NCAA March Madness 2008 (EA) – Playstation3

Project Gotham Racing (Microsoft) – Xbox360

15-466 : Game Programming

Spring 2008

(Lecture 7)

James Kuffner
Carnegie Mellon University

Today's Overview

- Object-Object Proximity Queries (a.k.a "Collision Checking")
  - Binary Overlap Tests (pure collision checking)
  - Minimum Distance Computations
- Bounding Volume Hierarchies (BVHs)
  - Spheres, AABBs, OBBs, Swept Spheres
  - Convex Hulls

Geometric Proximity Queries Frequently Encountered

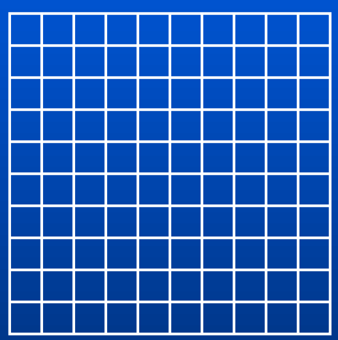
Given two geometric objects, determine:

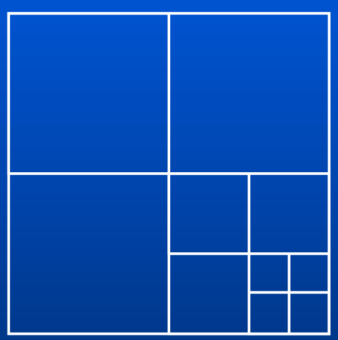
- If they intersect with each other?
- If they intersect, at what point(s) do they intersect?
- If they do not interpenetrate each other, how far are they apart? (minimum distance query)
- If they define volumes, do they overlap?
- If they interpenetrate, what is the penetration distance?

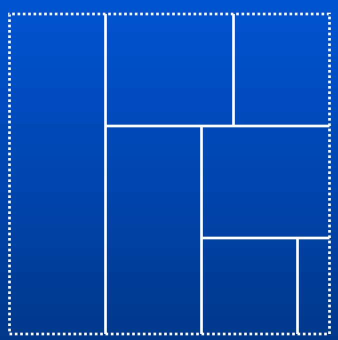
Source of Difficulty

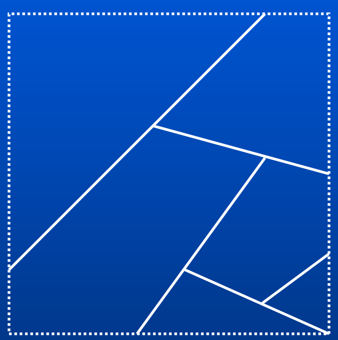
- The number, complexity, and arrangement of objects can greatly impact computational costs of proximity queries
- Number of Object pairs to be tested
  – N objects, naïve pairwise tests = $O(N^2)$
- For any pair of objects (e.g. two tri-meshes)
  – Geometric complexity of objects
    » F average object features, naïve pairwise tests = $O(F^2)$
  – Relative pose or motion over time

Spatial Data Structures & Subdivision

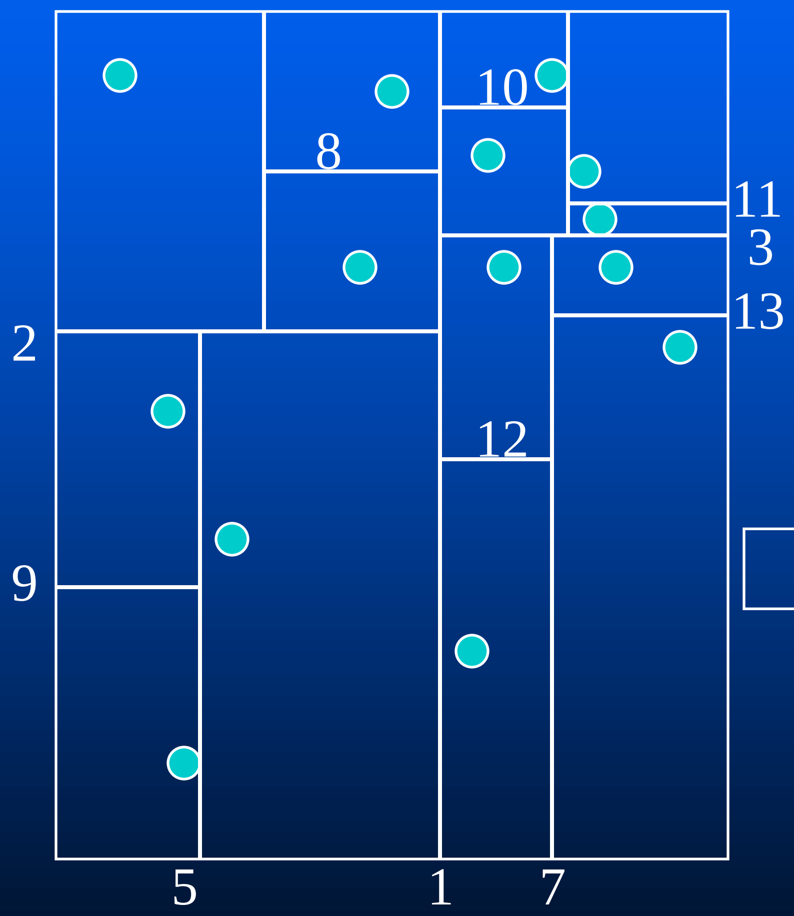

Uniform Spatial Sub

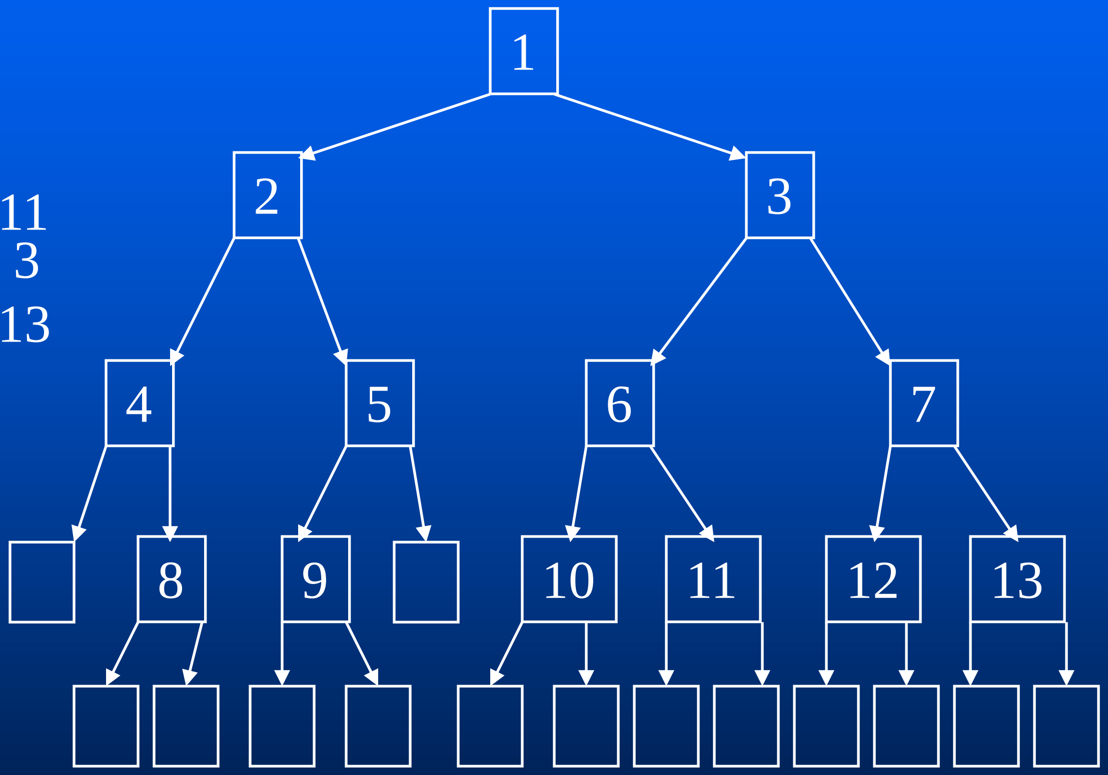

Quadtree/Octree


kd-tree

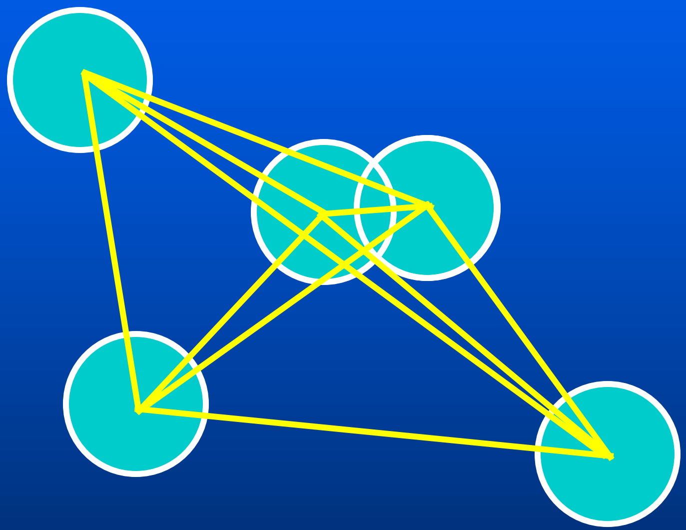

BSP-tree

kd-tree example

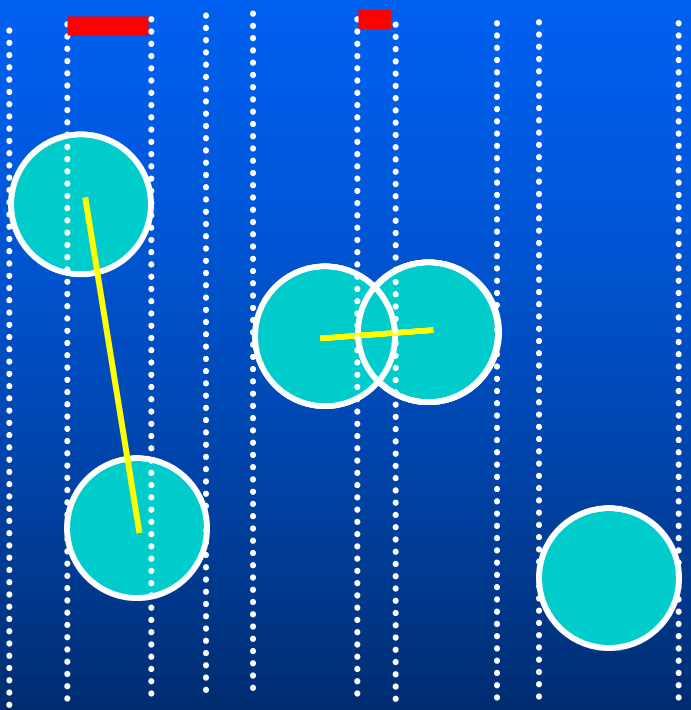




Collision Detection: Broad Phase vs Narrow Phase



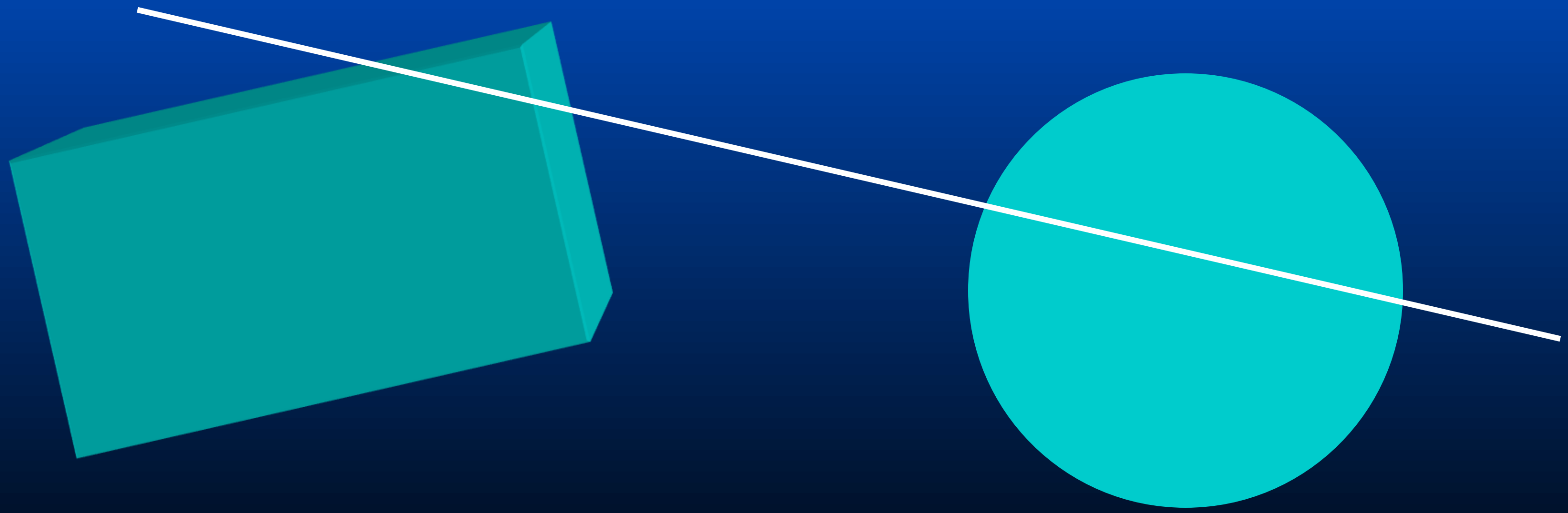
BROAD PHASE:
Determine Which
Pairs to Check



NARROW PHASE :
Actually Calculate
Individual Pair Tests

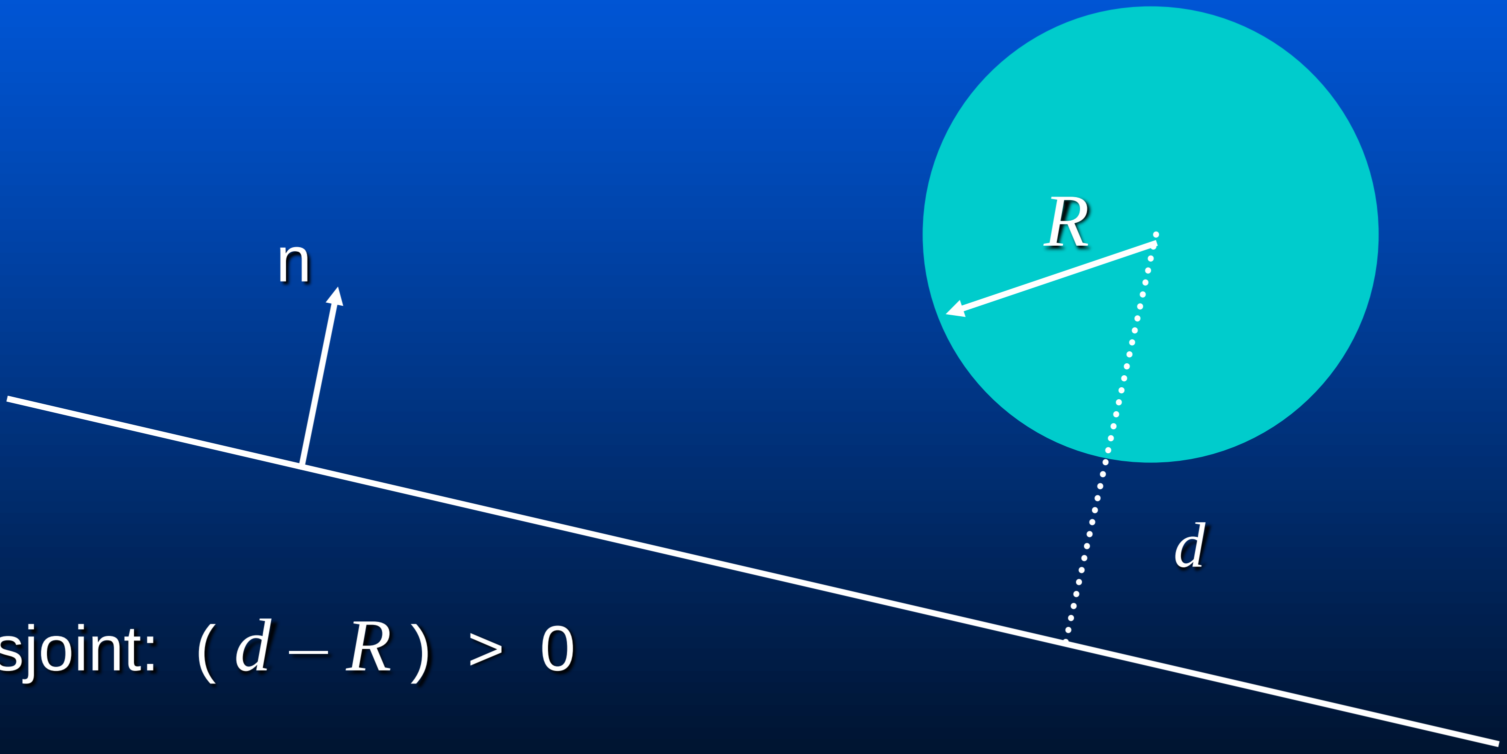
Intersection Tests for Boxes, Spheres, and Planes

- Frequently Used in Games
- Relatively Simple and Cheap to Compute



Sphere-Plane Intersection

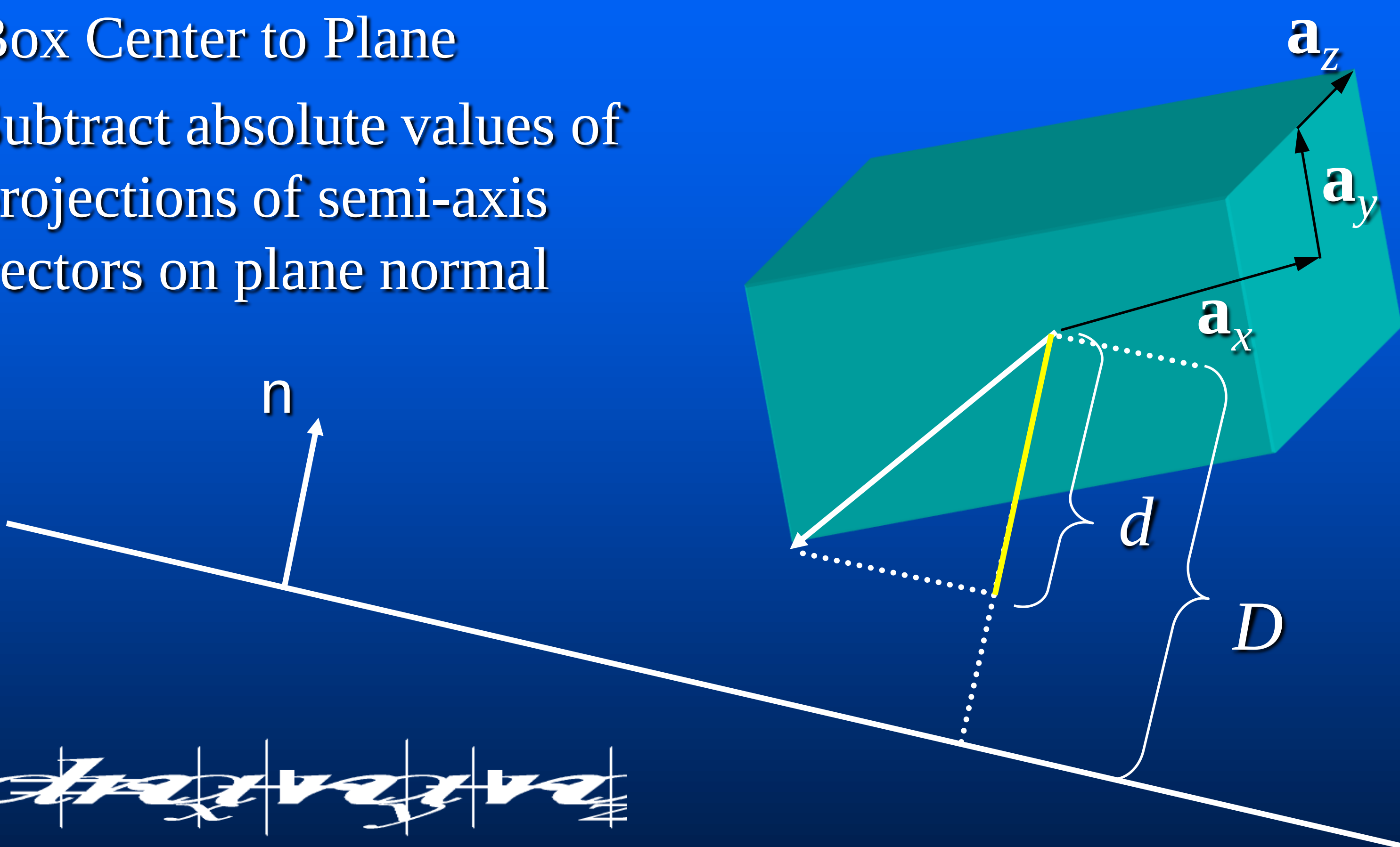
- Use distance from sphere center to plane



Disjoint: $(d - R) > 0$

Box-Plane Intersection

- Calculate Distance *D* from Box Center to Plane
- Subtract absolute values of projections of semi-axis vectors on plane normal

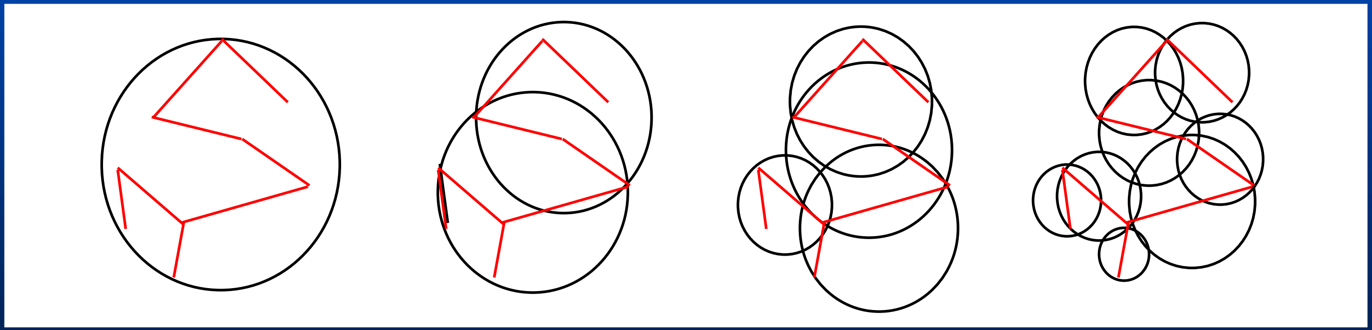


Disjoint: $(D - d) > 0$

Bounding Volume Hierarchies

- Model Hierarchy:
  - Simple volume that bounds a set of triangles
  - Nodes bound a subset of the parent's triangles
  - Leaves contain individual triangles

- Sample Binary BVH:



Can be built "top-down" or "bottom-up"

Example Bounding Volumes

- Spheres
- Ellipsoids
- Axis-Aligned Bounding Boxes (AABB)
- Oriented Bounding Boxes (OBBs)
- Convex Hulls
- *k*-Discrete Orientation Polytopes (*k*-dop)
- Spherical Shells
- Swept-Sphere Volumes (SSVs)

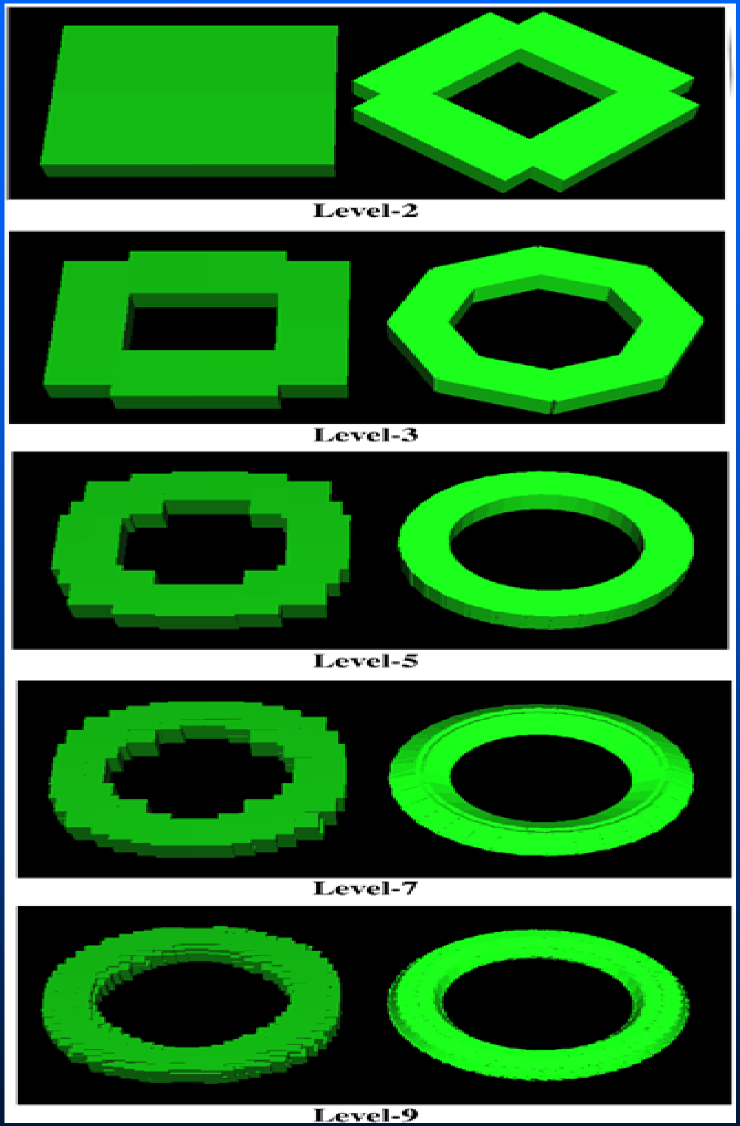
Trade-off in Choosing BV's

Sphere　AABB　OBB　6-dop　Convex Hull

Increasing:

- Complexity
- Tightness of Fit
- Cost of overlap test

Example: AABB's vs. OBB's

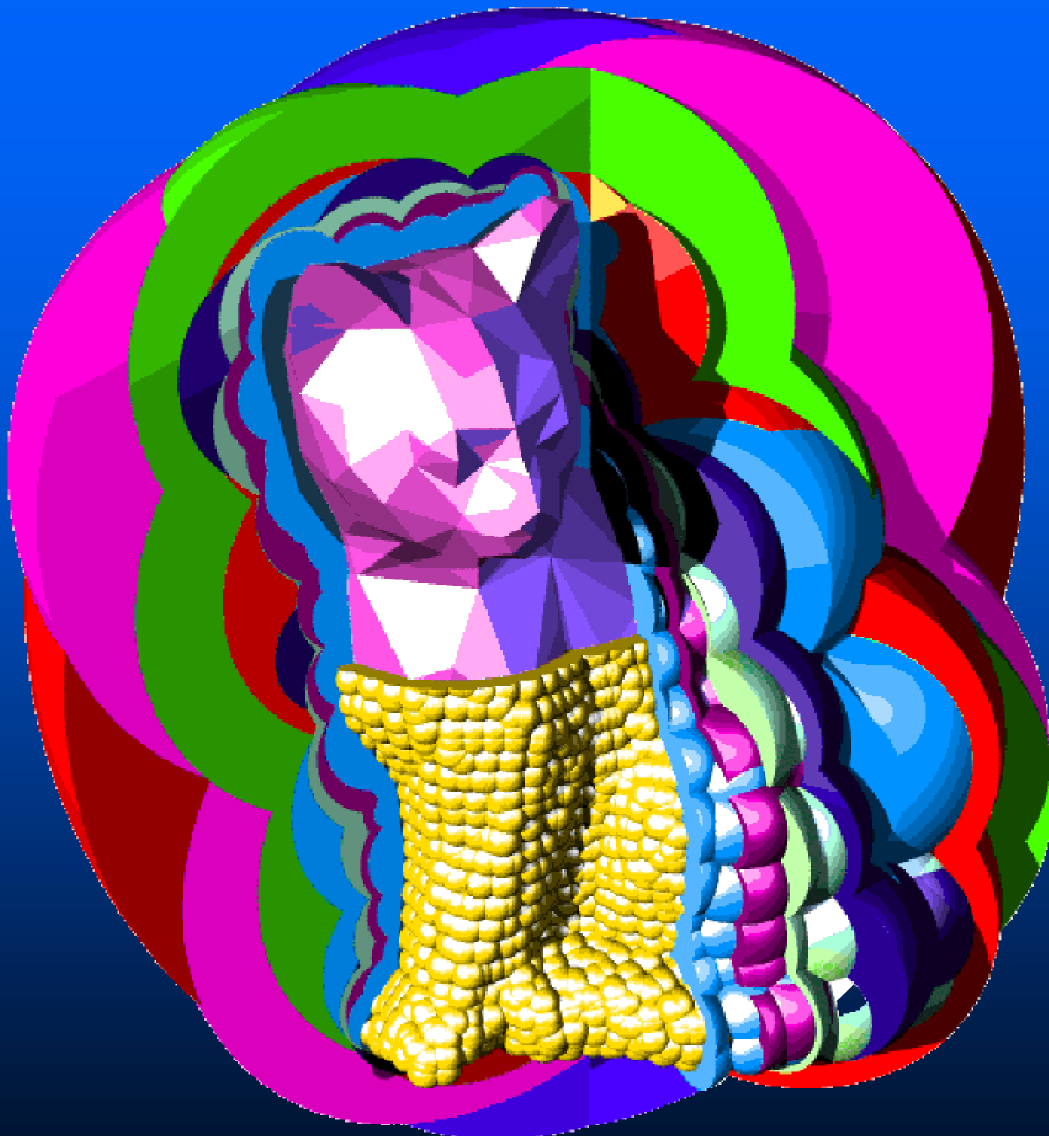


http://gamma.cs.unc.edu/

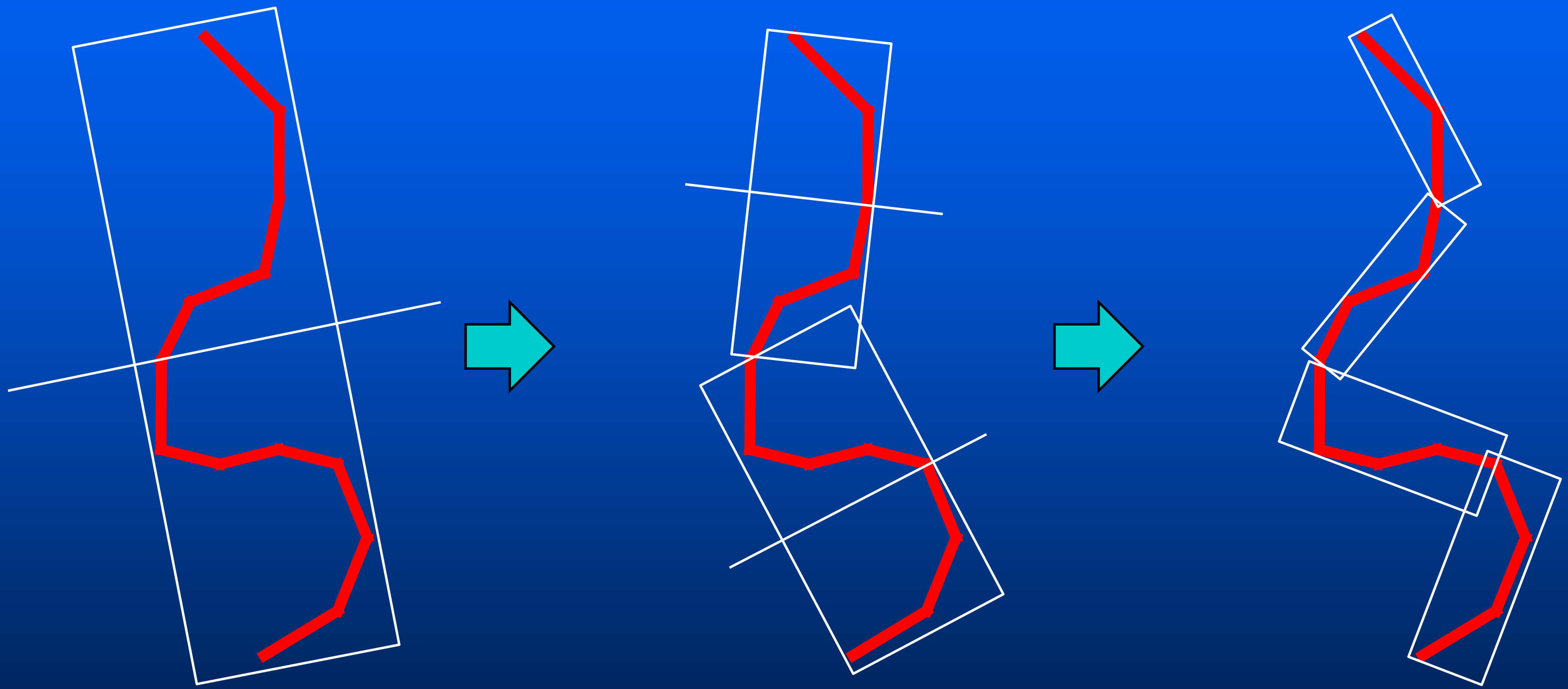
Approximation of a Torus

Image from:
OBB-Tree: A Hierarchical Structure for Rapid Interference Detection , *S. Gottschalk, M. C. Lin and D. Manocha* . Proc. of ACM Siggraph'96.

Sphere Tree Hierarchy

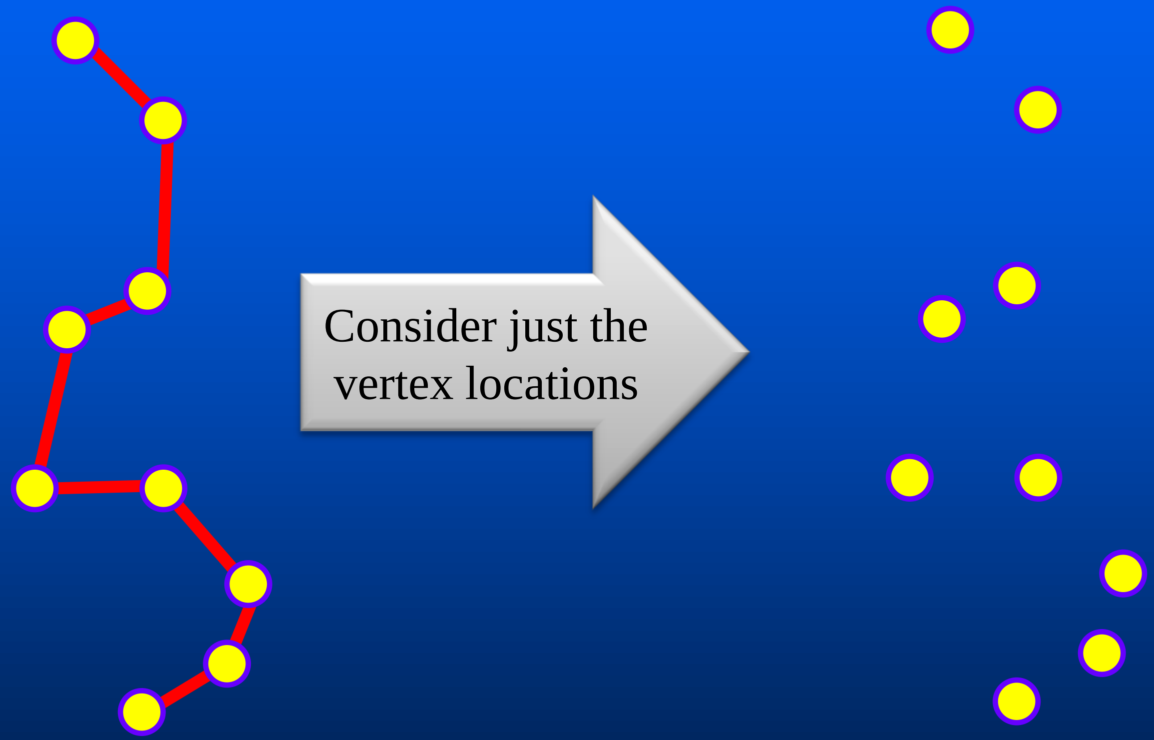


Building an OBBTree

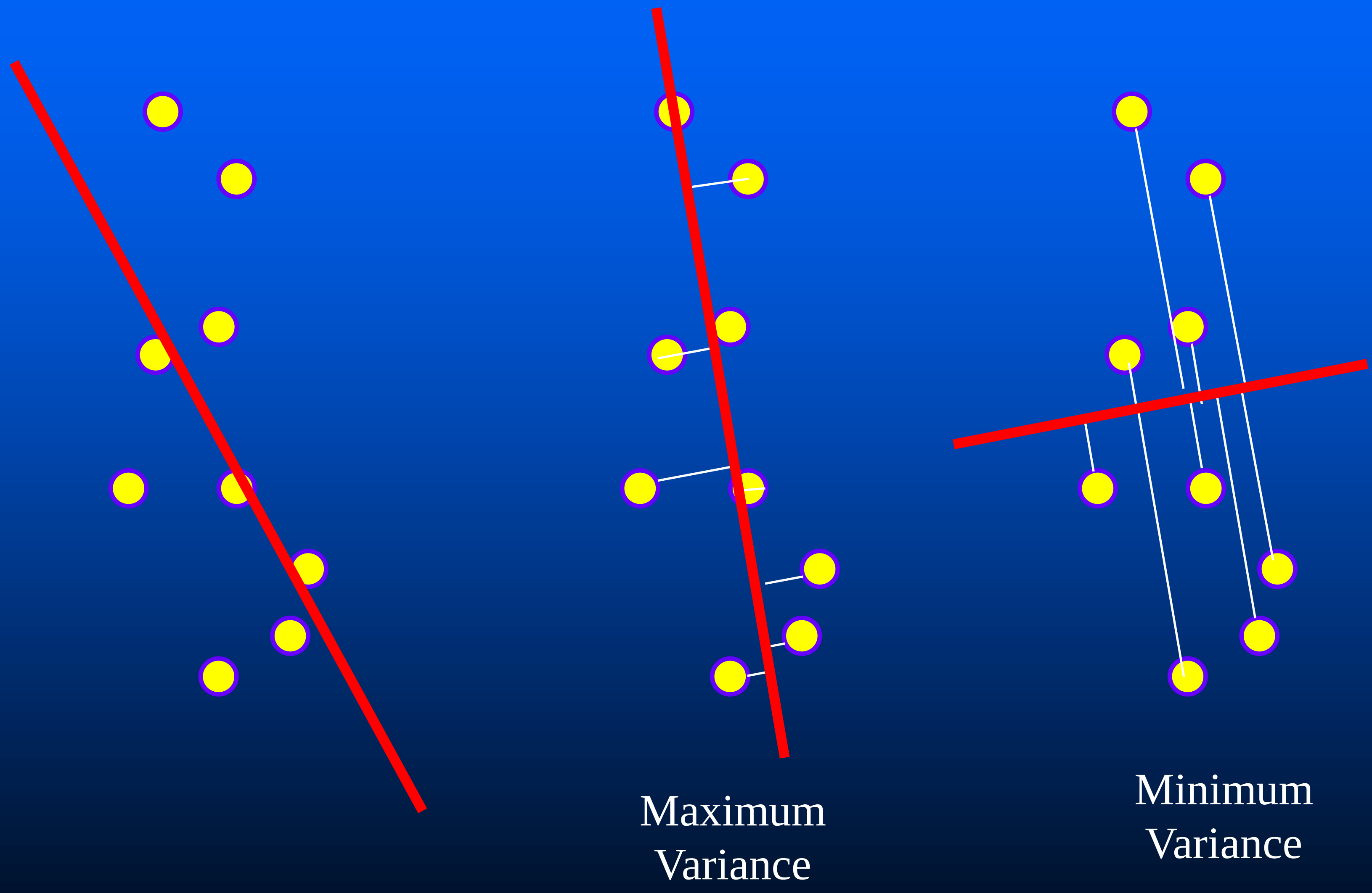


Recursive "top-down" construction by splitting the longest axis

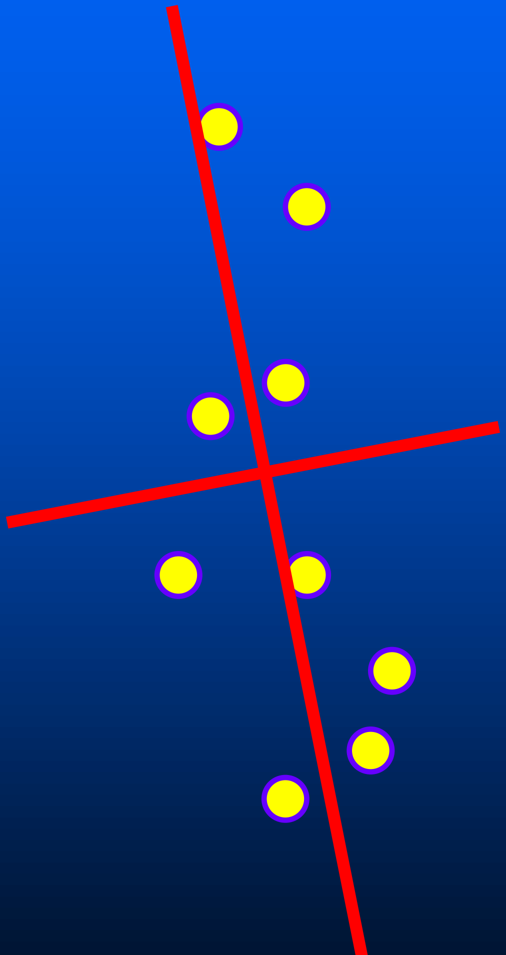
Building an OBB Tree



Fitting a line to the point distribtion

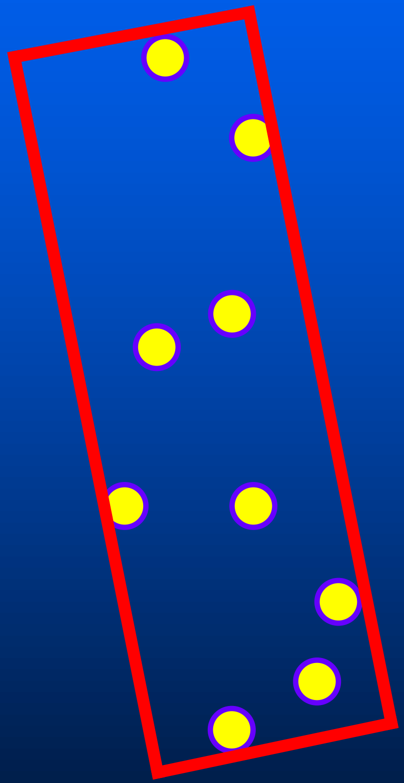


Eigenvectors of Covariance Matrix of Point Coordinates



Eigenvectors determine direction of OBB axes

Size of box determined by extremal points



Fitting OBBs

- Consider coordinates of a set of n points :

$$p^i = (p_x^i, p_y^i, p_z^i) \qquad i \in \{1, \cdots, n\}$$

- Let Covariance matrix C be defined as :

$$C = \begin{bmatrix} C_{xx} & C_{yx} & C_{zx} \\ C_{xy} & C_{yy} & C_{zy} \\ C_{xz} & C_{yz} & C_{zz} \end{bmatrix}$$

$$C_{xy} = \frac{1}{n} \sum_{i=1}^{n} (p_x^i - \mu_x)(p_y^i - \mu_y)$$

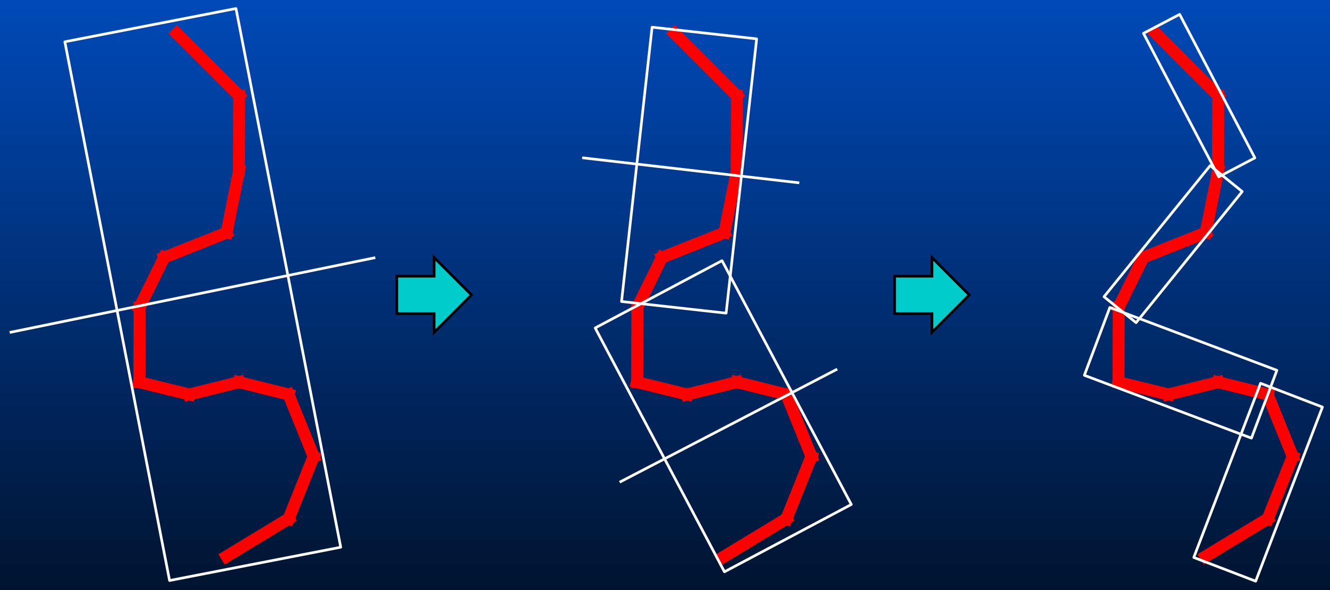
$C_{xx}, C_{xz}, C_{yz}, \cdots$ defined similarly

coordinate means:

$$\mu_x = \frac{1}{n} \sum_{i=1}^{n} p_x^i \qquad \mu_y = \frac{1}{n} \sum_{i=1}^{n} p_y^i \qquad \mu_z = \frac{1}{n} \sum_{i=1}^{n} p_z^i$$

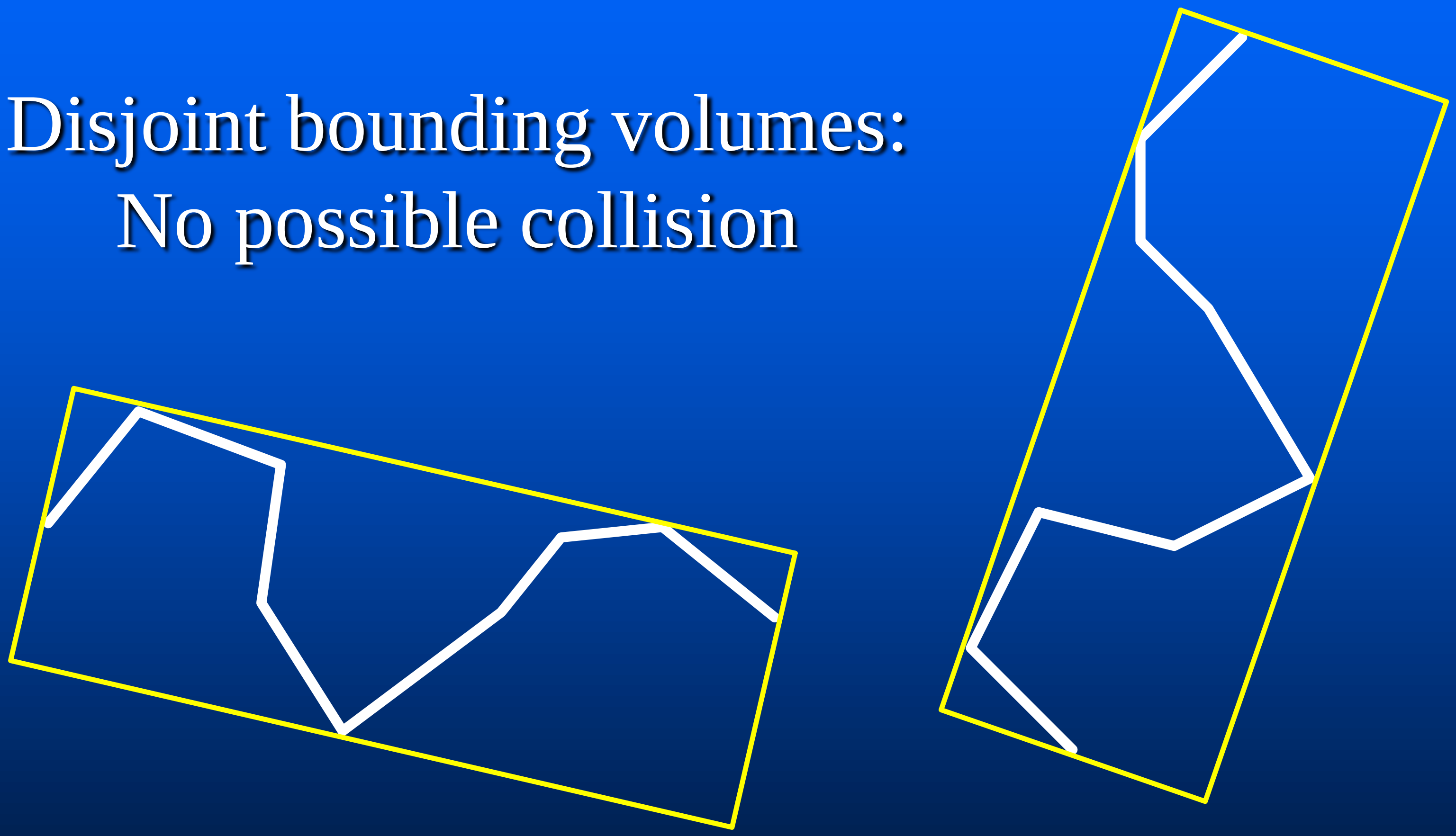
Building an OBB Tree: Summary

- Top-down construction
- Covariance based
- O(n log n) fitting time for single BV
- O(n log^2 n) fitting time for entire tree



Tree Overlap Testing : Tree Traversal

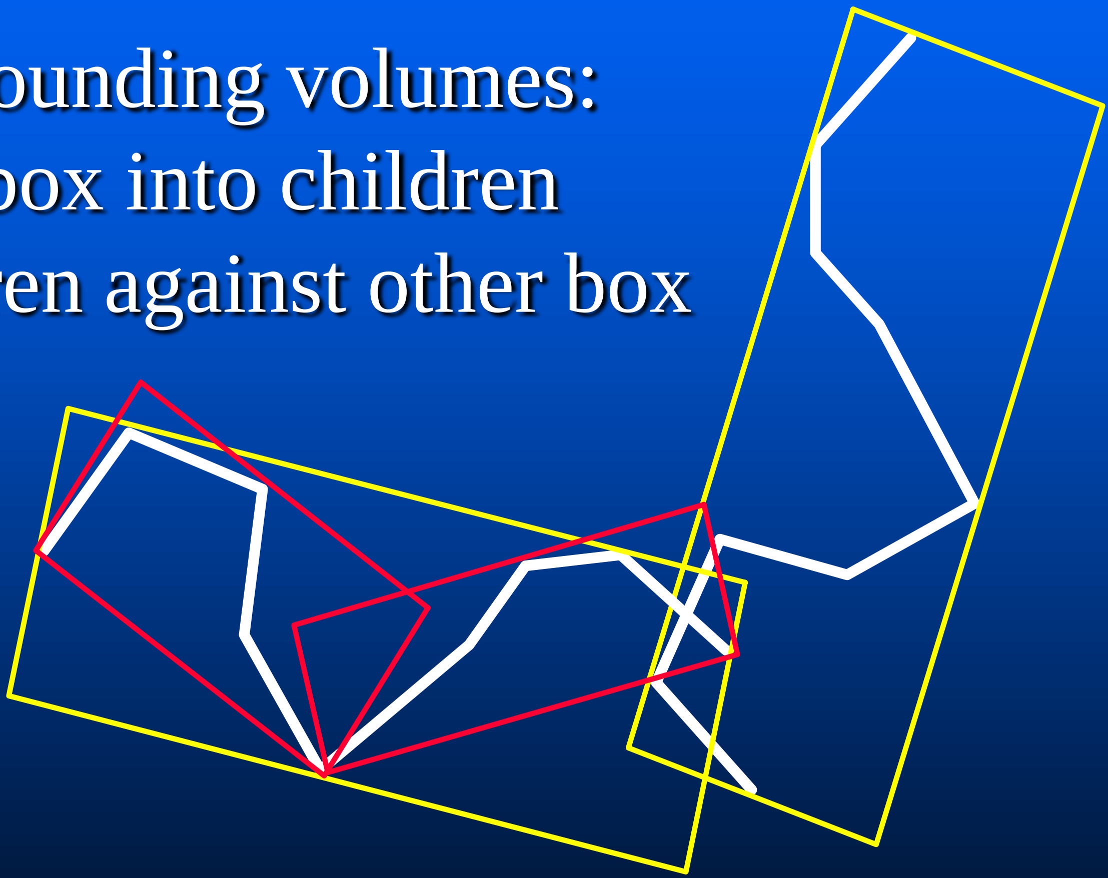
Disjoint bounding volumes:
No possible collision



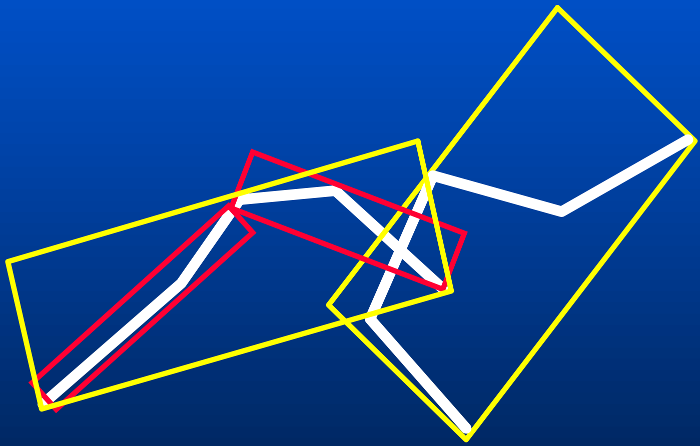
Tree Overlap Testing : Tree Traversal

Overlapping bounding volumes:

- split one box into children
- test children against other box

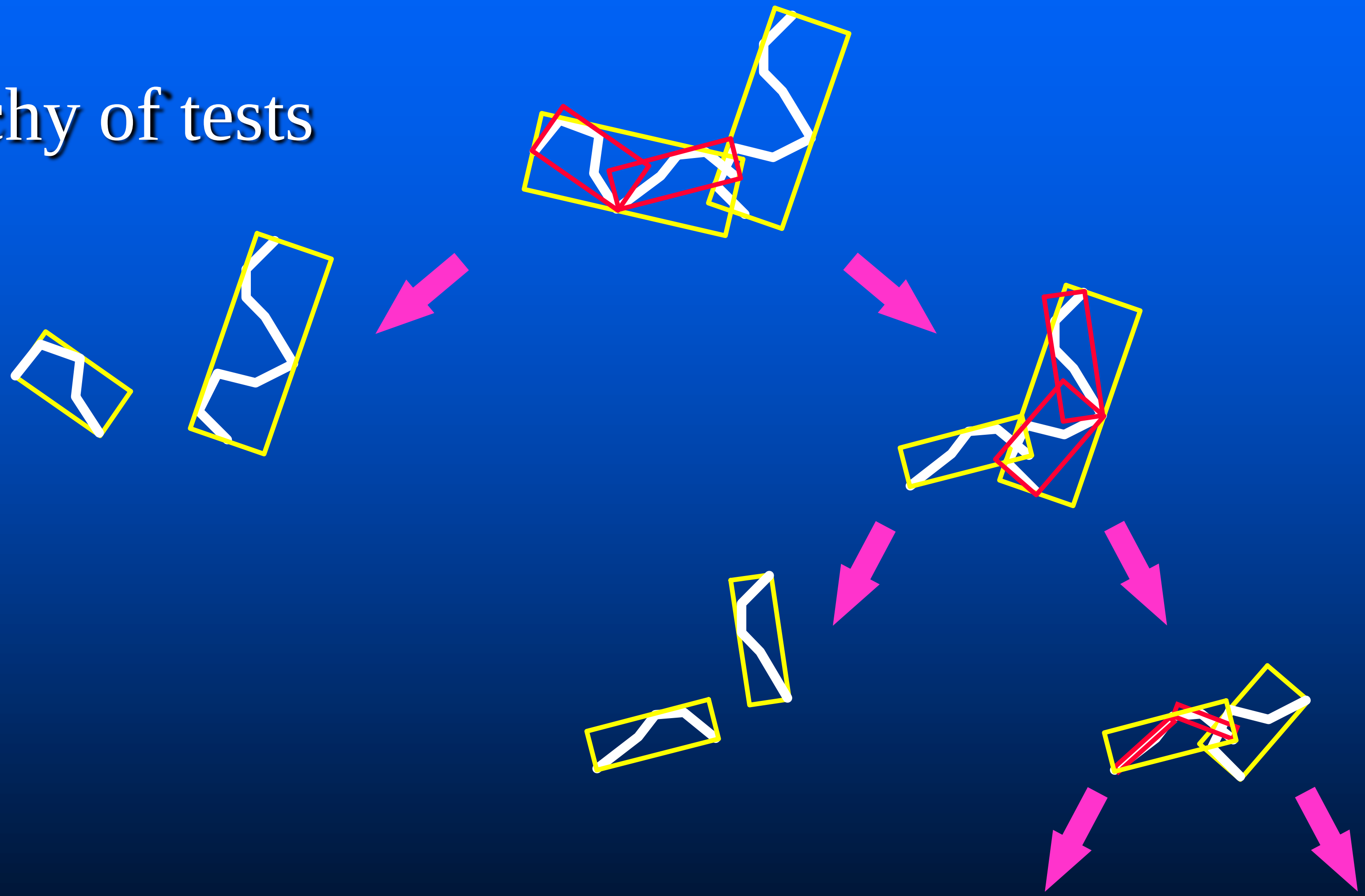


Tree Overlap Testing : Tree Traversal



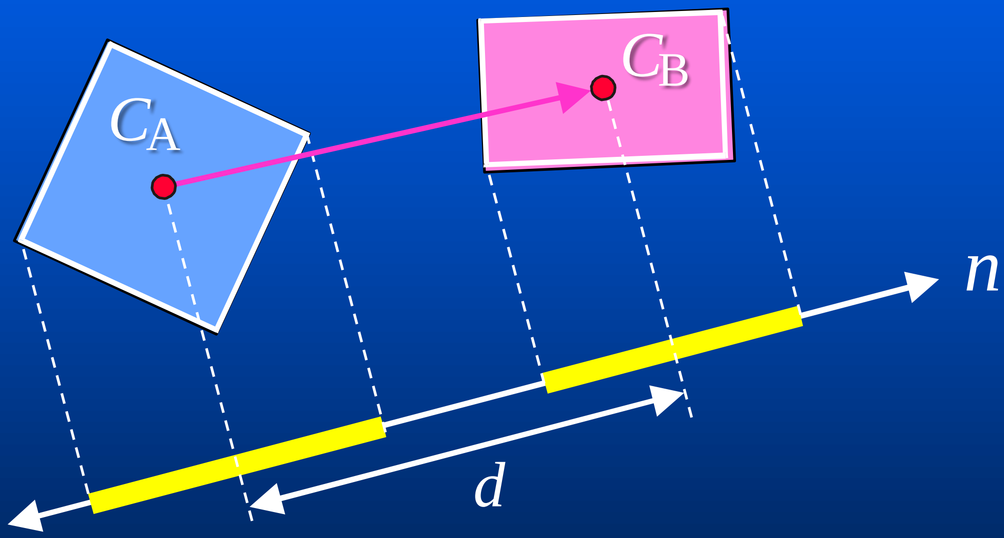
Tree Overlap Testing : Tree Traversal

Hierarchy of tests



OBB Separating Axis Theorem

- Test for overlap of projections of OBBs along line parallel to candidate separating plane normal *n*



$$d = \left| \left(C_B - C_A \right) \bullet n \right|$$

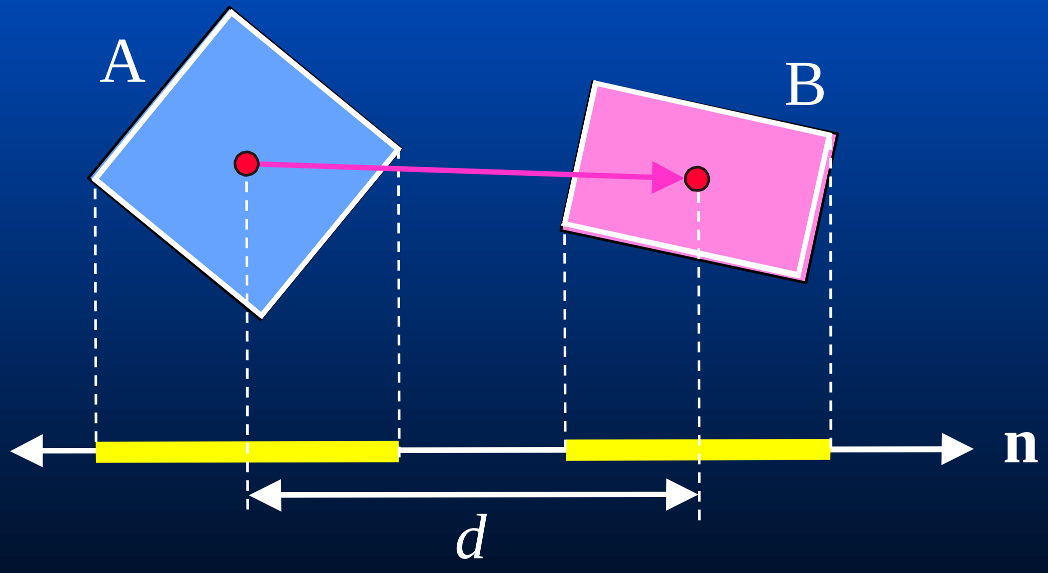
Separating Axis Theorem

Two polytopes A and B are disjoint iff there exists a separating axis which is :

1) perpendicular to a face normal from either

OR

2)perpendicular to an edge from each



Implications of Theorem

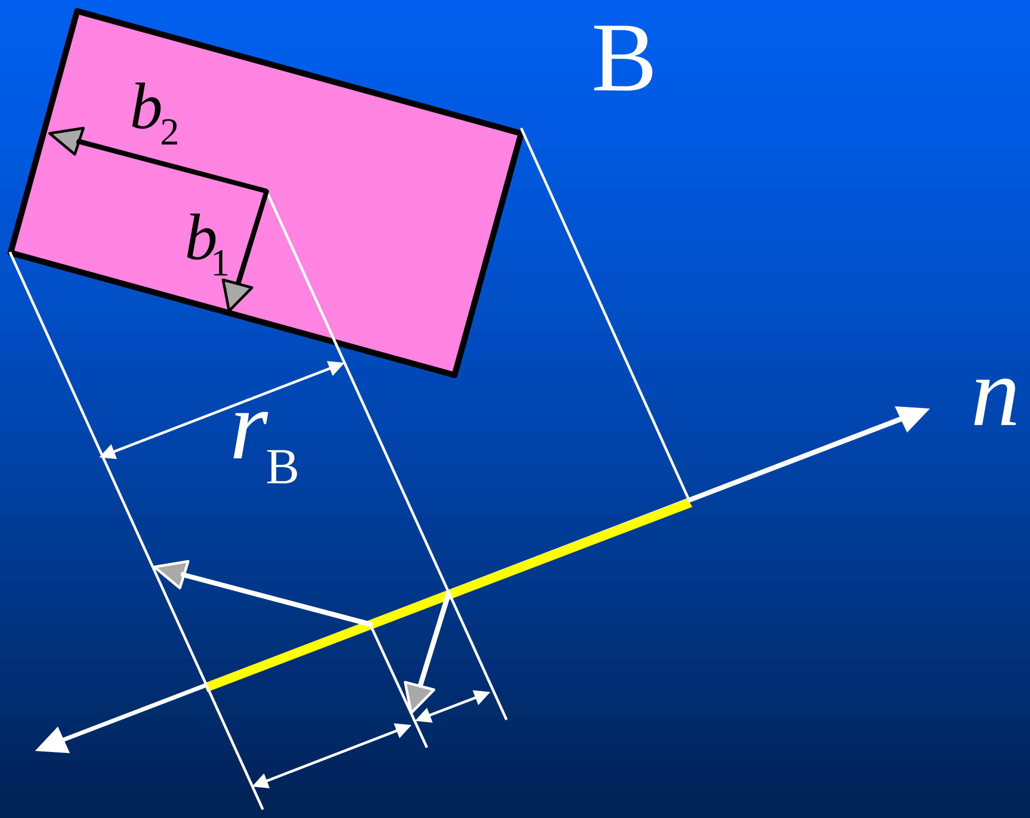
- Given two generic polytopes, each with E edges and F faces, number of candidate axes to test is:

$$2F + E^2$$

- OBBs have only $E = 3$ distinct edge directions, and only $F = 3$ distinct face normals. OBBs need at most $6 + 9 = $ **15** axis tests.
- Because OBB edge directions and normals each form orthogonal frames, the axis tests are rather simple.

OBB-OBB Overlap Test

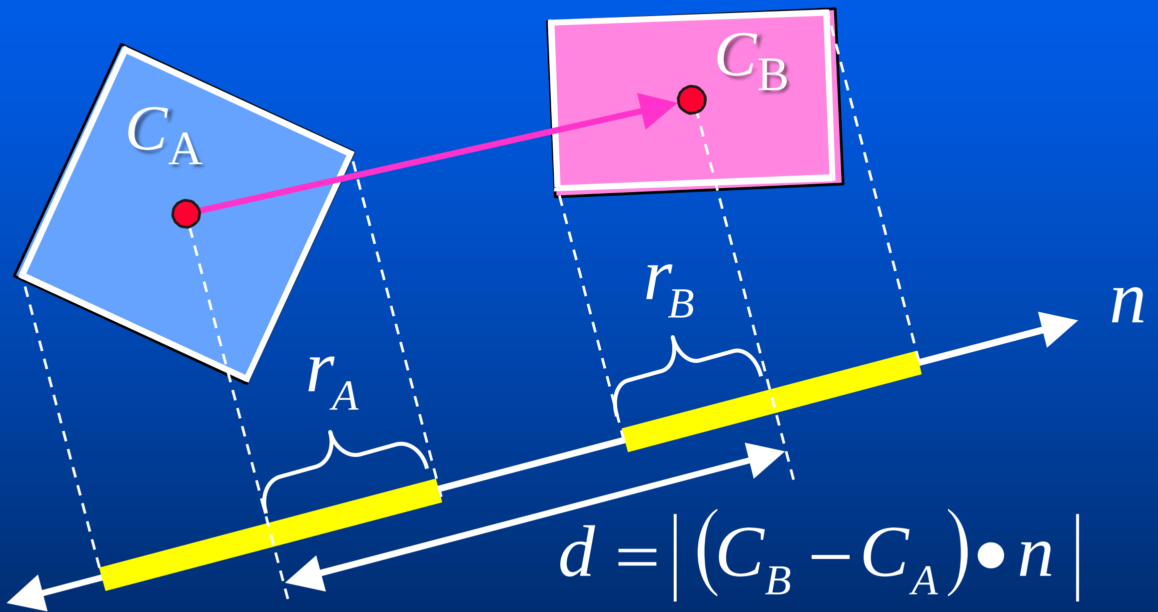
- Half-length of interval is sum of box axis projections.



$$r_B = |b_1 \bullet n| + |b_2 \bullet n| + |b_3 \bullet n|$$

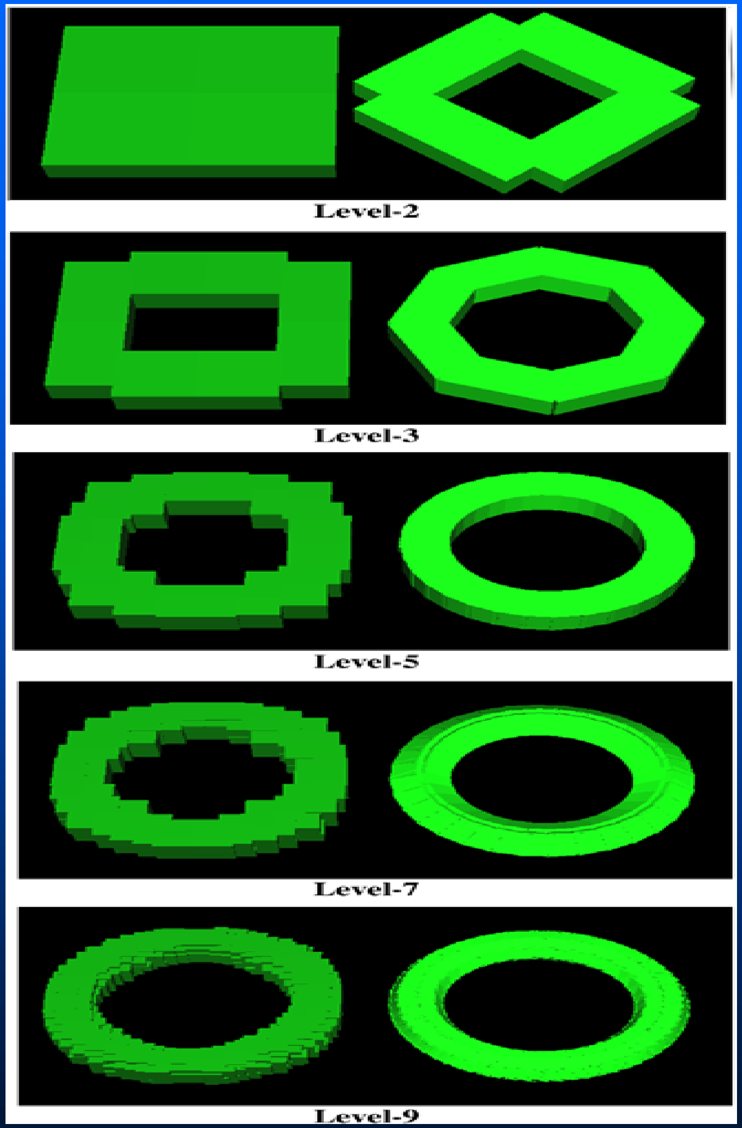
OBB Separating Axis Theorem

- Test for overlap of projections of OBBs along line parallel to candidate separating plane normal *n*



NO OVERLAP : $\left| \left(C_B - C_A\right) \bullet n \right| - r_A - r_B > 0$

Example: AABB's vs. OBB's



http://gamma.cs.unc.edu/

Approximation of a Torus

Image from:
OBB-Tree: A Hierarchical Structure for Rapid Interference Detection , *S. Gottschalk, M. C. Lin and D. Manocha* . Proc. of ACM Siggraph'96.

Sphere-Trees

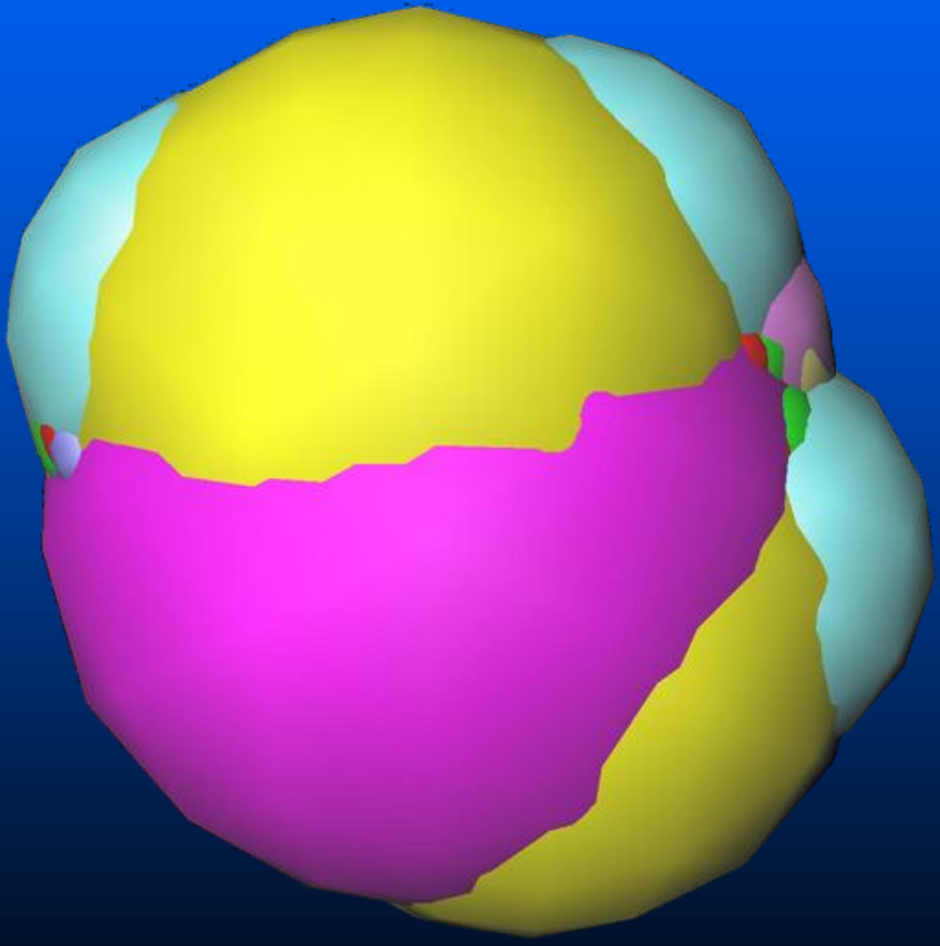
Hierarchy of sets of spheres

- Advantages:
  - Simplicity in checking overlaps between two bounding spheres
  - Invariant to rotations and can apply the same transformation to the centers, if objects are rigid

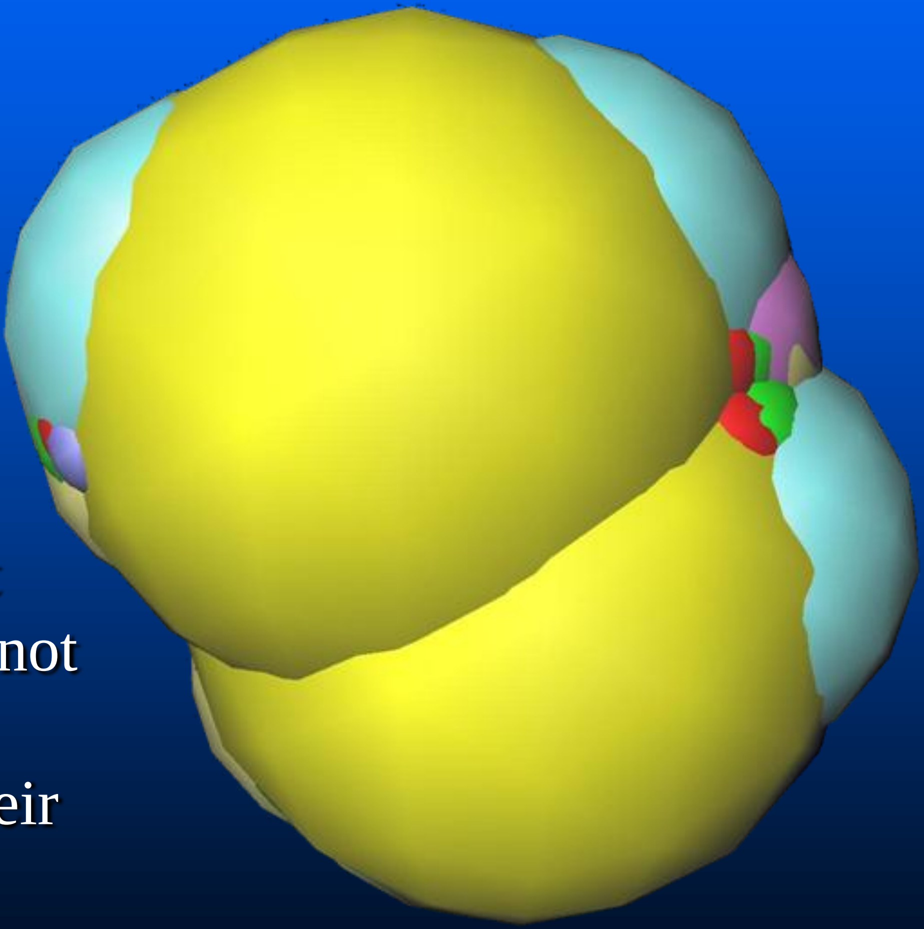
- Shortcomings:
  - Not always the best approximation (esp. bad for long, skinny objects)
  - Need good methods to build compact sphere-trees efficiently

Sphere Hierarchy (level 0)

Pink sphere (level 0) contains entire object geometry

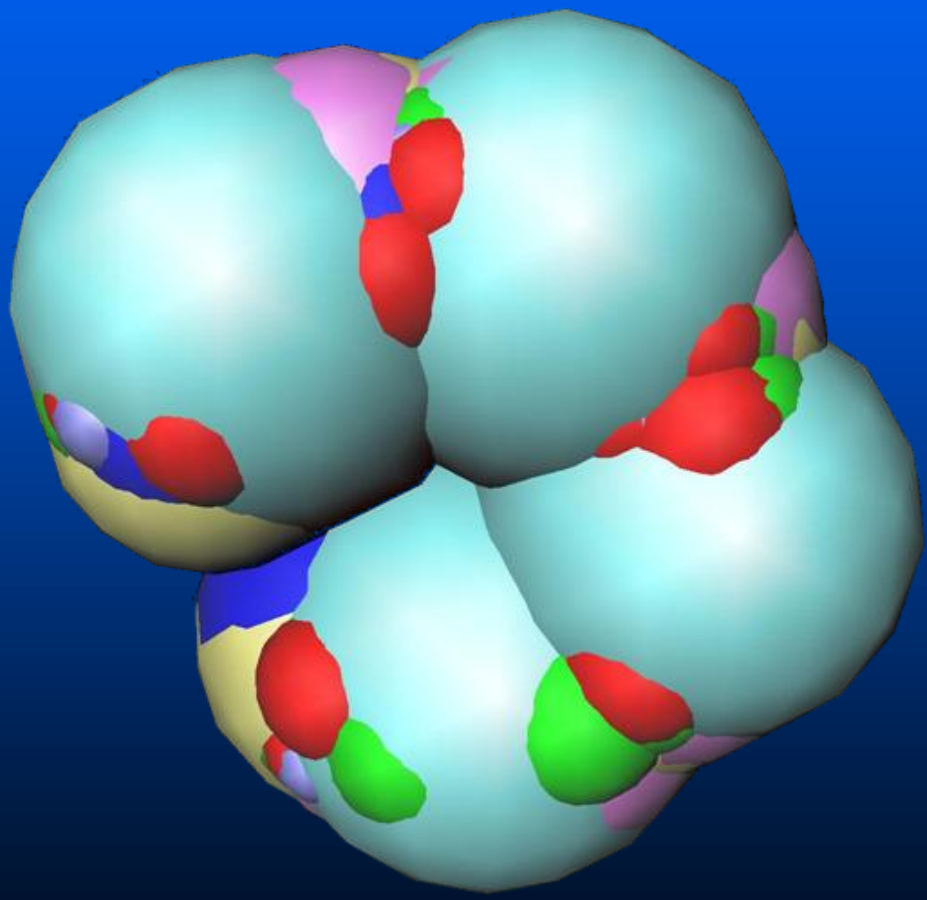


Sphere Hierarchy (level 1)

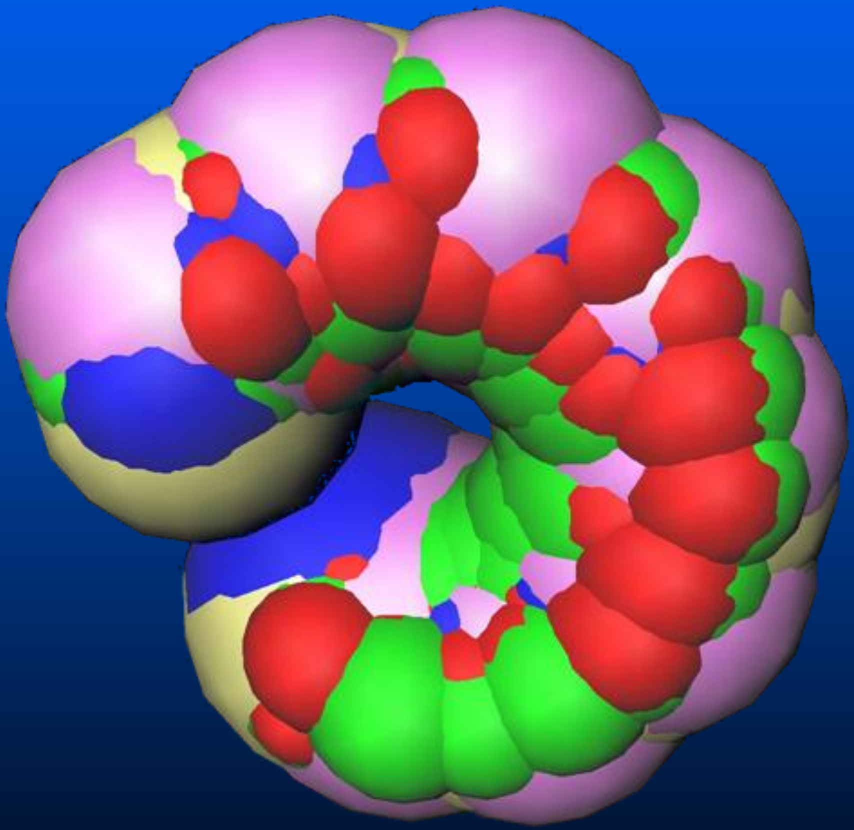


NOTE: spheres at each level may not be completely contained by their parent spheres

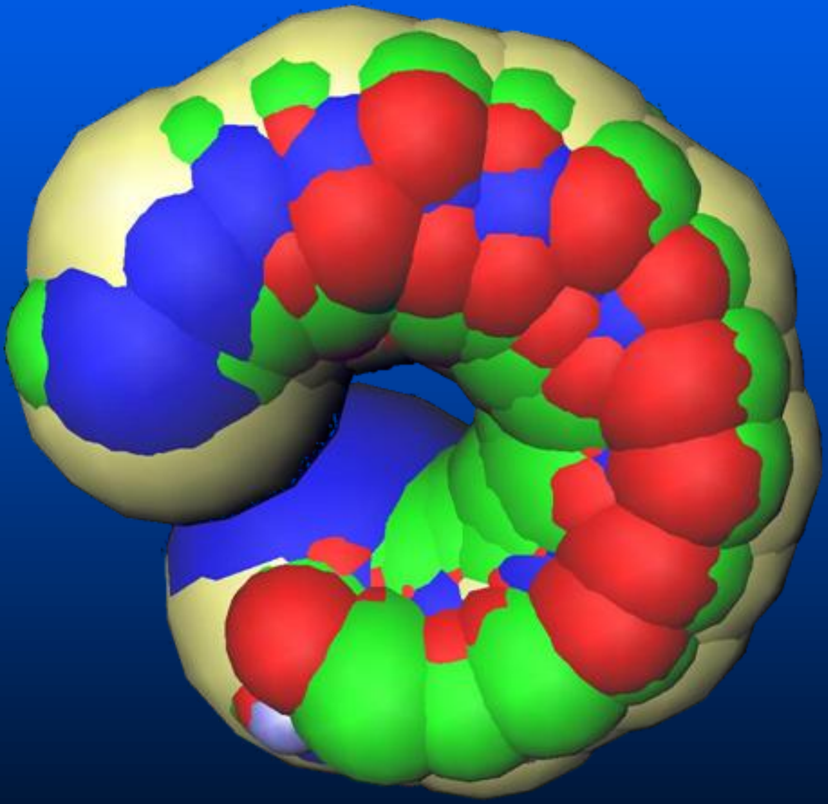
Sphere Hierarchy (level 2)



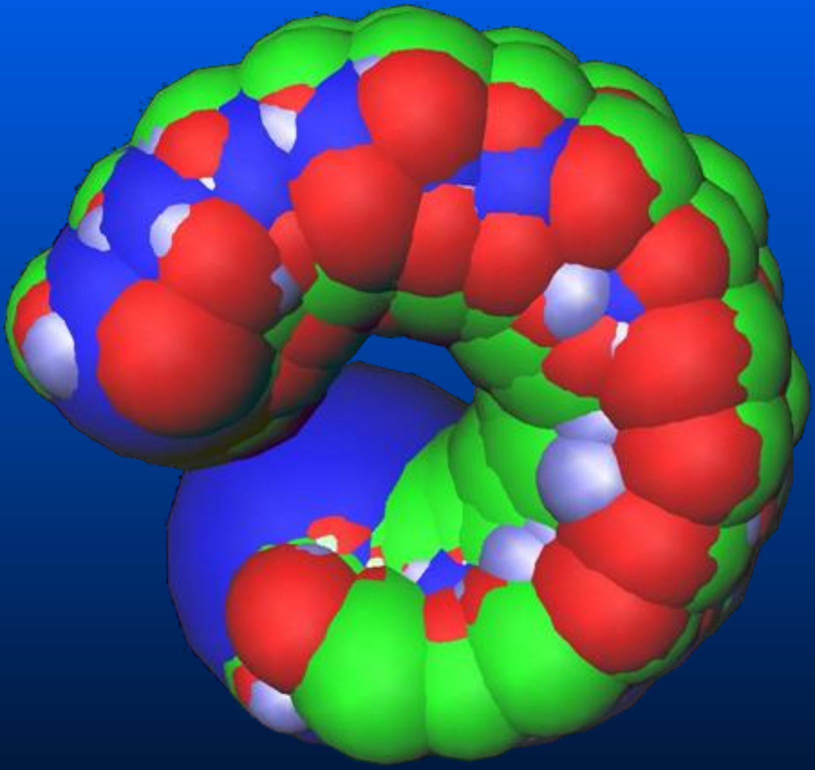
Sphere Hierarchy (level 3)



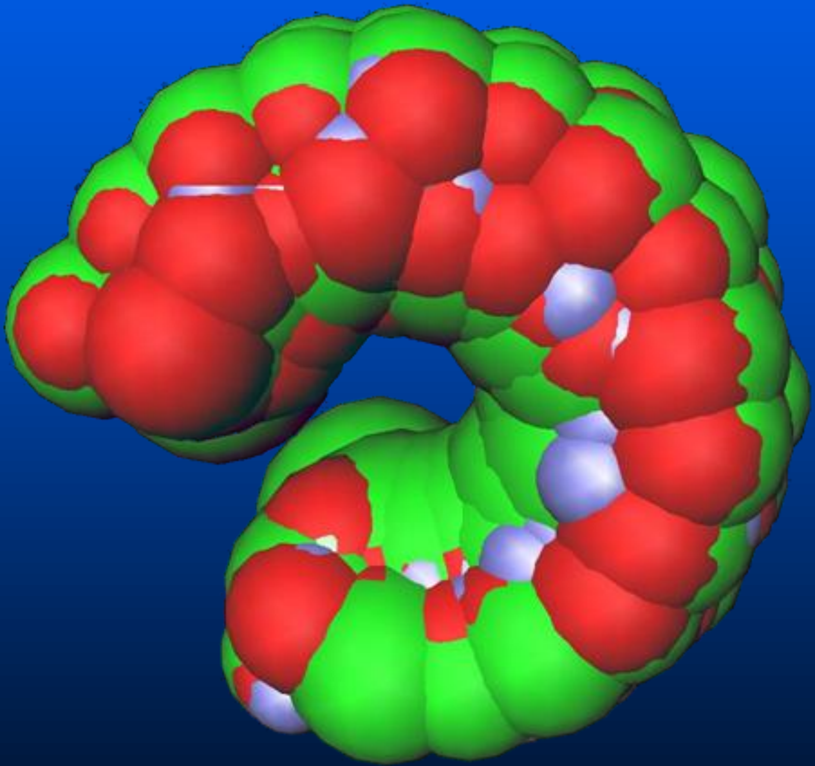
Sphere Hierarchy (level 4)



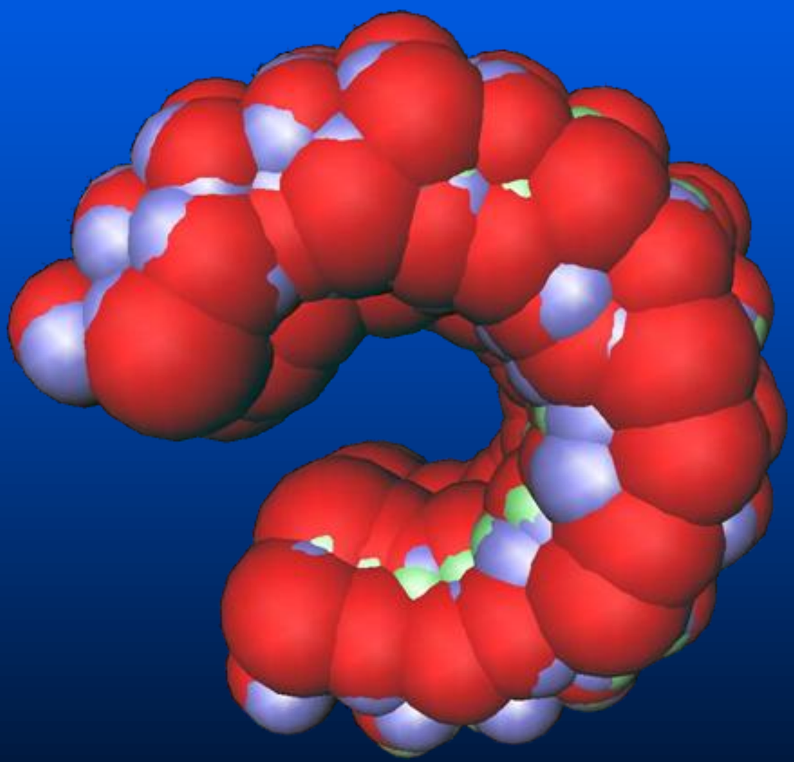
Sphere Hierarchy (level 5)



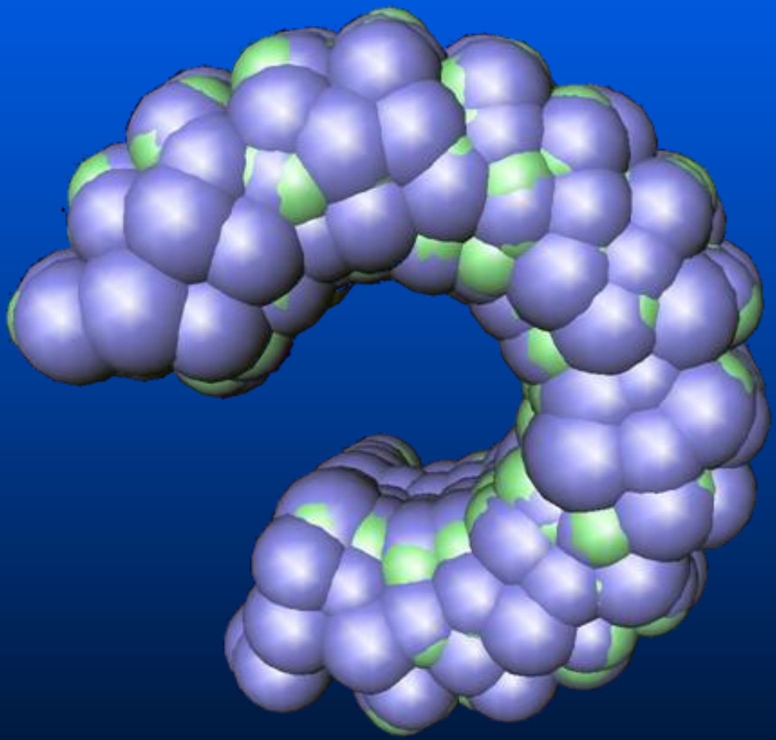
Sphere Hierarchy (level 6)



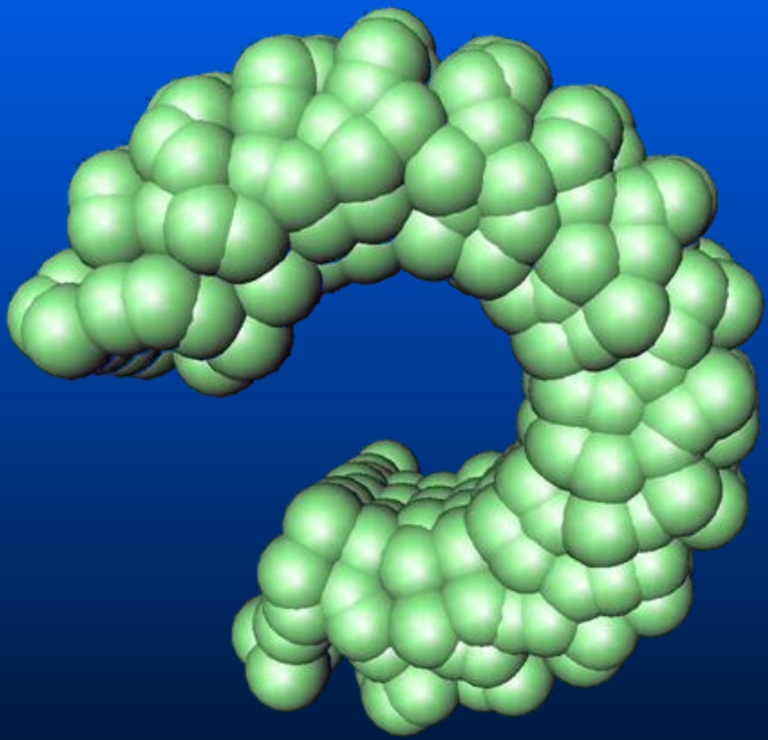
Sphere Hierarchy (level 7)



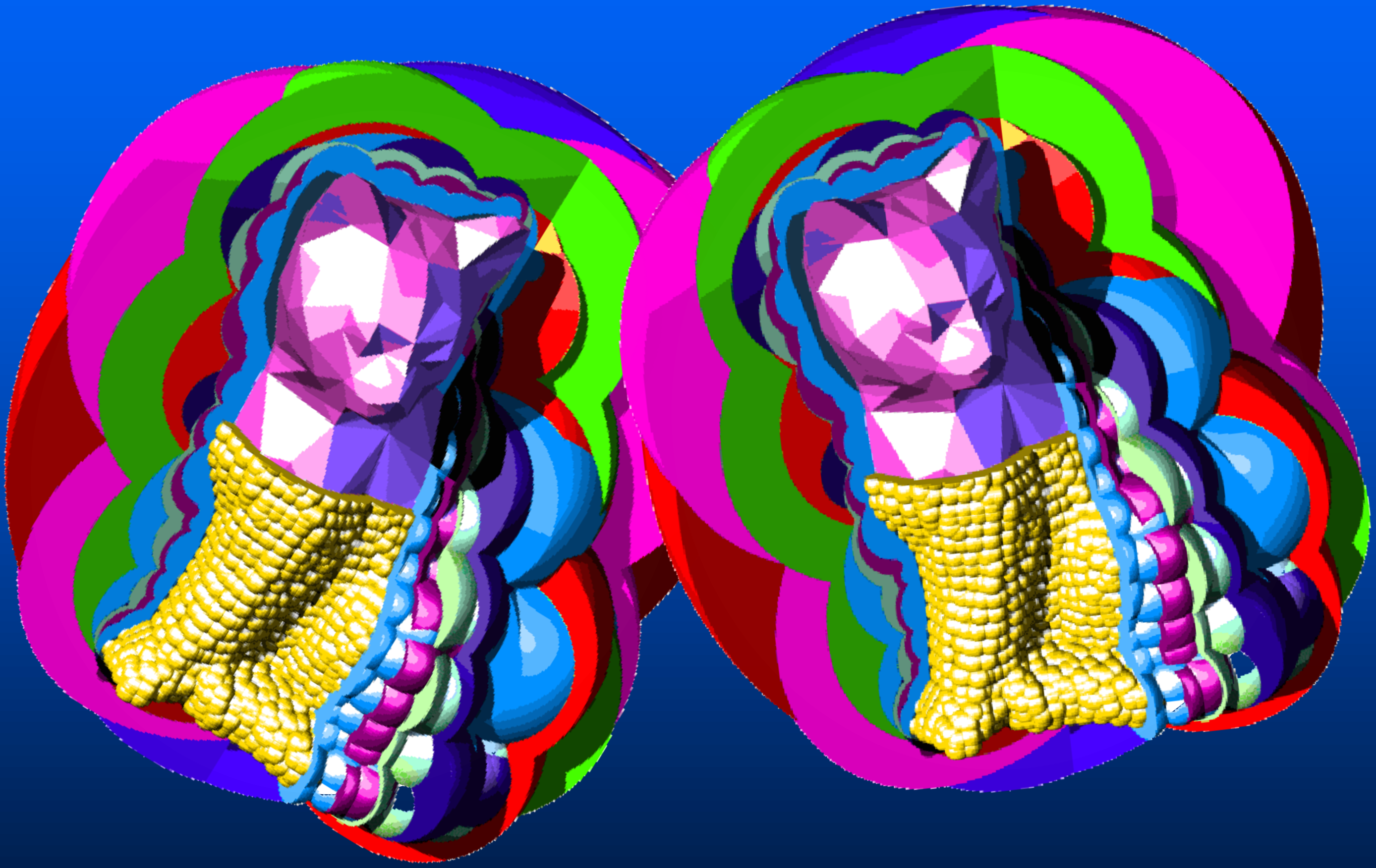
Sphere Hierarchy (level 8)



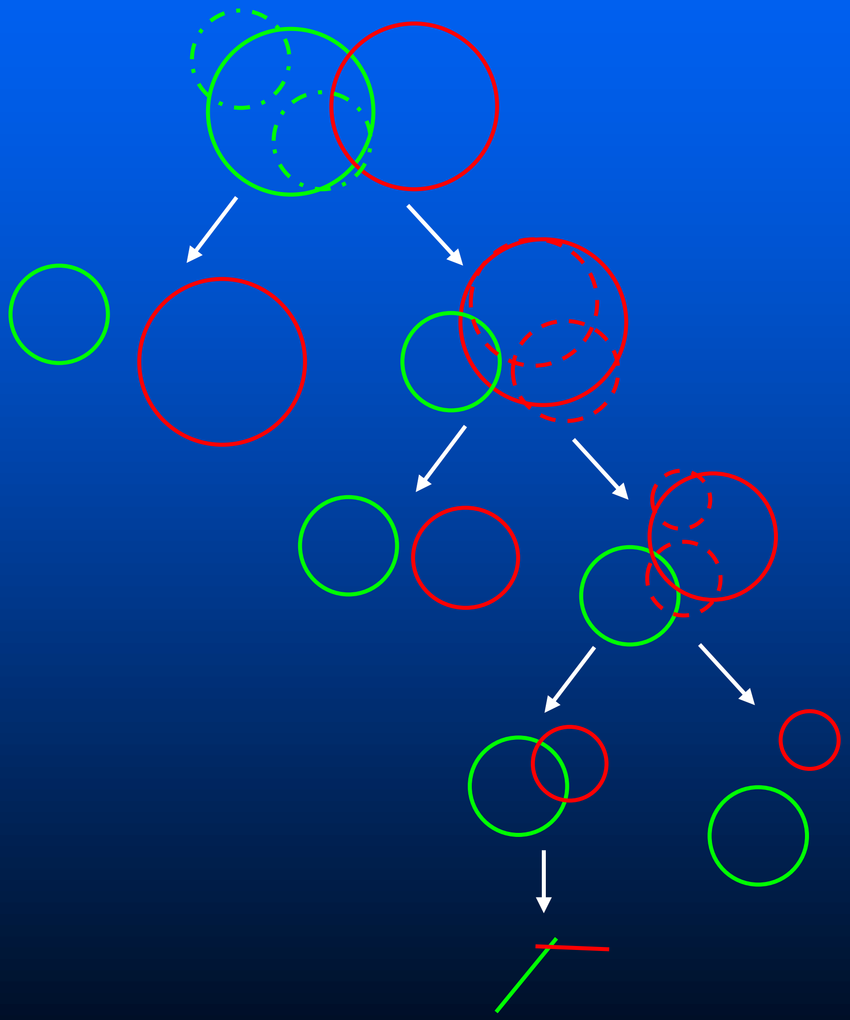
Sphere Hierarchy (level 9)



Colliding Two Object Hierarchies



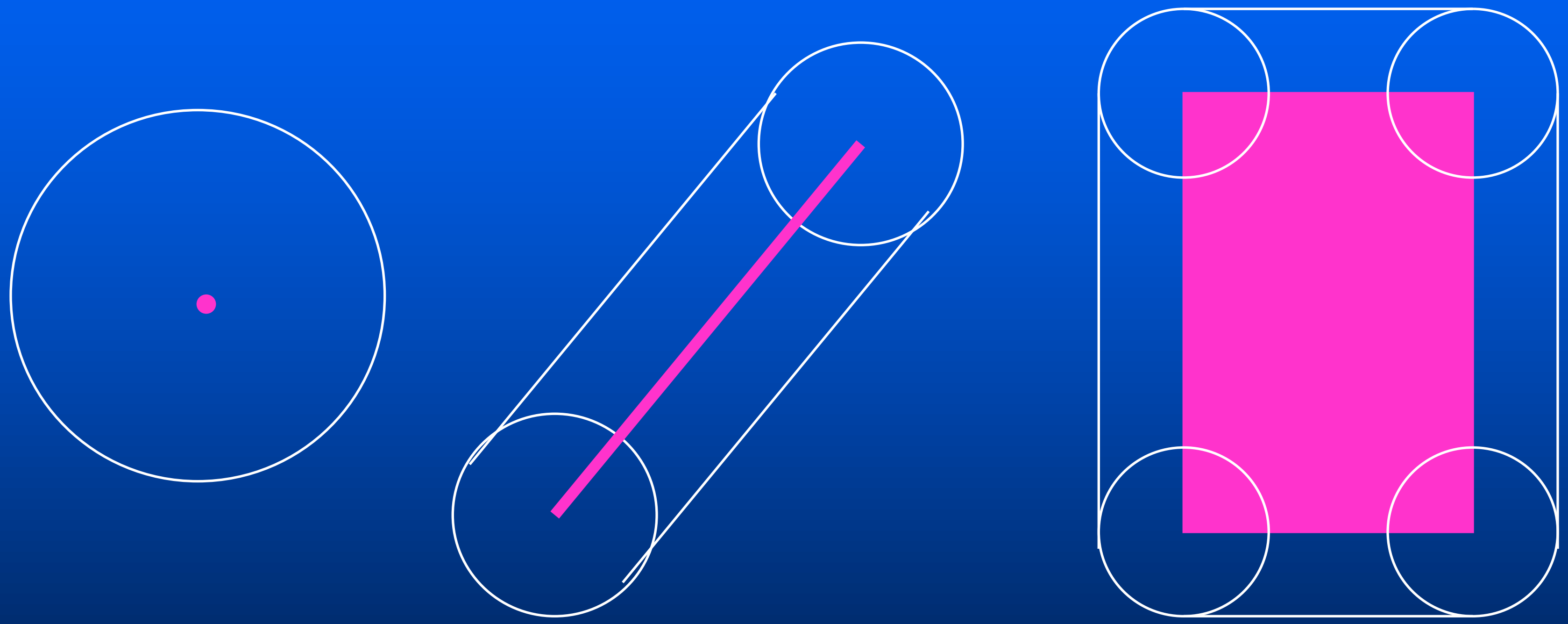
BVH-Based Collision Detection



Collision Detection using BVHs

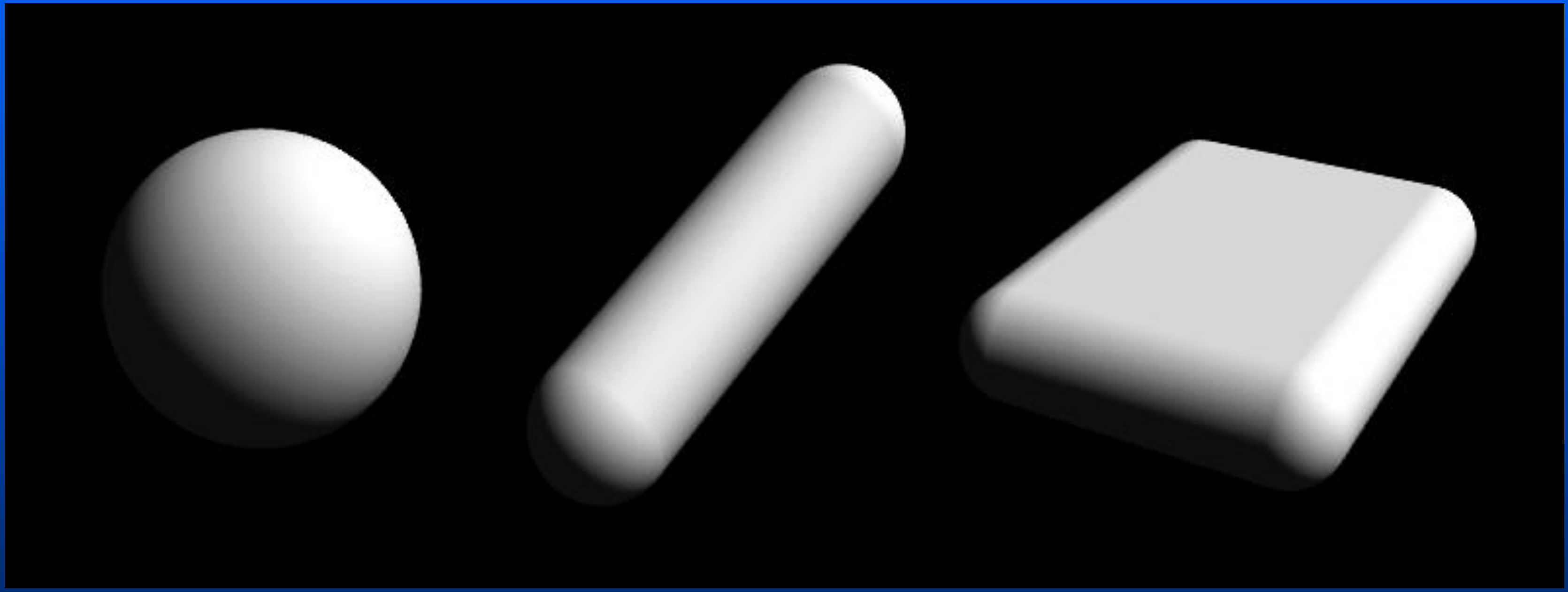
1. Check for collision between two parent nodes (starting from the roots of two given trees)
2. If there is no interference between two parents,
3. Then stop and report "no collision"
4. Else All children of one parent node are checked against all children of the other node
5. If there is a collision between the children
6. Then If at leaf nodes
7. Then report "collision"
8. Else go to Step 4
9. Else stop and report "no collision"

Hybrid Hierarchy of Swept Sphere Volumes



PSS LSS RSS

Swept Sphere Volumes (S-topes)



PSS **LSS** **RSS**

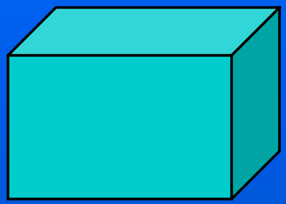
k-DOP's

- *k*-dop (*k*-discrete oriented polytope) : convex polytope whose facets are determined by half-spaces whose outward normals come from a small fixed set of *k* directions

- Examples:
  - In 2D, an 8-dop consists of the set of directions +/- {45,90,135,180} degrees
  - In 3D, an AABB is a 6-dop with direction vectors determined by the +/- coordinate axes.

Choices of k-dops in 3D

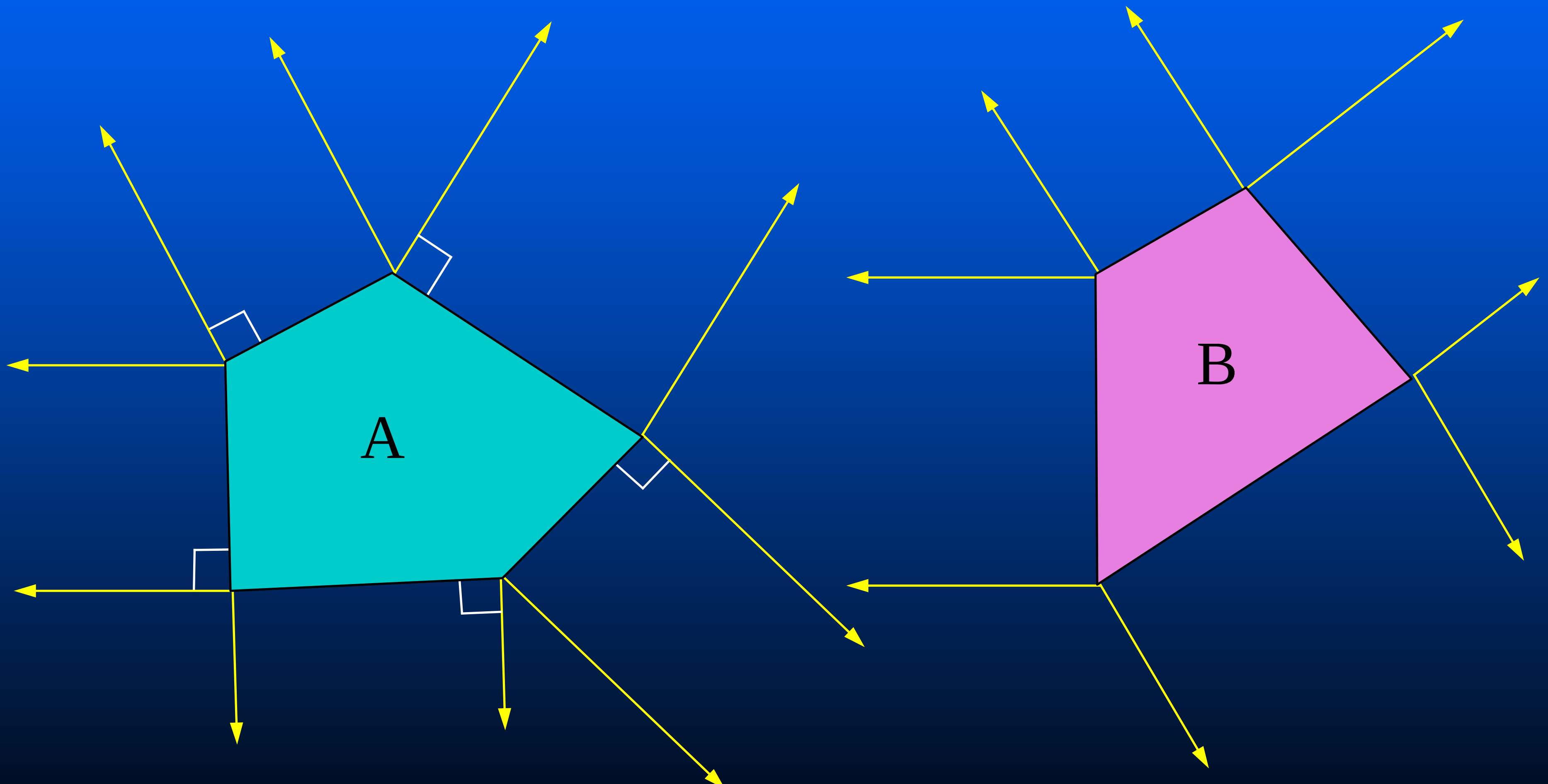
- 6-dop: defined by coordinate axes



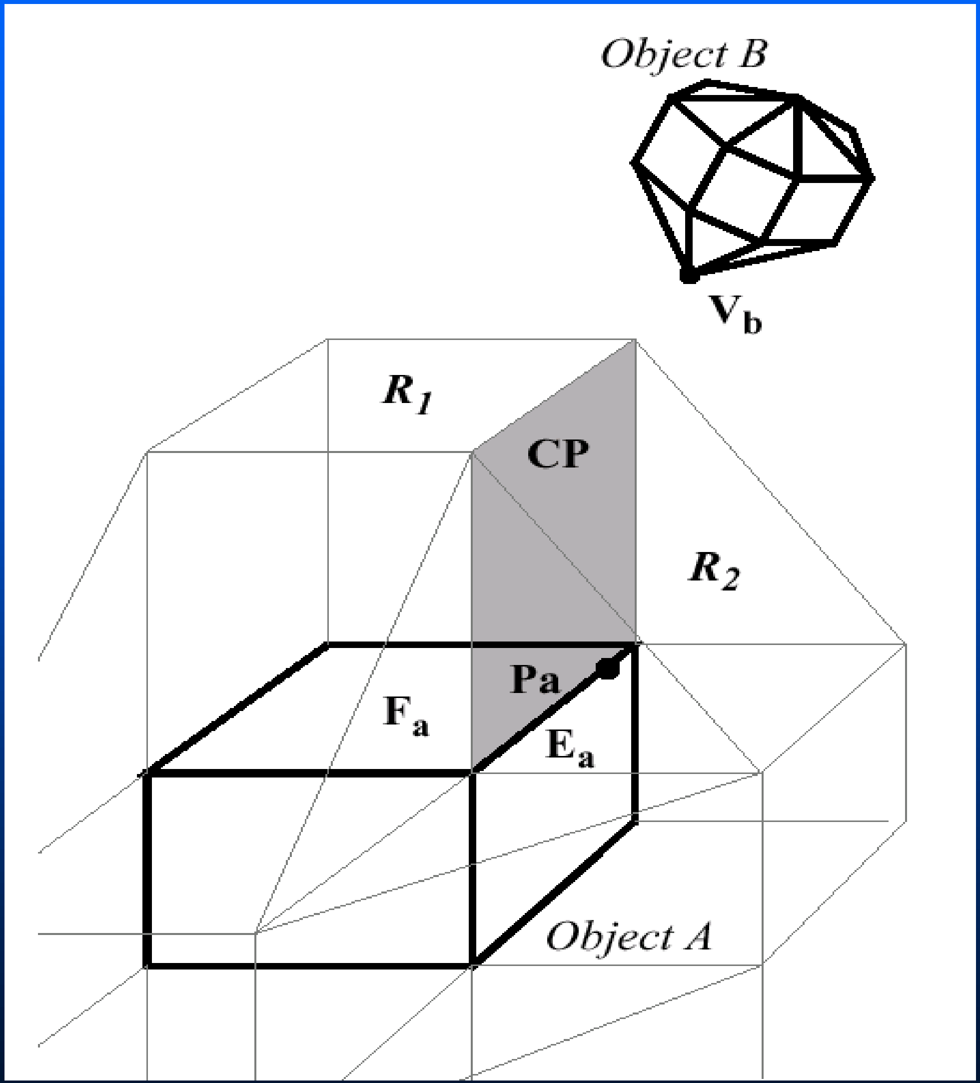
- 14-dop: defined by the vectors (1,0,0), (0,1,0), (0,0,1), (1,1,1), (1,-1,1), (1,1,-1) and (1,-1,-1)
- 18-dop: defined by the vectors (1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1), (1,-1,0), (1,0,-1) and (0,1,-1)
- 26-dop: defined by the vectors (1,0,0), (0,1,0), (0,0,1), (1,1,1), (1,-1,1), (1,1,-1), (1,-1,-1), (1,1,0), (1,0,1), (0,1,1), (1,-1,0), (1,0,-1) and (0,1,-1)

2D Voronoi Regions for Polygons

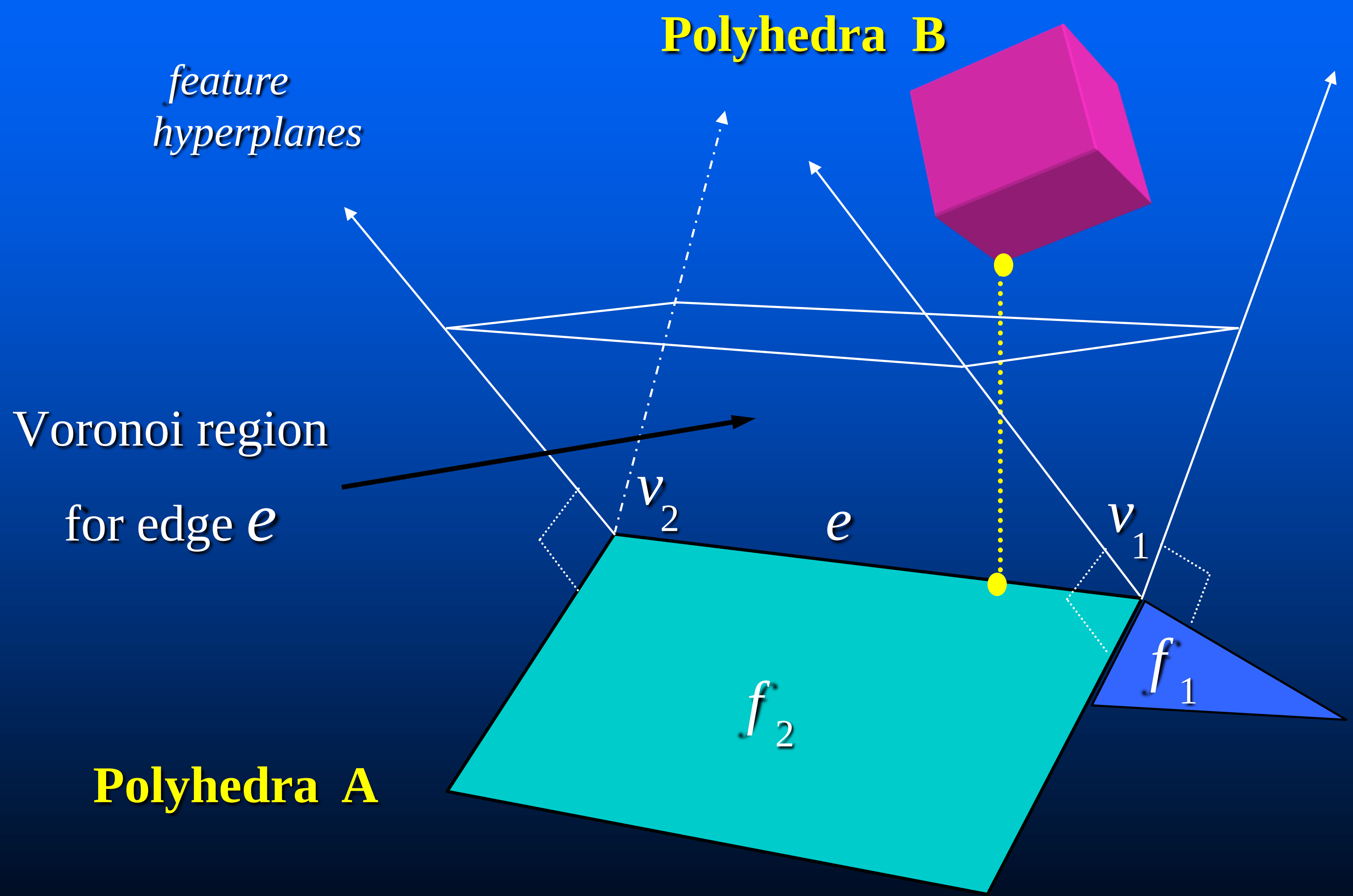
- Voronoi planes are always perpendicular to adjacent face planes for convex polygons



3D Voronoi Regions for Polyhedra

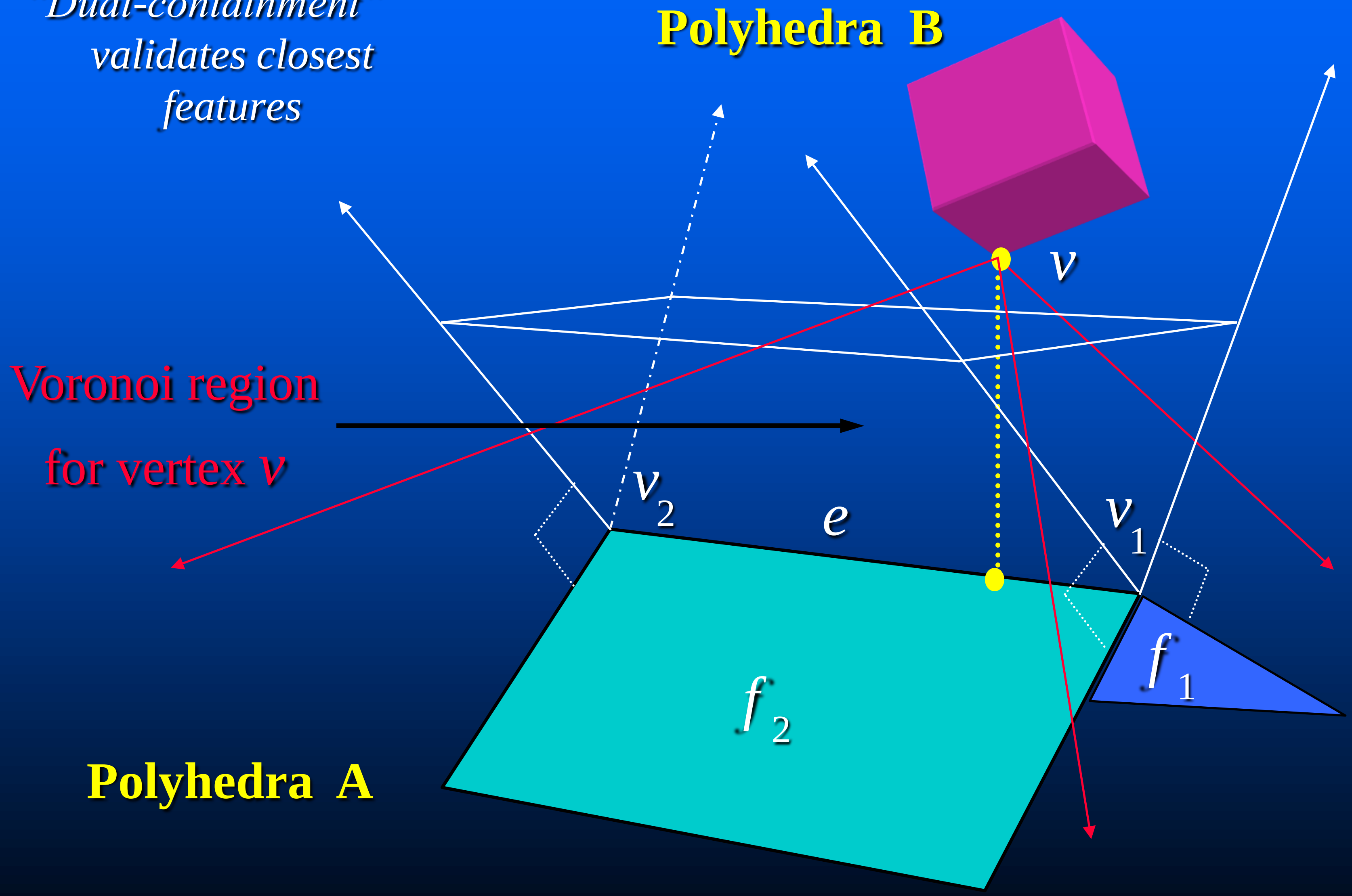


Voronoi-based Minimum Distances



Voronoi-based Minimum Distances

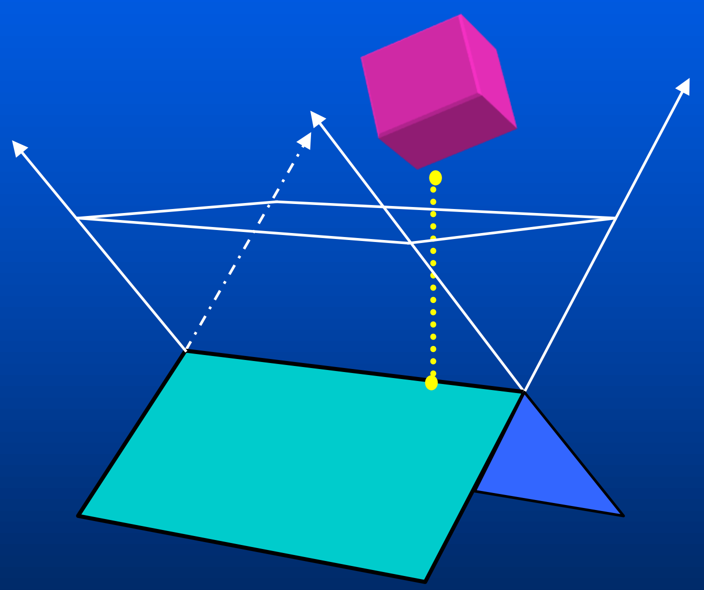
"Dual-containment" validates closest features



Voronoi-based Minimum Distances

POSSIBLE STATES:

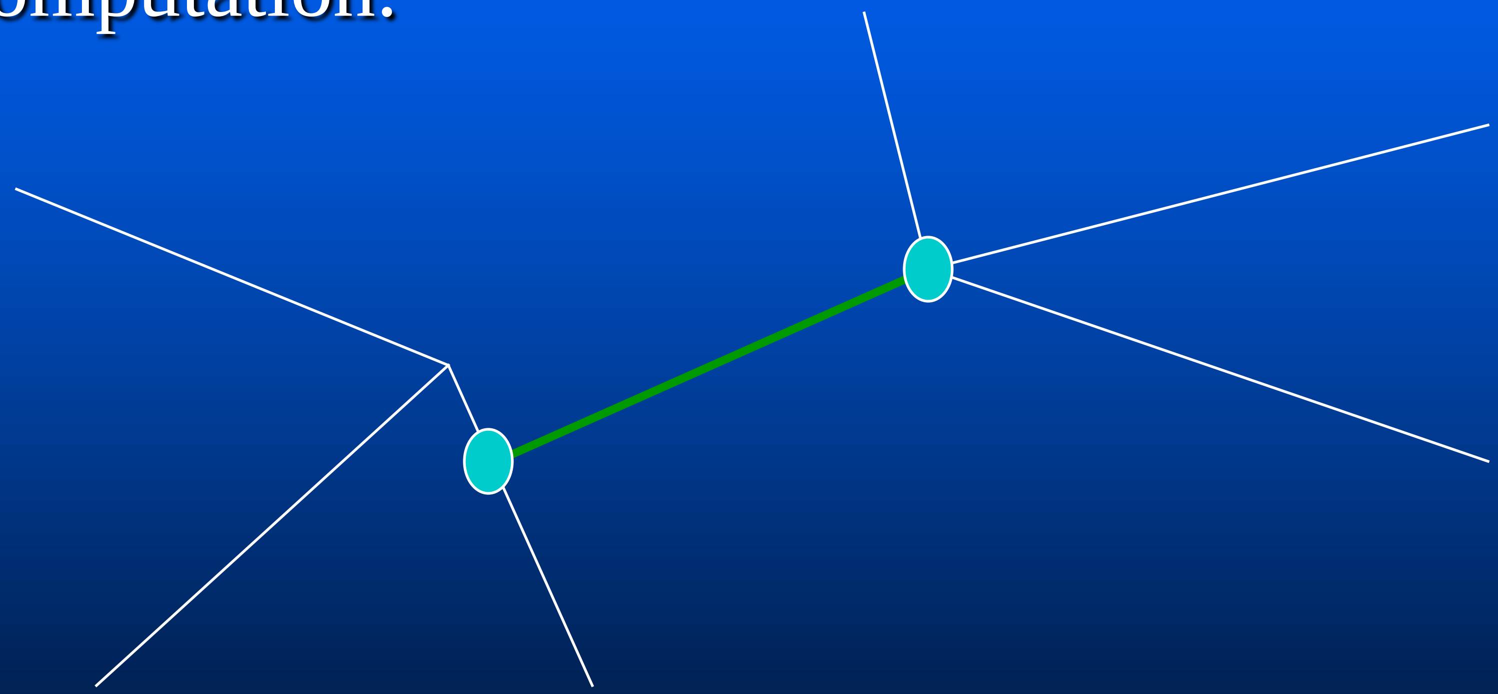
- Vertex – Vertex (V-V)
- Vertex – Edge (V-E)
- Vertex – Face (V-F)
- Edge – Edge (E-E)
- Edge – Face (E-F)



Algorithm keeps track of possible transitions between states

Coherency and Efficiency

- Closest feature pairs cached from previous computation:



Subsequent checks verify Voronoi containment constraints

Some Links and Resources

- University of North Carolina (UNC) at Chapel Hill is a leader in geometric proximity query research and has many software packages available to students and educators

  http://gamma.cs.unc.edu/

  http://www.cs.unc.edu/~geom/collide/

- Other Collision Detection Resources for games found here:

  http://www.gamedev.net/

  http://www.gamasutra.com/

Some materials in these slides adapted from lecture notes by Ming Lin and Dinesh Manocha at UNC Chapel Hill