Contract Theory

Lecture 7: Hidden Information - Optimal Contracts

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Winter Semester 2024–25

- Revelation principle: Restriction to direct incentive compatible mechanisms is w.l.o.g.!
- In case of incomplete information, the incentive compatibility constraints (IC) need to be considered in addition to the participation constraints:

$$\underline{t} - \underline{\theta}\underline{q} \geq \overline{t} - \underline{\theta}\overline{q} \tag{1}$$

$$\overline{t} - \overline{\theta}\overline{q} \geq \underline{t} - \overline{\theta}\underline{q} \tag{2}$$

- Hence, each type must weakly prefer the contract designed for him.

- Special case 1: Bunching (principal offers the same contract to both types).
- Special case 2: (Partial) Shut-Down (extreme case of screening): the inefficient type is offered the contract $(0, 0)$.
- In the following, by assumption, we rule out the uninteresting case of complete shut-down where both types are offered a quantity of 0.

IC constraints impose structure on optimal contracts

- Adding (1) and (2) yields

$$
\begin{aligned}
\underline{t} + \overline{t} - \underline{\theta}\underline{q} - \overline{\theta}\overline{q} &\geq \underline{t} + \overline{t} - \underline{\theta}\overline{q} - \overline{\theta}\underline{q} \Leftrightarrow \\
-\underline{\theta}\underline{q} - \overline{\theta}\overline{q} &\geq -\underline{\theta}\overline{q} - \overline{\theta}\underline{q} \Leftrightarrow \\
\underline{\theta}\overline{q} - \overline{\theta}\overline{q} &\geq \underline{\theta}\underline{q} - \overline{\theta}\underline{q} \Leftrightarrow \\
(\underline{\theta} - \overline{\theta})\overline{q} &\geq (\underline{\theta} - \overline{\theta})\underline{q} \Leftrightarrow \\
\overline{q} &\leq \underline{q} \text{ (due to } (\underline{\theta} - \overline{\theta}) < 0)
\end{aligned}
$$

- That is, from this we know that in any equilibrium outcome the low cost type will trade a weakly larger quantity.

- Such monotonicity results extend to more general settings (with more than two types), and are very useful for pinning down equilibrium transfers (see e.g., the textbook by Schweizer, 1999, Theorem 1).

Information rents

- As shown before, under complete information, P keeps all agent types at their reservation utility of zero.
- This will not be possible under hidden information. In particular, even under the optimal contract some types might obtain a strictly positive utility (which is called an **information rent**).
- To see this, note that if type $\underline{\theta}$ would imitate type $\overline{\theta}$ by choosing the contract $(\overline{q}, \overline{t})$, then his utility would be:

$$\overline{t} - \underline{\theta}\overline{q} = \overline{t} - \overline{\theta}\overline{q} + \Delta\theta\overline{q}$$

- So, even if the utility of type $\overline{\theta}$ could be reduced to zero $(\overline{t} - \overline{\theta}\overline{q} = 0)$, whenever $\overline{q} > 0$ holds, type $\underline{\theta}$ would receive an information rent of at least $\Delta\theta\overline{q} > 0$ (because his incentive-compatibility condition has to be satisfied).

Maximization Problem of the Principal

- The principal searches for a menu of contracts $\{(\underline{t}, \underline{q}), (\bar{t}, \bar{q})\}$, that maximizes his expected utility, subject to the incentive compatibility (IC) and participation constraints (PC):

$$\max_{\{(\underline{t},\underline{q}),(\bar{t},\bar{q})\}} [\nu(S(\underline{q}) - \underline{t}) + (1-\nu)(S(\bar{q}) - \bar{t})]$$

$$\text{s.t. } (\textbf{??}), (\textbf{??}), (1), (2)$$

- In order to gain an intuition, it will be helpful to make a "variable switch". Define:

$$\begin{aligned} \underline{U} &= \underline{t} - \underline{\theta}\underline{q} \\ \bar{U} &= \bar{t} - \bar{\theta}\bar{q} \end{aligned}$$

- In the following, we look at the maximization problem in (U, q)-space instead of (t, q)-space. This is helpful because it will highlight the tradeoff between allocation (q) and distribution (U).

- Using the definition $U = t - \theta q$ and replacing the respective t, the objective function can be written as

$$\underbrace{\nu(S(\underline{q}) - \underline{\theta}\underline{q}) + (1-\nu)(S(\overline{q}) - \overline{\theta}\overline{q})}_{\text{expected allocation efficiency}} - \underbrace{(\nu\underline{U} + (1-\nu)\overline{U})}_{\text{expected information rent}} \tag{3}$$

- Hence, P's objective boils down to maximizing social welfare minus expected information rent.

- Hence, P faces an objective function that is distorted relative to the complete information (first-best) benchmark.

- With the change of variables, also the constraints have to be re-written:

$$
\begin{aligned}
\underline{U} &\geq 0 && (4)\\
\overline{U} &\geq 0 && (5)\\
\underline{U} &\geq \overline{U} + \Delta\theta\overline{q} && (6)\\
\overline{U} &\geq \underline{U} - \Delta\theta\underline{q} && (7)
\end{aligned}
$$

- The modified maximization problem of the principal can then be written as

$$
\max_{\{(\underline{U},\underline{q}),(\overline{U},\overline{q})\}} \nu(S(\underline{q}) - \underline{\theta}\underline{q}) + (1-\nu)(S(\overline{q}) - \overline{\theta}\overline{q}) - (\nu\underline{U} + (1-\nu)\overline{U})
$$

subject to (4), (5), (6), (7)

- Let's first consider contracts without "shut-down", where $\overline{q} > 0$.

Characterizing the optimal second-best contract

- How do we solve this (apparently complicated) maximization problem?
- Not by "brute force" calculations, but by careful thinking.
- The best way to attack such a problem is trying to simplify the set of constraints (4), (5), (6), and (7):
- **Step 1:** The PC of the efficient type $\underline{\theta}$ (4) cannot be binding because his expected utility is strictly higher than that of type $\overline{\theta}$ (due to IC).
- **Step 2:** The IC of the inefficient type $\overline{\theta}$ (7) does not seem to be an issue, since the problem is that the efficient type imitates the inefficient one and not the other way round: neglect (7) for the moment and check it after having found the solution.
- **Step 3:** The remaining two constraints must be binding at the optimum, otherwise the principal could reduce the payments and increase his expected utility without violating one of the other constraints.

Optimal contract for type $\underline{\theta}$: No Distortion at the Top

- The two binding constraints (5) and (6) imply that the efficient type earns a (information) rent, while the inefficient type does not:

$$\begin{aligned} \underline{U} &= \Delta\theta\overline{q} \\ \overline{U} &= 0 \end{aligned}$$

- Substituting into the principal's objective function yields

$$\max_{\overline{q},\underline{q}} \nu(S(\underline{q}) - \underline{\theta}\underline{q}) + (1-\nu)(S(\overline{q}) - \overline{\theta}\overline{q}) - \nu\Delta\theta\overline{q}$$

- Obviously, the expected information rent $\nu\Delta\theta\overline{q}$ is independent of $\underline{q}$.
- It follows that there is no need to distort $\underline{q}$, and we have

$$S'(\underline{q}) = \underline{\theta} \Leftrightarrow \underline{q}^{SB} = \underline{q}^{*}$$

i.e., the quantity for the efficient type is the same as in the first-best solution (**no distortion at the top**).

Optimal contract for type $\overline{\theta}$

- By contrast, the output level of the inefficient type is distorted: From the first-order condition, we get

$$\begin{aligned}(1-\nu)(S'(\overline{q})-\overline{\theta}) &= \nu\Delta\theta \Leftrightarrow \\ S'(\overline{q})-\overline{\theta} &= \frac{\nu}{(1-\nu)}\Delta\theta \\ &\Rightarrow \overline{q}^{SB} < \overline{q}^{*}\end{aligned}$$

- On the left-hand side of the first row, we have the marginal efficiency gain of an increase of $\overline{q}$.
- On the right-hand side, we have the marginal increase of the information rent.

- For the second-best contract, we thus get the following ranking of output levels:

$$\underline{q}^{SB} = \underline{q}^{*} > \overline{q}^{*} > \overline{q}^{SB}$$

- It follows directly that (7) (ignored so far) is also satisfied:

$$\begin{array}{rcl} \overline{U} & \geq & \underline{U} - \Delta\theta\underline{q} \\ \Leftrightarrow \quad 0 & \geq & \Delta\theta\overline{q}^{SB} - \Delta\theta\underline{q}^{SB} \end{array}$$

- Note: We obtain the potentially surprising result, that the output level of the **inefficient** type is distorted in order to reduce the **efficient** type's incentive to misrepresent his type.

- Now the transfer payments can easily be derived:

$$\begin{array}{rcl} \underline{t}^{SB} & = & \underline{\theta}\underline{q}^{*} + \Delta\theta\overline{q}^{SB} \\ \overline{t}^{SB} & = & \overline{\theta}\overline{q}^{SB} \end{array}$$

- Only the efficient type receives an information rent $\Delta\theta\overline{q}^{SB} > 0$.

Shut-Down

- Under what circumstances is it optimal to let only the efficient type $\underline{\theta}$ produce, while there is shut-down of the inefficient type, i.e., $\overline{q}^{SB} = 0$?
- Advantage: No information rent because mimicking would lead to $\underline{U} = 0$.
- Disadvantage: Type $\overline{\theta}$ does not produce at all and thus there is no surplus generated from this type.
- Hence, the principal prefers "shut-down" if:

$$\overbrace{\nu \Delta\theta \overline{q}^{SB}}^{\text{expected information rent}} \geq \overbrace{(1-\nu)(S(\overline{q}^{SB}) - \overline{\theta}\overline{q}^{SB})}^{\text{expected utility from production with type } \overline{\theta}}$$

Implications

- In order to reduce information rent, the principal deviates from the first-best allocation.
- Even though we are in a setting of **"complete" contracts**, **"transaction costs"** arise endogenously due to asymmetric information.
- How could such information asymmetries be reduced?

Timing of contract offers

- So far, we have considered the case where the principal offers contracts at a time when the agent already knows his private information (sometimes, this is also referred to as **"interim contracting"**).

- As a consequence the participation constraints needed to be satisfied type-wise, i.e., for both type $\underline{\theta}$ and type $\overline{\theta}$.

- Consider now the case where the principal can offer contracts *ex-ante*, i.e., **before** the agent learns his type.

- In contrast to interim contracting, the participation constraint just needs to be satisfied in expectation:

$$\nu \underline{U} + (1 - \nu)\overline{U} \geq 0 \tag{8}$$

- It can easily be shown that in this case, the first-best can be implemented (as long as the agent is risk-neutral). The new participation constraint (8) will then be satisfied with equality.

- For example, this can be achieved with the following transfers:

$$\begin{aligned} \underline{U} &= (1-\nu)\Delta\theta\overline{q}^* > 0 \\ \overline{U} &= -\nu\Delta\theta\overline{q}^* < 0 \end{aligned}$$

- Thus, the agent is punished if his type turns out to be $\overline{\theta}$ and rewarded if it is $\underline{\theta}$.

Remarks

- Assumption: The principal can commit to stick to the contracts offered, although in equilibrium this leads to allocative inefficiencies with positive probability.

- This creates a strong incentive for renegotiation before the agent starts producing, because by implementing the first-best, ex post an additional surplus could be generated (and shared by the parties). This is anticipated by the agent, which would make it harder to satisfy the IC ex-ante because the agent receives a higher net utility ex-post.

- However, note that this issue arises with direct mechanisms. In the case of **indirect mechanisms** the principal in equilibrium learns the agent's type by observing q and therefore only after the production has already taken place.

Potential extensions

- More general utility function of the agent
- Information structure:
  - More than two types: IC-conditions “upwards” and “downwards” important. Bunching?
  - Multidimensional asymmetric information: Types cannot necessarily be ordered
- Outside option:
  - type-dependent: The IC of the inefficient type may be binding too (“countervailing incentives”)
  - stochastic
- Ex-ante contracting when transfer payments are constrained by limited liability
- Monitoring
- Two-sided asymmetric information (Myerson-Satterthwaite Theorem)