Contract Theory

Lecture 8: Hidden Information - Applications and Extensions

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Extension: Monitoring

2. Extension: Monitoring

- Again, same structure as in the basic model.
- However, now the principal has the possibility to verify the agent's message by using some (costly) monitoring technology.
- This way it is possible to punish the agent in case he gets caught lying.
- Implication: Even though the agent will not pay any penalty (in equilibrium, he will tell the truth), it becomes easier for P to satisfy the incentive-compatibility constraints.

Additional assumptions and notation

- The monitoring intensity p is endogenously chosen by the principal.
- $p \in [0, 1]$: Probability that the principal finds out the type of the agent.

- $c(p)$: Monitoring cost
- Assume: $c' > 0$ and $c'' > 0$, $c(0) = 0$, $c'(0) = 0$ and $c'(1) = \infty$.

- The latter two conditions are called “Inada”-conditions and ensure an interior solution for the optimal p.

Optimal Contracts

- Again, the revelation principle applies, and we can restrict attention to incentive-compatible direct mechanisms.
- In the present context, a direct mechanism is a mapping of the agent's message $\widetilde{\theta}$ into four variables: $t(\widetilde{\theta})$, $q(\widetilde{\theta})$, $p(\widetilde{\theta})$ and $P(\theta, \widetilde{\theta})$, where p is the monitoring probability, and $P(\theta, \widetilde{\theta}) > 0$ is the penalty imposed on the agent if he reports to be type $\widetilde{\theta}$ when his type is actually θ.
- Notation: $\underline{P} := P(\underline{\theta}, \overline{\theta})$ and $\overline{P} := P(\overline{\theta}, \underline{\theta})$.

IC and PC constraints

- The IC constraints are given by:

$$\begin{aligned} \underline{U} &= \underline{t} - \underline{\theta}\underline{q} \geq \overline{t} - \underline{\theta}\overline{q} - \overline{p}\underline{P} \\ \overline{U} &= \overline{t} - \overline{\theta}\overline{q} \geq \underline{t} - \overline{\theta}\underline{q} - \underline{p}\overline{P} \end{aligned}$$

- The penalties $\overline{P}$ and $\underline{P}$ make it easier to satisfy the IC constraints.
- The PCs are given by: $\underline{U} \geq 0$ and $\overline{U} \geq 0$.

Two types of penalties

1. Exogenous penalty: Upper limit l. Possible interpretation: limited liability or wealth constraint of the agent:

$$\begin{aligned} \underline{P} &\leq l \\ \overline{P} &\leq l \end{aligned}$$

2. Endogenous penalty: Penalty cannot exceed the maximum utility that the agent extracts from misrepresenting his type:

$$\begin{aligned} \underline{P} &\leq \bar{t} - \underline{\theta}\bar{q} \\ \overline{P} &\leq \underline{t} - \bar{\theta}\underline{q} \end{aligned}$$

Maximization problem of the principal

- The principal's objective is

$$\max\left\{\nu(S(\underline{q})-\underline{\theta}\underline{q}-\underline{U}-c(\underline{p}))+(1-\nu)(S(\overline{q})-\overline{\theta}\overline{q}-\overline{U}-c(\overline{p}))\right\}$$

  subject to the PC and IC constraints and the constraints on $\overline{P}$ and $\underline{P}$.

- Note that the penalties do not enter P's objective function because in equilibrium the agent will tell the truth.

Simplifying P's maximization problem

- Again, we suppose that the incentive-compatibility constraint of type $\underline{\theta}$ and the participation constraint of type $\overline{\theta}$ are binding.

- At the optimum, the penalties $\overline{P}$ and $\underline{P}$ are at their maximal levels ("Maximum-Punishment-Principle").
- As the incentive-compatibility constraint of type $\overline{\theta}$ will not be binding and $c(p) > 0$ for all $p > 0$, we have $\underline{p} = 0$ at the optimum ($\overline{P}$ is then irrelevant).
- Thus it follows:

$$\begin{aligned}\overline{U} &= 0 \\ \underline{U} &= \Delta\theta\overline{q} - \overline{p}\underline{P}\end{aligned}$$

Simplified maximization problem of the principal

- P's problem can now be simplified to:

$$\max\left\{\nu(S(\underline{q})-\underline{\theta}\underline{q}-\Delta\theta\overline{q}+\overline{p}\underline{P})+(1-\nu)(S(\overline{q})-\overline{\theta}\overline{q}-c(\overline{p}))\right\}$$

subject to the constraint of either $\underline{P}\leq\Delta\theta\overline{q}$ or $\underline{P}\leq l$ (both of which will be binding in the optimum).

- First-order condition for type $\underline{\theta}$:

$$\nu(S'(\underline{q})-\underline{\theta}))=0\Rightarrow\underline{q}^{SB}=\underline{q}^{*}\text{ (no distortion at the top)}$$

First-order condition for type $\overline{\theta}$

- With exogenous penalty ($\underline{P} = l$):

$$\begin{aligned} -\nu\Delta\theta + (1-\nu)(S'(\overline{q}) - \overline{\theta}) &= 0 \Leftrightarrow \\ S'(\overline{q}) &= \overline{\theta} + \frac{\nu}{(1-\nu)}\Delta\theta \\ &\Rightarrow \overline{q}^{SB} < \overline{q}^{*} \end{aligned}$$

- With endogenous penalty ($\underline{P} = \Delta\theta\overline{q}$):

$$\begin{aligned} -\nu\Delta\theta + \overline{p}\nu\Delta\theta + (1-\nu)(S'(\overline{q}) - \overline{\theta}) &= 0 \Leftrightarrow \\ S'(\overline{q}) &= \overline{\theta} + \frac{(1-\overline{p})\nu}{(1-\nu)}\Delta\theta \\ &\Rightarrow \overline{q}^{SB} < \overline{q}^{*} \end{aligned}$$

Monitoring of type $\overline{\theta}$

- With exogenous penalty ($\underline{P} = l$):

$$c'(\overline{p}) = \frac{\nu}{1-\nu} l$$

- With endogenous penalty ($\underline{P} = \Delta\theta\overline{q}$):

$$c'(\overline{p}) = \frac{\nu}{1-\nu} \Delta\theta\overline{q}$$

Summary

- With exogenous as well as with endogenous penalty the inefficient type $\overline{\theta}$ will be monitored with positive probability.
- There is a trade-off between monitoring costs and the reduction of the information rent of the efficient type $\underline{\theta}$.
- With exogenous penalty there is no additional distortion of the output level.
- With endogenous penalty the distortion of the output level is narrowed by the possibility of monitoring.
- If l becomes too large, the participation constraint of type $\underline{\theta}$ may also be binding at the optimum.
- Note: Ex-post, P has no incentive to actually carry out the (costly) monitoring because in equilibrium the agent tells the truth (i.e., the commitment assumption is crucial).

Application: Price Discriminating Monopolist (Maskin and Riley, 1984)

1. Application: Price Discriminating Monopolist (Maskin and Riley, 1984)

- A monopolist can produce $q \geq 0$ units of a good.
- Marginal costs $c > 0$, no fixed costs.
- Consumers have private information regarding their valuation of the good: the valuation parameter θ is private information of each consumer and can take on two values, $\theta \in \{\underline{\theta}, \overline{\theta}\}$ where $\underline{\theta} < \overline{\theta}$.
- Consumers' utility function is given by $v(q, t, \theta) = \theta \cdot u(q) - t$, where $u' > 0$ and $u'' < 0$.
- t denotes a payment from the consumer to the monopolist.

- The fractions of type $\underline{\theta}$- and type $\overline{\theta}$-consumers in the total population (normalized to 1) are given by $(1-\nu)$ and ν, respectively.

- Note: In contrast to the basic model here $\overline{\theta}$ is the "efficient" type, because he has a higher valuation, and $\underline{\theta}$ is the "inefficient" type.

- The profit function of the monopolist is given by $\pi(q,t) = t - c \cdot q$

- Here, the principal is the monopolist who offers contracts (t,q), and agents can either accept or reject. Their outside option is normalized to 0.

1.1. First-Best Solution

- As a benchmark, consider the case where each consumer's θ is observed by the monopolist.
- The maximization problem of the monopolist is then given by:

$$\max_{q,t} \{t - cq\}$$
$$\text{s.t.} \quad \theta u(q) - t \geq 0$$

- In the optimum, the participation constraint has to be satisfied with equality, and hence we arrive at:

$$\theta u'(q^*) = c$$

- Thus, the optimal output level q^* is where the marginal utility $\theta u'(q^*)$ equals marginal costs c.
- Due to the payment $t^* = \theta u(q^*)$ the entire surplus goes to the monopolist.

- Now assume that each consumer's θ is private information
- Maximization problem of the monopolist:

$$\max_{\overline{q},\underline{q},\overline{t},\underline{t}} \left\{(1-\nu)(\underline{t}-c\underline{q})+\nu(\overline{t}-c\overline{q})\right\} \tag{1}$$

$$\text{subject to} \quad \underline{\theta}u(\underline{q})-\underline{t} \geq \underline{\theta}u(\overline{q})-\overline{t} \tag{2}$$

$$\overline{\theta}u(\overline{q})-\overline{t} \geq \overline{\theta}u(\underline{q})-\underline{t} \tag{3}$$

$$\underline{\theta}u(\underline{q})-\underline{t} \geq 0 \tag{4}$$

$$\overline{\theta}u(\overline{q})-\overline{t} \geq 0 \tag{5}$$

- The first two constraints are the incentive compatibility constraints (IC), the latter two are the participation constraints (PC).

Characterization of the optimal contract

- Deriving the solution is analogous to the basic model.
- Which of the constraints are binding?
- **Step 1:** The PC of type $\overline{\theta}$ (5) cannot be binding because his payoff always (weakly) exceeds that of type $\underline{\theta}$:

$$\overline{\theta}u(\overline{q}) - \overline{t} \underbrace{\geq}_{=(3)} \overline{\theta}u(\underline{q}) - \underline{t} \underbrace{\qquad\geq\qquad}_{(\text{because } \overline{\theta} > \underline{\theta})} \underline{\theta}u(\underline{q}) - \underline{t} \underbrace{\geq}_{(4)} 0$$

- **Step 2:** The PC of type $\underline{\theta}$ (4) must be binding because otherwise the monopolist could (somewhat) increase both $\overline{t}$ and $\underline{t}$ without violating the ICs, and thereby she could increase her profit.

- **Step 3:** The IC of type $\overline{\theta}$ (3) must be binding: otherwise the monopolist could increase $\overline{t}$ without violating any other constraint, and thereby she could increase her profit.

- **Step 4:** For the moment, we suppose that (2) is not binding and check this after having found the candidate solution.

- Thus, we have:

$$\begin{aligned}
\underline{t} &= \underline{\theta} u(\underline{q}) \\
\overline{\theta} u(\overline{q}) - \overline{t} &= \overline{\theta} u(\underline{q}) - \underline{t} \Leftrightarrow \\
\overline{t} &= \underline{t} + \overline{\theta} u(\overline{q}) - \overline{\theta} u(\underline{q}) \\
&= \overline{\theta} u(\overline{q}) - \Delta\theta u(\underline{q})
\end{aligned}$$

- Again, the efficient type receives an information rent.

- Substituting into the objective function of the monopolist yields:

$$\max_{\overline{q},\underline{q}} \left\{ (1-\nu)(\underline{\theta} u(\underline{q}) - c\underline{q}) + \nu(\overline{\theta} u(\overline{q}) - \Delta\theta u(\underline{q}) - c\overline{q}) \right\}$$

- The first-order condition for $\underline{q}$ is given by:

$$
\begin{aligned}
u'(\underline{q})((1-\nu)\underline{\theta} - \nu\Delta\theta) &= (1-v)c \Leftrightarrow \\
u'(\underline{q})(\underline{\theta} - \frac{\nu}{(1-\nu)}\Delta\theta) &= c
\end{aligned}
$$

- The first-order condition for $\overline{q}$ is given by:

$$\nu(\overline{\theta}u'(\overline{q}) - c)) = 0$$

Summary

1. The output level offered to type $\overline{\theta}$ is the same as in the first-best: $\overline{q}^{SB} = \overline{q}^*$. Again "no distortion at the top".
2. The output level offered to type $\underline{\theta}$ is lower compared to the first-best ($\underline{q}^{SB} < \underline{q}^*$). The monopolist wants to reduce the incentive for type $\overline{\theta}$ to mimic type $\underline{\theta}$ in order to limit type $\overline{\theta}$'s information rent.
3. The monopolist engages in "price discrimination" because in the optimum both consumer types pay different prices per unit:

$$\begin{aligned} \underline{t}^{SB} &= \underline{\theta} u(\underline{q}^{SB}) \\ \overline{t}^{SB} &= \overline{\theta} u(\overline{q}^*) - \Delta\theta u(\underline{q}^{SB}) \end{aligned}$$

so that prices per unit $\underline{p}$ and $\overline{p}$ are given by

$$\begin{aligned} \underline{p} &= \frac{\underline{\theta} u(\underline{q}^{SB})}{\underline{q}^{SB}} \\ \overline{p} &= \frac{\overline{\theta} u(\overline{q}^*) - \Delta\theta u(\underline{q}^{SB})}{\overline{q}^{SB}} \end{aligned}$$