Graded Assignment 4

Math Basics for Machine Learning

Fall 2022

Instructions

This is the second graded assignment for the Math Basics for Machine Learning course. It contains three tasks. The instructions, as well as links to supplementary material, are given in the task descriptions below.

You should submit the **detailed solutions** to the tasks, as well as your code by filling in the corresponding Google form.

You can earn 10 points for this assignment. This score will contribute to your final score for this course.

You must submit your answers by **Monday, November 14, 23:59 Anywhere on Earth**. *Late submissions will* ***not*** *be accepted.*

It is the idea that you complete this assignment individually. Do not collaborate or copy answers of somebody else.

Have fun!

1 Extrema of a function *(2 points)*

Find and classify all the critical points of the following function

$$f(x, y) = 7x - 8y + 2xy - x^2 + y^3$$

2 Gradient of a loss function *(2 points)*

Fitting a machine learning model means finding the optimal values of its parameters, which comes down to optimizing some loss function $\mathcal{L}$.

In class, we saw the least-squares example. Now, let's consider a so-called *logistic loss*:

$$\mathcal{L} = \sum_{i=1}^{n} \left[y_i \log \sigma_i + (1 - y_i) \log (1 - \sigma_i) \right],$$

where $\sigma_i = \sigma_i(w_0, w_1) = \frac{1}{1+\exp -(w_0 + w_1 x_i)}$.

Here, $(x_i, y_i),\ i = 1, ..., n$ are the observed data points, and w_0 and w_1 are the parameters of the model.

Find the gradient of the loss function above.

Hint: gradient of the loss function $\mathcal{L}(w_0, w_1)$ is a vector of its partial derivatives with respect to the model parameters w_0 and w_1.

3 Numerical integration *(6 points)*

Consider the following integral:

$$\int_{-2}^{4} \sin x^2 dx$$

How would you approach computing it? If you try applying some of the standard techniques we've reviewed in class, you'll notice that they don't do the trick here.

It turns out not every function has a simple closed-form antiderivative, and $f(x) = \sin x^2$ is one of them. So, to compute the definite integral above, we need to learn how to *approximate* it.

There are several numerical integration techniques, among which are **the midpoint rule** and **the trapezoid rule**. Based on very straightforward and simple ideas, they are powerful tools used in many real-world applications.

Your first task is to get familiar with these two approaches to numeric integration. Here are some resources that you might find useful:

- Theoretical summary + interactive demo [link]
- Khan's Academy (videos):
  - Midpoint rule [link]
  - Trapezoid rule [link]
- Paul's Online Notes:
  - Theoretical summary + a worked-out example [link]
  - More examples with solutions [link]

Now, approximate the value of the integral above

1. using the midpoint rule *(3 points)*
2. using the trapezoid rule *(3 points)*

In both cases, base your approximation on $n = 6$ points.

Hint 1: You can do this by hand, but it's easier to use Python for computations. If you do so, please attach your code as usual. The idea is that you implement the methods yourself, do not use any readily-available implementation.

Hint 2: To make sure that your computations are correct, you can compare the result that you are getting with the answer provided by Wolfram.