HIII

by

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Abstract

This thesis investigates the invariant extended Kalman filter (IEKF), a recently introduced method for nonlinear state estimation on matrix Lie groups. The IEKF is well suited to a particular class of systems, namely those with group-affine process models and invariant measurement models. In fact, when these conditions are met, the IEKF is a locally asymptotically convergent observer. However, in practice, process models are often not group-affine, and measurement models are often not invariant. The effect of removing these assumptions is investigated in this thesis. In particular, a 3D example is considered, with and without bias estimation. Estimating bias renders the process model not group affine. Then, a non-invariant measurement model is considered. Two different techniques are proposed to incorporate this measurement model into an IEKF, a standard approach using the non-invariant model and a novel approach in which the measurement is preprocessed to force the preprocessed measurement to be invariant. These practical extensions of the IEKF are tested in simulation to determine the effectiveness of the IEKF for more general state estimation problems. Lastly, batch estimation in the invariant framework is formulated. The problem of interest is the simultaneous localization and mapping (SLAM) problem. A general derivation of the SLAM problem on matrix Lie groups is presented. Invariant estimation theory is then leveraged. An inertial navigation example with bias estimation is then presented, with testing done in simulation.

Résumé

Cette thèse étudie le filter de Kalman invariant (IEKF), une méthode récemment introduite pour l'estimation d'état non linéaire sur des groupes de Lie matriciels. L'IEKF est bien adapté à une classe particulière de systèmes, plus précisément ceux dotés de fonctions affines et de fonctions d'observation invariantes. En fait, lorsque ces conditions sont satisfaites, l'IEKF est un observateur localement asymptotiquement convergent. Cependant, en situation pratique, ces conditions ne sont souvent pas satisfaites. Des scénarios ou ces conditions ne sont pas satisfaites sont étudié ici. En particulier, un exemple 3D est considéré, avec et sans estimation de biais dans le gyroscope. L'estimation du biais rend la fonction non-affinée. Ensuite, une fonction d'observation non-invariante est considéré. Deux techniques différentes sont proposées pour incorporer cette fonction d'observation dans un IEKF, une approche standard utilisant la fonction non-invariante et une nouvelle approche dans laquelle la mesure est prétraitée pour la forcer être invariante. Ces extensions pratiques du IEKF sont testées en simulation pour déterminer son efficacités pour des problèmes d'estimation d'état plus généraux. Enfin, une technique d'estimation par lot dans le cadre invariant est formulée. Un intérêt particulier est porté au problème de la localisation et de la cartographie simultanées (SLAM). Une dérivation générale du problème SLAM sur les groupes de Lie matriciels est présentée. La théorie de l'estimation invariante est ensuite mise à profit. Un exemple de navigation inertielle avec estimation du biais est ensuite présenté, avec des tests effectués en simulation.

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Preface

The contributions of this thesis that are original to the author's knowledge are as follows.

- Chapter **??**
  - Solving the batch SLAM problem using a right-invariant framework while explicitly considering bias states.

All text, plots, figures and results in this thesis are produced by Jonathan Arsenault. The IEKF was originally introduced by Silvère Bonnabel and Axel Barrau in continuous time in [1], and in discrete time in [2].

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Chapter 1

Introduction

The onboard computers of autonomous robots, such as unmanned aerial vehicles (UAV), mobile robots, or autonomous underwater vehicles (AUV), run navigation, guidance, and control algorithms that enable the robotic system to perform desired tasks. The navigation algorithm is responsible for estimating the states of the robot. The guidance algorithm considers planning the path the robot will take to complete its task. Lastly, the controller computes control inputs, such as forces and torques, to be applied so that the robot follows the desired trajectory. These three modules are of equal importance, and are intrinsically linked.

This thesis is focused on the navigation problem, also commonly called the state estimation problem. State estimation is the process of estimating the states of a system given noisy and biased sensor data. For example, an UAV must typically maintain a robust and accurate estimate of its position, velocity, and attitude in order to perform precision tasks, such as parcel delivery or surveillance. However, the sensors onboard UAVs are often of lower quality, to minimize the cost of the system, necessitating a state estimation algorithm that can reliably estimate the position, velocity, and attitude of the UAV given low-qulity senor data.

Several different state estimation techniques exist, each with their advantages and disadvantages. Roughly speaking, they can be separated into batch algorithms, which typically run offline, and sequential algorithms, which typically run in real time. Batch algorithms use sensor data over the entire trajectory to in turn provide an estimate of the states over the entire trajectory. Traditional batch algorithms include the (nonlinear) least-squares formulation [3, Sec. 4.3] and the forward-backward smoother [3, Sec. 3.2.2] and Rauch-Tung-Striebel smoother [3, Sec. 3.2.3]. Batch algorithms are especially useful when reconstructing scenes for metrology or photogrammetry applications, for example. In addition, simultaneous localization and mapping (SLAM) algorithms are often batch algorithms that do not run in real time.

In real-time applications, sequential state estimation methods are often preferred. The most commonly used algorithms for real-time state estimation are approximations of the Bayes filter [4], such as the Kalman filter, extended Kalman filter (EKF), or sigma-point Kalman filter. Other real-time state estimation methods that leverage concepts from the batch formulation, such as using a bundle of sensor data or iteration, include the sliding window filter [5], iterative extended Kalman filter [3, Sec. 4.2.5], and iterative sigma-point Kalman filter [3, Sec. 4.2.10].

In industry, the EKF is often the algorithm of choice, due to its relative simplicity and its track record of effectiveness. However, it does have its deficiencies. In this thesis, a variant of the EKF, the invariant extended Kalman filter (IEKF) is considered. For a review of the IEKF, see Chapter **??**. The main idea behind the invariant filtering framework is that certain problems (i.e., so-called "left-invariant" problems) do not explicitly depend on a particular inertial frame, and others (i.e., so-called "right-invariant" problems) do not explicitly depend on a particular body-fixed frame. Not all estimation problems fit the invariant filtering framework, but when an estimation problem does, extremely appealing properties appear.

1.1 Thesis Objective

The objective of this thesis is to determine how the invariant filtering framework can be used to improve existing state estimation methods. In particular, the contribution of this thesis is an overview of practical considerations of the IEKF and an extension of the invariant estimation theory to the SLAM problem posed in a batch framework.

Another contribution of this thesis is to thoroughly summarize the theory behind the IEKF. This includes some proofs that are missing from the literature. A major contribution of this thesis is to compare the various error definitions that can be used in solving the SLAM problem. This includes a general formulation for performing SLAM when the state can be formulated as an element of a matrix Lie group. Modifying the error definitions leads to Jacobians that may depend less, or not at all, on the state estimate. Lastly, this thesis provides a thorough analysis of the practical implications of the IEKF. The theory behind the IEKF is sound, but the assumptions made often do not hold in practice. Of note, a novel method of using the IEKF in conjunction with a stereo camera is presented.

1.2 Thesis Overview

This thesis is structured as follows.

Chapter **??** summarizes mathematical concepts and notation that are used throughout this thesis.

Chapter **??** outlines the IEKF. The relevant theorems and proofs are presented in continuous and discrete-time. The left-invariant extended Kalman filter and right-invariant extended Kalman filter are then detailed.

In Chapter **??**, several examples of the IEKF are presented to illustrate how to practically implement an IEKF and to compare its performance to that of a standard multiplicative extended Kalman filter (MEKF).

In Chapter **??**, a solution to the SLAM problem in the invariant framework is presented. Simulation results are shown comparing the novel formulation to more traditional batch-based solutions to the SLAM problem.

This thesis is concluded in Chapter 2, where a summary of the findings are presented, along with recommended future work.

Chapter 2

Closing Remarks and Future Work

2.1 Conclusions

In this thesis, an in-depth analysis of state estimation in an invariant framework is presented. Through rigorous testing, the advantages and limitations of these different techniques are determined. Furthermore, an extension of invariant filtering theory to the problem of a batch solution to the SLAM problem is presented.

The IEKF is superior to the traditional MEKF in certain situations. It is better suited to problems where the state can be defined on matrix Lie groups, which is the case for many robotics problems. Throughout the simulations presented herein, the performance of the IEKF is on average better than that of the MEKF. However, only particular sample problems are used to illustrate this. It would therefore be irresponsible to state that the IEKF would always perform better than the MEKF. However, certain clear conclusions can be drawn.

First, state-independent Jacobians, such as those obtained in an IEKF, are advantageous in cases where the best estimate of the state is far from the true value. In most situations, this is seen when the initialization is poor. The IEKF's better performance is therefore mostly attributed to better performance in the transient period before the filter reaches steady state. Thus, the IEKF should be the state estimator of choice in applications where the initial state is unknown, and no other initialization scheme is available.

Second, leveraging the invariant framework in batch estimation only has limited advantages. In standard batch estimation, the Jacobians may initially be inaccurate if they depend on the state. However, as the solution converges, the Jacobians will be closer to the true Jacobians, as the error in the state estimate decreases. At this stage, there is minimal difference between a state-independent Jacobian and a Jacobian computed using an accurate state estimate.

2.2 Future Work

In Chapter **??**, the IEKF is compared to the MEKF. The MEKF was used a baseline as it is commonly used. However, comparing the IEKF to an iterative version of the MEKF may yield different results. The iterative MEKF improves upon the MEKF by recomputing the Jacobians at each time step until convergence. Furthermore, an iterative version of the IEKF could also be developed. This iterative IEKF would only be useful in scenarios where the process model is not group affine, or the measurement model is not invariant, leading to state dependent Jacobians like the MEKF.

Another avenue to explore would involve using a realistic sensor model in invariant batch SLAM. A future study using a stereo camera model or LIDAR model would also allow the invariant batch SLAM algorithms to be tested on experimental data.

Lastly, a study analyzing the consistency of the IEKF versus other other filtering techniques should be conducted. The IEKF should theoretically be more consistent, as its more accurate Jacobians mean the covariance better captures the underlying distribution. In a similar vein, the impact of unknown disturbances should be studied. The IEKF may be better suited to handle these, once again, due to theoretically exact Jacobians.

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