PPLATO PROMOTING PHYSICS LEARNING & TEACHING OPPORTUNITIES

Basic Mathematics

The UNIVERSITY of PLYMOUTH

Introduction to Vectors

R Horan & M Lavelle

The aim of this document is to provide a short, self assessment programme for students who wish to acquire a basic understanding of vectors.

Last Revision Date: December 21, 2004 Version 1.0

Table of Contents

The full range of these packages and some instructions, should they be required, can be obtained from our web page Mathematics Support Materials.

1. Vectors (Introduction)

A vector is a combination of three things:

- a positive number called its *magnitude*,
- a *direction* in space,
- a *sense* making more precise the idea of direction.

Typically a vector is illustrated as a directed straight line.

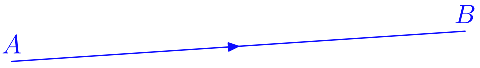


Diagram 1

The vector in the above diagram would be written as $\overrightarrow{AB}$ with:

- the direction of the arrow, from the point A to the point B, indicating the *sense* of the vector,
- the *magnitude* of $\overrightarrow{AB}$ given by the length of AB.

The *magnitude* of $\overrightarrow{AB}$ is written $|\overrightarrow{AB}|$.

There are very many physical quantities which are best described as vectors; velocity, acceleration and force are all *vector* quantities.

Two vectors are *equal* if they have the same *magnitude*, the same *direction* (i.e. they are *parallel*) and the same *sense*.

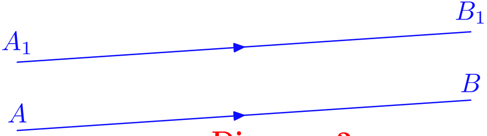


Diagram 2

In **diagram 2** the vectors $\overrightarrow{AB}$ and $\overrightarrow{A_1B_1}$ are equal, i.e. $\overrightarrow{AB}=\overrightarrow{A_1B_1}$. If two vectors have the same length, are parallel but have *opposite* senses then one is the negative of the other.

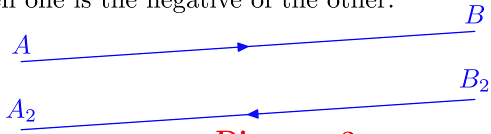


Diagram 3

In **diagram 3** the vectors $\overrightarrow{AB}$ and $\overrightarrow{B_2A_2}$ are of equal length, are parallel but are *opposite* in sense, so $\overrightarrow{AB}=-\overrightarrow{B_2A_2}$.

Quiz

Diagram 4 shows a parallelogram. Which of the following equations is the correct one?

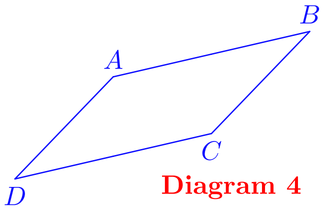


Diagram 4

(a) $\vec{DA} = \vec{BC}$, (b) $\vec{AD} = -\vec{CB}$, (c) $\vec{AD} = \vec{CB}$, (d) $\vec{DA} = -\vec{CB}$.

If two vectors are parallel, have the same sense but different magnitudes then one vector is a *scalar* (i.e. numeric) multiple of the other.

In **diagram 5** the vector $\vec{AB}$ is parallel to $\vec{A_3B_3}$, has the same sense but is twice as long, so $\vec{AB} = 2\ \vec{A_3B_3}$.

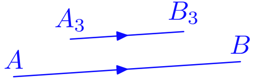


Diagram 5

In general *multiplying a vector by a positive number λ gives a vector parallel to the original vector, with the same sense but with magnitude λ times that of the original*. If λ is *negative* then the sense is reversed. Thus from **diagram 5** for example, $\vec{A_3B_3} = -\frac{1}{2}\ \vec{BA}$.

2. Addition of Vectors

In **diagram 6** the three vectors given by $\overrightarrow{AB}$, $\overrightarrow{BC}$, and $\overrightarrow{AC}$, make up the sides of a triangle as shown. Referring to this diagram, the law of addition for vectors, which is usually known as the *triangle law of addition*, is

$$\boxed{\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}\ .}$$

The vector $\overrightarrow{AC}$ is called the *resultant vector*.

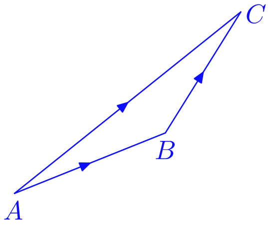


Diagram 6

Physical quantities which can be described as vectors satisfy such a law. One such example is the action of forces. If two forces are represented by the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$ then the effect of applying *both of these forces together* is equivalent to a single force, the *resultant force*, represented by the vector $\overrightarrow{AC}$.

One further vector is required, the *zero vector*. This has *no direction* and *zero magnitude*. It will be written as $\mathbf{0}$.

Example 1 (**The mid-points theorem**)

Let ABC be a triangle and let D be the midpoint of AC and E be the midpoint of BC. Prove that DE is parallel to AB and half its length i.e. $|AB| = 2|DE|$.

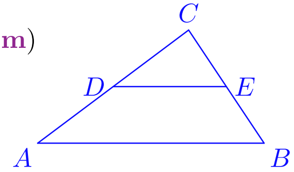


Diagram 7

Proof

Since D is the midpoint of $\vec{AC}$, it follows that $\vec{AC} = 2\,\vec{DC}$. Similarly $\vec{CB} = 2\,\vec{CE}$. Then

$$\begin{aligned}\vec{AC} + \vec{CB} &= 2\,\vec{DC} + 2\,\vec{CE} \\ &= 2(\vec{DC} + \vec{CE})\,.\end{aligned}$$

Now $\quad \vec{AC} + \vec{CB} = \vec{AB} \quad$ and $\quad \vec{DC} + \vec{CE} = \vec{DE}$.

Substituting these into the equation above gives $\vec{AB} = 2\,\vec{DE}$.

Since these are vectors, AB must be parallel to DE and the length of AB is twice that of DE, i.e. $|\vec{AB}| = 2|\vec{DE}|$.

3. Component Form of Vectors

The diagram shows a vector $\vec{OC}$ at an angle to the x axis. Take $\mathbf{i}$ to be a vector of length 1 (called a *unit vector*) parallel to the x axis and in the positive direction, and $\mathbf{j}$ to be a vector of length 1 (another *unit vector*) parallel to the y axis and in the positive direction.

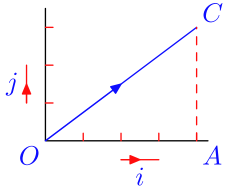


Diagram 8

From **diagram 8**, $\vec{OC} = \vec{OA} + \vec{AC}$. The vector $\vec{OA}$ is parallel to the vector $\mathbf{i}$ and four times its length so $\vec{OA} = 4\mathbf{i}$. Similarly $\vec{AC} = 3\mathbf{j}$. Thus the vector $\vec{OC}$ may be written as

$$\vec{OC} = 4\mathbf{i} + 3\mathbf{j}\,.$$

This is known as the *2-dimensional component form* of the vector. In general every vector can be written in component form. This package will consider only 2-dimensional vectors.

EXERCISE 1. From **diagram 9**, write down the component form of the following vectors: (Click on the green letters for solutions.)

(a) $\overrightarrow{OA}$, (b) $\overrightarrow{OB}$,

(c) $\overrightarrow{OC}$, (d) $\overrightarrow{OD}$,

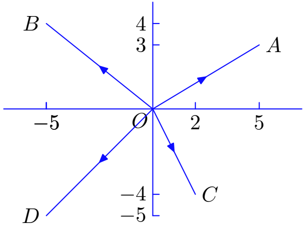


Diagram 9

In this package, the following properties of vectors are used.

- To add two or more vectors in component form, add the corresponding components.
- To multiply a vector in component form by a scalar, multiply each of the components by the scalar.
- If a vector in component form is $a\mathbf{i} + b\mathbf{j}$ then its magnitude is $\sqrt{a^2 + b^2}$. (*Pythagoras' theorem*)

Example 3

If $\vec{AB} = 2\mathbf{i} + 2\mathbf{j}$ and $\vec{BC} = \mathbf{i} + 2\mathbf{j}$, prove that the magnitude of $\vec{AC}$ is 5.

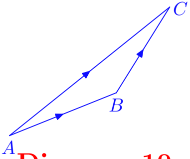


Diagram 10

Proof

The three vectors form three sides of a triangle (see **diagram 10** which is NOT to scale) so

$$\begin{aligned} \vec{AC} = \vec{AB} + \vec{BC} &= (2\mathbf{i} + 2\mathbf{j}) + (1\mathbf{i} + 2\mathbf{j}) \\ &= (2\mathbf{i} + 1\mathbf{i}) + (2\mathbf{j} + 2\mathbf{j}) = 3\mathbf{i} + 4\mathbf{j}\,. \end{aligned}$$

Thus $|\ \vec{AC}\ | = \sqrt{3^2 + 4^2} = 5$.

NB Vectors are often printed as boldface lower case letters such as $\mathbf{a}$.

EXERCISE 2. If $\mathbf{a} = -\mathbf{i} + 3\mathbf{j}$, $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{c} = \mathbf{i} - 2\mathbf{j}$, calculate:

(a) $\mathbf{a} + \mathbf{b}$, (b) $\mathbf{b} + \mathbf{c}$, (c) $\mathbf{a} + \mathbf{b} + \mathbf{c}$,

(d) $\mathbf{a} + 2\mathbf{b}$, (e) $2\mathbf{b} - 3\mathbf{a}$, (f) $|\mathbf{a}|$,

(g) $|\mathbf{a} + \mathbf{b}|$, (h) $|\mathbf{a}| + |\mathbf{b}|$, (i) $|2\mathbf{a} - \mathbf{b}|$,

Example 4 Two vectors are $\vec{AB} = \mathbf{i} + \mathbf{j}$ and $\vec{CD} = 2\mathbf{i} + 3\mathbf{j}$. Find

(a) The value of λ such that $\lambda\,\vec{AB} + \vec{CD}$ is parallel to $\mathbf{i}$,

(b) The value of λ such that $\lambda\,\vec{AB} + \vec{CD}$ is parallel to $4\mathbf{i} + 3\mathbf{j}$.

Solution First find $\lambda\,\vec{AB} + \vec{CD}$ in component form.

$$\begin{aligned} \lambda\,\vec{AB} + \vec{CD} &= \lambda(\mathbf{i} + \mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) \\ &= (\lambda\mathbf{i} + \lambda\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) \\ &= (\lambda + 2)\mathbf{i} + (\lambda + 3)\mathbf{j}\,. \end{aligned}$$

(a) If $\lambda\,\vec{AB} + \vec{CD}$ is parallel to $\mathbf{i}$ then the $\mathbf{j}$ component must be zero, i.e. $\lambda + 3 = 0$. Thus $\lambda = -3$ and we have $-3\,\vec{AB} + \vec{CD} = -\mathbf{i}$.

(b) If $\lambda\,\vec{AB} + \vec{CD}$ is parallel to $4\mathbf{i} + 3\mathbf{j}$ then there is a number κ such that

$$\begin{aligned} (\lambda + 2)\mathbf{i} + (\lambda + 3)\mathbf{j} &= \kappa(4\mathbf{i} + 3\mathbf{j}) \\ \therefore (\lambda + 2)\mathbf{i} + (\lambda + 3)\mathbf{j} &= 4\kappa\mathbf{i} + 3k\mathbf{j} \\ \text{so} \quad \lambda + 2 = 4\kappa \quad &\text{and} \quad \lambda + 3 = 3\kappa\,. \end{aligned}$$

$$\begin{aligned} \text{Then} \qquad \frac{\lambda+2}{\lambda+3} &= \frac{4\kappa}{3\kappa} = \frac{4}{3} \\ \therefore 3(\lambda+2) &= 4(\lambda+3) \\ 3\lambda+6 &= 4\lambda+12 \\ 6-12 &= 4\lambda-3\lambda \\ \text{i.e.} \qquad \lambda &= -6\,, \end{aligned}$$

and the vector is $-6(\mathbf{i}+\mathbf{j})+(2\mathbf{i}+3\mathbf{j}) = -4\mathbf{i}-3\mathbf{j} = -(4\mathbf{i}+3\mathbf{j})$.

Quiz If $\mathbf{a} = 2\mathbf{i}+3\mathbf{j}$, $\mathbf{b} = -3\mathbf{i}+2\mathbf{j}$ and $\mathbf{c} = 2\mathbf{i}-\mathbf{j}$, which of the following vectors is parallel to the resultant of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$, i.e. $\mathbf{a}+\mathbf{b}+\mathbf{c}$?

(a) $-2\mathbf{i}-6\mathbf{j}$, (b) $2\mathbf{i}-6\mathbf{j}$, (c) $2\mathbf{i}+8\mathbf{j}$, (d) $2\mathbf{i}-8\mathbf{j}$.

Quiz If $\mathbf{a} = \mathbf{i}+\mathbf{j}$ and $\mathbf{b} = \mathbf{i}-\mathbf{j}$, for which of the following values of λ is the vector $\lambda\mathbf{a}+\mathbf{b}$ parallel to $\mathbf{c} = 2\mathbf{i}-3\mathbf{j}$?

(a) $\lambda = \frac{1}{5}$, (b) $\lambda = -\frac{1}{5}$, (c) $\lambda = 5$, (d) $\lambda = -5$.

4. Quiz on Vectors

Choose the correct option for each of the following.

Begin Quiz

1. If $\mathbf{a} = -2\mathbf{i} + 4\mathbf{j}$, $\mathbf{b} = 3\mathbf{i} - 2\mathbf{j}$, $\mathbf{c} = 4\mathbf{i} + 5\mathbf{j}$ then $\mathbf{a} + \mathbf{b} + \mathbf{c}$ is

(a) $-5\mathbf{i} - 7\mathbf{j}$, (b) $5\mathbf{i} - 7\mathbf{j}$, (c) $-5\mathbf{i} + 7\mathbf{j}$, (d) $5\mathbf{i} + 7\mathbf{j}$.

2. If $\mathbf{u} = -2\mathbf{i} + 4\mathbf{j}$, $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j}$, $\mathbf{w} = 4\mathbf{i} + 6\mathbf{j}$ then $|\mathbf{u} + \mathbf{v} + \mathbf{w}|$ is

(a) 5, (b) 13, (c) 4, (d) 15.

3. If $\mathbf{u} = -\mathbf{i} + 3\mathbf{j}$ and $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$, then $\lambda\mathbf{u} + \mathbf{v}$ is parallel to $\mathbf{w} = -\mathbf{i} + 4\mathbf{j}$ if λ is

(a) -6, (b) 6, (c) -5, (d) 5.

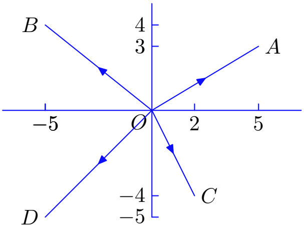
End Quiz Score: Correct

Solutions to Exercises

Exercise 1(a)

For the vector $\vec{OA}$ shown on the diagram the component in the direction given by the unit vector $\mathbf{i}$ is 5 and the component in the direction $\mathbf{j}$ is 3. Therefore the 2-dimensional vector $\vec{OA}$ is, in component form, written as

$$\vec{OA} = 5\mathbf{i} + 3\mathbf{j}\,.$$

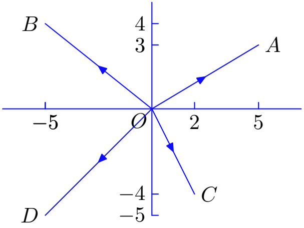


Click on the green square to return

Exercise 1(b)

The vector $\vec{OB}$ shown on the diagram has the component -5 in the $\mathbf{i}$ direction while the component in the $\mathbf{j}$ direction is 4. Thus the 2-dimensional vector $\vec{OB}$ in component form is written as

$$\vec{OB} = -5\mathbf{i} + 4\mathbf{j}\,.$$

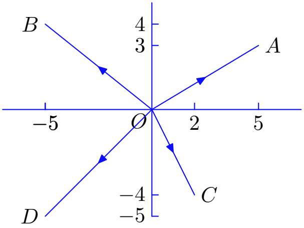


Click on the green square to return

Exercise 1(c)

For the vector $\vec{OC}$ shown on the diagram the component in the direction given by the unit vector $\mathbf{i}$ is 2 while the component in the direction given by $\mathbf{j}$ is -4. Therefore the component form of the 2-dimensional vector $\vec{OC}$ is

$$\vec{OC} = 2\mathbf{i} - 4\mathbf{j}\,.$$

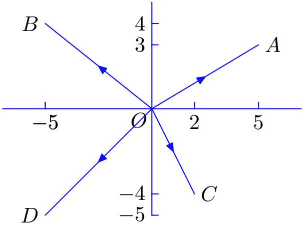


Click on the green square to return

Exercise 1(d)

For the vector $\overrightarrow{OD}$ shown on the diagram the component in the direction given by the unit vector $\mathbf{i}$ is -5 and the component in the direction given by $\mathbf{j}$ is also -5. The component form of the 2-dimensional vector $\overrightarrow{OD}$ is therefore

$$\overrightarrow{OC} = -5\mathbf{i} - 5\mathbf{j}\,.$$



Click on the green square to return

Exercise 2(a)
The sum of the two vectors

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j} \qquad \text{and} \qquad \mathbf{b} = 2\mathbf{i} + 3\mathbf{j}$$

is found by summing up the corresponding components of each vector. Thus

$$\mathbf{a} + \mathbf{b} = (-\mathbf{i} + 3\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) = (-1 + 2)\mathbf{i} + (3 + 3)\mathbf{j} = \mathbf{i} + 6\mathbf{j}\,.$$

Click on the green square to return

Exercise 2(b)

The sum of the two vectors

$$\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} \qquad \text{and} \qquad \mathbf{c} = \mathbf{i} - 2\mathbf{j}$$

is found by adding the corresponding components of each vector. Thus

$$\mathbf{b} + \mathbf{c} = (2\mathbf{i} + 3\mathbf{j}) + (\mathbf{i} - 2\mathbf{j}) = (2+1)\mathbf{i} + (3-2)\mathbf{j} = 3\mathbf{i} + \mathbf{j}\,.$$

Click on the green square to return

Exercise 2(c)
To find the sum of the three vectors

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j}\,, \qquad \mathbf{b} = 2\mathbf{i} + 3\mathbf{j} \qquad \text{and} \qquad \mathbf{c} = \mathbf{i} - 2\mathbf{j}\ ,$$

add the corresponding components of each vector. The resulting vector is thus

$$\begin{aligned} \mathbf{a} + \mathbf{b} + \mathbf{c} &= (-\mathbf{i} + 3\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) + (\mathbf{i} - 2\mathbf{j}) \\ &= (-1 + 2 + 1)\mathbf{i} + (3 + 3 - 2)\mathbf{j} = 2\mathbf{i} + 4\mathbf{j}\,. \end{aligned}$$

Click on the green square to return

Exercise 2(d)

To find the sum $\mathbf{a}+2\mathbf{b}$ with

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j} \qquad \text{and} \qquad \mathbf{b} = 2\mathbf{i} + 3\mathbf{j}\,,$$

first find the vector $2\mathbf{b}$:

$$2\,\mathbf{b} = 2 \times (2\mathbf{i} + 3\mathbf{j}) = 4\mathbf{i} + 6\mathbf{j}\,.$$

The vector $\mathbf{a}+2\mathbf{b}$ is now found by adding the corresponding components of each vector. The resulting vector is thus

$$\begin{aligned} \mathbf{a}+2\mathbf{b} &= (-\mathbf{i}+3\mathbf{j}) + (4\mathbf{i}+6\mathbf{j}) \\ &= (-1+4)\mathbf{i} + (3+6)\mathbf{j} = 3\mathbf{i} + 9\mathbf{j}\,. \end{aligned}$$

Click on the green square to return

$\square$

Exercise 2(e)

To find the vector $\mathbf{2b}-\mathbf{3a}$ with

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j} \qquad \text{and} \qquad \mathbf{b} = 2\mathbf{i} + 3\mathbf{j}\,,$$

first find the vectors $\mathbf{2b}$ and $\mathbf{3a}$:

$$\begin{aligned} 2\,\mathbf{b} &= 2\times(2\mathbf{i}+3\mathbf{j}) = 4\mathbf{i}+6\mathbf{j}\,, \\ 3\,\mathbf{a} &= 3\times(-\mathbf{i}+3\mathbf{j}) = -3\mathbf{i}+9\mathbf{j}\,, \end{aligned}$$

The vector $\mathbf{2b}-\mathbf{3a}$ is now easily found by subtracting the components of these vectors:

$$\begin{aligned} \mathbf{2b}-\mathbf{3a} &= (4\mathbf{i}+6\mathbf{j}) - (-3\mathbf{i}+9\mathbf{j}) \\ &= (4+3)\mathbf{i} + (6-9)\mathbf{j} = \mathbf{7i}-\mathbf{3j}\,. \end{aligned}$$

Click on the green square to return

Exercise 2(f)
The magnitude of the vector

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j}$$

is given by

$$|\mathbf{a}| = \sqrt{(-1)^2 + 3^2} = \sqrt{1+9} = \sqrt{10}\,.$$

Click on the green square to return

Exercise 2(g)

To find the magnitude of the vector $\mathbf{a}+\mathbf{b}$, first find the sum of the two vectors

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j} \qquad \text{and} \qquad \mathbf{b} = 2\mathbf{i} + 3\mathbf{j}\,.$$

The resulting vector is

$$\mathbf{a}+\mathbf{b} \;=\; (-\mathbf{i}+3\mathbf{j}) + (2\mathbf{i}+3\mathbf{j}) = (-1+2)\mathbf{i} + (3+3)\mathbf{j} = \mathbf{i} + 6\mathbf{j}\,.$$

The magnitude of this vector is given by

$$|\mathbf{a}+\mathbf{b}| = \sqrt{1^2+6^2} = \sqrt{37}\,.$$

Click on the green square to return

Exercise 2(h)
To find $|\mathbf{a}| + |\mathbf{b}|$, first find the magnitude of each of the vectors $\mathbf{a} = -\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j}$.
The magnitude of the vector $\mathbf{a}$ is

$$|\mathbf{a}| = \sqrt{(-1)^2 + 3^2} = \sqrt{10}\,.$$

The magnitude of the vector $\mathbf{b}$ is

$$|\mathbf{b}| = \sqrt{2^2 + 3^2} = \sqrt{13}\,.$$

Therefore

$$|\mathbf{a}| + |\mathbf{b}| = \sqrt{10} + \sqrt{13}\,.$$

Click on the green square to return

Exercise 2(i)

To find $|\mathbf{2a-b}|$, first find $\mathbf{2a-b}$. The vector $\mathbf{a}$ in component form is given as

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j}$$

so the component form of the vector $\mathbf{2a}$ is

$$\mathbf{2a} = 2\times(-1)\mathbf{i} + 2\times 3\mathbf{j} = -2\mathbf{i} + 6\mathbf{j}\,.$$

The difference between $\mathbf{2a}$ and $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j}$ is the vector

$$\mathbf{2a-b} = (-2\mathbf{i}+6\mathbf{j}) - (2\mathbf{i}+3\mathbf{j}) = (-2-2)\mathbf{i} + (6-3)\mathbf{j} = -4\mathbf{i} + 3\mathbf{j}\,.$$

The magnitude of the resulting vector $\mathbf{2a-b}$ is therefore

$$|\mathbf{2a-b}| = \sqrt{(-4)^2+3^2} = \sqrt{25} = 5\,.$$

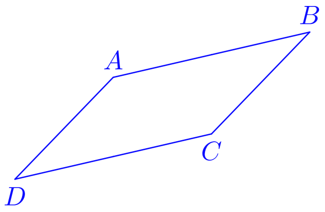
Click on the green square to return

□

Solutions to Quizzes

Solution to Quiz:

According to the diagram shown opposite the magnitudes of the vectors $\vec{AD}$ and $\vec{CB}$ are equal, but the direction of the vector $\vec{AD}$ is from the point A to the point D, while the direction of the vector $\vec{CB}$ is opposite, from the point B to the point C. Therefore $\vec{AD} = -\vec{CB}$.



If checked, the other solutions will be found to be false. End Quiz

Solution to Quiz:
In order to determine which of the vectors is parallel to the resultant of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$, the resultant must first be calculated.

The resultant of the three vectors

$$\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}\,, \qquad \mathbf{b} = -3\mathbf{i} + 2\mathbf{j} \qquad \text{and} \qquad \mathbf{c} = 2\mathbf{i} - \mathbf{j}\,.$$

is

$$\begin{aligned} \mathbf{a} + \mathbf{b} + \mathbf{c} &= (2\mathbf{i} + 3\mathbf{j}) + (-3\mathbf{i} + 2\mathbf{j}) + (2\mathbf{i} - \mathbf{j}) \\ &= (2 - 3 + 2)\mathbf{i} + (3 + 2 - 1)\mathbf{j} = \mathbf{i} + 4\mathbf{j}\,. \end{aligned}$$

Next note that the vector $2\mathbf{i}+8\mathbf{j}$ given in the answer **(c)** can be written as

$$2\mathbf{i} + 8\mathbf{j} = 2 \times (\mathbf{i} + 4\mathbf{j}) = 2(\mathbf{a} + \mathbf{b} + \mathbf{c})\,,$$

so the resultant is parallel to the vector $2\mathbf{i} + 8\mathbf{j}$. End Quiz

Solution to Quiz: To find the value of λ for which $\lambda\mathbf{a} + \mathbf{b}$ parallel to $\mathbf{c} = 2\mathbf{i} - 3\mathbf{j}$, first calculate the former. If $\mathbf{a} = \mathbf{i} + \mathbf{j}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j}$ then

$$\lambda\mathbf{a} + \mathbf{b} = \lambda(\mathbf{i} + \mathbf{j}) + (\mathbf{i} - \mathbf{j}) = (\lambda + 1)\mathbf{i} + (\lambda - 1)\mathbf{j}\,.$$

If this vector is parallel to the vector $\mathbf{c} = 2\mathbf{i} - 3\mathbf{j}$ then there is a number k such that

$$(\lambda + 1)\mathbf{i} + (\lambda - 1)\mathbf{j} = k(2\mathbf{i} - 3\mathbf{j})\,.$$

This holds when $\lambda + 1 = 2k$ and $\lambda - 1 = -3k\,.$
Multiply the first equation by 3

$$3\lambda + 3 = 6k\,,$$

and the second one by 2

$$2\lambda - 2 = -6k\,.$$

Now add the left and right sides of these equations to obtain:

$$5\lambda + 1 = 0\,, \qquad \text{thus} \qquad \lambda = -\frac{1}{5}\,.$$

End Quiz