COM1002: Foundations of Computer Science
Problem Sheet 4:
Matrices and Systems of Linear Equations

1. Solve the system of linear equations

$$\begin{aligned} x + y + z &= 1 \\ x + 2y + 3z &= 6 \\ x - y - z &= 0 \end{aligned}$$

2. Solve the system of linear equations

$$\begin{aligned} 5x + 3y + 2z &= 19 \\ x + y + z &= 4 \\ 3x + 2y + z &= 12 \end{aligned}$$

3. Let

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & -1 \\ 2 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

   (a) Transform A by elementary row operations to reduced row echelon form.

   (b) Find the general solution to the system of linear equations

$$\begin{aligned} w + y &= 2 \\ w + x + y + z &= 1 \\ x + z &= -1 \\ 2w + x + y + z &= 0 \end{aligned}$$

4. Let

$$B = \begin{pmatrix} 1 & 2 & 1 & 2 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 2 \\ 2 & 2 & 1 & 1 & 1 & 4 \\ 1 & 1 & 1 & 0 & 0 & 6 \end{pmatrix}.$$

(a) Transform B by elementary row operations to reduced row echelon form.

(b) Find the general solution to the system of linear equations

$$\begin{aligned} v + 2w + x + 2y + z &= 0 \\ v + w + x + y + z &= 2 \\ 2v + 2w + x + y + z &= 4 \\ v + w + x &= 6 \end{aligned}$$

(c) Find the solution to the system of linear equations

$$\begin{aligned} w + x + 2y + z &= 0 \\ w + x + y + z &= 2 \\ 2w + x + y + z &= 4 \\ w + x &= 6 \end{aligned}$$

5. Find the general solution to the system of linear equations

$$\begin{aligned} w + x + 2y + z &= 1 \\ w + x - y + z &= -1 \\ x + y + z &= 0 \\ w + 3x + 2y + 3z &= 2 \end{aligned}$$

6. Alice is now twice as old as Paul. Three years ago she was two years younger than three times Paul's age. How old are Alice and Paul now?