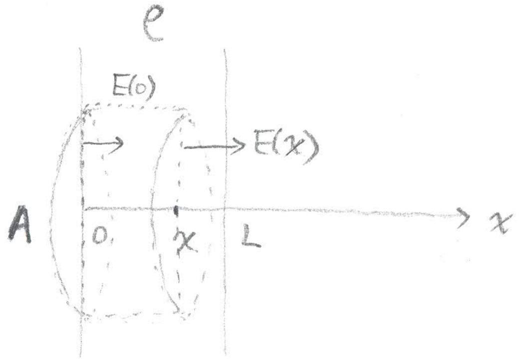
ECE 3150 Solutions HW #1 (Changjian Zhang)

1. (a) A nice way to solve this problem is to apply the integral form of Gauss's Law.



We choose a cylinder in the space as indicated on the left. The area of top/bottom is A.

According to Gauss's Law,

$$-E(0)A + E(x)A = \frac{\rho x A}{\varepsilon_0} \quad \text{for } 0<x<L.$$

$$\Rightarrow \quad E(x) = \frac{\rho x}{\varepsilon_0} + E(0), \quad 0<x<L$$

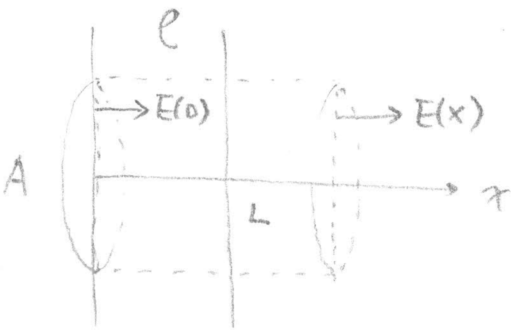
$$-\frac{d\phi}{dx} = E(x) \Rightarrow \phi(x) = -\frac{\rho x^2}{2\varepsilon_0} - E(0)x + \phi(0), \quad 0<x<L$$

$$\phi(L) = -\frac{\rho L^2}{2\varepsilon_0} - E(0)L + \phi(0)$$

(b) Choose a slightly different cylinder for $x>L$

According to Gauss's Law,

$$-E(0)A + E(x)A = \frac{\rho L}{\varepsilon_0}A \quad \text{for } x>L$$



$$\Rightarrow \quad E(x) = \frac{\rho L}{\varepsilon_0} + E(0), \quad x>L$$

$$-\frac{d\phi}{dx} = E(x) \Rightarrow \int_{\phi(L)}^{\phi(x)} d\phi = -\int_L^x E\,dx$$

$$\Rightarrow \phi(x) = -\frac{\rho L^2}{2\varepsilon_0} - \frac{\rho L}{\varepsilon_0}x + \phi(0), \quad x>L$$

(c) According to the question, we have boundary conditions for the new system that the potential and electric field at $x=0$ are $\phi'(0)$ and $E'(0)$.

Following the same arguments in (a) and (b), we have

$$E(x) = \frac{\rho}{\varepsilon_0} x + E'(0), \quad 0 < x < L$$

$$E(x) = \frac{\rho L + \sigma}{\varepsilon_0} + E'(0), \quad x > L$$

$$\phi(x) = -\frac{\rho x^2}{2\varepsilon_0} - E'(0)\, x + \phi'(0), \quad 0 < x < L$$

$$\phi(x) = -\frac{\rho L^2}{2\varepsilon_0} - \frac{\rho L + \sigma}{\varepsilon_0}(x-L) - E'(0)\, x + \phi'(0), \quad x > L$$

2. Sample A: N type. $n_0 = 10^{17}\ cm^{-3}$, $P_0 = 10^{3}\ cm^{-3}$.

$\rho = 0.0391\ \Omega/m$, $\phi_n = 419\ mV$

Sample B: P type. $n_0 = 2\times10^{4}\ cm^{-3}$, $P_0 = 5\times10^{15}\ cm^{-3}$

$\rho = 2.0833\ \Omega/m$, $\phi_p = -341\ mV$

Sample C: Intrinsic. $n_0 = 10^{10}\ cm^{-3}$, $P_0 = 10^{10}\ cm^{-3}$

$\rho = 2.84\times10^{5}\ \Omega/m$, $\phi = 0$

3.

(a) According to the plot

$\phi_n = -0.03x + 0.42$. in Volts x is in μm

$n = n_i e^{\frac{q\phi}{KT}}$. at room temperature, $KT \sim 26meV$.

$n = n_i e^{\frac{\phi}{0.026}}$, $n_i = 10^{10}\ cm^{-3}$

$P = \frac{n_i^2}{n}$. in cm^{-3}

Matlab plots according to these fomulas are attached in the end.

(b) $E = -\frac{d\phi}{dx} = 300\ V/cm$.

(c) $J_{drif} = n q_e \mu E$. where $n = n_i e^{\frac{\phi}{0.026}}$. $q_e = 1.6 \times 10^{-19} C$.

$\mu = 1000\ cm^2/Vs$. $E = 300\ V/cm$

(d) $J_{diff} = q_e D_e \frac{dn}{dx}$. $D_e = \frac{\mu KT}{q_e}$. where

$\mu = 1000\ cm^2/Vs$. $KT = 0.026eV$. $q_e = 1.6 \times 10^{-19} C$

Matlab plots are attached in the end.

4

$$\frac{kT}{q} = \frac{D}{\mu} \Rightarrow \mu = \frac{qD}{kT}$$

$$R = \rho\frac{l}{A} \Rightarrow \rho = \frac{RA}{l}. \qquad \sigma = \frac{1}{\rho} = \frac{l}{RA}$$

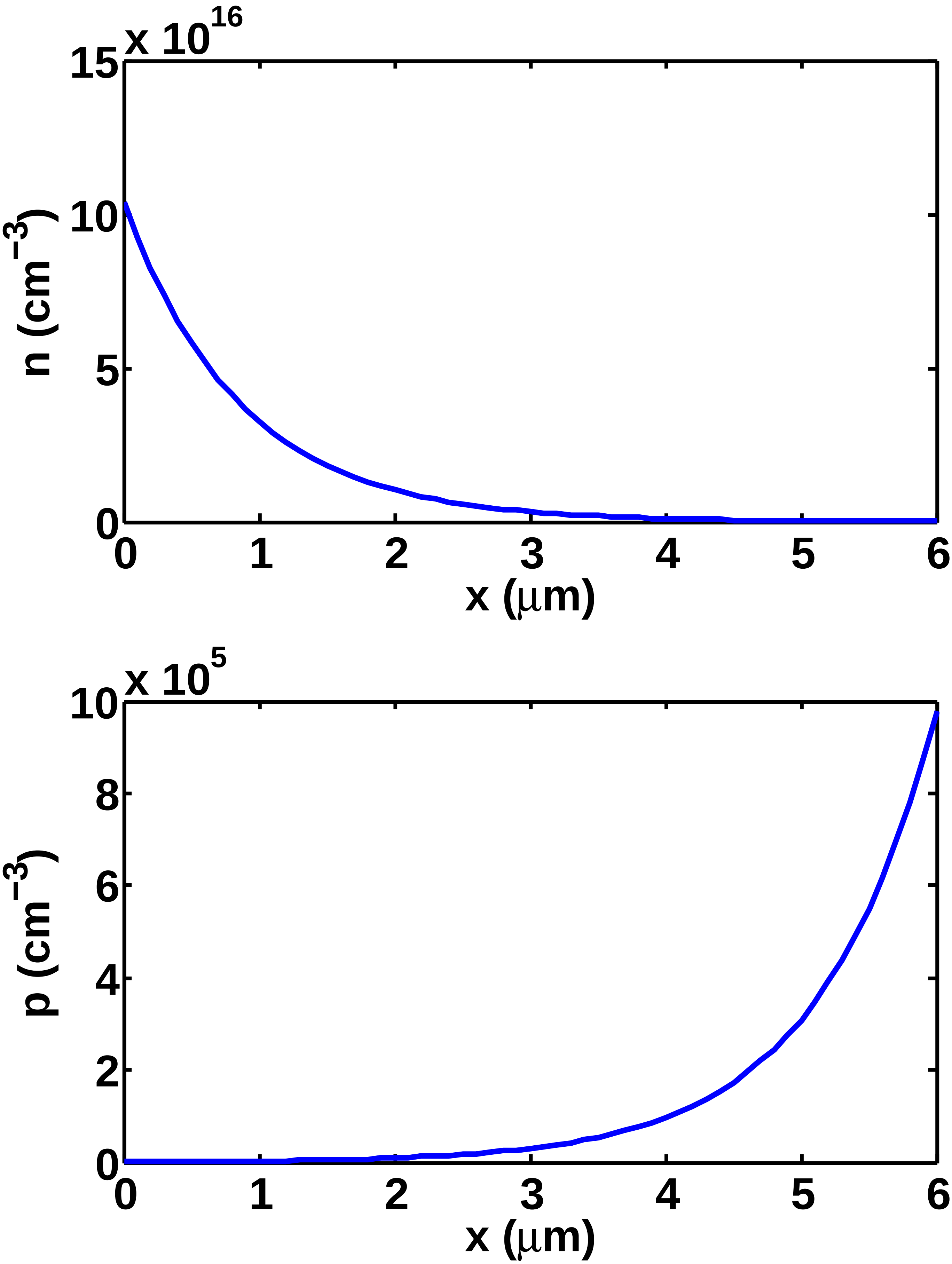
$$\sigma = p\mu q \Rightarrow p = \frac{\sigma}{\mu q} = \frac{\frac{l}{RA}}{\frac{qD}{kT}\cdot q}$$

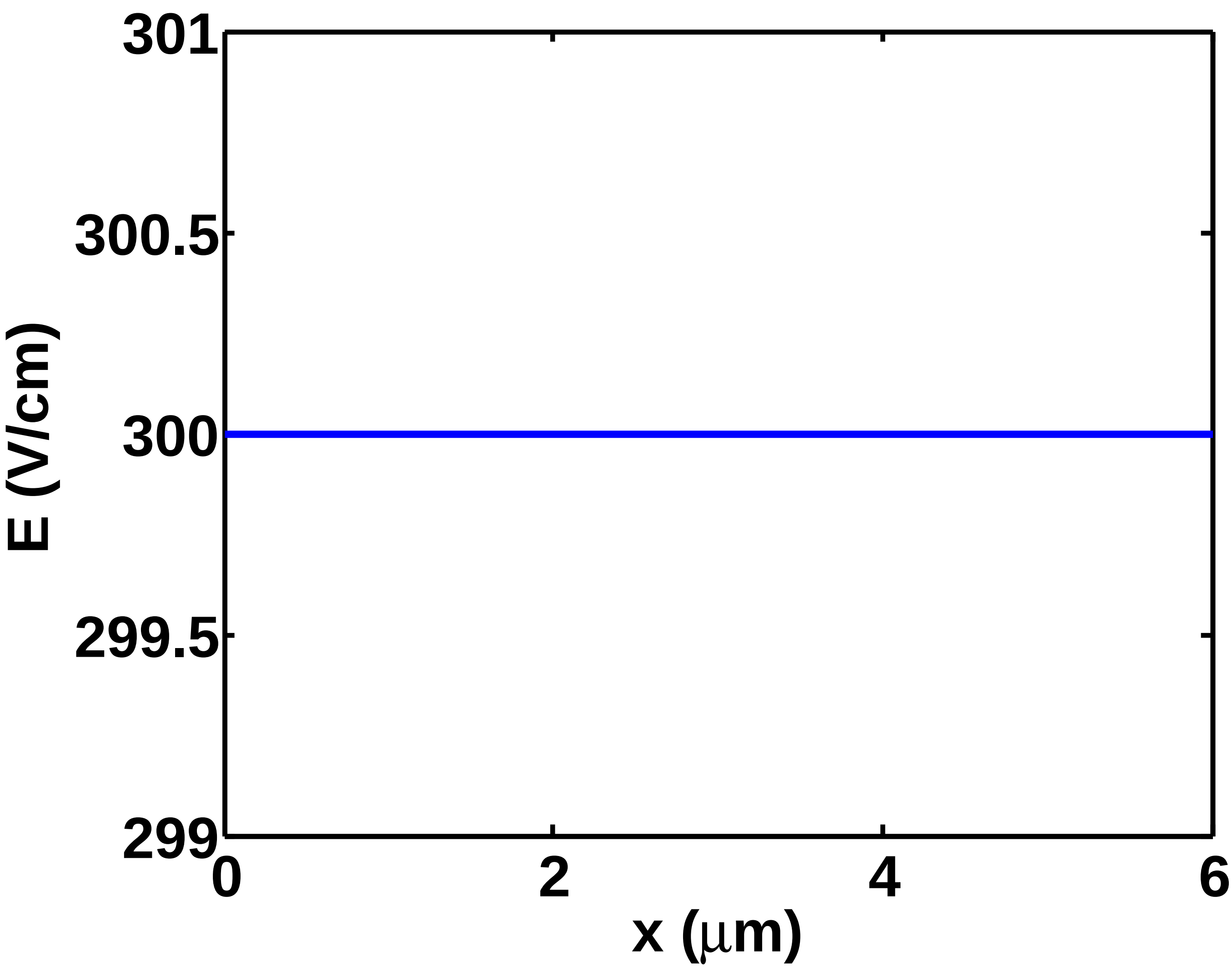
where $l = 10\times10^{-6}$ m. $R = 100\,\Omega$. $A = 10^{-12}\ m^2$. $q = 1.6\times10^{-19}$ C

$D = 27\times10^{-4}\ m^2/s$. $\frac{kT}{q} = 0.026$ V

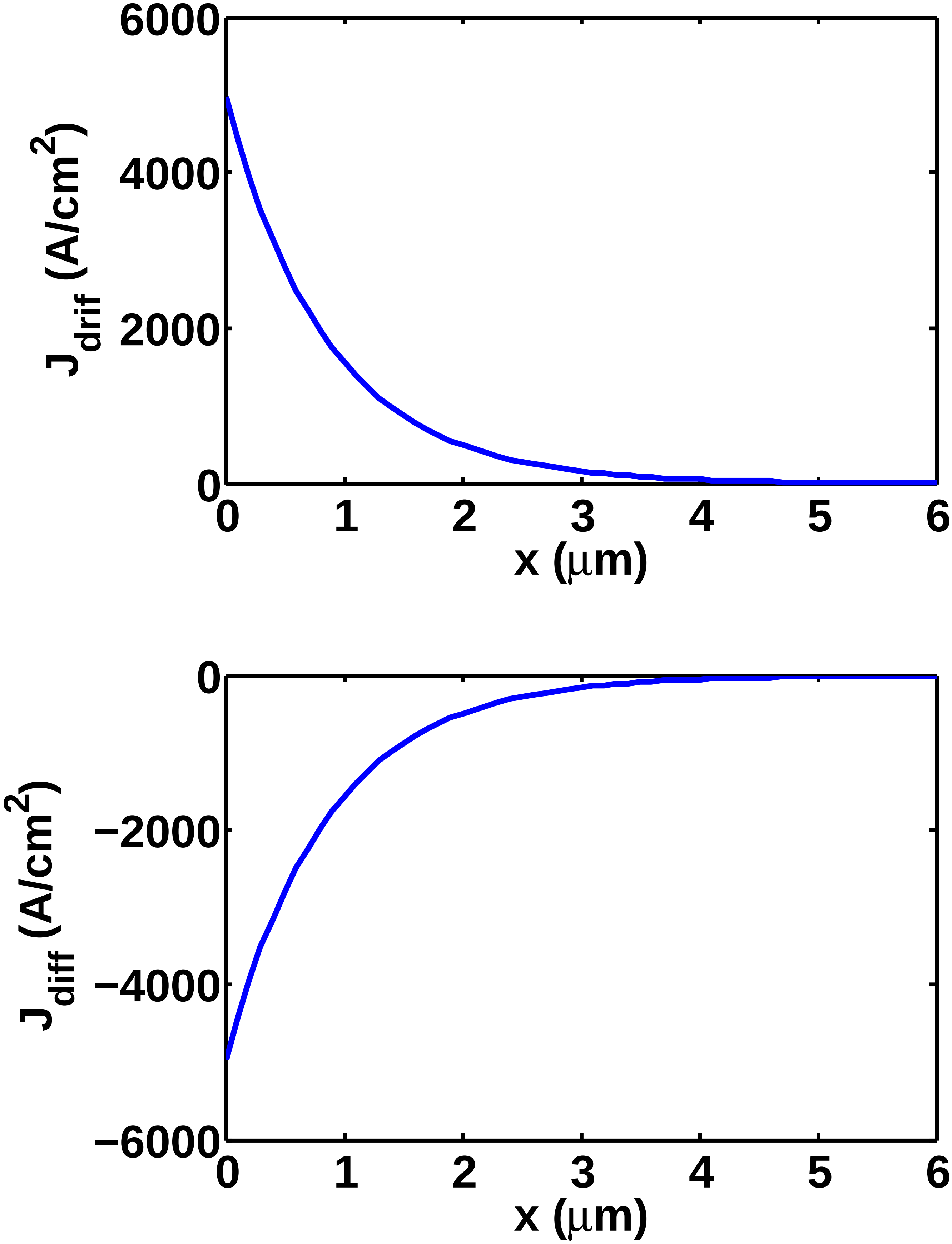
By putting in all these numbers, we get

$$p = 6\times10^{24}\ m^{-3} = 6\times10^{18}\ cm^{-3}$$


x 10^16
15
10
5
0
n (cm^−3)
0
1
2
3
4
5
6
x (μm)
x 10^5
10
8
6
4
2
0
p (cm^−3)
0
1
2
3
4
5
6
x (μm)



301
300.5
300
299.5
299
E (V/cm)
0
2
4
6
x (μm)


6000
4000
2000
0
J_{drif} (A/cm^2)
0
1
2
3
4
5
6
x (μm)
0
−2000
−4000
−6000
J_{diff} (A/cm^2)
0
1
2
3
4
5
6
x (μm)