Math Basics for Machine Learning
Graded Assignment 1

Your Name Here

Fall 2023

Instructions

This is the first graded assignment for the Math Basics for Machine Learning course. It contains three tasks. The instructions, as well as links to supplementary material, are given in the task descriptions.

Provide **detailed solutions** to the tasks in this assignment. Then, save your solution document as a .pdf file and submit it by filling in the corresponding Google form.

For some tasks, it might be convenient to use Python rather than performing the computations by hand. If you do so, attach your code as well (e.g., attach link to the notebook with code, save it to `.pdf` or simply make screenshots and add them to the file with other solutions).

In total, you can earn 10 points for this assignment. This score will contribute to you final score for this course.

You must submit your answers by **Monday, October 9, 18:59 Moscow Time**. *Late submissions will **not** be accepted.*

It is the idea that you complete this assignment individually. Do not collaborate or copy answers of somebody else.

Have fun!

1. (2 points) Given a vector a and a set of vectors $X = x_1, ..., x_m$, we say that x_i is the *nearest neighbor* of a if it's the closest vector to a from the set X. In ML, the idea of the nearest neighbour and its generalisation, k-nearest neighbours, is used for solving classification and regression problems (see k-nearest neighbours algorithm).

   Find the nearest neighbour of $a = [3, 1, 4]$ from the set of vectors $x_1, ..., x_4$ below.

$$x_1 = \begin{bmatrix} 4 \\ 3 \\ 6 \end{bmatrix}, \ x_2 = \begin{bmatrix} 3 \\ 1 \\ 9 \end{bmatrix}, \ x_3 = \begin{bmatrix} 1 \\ 4 \\ 10 \end{bmatrix}, \ x_4 = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$$

   If there is more than one nearest neighbour, mark all of them. Use

   (a) (1 point) Euclidean distance

   Solution: Type your solution here

   (b) (1 point) cosine similarity

   Solution: Type your solution here

2. (2 points) Consider a linear classifier with the following separating hyperplane:

$$x_1 + x_2 - x_3 + x_4 - x_5 = 2$$

   According to this classifier, which class labels will be assigned to the following examples?

	x_1	x_2	x_3	x_4	x_5
a	1	0	1	0	0
b	0	1	1	0	1
c	1	1	1	1	1
d	2	0	1	0	1
e	2	2	0	0	0

   Solution: Type your solution here

3. (3 points) During the lectures, we discussed the concept of linear (in)dependence and saw examples of linearly (in)dependent sets of vectors.

   A general way to check linear independence of a given set of vectors is to combine those vectors in a matrix and bring it to reduced row echelon form (RREF) with the help of elementary row operations.

   If you are not yet familiar with this procedure, you can learn more about bringing a matrix to its RREF in this video. There is also another video that demonstrates how we can test a set of vectors for linear independence.

   Once you are ready you can solve the questions in this section.

(a) (1 point) Consider vectors v_1, v_2, v_3 and v_4 defined below:

$$v_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 2 \\ 4 \\ 1 \\ 3 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

What is the dimensionality of the subspace $V = span\{v_1, v_2, v_3, v_4\}$ spanned by those vectors?

Solution: Type your solution here

(b) (2 points) Consider subspace $V = span\{v_1, v_2, v_3, v_4\}$, where v_1, v_2, v_3 and v_4 are vectors from the previous question. List **all** subsets of $\{v_1, v_2, v_3, v_4\}$ that form a basis in the vector space V. Explain.

Solution: Type your solution here

4. (3 points) Consider two sets of vectors, B and S:

$$B = \left\{ b_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}, \quad b_3 = \begin{bmatrix} 3 \\ 8 \\ 2 \end{bmatrix} \right\}$$

$$S = \left\{ s_1 = \begin{bmatrix} 3 \\ 5 \\ 8 \end{bmatrix}, \quad s_2 = \begin{bmatrix} 5 \\ 14 \\ 13 \end{bmatrix}, \quad s_3 = \begin{bmatrix} 1 \\ 9 \\ 2 \end{bmatrix} \right\}.$$

Both S and B form valid bases in $\mathbb{R}^3$. Find a transition matrix $A_{B \to S}$ from basis B to basis S.

Solution: Type your solution here