ECE 3150 Homework #6 Solutions
(Farhan Rana)

5.1

a) $R_{min} = 0$ (trivial)

In saturation: $\frac{W}{2L}\mu_n C_{ox}\left(V_{IN}-V_{TN}\right)^2\left(1+\lambda_n V_{OUT}\right) = \frac{V_{DD}-V_{OUT}}{R}$

And: $V_{DS} \geq V_{GS}-V_{TN} \Rightarrow V_{OUT} \geq V_{IN}-V_{TN}$.

The former gives:

$$R = \frac{V_{DD}-V_{OUT}}{\frac{W}{2L}\mu_n C_{ox}\left(V_{IN}-V_{TN}\right)^2\left(1+\lambda_n V_{OUT}\right)}.$$

Combined with $V_{OUT} \geq V_{IN}-V_{TN}$ gives.

$$R \leq R_{max} = \frac{V_{DD}-\left(V_{IN}-V_{TN}\right)}{\frac{W}{2L}\mu_n C_{ox}\left(V_{IN}-V_{TN}\right)^2\left(1+\lambda_n\left(V_{IN}-V_{TN}\right)\right)}.$$

See the plot on attached page.

b) $I_D = 200\,\mu A = \frac{W}{2L}\mu_n C_{ox}\left(V_{IN}-V_{TN}\right)^2\left(1+\lambda_n V_{OUT}\right)$

$V_{OUT} = 1.5\,V \Rightarrow V_{IN} = 0.917\,V$

Also, $R = \frac{V_{DD}-V_{OUT}}{I_D} = \frac{2.5-1.5}{200\,\mu A} = 5\,k\Omega$

c) + d) As V_{IN} is increased, V_{OUT} decreases. When $V_{OUT} < V_{IN}-V_{TN}$ the FET goes out of saturation. At the boundary b/w saturation and linear region $V_{OUT} = V_{IN}-V_{TN}$

$$\Rightarrow \frac{W}{2L}\mu_n C_{ox}\left(V_{OUT}\right)^2\left(1+\lambda_n V_{OUT}\right) = \frac{V_{DD}-V_{OUT}}{R} \quad \{R = 5\,k\Omega\}$$

$$\Rightarrow V_{OUT} = 0.599\,V \quad \Rightarrow V_{IN} = 0.599 + V_{TN} = 1.099\,V$$

On the other hand, as V_{IN} is decreased, V_{OUT} increases. When $V_{IN} < V_{TN}$, FET goes into cut-off and $V_{OUT} = V_{DD} = 2.5\,V$.

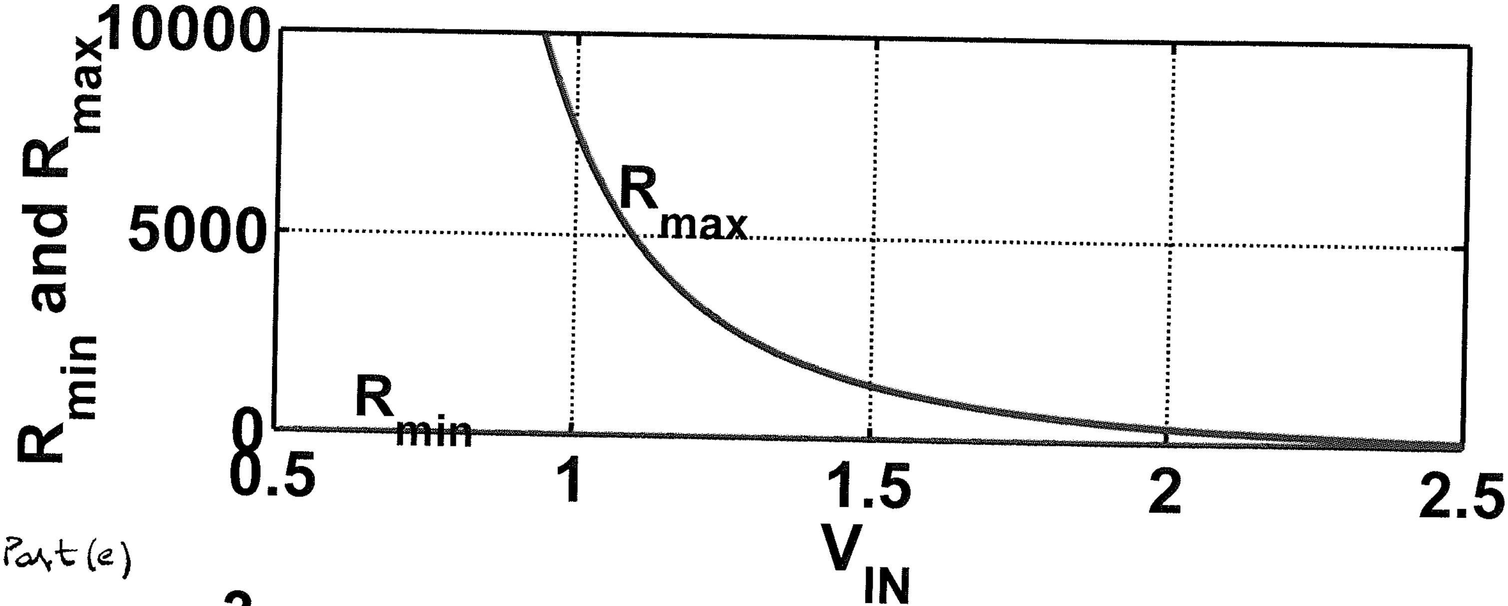
So for the FET ~~to~~ be in saturation:

$$0.599 < V_{OUT} < 2.5 \text{ Volts}$$

& $$V_{TN} < V_{IN} < 1.099 \text{ Volts}.$$

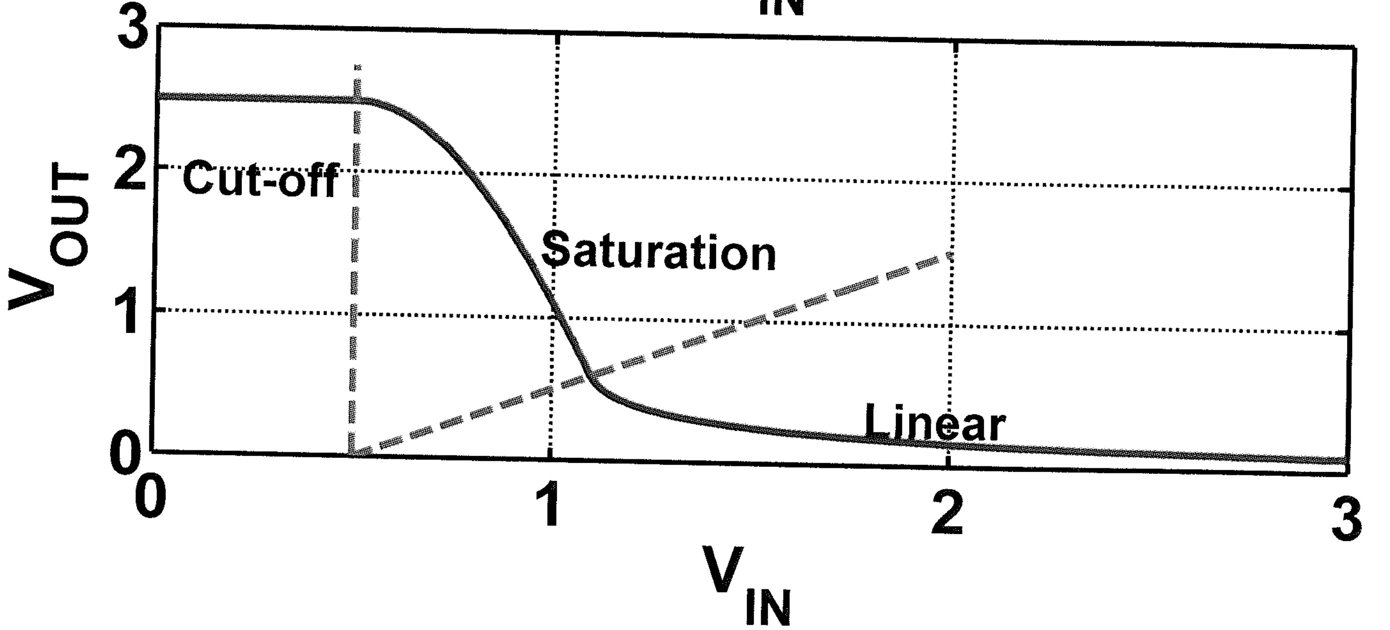
e) See attached plot.

Part (a)

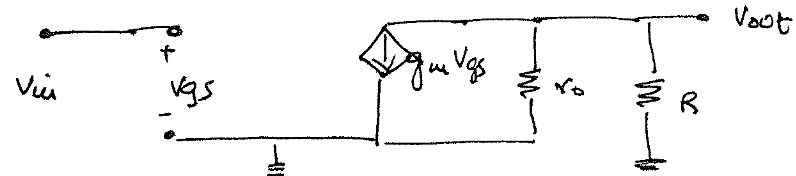


10000
5000
0
0.5
1
1.5
2
2.5
R_{min} and R_{max}
R_{max}
R_{min}
V_{IN}

Part (e)



3
2
1
0
0
1
2
3
V_{OUT}
V_{IN}
Cut-off
Saturation
Linear

f) 

$$A_V = \frac{V_{out}}{V_{in}} = -g_m \left(r_o \| R \right).$$

$$g_m = \frac{W}{L} \mu_n C_{ox} \left(V_{IN} - V_{TN} \right) \left(1 + \lambda_n V_{OUT} \right) = 9.59 \times 10^{-4} \ \Omega^{-1}$$

$$g_o = \frac{1}{r_o} = \frac{W}{2L} \mu_n C_{ox} \left(V_{IN} - V_{TN} \right)^2 \lambda_n = 1.739 \times 10^{-5} \ \Omega^{-1}$$

$$\Rightarrow A_V = -4.41$$

g) Now we have:

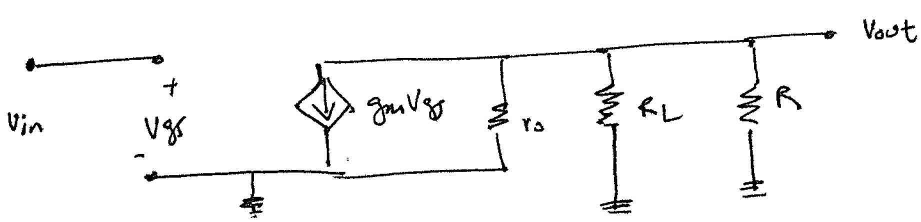
$$\frac{W}{2L} \mu_n C_{ox} \left(V_{IN} - V_{TN} \right)^2 \left(1 + \lambda_n V_{OUT} \right) = \frac{V_{DD} - V_{OUT}}{R} - \frac{V_{OUT}}{R_L}.$$

$\Rightarrow V_{OUT} = 1.028$ Volts $\qquad I_{OUT} = 102.75\ \mu A$.

h) Now $g_m = 9.19 \times 10^{-4} \ \Omega^{-1}$

$$g_o = \frac{1}{r_o} = 1.739 \times 10^{-5} \ \Omega^{-1}$$

$$A_V = -g_m \left(r_o \| R \| R_L \right) = -2.895$$



6.2

a) Suppose both FETs are in saturation. Then

$$\frac{W}{2L}\mu_n C_{ox}\left(V_{IN}-V_{TN}\right)^2\left(1+\lambda_n V_{OUT}\right)=\frac{W}{2L}\mu_n C_{ox}\left(V_{DD}-V_{OUT}-V_{TN}\right)^2\left(1+\lambda_n\left(V_{DD}-V_{OUT}\right)\right)$$

$$\Rightarrow \quad (0.75)^2\left(1+0.1V_{OUT}\right)=\left(2-V_{OUT}\right)^2\left(1+0.1\left(2.5-V_{OUT}\right)\right)$$

$\Rightarrow V_{OUT}=1.25\text{ V}$ $\Rightarrow$ For this V_{OUT} bottom FET is in saturation.
And the top FET is always in saturation (or cut off)

b) Again suppose both FETs are in saturation. Use eq. from part (a):

$$\Rightarrow \quad (1.5)^2\left(1+0.1V_{OUT}\right)=\left(2-V_{OUT}\right)^2\left(1+0.1\left(2.5-V_{OUT}\right)\right)$$

$$\Rightarrow \quad V_{OUT}=0.586\text{ V}$$

BUT now $V_{OUT} < V_{IN}-V_{TN}=1.5\text{ V}$ $\Rightarrow$ bottom FET must be in linear or triode region. So we have now:

$$\frac{W}{L}\mu_n C_{ox}\left(V_{IN}-V_{TN}-\frac{V_{OUT}}{2}\right)V_{OUT}\left(1+\lambda_n V_{OUT}\right)=\frac{W}{2L}\mu_n C_{ox}\left(V_{DD}-V_{OUT}-V_{TN}\right)^2\left(1+\lambda_n\left(V_{DD}-V_{OUT}\right)\right)$$

$$\Rightarrow \quad \left(1.5-\frac{V_{OUT}}{2}\right)V_{OUT}\left(1+0.1V_{OUT}\right)=0.5\left(2-V_{OUT}\right)^2\left(1+0.1\left(2.5-V_{OUT}\right)\right)$$

$\Rightarrow V_{OUT}=0.755\text{ V}$ $\Rightarrow$ bottom FET is in linear region.

c) If $V_{OUT}\geq 2.0\text{ V}$, the top FET will go into cut-off.

So V_{OUT} needs to be less than 2.0 Volts.

V_{IN} can be found from the eq. in part (a): $V_{IN}=0.5$ Volts.

So if $V_{IN} > 0.5$ Volts, $V_{OUT} < 2.0$ Volts and none of the FETs will be in cut-off.

d) If V_{IN} is too large, V_{OUT} will be so small that the bottom

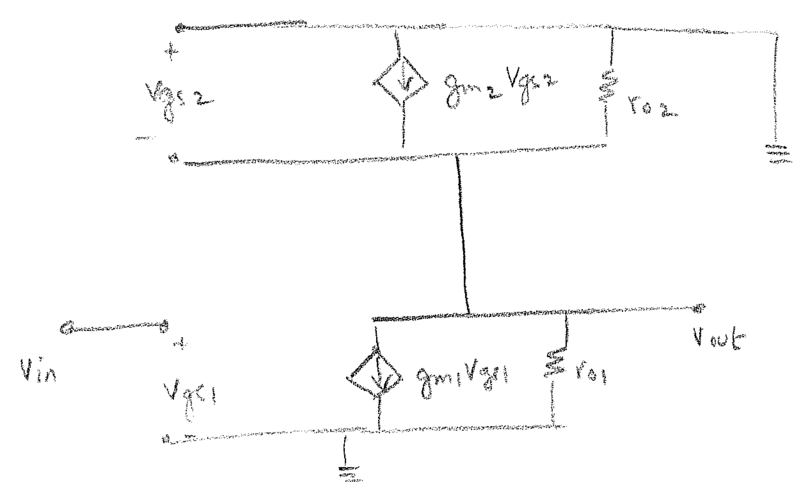
FET will go into the linear region (as was the case in part (b)).

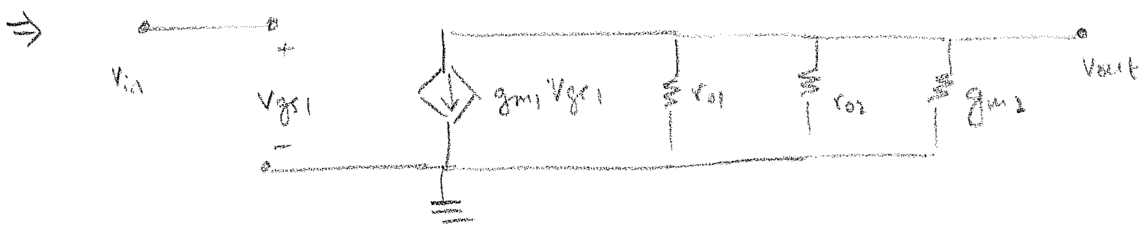
At the boundary b/w linear and saturation region $V_{OUT} = V_{IN} - V_{TN}$.

$$\Rightarrow \quad \frac{W}{2L}\mu_n C_{ox}\left(V_{OUT}\right)^2\left(1+\lambda_n V_{OUT}\right) = \frac{W}{2L}\mu_n C_{ox}\left(V_{DD}-V_{OUT}-V_{TN}\right)^2\left(1+\lambda_n\left(V_{DD}-V_{OUT}\right)\right)$$

$\Rightarrow \quad V_{OUT} = 1.01$ Volts and $V_{IN} = 1.51$ Volts.

e)





$$A_V = -g_{m1}\left(r_{o1} \,\|\, r_{o2} \,\|\, \frac{1}{g_{m2}}\right)$$

$\left\{\begin{array}{l} \text{Since } r_{o1} \sim r_{o2} \gg \frac{1}{g_{m2}} \\ A_V \sim -\frac{g_{m1}}{g_{m2}} \sim -1 \longrightarrow \text{rough estimate} \end{array}\right.$

f) $V_{IN} = 1.25\,V \Rightarrow g_{m1} = g_{m2} = \frac{W}{L}\mu_n C_{ox}(0.75)(1+0.1\times 1.25) = 1.7\times10^{-3}\ \Omega^{-1}$

$$g_{o1} = g_{o2} = r_{o1}^{-1} = r_{o2}^{-1} = \frac{W}{2L}\mu_n C_{ox}(0.75)^2\, 0.1 = 5.63\times10^{-5}\ \Omega^{-1}$$

$\Rightarrow \quad A_V = -0.9379$

6.3

a) Just by the symmetry of PFET and NFET ($k_n = k_p$) one can guess that $V_{OUT} = 1.25$ V and both FETs are in saturation.

b) If V_{OUT} becomes too large the PFET will go into the linear region. This would happen when $V_{OUT} - V_{DD} > V_B - V_{DD} - V_{TP}$

$\Rightarrow$ $V_{OUT} > +1.75$ Volts.

When the PFET is at the boundary b/w linear and saturation regions $V_{OUT} = 1.75$ Volts

$$\Rightarrow \quad \frac{k_n}{2}(V_{IN} - V_{TN})^2 (1 + \lambda_n V_{OUT}) = \frac{k_p}{2}(V_B - V_{DD} - V_{TP})^2 (1 - 0.1(V_{OUT} - V_{DD}))$$

$\Rightarrow$ $V_{IN} = 1.218$ Volts

c) If V_{OUT} becomes too small, NFET will go into the linear region. This would happen when $V_{OUT} = V_{IN} - V_{TN}$

$$\Rightarrow \quad \frac{k_n}{2}(V_{OUT})^2 (1 + \lambda_n V_{OUT}) = \frac{k_p}{2}(V_B - V_{DD} - V_{TP})^2 (1 - 0.1(V_{OUT} - V_{DD}))$$

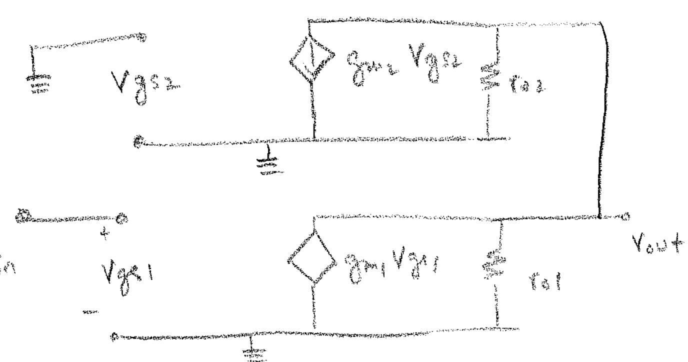
$\Rightarrow$ $V_{OUT} = 0.782$ Volts.

$\Rightarrow$ $V_{IN} = 1.282$ Volts

b) + c) $\Rightarrow$ Output Voltage swing : $0.782 < V_{OUT} < 1.75$

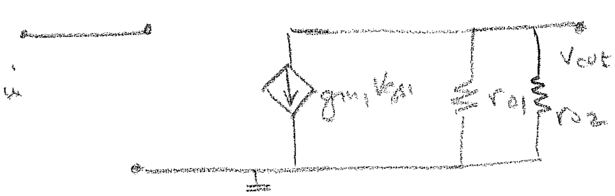
Input Voltage swing : $1.218 < V_{IN} < 1.282$

d)



$A_v = -g_{m1}(r_{o1} \| r_{o2})$.

$\Rightarrow$



e) $g_{m1} = g_{m2} = k_n (V_{IN} - V_{TN})(1 + \lambda_n V_{OUT}) = 1.7 \times 10^{-3}\ \Omega^{-1}$

$r_{o1}^{-1} = r_{o2}^{-1} = \frac{k_n}{2} (V_{IN} - V_{TN})^2 \lambda_n = 5.63 \times 10^{-5}\ \Omega^{-1}$

$AV = -15$ (pretty large!)

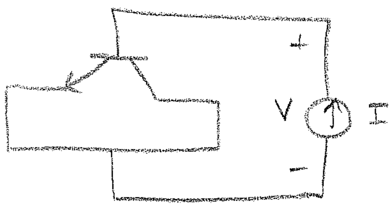
f) 6.1(f) the gain Av was modest (-2.895)

6.2(f) the gain was less than unity (-0.9379)

So 6.3(e) wins in terms of the gain!

6.4

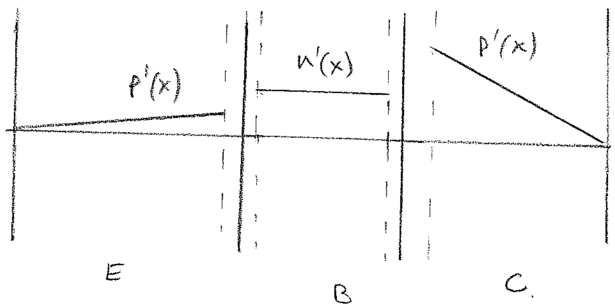
(i)



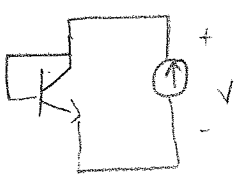
$\Rightarrow V = V_{BE} > 0$

$V_{CE} = V_{CB} + V_{BE} = 0 \Rightarrow V_{CB} = -V_{BE} = -V < 0.$

$\Rightarrow$ Both BE and BC PN junction are forward biased $\Rightarrow$ Saturation.



(ii)

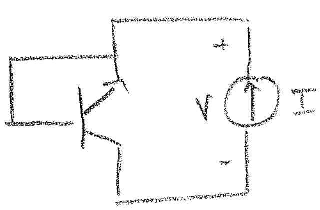


$V = V_{BE} > 0 \qquad V_{CE} = V_{CB} + V_{BE} = 0 + V_{BE} = V_{BE} \Rightarrow V_{CB} = 0$

$\Rightarrow$ forward active.

and carrier density is sketched in the homework question.

(iii)



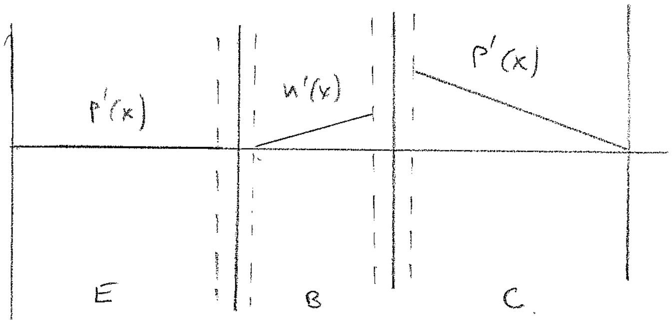
$V_{BE} = 0$

$V_{CE} = -V = V_{CB} + V_{BE} = V_{CB}$

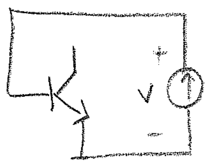
$\Rightarrow V_{CB} < 0.$

BC junction is forward biased and BE junction is unbiased.

So the BJT is in the reverse active mode of operation.



(iv).

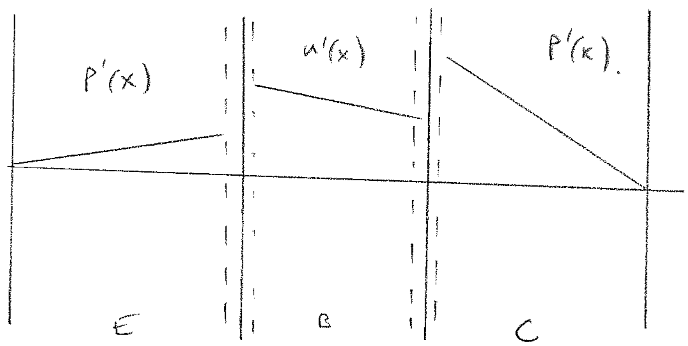


$V_{BE} = V > 0$

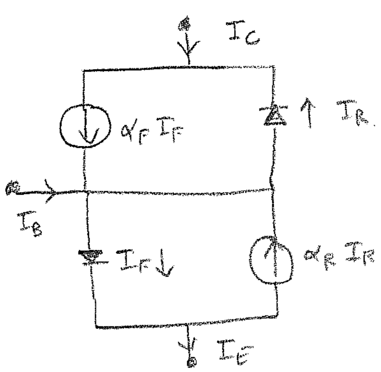
But $I_C = 0$ therefore $V_{BC} > 0$. or $V_{CB} < 0$.

(see lecture notes)

Both PN junction are forward biased $\Rightarrow$ saturation.



b)



(i) $-I_C = I_R - \alpha_F I_F$

$$-I_C = I_{CS}\left(e^{qV/kT}-1\right) - \alpha_F I_{ES}\left(e^{qV/kT}-1\right)$$

$$I_E = I_F - \alpha_R I_R = I_{ES}\left(e^{qV/kT}-1\right) - \alpha_R I_{CS}\left(e^{qV/kT}-1\right)$$

$$\Rightarrow I = I_B = I_E - I_C = \left\{I_{ES}(1-\alpha_F) + I_{CS}(1-\alpha_R)\right\}\left(e^{qV/kT}-1\right)$$

(ii) $I = I_C + I_B = I_E = I_{ES}\left(e^{qV/kT}-1\right)$.

(iii) $-I_E + I_B = I = \alpha_R I_R - I_F + I_B = -I_C = I_R - \alpha_F I_F$

Since $V_{BE} = 0 \Rightarrow I_F = 0$. $\Rightarrow \alpha_R I_R + I_B = I_R$.

$\Rightarrow I_B = (1-\alpha_R) I_R$. $\Rightarrow -I_E = \alpha_R I_R$. $\Rightarrow I = I_C = I_R = I_{CS}\left(e^{qV/kT}-1\right)$

(iv) See the lecture note for solution.