COM1002: Foundations of Computer Science
Problem Sheet 1: Prime numbers and factorisation

1. Which of the following are prime numbers?

   (a) 221
   (b) 223
   (c) 503
   (d) 621

2. Find the following highest common factors.

   (a) $hcf(10, 30)$.
   (b) $hcf(22, 6)$.
   (c) $hcf(14, 25)$.
   (d) $hcf(28, 8)$.

3. Find three integers that have no mutual common factor (except ± 1), but where any two of them do share a common factor bigger than 1.

4. Prove by induction that
$$1 + 2 + \cdots + n = \frac{1}{2}n(n + 1)$$
for all integers $n \geq 1$.

5. Let $f(x) = 5x + 3$ and $g(x) = 5(x - 2) + 8$. Try to prove that $f(n) = g(n)$ by assuming it is true for $n = k$ and then showing it is true for $n = k + 1$. What is wrong here?

6. Prove that $n! > 2^n$ for all $n \geq 4$. Recall that $n! = 1 \times 2 \times 3 \times \cdots \times n$ for a positive integer n.

7. A gardener has been supplied with 70 square slabs and told to construct a single rectangular patio using all of the slabs. How many possible ways are there to do this?