COM1002: Foundations of Computer Science
Problem Sheet 6: Eigenvectors and Diagonalisation

1. Find the characteristic polynomial, eigenvalues and all the corresponding eigenvectors of the matrix
$$A = \begin{pmatrix} 1 & 4 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{pmatrix}.$$

2. Find the characteristic polynomial, eigenvalues and all the corresponding eigenvectors of the matrix
$$B = \begin{pmatrix} 3 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & -1 \end{pmatrix}.$$

3. Show, directly from the definition of eigenvalue, that 0 is an eigenvalue of the matrix
$$N := \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$
Show, also directly from the definition of eigenvalue, that an arbitrary non-zero number k is not an eigenvalue of N.

Find all the eigenvectors of N.

4. Let
$$A = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$$
   (a) Find the eigenvalues of A, and for each eigenvalue a corresponding eigenvector.
   (b) Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

5. Let
$$B = \begin{pmatrix} 1 & 0 & 0 \\ 3 & -1 & 1 \\ -3 & 0 & 2 \end{pmatrix}.$$
   (a) Find the eigenvalues of B, and for each eigenvalue a corresponding eigenvector.
   (b) Find an invertible matrix P and a diagonal matrix D such that $P^{-1}BP = D$.

6. For the matrices A and B in the previous two questions, work out A^6 and B^6.