# COM1002: Foundations of Computer Science
# Problem Sheet 7: Combinatorics and Probability

1. In arranging people around a circular table, we take into account their seats relative to each other, not the actual position of any one person. Show that $n$ people can be arranged around a circular table in $(n-1)!$ ways.

2. Prove that at least two people in Sheffield (population 534,500) have the same initials, assuming no one has more than four initials.

3. A poker hand consists of 5 cards from an ordinary deck of 52 cards.

   (a) How many different poker hands are there?

   (b) A *flush* is a poker hand where all cards have the same suit. How many flushes are there?

   (c) *Three of a kind* is a poker hand where three cards show the same number. How many three of a kind hands are there?

4. When drawing three cards from an ordinary 52 card deck in order, how many draws are there where the first card is an ace, the second card a two, and the third card a three?

5. When rolling an ordinary fair six-sided dice twice, work out the following probabilities:

   (a) The probability that the total of the two rolls is 8 or higher.

   (b) The probability that the first die roll is odd.

   (c) The probability that the total of the two rolls is 7 or lower.

   (d) The probability that the first die roll is odd, and the total of the two rolls is 8 or higher.

6. Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a prize; behind the other two, no prize.

   You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which reveals no prize. He then says to you, "Do you want to pick door No. 2?"

   Is it to your advantage to switch your choice?

7. What is the probability that in a group of $n$ people there will be at least one pair with the same birthday?

   How large must $n$ be so that this probability is at least $1/2$. Assume that all birthdays are equally likely, and that the birthdays of different people are independent (so no twins, triplets and quadruplets in the group!)

8. To play the national lottery, you pick 6 different numbers from $1$ to $49$. In the prize draw, six numbers are drawn from a set of individually numbered balls with numbers in the range $1$ to $49$. Balls, once drawn are not returned to the draw machine. Prizes are awarded to players who match at least three of the six drawn numbers with increasing prize value for matching more of the drawn numbers.

   (a) Calculate the probability of matching three numbers.

   (b) Calculate the probability of matching four numbers.

   (c) Calculate the probability of matching five numbers.

   (d) Calculate the probability of matching all six numbers.

   (e) There is also a bonus ball, which is drawn after the six main numbers. A player wins a special prize if their numbers match five of those drawn initially, along with the bonus ball. Calculate the probability of this.

9. A population consists of $53\%$ men. The probability of colour blindness is $0.02$ for a man and $0.001$ for a woman. Find the probability that a person picked at random is colour blind.

10. On a particular journey, the probability of a sober driver having an accident is $0.001$, and the probability of a drunk driver having an accident is $0.1$. The probability that a given driver is drunk is $0.08$.

    If a driver has an accident, how likely is it that they were drunk?