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Collaborative Robotic Conveyor System

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Abstract

The main concentration of this project is on understanding and implementation of core concepts of robot modeling, including forward kinematic, inverse kinematics, velocity kinematics and motion planning to avoid collisions for manipulators and robots. For the scope of this project kinematics equations including forward kinematic equations, inverse kinematic equations and velocity kinematics equation would be manually calculated for a 7 degree of freedom robot namely, Baxter. Additionally, the calculated equations would be validated through simulation in V-rep (robot simulation environment). Using the kinematic concepts mentioned we would design a Collaborative Robotics Conveyor System to pick and place object to the desired location avoiding collision with obstacles.

Key words: Forward kinematic, Inverse kinematics, Velocity kinematics, Motion planning, Degree of Freedom(dof), Baxter

Project Goal

1. Understanding and implementation of core concepts of robot modeling, including forward kinematic, inverse kinematics, velocity kinematics and motion planning.
2. Calculate kinematics equations mainly forward kinematic equations, inverse kinematic equations and velocity kinematics equation for Baxter.
3. Learn Lua programming and design simulation environment in V-rep.
4. Validate the kinematic equations using simulation in V-rep.
5. Design a Collaborative Robotics Conveyor System to pick and place object to the desired location avoiding collision with obstacles.

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1 Motivation & Introduction

In 18th and 19th century "Industrial Revolution" introduced new manufacturing technologies which fueled the economic growth, and introduced a smarter way of production. At present 'Industry 4.0' is considered to be the new industrial revolution which focuses on automation, integration and optimization of manufacturing process. Thus, resulting in reduced production cycle, improved production quality, reducing human interaction and increasing adaptability to different production processes. It is observed that autonomous robots are an integral part of 'Industry 4.0', as they can complete the tasks intelligently, and in an orchestrated manner with minimal human input. So, in-depth understanding of robot model and work space is inevitable for efficient production process.[1]

Moreover, a survey by RIA illustrated that articulated robots are the most popular industrial robots; and account to 35% usage by industry, followed by Cartesian robots at 19% in the robot use-age. A keen look into robot application in industry depicts that palletizing & depalletizing leads as the top application, at 51%; followed closely by pick-and-place, at 48.2%; case packing, at 40.8%; material handling, at 32.2%; and assembly of mixed load pallets, at 16.3%. [2]



(a) Robot Application in Industry

(b) Robot Type

Figure 1.1: Current Industrial Trend

Thus if we consider all the above suggested data we can conclude that pick and place task with articulated robot is an important task for the present day industry. Hence, the main motivation of the project is to design a collaborative robotic conveyor system using Baxter robot in V-rep for packaging industry.

The report is organized in 7 sections. Section 2 provides all details about the Baxter robot including robot structure, specifications and appropriateness for the task. Followed by main assumption made while designing the system in section 3. However, section 4 deals with manual calculation for forward kinematics, inverse kinematics, and velocity kinematics. Section 5 covers path planning and obstacle avoidance. In section 6 validation for the kinematics equation is discussed briefly with appropriate examples and diagrams. Finally, future work, conclusion and goal accomplishments is provided in section 7.

2 All about Baxter

Baxter is world's first two-arm collaborative robot developed in 2012, it revolutionized the manufacturing world and literally became the face of the collaborative robotics category. Today, leading companies of all sizes, industries and specialties rely on Baxter to handle their repetitive production tasks, gaining a significant competitive advantage for their business. Thus, it is important to have a good understanding of Baxter.

This section provides all the necessary specifications and structural details which would help in efficient use of Baxter.

2.1 Baxter Specification

Specification	Details
Weight	165 lbs. without pedestal 306 lbs. with optional pedestal
Degrees of Freedom	7 dof
Maximum Reach	1210 mm per arm
Payload	5 lb. (2.2 kg) per arm
Applications	Packaging, kitting, line loading, material handling, and more.
Embedded Vision	Camera in each arm (1280 x 800 pixels)
Safety by Inherent Design	Power and force limited compliant arm with series elastic actuators and embedded sensors
Embedded Force Sensing and other sensors	Force sensors embedded at each joint (standard), plus sonar, accelerometers and range-finding sensors
IP Classification	IP50 rating
Power Requirement	Standard power outlet (120V, 6 amps)
Operating Software and Onboard computer	Intera, intel core i7 3.4GHz processor, 4GB memory

Table 2.1: Baxter Specification

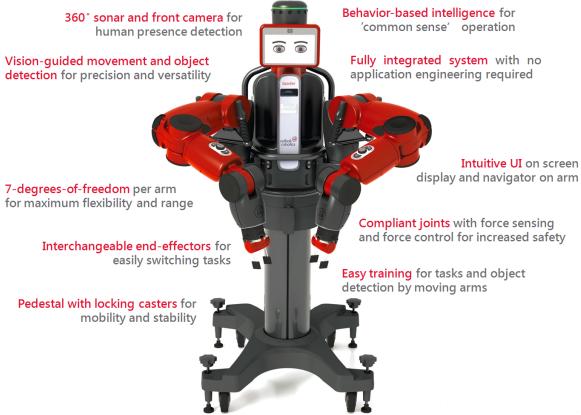


Figure 2.1: Baxter features[10]

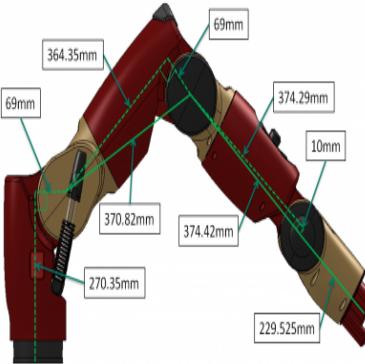
2.2 Structural Description

Specification	Details
Height	3' tall (around 6' tall with stationary pedestal)
Wingspan	103"
Pedestal base	32" x 36"
Head (dof)	2-dof
Shoulder joint	2-dof (offset-U-joint)
Elbow joint	2-dof (offset-U-joint)
Wrist joint	3-dof (offset-S-joint)
L0	270.35 mm
L1	69.00 mm
L2	364.35 mm
L3	69.00 mm
L4	374.29 mm
L5	10.00 mm
L6	368.30 mm

Table 2.2: Structural Description

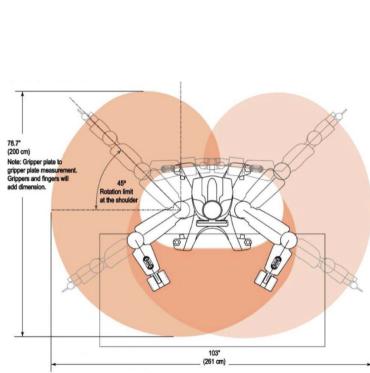


(a) Baxter Joints[8]

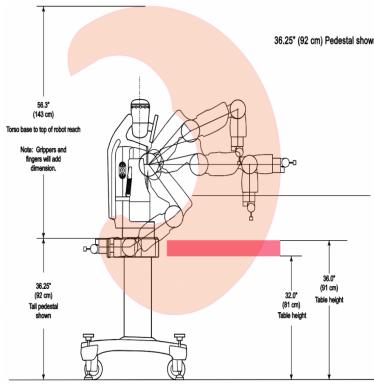


(b) Baxter link and offset length[9]

Figure 2.2: Baxter structure



(a) Top view



(b) Side view

Figure 2.3: Baxter Workspace [7]

2.3 Appropriateness for the task

In this sub-section we will briefly discuss how Baxter has a upper hand over other robot for the task under consideration. There are several reasons for selecting Baxter Robot for the task few are illustrated below:

1. There are several industrial robots which are capable for doing the pick and place or packaging task in the industry, but they are costly. Whereas, Baxter on the other side is about 4 times less expensive (due to its plastic body) which makes it affordable even for small to medium businesses."Rethink estimates that the cost of operation works out to about \$4.00 an hour".
2. Baxter can be easily installed in human-occupied environment. Other Industrial robots are difficult to incorporate in the working environment and requires specially designed environment(caged) to protect workers from fast, rigid and powerful movements of the robot.Thus, Baxter has in build safety feature.

3. Baxter can be reprogrammed to perform different tasks even by a non technical worker. Thus, it is highly flexible.
4. It can adapt to the surrounding quickly. For example, it knows that if it drops an object, it needs to stop and retrieve another before continuing its motion. Moreover, it can adjust according to the conveyor belt speeds variations. [4]

3 Project Assumptions

To design a collaborative robotics conveyor system we made the following assumptions:

1. All the joints and objects are considered to be rigid.
2. External disturbances are not taken into account while modeling.
3. There is no (inverse kinematic)analytical solution for general 7 dof Baxter arm with nonzero offset. Thus, we lock the joint angle θ_3 to 0 and assumed link lengths L_5 to be 0 to obtain a solution.
4. The path generated using path planner is just one solution among the infinite possible scenarios and may not be the optimal solution.
5. Robot self collision is not considered i.e. collision with obstacle is only taken under consideration for the scope of this project.
6. The Path Planning module is not available in the V-rep version and thus, path planning is done using forward kinematics by selection of appropriate way points.

4 Baxter Kinematics

4.1 Forward Kinematics

In this section we will discuss about the Modified Denavit–Hartenberg convention and forward kinematic for Baxter robot.

4.1.1 Modified Denavit–Hartenberg convention

Modified Denavit–Hartenberg convention is a commonly used convention for selecting frames of reference in robotics applications which was introduced by Craig, in 2005. In this convention, coordinate frames are attached to the joints between two links such that one transformation is associated with the joint X axes, and the second is associated with the link Z axes.[7]

The modified DH parameters are a set of four transformation parameters which form a relationship between two adjacent frames defined by modified DH convention. Four parameters defined for modified DH are:

1. α_{i-1} : Rotation about x_{i-1} ,axis.
2. a_{i-1} : Offset along previous x to the common normal i.e. x_{i-1} ,axis.
3. θ_i : Rotation about the new z axis i.e. z_i
4. d_i : Length of the offset along new z to the common normal i.e z_i .

4.1.2 Forward Kinematics For Baxter using Modified Denavit–Hartenberg convention

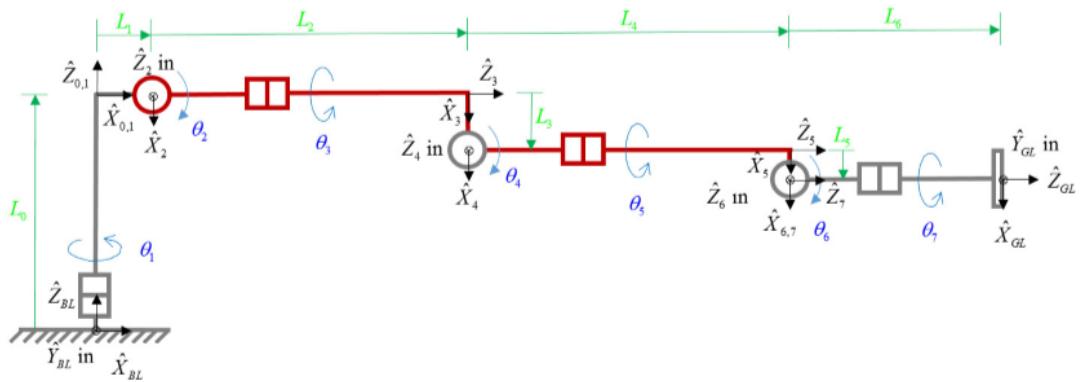


Figure 4.1: Forward Kinematics Frame Diagram [7]

Frames	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90	L_1	0	$\theta_2 + 90$
3	90	0	L_2	θ_3
4	-90	L_3	0	θ_4
5	90	0	L_4	θ_5
6	-90	L_5	0	θ_6
7	90	0	0	θ_7

Table 4.1: Modified DH Table for Forward Kinematics

The transformation matrix are formulate by using:

$$[{}^{i-1}T] = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i) & 0 & a_{i-1} \\ \sin(\theta_i)\cos(\alpha_{i-1}) & \cos(\theta_i)\cos(\alpha_{i-1}) & -\sin(\alpha_{i-1}) & -d_i \sin(\alpha_{i-1}) \\ \sin(\theta_i)\sin(\alpha_{i-1}) & \cos(\theta_i)\sin(\alpha_{i-1}) & \cos(\alpha_{i-1}) & d_i \cos(\alpha_{i-1}) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.1)$$

$$T_1 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_2 = \begin{bmatrix} -\sin(\theta_2) & -\cos(\theta_2) & 0 & L_1 \\ 0 & 0 & 1 & 0 \\ -\cos(\theta_2) & \sin(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.2)$$

$$T_3 = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 0 \\ 0 & 0 & -1 & -L_2 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_4 = \begin{bmatrix} \cos(\theta_4) & -\sin(\theta_4) & 0 & L_3 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_4) & -\cos(\theta_4) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.3)$$

$$T_5 = \begin{bmatrix} \cos(\theta_5) & -\sin(\theta_5) & 0 & 0 \\ 0 & 0 & -1 & -L_4 \\ \sin(\theta_5) & \cos(\theta_5) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_6 = \begin{bmatrix} \cos(\theta_6) & -\sin(\theta_6) & 0 & L_5 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_6) & -\cos(\theta_6) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.4)$$

$$T_7 = \begin{bmatrix} \cos(\theta_7) & -\sin(\theta_7) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin(\theta_7) & \cos(\theta_7) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_{endeff} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.5)$$

fixed frame needed to get end-effector position in world co-ordinate frame.
where L=278mm, H=1104mm, and h=64mm

$$T_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_{base} = \begin{bmatrix} 0.707 & 0.707 & 0 & L \\ -0.707 & 0.707 & 0 & -h \\ 0 & 0 & 1 & H \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.6)$$

Length	Value (mm)
L	278
h	64
H	1104

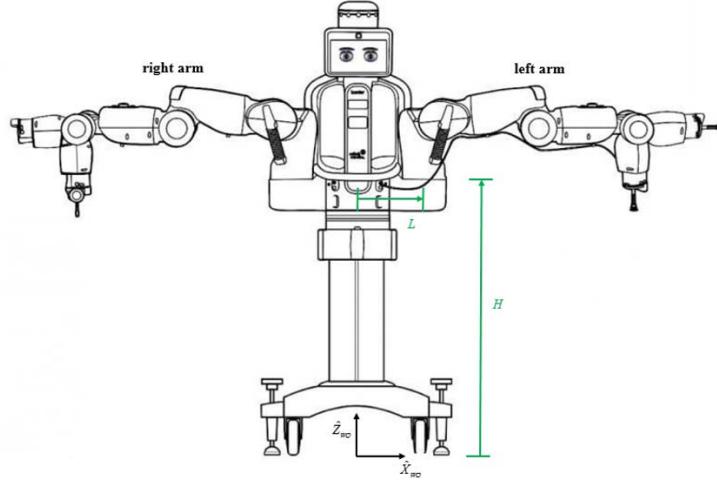


Figure 4.2: Fixed frames [7]

Thus the end effector position can be given by:

$$\begin{aligned}
 T_{final} &= T_{base} * T_0 * T_1 * T_2 * T_3 * T_4 * T_5 * T_6 * T_7 * T_{endeff} \\
 X_{final} &= l6 * (\sin(t6) * (2^{1/2}) * \sin(t5) * ((707 * \cos(t3) * \cos(t1 + pi/4)) / 1000 + (707 * \sin(t2) * \sin(t3) * \sin(t1 + pi/4)) / 1000 - 2^{1/2} * \cos(t5) * ((707 * \cos(t2) * \sin(t4) * \sin(t1 + pi/4)) / 1000 - (707 * \cos(t4) * \cos(t1 + pi/4) * \sin(t3)) / 1000 + (707 * \cos(t3) * \cos(t4) * \sin(t2) * \sin(t1 + pi/4)) / 1000) + 2^{1/2} * \cos(t6) * ((707 * \cos(t2) * \cos(t4) * \sin(t1 + pi/4)) / 1000 + (707 * \cos(t1 + pi/4) * \sin(t3) * \sin(t4)) / 1000 - (707 * \cos(t3) * \sin(t2) * \sin(t4) * \sin(t1 + pi/4)) / 1000) + l5 * (2^{1/2}) * \sin(t5) * ((707 * \cos(t3) * \cos(t1 + pi/4)) / 1000 + (707 * \sin(t2) * \sin(t3) * \sin(t1 + pi/4)) / 1000 - 2^{1/2} * \cos(t5) * ((707 * \cos(t2) * \sin(t4) * \sin(t1 + pi/4)) / 1000 - (707 * \cos(t4) * \cos(t1 + pi/4) * \sin(t3)) / 1000 + (707 * \cos(t3) * \cos(t4) * \sin(t2) * \sin(t1 + pi/4)) / 1000) + (707 * 2^{1/2}) * l1 * \sin(t1 + pi/4)) / 1000 + 2^{1/2} * l4 * ((707 * \cos(t2) * \cos(t4) * \sin(t1 + pi/4)) / 1000 + (707 * \cos(t1 + pi/4) * \sin(t3) * \sin(t4)) / 1000 - (707 * \cos(t3) * \sin(t2) * \sin(t4) * \sin(t1 + pi/4)) / 1000) + (707 * 2^{1/2}) * l3 * ((707 * \cos(t1 + pi/4) * \sin(t3)) / 1000 - (707 * \cos(t3) * \sin(t2) * \sin(t1 + pi/4)) / 1000) + (707 * 2^{1/2}) * l2 * \cos(t2) * \sin(t1 + pi/4)) / 1000 + 139 / 500 \\
 Y_{final} &= l6 * (\sin(t6) * (2^{1/2}) * \sin(t5) * ((707 * \cos(t3) * \sin(t1 + pi/4)) / 1000 - (707 * \cos(t1 + pi/4) * \sin(t2) * \sin(t3)) / 1000) + 2^{1/2} * \cos(t5) * ((707 * \cos(t2) * \cos(t1 + pi/4) * \sin(t4)) / 1000 + (707 * \cos(t4) * \sin(t3) * \sin(t1 + pi/4)) / 1000 + (707 * \cos(t3) * \cos(t4) * \cos(t1 + pi/4) * \sin(t2)) / 1000) + 2^{1/2} * \cos(t6) * ((707 * \sin(t3) * \sin(t4) * \sin(t1 + pi/4)) / 1000 - (707 * \cos(t2) * \cos(t4) * \cos(t1 + pi/4)) / 1000 + (707 * \cos(t3) * \cos(t1 + pi/4) * \sin(t2) * \sin(t4)) / 1000) + l5 * (2^{1/2}) * \sin(t5) * ((707 * \cos(t3) * \sin(t1 + pi/4)) / 1000 - (707 * \cos(t1 + pi/4) * \sin(t2) * \sin(t3)) / 1000) + 2^{1/2} * \cos(t5) * ((707 * \cos(t2) * \cos(t1 + pi/4) * \sin(t4)) / 1000 + (707 * \cos(t4) * \sin(t3) * \sin(t1 + pi/4)) / 1000 + (707 * \cos(t3) * \cos(t4) * \cos(t1 + pi/4) * \sin(t2)) / 1000) - (707 * 2^{1/2}) * l1 * \cos(t1 + pi/4)) / 1000 + 2^{1/2} * l4 * ((707 * \sin(t3) * \sin(t4) * \sin(t1 + pi/4)) / 1000 - (707 * \cos(t2) * \cos(t4) * \cos(t1 + pi/4) * \sin(t2)) / 1000) - (707 * \cos(t1 + pi/4) * \sin(t3) * \sin(t4)) / 1000
 \end{aligned}$$

$$\begin{aligned}
& \sin(t2) * \sin(t4)) / 1000 + 2^{(1/2)} * l3 * ((707 * \sin(t3) * \sin(t1 + pi/4)) / 1000 + (707 * \cos(t3) * \\
& \cos(t1 + pi/4) * \sin(t2)) / 1000 - (707 * 2^{(1/2)} * l2 * \cos(t2) * \cos(t1 + pi/4)) / 1000 - 8 / 125 \\
Z_{final} = & l0 + l6 * (\sin(t6) * (\cos(t5) * (\sin(t2) * \sin(t4) - \cos(t2) * \cos(t3) * \cos(t4)) + \cos(t2) * \\
& \sin(t3) * \sin(t5)) - \cos(t6) * (\cos(t4) * \sin(t2) + \cos(t2) * \cos(t3) * \sin(t4))) - l4 * (\cos(t4) * \\
& \sin(t2) + \cos(t2) * \cos(t3) * \sin(t4)) - l2 * \sin(t2) + l5 * (\cos(t5) * (\sin(t2) * \sin(t4) - \cos(t2) * \\
& \cos(t3) * \cos(t4)) + \cos(t2) * \sin(t3) * \sin(t5)) - l3 * \cos
\end{aligned}$$

4.2 Inverse Kinematics For Baxter using Modified Denavit–Hartenberg convention

This section covers inverse kinematic for Baxter. Now we will consider that we know the final end-effector position, but are unaware of the joint variables. Thus, we need to calculate the joint variable that could take us to the desired orientation. It is interesting to note that there is no analytical solution for Baxter. Therefore, to obtain an analytical solution we will consider that θ_3 is locked and the length of link L_5 is negligible almost 0. Moreover, we will fabricate a new link called L_h by combining L_2 and L_3 given as :

$$L_h = \sqrt{L_2^2 + L_3^2} \quad (4.7)$$

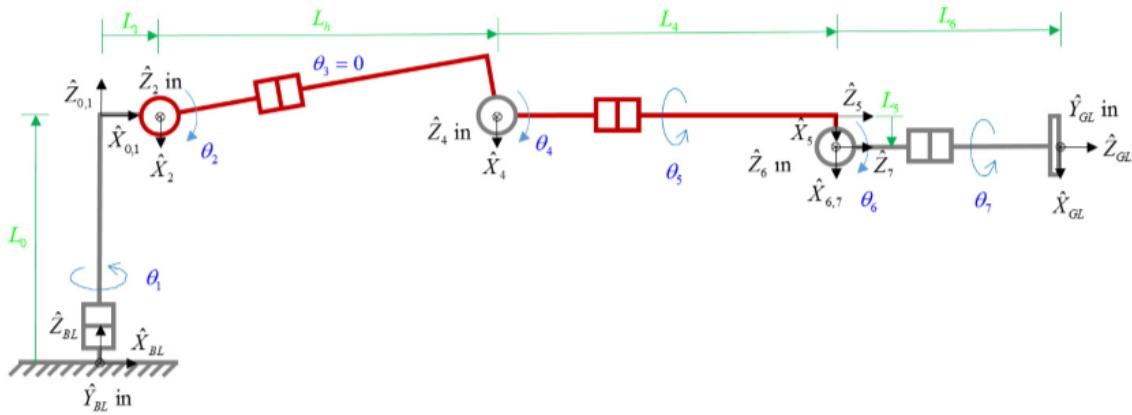


Figure 4.3: Inverse Kinematics Frame Diagram [7]

Frames	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90	L_1	0	θ_2
3	0	L_h	0	$\theta_4 + 90$
4	90	0	L_4	θ_5
5	-90	L_5	0	θ_6
6	90	0	0	θ_7

Table 4.2: Modified DH Table for Inverse Kinematics

$$T_1 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_2 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & L_1 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_2) & -\cos(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.8)$$

$$T_4 = \begin{bmatrix} -\sin(\theta_4) & -\cos(\theta_4) & 0 & L_h \\ \cos(\theta_4) & -\sin(\theta_4) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_5 = \begin{bmatrix} \cos(\theta_5) & -\sin(\theta_5) & 0 & 0 \\ 0 & 0 & -1 & -L_4 \\ \sin(\theta_5) & \cos(\theta_5) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.9)$$

$$T_6 = \begin{bmatrix} \cos(\theta_6) & -\sin(\theta_6) & 0 & L_5 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_6) & -\cos(\theta_6) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_7 = \begin{bmatrix} \cos(\theta_7) & -\sin(\theta_7) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin(\theta_7) & \cos(\theta_7) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.10)$$

We can group the transformation into two parts :

1. Elbow Joint
2. Wrist Joint

Transformation for Elbow Joint is T_{elbow} :

$$T_{elbow} = T_1 * T_2 * T_4$$

$$T_{elbow} = \begin{bmatrix} -\cos(\theta_1)\sin(\theta_2 + \theta_4) & -\cos(\theta_1)\cos(\theta_2 + \theta_4) & -\sin(\theta_1) & \cos(\theta_1)(L_1 + L_h\cos(\theta_2)) \\ -\sin(\theta_1)\sin(\theta_2 + \theta_4) & -\sin(\theta_1)\cos(\theta_2 + \theta_4) & \cos(\theta_1) & \sin(\theta_1)(L_1 + L_h\cos(\theta_2)) \\ -\cos(\theta_2 + \theta_4) & \sin(\theta_2 + \theta_4) & 0 & -L_h\sin(\theta_2) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.11)$$

Transformation for Wrist Joint is T_{wrist} :

$$T_{wrist} = T_5 * T_6 * T_7$$

$$T_{wrist} = \begin{bmatrix} -\sin(\theta_5)\sin(\theta_7) + \cos(\theta_5)\cos(\theta_6)\cos(\theta_7) & -\sin(\theta_5)\cos(\theta_7) - \cos(\theta_5)\cos(\theta_6)\sin(\theta_7) & \cos(\theta_5)\sin(\theta_6) & L_5\cos(\theta_5) \\ \cos(\theta_7)\sin(\theta_6) & -\sin(\theta_6)\sin(\theta_7) & -\cos(\theta_6) & -L_4 \\ \cos(\theta_5)\sin(\theta_7) + \sin(\theta_5)\cos(\theta_6)\cos(\theta_7) & \cos(\theta_5)\cos(\theta_7) - \sin(\theta_5)\cos(\theta_6)\sin(\theta_7) & \sin(\theta_5)\sin(\theta_6) & L_5\sin(\theta_5) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.12)$$

Thus, we will use T_{elbow} and T_{wrist} to form the final transformation matrix that will be used to find the angles. To find θ_1 , θ_2 and θ_4 we will use the translation component of the transformation matrix. To evaluate θ_5 , θ_6 and θ_7 we will use the rotational component of the wrist matrix.

Joint angle θ_1 is found from a ratio of the y to x :

$$\theta_1 = \frac{\sin(\theta_1)(L_1 + L_h\cos(\theta_2) + L_4\cos(\theta_2 + \theta_4))}{\cos(\theta_1)(L_1 + L_h\cos(\theta_2) + L_4\cos(\theta_2 + \theta_4))} \quad (4.13)$$

thus,

$$\theta_1 = \text{atan2}(y, x) \quad (4.14)$$

solving for θ_2 :

we know that:

$$L_4\cos(\theta_2 + \theta_4) = \frac{x}{\cos(\theta_1)} - L_1 - L_h\cos(\theta_2) \quad (4.15)$$

$$L_4\sin(\theta_2 + \theta_4) = -z - L_h\sin(\theta_2) \quad (4.16)$$

Thus, using $\cos(\theta_2 + \theta_4)^2 + \sin(\theta_2 + \theta_4)^2 = 1$ and above stated equations we can write

$$2L_h(L_1 - \frac{x}{\cos(\theta_1)})\cos(\theta_2) + 2L_h z \sin(\theta_2) + \frac{x^2}{\cos(\theta_1)^2} + L_1^2 + L_h^2 - L_4^2 + z^2 - 2\frac{L_1 x}{\cos(\theta_1)} = 0 \quad (4.17)$$

Now by the method of substitution we solve equation(4.16).

where $t = \tan(\theta_2/2)$, $\cos(\theta_2) = \frac{1-t^2}{1+t^2}$ and $\sin(\theta_2) = \frac{2t}{1+t^2}$
and solving the equation (4.15) we get:

$$\theta_2 = 2\text{atan2}(t) \quad (4.18)$$

thus,

$$\theta_4 = \text{atan2}(-z - L_h \sin(\theta_2), \frac{x}{\cos(\theta_1)} - L_1 - L_h \cos(\theta_2)) - \theta_2 \quad (4.19)$$

Now considering the rotational part of wrist matrix:

$$R = \begin{bmatrix} -\sin(\theta_5)\sin(\theta_7) + \cos(\theta_5)\cos(\theta_6)\cos(\theta_7) & -\sin(\theta_5)\cos(\theta_7) - \cos(\theta_5)\cos(\theta_6)\sin(\theta_7) & \cos(\theta_5)\sin(\theta_6) \\ \sin(\theta_6)\cos(\theta_7) & -\sin(\theta_6)\sin(\theta_7) & -\cos(\theta_6) \\ \cos(\theta_5)\sin(\theta_7) + \sin(\theta_5)\cos(\theta_6)\cos(\theta_7) & \cos(\theta_5)\cos(\theta_7) - \sin(\theta_5)\cos(\theta_6)\sin(\theta_7) & \sin(\theta_5)\sin(\theta_6) \end{bmatrix} \quad (4.20)$$

So,

$$\theta_5 = \text{atan2}(R_{33}, R_{13}) \quad (4.21)$$

$$\theta_7 = \text{atan2}(-R_{22}, R_{21}) \quad (4.22)$$

$$\theta_6 = \text{atan2}(R_{21}/(\cos(\theta_7)), -R_{23}) \quad (4.23)$$

4.3 Velocity Kinematics & Jacobian

Jacobian is a multi-dimensional form of the derivative :

$$[J] = \left[\frac{\partial f_i}{\partial \theta_j} \right] \quad (4.24)$$

where f_i are the pose functions and θ_j represents the 7 joints angles . The i^{th} Jacobian matrix column is the end-effector translational and rotational velocity due to joint i, with the joint rate factored out. Then by linear superposition, the overall end-effector Cartesian velocity is the sum of all n columns (each multiplied by the respective joint rate). Each Jacobian matrix column i is the absolute Cartesian velocity vector of the last active joint frame n with respect to the base frame, due to joint i only, and with the variable joint rate $\dot{\theta}_i$ factored out.[7]

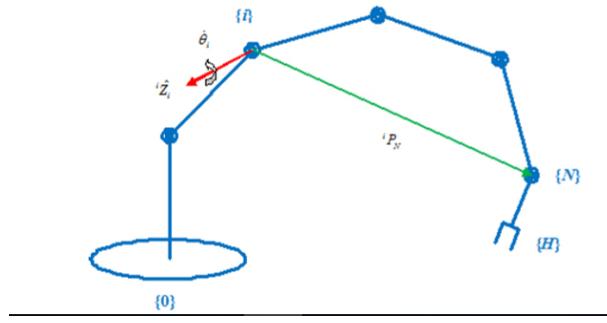


Figure 4.4: Jacobian Frame Diagram[7]

Jacobian is denoted by J and is evaluated by the below mentioned equation:
Here is the jacobian column i, for a revolute joint :

$$\{J\}_i = \begin{Bmatrix} \{{}^k \hat{Z}_i\} \times {}^k \{{}^i \hat{P}_N\} \\ \{{}^k \hat{Z}_i\} \end{Bmatrix} = \begin{Bmatrix} [{}^k R] \{{}^i \hat{Z} \times {}^i P_N\} \\ [{}^k R] \{{}^i \hat{Z}\} \end{Bmatrix} \quad (4.25)$$

where :

$$\{{}^k \hat{Z}_i\} = [{}^k R] \{{}^i \hat{Z}_i\} \quad (4.26)$$

is the 3rd column of orthonormal matrix

$$[{}^k R]$$

and :

$${}^k \{{}^i \hat{P}_N\} = [{}^k R]^i \{{}^i P_N\} \quad (4.27)$$

$$[{}^k J] = {}^k \left[\begin{array}{cccc} \left\{ \begin{array}{c} \{{}^k \hat{Z}_1\} \times {}^k \{{}^1 \hat{P}_N\} \\ \{{}^k \hat{Z}_1\} \end{array} \right\} \cdots & \left\{ \begin{array}{c} \{{}^k \hat{Z}_i\} \times {}^k \{{}^i \hat{P}_N\} \\ \{{}^k \hat{Z}_i\} \end{array} \right\} \cdots \cdots \cdots \\ & \left\{ \begin{array}{c} \{{}^k \hat{Z}_N\} \times {}^k \{{}^N \hat{P}_N\} \\ \{{}^k \hat{Z}_N\} \end{array} \right\} \end{array} \right] \quad (4.28)$$

The Baxter arm Jacobian matrix is given below for the general 7 dof case with $L_5 \neq 0$.

Note that :

$$\begin{aligned} C_1 &= \cos(\theta_1), C_2 = \cos(\theta_2), C_3 = \cos(\theta_3), C_4 = \cos(\theta_4), C_5 = \cos(\theta_5), \\ C_6 &= \cos(\theta_6), C_7 = \cos(\theta_7), S_1 = \sin(\theta_1), S_2 = \sin(\theta_2), S_3 = \sin(\theta_3), \\ S_4 &= \sin(\theta_4), S_5 = \sin(\theta_5), S_6 = \sin(\theta_6), S_7 = \sin(\theta_7) \end{aligned}$$

$$[{}^k J] = \begin{bmatrix} j_{11} & (L_2 C_4 + L_3 S_4 + L_4) C_3 - L_5 S_3 S_4 S_5 & -L_5 C_4 S_5 & L_4 & -L_5 S_5 & 0 & 0 \\ j_{21} & (-L_2 S_4 + L_3 C_4 + L_5 C_5) C_3 - L_5 S_3 C_4 S_5 & L_5 S_4 S_5 & L_5 C_5 & 0 & 0 & 0 \\ j_{31} & -(L_2 + L_4 C_4 - L_5 S_4 C_5) S_3 & L_3 + L_4 S_4 + L_5 C_4 C_5 & 0 & L_5 C_5 & 0 & 0 \\ S_2 S_4 - C_2 C_3 C_4 & S_3 C_4 & -S_4 & 0 & 0 & -S_5 & C_5 S_6 \\ S_2 C_4 + C_2 C_3 S_4 & -S_3 S_4 & -C_4 & 0 & -1 & 0 & -C_6 \\ C_2 S_3 & C_3 & 0 & 1 & 0 & C_5 & S_5 S_6 \end{bmatrix} \quad (4.29)$$

where :

$$j_{11} = ((L_1 + L_2 C_2) C_4 + (L_3 S_4 + L_4) C_2) S_3 + L_5 (S_2 C_4 + C_2 C_3 S_4) S_5 \quad (4.30)$$

$$j_{21} = -((L_1 + L_2 C_2) S_4 - L_3 C_2 C_4) S_3 - L_5 (S_2 S_4 - C_2 C_3 C_4) S_5 + L_5 C_2 S_3 C_5 \quad (4.31)$$

$$j_{31} = (L_1 + L_2 C_2) C_3 - L_3 S_2 - L_4 (S_2 S_4 - C_2 C_3 C_4) - L_5 (S_2 C_4 + C_2 C_3 S_4) C_5 \quad (4.32)$$

5 Path Planning & Obstacle Avoidance

The whole path is formulated by choice of the following sequence of position formed by selection of way points in the robot work-space to pick and place the object.

Step 1 to 4 takes the left arm from the start position to object location on conveyor belt, this is followed by closing of gripper to grab the object.

1. $\theta_1 = 15 * \pi / 180, \theta_2 = 0 * \pi / 180, \theta_3 = 0 * \pi / 180, \theta_4 = 0 * \pi / 180,$
 $\theta_5 = 0 * \pi / 180, \theta_6 = 0 * \pi / 180, \theta_7 = 0 * \pi / 180$
2. $\theta_1 = 15 * \pi / 180, \theta_2 = 0 * \pi / 180, \theta_3 = 0 * \pi / 180, \theta_4 = 0 * \pi / 180,$
 $\theta_5 = 0 * \pi / 180, \theta_6 = 70 * \pi / 180, \theta_7 = 0 * \pi / 180$
3. $\theta_1 = 15 * \pi / 180, \theta_2 = 0 * \pi / 180, \theta_3 = 0 * \pi / 180, \theta_4 = 0 * \pi / 180,$
 $\theta_5 = \pi / 180, \theta_6 = 70 * \pi / 180, \theta_7 = 9 * \pi / 180$
4. $\theta_1 = 15 * \pi / 180, \theta_2 = 9 * \pi / 180, \theta_3 = 0 * \pi / 180, \theta_4 = 0 * \pi / 180,$
 $\theta_5 = 0 * \pi / 180, \theta_6 = 70 * \pi / 180, \theta_7 = 9 * \pi / 180$

Step 5 to 7 moves the left arms from conveyor belt to end location in space avoiding obstacle and thus reaching the goal position:

5. $\theta_1 = 0 * \pi / 180, \theta_2 = 0 * \pi / 180, \theta_3 = 0 * \pi / 180, \theta_4 = 0 * \pi / 180,$
 $\theta_5 = 0 * \pi / 180, \theta_6 = 0 * \pi / 180, \theta_7 = 0 * \pi / 180$
6. $\theta_1 = -70 * \pi / 180, \theta_2 = 9 * \pi / 180, \theta_3 = 0 * \pi / 180, \theta_4 = 0 * \pi / 180,$
 $\theta_5 = 0 * \pi / 180, \theta_6 = 0 * \pi / 180, \theta_7 = 0 * \pi / 180$
7. $\theta_1 = -70 * \pi / 180, \theta_2 = 9 * \pi / 180, \theta_3 = 0 * \pi / 180, \theta_4 = 0 * \pi / 180,$
 $\theta_5 = \pi / 180, \theta_6 = 50 * \pi / 180, \theta_7 = 0 * \pi / 180$

Step 8 to 9 moves the left arms from end location to home position in space avoiding obstacle.

8. $\theta_1 = -70 * \pi / 180, \theta_2 = 0 * \pi / 180, \theta_3 = 0 * \pi / 180, \theta_4 = 0 * \pi / 180,$
 $\theta_5 = 0 * \pi / 180, \theta_6 = 0 * \pi / 180, \theta_7 = 0 * \pi / 180$
9. $\theta_1 = 0 * \pi / 180, \theta_2 = 0 * \pi / 180, \theta_3 = 0 * \pi / 180, \theta_4 = 0 * \pi / 180,$
 $\theta_5 = 0 * \pi / 180, \theta_6 = 0 * \pi / 180, \theta_7 = 0 * \pi / 180$

6 Verification of Equations and Simulation Results

In this section of the report we mainly concentrate on the verification of the manually calculated forward kinematics equations, inverse kinematics equations, and path planning to avoid collision with obstacles. There are three subsections which individually verify forward kinematics, inverse kinematics and path planning.

6.1 Forward Kinematics

For validating the forward kinematics equation, we follow the below mentioned procedure:

1. Select 7 angles corresponding to 7 joint angles namely $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$ & θ_7 .
2. Feed the values of the joint angles to the equation T_X, T_Y & T_Z obtained in the Forward Kinematics to get the x, y, and z co-ordinate of the gripper(end-effector).
3. Feed the values of the joint angles to the Lua script for Baxter in V-rep and run the code.
4. Select the gripper using the Object/item shift option to get the x, y, and z co-ordinate of the gripper(end-effector) in V-rep.
5. Match the calculated gripper(end-effector) co-ordinates to that obtained in V-rep.

Three different positions of Baxter arm in admissible work-space have been validated.

6.1.1 Case 1:

This is the most simple case. Here all the joint angles are considered to be equal to 0. On plugging in the joint angles equal to 0, in mat-lab code for forward kinematics we obtained the following results for the gripper(end-effector) co-ordinates $x=1.110m, y=-0.896m$, and $z=1.295m$. In V-rep we received nearly the same values for the gripper(end-effector) co-ordinates that is $x=1.0139m, y=-0.8332m$ and $z=1.2353m$.

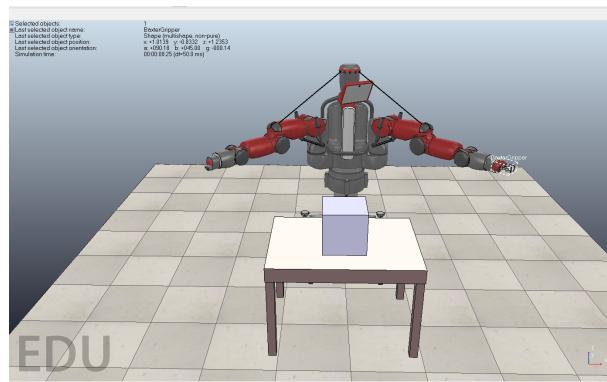


Figure 6.1: Case 1 Forward Kinematics Validation

6.1.2 Case 2:

In this case we tried to simulate an orientation that is most common for pick and place task. For the case we considered joint angles to be $\theta_1=0, \theta_2=-31, \theta_3=0, \theta_4=43, \theta_5=0, \theta_6=72, \theta_7=0$, in mat-lab code for forward kinematics we obtained the following results for the gripper(end-effector) co-ordinates x=0.857m,y=-0.643m, and z=1.049m. In V-rep we received nearly the same values for co-ordinates that is x=0.8385m, y=-0.6578m and z=1.0951m.

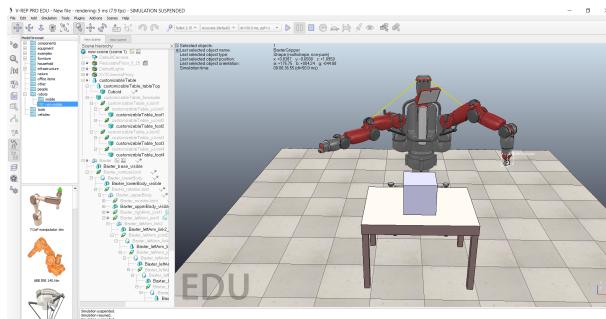


Figure 6.2: Case 2 Forward Kinematics Validation

6.1.3 Case 3:

In this case we modified the joint angles $\theta_1=10, \theta_2=20, \theta_3=30, \theta_4=40, \theta_5=50, \theta_6=60, \theta_7=70$, on plugging in the joint angles, in mat-lab code we obtained the following results x=1.026m,y=0.039m, and z=0.788m. In V-rep we received nearly the same values for co-ordinates that is x=0.9521m, y=-0.07m and z=0.7493m.

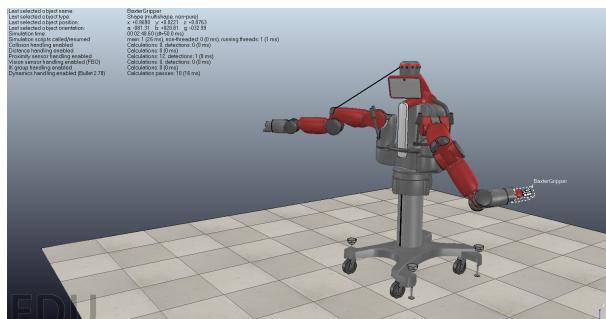


Figure 6.3: Case 3 Forward Kinematics Validation

6.2 Inverse Kinematics

For validation of inverse kinematics we consider the case 3 of forward kinematics. The transformation matrix T_{world} is given as below:

$$T_{world} = \begin{bmatrix} 0.733 & 0.653 & 0.190 & 0.843 \\ 0.384 & -0.628 & 0.677 & -0.162 \\ 0.562 & -0.423 & -0.711 & 0.661 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6.1)$$

Thus,multiplying with fixed transformation matrix inverse to get T_{endeff} :

$$T_{endeff} = \begin{bmatrix} 0.247 & 0.906 & -0.344 & 0.595 \\ 0.790 & 0.018 & 0.613 & 0.105 \\ 0.562 & -0.423 & -0.711 & -0.451 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6.2)$$

Applying the derived equations we get two configuration :

Elbow up:

$$\theta_1 = 10 \quad \theta_2 = 20 \quad \theta_3 = 0 \quad \theta_4 = 40 \quad \theta_5 = 50 \quad \theta_6 = 60 \quad \theta_7 = 70$$

Elbow down:

$$\theta_1 = 10 \quad \theta_2 = 60.2 \quad \theta_3 = 0 \quad \theta_4 = -40 \quad \theta_5 = 41.6 \quad \theta_6 = 88.4 \quad \theta_7 = 99.4$$

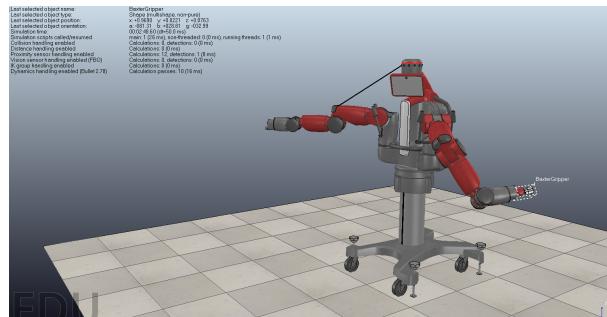


Figure 6.4: Inverse Kinematics Validation

6.3 Path Planning

Validation for path planning and obstacle avoidance is done by simulation in V-rep. All the way points and commands to close and open the gripper are given to the Baxter by Lua script from start to end position to move without collision with the obstacle kept on the table. All the generated position are displayed in the diagram below:

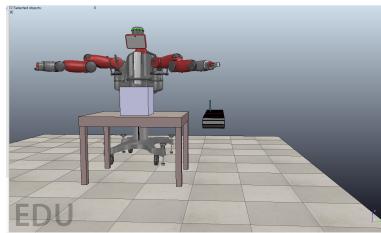


Figure 6.5: (a) Start configuration

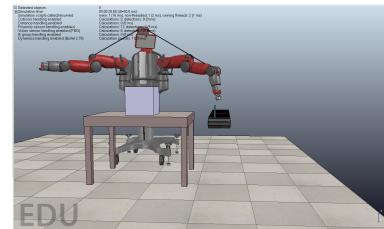


Figure 6.6: (b) Reaching the conveyor belt

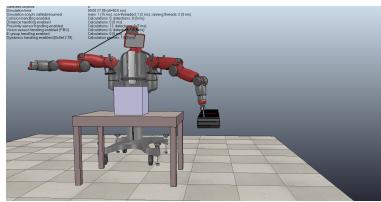


Figure 6.7: (c) Closing gripper to grasp the object

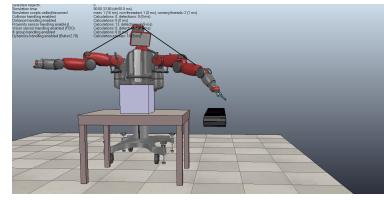


Figure 6.8: (d) Picking Object form conveyor belt

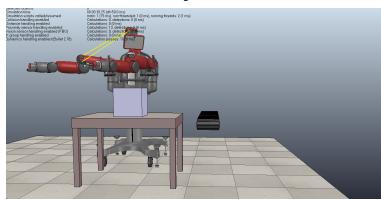


Figure 6.9: (e) Moving Object

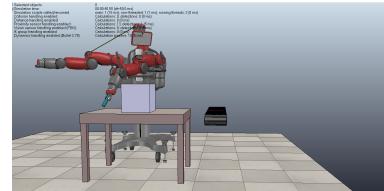


Figure 6.10: (f) Placing Object

Figure 6.11: Path planning avoiding obstacles

7 Future Works & Conclusion

7.1 Future Works

1. Would do grasp analysis for Baxter Robot.
2. Multi tasking using both Baxter arms simultaneously for pick and place.
3. Expanding the scope of the project to multi robot system and more complex and real scenarios considering external disturbances and task agility.
4. We would also like to analyse the performance of pick and place when object selection needs to be made from a mixture of objects which are placed very close to each other and are similar in some structural properties and color.

7.2 Conclusion

1. The project provided valuable understanding of present trend in the industry and how robots can collaboratively work with humans.
2. Through this project we were able to gain in depth knowledge about robot kinematics and learned new convention namely Modified DH convention.
3. Worked on path planning and obstacle avoidance for the Baxter robot to achieve pick and place task.
4. Learned new simulation methods for validation of robot kinematics in V-rep.
5. Implemented robot kinematic equation to form Collaborative Robotic Conveyor System.

7.3 Goal Accomplishments

Goal	Accomplishment
Understanding Baxter Kinematics	Achieved
Calculation of Baxter Kinematics	Achieved
Learning Lua programming for V-rep	Achieved
Path Planning and Obstacle avoidance	Achieved
Validation of Baxter Kinematics	Achieved
Designing of Collaborative Robotic Conveyor system	Achieved

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