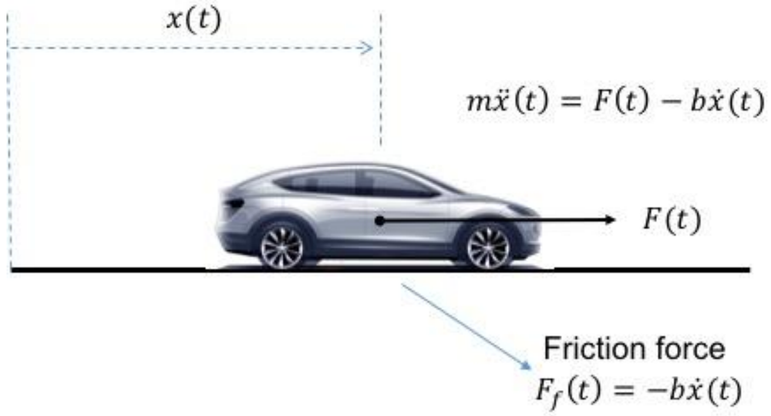
# Week 2 excersises

HiOA ELTS2300 Dynamic systems

## Simplified car model

Consider a simplified model of a car moving due to the action of motor generated force $F(t)$. The position of the car is given by $x(t)$. The combined air and road friction force $F_f(t)$ is assumed linear and proportional to car velocity with friction constant $b$. That is $F_f(t) = -b\dot{x}(t)$.




1) Write the system equations in state space form $\dot{\boldsymbol{x}} = f(\boldsymbol{x}, u)$, where $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2$, $x_1 = x$, $x_2 = \dot{x}$, and $u = F$. (Note the difference between bold symbol $\boldsymbol{x}$, used to represent the state vector, and position $x$ which does not use bold). (Note: Remember that we will try to use this notation during the course: bold symbols represent vector or matrix variables, whereas non bold symbols represent scalar variables).

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = f(\boldsymbol{x}, u) = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{u}{m} - \frac{b}{m} x_2 \end{bmatrix}$$

$$u = F$$

2) What are the units of all the involved variables and parameters $m, F, b, x, \dot{x}, \ddot{x}$ ? Verify that the simplified mathematical model is consistent with respect to the units.
3) Write the system equations in state form as a linear system $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u$, where $\boldsymbol{A} \in \mathbb{R}^{2\times 2}$, $\boldsymbol{B} \in \mathbb{R}^{2\times 1}$

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{F}{m} - \frac{b}{m} x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{m} \end{bmatrix} \in \mathbb{R}^{2\times 2}$$

$$\boldsymbol{B} = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \in \mathbb{R}^{2\times 1}$$

4) Later in the course you will learn to use the inverse Laplace transform to obtain the closed form solution of this system for some types of simple input functions. For example, when the motor force is constant, and the car starts at rest (zero velocity) it is possible to find a closed for solution for the car velocity given by:

$$v(t) = \dot{x}(t) = \frac{F}{b}\left(1 - e^{-\frac{b}{m}t}\right)$$

   a. Write a Matlab script to plot the response of the car velocity for different values of $m$, $F$, and $b$. Plot velocity as a function of time.
   b. What is the effect of the different parameters? How do they affect maximum speed, and acceleration? Can you guess what is the effect of the quotients $\frac{F}{b}$ and $\frac{b}{m}$ on the car model velocity response?

5) Suppose that we are trying to obtain a simple model of a Tesla model X which has an empty weight of approximately 2400kg, a maximum speed of 250km/h and accelerates between 0-100km/h in 4 seconds.
   a. Can you try to find values of $F$, and $b$ that produce similar results?
   b. According to the value of $F$ that you found, what is the power of the car at its maximum speed? How does it compare with the car specifications of 386kW? Hint: Power can be computed by $P = Fv$, where $F$ is the force in Newtons, $v$ is the speed in m/s, and $P$ is power in Watts.

6) Given a dynamic system state space form

$$\dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t), u(t))$$

The solution can be found by numeric integration. One of the simplest integration methods is the Forward Euler method. Starting from a given initial state $\boldsymbol{x}(0)$, the method computes the following states iteratively as

$$\boldsymbol{x}(t+h) = \boldsymbol{x}(t) + h\dot{\boldsymbol{x}}(t) = \boldsymbol{x}(t) + hf\big(\boldsymbol{x}(t), u(t)\big)$$

where $h$ is the integration step (in seconds).
   a. Implement in Matlab a Forward-Euler integrator to simulate the system. Plot the results. Plot position, velocity and acceleration as a function of time. Use $h = 0.01s$
   b. Experiment with different time steps $h$. Does it make any difference? What happens if you use a too large time step?
   c. Compare the results with the closed form solution in point 4. Do the results agree?

7) Use Matlab ode45 function to simulate the system. Compare the results with the closed form solution in point 4. and the forward Euler method. Do the results agree?

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