UNIVERSITY OF MARYLAND,
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Double Pendulum On Cart

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ENPM 667- Robot Control System

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Abstract

The main concentration of this project is on understanding and implementation of core concepts of controls, including state space representation, non-linear system design,LQR and LQG controller and Luenberger Observer for Double pendulum on cart. We will use concepts like controllability, observibility for the sytem to develop a roboust controller .For the scope of this project we will use mat-lab and simulink to model our system.
Key words: LQR, LQG, Luenberger observer,State space .

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1 Problem Statement

For this project we will consider a friction-less crane of mass M moving on a one dimensional track which is acted upon by an external force F. The crane has two loads suspended from cables attached to its base of mass m1 and m2 respectively, and the lengths of the cables are l1 and l2, respectively. The diagram below shows the pictorial representation of the problem statement.

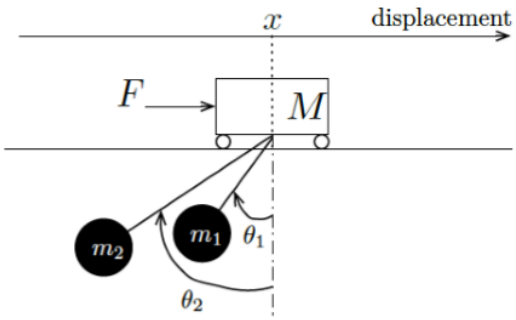

Figure 1.1: Problem statement

2 Part A
Motion Equation & Non-linear State Space

We will use the euler-Lagrange equation to formulate the motion equations and use it to fabricate the non linear state space representation. To compute the euler-Lagrange equations we need to calculate the kinetic and potential energy of the system.
Thus,the kinetic energy of the system is represented by KE and is given as :

$$KE = \frac{1}{2}\dot{x}^2(M + m_1 + m_2) - m_1\dot{x}l_1\dot{\theta}_1 cos(\theta_1) - m_2\dot{x}l_2\dot{\theta}_2 cos(\theta_2) + \frac{1}{2}m_1 l_1^2\dot{\theta_1}^2 + \frac{1}{2}m_2 l_2^2\dot{\theta_2}^2 \quad (2.1)$$

The potential energy is given by PE and is :

$$PE = m_1 g l_1(1 - cos(\theta_1) + m_2 g l_2(1 - cos(\theta_2) \quad (2.2)$$

According to the Eular-Lagrange equation the Lagrangian L is given as: $L = KE - PE$.

$$L = \frac{1}{2}\dot{x}^2(M + m_1 + m_2) - m_1\dot{x}l_1\dot{\theta}_1 cos(\theta_1) - m_2\dot{x}l_2\dot{\theta}_2 cos(\theta_2) + \frac{1}{2}m_1 l_1^2\dot{\theta_1}^2 + \frac{1}{2}m_2 l_2^2\dot{\theta_2}^2 - m_1 g l_1(1 - cos(\theta_1) - m_2 g l_2(1 - cos(\theta_2) \quad (2.3)$$

We will use the following equations to evaluate the dynamics of the system:

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{x}}) - \frac{\partial L}{\partial x} = F \quad (2.4)$$

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}_1}) - \frac{\partial L}{\partial \theta_1} = 0 \quad (2.5)$$

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}_2}) - \frac{\partial L}{\partial \theta_2} = 0 \quad (2.6)$$

First finding the value of equation 2.4 :

$$\frac{\partial L}{\partial x} = 0 \quad (2.7)$$

$$\frac{\partial L}{\partial \dot{x}} = \dot{x}(M + m_1 + m_2) - m_1 l_1\dot{\theta}_1 cos(\theta_1) - m_2 l_2\dot{\theta}_2 cos(\theta_2) \quad (2.8)$$

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{x}}) = \ddot{x}(M+m_1+m_2) - m_1 l_1 \ddot{\theta}_1 cos(\theta_1) + m_1 l_1 \dot{\theta}_1^2 sin(\theta_1) - m_2 l_2 \ddot{\theta}_2 cos(\theta_2) + m_2 l_2 \dot{\theta}_2^2 sin(\theta) \tag{2.9}$$

Now, substituting values of 2.7 and 2.9 in 2.4 :

$$\boldsymbol{\ddot{x} = \frac{F + m_1 l_1 \ddot{\theta}_1 cos(\theta_1) - m_1 l_1 \dot{\theta}_1^2 sin(\theta_1) + m_2 l_2 \ddot{\theta}_2 cos(\theta_2) - m_2 l_2 \dot{\theta}_2^2 sin(\theta_2)}{(M + m_1 + m_2)}} \tag{2.10}$$

Now to find the value of equation 2.5 :

$$\frac{\partial L}{\partial \theta_1} = m_1 \dot{x} l_1 \dot{\theta}_1 sin(\theta_1) - m_1 g l_1 sin(\theta_1) \tag{2.11}$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = -m_1 \dot{x} l_1 cos(\theta_1) + m_1 l_1^2 \dot{\theta}_1 \tag{2.12}$$

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}_1}) = -m_1 \ddot{x} l_1 cos(\theta_1) + m_1 \dot{x} l_1 \dot{\theta}_1 sin(\theta_1) + m_1 l_1^2 \ddot{\theta}_1 \tag{2.13}$$

Substituting the values of 2.13 and 2.11 into 2.5 and simplifying, we get :

$$\boldsymbol{\ddot{\theta}_1 = \frac{\ddot{x} cos(\theta_1) - g sin(\theta_1)}{l_1}} \tag{2.14}$$

Now to evaluate equation 2.6:

$$\frac{\partial L}{\partial \theta_2} = m_2 \dot{x} l_2 \dot{\theta}_2 sin(\theta_2) - m_2 g l_2 sin(\theta_2) \tag{2.15}$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = -m_2 \dot{x} l_2 cos(\theta_2) + m_2 l_2^2 \dot{\theta}_2 \tag{2.16}$$

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}_2}) = -m_2 \ddot{x} l_2 cos(\theta_2) + m_2 \dot{x} l_2 \dot{\theta}_2 sin(\theta_2) + m_2 l_2^2 \ddot{\theta}_2 \tag{2.17}$$

Substituting the values of 2.15 and 2.17 into 2.6 and simplifying, we get :

$$\boldsymbol{\ddot{\theta}_2 = \frac{\ddot{x} cos(\theta_2) - g sin(\theta_2)}{l_2}} \tag{2.18}$$

3 Part B
Linearized State Space Representation

The non-linear state space representation of the double pendulum on a cart is given in the previous section. As we know that the system can be linearized around the equilibrium point, for our case the value corresponding to equilibrium point are:

1. $\theta_1 = 0$
2. $\theta_2 = 0$
3. x=0

Thus, we can simplify the state space equation by ignoring the higher order terms and considering the following assumption at equilibrium:

1. $\sin(\theta) \approx \theta$
2. $\cos(\theta) \approx 1$
3. $\sin(\theta)^2 \approx 0$
4. $\cos(\theta)^2 \approx 1$

Thus the linearized state space representation is illustrated below:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-gm_1}{M} & 0 & \frac{-gm_2}{M} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-g}{l_1}\frac{-gm_1}{Ml_1} & 0 & \frac{-gm_2}{Ml_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{-gm_1}{Ml_2} & 0 & \frac{-g}{l_2}\frac{-gm_2}{Ml_2} & 0 \end{bmatrix} \tag{3.1}$$

$$B = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{1}{Ml_1} \\ 0 \\ \frac{1}{Ml_2} \end{bmatrix} \tag{3.2}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.3}$$

$$D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.4}$$

Thus, we can write the state space for system as

$$dX(t)/dt = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-gm_1}{M} & 0 & \frac{-gm_2}{M} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-g}{l_1}\frac{-gm_1}{Ml_1} & 0 & \frac{-gm_2}{Ml_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{-gm_1}{Ml_2} & 0 & \frac{-g}{l_2}\frac{-gm_2}{Ml_2} & 0 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{1}{Ml_1} \\ 0 \\ \frac{1}{Ml_2} \end{bmatrix} U(t) \tag{3.5}$$

4 Part C
Condition for Controllablility

As we have now linearized the system. we can check for the controlablilty of the system by using the follwing condition:

1. The rank of control-ability matrix C should be equal to the rank of A matrix i.e

$$C = \begin{bmatrix} B & AB & A^2B & A^3B & .... & A^{n-1}B \end{bmatrix} \tag{4.1}$$

$$rank([C]) = n \tag{4.2}$$

2. The determinant of the control-ability matrix must be non zero.

Thus checking the second condition we observe that

$$det|[C]| = -\frac{g^6(l_1^2 - 2l1l_2 - l_2^2)}{M^6 l_1^6 l_2^6} \tag{4.3}$$

Thus, on simplification of equation (4.3) we get

$$det|[C]| = -\frac{g^6(l_1 - l_2)^2}{M^6 l_1^6 l_2^6} \tag{4.4}$$

Thus the system would be controllable if and only if $l_1 \neq l_2$.

5 Part D
LQR controller for the system

In this section we will design a LQR controller for both linear model and non-linear model of the system. We will feed in the given values for M, m_1,m_2,g, l_1, and l_2 to form A and B matrices and using above conditions validate the control-ability.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.9800 & 0 & -0.9800 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -0.5390 & 0 & -0.0490 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -0.0980 & 0 & -1.0780 & 0 \end{bmatrix} \tag{5.1}$$

$$B = 1.0e-03 * \begin{bmatrix} 0 \\ 1.0000 \\ 0 \\ 0.0500 \\ 0 \\ 0.1000 \end{bmatrix} \tag{5.2}$$

using

$$\text{C=} \begin{bmatrix} B & AB & A^2B & A^3B & .... & A^{n-1}B \end{bmatrix}$$

$$C = 1.0e-03 * \begin{bmatrix} 0 & 1.0000 & 0 & -0.1470 & 0 & 0.1417 \\ 1.0000 & 0 & -0.1470 & 0 & 0.1417 & 0 \\ 0 & 0.0500 & 0 & -0.0319 & 0 & 0.0227 \\ 0.0500 & 0 & -0.0319 & 0 & 0.0227 & 0 \\ 0 & 0.1000 & 0 & -0.1127 & 0 & 0.1246 \\ 0.1000 & 0 & -0.1127 & 0 & 0.1246 & 0 \end{bmatrix} \tag{5.3}$$

We get the following result
$rank(C)$ =6
$Det(C)$ =$-1.3841e-24$
Thus, system is controllable

5.1 LQR controller for Linear System

We will use the linear model developed for the system in the previous section and try to fabricate a LQR controller. Following this we will try to find the best value of the LQR parameters to obtain a good result. The case 1 below shows the case where the parameters are not suitable and therefore the state of the system is not converging to the desired state.

After a process of fine tuning, we were able to obtain an appropriate controller, the details of which are provided in case 2 below.

5.1.1 Case 1

Here we consider the below mentioned Q and R

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} R = \begin{bmatrix} 0.001 \end{bmatrix} \tag{5.4}$$

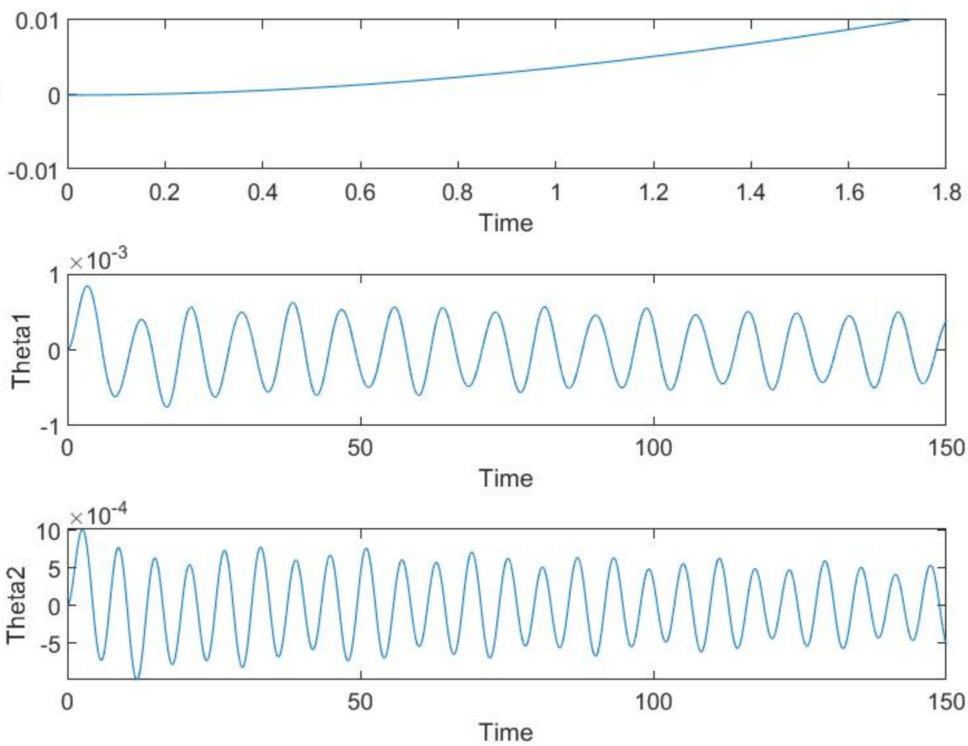

Figure 5.1: LQR response for Initial Condition [0,0,0,0,0,0]

5.1.2 Case 2

We observed that to develop good controller we need to provide high cost to $\dot{\theta}_1$ $\dot{\theta}_2$ $\dot{x}$ and x. The final values of Q and R as follows :

$$Q = \begin{bmatrix} 3500 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5000 \end{bmatrix} R = \begin{bmatrix} 0.001 \end{bmatrix} \tag{5.5}$$

We tested the system for a number of initial conditions, two of which are shown below:

1. initial state=[0,0,0,0,0,0]
2. initial state=[1,0,0.5,0,0.5,0]

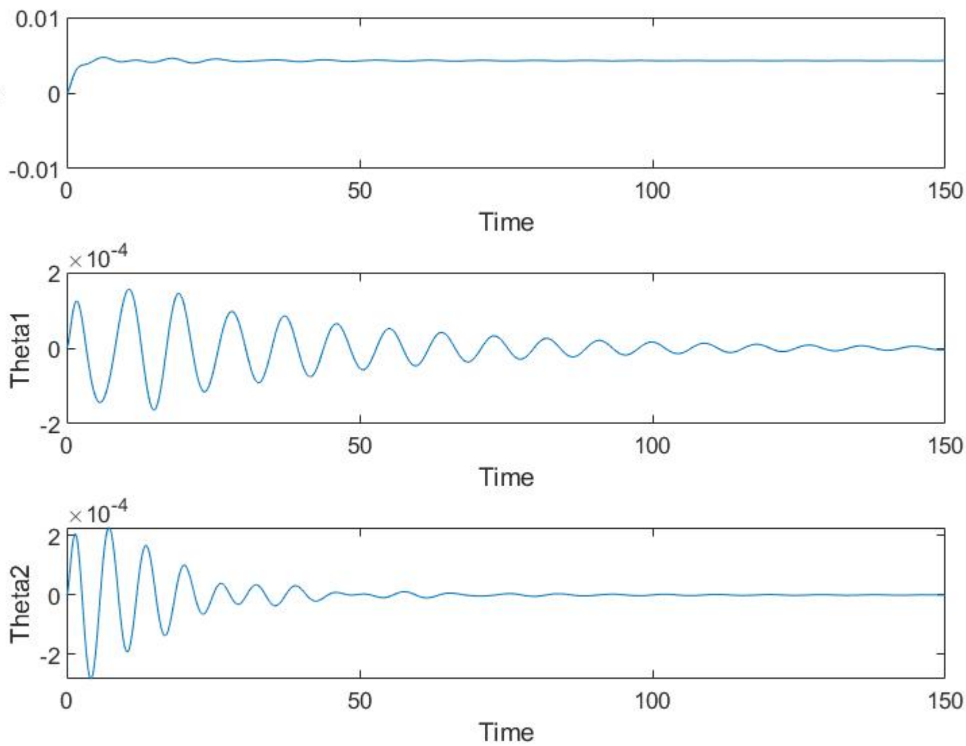

Figure 5.2: LQR response for Initial Condition [0,0,0,0,0,0]

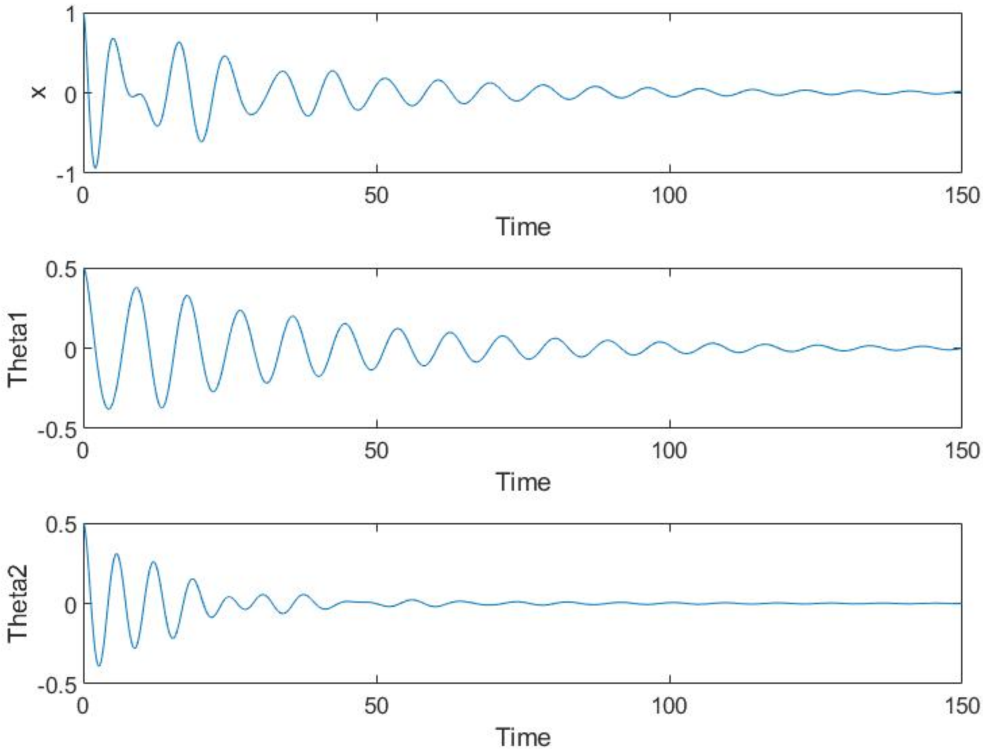

Figure 5.3: LQR response for Initial Condition [1,0,0.5,0,0.5,0]

5.2 LQR controller for Non Linear System

We will use simulink to simulate a non linear model for the system and test it for two initial conditions,

1. Case 1: Initial condition [0,0,0,0,0,0]
2. Case 2: Initial condition [1,0,0.5,0,0.5,0].

The simulink block diagram is illustrated below.

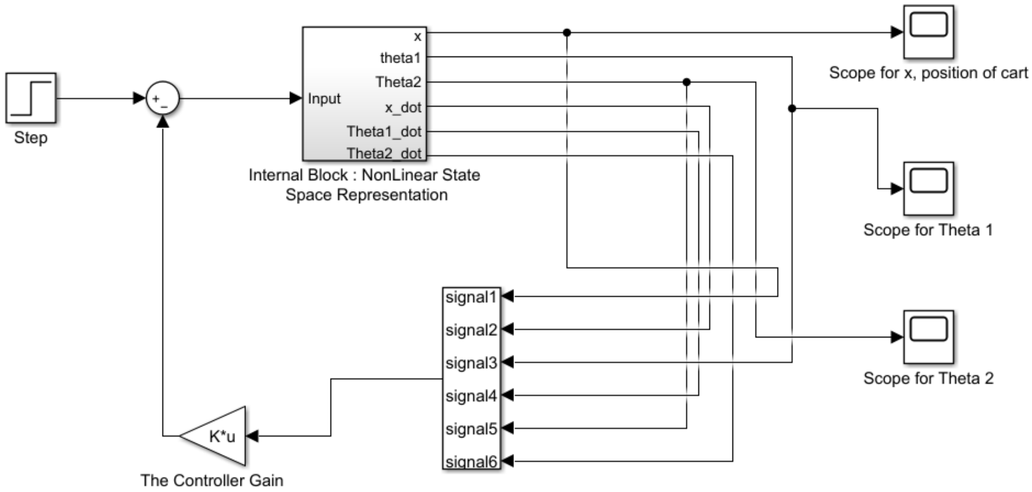

Figure 5.4: LQR for Non Linear System Block diagram

The system is locally stable using the indirect Lyapunov method as all the eigen values are negative.

$$E = \begin{bmatrix} -0.9298 + 0.9923i \\ -0.9298 - 0.9923i \\ -0.0800 + 0.9902i \\ -0.0800 - 0.9902i \\ -0.0259 + 0.7012i \\ -0.0259 - 0.7012i \end{bmatrix} \tag{5.6}$$

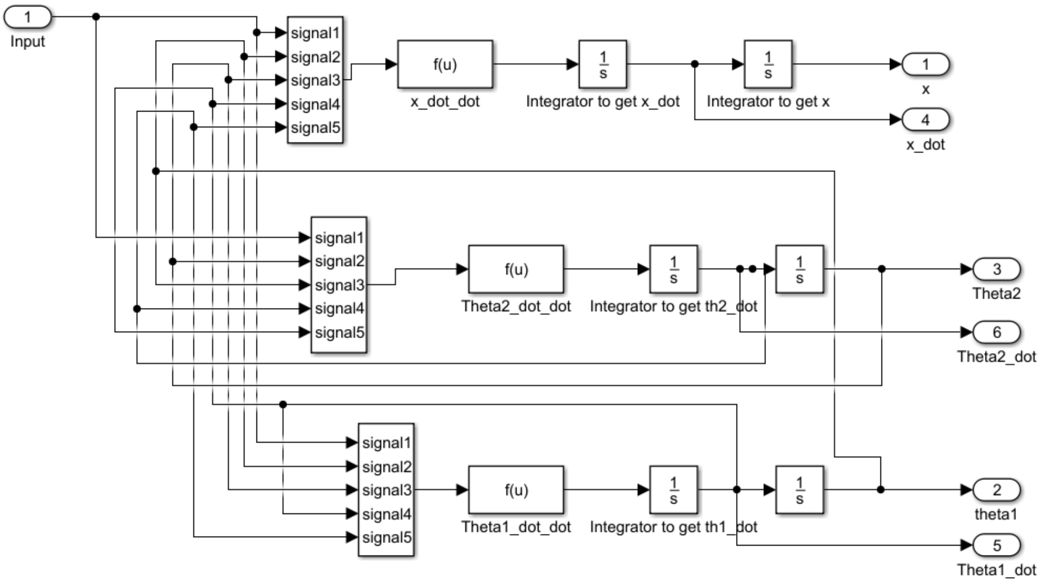

Figure 5.5: Internal Block diagram of LQR for Non Linear System

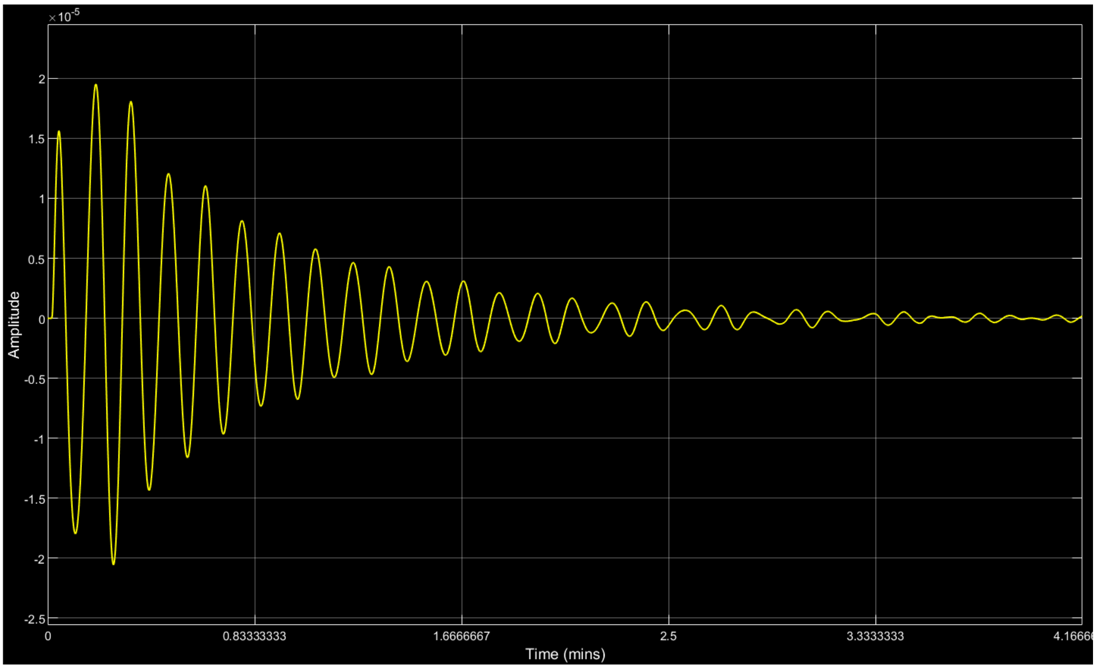

Figure 5.6: Non Linear LQR Output for θ_1 for initial condition : [0,0,0,0,0,0]

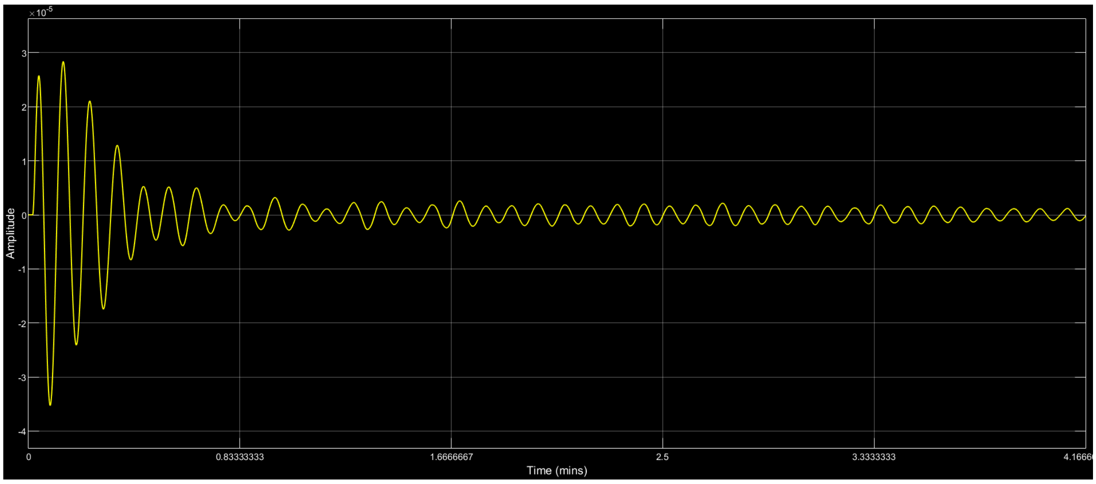

Figure 5.7: Non Linear LQR Output for θ_2 for initial condition : [0,0,0,0,0,0]

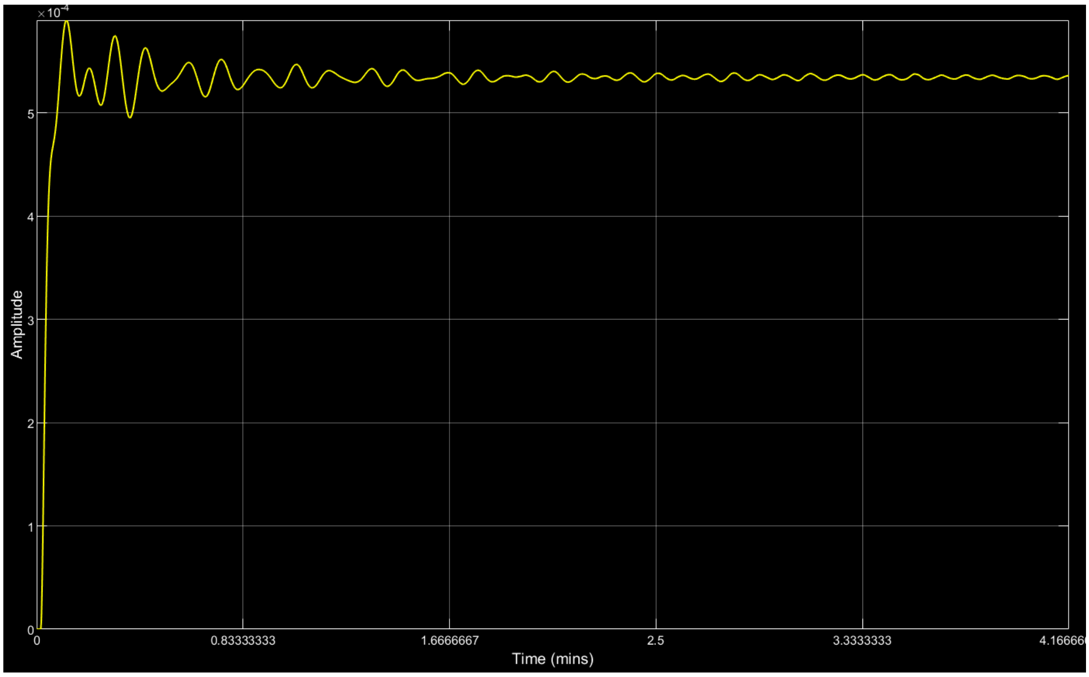

Figure 5.8: Non Linear LQR Output for cart position x for initial condition : [0,0,0,0,0,0]

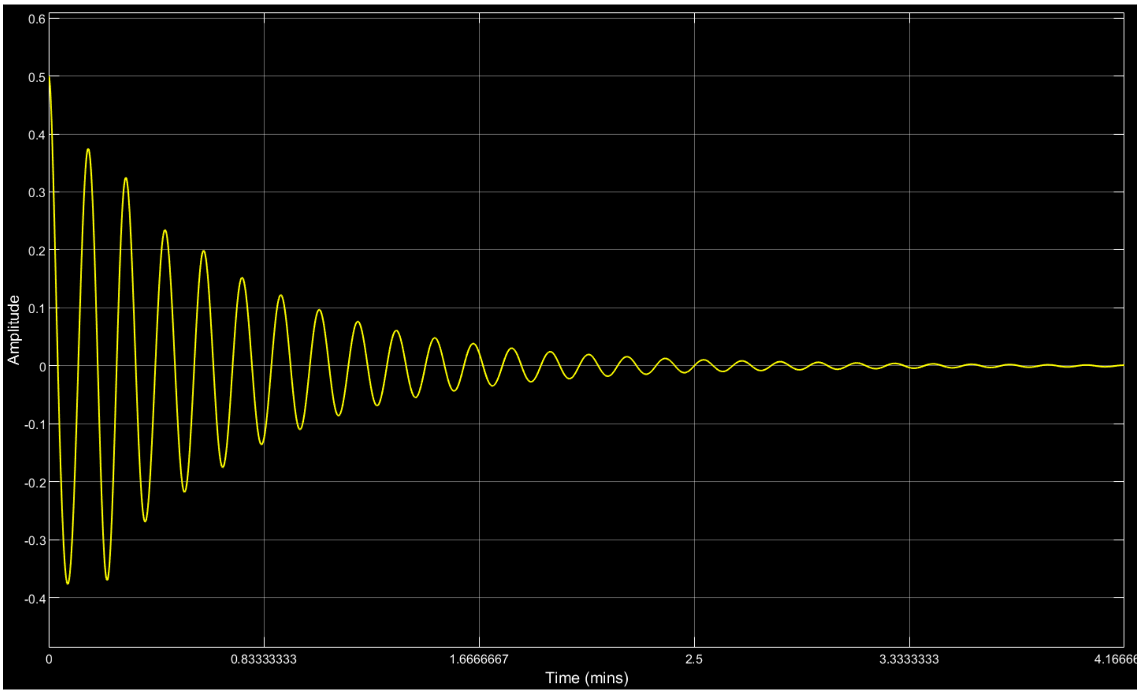

Figure 5.9: Non Linear LQR Output for θ_1 for initial condition : [1,0,0.5,0,0.5,0]

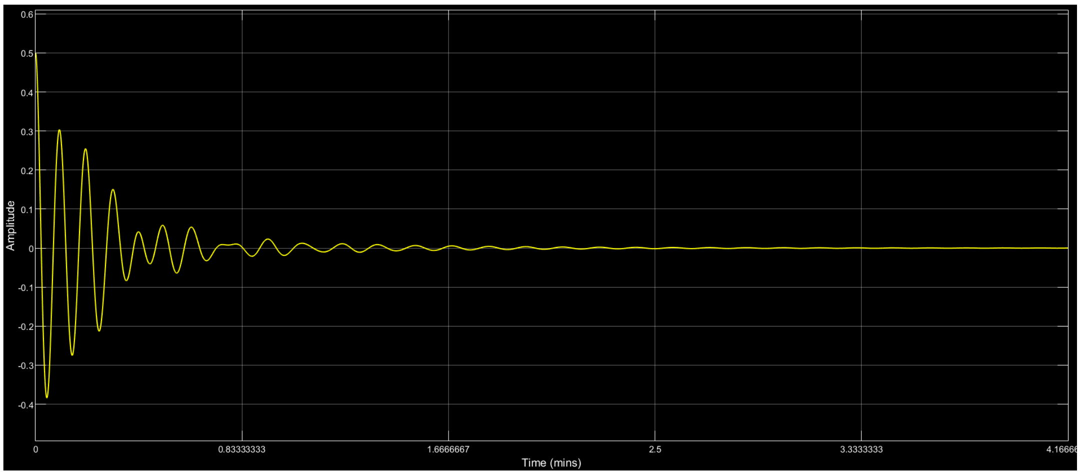

Figure 5.10: Non Linear LQR Output for θ_2 for initial condition : [1,0,0.5,0,0.5,0]

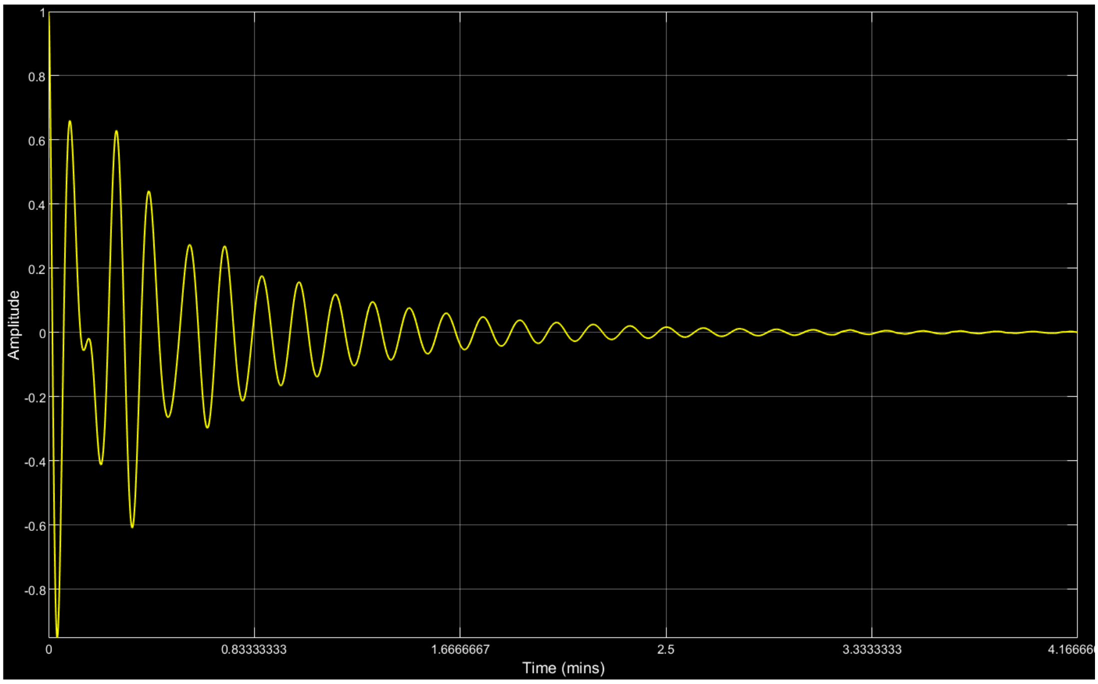

Figure 5.11: Non Linear LQR Output for x for initial condition : [1,0,0.5,0,0.5,0]

6 Part E
Observability Test for Linear system states

In this part we will check the observability of given states combinaion. We will use the observability matrix O given by

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \\ . \\ . \\ . \\ . \\ CA^{n-1} \end{bmatrix} \tag{6.1}$$

the condition used is Rank([O])=n

6.0.1 Case1: x(t)

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6.2}$$

$$O = \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.9800 & 0 & -0.9800 & 0 \\ 0 & 0 & 0 & -0.9800 & 0 & -0.9800 \\ 0 & 0 & 0.6243 & 0 & 1.1045 & 0 \\ 0 & 0 & 0 & 0.6243 & 0 & 1.1045 \end{bmatrix} \tag{6.3}$$

rank(O) =6 Thus, Observable

6.0.2 Case2: $\theta_1$and θ_2

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6.4}$$

$$
O = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1.0000 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.0000 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1.0000 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1.0000 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.5390 & 0 & -0.0490 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.0980 & 0 & -1.0780 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.5390 & 0 & -0.0490 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.0980 & 0 & -1.0780 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.2953 & 0 & 0.0792 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.1585 & 0 & 1.1669 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.2953 & 0 & 0.0792 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.1585 & 0 & 1.1669 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \tag{6.5}
$$

Thus rank(O)=4 Not Observable.

6.0.3 Case3: x(t) and θ_2

$$
C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{6.6}
$$

$$O = \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 \\ 0 & 0 & -0.9800 & 0 & -0.9800 & 0 \\ 0 & 0 & -0.0980 & 0 & -1.0780 & 0 \\ 0 & 0 & 0 & -0.9800 & 0 & -0.9800 \\ 0 & 0 & 0 & -0.0980 & 0 & -1.0780 \\ 0 & 0 & 0.6243 & 0 & 1.1045 & 0 \\ 0 & 0 & 0.1585 & 0 & 1.1669 & 0 \\ 0 & 0 & 0 & 0.6243 & 0 & 1.1045 \\ 0 & 0 & 0 & 0.1585 & 0 & 1.1669 \end{bmatrix} \tag{6.7}$$

rank(O)=6 Thus, Observable.

6.0.4 Case4: x(t), θ_1 and θ_2

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{6.8}$$

$$O = \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 \\ 0 & 0 & -0.9800 & 0 & -0.9800 & 0 \\ 0 & 0 & -0.5390 & 0 & -0.0490 & 0 \\ 0 & 0 & -0.0980 & 0 & -1.0780 & 0 \\ 0 & 0 & 0 & -0.9800 & 0 & -0.9800 \\ 0 & 0 & 0 & -0.5390 & 0 & -0.0490 \\ 0 & 0 & 0 & -0.0980 & 0 & -1.0780 \\ 0 & 0 & 0.6243 & 0 & 1.1045 & 0 \\ 0 & 0 & 0.2953 & 0 & 0.0792 & 0 \\ 0 & 0 & 0.1585 & 0 & 1.1669 & 0 \\ 0 & 0 & 0 & 0.6243 & 0 & 1.1045 \\ 0 & 0 & 0 & 0.2953 & 0 & 0.0792 \\ 0 & 0 & 0 & 0.1585 & 0 & 1.1669 \end{bmatrix} \tag{6.9}$$

rank(O)=6 Thus, Observable.

7 Part F
Luenberger Observer

We have implemented Luenberger observer for all the observable cases for both non-linear and linear system. We followed the following process for desinging of the Luenberger observer:

1. Calculate the gain for the system K and obtain poles by the formula eig(A-BK) in matlab so that we can select poles for luenberger observer accordingly.
2. Initial condition is provided for system and observer, to get the error between the real state value and observer value and check its convergence.
3. We would take observer poles to be greater than system poles and calculate the Luenberger observer gain L.
4. We would reformulate the A, B , C and D matrix of the system by using the luenberger gain L.
5. Run the system and analyses the response for different observable cases.

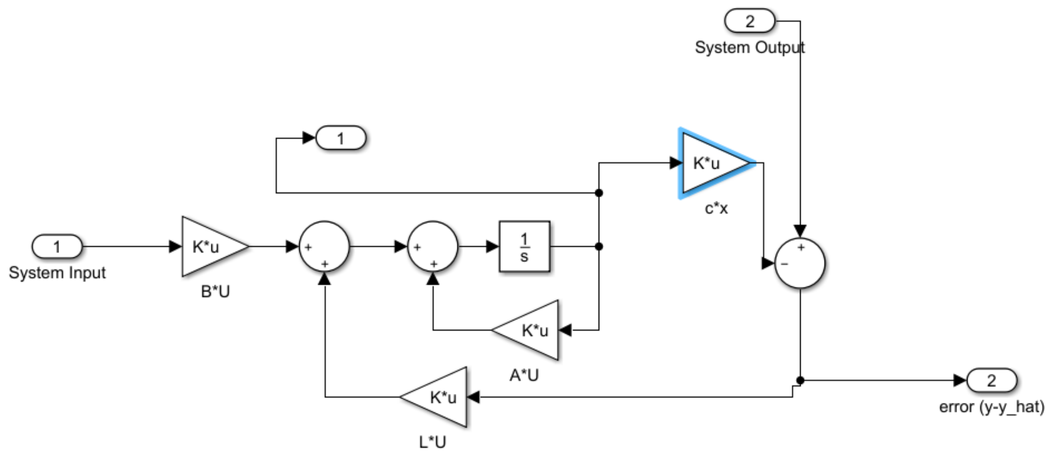

Figure 7.1: Luenberger Internal Block diagram

7.1 Luenberger Observer for Linear System

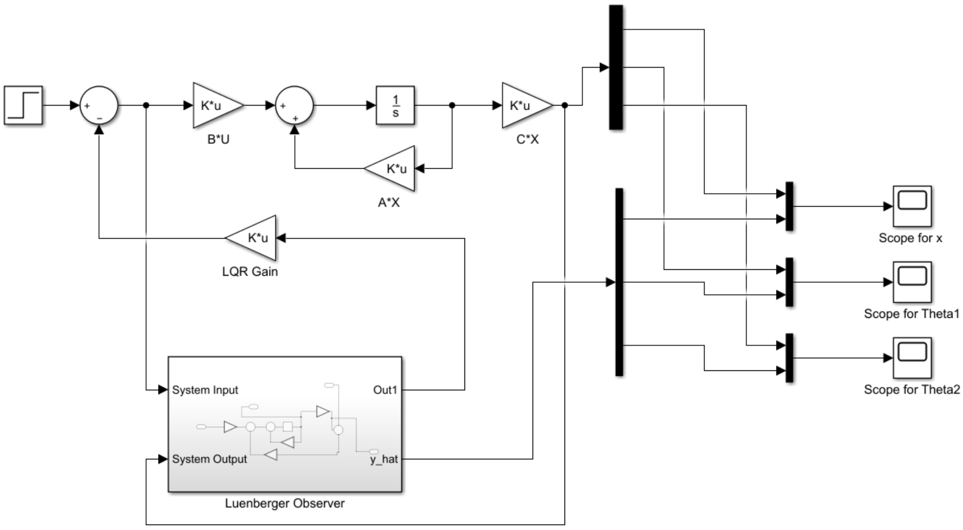

Figure 7.2: Linear System Luenberger Observer

7.1.1 Case 1: Output is x(t)

The plot of error between x(t) and x'(t) (the value of x estimated by the observer) is shown below. It can be seen from the figure that the error value converges to zero.

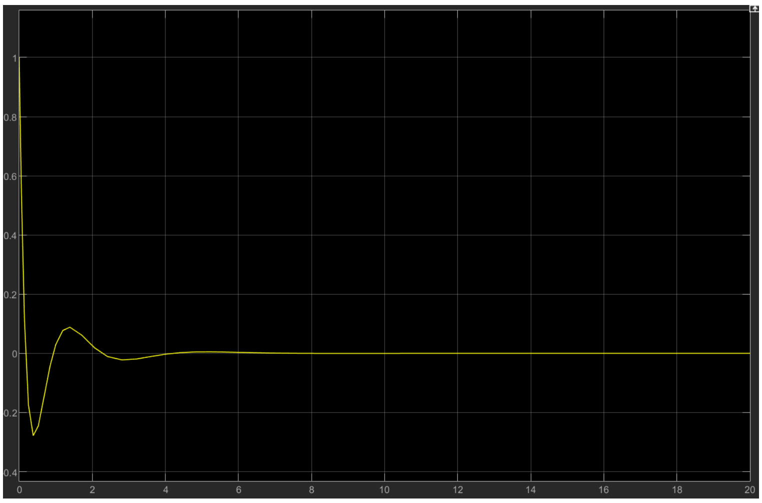

Figure 7.3: Case 1: Luenberger Observer Tracking Error for cart postion

7.1.2 Case 3: Output Vectors are (x(t),θ_2)

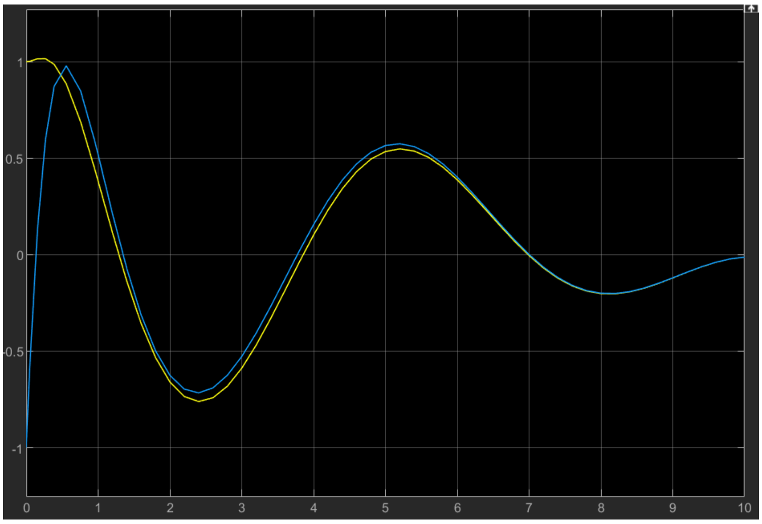

Figure 7.4: Case 3: Observer tracking output for X

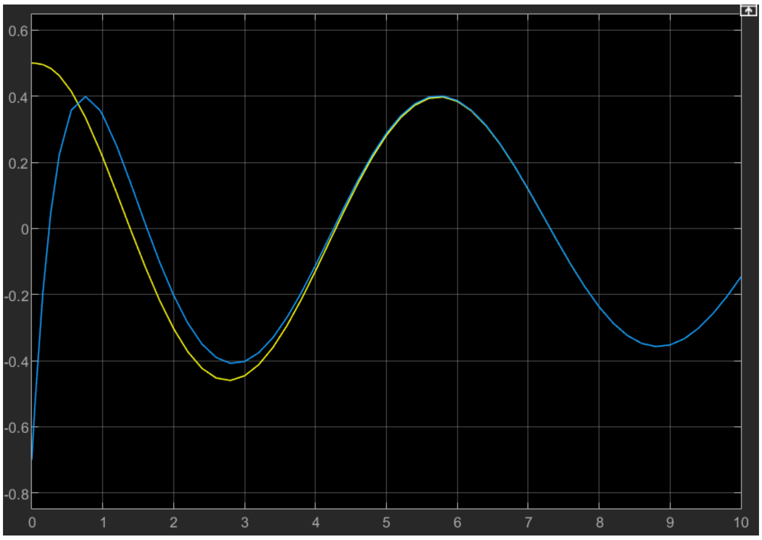

Figure 7.5: Case 3: Observer tracking output for θ_2

7.1.3 Case 4: Output Vectors are (x(t),θ_1(t),θ_2(t))

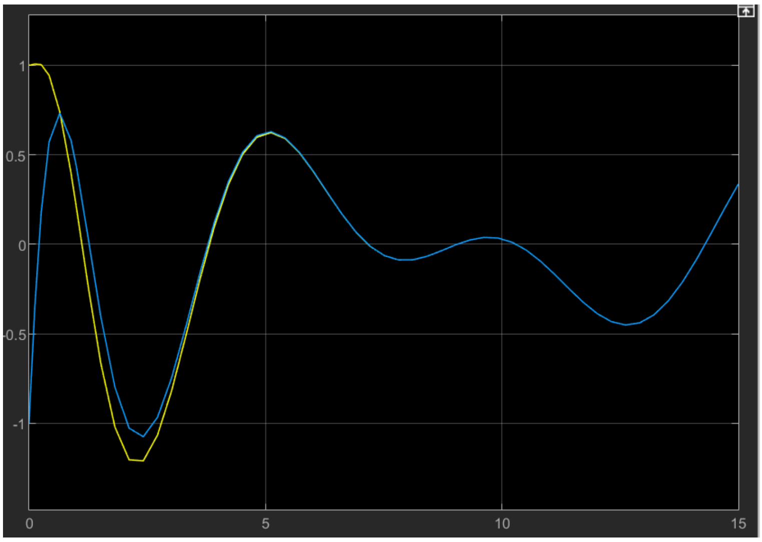

Figure 7.6: Case 4: Observer tracking output for X

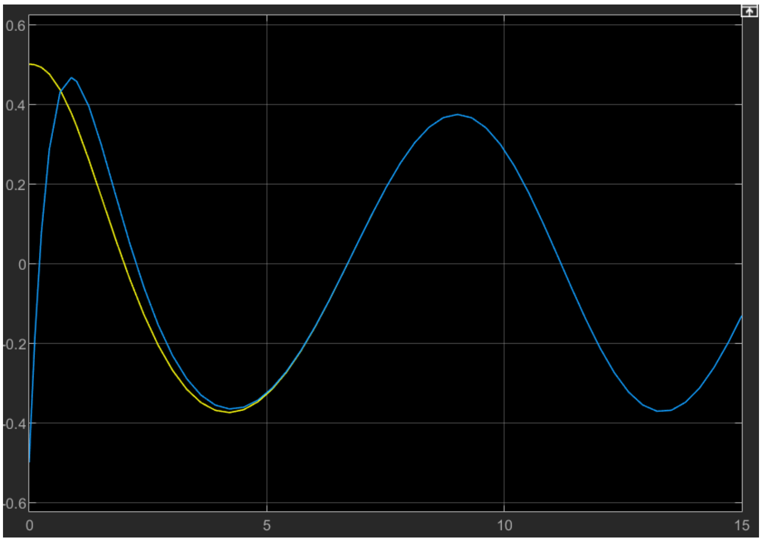

Figure 7.7: Case 4:Observer tracking output for θ_1

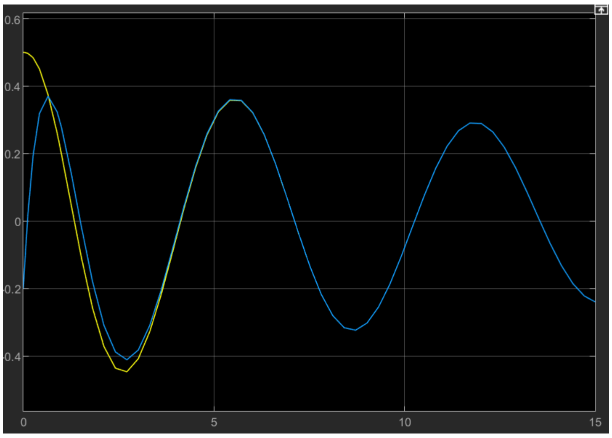

Figure 7.8: Case 4: Observer tracking output for θ_2

7.1.4 Luenberger Observer for Non-Linear System

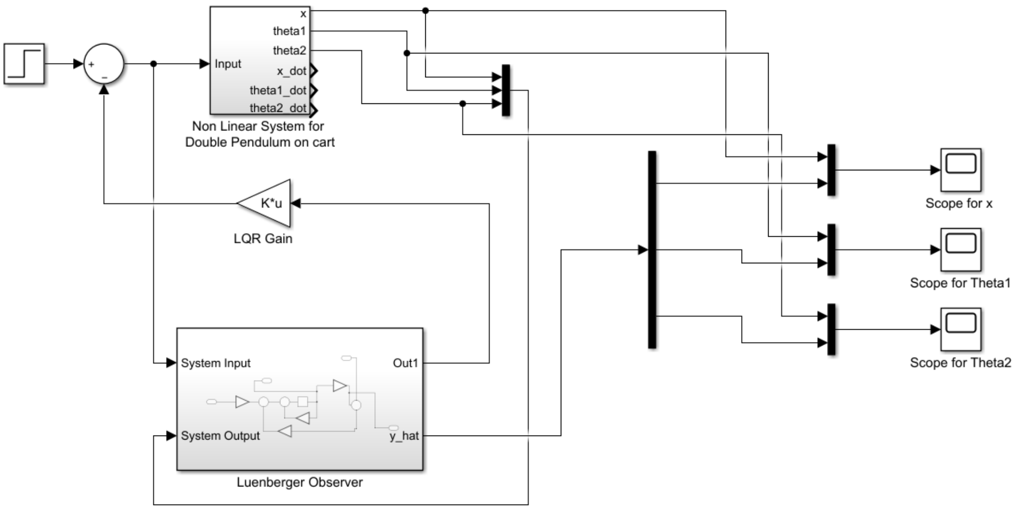

Figure 7.9: Non Linear system Luenberger Observer

7.1.5 Case 1: Output vector x(t)

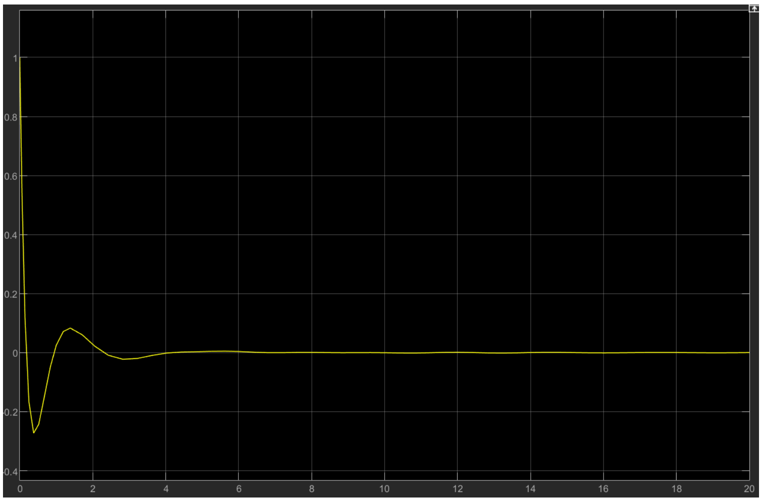

Figure 7.10: Case 1: Luenberger Observer Tracking Error

7.1.6 Case 3: Output vector (x(t),θ_2)

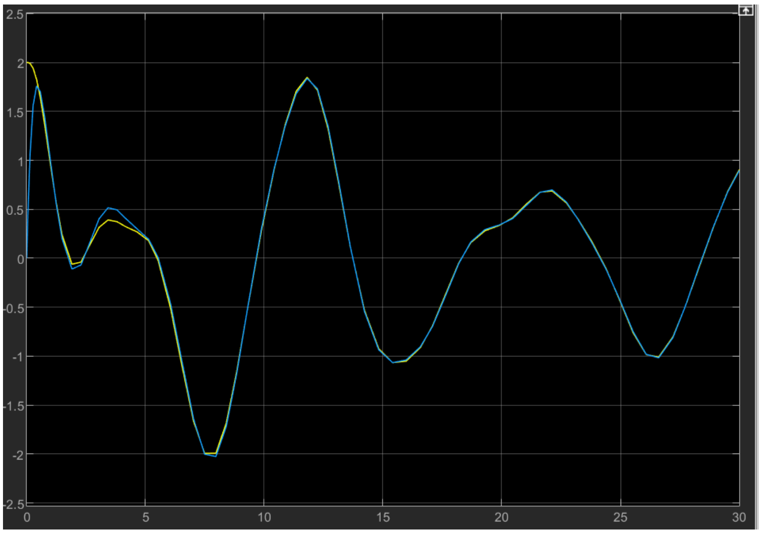

Figure 7.11: Observer tracking error for position of cart

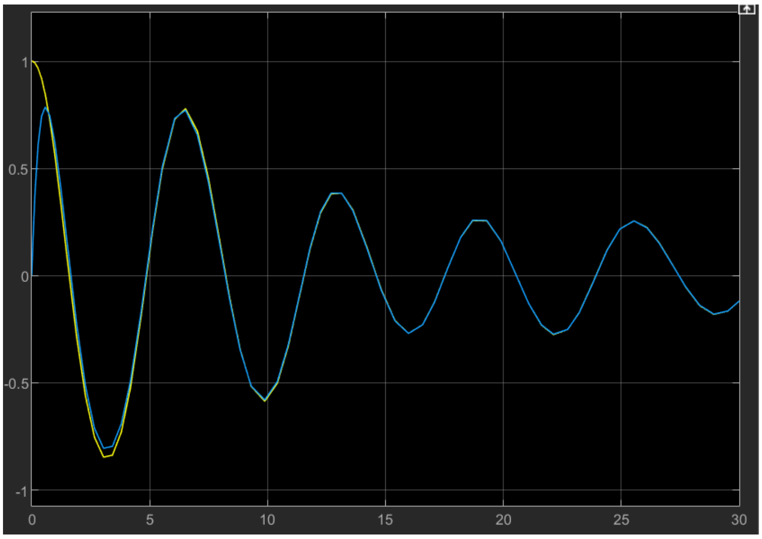

Figure 7.12: Case 2: Observer tracking output for θ_2

7.1.7 Case 4:

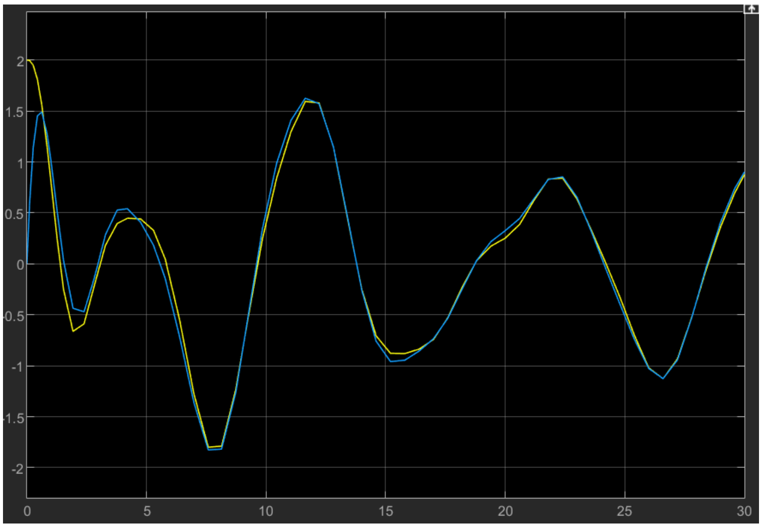

Figure 7.13: Case 4: Observer tracking output for X

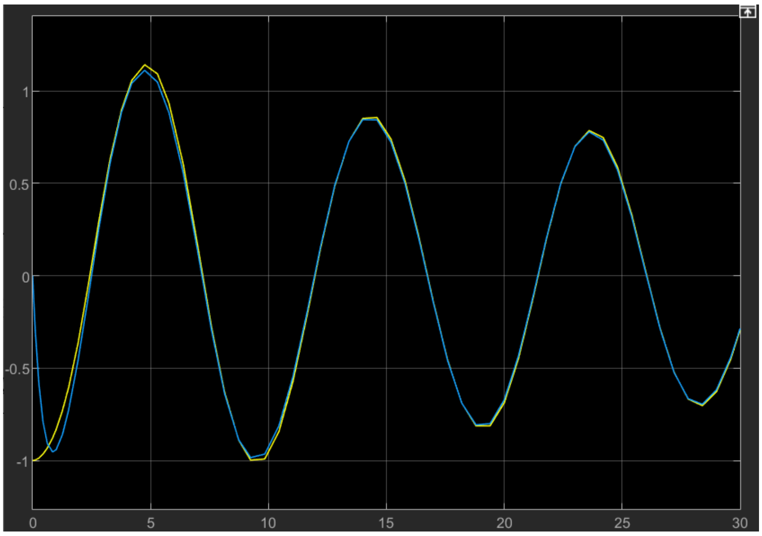

Figure 7.14: Case 4: Observer tracking output for $Theta_1$

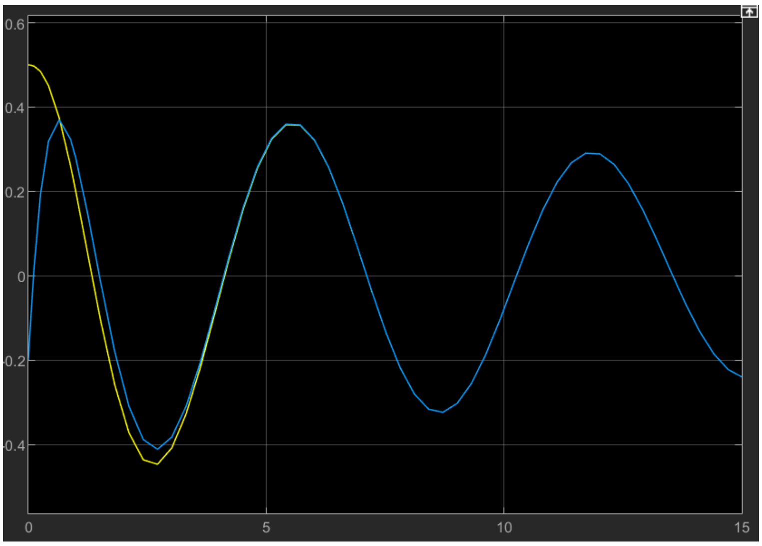

Figure 7.15: Case 4: Observer tracking output for $Theta_2$

8 Part G
LQG for the System

LQG is a combination of LQR and State estimator in or case it is the Kalman Filter. Thus, here kalman filter is used to make prediction for the states of the system considering the gaussian white noise disturbances and measurement noise. The system is redesigned to consider Kalman filter gain (kf) and LQR (K) gain and both are separately designed and combined together to give LQG. The block diagram and output for the new system is shown below:

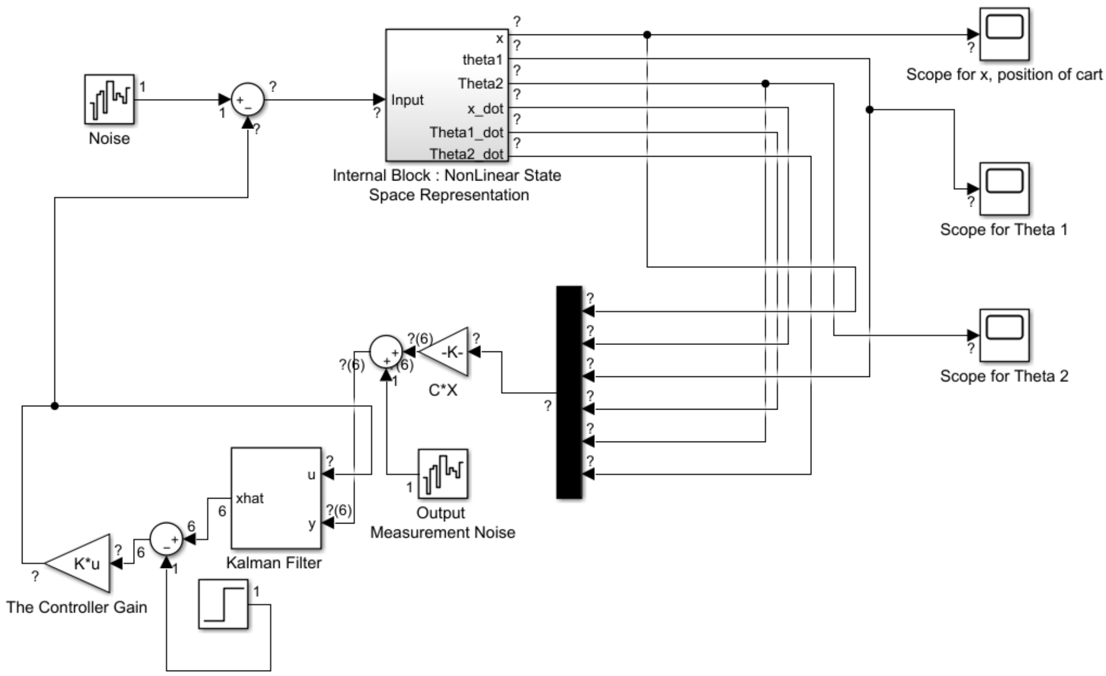

Figure 8.1: Block Diagram for LQG Non linear System in Simulink

This LQG controller will not be able to reject constant forces and external disturbances.The solution is augmenting a new state to consider an integral term and thus modify the LQR to LQRI.

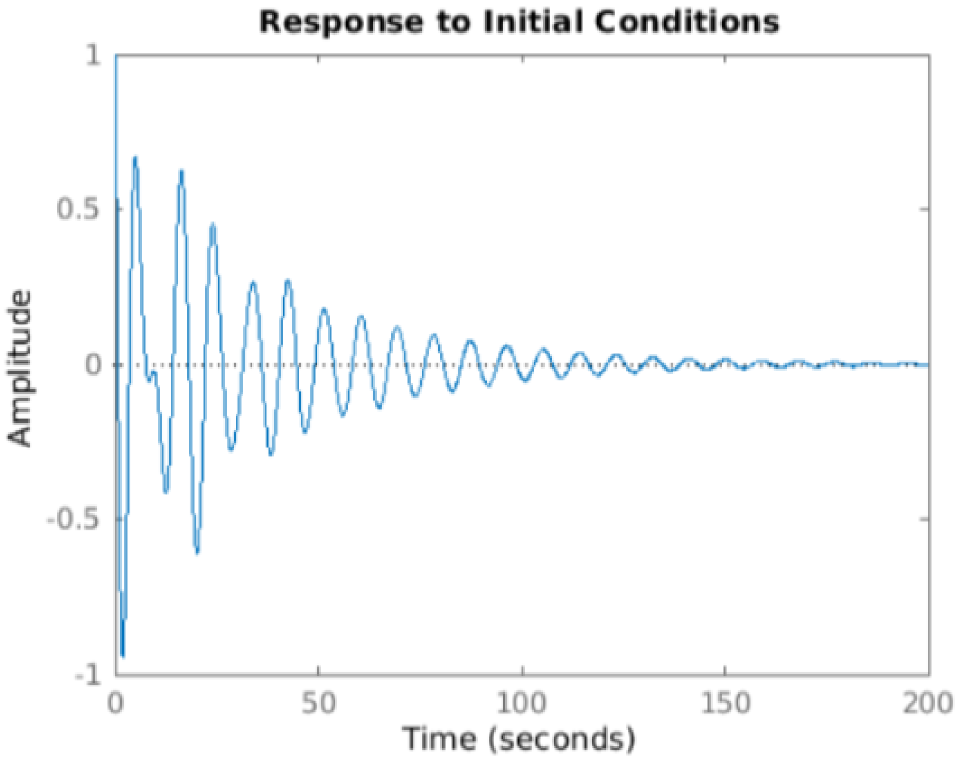

Figure 8.2: Response of LQG for Linear System

9 Conclusion

We can conclude the following by :

1. We were able to form system model using the Eular-Lagrange equations for double pendulum on cart problem.
2. The nonlinear model was linearized at the equilibrium point and condition for controllability were evaluated.
3. checked observanility and controllability for system.
4. Luneberger observer was implemented for both non-linear and linear system and all observable cases were analysed using simulink model.
5. We designed LQR for linear and non linear system modle and LQG for non linear system.

References

[1] Class notes

[2] Control bootcamp-Steve Brunton