ECE 3150 Homework 2 Solutions (Changjian Zhang)

2.1

a) $\sigma \simeq q\mu_p p_o = 4.8$ S/m or $.048$ S/cm $\quad \{ p_o \approx N_a = 10^{15}\ 1/\text{cm}^3$

$\Rightarrow R = \dfrac{L}{\sigma A} = 2.08 \times 10^5\ \Omega$

b) $G_L = 10^{23} \dfrac{1}{\text{cm}^3\text{-s}}$ $\quad n = n_o + n'$ $\quad p = p_o + p'$

$n \approx n'$

$$\frac{dn'}{dt} = -\frac{n'}{\tau_n} + G_L$$

In steady state $\dfrac{dn'}{dt} = 0 \Rightarrow n' = G_L \tau_n = 10^{17}\ 1/\text{cm}^3$

$$\sigma = q\left[\mu_p (p_o + p') + \mu_n (n_o + n')\right] \approx q(\mu_p + \mu_n) n' \qquad \{ p \approx n \approx n'$$

≈ 2080 S/m or 20.8 S/cm

$$R = \frac{L}{\sigma A} = 480\ \Omega$$

c) $G_L(t) = g_l + \text{Re}\{ g_l e^{j\omega t} \}$

Let $n = n_o + n'(t)$ $\quad p = p_o + p'(t)$

and let
$$n'(t) = n'_{DC} + \text{Re}\{ n'(\omega) e^{j\omega t} \}$$
$$p'(t) = \underbrace{p'_{DC}}_{\text{DC part}} + \underbrace{\text{Re}\{ p'(\omega) e^{j\omega t} \}}_{\text{AC part}}$$

$$\frac{dn'(t)}{dt} = -\frac{n'(t)}{\tau_n} + G_L(t).$$

Plug the assumed form of the solution in the equation to get:

$$\mathrm{Re}\{j\omega\, n'(\omega) e^{j\omega t}\} = -\frac{\left(n'_{DC} + \mathrm{Re}\{n'(\omega) e^{j\omega t}\}\right)}{\tau_n} + g_e + \mathrm{Re}\{g_e e^{j\omega t}\}.$$

Matching time independent terms gives:

$$n'_{DC} = g_e \tau_n$$

Matching time dependent terms give:

$$n'(\omega) = \frac{g_e}{j\omega + \frac{1}{\tau_n}}.$$

$$\Rightarrow \quad p'(t) = n'(t) = g_e \tau_n + \mathrm{Re}\left\{ \frac{g_e}{j\omega + \frac{1}{\tau_n}} e^{j\omega t} \right\}.$$

d) $$I(t) = \frac{V}{R(t)} = \frac{AV}{L}\sigma(t) = \frac{AV}{L} q\left[\mu_p(P_o + g_e\tau_n) + \mu_n(n_o + g_e\tau_n)\right]$$
$$+ \frac{AV}{L} q (\mu_p + \mu_n)\, \mathrm{Re}\left\{ \frac{g_e}{j\omega + \frac{1}{\tau_n}} e^{j\omega t} \right\}.$$

$$\Rightarrow \quad I_{DC} = \frac{AV}{L} q\left[\mu_p(P_o + g_e\tau_n) + \mu_n(n_o + g_e\tau_n)\right]$$

e) $$i(\omega) = \frac{AV}{L} q (\mu_p + \mu_n) \frac{g_e}{j\omega + \frac{1}{\tau_n}}.$$

f) $$|i(\omega)|^2 = \frac{|i(\omega=0)|^2}{1 + \omega^2\tau_n^2} \quad \Rightarrow \quad |i(\omega)|^2 = \frac{|i(0)|^2}{2} \text{ when } \omega = \frac{1}{\tau_n}.$$

$$\omega = \frac{1}{\tau_n} = 10^6 \text{ rad/s}$$

Problem 2.2

(a) A is wider.

$$x_{po} = \sqrt{\frac{2\varepsilon_s \phi_B}{q N_a}\left(\frac{N_d}{N_a+N_d}\right)} \qquad x_{no} = \sqrt{\frac{2\varepsilon_s \phi_B}{q N_d}\left(\frac{N_a}{N_a+N_d}\right)} , \quad \varepsilon_s = 11.7\,\varepsilon_0 .$$

$$\phi_B = \frac{kT}{q} \log\left(\frac{N_a N_d}{N_i^2}\right) \qquad N_i = 10^{10}/cm^3 .$$

$\Rightarrow$ $x_{po} = 0.3\mu m$ $\quad x_{no} = 0.03\mu m$. $\quad x_{po} + x_{no} = 0.33\mu m$

(b) B's E is the largest.

$$|E_{max}| = \sqrt{\frac{2 q \phi_B}{\varepsilon_s}\left(\frac{N_a N_d}{N_a+N_d}\right)} . \qquad N_a = 10^{18}/cm^3 . \quad N_d = 10^{17}/cm^3 .$$

$$= 1.6 \times 10^5 \ V/cm$$

(c) B.

Sample B has higher doping level and thinner depletion region.

At a certain reverse bias, the electric field inside B is higher.

Problem 2.3

(a)

$$\frac{A^2}{C_j^2} = \frac{2}{q_e N_a \varepsilon_s}(\phi_B - V_D)$$ since N side is heavily doped

So we only consider the depletion region on P side.

Reading from the plot.

$$\begin{cases} 7 \times 10^{26} = \dfrac{2}{q_e N_a \varepsilon_s A^2}(\phi_B + 5) \\ 1.06 \times 10^{26} = \dfrac{2}{q_e N_a \varepsilon_s A^2}(\phi_B - 0) \end{cases}$$

$\Rightarrow$ $\phi_B = 0.89\ V.$

(b) $q_e = 1.6 \times 10^{-19}\ C$ $\quad \varepsilon_s = 11.7\ \varepsilon_0$ $\quad \varepsilon_0 = 8.85 \times 10^{-12}\ F/m$

$A = 100\ \mu m^2.$

$N_a = 10^{17}/cm^3.$

$$\phi_B = \frac{KT}{q_e}\log\left(\frac{N_a N_d}{N_i^2}\right)$$ $N_i = 10^{10}/cm^3,$ $\quad KT = 25.8\,meV$

$\Rightarrow$ $N_d = 9.4 \times 10^{17}/cm^3$

(c)

$$Q = \int_0^{x_a} q N_a(x)\, dx \quad \Rightarrow \quad dQ = q N_a(x)\, dx$$

$$\begin{cases} dQ = q N_a(x)\, dx \\ dQ = -C_{dep}\, dV_D \end{cases} , \quad C_{dep} = \frac{C_j}{A}$$

$$\Rightarrow N_a(x) = -\frac{C_{dep}}{q}\frac{dV_D}{dx} \qquad C_{dep} = \frac{\varepsilon_s}{x} \Rightarrow x = \frac{\varepsilon_s}{C_{dep}} \Rightarrow dx = -\frac{\varepsilon_s}{C_{dep}^2}\, dC_{dep}$$

$$\Rightarrow N_a(x) = -\frac{C_{dep}}{q}\frac{dV_D}{\varepsilon_s}\cdot\left(-C_{dep}^2\frac{1}{dC_{dep}}\right) = \frac{C_{dep}^3}{q\varepsilon_s}\frac{dV_D}{dC_{dep}}$$

$$\Rightarrow \frac{1}{C_{dep}^3}\frac{dC_{dep}}{dV_D} = \frac{1}{q N_a(x)\varepsilon_s}$$

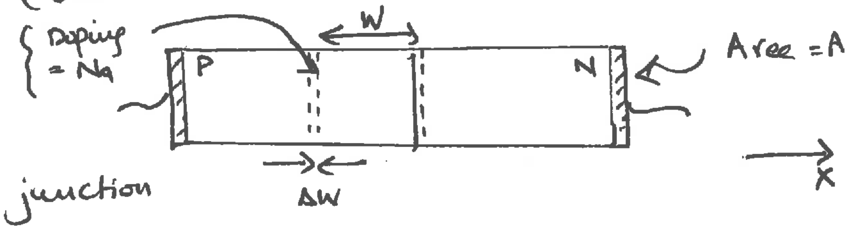
$$\Rightarrow \frac{d\left(\frac{1}{C_{dep}^2}\right)}{dV_D} = -\frac{2}{q N_a(x)\varepsilon_s} \Rightarrow \frac{d\left(\frac{A^2}{C_j^2}\right)}{dV_D} = -\frac{2}{q N_a(x)\varepsilon_s}$$

$$\Rightarrow \frac{d\left(\frac{1}{C_j^2}\right)}{dV_D} = -\frac{2}{q N_a(x)\varepsilon_s A^2}$$

So keeping q ε_s A^2 the same, when the slope increases by a factor of 2. $N_a(x)$ decreases by a factor of 2. This indicates that doping level changes at some position corresponding to the edge of depletion region at -5 volts.

Another look at part (c)

c) Consider a PN junction (one-sided) in which the depletion region width is W.



If the voltage across the junction is changed by ΔV_D, the depletion region width would change by ΔW, then the charge per unit area would change by $-qN_a\Delta W = \Delta Q$. The junction Electric field would ~~change~~ change by $\Delta E_x = \frac{\Delta Q}{\varepsilon_s} = -\frac{qN_a\Delta W}{\varepsilon_s}$. But $\Delta V_D = \Delta E_x W$. Therefore,

$$W\frac{\partial W}{\partial V_D} = W\frac{\Delta W}{\Delta V_D} = W\frac{\Delta W}{\Delta E_x W} = \frac{\Delta W}{\Delta E_x} = \frac{\Delta W\,\varepsilon_s}{-qN_A\Delta W} = -\frac{\varepsilon_s}{qN_A}$$

Note that the doping N_a above is the doping right at the edge of the depletion region on the P-side. Therefore,

$$\frac{\partial(1/C_j^2)}{\partial V_D} = \frac{2}{\varepsilon_s^2 A^2}W\frac{\partial W}{\partial V_D} = -\frac{2}{\varepsilon_s^2 A^2}\frac{\varepsilon_s}{qN_a} = -\frac{2}{q\varepsilon_s A^2 N_a}$$

The slope of the curve $\frac{1}{C_j^2}$ vs V_D is inversely related to the doping at the edge of the depletion region on the P-side (the lightly doped side).

Therefore, when V_D is decreased below $-5V$ the depletion region edge on the P-side moves into a region in which the doping is less than $10^{17}\frac{1}{cm^3}$ by a factor of 2 — so it is $5\times10^{16}\frac{1}{cm^3}$.

Problem 2.4

(a) At zero reverse bias, we have

$$\frac{A^2}{C^2} = \frac{2}{q \varepsilon_s N_a} \phi_B \quad \text{since} \quad N_a \ll N_d.$$

and

$$\phi_B = \frac{kT}{q} \log \frac{N_a N_d}{N_i^2}$$

In order to know A from the above expression, we need to know C as well.

For the RLC circuit at resonance frequency 1 GHz, the capacitance needs to satisfy

$$f = \frac{1}{2\pi\sqrt{LC}} = 1\ \text{GHz} \Rightarrow C = 5 \times 10^{-12}\ \text{F}$$

Then we get

$$A = 1.3 \times 10^{5}\ \mu m^2$$

(b) The breakdown voltage of silicon $E = 3 \times 10^5$ V/cm. According to the expression of electric field inside depletion region.

$$|E_{max}| = \sqrt{\frac{2q(\phi_B - V_D)}{\varepsilon_s}\left(\frac{N_a N_d}{N_a + N_d}\right)},$$

The maximum reverse bias is $V_d = -2.94 \times 10^3$ V

$$\frac{A^2}{C'^2} = \frac{2}{q \varepsilon_s N_a}(\phi_B - V_D)$$

$$\Rightarrow C' = 7.1 \times 10^{-14}\ \text{F}. \quad f' = \frac{1}{2\pi\sqrt{LC'}} = 8.4\ \text{GHz}$$