Problem 8.1

a)
$$I_{REF} = I_{D1} = I_{D2} \Rightarrow I_{REF} = \frac{k_n}{2}(V_1 - V_2 - V_{TN})^2\left[1+\lambda_n (V_1 - V_2)\right]$$
$$= \frac{k_n}{2}(V_2 - V_{TN})^2 (1+\lambda_n V_2)$$

$$\Rightarrow V_1 = 2V_2, \quad V_2 = 1.4351V, \quad V_1 = 2.8703V$$

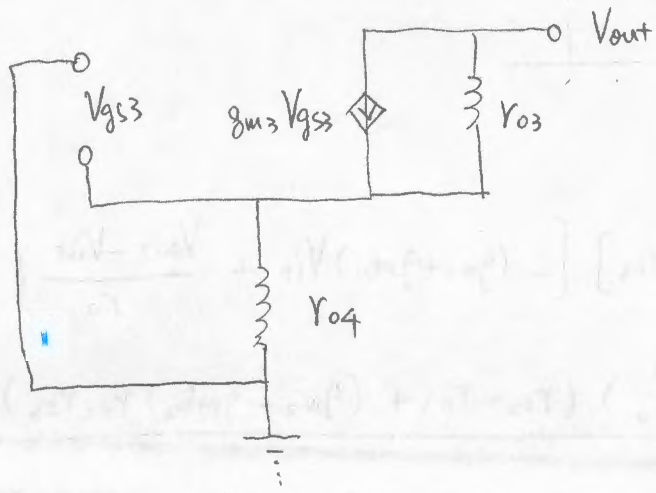
b)
$$I_{D3} = I_{D4} \Rightarrow \frac{k_n}{2}(V_1 - V_3 - V_{TN})^2 (1+\lambda_n (V_{OUT} - V_3))$$
$$= \frac{k_n}{2}(V_2 - V_{TN})^2 (1+\lambda_n V_3)$$

$$\Rightarrow V_{OUT} = \frac{1}{\lambda_n}\left[\frac{(V_2 - V_{TN})^2 (1+\lambda_n V_3)}{(V_1 - V_3 - V_{TN})^2} - 1\right] + V_3$$

c)
$M_3: \; V_1 - V_{TN} < V_{OUT}$ $\quad$ $M_4: \; V_2 - V_{TN} < V_3$

$$\Rightarrow V_{OUT} > 2.3703V$$

e)

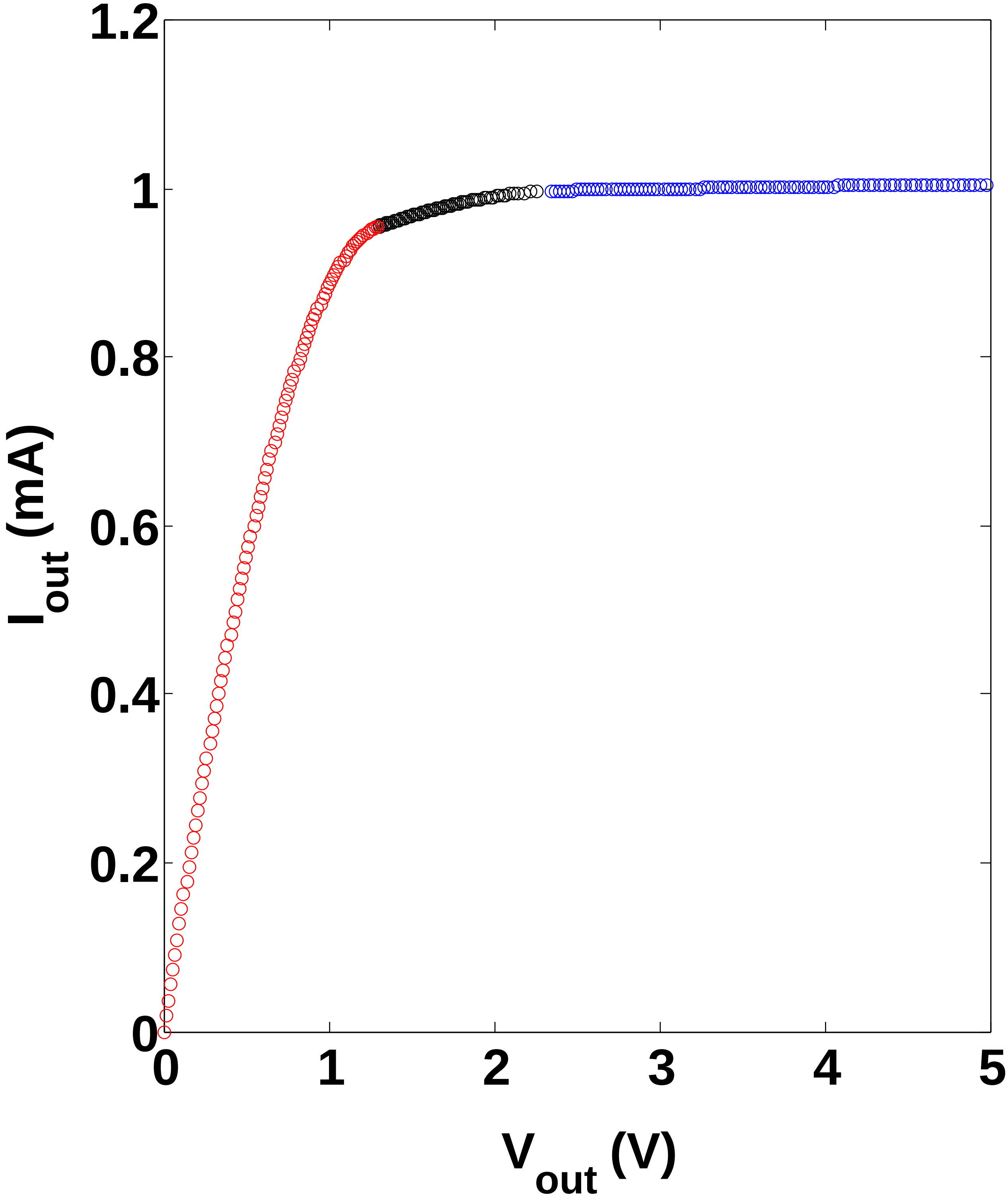


f) $r_{oc} = r_{o3} + r_{o4} + g_{m3} r_{o3} r_{o4}$

g) $g_o = \lambda_n I_D \Rightarrow r_{o3} = r_{o4} = 100000\,\Omega$.

$$g_{m3} = \sqrt{2 k_n I_D (1+\lambda_n V_{DS})} = 0.0021\,S$$

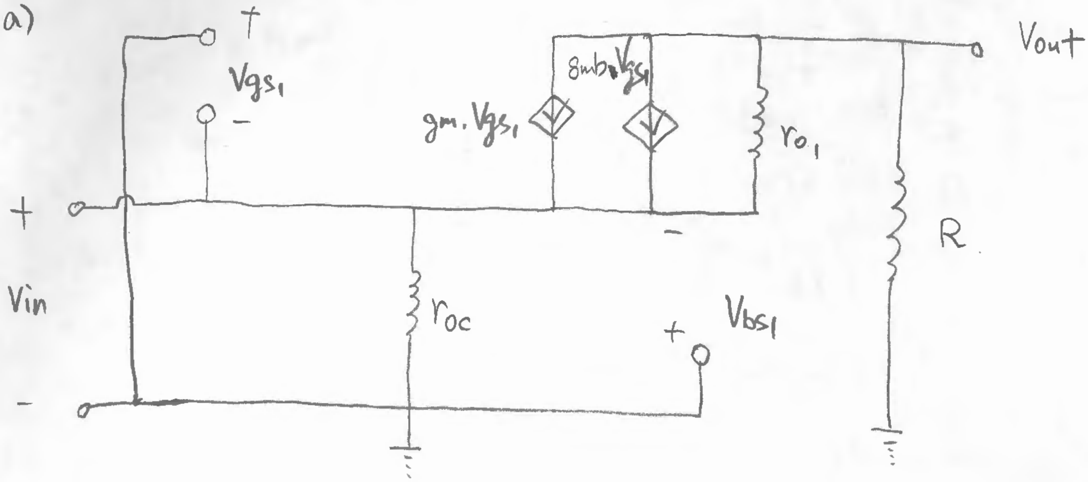
$$r_{oc} = 2.3387 \times 10^5\,\Omega$$



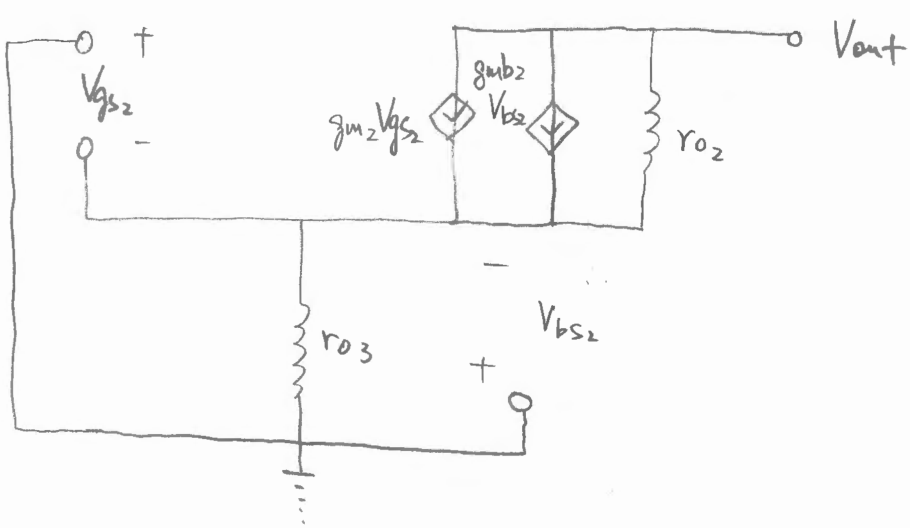
1.2
1
0.8
0.6
0.4
0.2
0
I_{out} (mA)
0
1
2
3
4
5
V_{out} (V)

Problem 8.2

a)



The circuit for R:

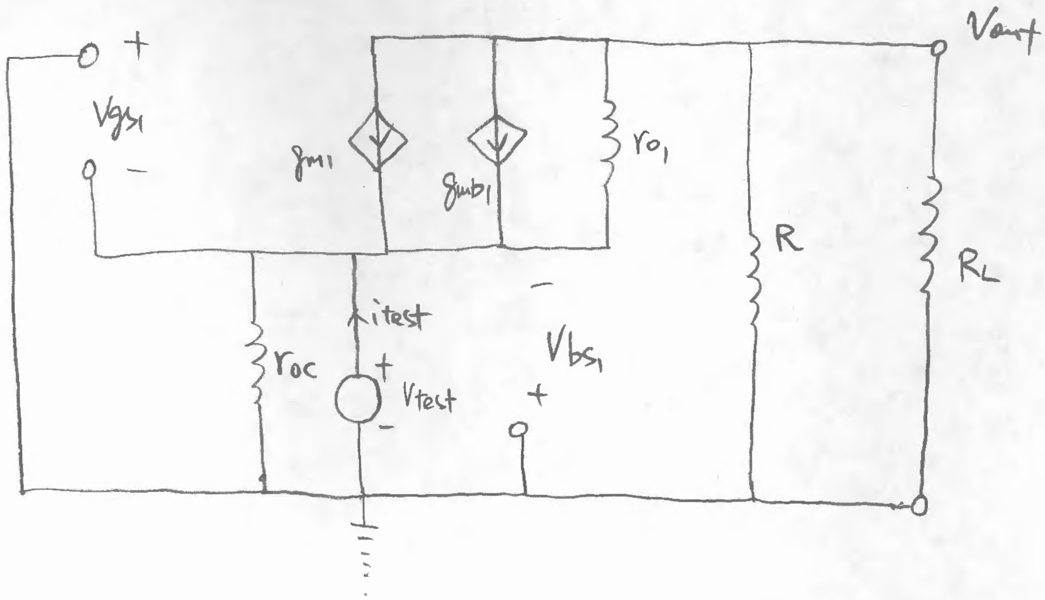


$$R = r_{o_2} + r_{o_3} + (g_{m_2} + g_{mb_2}) r_{o_2} r_{o_3}$$

$$V_{out} = -i_d'' R = -R\left[-(g_{m_1} + g_{mb_2})V_{in} + \frac{V_{out} - V_{in}}{r_o}\right]$$

$$\Rightarrow \quad A_v = \frac{V_{out}}{V_{in}} = \frac{\left(g_{m_1} + g_{mb_1} + \frac{1}{r_{o_1}}\right)R}{1 + \frac{R}{r_o}}$$

b)

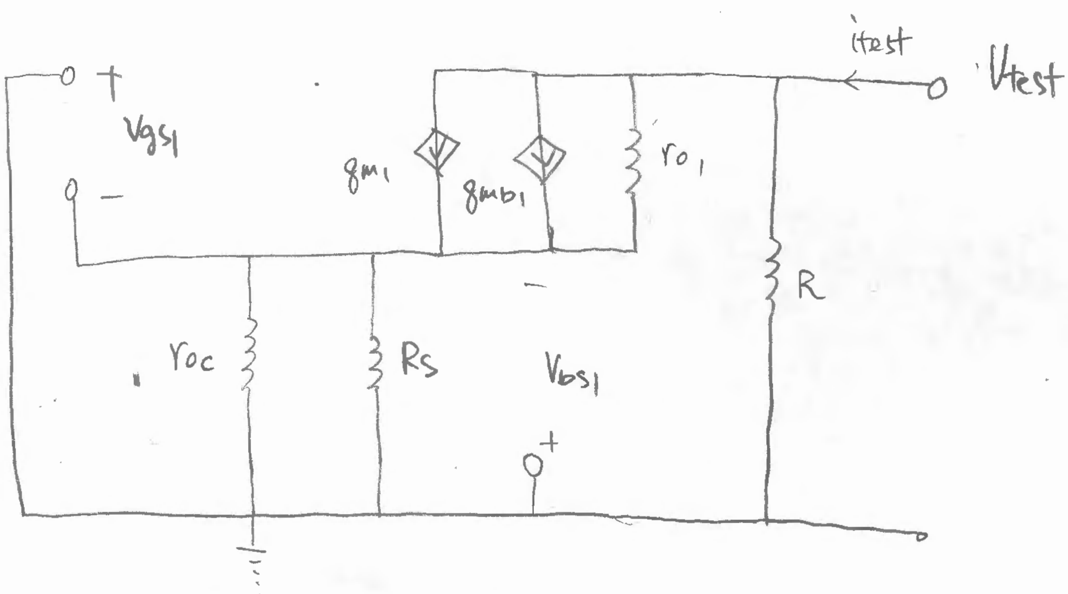


$$i_{test} = (g_{m1} + g_{mb1})\, V_{test} - \frac{V_{out} - V_{test}}{r_{o1}} + \frac{V_{test}}{r_{oc}}$$

$$= \left(g_{m1} + g_{mb1} + \frac{1}{r_{oc}}\right) V_{test} - \frac{1}{r_{o1}} \left[\frac{\left(g_{m1} + g_{mb1} + \frac{1}{r_{o1}}\right)(R // R_L)}{1 + \frac{R // R_L}{r_{o1}}} - 1 \right] V_{test}$$

$$\Rightarrow R_{in} = \frac{V_{test}}{i_{test}} = \frac{(g_{m1} + g_{mb1}) r_{o1} + \frac{r_{o1} + R // R_L}{r_{oc}} + 1}{r_{o1} + R // R_L}$$

c)

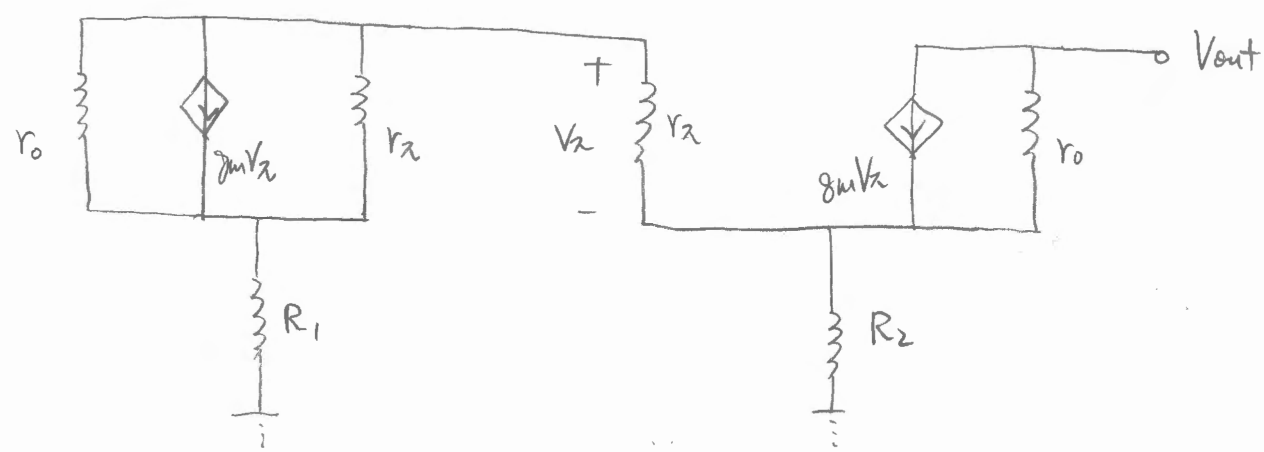


$$i_{test} = i_d + \frac{V_{test}}{R} = \frac{V_{test}}{R} + \frac{V_{test}}{r_{o1}} \Big/ \left[1 + \frac{r_{oc} // R_S}{r_{o1}} + (g_{m1} + g_{mb1})(r_{oc} // R_S) \right]$$

$$\Rightarrow R_{out} = \frac{V_{test}}{i_{test}} = \frac{1}{R} + 1 \Big/ \left[r_{o1} + r_{oc} // R_S + (g_{m1} + g_{mb1}) r_{o1} (r_{oc} // R_S) \right]$$

Problem 8.4

a)



According to the lecture notes,

$$r_{oc} = \frac{V_{test}}{i_{test}} = \frac{\frac{r_o}{R_2} + \frac{(1+g_m r_\pi) r_o}{R + r_\pi} + 1}{\frac{1}{R_2} + \frac{1}{R + r_\pi}}.$$

where $R = \left[\left(r_o \,//\, \frac{1}{g_m} \,//\, \frac{1}{r_\pi} \right) + R_1 \right]$

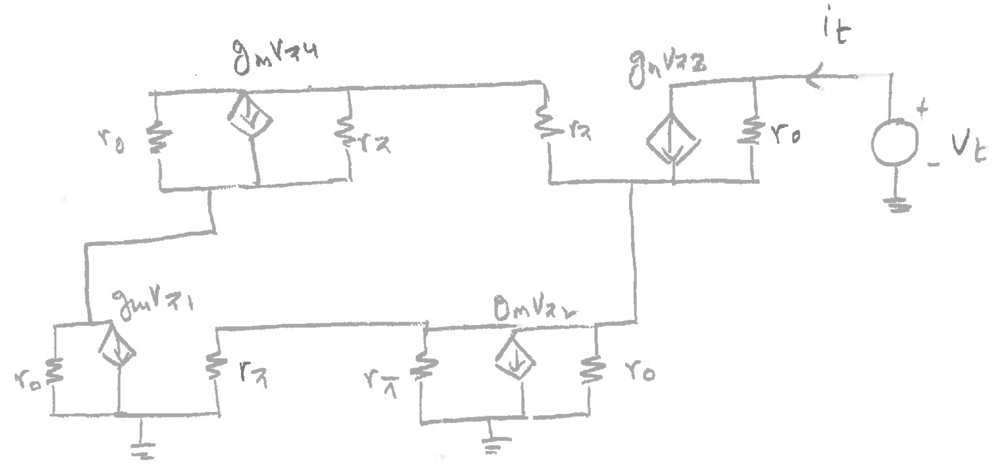
b) $V_{CE} > V_{CE\_SAT} \Rightarrow V_{out} - (1 + \frac{1}{\beta_F}) I_{out} R_2 > V_{CE\_SAT} + V_{BE\_ON}$

$\Rightarrow V_{out} > V_{CE\_SAT} + V_{BE\_ON} + (1 + \frac{1}{\beta_F}) I_{out} R_2$

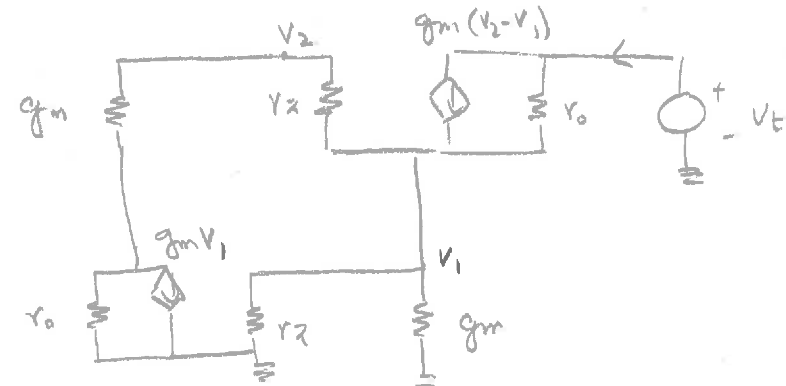
8.3

a) The small signal model becomes (assuming the output resistance of the current source is ∞)

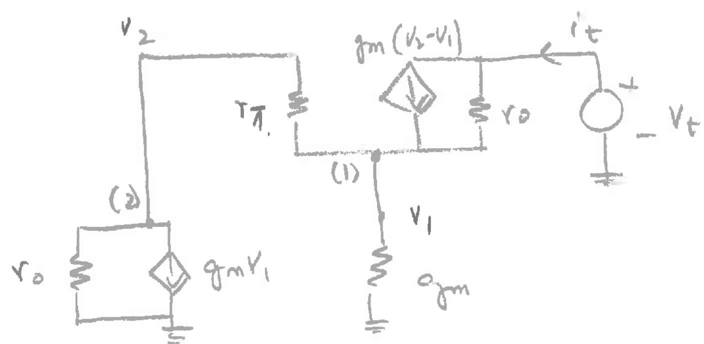
$$r_{oc} = \frac{v_t}{i_t}$$



The BJTs B4 and B2 are diode connected and their circuits can be replaced by conductances g_m to get.



This can be further simplified (since $g_m >> g_\pi = \frac{1}{r_\pi}$):



KCL at (1) gives:

$$i_t = \frac{v_t - v_1}{r_o} + g_m(v_2 - v_1) = g_m v_1 + \frac{v_1 - v_2}{r_\pi} \Rightarrow \frac{v_t - v_1}{r_o} + g_m(v_2 - v_1) \approx g_m v_1 .$$

KCL at (2) gives:

$$\frac{v_1 - v_2}{r_\pi} = g_m v_1 + \frac{v_2}{r_o} .$$

$$\Rightarrow \quad v_2 \cong -g_m (r_\pi \| r_o) v_1 .$$

$$\Rightarrow \quad v_2 - v_1 \approx -\left[g_m (r_\pi \| r_o) + 1\right] v_1 .$$

$\Rightarrow$

$$\frac{v_t}{r_o} = \frac{v_1}{r_o} - g_m (v_2 - v_1) + g_m v_1$$

$$\Rightarrow \quad v_t \approx v_1 \left[1 + g_m r_o + g_m r_o \left[g_m (r_\pi \| r_o) + 1\right]\right]$$

$$i_t = g_m v_1 + \frac{v_1 - v_2}{r_\pi} \cong \left[g_m + \frac{g_m (r_\pi \| r_o) + 1}{r_\pi}\right] v_1$$

$$\Rightarrow i_t \cong g_m \left[1 + \frac{(r_\pi \| r_o)}{r_\pi}\right] v_1 = g_m \left[1 + \frac{(r_\pi \| r_o)}{r_\pi}\right] \frac{v_t}{\left[1 + 2 g_m r_o + g_m r_o \left[g_m (r_\pi \| r_o)\right]\right]}$$

$$\Rightarrow r_{oc} \approx \frac{\left[1 + 2 g_m r_o + g_m r_o \left[g_m (r_\pi \| r_o)\right]\right]}{g_m \left[1 + \frac{(r_\pi \| r_o)}{r_\pi}\right]} \approx \frac{g_m (r_\pi \| r_o)}{\left[1 + \frac{(r_\pi \| r_o)}{r_\pi}\right]} r_o \approx \frac{\beta_F r_o}{2}$$

(if $r_o \gg r_\pi$).

b) $V_{OUT} > V_{BE\text{-}ON} + V_{CE\text{-}SAT}$