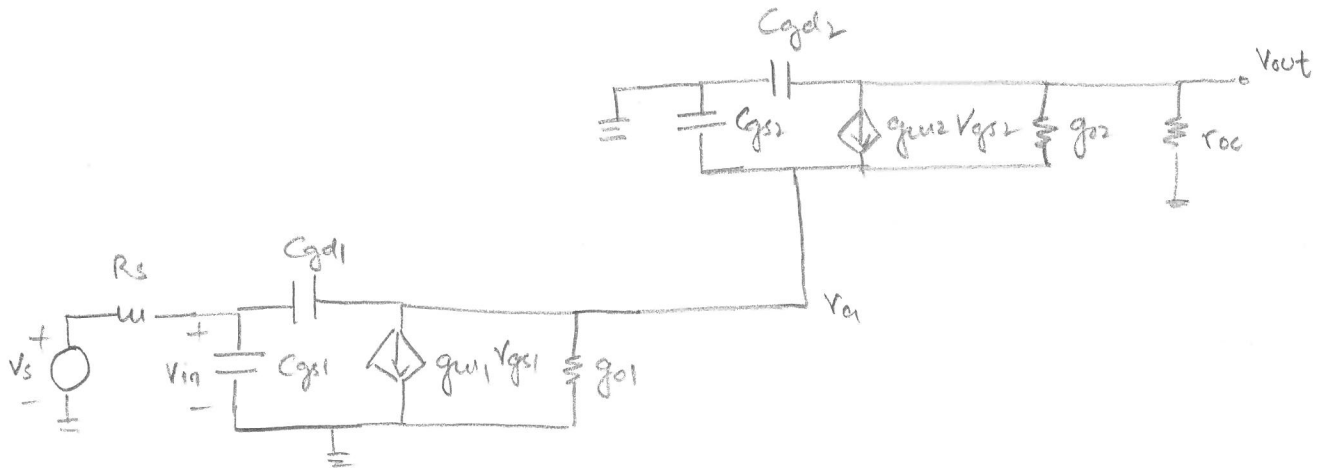


ECE Homework #11 Solutions (Forham Rams)

11.1

a)



b) $g_{m1} \approx \sqrt{2knI_D} = .0057 \text{ S}$ $g_o \approx \lambda nI_D = 8 \times 10^{-5} \text{ S}$

g_{m1} is ~ 70 times larger than g_o .

c) $\frac{V_{out}}{r_{oc}} + (V_{out} - V_a)g_{o2} - g_{m2}V_a + V_{out}j\omega C_{gd2} = 0$

$$\Rightarrow \frac{V_{out}}{V_a} = \frac{g_{m2} + g_{o2}}{\frac{1}{r_{oc}} + g_{o2} + j\omega C_{gd2}} \approx \frac{g_{m2}}{\frac{1}{r_{oc}} + g_{o2} + j\omega C_{gd2}} = \frac{g_{m2}(r_{oc} \parallel r_{o2})}{1 + j\omega C_{gd2}(r_{oc} \parallel r_{o2})} = H_2(\omega)$$

d) see attached plot.

e) $V_a g_{o1} + g_{m1}V_{in} + j\omega C_{gd1}(V_a - V_{in}) + V_a j\omega C_{gs2} + g_{m2}V_a + (V_a - V_{out})g_{o2} = 0$

$$\Rightarrow \frac{V_a}{V_{in}} = \frac{-g_{m1} + j\omega C_{gd1}}{g_{o1} + g_{o2} + g_{m2} + j\omega(C_{gd1} + C_{gs2}) - g_{o2}H_2(\omega)} = H_1(\omega)$$

f) see attached plot.

$$g) \quad \frac{V_{in} - V_s}{R_s} + V_{in} j\omega C_{gs1} + (V_{in} - V_s) j\omega C_{gd1} = 0.$$

$$\frac{V_{in}}{V_s} = \frac{1}{1 + j\omega (C_{gs1} + C_{gd1}) R_s - j\omega C_{gd1} R_s H_1(\omega)} = H_0(\omega).$$

h) See the attached plot

i). $|H_2(\omega)|^2$ has a 3dB freq of ~ 2.5 GHz.

$|H_1(\omega)|^2$ has a 3dB freq of ~ 1.58 GHz.

$|H_0(\omega)|^2$ has a 3dB freq of ~ 1.29 GHz.

So the first and the second stages limit the freq. bandwidth of the amplifier.

$$j) \quad \frac{V_{out}}{V_s} = \frac{V_{out}}{V_a} \cdot \frac{V_a}{V_{in}} \cdot \frac{V_{in}}{V_s} = H_2(\omega) H_1(\omega) H_0(\omega) = H_T(\omega)$$

See the attached plot. The 3dB frequency for $|H_T(\omega)|^2$ is ~ 0.80 GHz.

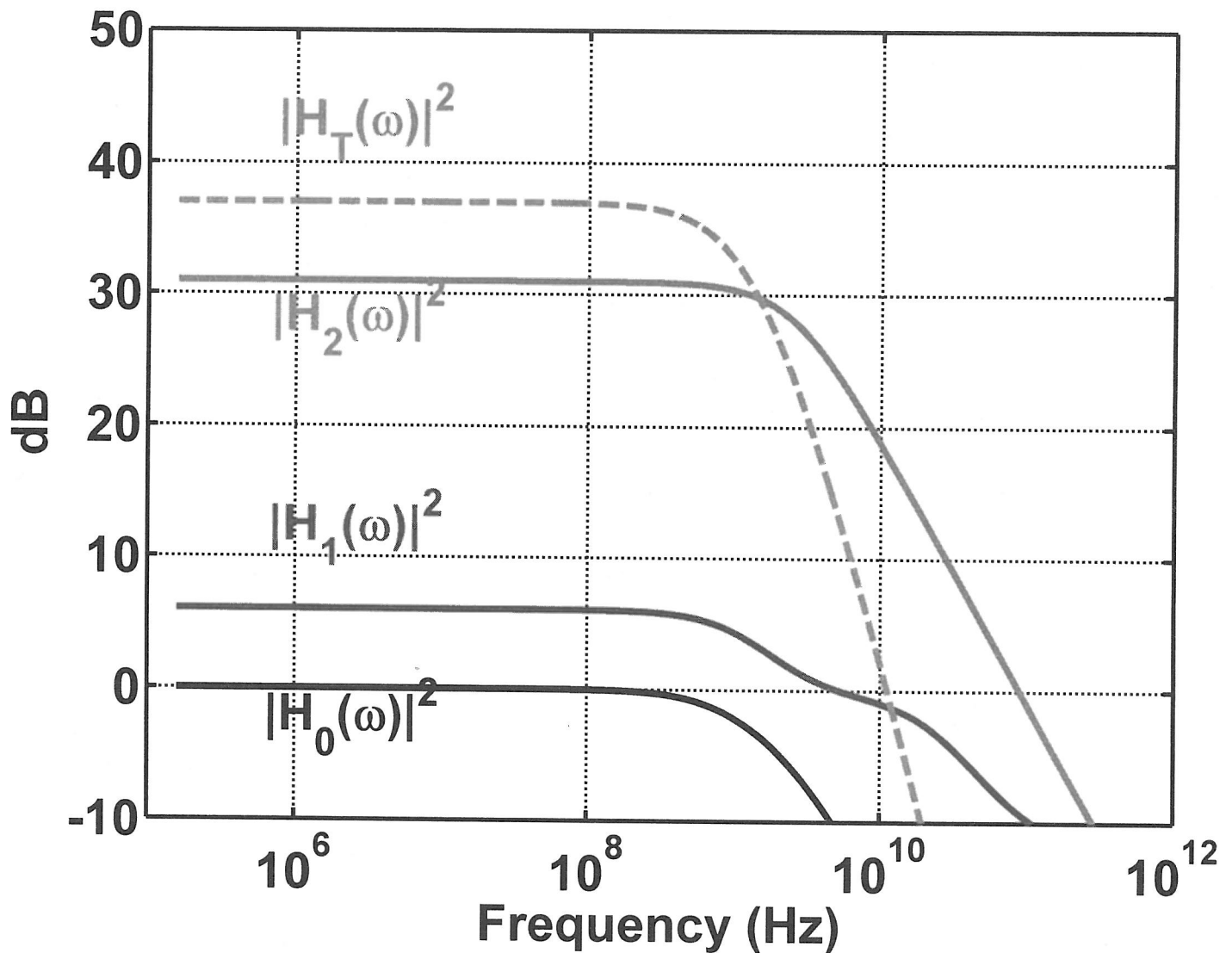
k) See the attached plot.

The 3dB freq. of the Common Source amp is ~ 0.18 GHz.

The Cascode has ~ 4.4 times larger 3dB frequency, compared to the Common Source.

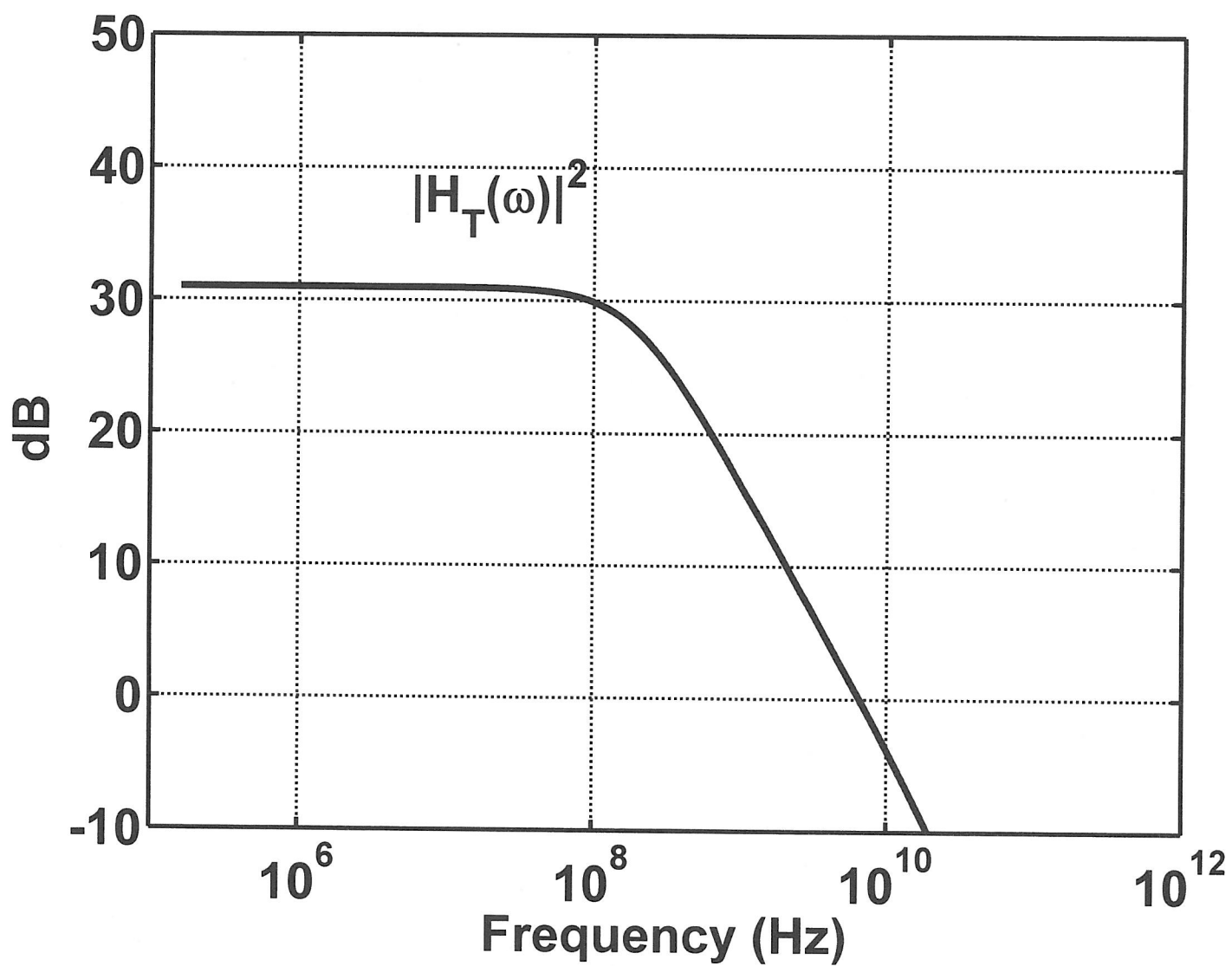
e) Common drain does not suffer from the Miller effect as there is no gain provided by this stage.

{ Cascode }



see (i)(j) for the 3dB frequencies of each curve

(Common Source)



11.2

a) As long as $V_{DS} > V_{GS} - V_{TN}$ for the NFET, or $V_{OUT} > V_{DD} - V_{TN}$, the NFET will be in saturation. In saturation,

$$C_L \frac{dV_{OUT}}{dt} = -I_D = -\frac{k_n}{2} (V_{DD} - V_{TN})^2$$

$$\Rightarrow V_{OUT}(t) = V_{OUT}(t=0) - \frac{k_n}{2C_L} (V_{DD} - V_{TN})^2 t$$

$$= V_{DD} - \frac{k_n}{2C_L} (V_{DD} - V_{TN})^2 t$$

The time it takes for the output to reach $V_{DD} - V_{TN}$

$$\text{is } t_1 = \frac{2C_L}{k_n} \frac{V_{TN}}{(V_{DD} - V_{TN})^2}$$

For $t > t_1$, NFET will be in the linear region, where,

$$C_L \frac{dV_{OUT}}{dt} = I_D = -k_n \left(V_{DD} - V_{TN} - \frac{V_{OUT}}{2} \right) V_{OUT} \quad \left\{ \begin{array}{l} \text{boundary condition} \\ V_{OUT}(t=0) = V_{DD} - V_{TN} \end{array} \right.$$

$$\int_{V_{DD}-V_{TN}}^{0.1V_{DD}} \frac{dV_{OUT}}{(2(V_{DD}-V_{TN}) - V_{OUT})V_{OUT}} = -\frac{k_n}{2C_L} t$$

$$\frac{1}{2(V_{DD}-V_{TN})} \log \left[\frac{0.1V_{DD}}{2(V_{DD}-V_{TN}) - 0.1V_{DD}} \right] = -\frac{k_n}{2C_L} t$$

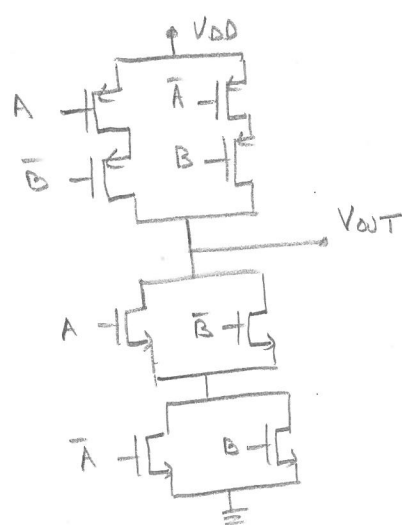
$$t = -\frac{2C_L}{k_n(V_{DD}-V_{TN})} \times \frac{1}{2} \log \left[\frac{0.1V_{DD}}{2(V_{DD}-V_{TN}) - 0.1V_{DD}} \right]$$

$$\text{Total time} = \frac{2C_L}{k_n(V_{DD}-V_{TN})} \left\{ \frac{V_{TN}}{V_{DD}-V_{TN}} - \frac{1}{2} \log \left[\frac{0.1V_{DD}}{2(V_{DD}-V_{TN}) - 0.1V_{DD}} \right] \right\}$$

- b) The expression for fall time will be the same as in part (a) if we replace k_n in there by $\frac{k_n}{2}$ because two NFETs in series with the same gate voltage is like one longer NFET that is twice as long.
- c) If we consider two NFETs as one longer FET then the bottom FET never operates in saturation. It always operates in the linear region.
- d) Yes. The top FET goes from saturation into the linear region when $V_{out} < V_{DD} - V_{TN}$.

11.3

$A \oplus B = A\bar{B} + \bar{A}B$, so the pull-up network could be:



The pull-down network is just the dual of it.

We will need to generate \bar{A} and \bar{B} using inverters. So we will need total of 12 FETs.