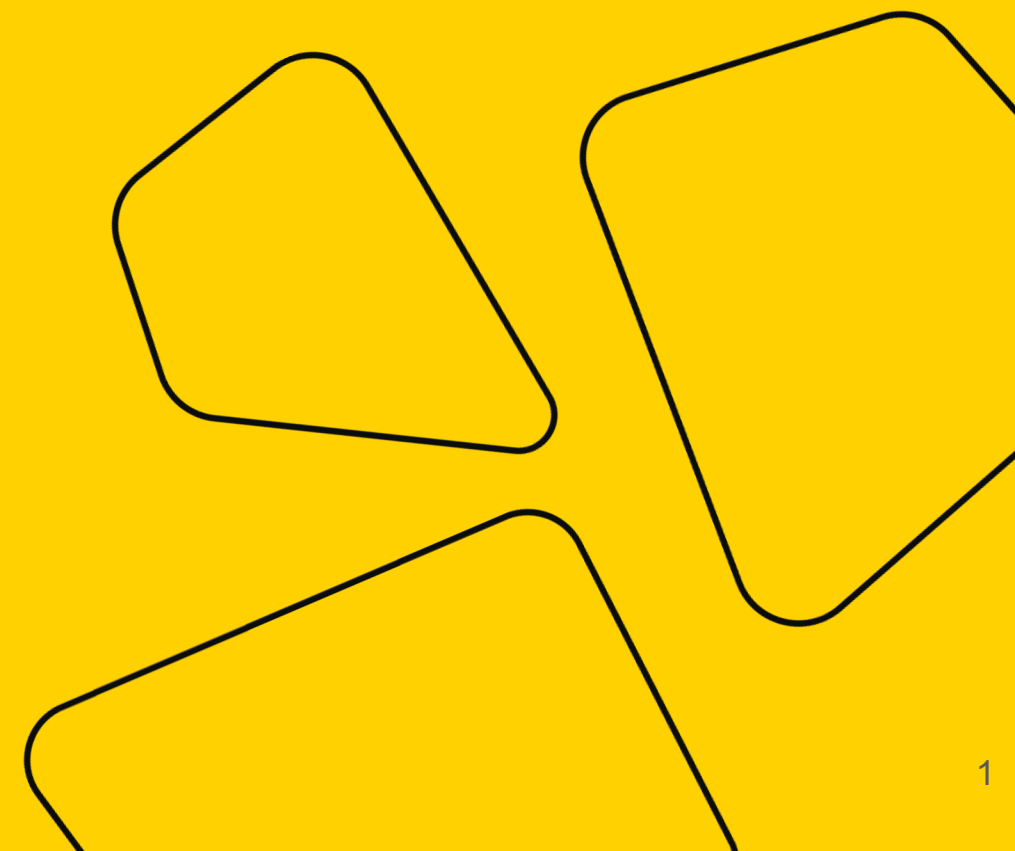




Math Basics for DS

Practical Session 7



Today

- Short quiz + review univariate calculus

Quiz

- <https://forms.gle/UFpg6aEhq8CQfCQt5>

Extrema of a function



Example

- Which value of $x \in [0, 1]$ maximizes the following function:

$$f(x) = x^a(1 - x)^b, \quad a, b \in \mathbb{N}$$

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