Exercise Session 2

1 Linear independence

Consider the following vectors:

$$e_1 = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix},\ e_2 = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix},\ e_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

(a) Prove that these vectors form basis in $\mathbb{R}^3$.

(b) Find the coordinates of $x = \begin{bmatrix} 6 \\ 2 \\ -7 \end{bmatrix}$ in this basis.

2 Change of coordinates

Consider two bases in $\mathbb{R}^4$:

$$S = \left\{ s_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} s_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix} s_3 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix} s_4 = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 3 \end{bmatrix} \right\}$$

$$B = \left\{ b_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix} b_2 = \begin{bmatrix} -2 \\ -3 \\ -5 \\ -4 \end{bmatrix} b_3 = \begin{bmatrix} 2 \\ 2 \\ 5 \\ 4 \end{bmatrix} b_4 = \begin{bmatrix} -2 \\ -3 \\ -4 \\ -4 \end{bmatrix} \right\}$$

Find the transition matrix from S to B.