ECE 3150 Homework #4 Solutions
(Farhan Rana)

<u>4.1</u>

a) charge stored in the N-side $= qA\int_{x_n}^{W_n+x_n} p'(x)\,dx = Q_d A$

$$= qA\frac{n_i^2}{N_d}\frac{W_n}{2}\left(e^{\frac{qV_D}{KT}}-1\right)$$

$$\Rightarrow C_d = \frac{q^2 A}{KT}\frac{n_i^2}{N_d}\frac{W_n}{2}e^{\frac{qV_D}{KT}}$$

current $= I_D = qA\frac{n_i^2}{N_d}\frac{D_p}{W_n}\left(e^{\frac{qV_D}{KT}}-1\right)$

$$\Rightarrow r_d^{-1} = \frac{q^2 A}{KT}\frac{n_i^2}{N_d}\frac{D_p}{W_n}\left(e^{\frac{qV_D}{KT}}-1\right)$$

b) $i_d(\omega) = \left(j\omega C_d + \frac{1}{r_d}\right)v_d(\omega) \Rightarrow v_d(\omega) = \frac{r_d\, i_d(\omega)}{1+j\omega C_d r_d}$

c) $\omega_{3dB} = \frac{1}{C_d r_d}$

d) $\omega_{3dB} = \frac{2D_p}{W_n^2}$

e) 4 mice

f) $N = RT$

g) $J_p(x=x_n) = q\frac{n_i^2}{N_d}\frac{D_p}{W_n}\left(e^{\frac{qV_D}{KT}}-1\right)$

$$Q_d = q\frac{n_i^2}{N_d}\frac{W_n}{2}\left(e^{\frac{qV_D}{KT}}-1\right)$$

$$\tau_{p\text{-}diff} = \frac{Q_d}{J_p(x=x_n)} = \frac{W_n^2}{2D}$$

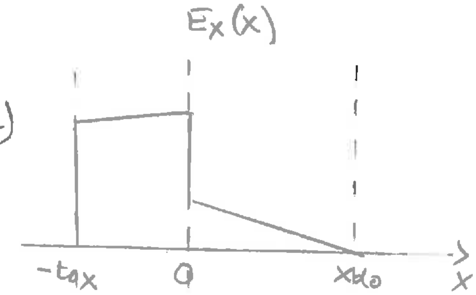
h) $\omega_{3dB} = \frac{1}{\tau_{p\text{-}diff}}$

4.2

a) – $\phi_B = \phi_m - \phi_p = \phi_n - \phi_p = kT\log\left(\frac{N_a N_d}{n_i^2}\right) = 0.89 \text{ V}$

b) $E_x(x=0^-) = \frac{\varepsilon_s}{\varepsilon_{ox}} E_x(x=0^+) = +\, q\frac{N_a x_{do}}{\varepsilon_{ox}}$

c)



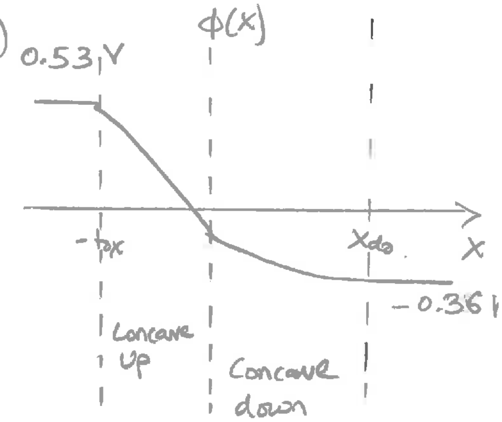
$$\frac{dE_x}{dx} = \frac{\rho_o}{\varepsilon_{ox}} \Rightarrow E_x(x) = q\frac{N_a x_{do}}{\varepsilon_{ox}} + \frac{\rho_o x}{\varepsilon_{ox}}$$

d) $\frac{d\phi}{dx} = -E_x(x) = -q\frac{N_a x_{do}}{\varepsilon_{ox}} - \frac{\rho_o x}{\varepsilon_{ox}}$

BC: $\phi(x=0) = \phi_p + q\frac{N_a}{2\varepsilon_s}x_{do}^2$

$$\Rightarrow \phi(x) = \phi_p + q\frac{N_a}{2\varepsilon_s}x_{do}^2 - q\frac{N_a x_{do}}{\varepsilon_{ox}}x - \frac{\rho_o x^2}{2\varepsilon_{ox}}$$

e)



f) $\phi(x=-t_{ox}) = \phi_M$

$$\Rightarrow \phi_p + q\frac{N_a}{2\varepsilon_s}x_{do}^2 + q\frac{N_a}{\varepsilon_{ox}}x_{do}t_{ox} - \frac{\rho_o t_{ox}^2}{2\varepsilon_{ox}} = \phi_M$$

$$\Rightarrow q\frac{N_a}{2\varepsilon_s}x_{do}^2 + q\frac{N_a}{\varepsilon_{ox}}x_{do}t_{ox} = \phi_B + \frac{\rho_o t_{ox}^2}{2\varepsilon_{ox}}$$

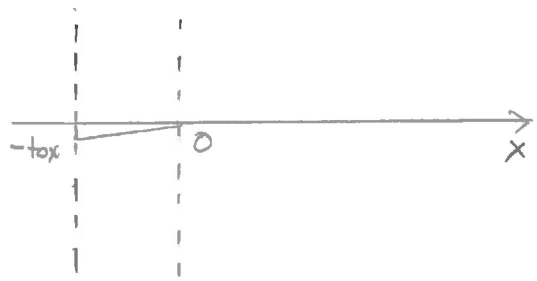
$$\Rightarrow x_{do} = -\frac{\varepsilon_s}{C_{ox}} + \sqrt{\left(\frac{\varepsilon_s}{C_{ox}}\right)^2 + \left(\frac{2\varepsilon_s}{qN_a}\right)\left(\phi_B + \frac{\rho_o t_{ox}^2}{2\varepsilon_{ox}}\right)}$$

g) when $V_{GB} \neq 0$ $\quad x_{do} = -\frac{\varepsilon_s}{C_{ox}} + \sqrt{\left(\frac{\varepsilon_s}{C_{ox}}\right)^2 + \left(\frac{2\varepsilon_s}{qN_a}\right)\left(\phi_B + V_{GB} + \frac{\rho_o t_{ox}^2}{2\varepsilon_{ox}}\right)}$

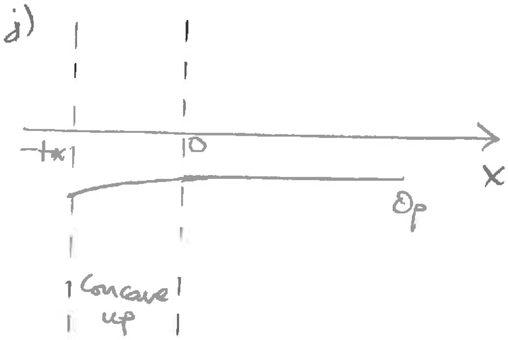
$x_{do} = 0$ when $V_{GB} = -\phi_B - \frac{\rho_o t_{ox}^2}{2\varepsilon_{ox}} = V_{FB}$

h) In flatband, since there is no charge in the semiconductor, the gate charge must be equal and opposite to the oxide charge $\Rightarrow Q_G = -\rho_o t_{ox}$

i)



j)



k) $Q_G = -\rho_o t_{ox} + C_{ox}(V_{GB} - V_{FB})$

l) $X_{do} = -\frac{\varepsilon_s}{C_{ox}} + \sqrt{\left(\frac{\varepsilon_s}{C_{ox}}\right)^2 + \left(\frac{2\varepsilon_s}{qN_a}\right)(V_{GB}-V_{FB})}$

m) $Q_G = -\rho_o t_{ox} + qN_aX_{do}$

n) From part (f) $V_{GB}-V_{FB} = \frac{qN_a}{2\varepsilon_s}X_{do}^2 + \frac{qN_a}{\varepsilon_{ox}}X_{do}t_{ox}$

$$= (\phi_s - \phi_p) + \frac{\sqrt{2q\varepsilon_s N_a(\phi_s-\phi_p)}}{C_{ox}}$$

Same as when $\rho_o = 0$

$$\Rightarrow V_{TN} = \underbrace{V_{FB}}_{\text{depends on } \rho_o} - 2\phi_p + \frac{\sqrt{2q\varepsilon_s N_a(-2\phi_p)}}{C_{ox}}$$

o) $V_{TN}(\rho_o) - V_{TN}(\rho_o=0) = -\frac{\rho_o t_{ox}^2}{2\varepsilon_{ox}}$

p) $Q_G = \sqrt{2q\varepsilon_s N_a(-2\phi_p)} - \rho_o t_{ox} + C_{ox}(V_{GB}-V_{TN})$

4.3

a) $V_{FB} = +0.5$ V

b) $V_{TN} = -0.6$ V

c) $C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = 8 \times 10^{-15}\ F/\mu m^2 = 8 \times 10^{-3}\ F/m^2$

$\Rightarrow t_{ox} = 4.3\ nm = 43\ °A$

d) N-type (PMOS capacitor)

e) Just before threshold; $C \approx 2\ fF/\mu m^2$

$\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_b} \Rightarrow C_b = \frac{8}{3}\ fF/\mu m^2 \qquad C_b = \frac{\varepsilon_s}{X_{dmax}}$

$X_{dmax} = \sqrt{\frac{2\varepsilon_s(+2\phi_n)}{qN_d}} \Rightarrow \sqrt{\frac{qN_d\varepsilon_s}{2(2\phi_n)}} = C_b = \frac{8}{3}\ fF/\mu m^2$

$\Rightarrow N_d \approx 8 \times 10^{17}\ 1/cm^3$

f) Analysis similar to that in problem 4.3 gives the result: $V_{TP}(\rho_o) - V_{TP}(\rho_o = 0) = -\frac{\rho_o t_{ox}^2}{2\varepsilon_{ox}} = -0.25$ V

$\Rightarrow \rho_o = +9.33 \times 10^{-1}\ C/cm^3$

4.4

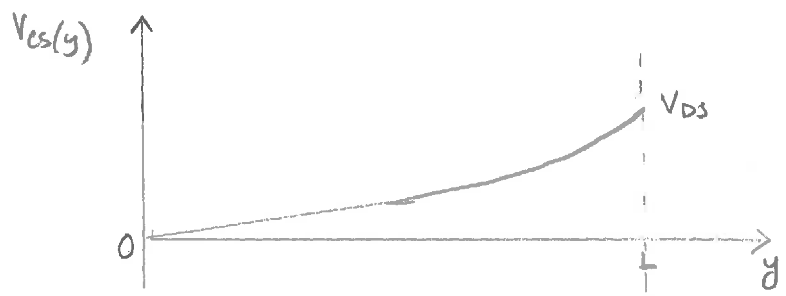
a) $$I_D = W\mu_n C_{ox}\left(V_{GS} - V_{TN} - V_{CS}(y)\right)\frac{dV_{CS}(y)}{dy}$$

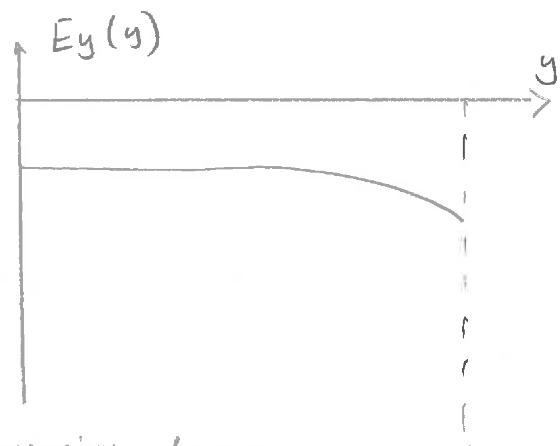
$$\Rightarrow \int_0^y I_D\,dy = \int_0^{V_{CS}} W\mu_n C_{ox}\left(V_{GS} - V_{TN} - V_{CS}\right) dV_{CS}$$

$$I_D y = W\mu_n C_{ox}\left[V_{GS} - V_{TN} - \frac{V_{CS}(y)}{2}\right]V_{CS}(y)$$

$$\Rightarrow V_{CS}(y) = (V_{GS} - V_{TN}) - \sqrt{(V_{GS} - V_{TN})^2 - \frac{2y\,I_D}{W\mu_n C_{ox}}}$$

b) $$E_y = -\frac{dV_{CS}}{dy} = \frac{-\dfrac{I_D}{W\mu_n C_{ox}}}{\sqrt{(V_{GS} - V_{TN})^2 - \dfrac{2yI_D}{W\mu_n C_{ox}}}}$$





The field is not uniform because the inversion layer charge density is not uniform along the channel. As the inversion layer charge density decreases going towards the drain end, the field increases, but the product gives the constant current (i.e. independent of position).

c) As $V_{DS} \to V_{GS} - V_{TN}$ and $I_D \to \frac{W}{2L}\mu_n C_{ox}(V_{GS} - V_{TN})^2$

the field at $y = L$, $E_y(y=L)$, approaches infinity — because the inversion layer charge density at $y = L$ is approaching zero.