2 \uparrow$

PART C – MATHEMATICS

61. $(y^2 - x^3)dx - xy\,dy = 0 \qquad x \neq 0$

$\Rightarrow y^2 - x^3 - xy\frac{dy}{dx} = 0$

or, $xy\frac{dy}{dx} - y^2 = -x^3$

$y\frac{dy}{dx} - \frac{1}{x}y^2 = -x^2 \qquad$ ....(i)

Let $y^2 = \mu$

$2y\frac{dy}{dx} = \frac{d\mu}{dx}$

Putting this value in equation (i)

$\frac{1}{2}\frac{d\mu}{dx} - \frac{1}{x}\mu = -x^2$

$\frac{d\mu}{dx} + \left(-\frac{2}{x}\right)\mu = -2x^2$ (ii)

I.F. $= e^{\int \frac{-2}{x}dx} = e^{-2\ell n x} = \frac{1}{x^2}$

Sol. of equation (ii)

$\mu \times \frac{1}{x^3} = \int -2x^2 \times \frac{1}{2}dx - C$

$\frac{\mu}{x^2} = -2x - C$

$y^2 = -2x^3 - cx^2$

$y^2 + 2x^3 + cx^2 = 0$

Hence correct answer is option B.

62. $\left[\vec{a}.\vec{b}.\vec{c}\right] = 0$

$$\begin{vmatrix} \alpha & 3 & 1 \\ 2 & 1 & -\alpha \\ \alpha & -2 & 3 \end{vmatrix} = 0$$

$\alpha(3-2\alpha) + 1(-\alpha^2 - 6) + 3(-4-\alpha) = 0$

$3\alpha - 2\alpha^2 - \alpha^2 - 6 - 12 - 3\alpha = 0$

$-3\alpha^2 - 18 = 0$

$\alpha^2 + 6 = 0$ not possible for real α

S is empty set

63. $\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha} dx = \int \frac{\sin(x+\alpha)}{\sin(x-\alpha)} dx$

Let, $x - \alpha = t$

$\Rightarrow \int \frac{\sin(t+2\alpha)}{\sin t} dt = \int \cos 2\alpha dt + \int \cot(t) \sin 2\alpha dt$

$= t.\cos 2\alpha + \ell n|\sin t|.\sin 2\alpha + C$

$= (x-\alpha)\cos 2\alpha + \ln|\sin(x-\alpha)|.\sin 2\alpha + C$

Hence the correct answer is option (C)

64. Given, $\cos 2x + 2\sin x = 2\alpha - 7$

$\Rightarrow 1 - 2\sin^2 x + \alpha \sin x = 2\alpha - 7$

$\Rightarrow 2\sin^2 x - \alpha \sin x + 2\alpha - 8 = 0$

$\Rightarrow \sin x = \frac{\alpha \pm \sqrt{\alpha^2 - 8(2\alpha - 8)}}{4}$

$\Rightarrow \sin x = \frac{\alpha \pm (\alpha - 8)}{4}$

$\Rightarrow \sin x = \frac{\alpha + \alpha - 8}{4}, \frac{\alpha - \alpha + 8}{4}$

$\sin x = 2$ (Not possible)

For solution

$-1 \le \frac{2\alpha - 8}{4} \le 1$

$-4 \le 2\alpha - 8 \le 4$

$\Rightarrow 4 \le 2\alpha \le 12$

$\Rightarrow \alpha \in [2, 6]$

65. $f(x) = 5 - |x - 2|$

$f(x)$ attains maximum value when $|x-2| = 0 \Rightarrow x = 2 = \alpha$

$g(x) = |x+1|$

$g(x)$ attains minimum value of $x = -1 = \beta$

$$\lim_{x \to -\alpha\beta} \frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$$

$$= \lim_{x \to 2} \frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)}$$

$$= \frac{(2-1)(2-3)}{(2-4)} = \frac{1}{2}$$

66. Let $z = x + 10i$

given $\frac{2z-n}{2z+n} = 2i - 1$

$\Rightarrow \frac{2(x+10i)-n}{2(x+10i)+n} = 2i - 1$

$\Rightarrow (2x-n) + 20i = (2i-1)\left[(2x+n) + 20i\right]$

Comparing real and imaginary part

$\Rightarrow 2x - n = 2(-20) - (2x+n)$ and $20 = 2(2x+n) - 20$

$\Rightarrow 2x - n = -40 - 2x - n$ and $20 = 4x + 2n - 20$

$\Rightarrow 4x = -40$ and $4x + 2n = 40$

$\Rightarrow x = -10$ and $-40 + 2n = 40$

$\Rightarrow n = 40$

$\Rightarrow n = 40$ and $\operatorname{Re}(z) = -10$

67. $\left(\frac{1}{60} - \frac{x^8}{81}\right)\left(2x^2 - \frac{3}{x^2}\right)^6$

Term independent of x will be $\frac{1}{60} \times$ independent of x in

$\left(2x^2 - \frac{3}{x^2}\right)^6 - \frac{1}{8} \times$ Term of x^{-8} in $\left(2x^2 \frac{3}{x^3}\right)^6$

T_{r+1} in $\left(2x^2 - \frac{3}{x^2}\right)^6$ will be $T_{r+1} = {}^6C_r \left(2x^2\right)^{6-r} \left(-\frac{3}{x^2}\right)^r$

$= {}^6C_r \, 2^{6-r} (-1)^r \times 3^r \times x^{12-2r-2r}$

Case I : For term independent of x, $12 - 4r = 0 \Rightarrow r = 3$

$T_4 = {}^6C_3 \times 2^3 \times 3^3 \times^6 = -20 \times 2^3 \times 3^3$

Case II : For term of x^{-8}

$12 - 4r = -8$

$4r = 20 \Rightarrow r = 5$

$T_6 = {}^6C_5 . 2^1 (-1) . 3^5 . x^{-8}$

Required Answer $= \frac{1}{60} \times (-20) 2^3 \times 3^3 - \frac{1}{81} \times 6 \times 2 \times (-1) \times 3^5$

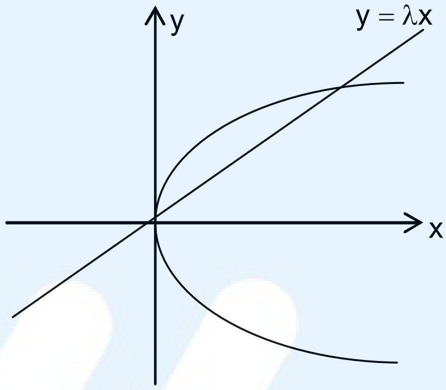
$= -72 + 36 = -36$

Hence the correct answer is option (B).

68. $y^2 = 4\lambda x$ and $y = \lambda x$

$\lambda^2 x^2 = 4\lambda x$

$x = 0$ and $x = \frac{4}{\lambda}$



Area $= \int_0^{4/\lambda} \left(\sqrt{4\lambda x} - \lambda x\right) dx = \frac{1}{9}$

$\Rightarrow 2\sqrt{\lambda} \times \left(\frac{x^{3/2}}{3/2}\right)_0^{4/\lambda} - \lambda \left(\frac{x^2}{2}\right)_0^{4/\lambda} = \frac{1}{9}$

$\frac{4}{3}\sqrt{\lambda} \times \left(2^2\right)^{3/4} \frac{x}{\lambda^{3/2}} - \frac{x}{2} \times \frac{16}{\lambda} = \frac{1}{9}$

$\Rightarrow \frac{32}{3\lambda} - \frac{8}{\lambda} = \frac{1}{9}$

$\Rightarrow \frac{8}{3\lambda} = \frac{1}{9}$

$\lambda = 24$

69. $[\sin\theta] x + [-\cos\theta] y = 0$ ....(1)

$[\cot\theta] x + y = 0$ ....(2)

Case I

When $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

$\sin\theta \in \left(\frac{\sqrt{3}}{2}, 1\right)$

$\cos\theta \in \left(-\frac{1}{2}, 0\right) - \cos\theta \in \left(0, \frac{1}{2}\right)$

$\cot\theta \in \left(-\frac{1}{\sqrt{3}}, 0\right)$

$[\sin\theta] = 0 \quad [-\cos\theta] = 0 \quad [\cot\theta] = -1$

Equation (1) and (2) will

$\left.\begin{matrix} 0x + 0y = 0 \\ -x + y = 0 \end{matrix}\right]$ system will have infinitely many solution

Case II

When $\theta \in \left(\pi, \frac{7\pi}{6}\right)$ $\sin\theta \in \left(-\frac{1}{2}, 0\right)$

$\cos\theta \in \left(-1, \frac{-\sqrt{3}}{2}\right)$

$\cot\theta \in \left(\sqrt{3}, \infty\right)$

$[\sin\theta] = -1, [\cos\theta] = -1$

$[\cot\theta] = \{1, 2, 3, ...\}$

$-x - y = 0$

$lx + y = 0 \qquad l = \{1, 2, ...\}$

It will have unique solution in all cases $x = 0, y = 0$

70. Total problems = 50

P (Solving) $= \frac{4}{5}$

P (Not solving) $= \frac{1}{5}$

P (unable to solve less than two problems)
= P (not solving one problem) + P (not solving zero problem)

$= {}^{50}C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^{50} + {}^{50}C_1 \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^{49}$

$= \frac{4^{50}}{5^{50}} + 50.\frac{4^{49}}{5.5^{49}}$

$= \left(\frac{4}{5}\right)^{50} + 10.\left(\frac{4}{5}\right)^{49}$

$= \left(\frac{4}{5}\right)^{49} \left(\frac{4}{5} + 10\right)$

$= \frac{54}{5}.\left(\frac{4}{5}\right)^{49}$

71. $a_1, a_2, ...a_n$ are in A.P.

$a_1 + a_7 + a_{16} = 40$

$\Rightarrow a + a + 6d + a + 15d = 40$

$\Rightarrow 3a + 21d = 40$

$\Rightarrow a + 7d = \frac{40}{3}$

$515 = \frac{15}{2}[2a + 14d]$

$= 15[a + 7d]$

$= 15 \times \frac{40}{3}$

= 200

72. Given $2a = 4$ and $2be = 4$

$\Rightarrow a = 2, be = 2$

$\Rightarrow b^2e^2 = 4$

$\Rightarrow b^2 - a^2 = 4$

$\Rightarrow b^2 = 8$

$\Rightarrow$ equation of ellipse

$\frac{x^2}{4} + \frac{y^2}{8} = 1$

Clearly option (D) satisfy the given curve.

73. Equation of plane containing both lines is

$$\begin{vmatrix} x-1 & y-1 & z \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = 0$$

$(x-1)(-4+1) + (y-1)(1+2) + z(1+2) = 0$

$-3(x-1) + 3(y-1) + 3z = 0$

$-x + 1 + y - 1 + z = 0$

$-x + y + z = 0$ distance from point (2, 1, 4) is $\left|\frac{-2+1+4}{\sqrt{1^2+1^2+1^2}}\right| = \sqrt{3}$

74. For $A = C, A - C = \phi$

$\Rightarrow \phi \subseteq B$

But $A \not\subseteq B$

$\Rightarrow$ option A is NOT true

Let $x \in (Cx \in (C \cup A) \cap (C \cup B))$

$\Rightarrow x(C \cup A)$ and $x \in (C \cup B)$

$\Rightarrow (x \in C \text{ or } x \in A)$ and

$(x \in C \text{ or } x \in B)$

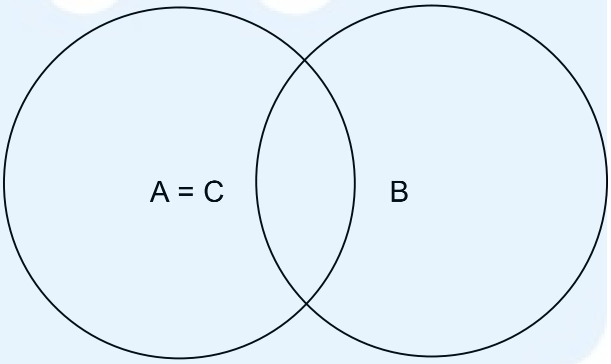
$\Rightarrow x \in C$ or $x \in (A \cap B)$

$\Rightarrow x \in C$ or $x \in C$ (as $A \cup B \subseteq C$)

$\Rightarrow x \in C$

$\Rightarrow (C \cup A) \cap (C \cup B) \subseteq C$ (1)

Now $x \in C \Rightarrow x \in (C \cup A)$ and $x \in (C \cup B)$



$\Rightarrow x \in (C \cup A) \cap (C \cup B)$

$\Rightarrow C \subseteq (C \cup A) \cap (C \cup B)$ (2)

$\Rightarrow$ from (1) and (2)

$C = (C \cup A) \cap (C \cup B)$

$\Rightarrow$ option B is true

Let $x \in A$ and $x \notin B$

$\Rightarrow x \in (A - B)$

$\Rightarrow x \in C$ (as $A - B \subseteq C$)

Let $x \in A$ and $x \in B$

$\Rightarrow x \in (A \cap B)$

$\Rightarrow x \in C$ (as $A \cap B \subseteq C$)

Hence $x \in A \Rightarrow x \in C$

$\Rightarrow A \subseteq C$

$\Rightarrow$ Option C is true

As $C \supseteq (A \cap B)$

$\Rightarrow B \cap C \supseteq (A \cap B)$

As $A \cap B \neq \phi$

$\Rightarrow B \cap C \neq \phi$

Hence the correct answer is option (A)

75. α, β, γ are in G.P.

$\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common roots.

Both roots will be common.

$$\frac{\alpha}{1} = \frac{2\beta}{1} = \frac{\gamma}{-1} = \lambda$$

$$\alpha(\beta + \gamma) = \lambda\left(\frac{\lambda}{2} - \lambda\right) = \frac{-\lambda^2}{2} = \beta\gamma$$

76. $x - y - 3 = 0$ ....(i)

will be chord of contact of parabola

Let the required point is $P(x_1, y_1)$ chord of contact for point P is

$$\frac{y + y_1}{2} = xx_1 - 4\frac{(x + x_1)}{2} + 3$$

$y + y_1 = 2x_1x - 4x - 4x_1 + 6$

As equation (i) and (ii) are same line

$$\frac{2x_1 - 4}{1} = \frac{-1}{-1} = \frac{-4x_1 - y_1 + 6}{-3}$$

$\Rightarrow 2x_1 - 4 = 1$

$x_1 = \frac{5}{2}$

$-4x_1 - y_1 + 6 = -3$

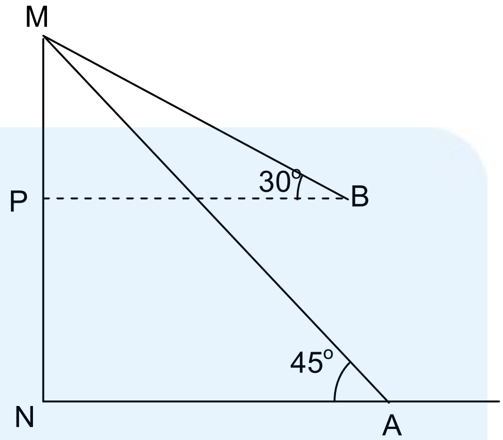
$-10 - y_1 + 9 = 0$

$y_1 = -1$

Hence correct answer is $\left(\frac{5}{2}, -1\right)$ which is option (D).

77. $AB = 30m = NP$

In ΔANM $\tan 45^o = \frac{MN}{AN} = 1$



$\Rightarrow MN = AN$

$PM = MN - 30$

$= AN - 30$

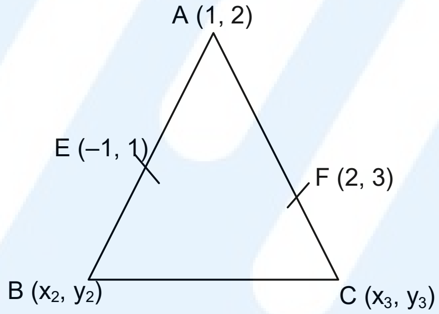
In ΔBPM $\tan 30^o = \frac{PM}{PB} = \frac{AN-30}{AN}$

$\frac{1}{\sqrt{3}} = \frac{AN-30}{AN}$

$AN = \sqrt{3}AN - 30\sqrt{3}$

$AN = \frac{30\sqrt{3}}{\sqrt{3}-1} = \frac{30\sqrt{3}\left(\sqrt{3}+1\right)}{2} = 15\left(3+\sqrt{3}\right)$

78.



$\frac{\alpha_2+1}{2} = -1, \frac{y_2+2}{2} = 1$

$x_2 = -3,\ y_2 = 0$

$B(-3, 0)$

$\frac{x_3+1}{2} = 2$ and $\frac{y_3+2}{2} = 3$

$x_3 = 3,\ y_3 = 4$

$C(3, 4)$

Centroid $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$\left(\frac{1-3+3}{3}, \frac{2+D+4}{3}\right) = \left(\frac{1}{3}, 2\right)$

79.

Coin	+15	+12	–6
Probability	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{26}{36}$

Probability of doublet $=\frac{6}{36}$

Probability of sum of 9 $=\frac{4}{36}$

Other probability $=\frac{26}{36}$

Expected gain/loss $=15\times\frac{6}{36}+12\times\frac{4}{36}-6\times\frac{26}{36}$

$=\frac{90}{36}+\frac{48}{36}-\frac{156}{36}=\frac{-1}{2}$

Hence correct answer is option (D).

80. $(1+x)^{20}={}^{20}C_0+{}^{20}C_1+{}^{20}C_2x^2+.......+{}^{20}C_{20}x^{20}$ ....(i)

Differential equation w.r.t. x

$20(1+x)^{19}={}^{20}C_1.1+2.\,{}^{20}C_2x+.......+20\,{}^{20}C_{20}x^{19}$ ....(ii)

Multiply equation (2) by x

$20x(1+x)^{19}={}^{20}C_1x+2.\,{}^{20}C_2x^2+......+20\,{}^{20}C_{20}x^{20}$ ....(iii)

Differential equation (3) w.r.t. x

$20\left[(1+x)^{19}+19x(1+x)^{18}\right]=1.\,{}^{20}C_1+2^2.\,{}^{20}C_2x+......(20^2)\,{}^{20}C_{20}x^{19}$ ....(iv)

Put $x=1$ in equation (iv)

$20\left(2^{19}+19.2^{18}\right)=1^2\,{}^{20}C_1+2^2\,{}^{20}C_2+......+\left(20^2\right){}^{20}C_{20}$

$=20\times2^{18}(2+19)=20\times21\times2^{18}$

$=420\times2^{18}$

$A=420,\ \beta=18$

Hence correct Option is (A).

81. Given, $y=\tan^{-1}\left(\frac{\sin x-\cos x}{\sin x+\cos x}\right)$

$\Rightarrow y=\tan^{-1}\left(\frac{\tan x-1}{\tan x+1}\right)$

$\Rightarrow y=-\tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$

$\Rightarrow y=-\tan^{-1}\left[\tan\left(\frac{\pi}{4}-x\right)\right]$

$\because 0<x<\frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{2} < -x < 0$

$\Rightarrow -\frac{\pi}{4} < \frac{\pi}{4} - x < 0$

$\Rightarrow y = -\left(\frac{\pi}{4} - x\right) \quad \left\{\because \tan^{-1}(\tan x) = x \ \forall \ x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right\}$

$\Rightarrow y = x - \frac{\pi}{4} \quad \frac{dy}{d\left(\frac{x}{2}\right)} = \frac{1}{\left(\frac{1}{2}\right)} = 2$

82. Equation of required circle will be $(x-3)^2 + (y \pm r)^2 = r^2$

$x^2 - 6x + 9 + y^2 \pm 2r\,y + r^2 = r^2$

$x^2 + y^2 - 6x \pm 2ry + 9 = 0 \quad \ldots\ldots\ldots(1)$

Length of y intercept $= 2\sqrt{f^2 - c} \quad = \pm r\,f$

$8 = 2\sqrt{r^2 - 9}$

$16 = r^2 - 9$

r = 5

So equation of required circle will be

$x^2 + y^2 - 6x \pm 10y + 9 = 0$ two circles

$x^2 + y^2 - 6x + 10y + 9 = 0 \quad \ldots\ldots\ldots..(2)$

$x^2 + y^2 - 6x - 10y + 9 = 0 \quad \ldots\ldots\ldots..(3)$

Given option (C) i.e. (3, 10) satisfy equation (3).

83. $y = mx + \frac{4}{m} \quad \ldots\ldots(i)$ is always tangent to $y^2 = 16x$

If it is tangent to the $xy = -4$

$x\left(mx + \frac{4}{m}\right) = -4$

$m^2x^2 + 4x = -4m$

$m^2x^2 + 4x = -4m$

$m^2x^2 + 4x + 4m = 0$

for tangent D = 0

$16 - 16m^3 = 0$

$\Rightarrow m = 1$ put in equation (i)

$y = x + 4$

So the correct answer is option (D)

84. Equation of angle bisectors

$\frac{2x - y + 2z - 4}{\sqrt{2^2 + (-1)^2 + 2^2}} = \pm\left(\frac{x + 2y + 2z - 2}{\sqrt{1^2 + 2^2 + 2^2}}\right) \quad \ldots\ldots..(1)$

Case I: take positive sign

$2x - y + 2z - 4 = x + 2y + 2z - 2$

$x - 3y - 2 = 0 \quad \ldots\ldots\ldots(2)$

Case II : take negative sign

$2x - y + 2z - 4 = -(x + 2y + 2z - 2)$

$2x - y + 2z - 4 = -x - 2y + 2z + 2$

$3x + y + 4z - 6 = 0 \ \ldots\ldots\ldots(3)$

Option (B) satisfy equation (3)

85. $\int_{\alpha}^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \log_e\left(\frac{9}{8}\right)$

$\Rightarrow \int_{\alpha}^{\alpha+1} \frac{(x+\alpha+1)-(x+\alpha)}{(x+\alpha)(x+\alpha+1)} dx = \log_e\left(\frac{9}{8}\right)$

$\Rightarrow \int_{\alpha}^{\alpha+1} \frac{dx}{x+\alpha} - \int_{\alpha}^{\alpha+1} \frac{dx}{x+\alpha+1} = \log_e\left(\frac{9}{8}\right)$

$\Rightarrow \log_e\left(\frac{x+\alpha}{x+\alpha+1}\right)\Bigg|_{\alpha}^{\alpha+1} = \log_e\left(\frac{9}{8}\right)$

$\Rightarrow \log_e\left(\frac{2\alpha+1}{2\alpha+2}\right) - \log\left(\frac{2\alpha}{2\alpha+1}\right) = \log_e\left(\frac{9}{8}\right)$

$\Rightarrow \log\left[\left(\frac{2\alpha+1}{2\alpha+2}\right)\left(\frac{2\alpha+1}{2\alpha}\right)\right] = \log_e \frac{9}{8}$

$\Rightarrow \frac{(2\alpha+1)^2}{4\alpha(\alpha+1)} = \frac{9}{8}$

$\Rightarrow 8\left[4\alpha^2 + 4\alpha + 1\right] = 9\left[4\alpha^2 + 4\alpha\right]$

$\Rightarrow 32\alpha^2 + 32\alpha + 8 = 36\alpha^2 + 36\alpha$

$\Rightarrow 4\alpha^2 + 4\alpha - 8 = 0$

$\Rightarrow \alpha^2 + \alpha - 2 = 0$

$= (\alpha+2)(\alpha-1) = 0$

$\Rightarrow \alpha = 1, -2$

Hence the correct answer is option (B).

86. Given 5 boys and n girls

Total ways of farming team of 3

Members under given condition

$= {}^5C_1 . {}^nC_2 + {}^5C_2 . {}^nC_1$

$\Rightarrow {}^5C_1 . {}^nC_2 + {}^5C_2 . {}^nC_1 = 1750$

$\Rightarrow \frac{5n(n-1)}{2} + 10n = 1750$

$\Rightarrow \frac{n(n-1)}{2} + 2n = 350$

$\Rightarrow n^2 + 3n = 700$

$\Rightarrow n^2 + 3n - 700 = 0$

$\Rightarrow n = 25$

87. $\theta \in \left(0, \frac{\pi}{3}\right)$

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2$

$$\Rightarrow \begin{vmatrix} 2 & \sin^2\theta & 4\cos 6\theta \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

expanding along first column

$\Rightarrow 2[1-0] - 1[-4\cos 6\theta] = 0$

$\Rightarrow 2 + 4\cos 6\theta = 0$

$\Rightarrow \cos 6\theta = -\frac{1}{2}$

$\Rightarrow 6\theta = \frac{2\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{9}$

88. OP = 4

given OP makes 60° with $x + y = 0$

Let slope of OP = m

$\Rightarrow \tan 60^\circ = \left|\frac{m+1}{1-m}\right|$

$\Rightarrow \frac{m+1}{m-1} = \sqrt{3}$ or $-\sqrt{3}$

$\Rightarrow m + 1 = \sqrt{3}m - \sqrt{3}$ or $m + 1 = \sqrt{3} - \sqrt{3}m$

$\Rightarrow m\left(\sqrt{3} - 1\right) = \sqrt{3} + 1$ or $m\left(1 + \sqrt{3}\right) = \sqrt{3} - 1$

$\Rightarrow m = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ or $m = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$\Rightarrow \tan\alpha = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ or $\tan\alpha = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$\Rightarrow$ equation of line $x\cos\alpha + y\sin\alpha = P$

$\Rightarrow \left(\sqrt{3}+1\right)x + \left(\sqrt{3}-1\right)y = 8\sqrt{2}$ or $\left(\sqrt{3}-1\right)x + \left(\sqrt{3}+1\right)y = 8\sqrt{2}$

89. $\lim_{x\to 0}\frac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$

$=\lim_{x\to 0}\frac{x+2\sin x}{x^2+2\sin x+1-\sin^2 x+x-1}\left(\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x-x+1}\right)$

$=\lim_{x\to 0}\frac{x+2\sin x}{x^2+2\sin x-\sin^2 x+x}.(2)$

Applying L'H Rule

$=\lim_{x\to 0}\frac{2.(1+2\cos x)}{2x+2\cos x-2\sin x\cos x+1}$

$=\frac{2(3)}{2+1}=2$

Hence the correct answer is option (A).

90. $\sim(p\to\sim q)=p\wedge q$

Hence the correct answer is option (C).