FIITJEE

Solutions to JEE(Main)-2020

Test Date: 3rd September 2020 (First Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

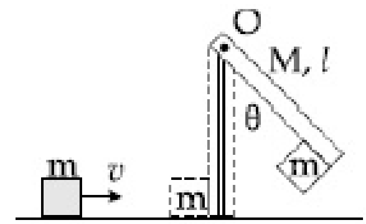
1. The test is of **3 hours** duration.
2. This **Test Paper** consists of **75** questions. The maximum marks are **300**.
3. There are ***three*** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 25 questions in each part of equal weightage out of which 20 questions are MCQs and 5 questions are numerical value based. Each question is allotted **4 (four)** marks for correct response.
4. **(Q. No. 01 – 20, 26 – 45, 51 – 70)** contains 60 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
5. **(Q. No. 21 – 25, 46 – 50, 71 – 75)** contains 15 Numerical based questions with answer as numerical value. Each question carries **+4 marks** for correct answer. There is no negative marking.
6. Candidates will be awarded marks as stated above in **instruction No.3** for correct response of each question. One mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer box.
7. There is only one correct response for each question. Marked up more than one response in any question will be treated as wrong response and marked up for wrong response will be deducted accordingly as per **instruction 6** above.

PART –A (PHYSICS)

1. A uniform thin rope of length 12 m and mass 6 kg hangs vertically from a rigid support and a block of mass 2 kg is attached to its free end. A transverse short wave-train of wavelength 6 cm is produced at the lower and the rope. What is the wavelength of the wave train (in cm) when it reaches the top of the tope?
(A) 6 (B) 12
(C) 3 (D) 9

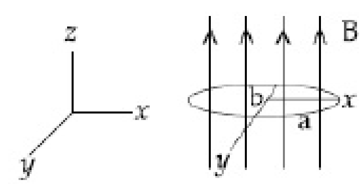
2. When the wavelength of radiation falling on a metal is charged from 500 nm to 200 nm the maximum kinetic energy of the photoelectrons becomes three times larger. The work function of the metals is close to:
(A) 1.02 eV (B) 0.61 eV
(C) 0.52 eV (D) 0.81 eV

3. A block of mass m = 1 kg slides with velocity v = 6 m/s on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to its as shown. The rod is pivoted about O and swings as a result of the collision making angle θ before momentarily coming to rest. If the rod has mass M = 2 kg, and length ℓ = 1 m, the value of θ is approximately: (take g = 10 m/s^2)



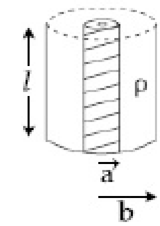
(A) 49° (B) 55°
(C) 69° (D) 63°

4 An elliptical loop having resistance R, of semi major axis a and semi minor axis b is placed in a magnetic field as shown in the figure. If the loop is rotated about the x-axis with angular frequency ω, the average power loss in the loop due to joule heating is:



(A) $\frac{\pi^2a^2b^2B^2\omega^2}{R}$
(B) zero
(C) $\frac{\pi abB\omega}{R}$
(D) $\frac{\pi^2a^2b^2B^2\omega^2}{2R}$

5 Model a torch battery of length ℓ to be made up of a thin cylindrical bar of radius 'a' and a concentric thin cylindrical shell of radius 'b' filled in between with an electrolyte of resistivity p (see figure). If the battery is connected to a resistance of value R, the maximum joule heating in R will take place for:



(A) $R = \frac{\rho}{\pi\ell}\ell n\left(\frac{b}{a}\right)$ (B) $R = \frac{\rho}{2\pi\ell}\left(\frac{b}{a}\right)$ (C) $R = \frac{2\rho}{\pi\ell}\ell n\left(\frac{b}{a}\right)$ (D) $R = \frac{\rho}{2\pi\ell}\ell n\left(\frac{b}{a}\right)$

6. In a radioactive material, fraction of active material remaining after time t is 9/16. The fraction that was remaining after t/2 is

(A) $\frac{3}{5}$ (B) $\frac{3}{4}$ (C) $\frac{7}{8}$ (D) $\frac{4}{5}$

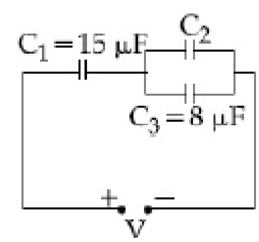
7. A charged particle carrying charge 1 μC is moving with velocity $\left(2\hat{i}+3\hat{j}+4j\hat{k}\right)$ ms^{-1}. If an external magnetic field of $\left(5\hat{i}+3\hat{j}-6j\hat{k}\right)\times 10^{-3}$ T exists in the region where the particle is moving then the force on the particle is $\vec{F}\times 10^{-9}$ N. The vector $\vec{F}$ is:

(A) $-30\hat{i}+32\hat{j}-9\hat{k}$
(B) $-3.0\hat{i}+3.2\hat{j}-0.9\hat{k}$
(C) $-300\hat{i}+320\hat{j}-90\hat{k}$
(D) $-0.30\hat{i}+0.32\hat{j}-0.09\hat{k}$

8. In a Young's double slit experiment, light of 500 nm is used to produce an interference pattern. When the distance between the slits is 0.05 mm, the angular width (in degree) of the fringes formed on the distance screen is close to:

(A) 1.7°
(B) 0.07°
(C) 0.57°
(D) 0.17°

9. In the circuit shown in the figure, the total charge is 750 μC and the voltage across capacitor C_2 is 20 V. Then the charge on capacitor C_2 is



(A) 160 μC
(B) 650 μC
(C) 590 μC
(D) 450 μC

10. A 750 Hz, 20 V (rms) source is connected to a resistance of 100 Ω an inductance of 0.1803 H and a capacitance of 10 μF all in series. The time in which the resistance (heat capacity
2 J/°C) will get heated by 10°C. (assume no loss of heat to the surroundings) is close to:

(A) 348 s
(B) 365 s
(C) 418 s
(D) 245 s

11. Pressure inside two soap bubbles are 1.01 and 1.02 atmosphere, respectively. The ratio of their volumes is

(A) 4 : 1
(B) 0.8 : 1
(C) 2 : 1
(D) 8 : 1

12. A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e. By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion so that it become $\sqrt{\frac{3}{2}}$ times larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is R. Value of R is:

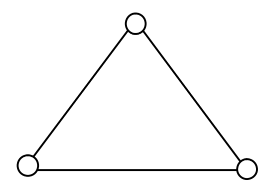
(A) $2.5R_e$
(B) $3R_e$
(C) $2R_e$
(D) $4R_e$

13. Magnitude of magnetic field (in SI units) at the centre of a hexagonal shape coil of side 10 cm. 50 turns and carrying current I (Ampere) in units of $\frac{\mu_0 I}{\pi}$ is:
(A) $500\sqrt{3}$ (B) $50\sqrt{3}$
(C) $5\sqrt{3}$ (D) $250\sqrt{3}$

14. The magnetic field of a plane electromagnetic wave is
$\vec{B} = 3 \times 10^{-8} \sin[200\,\pi(y+ct)]\,\hat{i}\,T$
where $c = 3 \times 10^8\ ms^{-1}$ is the speed of light.
The corresponding electric field is
(A) $\vec{E} = 9\sin[200\,\pi(y+ct)]\,\hat{k}\ V/m$ (B) $\vec{E} = -9\sin[200\,\pi(y+ct)]\,\hat{k}\ V/m$
(C) $\vec{E} = 3\times10^{-8}\sin[200\,\pi(y+ct)]\,\hat{k}\ V/m$ (D) $\vec{E} = -10^{-6}\sin[200\,\pi(y+ct)]\,\hat{k}\ V/m$

15. Using screw gauge of pitch 0.1 cm and 50 divisions on its circular scale, the thickness of an object is measured. It should correctly be recorded as
(A) 2.121 cm (B) 2.123 cm
(C) 2.124 cm (D) 2.125 cm

16. Consider a gas of triatomic molecules. The molecules are assumed to be triangular and made of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is:



(A) $\frac{5}{2}RT$ (B) $\frac{3}{2}RT$
(C) 3RT (D) $\frac{9}{2}RT$

17. Two isolated conducting spheres S_1 and S_2 of radius $\frac{2}{3}R$ and $\frac{1}{3}R$ have 12 μC and –3 μC charges, respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done the charges on S_1 and S_2 are respectively:
(A) +4.5 μC and –4.5 μC (B) 4.5 μC on both
(C) 6 μC and 3 μC (D) 3 μC and 6 μC

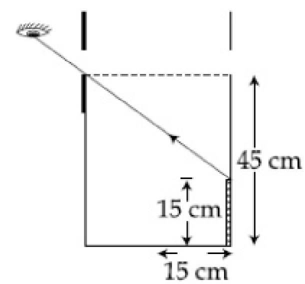
18. When a diode is forward biased, it has a voltage drop of 0.5 V. The safe limit of current through the diode is 10 mA. If a battery of emf 1.5 V is used in the circuit, the value of minimum resistance to be connected in series with the diode so that the current does not exceed the safe limit is
(A) 100 Ω (B) 200 Ω
(C) 50 Ω (D) 300 Ω

19. Moment of inertia of a cylinder of mass M, length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is $I = M\left(\frac{R^2}{4}+\frac{L^2}{12}\right)$. If such a cylinder is to be made for a given mass of a material, the ratio L/R for it to have minimum possible I is

(A) $\frac{2}{3}$ (B) $\sqrt{\frac{3}{2}}$ (C) $\frac{3}{2}$ (D) $\sqrt{\frac{2}{3}}$

20. A balloon filled with helium (32°C and 1.7 atm.) bursts. Immediately afterwards the expansion of helium can be considered as:
(A) irreversible adiabatic (B) reversible isothermal
(C) reversible adiabatic (D) irreversible isothermal

21. An observer can see through a small hole on the side of a jar (radius 15 cm) at a point at height of 15 cm from the bottom (see figure). The hole is at a height of 45 cm. When the jar is filled with a liquid up to a height of 30 cm the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid is N / 100, where N is an integer, the value of N is __________.



22. A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis at 5 revolutions per minute (rpm). The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre __________.

23. When a long glass capillary tube of radius 0.015 cm is dipped in a liquid, the liquid rises to a height of 15 cm within it. If the contact angle between the liquid and glass to close to 0°,
the surface tension of the liquid, in milli Newton m^{-1}, is [$\rho_{(lliquid)}$ = 900 kgm^{-3}, g = 10 ms^{-2}] (Give answer in closest integer)

24. A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizontally a distance of 0.2 m while launching the ball, the value of F (in N) is (g = 10 ms^{-2}) __________.

25. A bakelite beaker has volume capacity of 500 cc at 30°C. When it is partially filled with V_m volume (at 30°C) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{(beaker)} = 6 \times 10^{-6}$ °C^{-1} and $\gamma_{(mercury)} = 1.5 \times 10^{-4}$ °C^{-1}, where γ is the coefficient of volume expansion, then V_m (in cc) is close to __________.

PART –B (CHEMISTRY)

26. Aqua regia is used for dissolving noble metals (Au, Pt, etc.). The gas evolved in this process is:

(A) NO
(B) N_2O_3
(C) N_2
(D) N_2O_5

27. An acidic buffer is obtained on mixing:

(A) 100 mL of 0.1 M HCl and 200 mL of 0.1 M NaCl
(B) 100 mL of 0.1 M HCl and 200 mL of 0.1 M CH_3COONa
(C) 100 mL of 0.1 M CH_3COOH and 100 mL of 0.1 M NaOH
(D) 100 mL of 0.1 M CH_3COOH and 200 mL of 0.1 M NaOH

28. The mechanism of S_N1 reaction is given as:

$$\mathrm{R-X} \longrightarrow \underset{\text{Ion pair}}{\mathrm{R^{\oplus}X^{\ominus}}} \longrightarrow \underset{\substack{\text{Solvent}\\ \text{Separated ion}\\ \text{pair}}}{\mathrm{R^{\oplus}\|X^{\ominus}}} \xrightarrow{\mathrm{Y^{\ominus}}} \mathrm{R-Y+X^{\ominus}}$$

A student writes general characteristics based on the given mechanism as:

(a) The reaction is favoured by weak nucleophiles.
(b) $R^{\oplus}$ would be easily formed if the substituents are bulky.
(c) The reaction is accompanied by racemization.
(d) The reaction is favoured by non-polar solvents.

Which observations are correct?

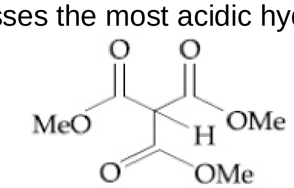
(A) (b) and (d)
(B) (a), (b) and (c)
(C) (a) and (c)
(D) (a) and (b)

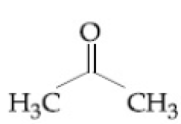
29. Of the species, NO, NO^+, NO^{2+} and NO^-, the one with minimum bond strength is

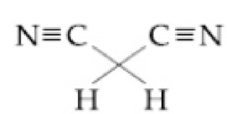
(A) NO^+
(B) NO
(C) NO^{2+}
(D) NO^-

30. Which one of the following compounds possesses the most acidic hydrogen?

(A) $H_3C-C\equiv C-H$

(B) 

(C) 

(D) 

31. The electronic spectrum of $[Ti(H_2O)_6]^{3+}$ shows a single broad peak with a maximum at 20,300 cm^{-1}. The crystal field stabilization energy (CFSE) of the complex ion, in kJ mol^{-1}, is: (1 kJ mol^{-1} = 83.7 cm^{-1})

(A) 242.5
(B) 97
(C) 83.7
(D) 145.5

32. The complex that can shows optical activity is:
(A) cis-$[CrCl_2(ox)_2]^{3-}$ (ox = oxalate)
(B) trans-$[Cr(Cl_2)(ox)_2]^{3-}$
(C) cis-$[Fe(NH_3)_2(CN)_4]^{-}$
(D) trans-$[Fe(NH_3)_2(CN)_4]^{-}$

33. Thermal power plants can lead to:
(A) Ozone layer depletion
(B) Acid rain
(C) Blue baby syndrome
(D) Eutrophication

34. In a molecule of pyrophosphoric acid, the number of P – OH, P = O and P – O – P bonds/ moiety(ies) respectively are
(A) 4, 2 and 1
(B) 4, 2 and 0
(C) 2, 4 and 1
(D) 3, 3 and 3

35. The atomic number of the element unnilennium is
(A) 102
(B) 108
(C) 119
(D) 109

36. Glycerol is separated in soap industries by:
(A) Differential extraction
(B) Fractional distillation
(C) Distillation under reduced pressure
(D) Steam distillation

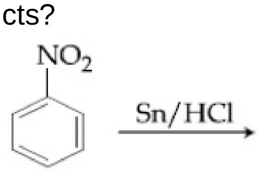
37. Let C_{NaCl} and C_{BaSO_4} be the conductances (in S) measured for saturated aqueous solution of NaCl and $BaSO_4$, respectively, at a temperature T.
Which of the following is false?
(A) $C_{NaCl}(T_2) > C_{NaCl}(T_1)$ for $T_2 > T_1$
(B) $C_{NaCl} >> C_{BaSO_4}$ at a given T
(C) $C_{BaSO_4}(T_2) > C_{BaSO_4}(T_1)$ for $T_2 > T_1$
(D) Ionic mobilities of ions from both salts increase with T

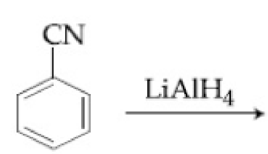
38. Henry's constant (in kbar) for four gases α, β, γ and δ in water at 298 K is given below:

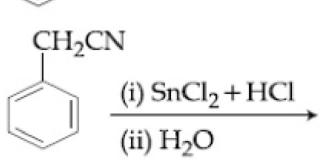
	α	β	γ	δ
K_H	50	2	2×10^{-5}	0.5

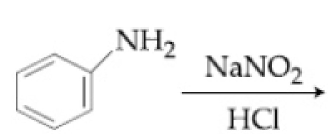
(density of water = 10^3 kg m^{-3} at 298 K)
This table implies that:
(A) α has the highest solubility in water at a given pressure
(B) solubility of γ at 308 K is lower than at 298 K.
(C) The pressure of a 55.5 molal solution of δ is 250 bar.
(D) The pressure of a 55.5 molal solution of γ is 1 bar.

39. The Khjeldahl method of Nitrogen estimation fails for which of the following reaction products?

(a) 

(b) 

(c) 

(d) 

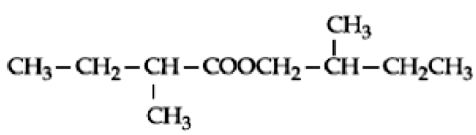
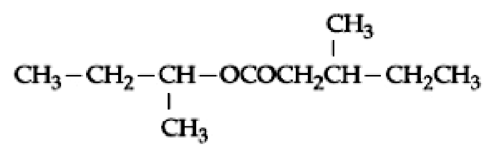
(A) (c) and (d) (B) (b) and (c)
(C) (a) and (d) (D) (a), (c) and (d)

40. The antifertility drug “Noverstrol” can react with:
(A) Alcoholic HCN; NaOCl; $ZnCl_2$/HCl
(B) Br_2/water; $ZnCl_2$/ HCl; NaOCl
(C) $ZnCl_2$/HCl ; $FeCl_3$; Alcoholic HCN
(D) Br_2/water; $ZnCl_2$/HCl; $FeCl_3$

41. If the boiling point of H_2O is 373 K, the boiling point of H_2S will be:
(A) more than 373 K
(B) less than 300 K
(C) greater than 300 K but less than 373 K
(D) equal to 373 K

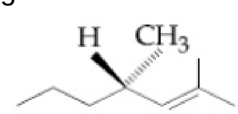
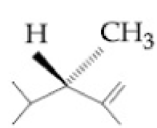
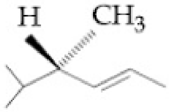
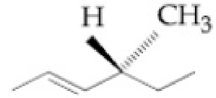
42. Tyndall effect is observed when
(A) The diameter of dispersed particles is similar to the wavelength of light used.
(B) The diameter of dispersed particles is much larger than the wavelength of light used.
(C) The refractive index of dispersed phase is greater than that of the dispersion medium.
(D) The diameter of dispersed particles is much smaller than the wavelength of light used.

43. An organic compound [A], molecular formula $C_{10}H_{20}O_2$ was hydrolyzed with dilute sulphuric acid to give a carboxylic acid [B] and an alcohol [C]. Oxidation of [C] with CrO_3–H_2SO_4 produced [B]. Which of the following structures are not possible for [A]?

(A) $(CH_2)_3$–C–$COOCH_2C(CH_3)_3$
(B) $CH_3CH_2CH_2COOCH_2CH_2CH_2CH_3$
(C) 
(D) 

44. It is true that:
(A) A first order reaction is always a single step reaction.
(B) A zero order reaction is a single step reaction.
(C) A second order reaction is always a multistep reaction.
(D) A zero order reaction is a multistep reaction.

45. Which of the following compounds produces an optically inactive compound on hydrogenation?

(A) 
(B) 
(C) 
(D) 

46. The volume strength of 8.9 M H_2O_2 solution calculated at 273 K and 1 atm is __________.
(R = 0.0821 L atm K^{-1} mol^{-1}) (rounded off to the nearest integer)

47. The photoelectric current from Na (work function, w_0 = 2.3 eV) is stopped by the output voltage of the cell

$Pt(s)/H_2(g, 1\ bar)\ |\ HCl\ (aq., pH = 1)\ |\ AgCl(s)\ |\ Ag(s)$.

The pH of aq. HCl required to stop the photoelectric current from K(w_0 = 2.25 eV), all other conditions remaining the same, is __________ $\times 10^{-2}$ (to the nearest integer).

Given,

$$2.303\frac{RT}{F} = 0.06\ V\ ;\ E^0_{AgCl\,|\,Ag\,|\,Cl^-} = 0.22\ V$$

48. The total number of monohalogenated organic products in the following (including stereoisomers) reaction is _________.

$$\underset{\text{(Simplest optically active alkene)}}{A} \xrightarrow[\text{(ii) } X_2/\Delta]{\text{(i) } H_2/Ni/\Delta}$$

49. An element with molar mass 2.7×10^{-2} kg mol^{-1} forms a cubic unit cell with edge length 405 pm. If its density is 2.7×10^3 kg m^{-3}, the radius of the element is approximately __________ $\times 10^{-12}$ m (to the nearest integer).

50. The mole fraction of glucose ($C_6H_{12}O_6$) in an aqueous binary solution is 0.1. The mass percentage of water in it, to the nearest integer, is _________.

PART–C (MATHEMATICS)

51. If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta - \frac{1}{\alpha}\right)$ is equal to:

(A) $\frac{9}{4}\left(9 - p^2\right)$ (B) $\frac{9}{4}\left(9 + q^2\right)$
(C) $\frac{9}{4}\left(9 - q^2\right)$ (D) $\frac{9}{4}\left(9 + p^2\right)$

52. A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2 + 4y^2 = 12$, then this hyperbola does not pass through which of the following points?

(A) $\left(1, -\frac{1}{\sqrt{2}}\right)$ (B) $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$
(C) $\left(-\sqrt{\frac{3}{2}}, 1\right)$ (D) $\left(\frac{1}{\sqrt{2}}, 0\right)$

53. Consider the two sets:
$A = \{m \in R$: both the roots of $x^2 - (m + 1)x + m + 4 = 0$ are real$\}$ and $B = [-3, 5)$.
Which of the following is not true?
(A) $A \cap B = \{-3\}$ (B) $B - A = (-3, 5)$
(C) $A - B = (-\infty, -3) \cup (5, \infty)$ (D) $A \cup B = R$

54. If $y^2 + \log_e(\cos^2 x) = y$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

Then:
(A) $|y'(0)| + |y''(0)| = 3$ (B) $|y''(0)| = 2$
(C) $|y'(0)| + |y''(0)| = 1$ (D) $|y''(0)| = 0$

55. Let P be a point on the parabola, $y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then:

(A) $MQ = \frac{1}{3}$ (B) $MQ = \frac{1}{4}$
(C) PN = 4 (D) PN = 3

56. The lines
$\vec{r} = \left(\hat{i} - \hat{j}\right) + \ell\left(2\hat{i} + \hat{k}\right)$ and
$\vec{r} = \left(2\hat{i} - \hat{j}\right) + m\left(\hat{i} + \hat{j} - \hat{k}\right)$
(A) intersect when $\ell = 2$ and $m = \frac{1}{2}$
(B) intersect when $\ell = 1$ and $m = 2$
(C) do not intersect for any values of ℓ and m
(D) intersect for all values of ℓ and m

57. The foot of the perpendicular drawn from the point (4, 2, 3) to the line joining the points (1, –2, 3) and (1, 1, 0) lies on the plane:
(A) $x - y - 2z = 1$
(B) $2x + y - z = 1$
(C) $x - 2y + z = 1$
(D) $x + 2y - z = 1$

58. The solution curve of the differential equation, $(1+e^{-x})(1+y^2)\frac{dy}{dx} = y^2$, which passes through the point (0, 1), is
(A) $y^2 + 1 = y\left(\log_e\left(\frac{1+e^x}{2}\right) + 2\right)$
(B) $y^2 + 1 = y\log_e\left(\frac{1+e^x}{2}\right)$
(C) $y^2 + 1 = y\left(\log_e\left(\frac{1+e^{-x}}{2}\right) + 2\right)$
(D) $y^2 = 1 + y\log_e\left(\frac{1+e^x}{2}\right)$

59. For the frequency distribution:

Variate (x):	x_1	x_2	$x_3 \ldots . x_{15}$
Frequency (f):	f_1	f_2	$f_3 \ldots . f_{15}$

Where $0 < x_1 < x_2 < x_3 < \ldots < x_{15} = 10$ and
$\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be:
(A) 4
(B) 2
(C) 6
(D) 1

60. If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} =$
$Ax^3 + Bx^2 + Cx + D$, then B + C is equal to:
(A) –3
(B) 9
(C) –1
(D) 1

61. The proposition $p \to \sim(p \wedge \sim q)$ is equivalent to:
(A) $(\sim p) \wedge q$
(B) q
(C) $(\sim p) \vee q$
(D) $(\sim p) \vee (\sim q)$

62. $\int_{-\pi}^{\pi} \left|\pi - |x|\right| dx$ is equal to:

(A) π^2
(B) $\frac{\pi^2}{2}$
(C) $\sqrt{2}\pi^2$
(D) $2\pi^2$

63. A die is thrown two times and the sum of the scorers appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is

(A) $\frac{1}{9}$
(B) $\frac{1}{3}$
(C) $\frac{1}{8}$
(D) $\frac{1}{4}$

64. If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is:

(A) $\frac{1}{4}$
(B) $\frac{1}{7}$
(C) $\frac{1}{6}$
(D) $\frac{1}{5}$

65. If the number of integral terms in the expansion of $\left(3^{\frac{1}{2}} + 5^{\frac{1}{8}}\right)^n$ is exactly 33, then the least value of 'n' is

(A) 264
(B) 248
(C) 256
(D) 128

66. The function, f(x) = $(3x - 7)x^{2/3}$, $x \in R$, is increasing for all x lying in:

(A) $\left(-\infty, \frac{14}{15}\right)$
(B) $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$
(C) $\left(-\infty, -\frac{14}{15}\right) \cup (0, \infty)$
(D) $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

67. $2\pi - \left(\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65}\right)$ is equal to:

(A) $\frac{3\pi}{2}$
(B) $\frac{5\pi}{4}$
(C) $\frac{7\pi}{4}$
(D) $\frac{\pi}{2}$

68. Let [t] denote the greatest integer $\le$ t. If for some $\lambda \in R - \{0, 1\}$, $\lim_{x \to 0} \left|\frac{1 - x + |x|}{\lambda - x + [x]}\right| = L$, then L is equal to:

(A) 0
(B) 1
(C) $\frac{1}{2}$
(D) 2

69. The area (in sq. units) of the region
$\{(x, y) : 0 \le y \le x^2 + 1,\ 0 \le y \le x + 1,\ \frac{1}{2} \le x \le 2\}$ is:
(A) $\frac{79}{16}$
(B) $\frac{23}{16}$
(C) $\frac{79}{24}$
(D) $\frac{23}{6}$

70. The value of $(2 \cdot {}^1P_0 - 3 \cdot {}^2P_1 + 4 \cdot {}^3P_2 - \ldots$ up to 51^{th} term) + (1! – 2! + 3! – … upt to 51^{th} term) is equal to:
(A) 1
(B) 1 + (51)!
(C) 1 + (52)!
(D) 1 – 51(51)!

71. Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}, x \in R$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then a_{22} is equal to _________.

72. The diameter of the circle, whose centre lies on the lines x + y = 2 in the first quadrant and which touches both the lines x = 3 and y = 2, is _________.

73. If $\lim\limits_{x \to 0} \left\{ \frac{1}{x^8} \left(1 - \cos\frac{x^2}{2} - \cos\frac{x^2}{4} + \cos\frac{x^2}{2}\cos\frac{x^2}{4} \right) \right\} = 2^{-k}$, then the value of k is _________.

74. The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots \text{ to } \infty\right)}$ is equal to _________.

75. If $\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{i-1}\right)^{n/3} = 1, \quad (m, n \in N)$
Then the greatest common divisor of the least values of m and n is _________.

FIITJEE
Solutions to JEE (Main)-2020
PART –A (PHYSICS)

1. **B**

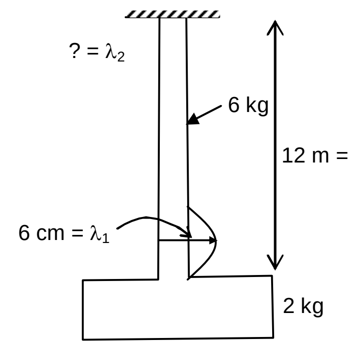
Sol. At lower end

$T_1 = 20 \Rightarrow v_1 = \sqrt{\frac{T_1}{\mu}} = \sqrt{\frac{20}{\mu}}$ ...(1)

At upper end

$T_2 = 80 \Rightarrow v_2 = \sqrt{\frac{T_2}{\mu}} = \sqrt{\frac{80}{\mu}}$

$= 2\,\mu$ ...(2)



$\therefore$ frequency remaining same

$f = f_2$

$\Rightarrow \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2} \Rightarrow \lambda_2 = \lambda_1 \frac{v_2}{v_1}$

$= 2\,\lambda = 12$ cm

2. **B**

Sol. If work function of metal be ϕ, then K.E. of emitted photo electron,

$k = h\nu\phi = \frac{hc}{\lambda} - \phi$ …(1)

$\therefore$ at $\lambda_1 = 500$ nm

$k_1 = \frac{hc}{\lambda_1} - \phi$ …(2)

At $\lambda_2 = 200$ nm

$k_2 = \frac{hc}{\lambda_2} - \phi = 3k_1$ (given)

$= 3\left(\frac{hc}{\lambda_1} - \phi\right)$

$\Rightarrow \frac{hc}{1}\left(\frac{3}{\lambda_1} - \frac{1}{\lambda_2}\right) = 2\phi$

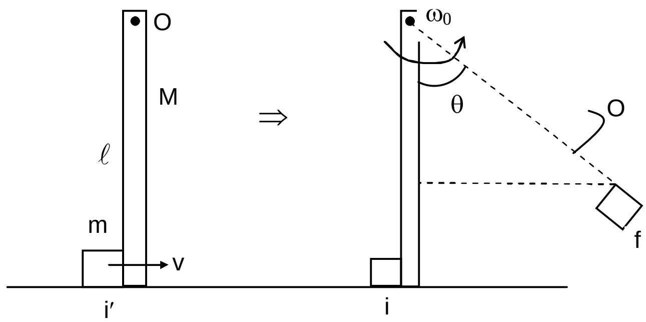
$\Rightarrow \phi = \frac{hc}{2}\left(\frac{3}{\lambda_1} - \frac{3}{\lambda_2}\right)$

$$= \frac{hc}{2\times 100\,nm}\left(\frac{3}{5}-\frac{1}{2}\right)$$

$$= \frac{1240}{2\times 100}\,\frac{1}{10} = 0.62\text{ eV.}$$

3. **D**

Sol.



COAM about O between (i) and (f)

$$mv\ell = \left(\frac{M\ell^2}{3}+m\ell^2\right)w_0$$

$$\frac{3mv}{(M\ell+3m\ell)} = ab \qquad ...(1)$$

COTME between (i) and (f) positrons

$$\frac{1}{2}\left(m\ell^2+\frac{M\ell^2}{3}\right)\left(\frac{3mv_o}{M\ell+3m\ell}\right)^2 = mg(\ell-\ell\cos\theta)+Mg\left(\frac{\ell}{2}-\frac{\ell}{2}\cos\theta\right)$$

$$\Rightarrow \frac{\ell}{2}\times\frac{1}{3}\,\frac{9m^2v_0^2}{(M\ell+3m\ell)} = \ell(1-\cos\theta)\left(mg+\frac{Mg}{2}\right)$$

$$\Rightarrow \frac{3}{2}\times\frac{(1)^2\times(1)\times 6^2}{(2\times 1+3\times 1\times 1)} = (1)\,(1-\cos\theta)\,(10+10)$$

$$\Rightarrow \frac{27\times 2}{20\times 5} = 0.54 = (1-\cos\theta)$$

$\Rightarrow$ $\cos\theta = 0.46$

$\Rightarrow$ $\theta \approx 63°$

4. **D**

Sol. For rotating loop

$\varepsilon_0 = BA\omega = B(\pi ab)\omega$

$\therefore$ Average power loss

$$P_{avg} = \frac{E_0^2}{2R} = \frac{\pi^2a^2b^2B^2\omega^2}{2R}$$

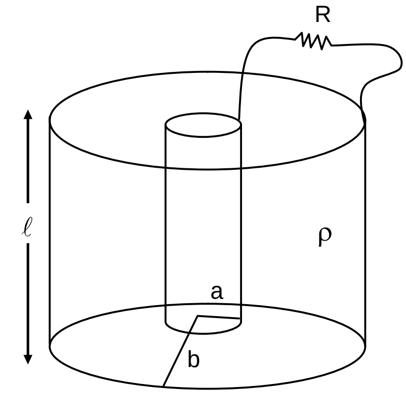
5. **D**

Sol. For maximum joule heating,

$R = R_{eq}$ of cylinder

$$= \int \frac{\rho dx}{(2\pi x \ell)}$$

$$= \frac{\rho}{2\pi\ell}\ell n\left(\frac{b}{a}\right)$$



6. **B**

Sol. $\frac{9}{16}N_o = N = N_o \quad e^{-\lambda t}$ ...(1)

$\therefore \quad N' = N_o\, e^{-\frac{\lambda t}{2}}$

$= N_o\left(e^{-\lambda t}\right)^{1/2}$

$\Rightarrow \quad \frac{N'}{N_o} = \left(\frac{9}{16}\right)^{1/2} = \frac{3}{4}$

7. **A**

Sol. Coulomb's law

$\vec{F} = q\vec{V} \times \vec{B}$

$= (10^{-6})\ (2\hat{i} + 3\hat{j} + 4\hat{k}) \times (5\,\hat{i} + 3\,\hat{j} - 6\,\hat{k}) \times 10^{-3}$

$= (10^{-9})\ \left[-30\hat{i} + 32\hat{j} - 9\hat{k}\right]$

8. **C**

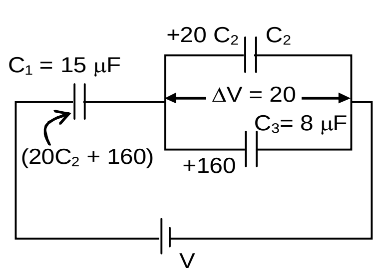
Sol. Angular width of a fringe is YDSE

$$= \frac{\lambda}{d} = \frac{500 \times 10^{-9}}{0.05 \times 10^{-3}}\text{rad}$$

$$= 10^{-2} \times \left(\frac{180}{3.14}\right) \approx 0.57^o$$

9. **C**

Sol.



Total charge on all cap (left plates)

$\Delta V = 20$ V

$\Rightarrow \quad 750 = (20\ C_2 + 660) \times 1$

$\Rightarrow \quad 20\ C_2 = 590\ \mu C$

10. **B**

Sol. Power loss in AC

$$P = \varepsilon_{rms}^2 R = \frac{\varepsilon_{rms}^2 R}{Z^2}$$

$\therefore \quad \Delta Q = Pt$

$\Rightarrow \quad 2 \times 10 = \dfrac{(20)^2 \times 100 \times t}{(715600)}$

$\Rightarrow \quad t = 358$ sec.

11. **D**

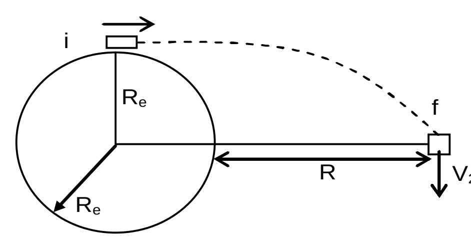
Sol. $\Delta P_1 = \dfrac{4T}{R_1} = 0.01 \qquad \ldots(1)$

& $\Delta P_2 = \dfrac{4T}{R_2} = 0.02 \qquad \ldots(2)$

$\therefore$ Ratio of volumes $\times \dfrac{R_1^3}{R_2^3} = \dfrac{1}{\left(\dfrac{1}{2}\right)^3} = 8:1$

12. **C**

Sol. $V = \sqrt{\dfrac{3}{2}}\sqrt{\dfrac{GMe}{R_e}} = \sqrt{\dfrac{36}{2R_e}}$



Between two positions

COAM

$$m\sqrt{\frac{36\,M_e}{2\,R_e}}\,R_e = m(R_e + R)V_2 \qquad ...(1)$$

COTME

$$-\frac{GM_e m}{R_e} + \frac{1}{2}m\frac{3GM_e}{2R_e} = \frac{-GMm}{(R_e + R)} + \frac{1}{2}mv_2^2 \qquad ...(2)$$

Solving:

$$\Rightarrow \quad -\frac{GM_e m}{4R_e} = -\frac{GM_e m}{(R_e + R)} + \frac{m}{2}\frac{\left(\dfrac{36\,M_e}{2\,R_e}\right)R_e^2}{(R_e + R)^2}$$

Let $R_e + R = x$

$$-\frac{1}{4R_e} = -\frac{1}{x} + \frac{3R_e}{4x^2}$$

$\Rightarrow \quad -x^2 = -4R_e n + 3R_e^2 \quad ; \quad x^2 + 4\,R_e x + R_e^2 = 0$

$$\Rightarrow \quad x = \frac{4\,R_e + \sqrt{16\,R_e^2 + 12\,R_e^2}}{2}$$

$= (2R_e + R_e)$

$= R = 2R_e$

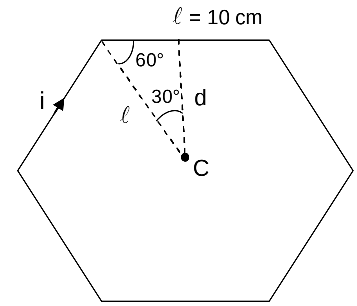
13. **A**

Sol.

$$\vec{B}(\text{at C}) = 6 \times \frac{\mu_o i}{4\pi d}(\sin 30^o \times 2) \times 50$$

$$= \frac{\mu_o i}{\pi} \times \frac{3}{2 \times \left(0.1 \times \frac{\sqrt{3}}{2}\right)} \times 50$$

$$= \frac{\mu_o i}{\pi} \times (500\sqrt{3})$$



14. **B**

Sol. $E_o = CB_o = 3 \times 10^8 \times 3 \times 10^{-8} = 9$ V/m

$\therefore \quad \vec{E} = -9 \sin[200\,\pi(y + ct)]\,\hat{k}$

Direction of travel

$\frac{d}{dt}(y + ct) = 0$

$\Rightarrow \quad \frac{ds}{dt} = -C \rightarrow$ along $(-\hat{j})$

$\because \quad \vec{E} \times \vec{B}$ should be along $(-\hat{j})$

15. **C**

Sol. $LC = \frac{p}{N} = \frac{0.1\,cm}{50}$ = 0.02 mm = 0.002 cm

Hence, measurement should be a multiple of LC

16. **C**

Sol. $U = \frac{nfRT}{2}$

$= (1)(6)\frac{RT}{2}$ = 3RT

17. **C**

Sol. Spheres are in parallel.

So, $C_{aq} = C1 + C2 = 4\pi e_0(R_1 + R_2)$

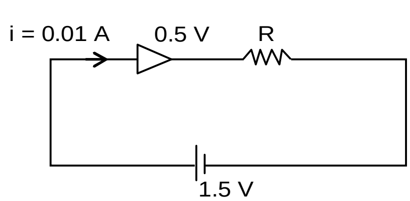
$\therefore \quad$ Potential, $V = \left(\frac{q_1 + q_2}{c_1 + c_2}\right)$

$\therefore \quad q_1 = Gv = \frac{2}{3}R \times \frac{(q)}{(R)} = 6\ \mu C$

& $q_2 = 3\ \mu C$

18. **A**

Sol.



KVL

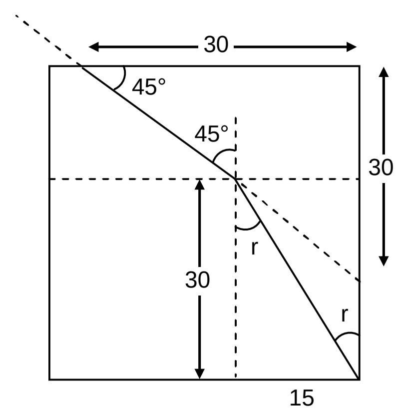
1.5 – 0.5 – 0.01 R = 0

$\Rightarrow$ R = 100 Ω

19. **B**
Sol. Mass of material = CONSTANT
$\Rightarrow$ $(\pi R^2 L\rho) = M$ …(1)
$\therefore$ $I = M\left(\frac{R^2}{4}+\frac{L^2}{12}\right) = M\left(\frac{M}{4\pi\rho L}+\frac{L^2}{12}\right)$
$\therefore$ For I maximum / minimum
$0 = \frac{dI}{DL} = M\left(-\frac{M}{4\pi\rho L^2}+\frac{L}{6}\right)$
$\Rightarrow$ $\frac{M}{\pi\rho L} = \frac{4}{6}L^2$
$\Rightarrow$ $R^2 = \frac{2}{3}L^2$ $\Rightarrow$ $\frac{L}{R} = \sqrt{\frac{3}{2}}$

20. **D**
Sol. After burning, heat exchange occurs between helium and atmospheric. Hence, irreversible, isothermal process.

21. **158.00**
Sol. (1) $\sin(45°) = n \sin r$
$= \frac{n\times 15}{\sqrt{15^2+30^2}}$
$\Rightarrow$ $n = \frac{1}{\sqrt{2}}\times\sqrt{5} = \sqrt{\frac{5}{2}}$
$= 1.58 = \frac{158}{100} = \frac{N}{100}$



$\Rightarrow$ N = 158.00

22. **9.00**
Sol. Conservation of angular momentum
$(5)\left(\frac{200\times R^2}{2}+80\,R^2\right) = \omega_f\left(\frac{200\times R^2}{2}\right)$
$\Rightarrow$ $\omega_p = \frac{5\times 180}{100}$ = 9.00

23. **101.25**
Sol. Height of capillary rise
$h = \frac{2T}{r\rho g}$
$\Rightarrow$ $T = \frac{hrg\rho}{2} = \frac{0.15\times 0.015\times 10\times 900\times 10^{-2}}{2}$
$= 10.125 \times 10^{-2}$ N/m
$= 101.25$ m N/m

24. **150.00**

Sol. $W = \Delta K = mgh$

$\Rightarrow \quad F \times 0.2 = 0.15 \times 10 \times 20$

$\Rightarrow \quad F = \dfrac{30}{0.2}$ = 150 Newton

25. **20.00**

Sol. Unfilled baker volume remains constant,

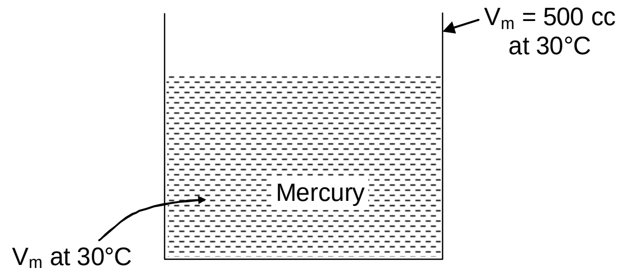
$\therefore \quad \Delta V_B = \Delta V_m$

$\Rightarrow \quad V_B\, Y_B\, \Delta T = V_M\, Y_M\, \Delta T$

$\Rightarrow \quad V_M = \dfrac{V_B Y_B}{Y_M} = \dfrac{500 \times 6 \times 10^{-6}}{1.5 \times 10^{-4}}$

$= 500 \times 4 \times 10^{-2}$

$= 20$ cc



PART –B (CHEMISTRY)

26. A

Sol. Aqua regia is HNO_3 : HCl
1 : 3

$$HNO_3 + 3HCl \longrightarrow 2H_2O + NO + 3[Cl]$$

(i) $Au + 3[Cl] \longrightarrow AuCl_3 \xrightarrow{HCl} HAuCl_4$

(ii) $Pt + 4[Cl] \longrightarrow PtCl_4 \xrightarrow{HCl} H_2PtCl_6$

27. B

Sol. Mixture of weak acid and its salt with strong base acts as buffer solution.

28. B

Sol. Above reaction is S_N1 reaction as it proceed via formation of carbocation. Polar solvent is more suitable for S_N1 and racemisation takes place.

29. D

Sol.

	Species	Bond order
(A)	NO^+	3
(B)	NO^{2+}	2.5
(C)	NO^-	2
(D)	NO	2.5

Bond order strength is proportional to bond order.

30. B

Sol. Acidic strength $\propto$–I, –M effect due to strong –I, –M effect of 3 – $COOCH_3$, it has most acidic Hydrogen.

31. B

Sol. $[Ti(H_2O)_6]^{3+}$ = $Ti^{3+} = 3d^1\ 4s^0$

$t_{2g}^{1,0,0,}, e_g^{0,0}$

CFSE = $\left[-0.4n_{t_{2g}} + 0.6n_{eg}\right]\Delta_0 + n(p)$

= $[-0.4 \times 1]20300 = -8120\ cm^{-1}$

$$= \frac{-8120}{83.7} kJ/mole = -97\,kJ/mole$$

32. A

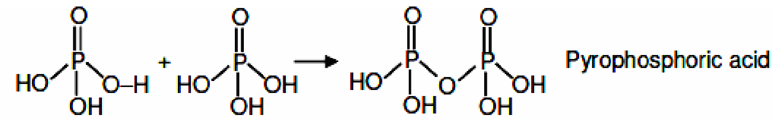
Sol. Only cis-$[CrCl_2(ox)_2]^{3-}$ show optical isomerism while its trans form do not show optical isomerism due to presence of plane of symmetry.

33. B

Sol. Burning of fossil fuels (which contain sulphur and nitrogenous matter) such as coal and oil in power stations and furnaces produce sulphur dioxide and nitrogen oxides which causes acid rain.

34. A

Sol.



No. of P =O bond = 2.
P–OH bond = 4.
P–O–P bond = 1.

35. D

Sol. un = 1
nil = 0
enn = 9
So atomic number = 109

36. C

Sol. Glycerol can be separated from spent-lye in soap industry by using reduce pressure distillation technique.

37. D

Sol. (i) Ionic mobilities decrease with increase in temperature due to increase in random motion and hence decrease in relaxation time so decrease in drift speed.
(ii) NaCl is completely soluble salt while $BaSO_4$ is sparingly soluble salt so $C_1 >> C_2$.
(iii) On increase in temperature conductance increase.

38. C

Sol. (i) Though solubility of gas will decrease with increase in temperature but this conclusion can not be drawn from the given table.
(ii) For γ ; $(P)_\gamma = (K_H)_\gamma.(X)_\gamma$

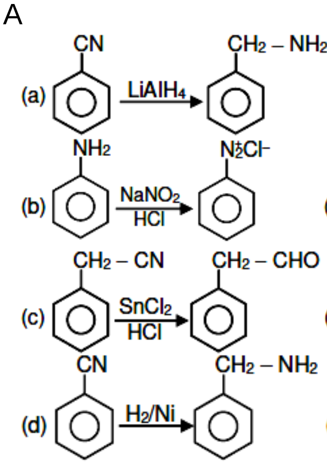
$$= 2\times10^{-2}\left[\frac{55.5}{55.5+\frac{1000}{18}}\right] = 10^{-2}\text{ bar}$$

(iii) For $\delta \Rightarrow P_\delta = (k_H)_\delta.(X)_\delta = 0.5 \times 10^3 \times\frac{1}{2} = 250$ bar

(iv) From Henry's law
$P = k_H(X)$
Higher the value of kH smaller will be solubility so γ is more soluble.

39. A

Sol.



(a) (N – present in product so will show Kjeldhal Test)

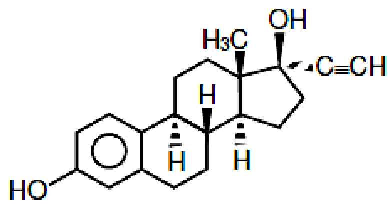
(b) ($-N_2^+$ Never show Kjeldhal Test)

(c) (N–abscent, wo will not show Kjeldhal Test)

(d) (will give positive Kjeldhal test due to presence of $-NH_2$ group)

40. D

Sol.



Novestrol (Anti Fertility Drugs)

Novestrol has phenolic functional group, alcoholic functional group and Terminal alkyne.

41. B

Sol. At room temperature water is liquid and has boiling point 373 K due to hydrogen bonding. Where as H_2S is gas and it has no hydrogen bonding. Hence boiling point of H_2S is less than 300 K [Boiling point of H_2S is –60ºC].

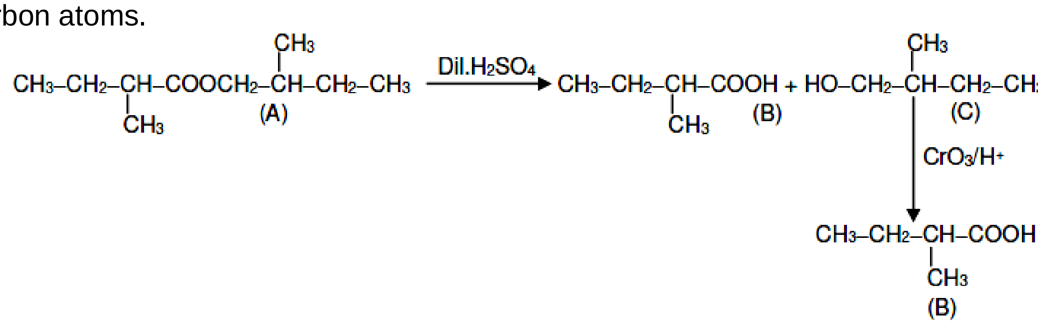
42. A

Sol. The diameter of the dispersed particles is not much smaller than the wavelength of the light used. The intensity of scattered light depends on the difference between the refractive indice of the D.P and D.M., In lyophobic colloids, this difference is appreciable and therefore the tyndal effect is quite well defined but in lyophilic sols the difference is very small and the tyndal effect is very weak. So, to show Tyndall effect the refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.

43. BD

Sol. (B) contains eight carbon atoms whereas the molecular formula $C_{10}H_{20}O_2$ contains ten carbon atoms.

(D)

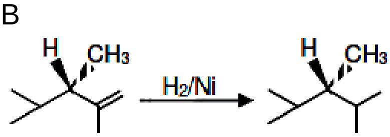


44. D

Sol. Zero order reaction is always multi step reaction.

45. B

Sol.



Optically inactive

46. 100

Sol. Molarity of H_2O_2 solution = $\left(\dfrac{\text{Volume strength}}{11.2}\right)$

Volume strength = $8.9 \times 11.2 = 99.68$ V

47. 142

Sol. Sodium metal :

$E = E_0 + (KE)_{max}$; $E^0_{cell} = 0.22$ V

Cell reaction

Cathode : $AgCl(s) + e^- \longrightarrow Ag(s) + Cl^-(aq)$

Anode : $\frac{1}{2}H_2(g) \longrightarrow H^+(aq) + e^-$

Overall : $AgCl(s) + \frac{1}{2}H_2(g) \longrightarrow Ag(s) + H^+(aq) + Cl^-(aq)$

$$E_{cell} = E^0_{cell} - \frac{0.06}{1}\log[H^+][Cl^-]$$

$$E_{cell} = 0.22 - \frac{0.06}{1}\log[10^{-1}][10^{-1}] = 0.22 + 0.12 = 0.34 \text{ V}$$

$(KE)_{max} = E_{cell} = 0.34$ eV

So $E = 2.3 + 0.34 = 2.64$ eV = Energy of photon incident

For potassium metal :

$E = E_0 + (KE)_{max}$

$2.64 = 2.25 + (KE)_{max}$

$(KE)_{max} = 0.39 = E_{cell}$

Cell reaction

Cathode : $AgCl(s) + e^- \longrightarrow Ag(s) + Cl^-(aq)$

Anode : $\frac{1}{2}H_2(g) \longrightarrow H^+(aq) + e^-$

Overall : $AgCl(s) + \frac{1}{2}H_2(g) \longrightarrow Ag(s) + H^+(aq) + Cl^-(aq)$

$$E_{cell} = E^0_{cell} - \frac{0.06}{1}\log[H^+][Cl^-]$$

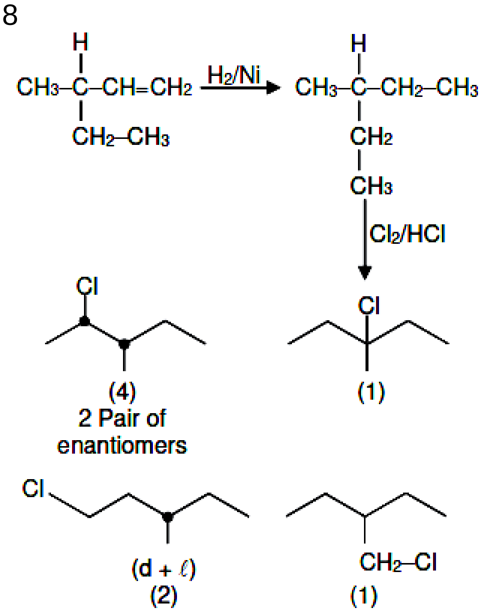
$0.39 = 0.22 - 0.12\log[H^+]$

$0.17 = 0.12 \times pH$

$pH = 17/12 = 1.4166 = 1.42$

48. 8

Sol.



49. 143

Sol. $d = \dfrac{Z \times M}{Na \times Volume}$

$2.7 = \dfrac{Z \times 27}{6.02 \times 10^{23} \times \left[4.05 \times 10^{-3}\right]^3}$

Z = 4 $\Rightarrow$ fcc unit cell

For fcc unit cell $4r = \sqrt{2}a$

$r = \dfrac{1.414 \times 405}{4}$ = 143.1675 pm = 143.17 pm

50. 47

Sol. Let total mole of solution = 1

So mole of glucose = 0.1

Mole of H_2O = 0.9

%(w/w) of H_2O = $\left[\dfrac{0.9 \times 18}{0.9 \times 18 + 0.1 \times 180}\right] \times 100 = 47.368 = 47.37$

PART–C (MATHEMATICS)

51. D

Sol. α, β are roots of $x^2 + px + 2 = 0$

$\Rightarrow \alpha^2 + p\alpha + 2 = 0$ and $\beta^2 + p\beta + 2 = 0$

$\Rightarrow \frac{1}{\alpha}, \frac{1}{\beta}$ are roots of $2x^2 + px + 1 = 0$

But $\frac{1}{\alpha}, \frac{1}{\beta}$ are roots of $2x^2 + 2qx + 1 = 0$

$\Rightarrow p = 2q$

Also $\alpha + \beta = -p \qquad \alpha\beta = 2$

$$\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$$

$$= \left(\frac{\alpha^2 - 1}{\alpha}\right)\left(\frac{\beta^2 - 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\alpha}\right)$$

$$= \frac{(-p\alpha - 3)(-p\beta - 3)(\alpha\beta + 1)^2}{(\alpha\beta)^2} =$$

$$= \frac{9}{4}\left(p^2\alpha\beta + 3p(\alpha + \beta) + 9\right)$$

$$= \frac{9}{4}\left(9 - p^2\right) = \frac{9}{4}\left(9 - 4q^2\right)$$

52. B

Sol. Ellipse : $\frac{x^2}{4} + \frac{y^2}{3} = 1$

eccentricity $= \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

$\therefore$ foci $= (\pm 1, 0)$

For hyperbola, given $2a = \sqrt{2} \Rightarrow a = \frac{1}{\sqrt{2}}$

$\therefore$ hyperbola will be $\frac{x^2}{1/2} - \frac{y^2}{b^2} = 1$

eccentricity $= \sqrt{1 + 2b^2}$

$\therefore$ foci $= \left(\pm\sqrt{\frac{1 + 2b^2}{2}}, 0\right)$

$\because$ Ellipse and hyperbola have same foci

$\Rightarrow \sqrt{\frac{1+2b^2}{2}} = 1$

$\Rightarrow b^2 = \frac{1}{2}$

$\therefore$ Equation of hyperbola: $\frac{x^2}{1/2} - \frac{y^2}{1/2} = 1$

$\Rightarrow x^2 - y^2 = \frac{1}{2}$

Clearly, $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$ does not lie on it.

53. C

Sol. $A : D \geq 0$

$\Rightarrow (m+1)^2 - 4(m+4) \geq 0$

$\Rightarrow m^2 + 2m + 1 - 4m - 16 \geq 0$

$\Rightarrow m^2 - 2m - 15 \geq 0$

$\Rightarrow (m-5)(m+3) \geq 0$

$\Rightarrow m \in (-\infty, -3] \cup [5, \infty)$

$\therefore A = (-\infty, -3] \cup [5, \infty)$

$B = [-3, 5)$

$A - B = (-\infty, -3) \cup [5, \infty)$

$A \cap B = \{-3\}$

$B - A = (-3, 5)$

$A \cup B = R$

54. B

Sol. $y^2 + \ln(\cos^2 x) = y \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

for $x = 0$ $\quad y = 0$ or 1

Differentiating wrt x

$\Rightarrow 2yy' - 2\tan x = y'$

At $(0,0)\ y' = 0$

At $(0,1)\ y' = 0$

Differentiating wrt x

$2yy'' + 2(y')^2 - 2\sec^2 x = y''$

At (0, 0) $y'' = -2$

At (0, 1) $y'' = 2$

$\therefore |y''(0)| = 2$

55. B

Sol. Let $P = \left(3t^2, 6t\right)$; $N = \left(3t^2, 0\right)$

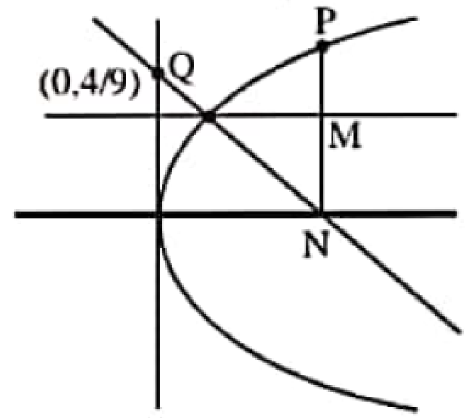
$M = \left(3t^2, 3t\right)$

Equation of MQ : y = 3t

$\therefore \quad Q = \left(\frac{3}{4}t^2, 3t\right)$

Equation of NQ

$$y = \frac{3t}{\left(\frac{3}{4}t^2 - 3t^2\right)}\left(x - 3t^2\right)$$



y – intercept of NQ $= 4t = \frac{4}{3} \Rightarrow t = \frac{1}{3}$

$\therefore \quad MQ = \frac{9}{4}t^2 = \frac{1}{4}$

$PN = 6t = 2$

56. C

Sol. $\vec{r} = \hat{i}\left(1 + 12\ell\right) + \hat{j}\left(-1\right) + \hat{k}\left(\ell\right)$

$\vec{r} = \hat{i}\left(2 + m\right) + \hat{j}\left(m - 1\right) + \hat{k}\left(-m\right)$

For intersection

$1 + 2\ell = 2 + m$ …………(i)

$-1 = m - 1$ …………(ii)

$\ell = -m$ …………(iii)

from (ii) m = 0

from (iii) $\ell = 0$

These values of m and ℓ do not satisfy equation (1).

Hence the two lines do not intersect for any values of ℓ and m.

57. B

Sol. Equation of

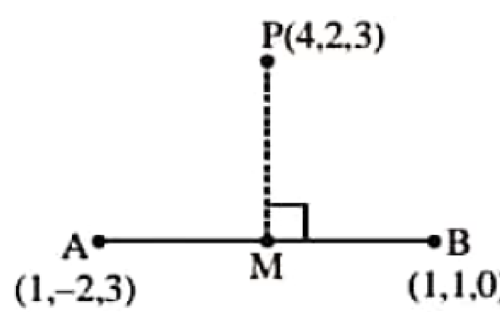
$AB = \vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(3\hat{j} - 3\hat{k}\right)$

Let coordinates of M

$= \left(1, \left(1 + 3\lambda\right), -3\lambda\right)$.

$\overrightarrow{PM} = -3\hat{i} + \left(3\lambda - 1\right)\hat{j} - 3\left(\lambda + 1\right)\hat{k}$

$\overrightarrow{AB} = 3\hat{j} - 3\hat{k}$



$\because \overrightarrow{PM} \perp \overrightarrow{AB} \Rightarrow \overrightarrow{PM}.\overrightarrow{AB} = 0$

$\Rightarrow 3\left(3\lambda - 1\right) + 9\left(\lambda + 1\right) = 0$

$\Rightarrow \lambda = -\frac{1}{3}$

$\therefore \; M = \left(1, 0, 1\right)$

Clearly M lies on $2x + y - z = 1$.

58. D

Sol. $\left(1+e^{-x}\right)\left(1+y^2\right)\frac{dy}{dx}=y^2$

$\Rightarrow \left(1+y^{-2}\right)dy=\left(\frac{e^x}{1+e^x}\right)dx$

$\Rightarrow \left(y-\frac{1}{y}\right)=\ell n\left(1+e^x\right)+c$

$\therefore$ It passes through (0, 1) $\Rightarrow c=-\ell n2$

$\Rightarrow y^2=1+y\ell n\left(\frac{1+e^x}{2}\right)$

59. C

Sol. $\therefore \sigma^2 \le \frac{1}{4}(M-m)^2$

Where M and m are upper and lower bounds of values of any random variable.

$\therefore \quad \sigma^2 < \frac{1}{4}(10-0)^2$

$\Rightarrow \quad 0<\sigma<5$

$\therefore \quad \sigma \ne 6$

60. A

Sol. $\Delta=\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix}$

$=Ax^3+Bx^2+Cx+D$.

$R_2 \to R_2-R_1 \qquad R_3 \to R_3-R_2$

$\Delta=\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-1 & x-1 & x-1 \\ x-2 & 2(x-2) & 6(x-2) \end{vmatrix}$

$=(x-1)(x-2)\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & 1 & 1 \\ 1 & 2 & 6 \end{vmatrix}$

$=-3(x-1)^2(x-2)=-3x^3+12x^2-15x+6$

$\therefore B+C=12-15=-3$

61. C

Sol. $p \to \sim(p \wedge \sim q)$

$=\sim p \vee \sim(p \wedge \sim q)$

$= \sim p \vee \sim p \vee q$
$= \sim p \vee q$

62. A

Sol. $\int_{-\pi}^{\pi} \left|\pi - |x|\right| dx = 2\int_{0}^{\pi} |\pi - x| dx$

$= 2\int_{0}^{\pi} (\pi - x) dx$

$= 2\left[\pi x - \frac{x^2}{2}\right]_0^{\pi} = \pi^2$

63. A

Sol. Sum obtained is a multiple of 4.

$A = \{(1,3),(2,2),(3,1),(2,6),(3,5),(4,4),(5,3)(6,2),(6,6)\}$

B : Score of 4 has appeared at least once.

$B = \{(1,4),(2,4),(3,4),(4,4),(5,4),(6,4),(4,1),(4,2),(4,3),(4,5),(4,6)\}$

Required probability $= P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

$= \frac{\frac{1}{36}}{\frac{9}{36}} = \frac{1}{9}$

64. C

Sol. Sum of 1st 25 terms = sum of its next 15 terms

$\Rightarrow (T_1 + ... + T_{25}) = (T_{26} + ... + T_{40})$

$\Rightarrow (T_1 + ... + T_{40}) = 2(T_1 + ... + T_{25})$

$\Rightarrow \frac{40}{2}[2 \times 3 + (39d)] = 2 \times \frac{25}{2}[2 \times 2 + 24d]$

$\Rightarrow d = \frac{1}{6}$

65. C

Sol. $T_{r+1} = {}^nC_r (3)^{\frac{n-r}{2}} (5)^{\frac{r}{8}} \qquad (n \geq r)$

Clearly r should be a multiple of 8.

$\because$ there are exactly 33 integral terms

Possible values of r can be

0, 8, 16, ………,32×8

$\therefore$ least value of n = 256

66. D

Sol. $f(x) = (3x - 7)x^{2/3}$

$\Rightarrow f(x) = 3x^{5/3} - 7x^{2/3}$

$\Rightarrow f'(x) = 5x^{2/3} - \dfrac{14}{3x^{1/3}}$

$= \dfrac{15x - 14}{3x^{1/3}} > 0$

$\therefore\ f'(x) > 0 \;\forall\, x \in (-\infty, 0) \cup \left(\dfrac{14}{15}, \infty\right)$

67. A

Sol. $2\pi - \left(\sin^{-1}\left(\dfrac{4}{5}\right) + \sin^{-1}\left(\dfrac{5}{13}\right) + \sin^{-1}\left(\dfrac{16}{65}\right)\right)$

$= 2\pi - \left(\tan^{-1}\left(\dfrac{4}{3}\right) + \tan^{-1}\left(\dfrac{5}{12}\right) + \tan^{-1}\left(\dfrac{16}{63}\right)\right)$

$= 2\pi - \left(\tan^{-1}\left(\dfrac{63}{16}\right) + \tan^{-1}\left(\dfrac{16}{63}\right)\right)$

$= 2\pi - \dfrac{\pi}{2} = \dfrac{3\pi}{2}$

68. D

Sol. LHL : $\lim\limits_{x \to 0^-} \left|\dfrac{1 - x - x}{\lambda - x - 1}\right| = \left|\dfrac{1}{\lambda - 1}\right|$

RHL : $\lim\limits_{x \to 0^+} \left|\dfrac{1 - x + x}{\lambda - x + 0}\right| = \left|\dfrac{1}{\lambda}\right|$

For existence of limit

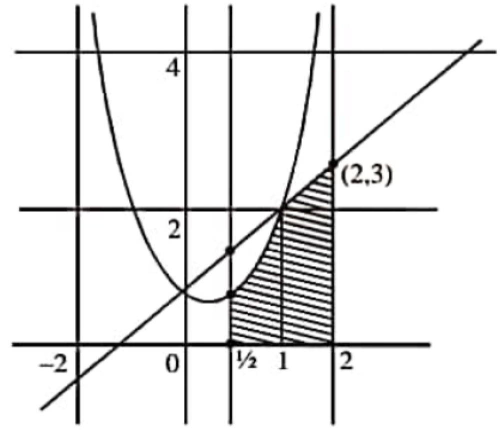
LHL = RHL

$\Rightarrow \dfrac{1}{|\lambda - 1|} = \dfrac{1}{|\lambda|} \Rightarrow \lambda = \dfrac{1}{2}$

$\therefore L = \dfrac{1}{|\lambda|} = 2$

69. C

Sol. $0 \le y \le x^2 + 1,\ 0 \le y \le x + 1,\ \frac{1}{2} \le x \le 2$



Required area

$= \frac{19}{24} + \frac{5}{2} = \frac{79}{24}$

70. C

Sol. $S = \left(2.\ {}^1P_0 - 3.\ {}^2P_1 + 4.\ {}^3P_2 \text{.......upto 51 terms}\right) + \left(1! - 2! + 3! - \text{.......upto 51 terms}\right)$

$\left[\because\ {}^nP_{n-1} = n!\right]$

$\therefore \quad S = \left(2 \times 1! - 3 \times 2! + 4 \times 3! \ldots\ldots + 52.51!\right) + \left(1! - 2! + 3! \ldots\ldots (51)!\right)$

$= \left(2! - 3! + 4! \ldots\ldots + 52!\right) + \left(1! - 2! + 3! - 4! + \ldots\ldots + (51)!\right)$

$= 1! + 52!$

71. 10

Sol. $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$

$A^4 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$

$= \begin{bmatrix} \left(x^2+1\right)^2 + x^2 & x\left(x^2+1\right) + x \\ x\left(x^2+1\right) + x & x^2+1 \end{bmatrix}$

$a_{11} = \left(x^2+1\right)^2 + x^2 = 109$

$\Rightarrow x = \pm 3$

$a_{22} = x^2 + 1 = 10$

72. 3

Sol. $\because$ center lies on $x+y=2$ and in 1st quadrant center $=(\alpha, 2-\alpha)$ where

$\alpha>0$ and $2-\alpha>0 \Rightarrow 0<\alpha<2$

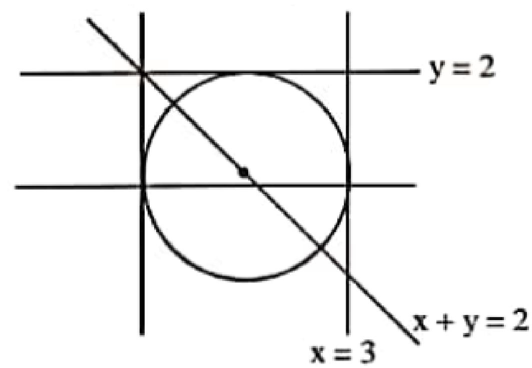
$\because$ circle touches x = 3 and y = 2

$\Rightarrow |3-\alpha| = |2-(2-\alpha)|$ = radius

$\Rightarrow |3-\alpha| = |\alpha| \Rightarrow \alpha = \frac{3}{2}$

$\therefore$ radius = α

$\Rightarrow$ Diameter $= 2\alpha = 3$.



73. 8

Sol. $\lim\limits_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\} = 2^{-k}$

$$\Rightarrow \lim_{x\to 0}\frac{\left(1-\cos\frac{x^2}{2}\right)}{4\left(\frac{x^2}{2}\right)^2}\frac{\left(1-\cos\frac{x^2}{4}\right)}{16\left(\frac{x^2}{4}\right)^2} = \frac{1}{8}\times\frac{1}{32} = 2^{-k}$$

$\Rightarrow 2^{-8} = 2^{-k} \Rightarrow k = 8$

74. 4

Sol. $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+.......\text{to}\infty\right)}$

$= \left(\frac{4}{24}\right)^{\log_{\left(\frac{5}{2}\right)}\left(\frac{1}{2}\right)}$

$= \left(\frac{1}{2}\right)^{\log_{\left(\frac{5}{2}\right)}\left(\frac{4}{25}\right)} = \left(\frac{1}{2}\right)^{-2} = 4$

75. 4

Sol. $\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{i-1}\right)^{n/3} = 1$

$\Rightarrow \left(\frac{(1+i)^2}{2}\right)^{m/2} = \left(\frac{(1+i)^2}{-2}\right)^{n/3} = 1$

$\Rightarrow (i)^{m/2} = (-i)^{n/3} = 1$

$\Rightarrow \frac{m}{2} = 4k_1$ and $\frac{n}{3} = 4k_2$

$\Rightarrow m = 8k_1$ and $n = 12k_2$

Least value of $m = 8$ and $n = 12$

$\therefore$ GCD = 4