SEPARATING AXIS THEOREM

CS 447/547

Basic Geometry

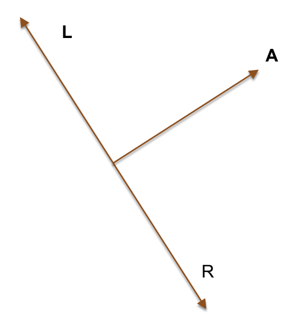
- Vectors
  - $\mathbf{A} = (x_a, y_a)$ (direction)
  - $\mathbf{P_0P_1}$, $\mathbf{P_1P_2}$ (start, end points)

Matrix Operations

- Rotation $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
- Scaling $\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$

Normals

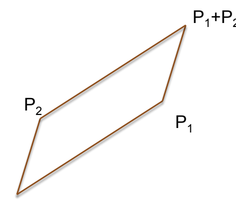
- $\mathbf{R} = \begin{bmatrix} -A_y \\ A_x \end{bmatrix}$
- $\mathbf{L} = \begin{bmatrix} A_y \\ -A_x \end{bmatrix}$



- Dot Product (Scalar Product)
  - $\mathbf{A} \bullet \mathbf{B} = x_a x_b + y_a y_b$
- Cross Product
  - $\mathbf{A} \times \mathbf{B} = x_a y_b - x_b y_a = -\mathbf{B} \times \mathbf{A}$
- Magnitude & Unit Vector
  - $|\mathbf{A}| = (x_a x_a + y_a y_a)^{1/2}$
  - $\hat{\mathbf{A}} = \mathbf{A} / (x_a x_a + y_a y_a)^{1/2}$

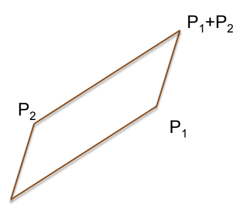
Geometric Interpretations

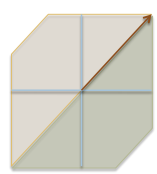
- Cross product is *signed area* parallelogram defined by two vectors



Geometric Interpretations

- $\mathbf{P_1} \times \mathbf{P_2}$ is positive if $\mathbf{P_1}$ is clockwise from $\mathbf{P_2}$ with respect to the origin





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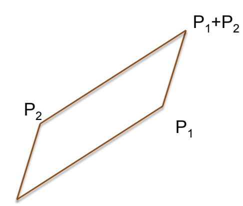
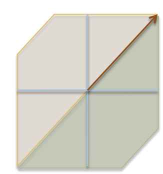
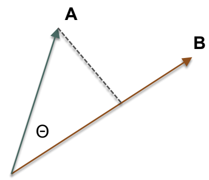




• Geometric Interpretations

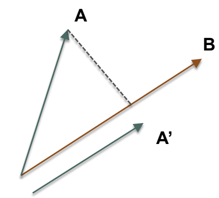
- $\mathbf{A} \bullet \mathbf{B} = |\mathbf{A}|\,|\mathbf{B}| \cos \Theta$



- $\mathbf{A} \bullet \mathbf{B} =$
  - Positive if they point in the 'same' direction
  - 0 if they are orthogonal
  - Negative if they point in the 'opposite' direction

Projection onto a Line

- $\mathbf{A'} = \begin{bmatrix} (\mathbf{A} \bullet \mathbf{B})\mathbf{B}_x \;/\; |\mathbf{B}|^2 \\ (\mathbf{A} \bullet \mathbf{B})\mathbf{B}_y \;/\; |\mathbf{B}|^2 \end{bmatrix}$

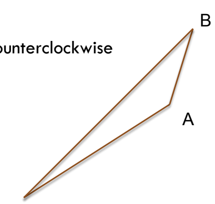


Determining Segment Turns

- Consider two consecutive line segments **A** and **B** that share an end point **p.** How can we tell if when travelling along **A** and then **B** we turn left or right?

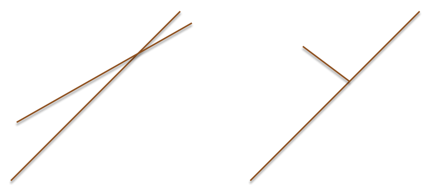
Determining Segment Turns

- Consider two consecutive line segments **A** and **B** that share an end point **p.** How can we tell if when travelling along **A** and then **B** we turn left or right?
- Check if directed segment:
  (origin, B) is clockwise or counterclockwise from (origin, A)



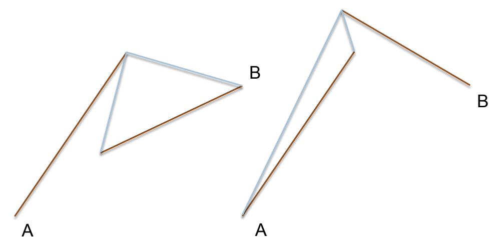
Line Intersection Test

- How can we use the cross product to check for line intersection?



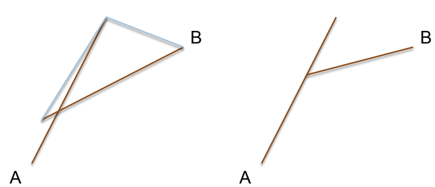
Line Intersection Test

- Check turns from **A** to end points of **B** (and vice versa)



Line Intersection Test

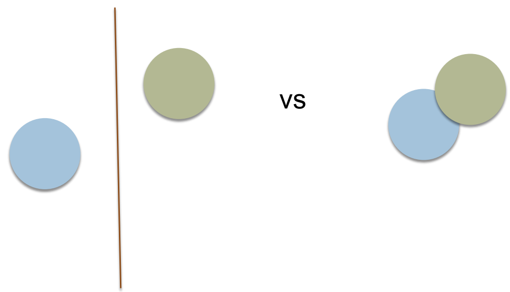
- Don't forget what to do if cross product is zero.



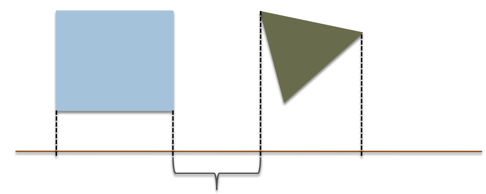
The Method of Separating Axes

Consider Some Objects

- If they don't intersect, we can draw a line between them. A *separating* line…

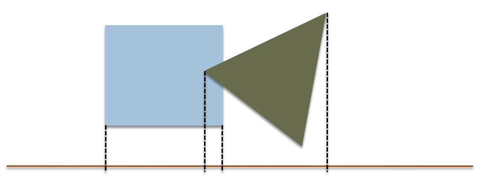


Consider Projections onto a Line



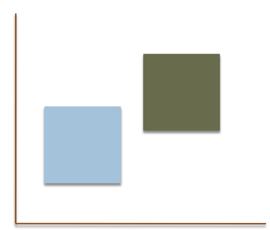
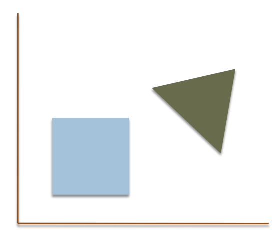
- Separation exists when projected onto axis if objects don't intersect

Consider Projections onto a Line



- No separation exists on any axis if objects do intersect

What Axes Need Checking?

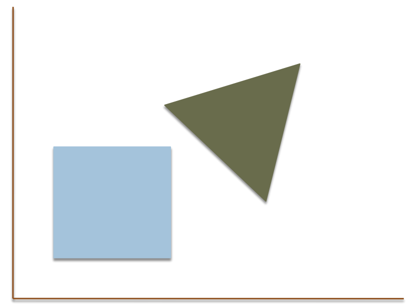
RECTANGLE / RECTANGLE	RECTANGLE / TRIANGLE
	

Axes Are Based on Each Object…

RECTANGLE

TRIANGLE

Axes normal to rectangle faces



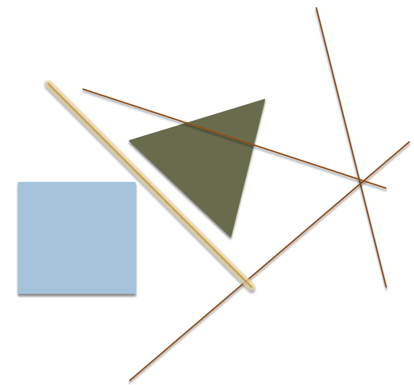
Axes Are Based on Each Object…

RECTANGLE

TRIANGLE

Axes are normal to triangles faces

Separation on one axis



Axes Are Based on Each Object…

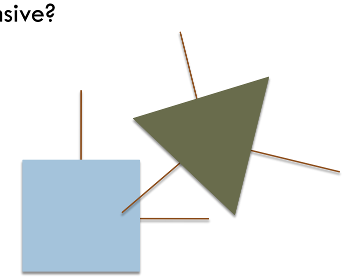
RECTANGLE

TRIANGLE

Axes are normal to triangles faces

Separation on one axis

- When is collision detection most expensive?



Minimum Separation

RECTANGLE

TRIANGLE

Axes are normal to triangles faces

Separation on one axis

- Which direction does the object need to move to resolve the collision with the least translation?