Homework 7 Solutions (Changjian Zhang)

Problem 7.1

a) For forward active region.

$V_{CE} \geq V_{CE-SAT}$

$V_{CE} = V_{DD} - I_C R$. $\quad I_C = \beta_F I_B = \beta_{F0}(1+\frac{V_{CE}}{V_A}) I_B$. $\quad I_B \in [5, 50]\mu A$

$\Rightarrow V_{CE} = V_{DD} - \beta_{F0} R(1+\frac{V_{CE}}{V_A}) I_B \Rightarrow V_{CE} = \dfrac{V_{DD} - \beta_{F0} I_B R}{1+\beta_{F0} I_B R/V_A}$

$\Rightarrow \dfrac{V_{DD} - \beta_{F0} I_B R}{1+\beta_{F0} I_B R/V_A} \geq V_{CE-SAT}$, So $R_{max} = \dfrac{V_{DD} - V_{CE\_SAT}}{\beta_{F0} I_B (1+\frac{V_{CE\_SAT}}{V_A})}$

$R_{min} = 0$.

The plot is attached in the end.

b) $V_{out} = 2.5V$. $\quad I_C = 1mA$

$I_C = \beta_F I_B = \beta_{F0}(1+\frac{V_{CE}}{V_A}) I_B$, $\quad V_{out} = V_{CE} \Rightarrow I_B = \dfrac{I_C}{\beta_{F0}(1+\frac{V_{OUT}}{V_A})} \approx 9.5\mu A$

$V_{CE} = V_{DD} - I_C R \Rightarrow R = \dfrac{V_{DD} - V_{CE}}{I_C} = 2500\Omega$

c) $R = 2500\Omega$. $\quad V_{CE} > V_{CE\_SAT}$. $\quad V_{CE} = V_{DD} - I_C R = V_{DD} - \beta_{F0}(1+\frac{V_{CE}}{V_A}) I_B R$

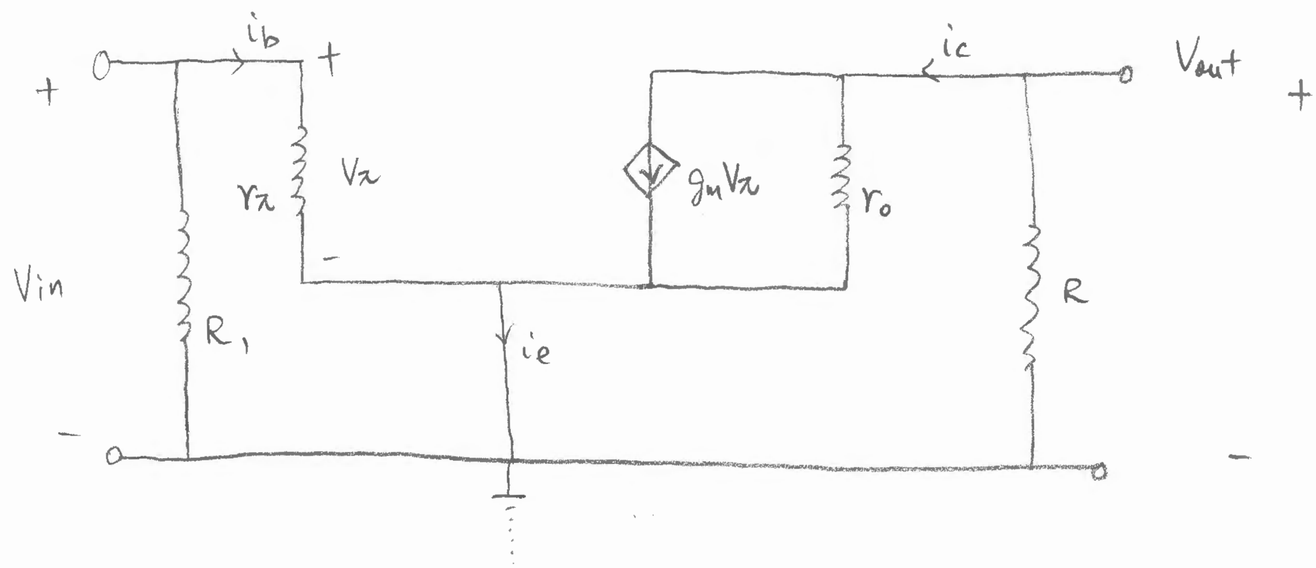
$\Rightarrow V_{CE} = \dfrac{V_{DD} - \beta_{F0} I_B R}{1+\beta_{F0} I_B R/V_A} > V_{CE\_SAT}$

$\Rightarrow I_{B\_max} = \dfrac{V_{DD} - V_{CE\_SAT}}{\beta_{F0} R(1+\frac{V_{CE\_SAT}}{V_A})} \approx 19\mu A$, $\quad I_B \in [0, 19]\mu A$

$V_{CE} = V_{out} > V_{CE\_SAT}$ so $V_{out} \in [0.2, 5]$ V

e) $$R_1 = \frac{V_{DD} - V_{BE\_ON}}{I_B} = 462000$$

f)



$$A_v = \frac{V_{out}}{V_{in}} = -g_m(r_o // R)$$

If $R_1 << r_\pi$, $V_{CE} < V_{BE}$, in saturation region, g_m is small, so A_v is small.

g) $A_v = -g_m(r_o // R)$, $g_m = \frac{q I_c}{kT}$, $r_o = \frac{1}{g_o}$, $g_o = \frac{I_c}{V_A}$

$I_c = 1mA$, $V_A = 50V$. $A_v \approx -92$

h) $$A_v = -\frac{q I_c}{kT} \frac{\frac{V_A}{I_c} R}{\left(\frac{V_A}{I_c} + R\right)} = -\frac{q V_A}{kT} \frac{1}{\left(\frac{V_A}{R I_c} + 1\right)}.$$ $V_{OUT} = V_{DD} - R I_c$.

If V_{out} is fixed at 2.5V, $R I_c$ is fixed, the A_v is fixed.

So no, we can't.

Problem 7.2

a)

$V_{OUT} = -I_C R \Rightarrow R = 2500\Omega$

In forward active region:

$$-\frac{V_B}{R_1} + \frac{V_{DD}-V_B}{R_2} = I_B \quad ①$$

$V_B = -V_{CE} + V_{BE\_ON} + V_{out}$, $V_{DD} + I_E R_E - V_{EC} - V_{OUT} = 0$

$$\Rightarrow V_B = V_{DD} + I_E R_E + V_{BE\_ON} \quad ②$$

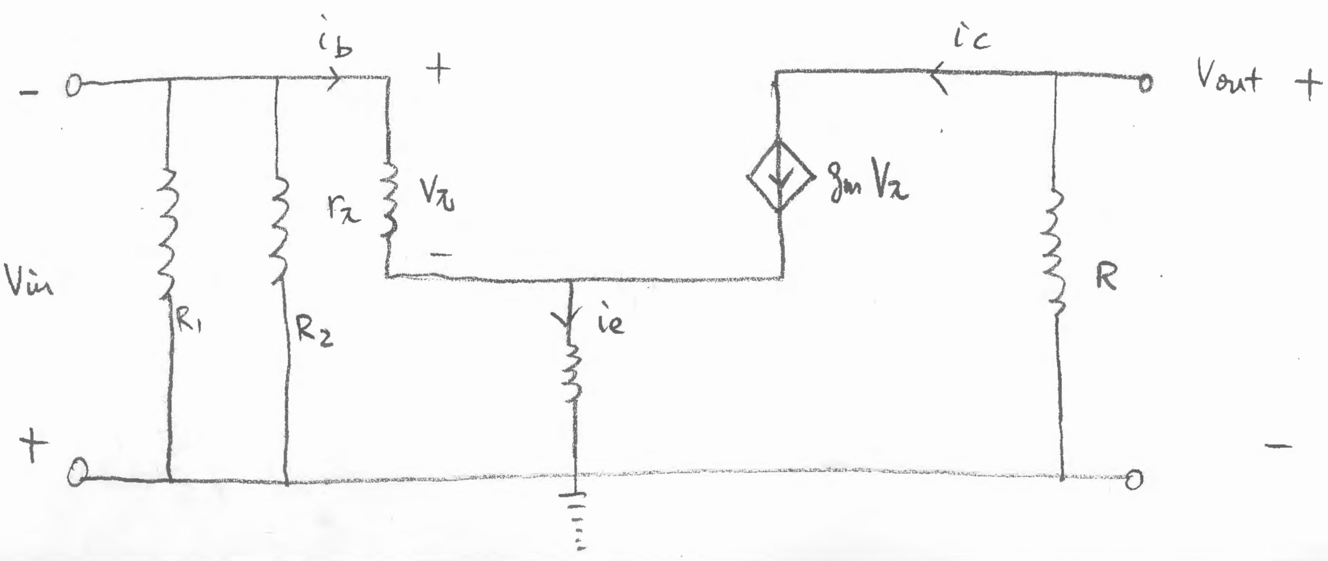
① and ②,

$$-\frac{V_{DD} + I_E R_E + V_{BE\_ON}}{R_1} + \frac{-I_E R_E - V_{BE\_ON}}{R_2} = I_B$$

$$I_E = \left(1+\frac{1}{\beta_o}\right) I_C, \quad I_B = \frac{I_C}{\beta_o}.$$

$$\Rightarrow \frac{R_1}{R_2} \sim 6$$

b)



$$V_{OUT} = -i_c R = -g_m V_\pi R \qquad \frac{-V_{in} - i_b r_\pi}{R_E} = i_b + i_c = (1+\beta_F) i_b$$

$$\Rightarrow -V_{in} = (1+\beta_F) i_b R_E + r_\pi i_b$$

$$\Rightarrow A_v = \frac{V_{out}}{V_{in}} = \frac{\beta_F R}{(1+\beta_F) R_E + r_\pi}, \quad r_\pi = \frac{1}{g_\pi}, \quad g_\pi = \frac{g_m}{\beta_F}$$

$$\Rightarrow A_v = \frac{\beta_F R}{(1+\beta_F) R_E + \frac{\beta_F}{g_m}} \approx \frac{g_m R}{1 + g_m R_E} \approx 33.3$$

c) $V_{CE} < V_{CE\_SAT}$, $V_{OUT} = -I_c R$, $V_{OUT} - V_{CE} - I_E R_E = V_{DD}$

$$V_{OUT} = V_{DD} + V_{CE} + I_E R_E = V_{DD} + V_{CE} - (1+\frac{1}{\beta_F}) \frac{R_E}{R} V_{OUT}$$

$$\Rightarrow V_{CE} = \left[1 + (1+\frac{1}{\beta_F})\frac{R_E}{R}\right] V_{OUT} - V_{DD} < V_{CE\_SAT}$$

$$\Rightarrow V_{OUT} < \frac{V_{CE\_SAT} + V_{DD}}{1 + (1+\frac{1}{\beta_F})\frac{R_E}{R}}$$

$I_c < 0 \Rightarrow V_{OUT} > 0$

So $\quad 0V < V_{OUT} < 4.61\ V$

Problem 7.3

a) $I_{C1} = -I_{C2} = 1mA$ $V_{OUT} = 2.5V$, $V_{CE} = V_{OUT}$

$$I_{B1}\beta_{FO}\left(1+\frac{V_{CE}}{V_A}\right) = I_{C1} \Rightarrow I_{B1} = \frac{I_{C1}}{\beta_{FO}\left(1+\frac{V_{out}}{V_A}\right)} \approx 9.5\mu A$$

$$-I_{B2}\beta_{FO}\left(1+\frac{|V_{OUT}-V_{DD}|}{V_A}\right) = I_{C2} \Rightarrow I_{B2} = \frac{-I_{C2}}{\beta_{FO}\left(1+\frac{|V_{OUT}-V_{DD}|}{V_A}\right)} \approx 9.5\mu A$$

b) & c)
For NPN, $V_{CE} > V_{CE\_SAT} \Rightarrow V_{OUT} > 0.2V$

For PNP, $V_{CE} < V_{CE\_SAT} \Rightarrow V_{OUT} - V_{DD} < -0.2V \Rightarrow V_{OUT} < 4.8V$

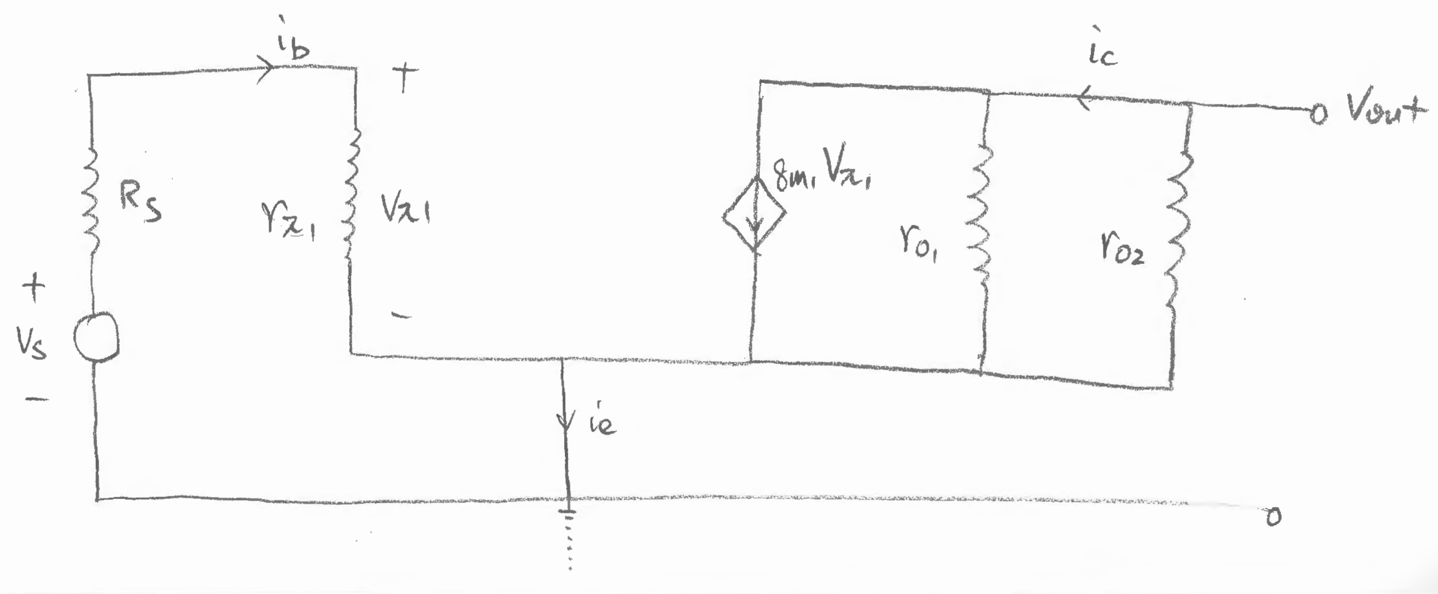
$\Rightarrow$ $0.2V < V_{OUT} < 4.8V$

d) For 7.1 $0.2V \sim 5V$

For 7.2 $0V \sim 4.61V$

For 7.3 $0.2V \sim 4.8V$

e)



$$A_V = \frac{V_{out}}{V_{in}}, \quad V_{out} = -g_{m1} V_{\pi 1} (r_{o1} // r_{o2}), \quad V_{\pi 1} = \frac{r_{\pi 1}}{R_S + r_{\pi 1}} V_S$$

$$\Rightarrow A_V = -g_{m1} \frac{r_{\pi 1}}{R_S + r_{\pi 1}} (r_{o1} // r_{o2}), \quad g_{m1} = \frac{q I_C}{kT}, \quad g_\pi = \frac{q I_B}{kT}$$

$$r_{\pi 1} = \frac{1}{g_{\pi 1}} = \frac{kT}{q I_{B1}}, \quad I_{B1} = \frac{I_{C1}}{\beta_{F0}\left(1 + \frac{V_{OUT}}{V_A}\right)}, \quad r_o = \frac{1}{g_o}, \quad g_o = \frac{I_C}{V_A}$$

f) For 7.1: $A_V = -g_m (r_o // R) \sim -92$ when $R = 2500\,\Omega$

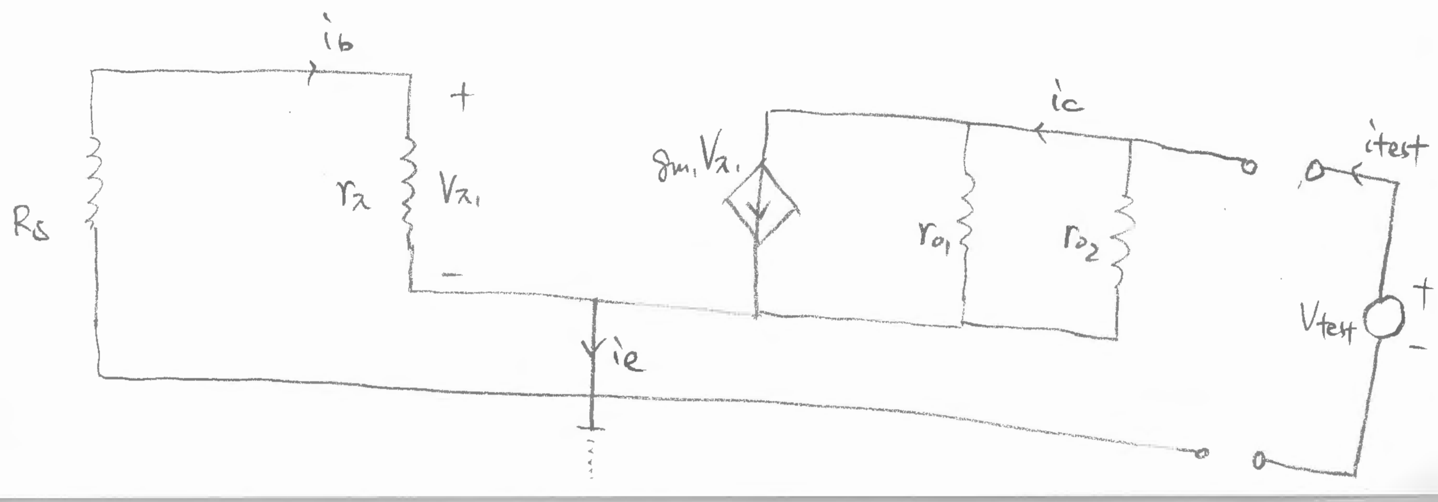
For 7.2: $A_V \approx \dfrac{g_m R}{1 + g_m R_E} \sim 33$

For 7.3: $A_V = -g_{m1} (r_{o1} // r_{o2}) \approx -961$, $r_{o1} = r_{o2} = 5 \times 10^4\,\Omega$

It's better Because in 7.3 there's no R_E, which reduces the gain. Also, the equavalent resistance r_{o2} is much larger than the resistor R used in 7.1.

g) $R_C = 0$, $A_V = -g_m (r_{o1} // r_{o2}) \approx -961$

h)

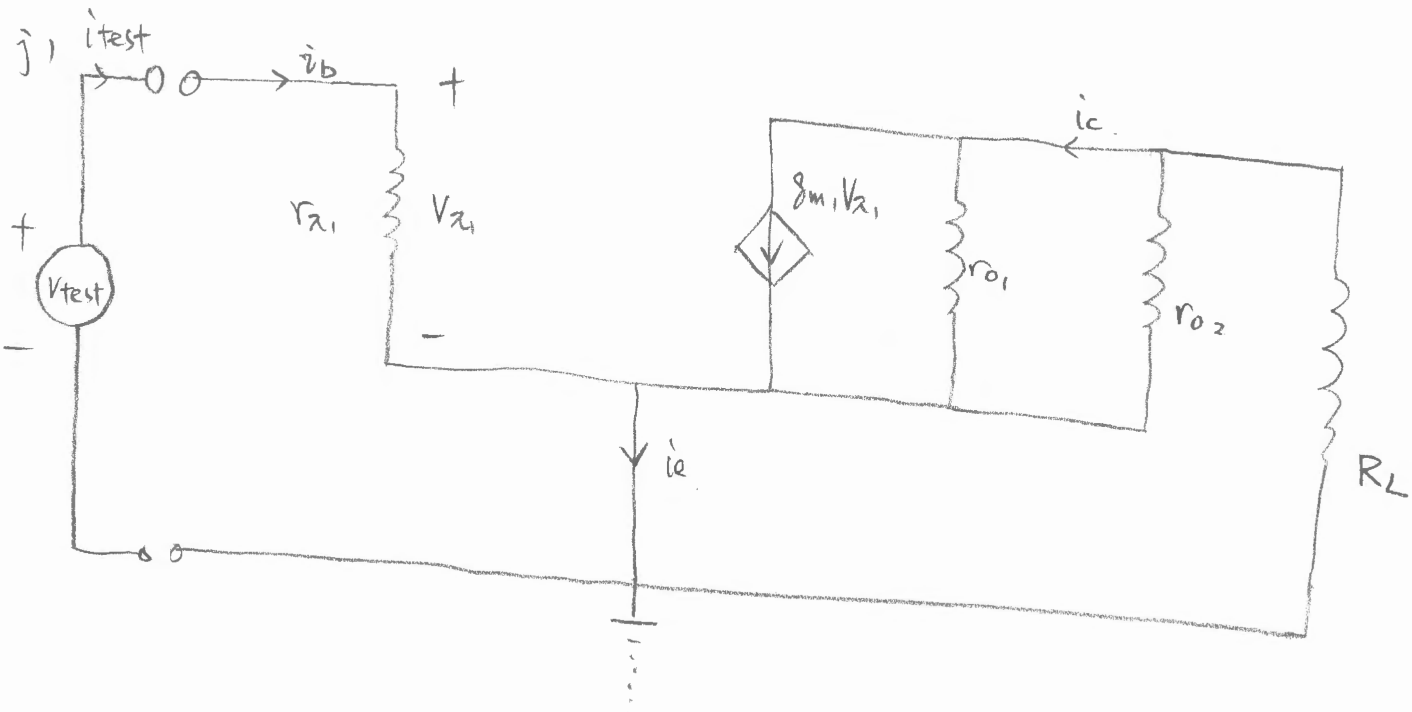


$$R_{out} = \frac{V_{test}}{i_{test}}$$

$V_{\pi 1} = 0$ $\quad g_{m1} V_{\pi 1} = 0$, so $R_{out} = r_{o1} // r_{o2} = \dfrac{r_{o1} r_{o2}}{r_{o1} + r_{o2}}$

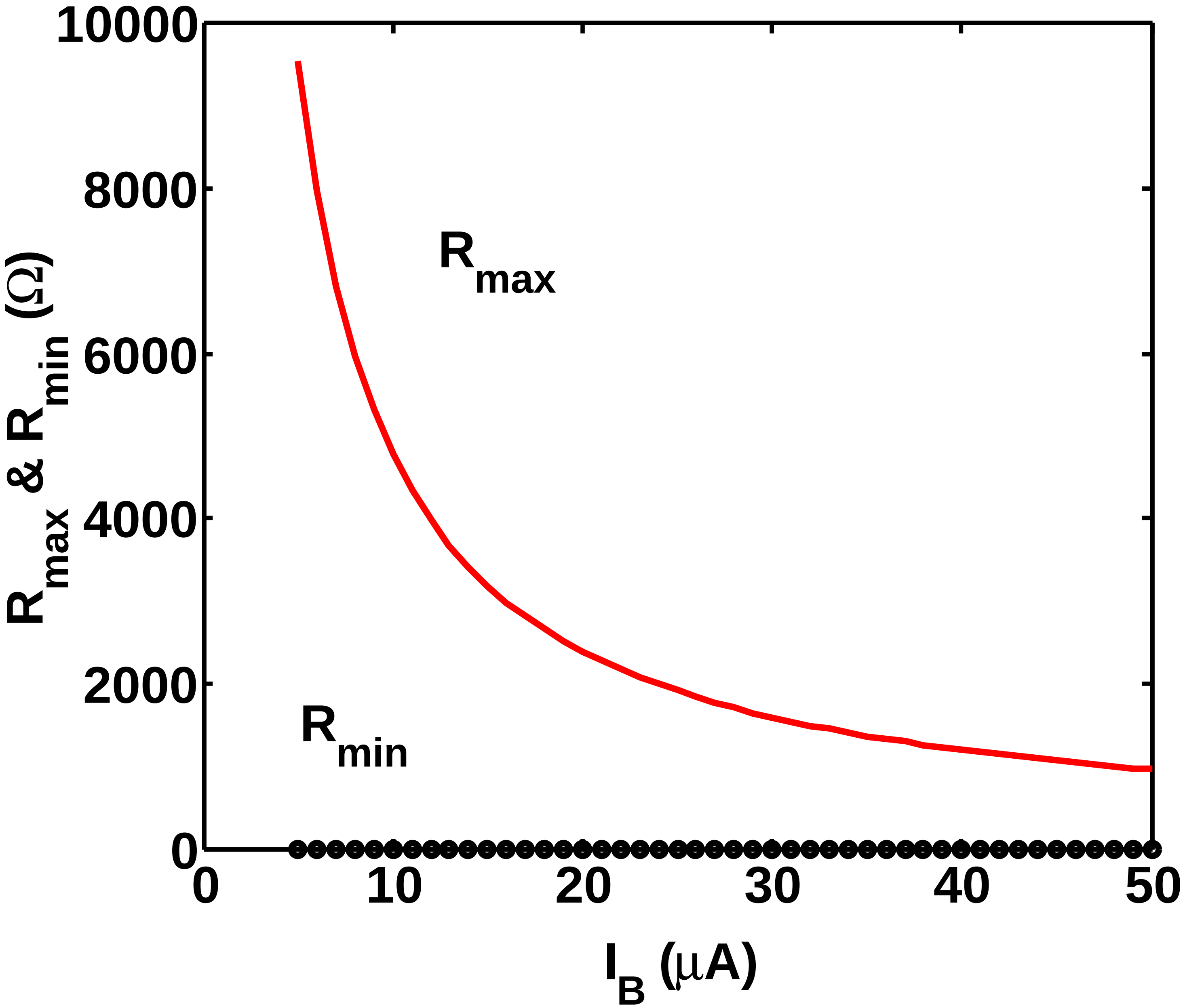
i) $r_{o1} = r_{o2} = 5 \times 10^4 \Omega$, $\quad R_{out} = 2.5 \times 10^4 \Omega$

j)



$$R_{in} = \frac{V_{test}}{i_{test}} = r_{\pi 1} = \frac{1}{g_{\pi 1}} = \frac{kT}{q I_B}$$

k) $R_{in} \approx 2.73 \times 10^3 \Omega$



10000
8000
6000
4000
2000
0
0
10
20
30
40
50
R max
R min
R max & R min (Ω)
I B (μA)