Perceptron Training: AND Gate with Ground Truth Highlight

August 6, 2025

Epoch 1

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	**0**	**0**	**0**
2	0	1	0
3	1	0	0
4	1	1	1

Sample 1

$$x_1 = 0$$
$$x_2 = 0$$
$$y = 0$$
$$\vec{w} = [0, 0]$$
$$b = 0.0$$
$$\eta = 1.0$$

Weighted Sum Calculation

General Formula:

$$z = \sum_{i=1}^{n} w_i x_i + b$$

Substitution: $z = 0 \cdot 0 + 0 \cdot 0 + 0.0 = 0.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 1$$

Error Calculation

$$\text{error} = y - \hat{y} = 0 - 1 = -1$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = 0.0 + 1.0 \cdot (-1) = -1.0$$

Bias Updated: Yes

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [0, 0] + 1.0 \cdot (-1) \cdot [0, 0] = [0.0, 0.0]$$

Weights Updated: No

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	0	0	0
2	**0**	**1**	**0**
3	1	0	0
4	1	1	1

Sample 2

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 1 \\ y &= 0 \\ \vec{w} &= [0.0, 0.0] \\ b &= -1.0 \\ \eta &= 1.0 \end{aligned}$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 0.0 \cdot 0 + 0.0 \cdot 1 + -1.0 = -1.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 0$$

Error Calculation

$$\text{error} = y - \hat{y} = 0 - 0 = 0$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -1.0 + 1.0 \cdot (0) = -1.0$$

Bias Updated: No

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [0.0, 0.0] + 1.0 \cdot (0) \cdot [0, 1] = [0.0, 0.0]$$

Weights Updated: No

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	0	0	0
2	0	1	0
3	**1**	**0**	**0**
4	1	1	1

Sample 3

$$
\begin{aligned}
x_1 &= 1 \\
x_2 &= 0 \\
y &= 0 \\
\vec{w} &= [0.0, 0.0] \\
b &= -1.0 \\
\eta &= 1.0
\end{aligned}
$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 0.0 \cdot 1 + 0.0 \cdot 0 + -1.0 = -1.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 0$$

Error Calculation

$$\text{error} = y - \hat{y} = 0 - 0 = 0$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -1.0 + 1.0 \cdot (0) = -1.0$$

Bias Updated: No

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [0.0, 0.0] + 1.0 \cdot (0) \cdot [1, 0] = [0.0, 0.0]$$

Weights Updated: No

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	0	0	0
2	0	1	0
3	1	0	0
4	**1**	**1**	**1**

Sample 4

$$
\begin{aligned}
x_1 &= 1 \\
x_2 &= 1 \\
y &= 1 \\
\vec{w} &= [0.0, 0.0] \\
b &= -1.0 \\
\eta &= 1.0
\end{aligned}
$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 0.0 \cdot 1 + 0.0 \cdot 1 + -1.0 = -1.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 0$$

Error Calculation

$$\text{error} = y - \hat{y} = 1 - 0 = 1$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -1.0 + 1.0 \cdot (1) = 0.0$$

Bias Updated: Yes

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [0.0, 0.0] + 1.0 \cdot (1) \cdot [1, 1] = [1.0, 1.0]$$

Weights Updated: Yes

Epoch 2

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	**0**	**0**	**0**
2	0	1	0
3	1	0	0
4	1	1	1

Sample 1

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 0 \\ y &= 0 \\ \vec{w} &= [1.0, 1.0] \\ b &= 0.0 \\ \eta &= 1.0 \end{aligned}$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 1.0 \cdot 0 + 1.0 \cdot 0 + 0.0 = 0.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 1$$

Error Calculation

$$\text{error} = y - \hat{y} = 0 - 1 = -1$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = 0.0 + 1.0 \cdot (-1) = -1.0$$

Bias Updated: Yes

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [1.0, 1.0] + 1.0 \cdot (-1) \cdot [0, 0] = [1.0, 1.0]$$

Weights Updated: No

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	0	0	0
2	**0**	**1**	**0**
3	1	0	0
4	1	1	1

Sample 2

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 1 \\ y &= 0 \\ \vec{w} &= [1.0, 1.0] \\ b &= -1.0 \\ \eta &= 1.0 \end{aligned}$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 1.0 \cdot 0 + 1.0 \cdot 1 + -1.0 = 0.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 1$$

Error Calculation

$$\text{error} = y - \hat{y} = 0 - 1 = -1$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -1.0 + 1.0 \cdot (-1) = -2.0$$

Bias Updated: Yes

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [1.0, 1.0] + 1.0 \cdot (-1) \cdot [0, 1] = [1.0, 0.0]$$

Weights Updated: Yes

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	0	0	0
2	0	1	0
3	**1**	**0**	**0**
4	1	1	1

Sample 3

$$\begin{aligned} x_1 &= 1 \\ x_2 &= 0 \\ y &= 0 \\ \vec{w} &= [1.0, 0.0] \\ b &= -2.0 \\ \eta &= 1.0 \end{aligned}$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 1.0 \cdot 1 + 0.0 \cdot 0 + -2.0 = -1.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 0$$

Error Calculation

$$\text{error} = y - \hat{y} = 0 - 0 = 0$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -2.0 + 1.0 \cdot (0) = -2.0$$

Bias Updated: No

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [1.0, 0.0] + 1.0 \cdot (0) \cdot [1, 0] = [1.0, 0.0]$$

Weights Updated: No

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	0	0	0
2	0	1	0
3	1	0	0
4	**1**	**1**	**1**

Sample 4

$$
\begin{aligned}
x_1 &= 1 \\
x_2 &= 1 \\
y &= 1 \\
\vec{w} &= [1.0, 0.0] \\
b &= -2.0 \\
\eta &= 1.0
\end{aligned}
$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 1.0 \cdot 1 + 0.0 \cdot 1 + -2.0 = -1.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 0$$

Error Calculation

$$\text{error} = y - \hat{y} = 1 - 0 = 1$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -2.0 + 1.0 \cdot (1) = -1.0$$

Bias Updated: Yes

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [1.0, 0.0] + 1.0 \cdot (1) \cdot [1, 1] = [2.0, 1.0]$$

Weights Updated: Yes

Epoch 3

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	**0**	**0**	**0**
2	0	1	0
3	1	0	0
4	1	1	1

Sample 1

$$
\begin{aligned}
x_1 &= 0 \\
x_2 &= 0 \\
y &= 0 \\
\vec{w} &= [2.0, 1.0] \\
b &= -1.0 \\
\eta &= 1.0
\end{aligned}
$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 2.0 \cdot 0 + 1.0 \cdot 0 + -1.0 = -1.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 0$$

Error Calculation

$$\text{error} = y - \hat{y} = 0 - 0 = 0$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -1.0 + 1.0 \cdot (0) = -1.0$$

Bias Updated: No

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [2.0, 1.0] + 1.0 \cdot (0) \cdot [0, 0] = [2.0, 1.0]$$

Weights Updated: No

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	0	0	0
2	**0**	**1**	**0**
3	1	0	0
4	1	1	1

Sample 2

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 1 \\ y &= 0 \\ \vec{w} &= [2.0, 1.0] \\ b &= -1.0 \\ \eta &= 1.0 \end{aligned}$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 2.0 \cdot 0 + 1.0 \cdot 1 + -1.0 = 0.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 1$$

Error Calculation

$$\text{error} = y - \hat{y} = 0 - 1 = -1$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -1.0 + 1.0 \cdot (-1) = -2.0$$

Bias Updated: Yes

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [2.0, 1.0] + 1.0 \cdot (-1) \cdot [0, 1] = [2.0, 0.0]$$

Weights Updated: Yes

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	0	0	0
2	0	1	0
3	**1**	**0**	**0**
4	1	1	1

Sample 3

$$\begin{aligned} x_1 &= 1 \\ x_2 &= 0 \\ y &= 0 \\ \vec{w} &= [2.0, 0.0] \\ b &= -2.0 \\ \eta &= 1.0 \end{aligned}$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 2.0 \cdot 1 + 0.0 \cdot 0 + -2.0 = 0.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 1$$

Error Calculation

$$\text{error} = y - \hat{y} = 0 - 1 = -1$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -2.0 + 1.0 \cdot (-1) = -3.0$$

Bias Updated: Yes

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [2.0, 0.0] + 1.0 \cdot (-1) \cdot [1, 0] = [1.0, 0.0]$$

Weights Updated: Yes

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	0	0	0
2	0	1	0
3	1	0	0
4	**1**	**1**	**1**

Sample 4

$$\begin{aligned} x_1 &= 1 \\ x_2 &= 1 \\ y &= 1 \\ \vec{w} &= [1.0, 0.0] \\ b &= -3.0 \\ \eta &= 1.0 \end{aligned}$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 1.0 \cdot 1 + 0.0 \cdot 1 + -3.0 = -2.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 0$$

Error Calculation

$$\text{error} = y - \hat{y} = 1 - 0 = 1$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -3.0 + 1.0 \cdot (1) = -2.0$$

Bias Updated: Yes

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [1.0, 0.0] + 1.0 \cdot (1) \cdot [1, 1] = [2.0, 1.0]$$

Weights Updated: Yes

Epoch 4

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	**0**	**0**	**0**
2	0	1	0
3	1	0	0
4	1	1	1

Sample 1

$$
\begin{aligned}
x_1 &= 0 \\
x_2 &= 0 \\
y &= 0 \\
\vec{w} &= [2.0, 1.0] \\
b &= -2.0 \\
\eta &= 1.0
\end{aligned}
$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 2.0 \cdot 0 + 1.0 \cdot 0 + -2.0 = -2.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 0$$

Error Calculation

$$\text{error} = y - \hat{y} = 0 - 0 = 0$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -2.0 + 1.0 \cdot (0) = -2.0$$

Bias Updated: No

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [2.0, 1.0] + 1.0 \cdot (0) \cdot [0, 0] = [2.0, 1.0]$$

Weights Updated: No

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	0	0	0
2	**0**	**1**	**0**
3	1	0	0
4	1	1	1

Sample 2

$$
\begin{aligned}
x_1 &= 0 \\
x_2 &= 1 \\
y &= 0 \\
\vec{w} &= [2.0, 1.0] \\
b &= -2.0 \\
\eta &= 1.0
\end{aligned}
$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 2.0 \cdot 0 + 1.0 \cdot 1 + -2.0 = -1.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 0$$

Error Calculation

$$\text{error} = y - \hat{y} = 0 - 0 = 0$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -2.0 + 1.0 \cdot (0) = -2.0$$

Bias Updated: No

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [2.0, 1.0] + 1.0 \cdot (0) \cdot [0, 1] = [2.0, 1.0]$$

Weights Updated: No

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	0	0	0
2	0	1	0
3	**1**	**0**	**0**
4	1	1	1

Sample 3

$$
\begin{aligned}
x_1 &= 1 \\
x_2 &= 0 \\
y &= 0 \\
\vec{w} &= [2.0, 1.0] \\
b &= -2.0 \\
\eta &= 1.0
\end{aligned}
$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 2.0 \cdot 1 + 1.0 \cdot 0 + -2.0 = 0.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 1$$

Error Calculation

$$\text{error} = y - \hat{y} = 0 - 1 = -1$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -2.0 + 1.0 \cdot (-1) = -3.0$$

Bias Updated: Yes

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [2.0, 1.0] + 1.0 \cdot (-1) \cdot [1, 0] = [1.0, 1.0]$$

Weights Updated: Yes

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	0	0	0
2	0	1	0
3	1	0	0
4	**1**	**1**	**1**

Sample 4

$$\begin{aligned} x_1 &= 1 \\ x_2 &= 1 \\ y &= 1 \\ \vec{w} &= [1.0, 1.0] \\ b &= -3.0 \\ \eta &= 1.0 \end{aligned}$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 1.0 \cdot 1 + 1.0 \cdot 1 + -3.0 = -1.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 0$$

Error Calculation

$$\text{error} = y - \hat{y} = 1 - 0 = 1$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -3.0 + 1.0 \cdot (1) = -2.0$$

Bias Updated: Yes

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [1.0, 1.0] + 1.0 \cdot (1) \cdot [1, 1] = [2.0, 2.0]$$

Weights Updated: Yes

Epoch 5

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	**0**	**0**	**0**
2	0	1	0
3	1	0	0
4	1	1	1

Sample 1

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 0 \\ y &= 0 \\ \vec{w} &= [2.0, 2.0] \\ b &= -2.0 \\ \eta &= 1.0 \end{aligned}$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 2.0 \cdot 0 + 2.0 \cdot 0 + -2.0 = -2.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 0$$

Error Calculation

$$\text{error} = y - \hat{y} = 0 - 0 = 0$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -2.0 + 1.0 \cdot (0) = -2.0$$

Bias Updated: No

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [2.0, 2.0] + 1.0 \cdot (0) \cdot [0, 0] = [2.0, 2.0]$$

Weights Updated: No

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	0	0	0
2	**0**	**1**	**0**
3	1	0	0
4	1	1	1

Sample 2

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 1 \\ y &= 0 \\ \vec{w} &= [2.0, 2.0] \\ b &= -2.0 \\ \eta &= 1.0 \end{aligned}$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 2.0 \cdot 0 + 2.0 \cdot 1 + -2.0 = 0.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 1$$

Error Calculation

$$\text{error} = y - \hat{y} = 0 - 1 = -1$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -2.0 + 1.0 \cdot (-1) = -3.0$$

Bias Updated: Yes

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [2.0, 2.0] + 1.0 \cdot (-1) \cdot [0, 1] = [2.0, 1.0]$$

Weights Updated: Yes

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	0	0	0
2	0	1	0
3	**1**	**0**	**0**
4	1	1	1

Sample 3

$$
\begin{aligned}
x_1 &= 1 \\
x_2 &= 0 \\
y &= 0 \\
\vec{w} &= [2.0, 1.0] \\
b &= -3.0 \\
\eta &= 1.0
\end{aligned}
$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 2.0 \cdot 1 + 1.0 \cdot 0 + -3.0 = -1.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 0$$

Error Calculation

$$\text{error} = y - \hat{y} = 0 - 0 = 0$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -3.0 + 1.0 \cdot (0) = -3.0$$

Bias Updated: No

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [2.0, 1.0] + 1.0 \cdot (0) \cdot [1, 0] = [2.0, 1.0]$$

Weights Updated: No

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	0	0	0
2	0	1	0
3	1	0	0
4	**1**	**1**	**1**

Sample 4

$$
\begin{aligned}
x_1 &= 1 \\
x_2 &= 1 \\
y &= 1 \\
\vec{w} &= [2.0, 1.0] \\
b &= -3.0 \\
\eta &= 1.0
\end{aligned}
$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 2.0 \cdot 1 + 1.0 \cdot 1 + -3.0 = 0.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 1$$

Error Calculation

$$\text{error} = y - \hat{y} = 1 - 1 = 0$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -3.0 + 1.0 \cdot (0) = -3.0$$

Bias Updated: No

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [2.0, 1.0] + 1.0 \cdot (0) \cdot [1, 1] = [2.0, 1.0]$$

Weights Updated: No

Epoch 6

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	**0**	**0**	**0**
2	0	1	0
3	1	0	0
4	1	1	1

Sample 1

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 0 \\ y &= 0 \\ \vec{w} &= [2.0, 1.0] \\ b &= -3.0 \\ \eta &= 1.0 \end{aligned}$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 2.0 \cdot 0 + 1.0 \cdot 0 + -3.0 = -3.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 0$$

Error Calculation

$$\text{error} = y - \hat{y} = 0 - 0 = 0$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -3.0 + 1.0 \cdot (0) = -3.0$$

Bias Updated: No

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [2.0, 1.0] + 1.0 \cdot (0) \cdot [0, 0] = [2.0, 1.0]$$

Weights Updated: No

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	0	0	0
2	**0**	**1**	**0**
3	1	0	0
4	1	1	1

Sample 2

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 1 \\ y &= 0 \\ \vec{w} &= [2.0, 1.0] \\ b &= -3.0 \\ \eta &= 1.0 \end{aligned}$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 2.0 \cdot 0 + 1.0 \cdot 1 + -3.0 = -2.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 0$$

Error Calculation

$$\text{error} = y - \hat{y} = 0 - 0 = 0$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -3.0 + 1.0 \cdot (0) = -3.0$$

Bias Updated: No

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [2.0, 1.0] + 1.0 \cdot (0) \cdot [0, 1] = [2.0, 1.0]$$

Weights Updated: No

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	0	0	0
2	0	1	0
3	**1**	**0**	**0**
4	1	1	1

Sample 3

$$\begin{aligned} x_1 &= 1 \\ x_2 &= 0 \\ y &= 0 \\ \vec{w} &= [2.0, 1.0] \\ b &= -3.0 \\ \eta &= 1.0 \end{aligned}$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 2.0 \cdot 1 + 1.0 \cdot 0 + -3.0 = -1.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 0$$

Error Calculation

$$\text{error} = y - \hat{y} = 0 - 0 = 0$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -3.0 + 1.0 \cdot (0) = -3.0$$

Bias Updated: No

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [2.0, 1.0] + 1.0 \cdot (0) \cdot [1, 0] = [2.0, 1.0]$$

Weights Updated: No

Ground Truth Table (AND Gate)

Sample	x_1	x_2	y
1	0	0	0
2	0	1	0
3	1	0	0
4	**1**	**1**	**1**

Sample 4

$$
\begin{aligned}
x_1 &= 1 \\
x_2 &= 1 \\
y &= 1 \\
\vec{w} &= [2.0, 1.0] \\
b &= -3.0 \\
\eta &= 1.0
\end{aligned}
$$

Weighted Sum Calculation

General Formula: $z = \sum_{i=1}^{n} w_i x_i + b$
Substitution: $\text{z} = 2.0 \cdot 1 + 1.0 \cdot 1 + -3.0 = 0.0$

Activation Output

$$\hat{y} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise} \end{cases} = 1$$

Error Calculation

$$\text{error} = y - \hat{y} = 1 - 1 = 0$$

Bias Update

$$b_{\text{new}} = b + \eta \cdot \text{error} = -3.0 + 1.0 \cdot (0) = -3.0$$

Bias Updated: No

Weights Update

$$\vec{w}_{\text{new}} = \vec{w} + \eta \cdot \text{error} \cdot \vec{x} = [2.0, 1.0] + 1.0 \cdot (0) \cdot [1, 1] = [2.0, 1.0]$$

Weights Updated: No