Contract Theory

Lecture 9: Hidden Action (Moral Hazard)

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Basic Problem

1. Moral Hazard: The Basic Problem

- A setting where there is complete information at the time of contracting.
- However, after the contract has been signed, the agent carries out an unobservable action that influence the social surplus.
- In contrast to models of incomplete information, the information structure is endogenous.

Some examples

- Insurance contracts: Insurance companies can usually not observe how careful drivers behave (but only whether a damage occurs or not).
- Employment contracts: Owners and bosses often delegate tasks to worker and managers, and they can typically only observe some (stochastic) performance signals (e.g., profits), but not the underlying effort choices.
- Bailing-out banks: Frequently, it is argued that if banks know that they will be bailed-out in case they get in trouble, this may adversely affect their risk-taking incentives.
- Grading: While I can observe your performance in an exam, I cannot observe how much effort you put in to prepare for the exam (where, in general, your performance will depend on both your effort and other (stochastic) factors).

Questions

- While the principal would like to control the agent's effort, she only has access to a noisy performance signal.
- This uncertainty is crucial: If the agent's action would lead to a (unique) deterministic outcome, the moral hazard problem would disappear.
- We examine the consequences of moral hazard on the optimal design of contracts.
- How "costly" is it for the principal to induce (unobservable) effort?

Basic Model

2. Basic Model

- A principal needs to delegate a task to an agent and can make a "take-it-or-leave-it" contract offer.

- If the agent accepts the offer, he can choose between two effort levels $e \in \{0, 1\}$.

- Effort e is associated with costs $\psi(e)$, where $\psi(0) = 0$ and $\psi(1) = \psi$.

- The agent receives a payment t from the principal.

- Utility function of the agent: $U = u(t) - \psi(e)$, where $u' > 0$ and $u'' < 0$ and $u(0) = 0$.

- Output $\widetilde{q} \in \{\overline{q}, \underline{q}\}$ stochastically depends on e:

$$\Pr(\widetilde{q} = \overline{q} \mid e = 1) = \pi_1 \quad \text{and} \quad \Pr(\widetilde{q} = \underline{q} \mid e = 1) = 1 - \pi_1$$

$$\Pr(\widetilde{q} = \overline{q} \mid e = 0) = \pi_0 \quad \text{and} \quad \Pr(\widetilde{q} = \underline{q} \mid e = 0) = 1 - \pi_0$$

- We have $\pi_1 > \pi_0$ so that for every q^*, $\Pr(\widetilde{q} \leq q^* \mid e)$ is decreasing in e (first-order stochastic dominance).

- Thus, higher effort makes low output less likely.

Information environment

- The principal can only observe $\widetilde{q}$, but not e. Thus, a contract can only condition on $\widetilde{q}$, but not on e.
- Therefore, a contract specifies output-dependent payments $t(\widetilde{q})$ from the principal to the agent, where $t(\overline{q}) = \overline{t}$ and $t(\underline{q}) = \underline{t}$.
- The principal's outcome-dependent revenues are given by $S(\overline{q}) = \overline{S}$ and $S(\underline{q}) = \underline{S}$, and hence, depending on the agent's effort, she obtains profits:

$$
\begin{aligned}
V(e=1) &= \pi_1(\overline{S} - \overline{t}) + (1-\pi_1)(\underline{S} - \underline{t}) \\
V(e=0) &= \pi_0(\overline{S} - \overline{t}) + (1-\pi_0)(\underline{S} - \underline{t})
\end{aligned}
$$

- Additional notation: $\Delta q = \overline{q} - \underline{q}$, $\Delta S = \overline{S} - \underline{S}$, $\Delta\pi = \pi_1 - \pi_0$.

Incentive-compatibility and participation constraints

- Should the principal offer a contract $\{\bar{t}, \underline{t}\}$ that induces $e = 1$, and if yes, what should this contract look like?
- To ensure that the agent accepts the offer and chooses $e = 1$, the contract needs to satisfy the following conditions.
- *Incentive compatibility constraint:* The agent's expected utility needs to be higher for $e = 1$ than for $e = 0$:

$$\pi_1 u(\bar{t}) + (1 - \pi_1)u(\underline{t}) - \psi \geq \pi_0 u(\bar{t}) + (1 - \pi_0)u(\underline{t}) \tag{1}$$

- *Participation constraint:* Ex ante, the agent's expected utility from the relationship with the principal must exceed his outside option (normalized to 0):

$$\pi_1 u(\bar{t}) + (1 - \pi_1)u(\underline{t}) - \psi \geq 0 \tag{2}$$

- A contract $\{\bar{t}, \underline{t}\}$ is called *incentive feasible*, if it satisfies conditions (1) and (2).

Timing

- Stage $t = 0$: The principal offers a contract $\{t(\overline{q}) = \overline{t},\ t(\underline{q}) = \underline{t}\}$.
- Stage $t = 1$: The agent accepts or rejects the offer.
- Stage $t = 2$: The agent exerts effort.
- Stage $t = 3$: The contract is executed.

2.1. First-Best Solution

- Assume that the effort levels e is observable and verifiable.
- In this benchmark case there is no problem of incentive compatibility.
- The first-best solution maximizes the expected utility of the principal subject to the constraint that the agent receives at least his outside option:

$$(e^*, \bar{t}^*, \underline{t}^*) \in \arg\max_{e, \bar{t}, \underline{t}} \left\{ \pi_e(\bar{S} - \bar{t}) + (1 - \pi_e)(\underline{S} - \underline{t}) \right\}$$

  subject to the constraint (2).
- How does the optimal contract $\{\bar{t}^*, \underline{t}^*\}$ look like?
- A priori it is not clear if $e = 1$ or $e = 0$ is optimal.
- In a first step, we examine which contract is optimal to induce a given effort level.

Inducing $e = 1$

- To induce $e = 1$, the principal maximizes

$$\max \pi_1(\overline{S} - \overline{t}) + (1 - \pi_1)(\underline{S} - \underline{t})$$

  subject to the PC constraint of the agent:

$$\pi_1 u(\overline{t}) + (1 - \pi_1)u(\underline{t}) - \psi \geq 0.$$

- Let μ be the Lagrange multiplier for the participation constraint.

- The first-order conditions are given by:

$$\begin{aligned} -\pi_1 + \mu\pi_1 u'(\overline{t}) &= 0 \Leftrightarrow \mu = \frac{1}{u'(\overline{t})} > 0, \text{ and} \\ -(1 - \pi_1) + \mu(1 - \pi_1)u'(\underline{t}) &= 0 \Leftrightarrow \mu = \frac{1}{u'(\underline{t})} > 0 \end{aligned}$$

- Since $\mu > 0$ the participation constraint needs to be binding at the optimum.
- Equating the first-order conditions yields:

$$\frac{1}{u'(\underline{t})} = \frac{1}{u'(\overline{t})} \Leftrightarrow \overline{t}^* = \underline{t}^* = t^*.$$

- Thus, the agent receives the same payment in both states of the world.

- We know that the participation constraint is binding at the optimum. Thus, we have:

$$\pi_1 u(t^*) + (1 - \pi_1) u(t^*) - \psi = 0 \Leftrightarrow u(t^*) - \psi = 0 \Leftrightarrow t^* = u^{-1}(\psi)$$

Summary for case $e = 1$

- To implement $e = 1$, optimality implies $\overline{t}^* = \underline{t}^* = u^{-1}(\psi)$.
- Complete insurance for the agent.
- The agent just receives his reservation utility, and therefore condition (2) is binding at the optimum.

Inducing $e = 0$

- $e = 0$ can optimally be induced by $\overline{t}^* = \underline{t}^* = 0$.

- In this case, we have:

$$V(e=1) \quad = \quad \pi_1 \overline{S} + (1-\pi_1)\underline{S} - u^{-1}(\psi), \text{ and}$$

$$V(e=0) \quad = \quad \pi_0 \overline{S} + (1-\pi_0)\underline{S}$$

- Hence, the principal prefers to induce $e = 1$ if:

$$\Delta\pi\Delta S \geq u^{-1}(\psi)$$

- Thus, the increase in expected utility must be larger than the additional costs that occur when inducing $e = 1$.

2.2. Risk Neutrality and No Limited Liability

- In the following, we go back to the the moral hazard environment where agent's effort is unobservable.
- Here, we deviate from the basic model by assuming that the agent is risk neutral: $u(t) = t$.

Second-best optimal contract

- The principal's maximization problem is given by:

$$\max_{\bar{t},\underline{t}} \ \pi_1(\bar{S} - \bar{t}) + (1 - \pi_1)(\underline{S} - \underline{t})$$

  subject to (1) and (2).
- Note that both the objective function and the constraints are linear in the transfer payments.
- Thus, we have "corner solutions" where both constraints are binding:

$$\underline{t}^* = -\frac{\pi_0}{\Delta\pi}\psi < 0, \text{ and} \quad (3)$$

$$\bar{t}^* = \frac{1-\pi_0}{\Delta\pi}\psi > 0. \quad (4)$$

- The agent is being punished with a negative payment $\underline{t}^* < 0$ if he produces low output and rewarded with a positive payment $\bar{t}^* > 0$ if he produces high output.

- How does his expected utility change if he chooses $e = 0$ instead of $e = 1$:

$$\Delta\pi(\bar{t}^* - \underline{t}^*) = \Delta\pi(\frac{1-\pi_0}{\Delta\pi}\psi \frac{\pi_0}{\Delta\pi}\psi) = \psi$$

- Therefore, the increase in the payment exactly equals the additional costs ψ that occur when choosing $e = 1$.
- The expected payoff of the principal is $\pi_1\bar{t}^* + (1-\pi_1)\underline{t}^* = \psi$, and thus the same as in the (first-best) case of complete information.
- So, under risk neutrality and in the absence of a limited liability constraint, there is no welfare loss due to moral hazard.

Alternative solution: Sell-the-Shop

- The following "sell-the-shop" contract also implements the first-best solution:
$$\overline{t}' = \overline{S} - T^* \text{ and } \underline{t}' = \underline{S} - T^*.$$
- In this case, the agent receives the complete revenue in every state of the world and pays the principal a fixed amount that is set such that constraint (2) is binding:
$$T^* = \pi_1 \overline{S} + (1 - \pi_1)\underline{S} - \psi.$$
- The principal thus again receives the entire surplus.