Contract Theory

Lecture 6: Hidden Information

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What is the Theory of Incentives (Contract Theory) About?

Part I: Introduction: What is the Theory of Incentives (and Contracts) About?

Incentive theory focuses on problems with the following characteristics:

- Several individuals (or firms) are in an economic relationship in which they can generate a surplus and divide it amongst them.
- The size of this surplus depends on the individuals' actions or characteristics.
- The problem of *opportunistic behavior* arises: Individuals choose the action that maximizes their private benefit, but not necessarily the social benefit of the group. That is, there is an "**incentive problem**".
- Signing a contract ex-ante may help the parties to overcome (or at least to reduce) incentive problems.

Asymmetric information as a source of incentive problems

- Important characteristics of individuals may be unobservable to other parties (i.e., there might be **hidden information** or **adverse selection**). As a result, the parties cannot directly condition on these hidden characteristics.

- In many cases, the actions which parties take might be unobservable to the other parties (i.e., there might be **hidden actions** or **moral hazard**). As a result, the parties cannot directly specify that certain actions should be taken.

- Finally, while certain variables might be perfectly observable to the interacting parties, some variables might not be verifiable by a court. Such problems of non-verifiability are at centre stage in the **theory of incomplete contracts**. As a result, so-called **hold-up problems** might arise.

Questions

- Contract theory considers situations, in which incentive problems of the kind described above arise.
- Contract theory studies how interacting parties should structure their relationship in the presence of such frictions in order to accomplish a given goal.
- Is it possible to induce individuals to reveal their private information voluntarily (through clever design of contracts)?
- How should a contract condition on observable (and verifiable) information?
- What form do optimal contracts take?
- What kind of inefficiencies do arise due to incentive problems?

Fields of Application

- Compensation schemes for workers and managers
- Regulation of natural monopolies
- Design of credit contracts
- Design of insurance contracts
- Provision of public goods
- Design of trade contracts
- Vertical integration and outsourcing
- And many more...

First-best versus second-best

- Contracting problems are often associated with inefficiencies in the sense that the first-best allocation will not be achieved.
- The first-best allocation is the one that would maximize the social surplus in the absence of incentive problems.
- If the first-best is unattainable in the presence of incentive problems, the parties will strive to achieve the "second-best" (i.e., the constrained optimum).

Contract theory versus economics of information

- Economics of Information was the first theory trying to explain inefficiencies arising from asymmetric information using partial equilibrium models.
- Nobel prize 2001:
  - George E. Akerlof: "Adverse selection in markets"
  - Michael Spence: "Signalling"
  - Joseph E. Stiglitz: "Screening"
- In contrast to contract theory, which focuses on the design of contracts, in information economics the (simple) form of contracts is taken as exogenously given.
- Information economics usually considers interaction in anonymous markets, whereas contract theory usually focuses on the (strategic) interaction of a small group of agents (e.g., a buyer and a seller).
- In contract theory, the role of markets is often reduced to simply determining the outside options of the contracting parties.

Contract theory versus game theory

- Strategic considerations also play an important role in contract theory. That is, we will again rely on game-theoretic solution concepts such as Nash equilibrium, subgame perfect equilibrium, Bayesian Nash equilibrium, etc...
- Game theory takes the game form as given and aims to predict the behavior of the parties.
- Contract theory takes a reverse approach: each contract induces a different game form (i.e., induces a specific non-cooperative game). We then derive the (second-best) contract that leads to the "optimal" outcome (according to some criterion, e.g., social welfare or private profit)
- In game theory there often exists the problem of multiple equilibria.
- In contract theory, we circumvent this problem by assuming that, in the case of multiple equilibria, the parties can coordinate on which equilibrium will be played.

The Informational Environment Determines Which Contracts are Feasible

2. The Informational Environment Determines Which Contracts are Feasible

- The informational environment ("Who knows what when?") determines the set of feasible contracts.
- By assumption, contracts can only condition on verifiable information.
- Also by assumption, contracting parties will always adhere to such a contract (i.e., we rule out legal disputes).

Canonical situations of asymmetric information

- Asymmetric information at the time of contracting:
  - Emerges as a consequence of unobservable characteristics of (some of) the contracting parties.
  - Synonyms: "hidden information", "incomplete information", "adverse selection".
  - In this case, the information structure is exogenously given.
- Symmetric information at the time of contracting:
  - Asymmetry emerges as a consequence of unobservable actions *after* contracting.
  - Synonyms: "Hidden Action", "Moral Hazard".
  - Here, the information structure ist endogenous.
- Observability versus verifiability
  - The contracting parties have symmetric information throughout.
  - However, some of the information that is observable to the parties cannot be verified by courts.
  - At centre stage in the *theory of incomplete contracts* and the *hold-up problem*.

Part II: Incomplete Information (Adverse Selection)

Basic Problem

Basic Model

- First-Best Solution
- Revelation Principle

Basic Problem

3. Adverse Selection: The Basic Problem

- Two parties - a principal and an agent - are in an economic relationship.
- "Adverse selection" describes situations in which the agent has, at the time of contracting, private information with respect to parameters that are relevant for the value of the relationship between the principal and the agent.
- In contrast to the moral hazard case (analyzed below), the information structure is exogenously given.

Standard Examples

- Product markets: Sellers typically have better information about the quality of a product than potential buyers (e.g., the lemons model of Akerlof).

- Health insurance: Individuals often have better information about their health status than the insurance company (earlier diseases, smoking and drinking habits, etc...).

- Employment contracts: Employees typically have better information about relevant aspects like their productivity or intrinsic motivation.

- Regulation of firms: A firm typically has better information about its own cost structure than a regulatory agency.

Issues considered

- Examining the impact of adverse selection on the design of contracts.
- Can the principal induce the agent to (truthfully) reveal his information?
- Does the principal **want** to induce the agent to reveal his information?
- How much of the principal's profit is lost due to asymmetric information?
- How large is the welfare loss due to asymmetric information?

Screening vs Signaling

- In **signaling** models the better informed party moves first.
  - Example: An employee tries to signal high productivity by foregoing employment protection or by a longer period of education (see Spence's model of "job market signaling").
- In **screening** models (on which we focus here), the less informed party may optimally offer a **menu of contracts**, and different **types** of agents may endogenously select into different contracts.
  - Example: A car insurance typically offers contracts with high deductible and low premiums and contracts with low deductible and high premiums.
  - The intention behind this is that drivers with a high accident risk ("bad types") tend to choose the second contract compared to those with low accident risk ("good types").
  - Drivers reveal (part of) their private information **in equilibrium**.
  - However, in general this will be "costly" for the principal, because often the agent can extract a rent due to his informational advantage (i.e., an **"information rent"** on top of his reservation utility).

Basic Model

4. Basic Model

- A principal needs an agent for the production of a good.
- The value of q units of the good is $S(q)$, where $S' > 0$, $S'' < 0$ and $S(0) = 0$.
- The agent's marginal cost of production θ is either "high" or "low", i.e., $\theta \in \{\underline{\theta}, \overline{\theta}\}$ where $\overline{\theta} > \underline{\theta}$. Thus, we call type $\underline{\theta}$ the "efficient" type and type $\overline{\theta}$ the "inefficient" type of the agent.
- Define $\Delta\theta = \overline{\theta} - \underline{\theta} > 0$.
- Transfer (payment) from the principal to the agent: $t(\theta)$, where we use the short-hand notation $\overline{t} = t(\overline{\theta})$ and $\underline{t} = t(\underline{\theta})$.
- No fixed costs. Both risk-neutral. Agent's reservation utility is 0.

Informational Environment

- The principal cannot observe the agent's cost type θ.
- The agent knows his type at the time of contracting whereas the principal only knows the probabilities v and $(1-v)$ for the types $\underline{\theta}$ and $\overline{\theta}$, respectively.
- Note: As before, it is important that this probability distribution is "common knowledge", it is known by the principal and this in turn is known by the agent etc....
- The variable q and some transfer t can be part of a contract (as they are assumed to be verifiable by a court of law), but the information parameter θ cannot.
- We assume that the principal has the entire bargaining power and can make a "take-it-or-leave-it" offer (in contrast to moral hazard models, this assumption is not without loss of generality).

Timing

- Stage 0: The agent (A) learns his type θ.
- Stage 1: The principal (P) may offer a "menu of contracts".
- Stage 2: A accepts some offered contract or rejects.
- Stage 3: The selected contract is executed.

4.1. First-Best Solution: Complete Information Benchmark

- The principal can observe the agent's type, and offers the respective type a **single** contract (q, t)
- P has to obey the respective type's participation constraint:

$$\underline{t} - \underline{\theta}\underline{q} \geq 0 \quad or \tag{1}$$

$$\overline{t} - \overline{\theta}\overline{q} \geq 0 \tag{2}$$

- In the optimum, for a given quantity q, this leads to the following transfers (which lead each type to just accept the respective contract):

$$\underline{t} = \underline{\theta}\underline{q} \quad or \tag{3}$$

$$\overline{t} = \overline{\theta}\overline{q} \tag{4}$$

- For a given quantity, the agent is only compensated for her cost of production and just receives his reservation utility.
- The principal is **residual claimant** on the social surplus, and hence, in the initial contract, she will choose the (type-dependent) quantity that maximizes the social surplus $S(q) - \theta q$.

- For the first-best production levels, the respective first-order conditions are:

$$S'\left(\underline{q}^*\right) = \underline{\theta} \quad \textit{and} \quad S'\left(\overline{q}^*\right) = \overline{\theta} \tag{5}$$

- Obviously, $\underline{q}^* > \overline{q}^*$ at the optimum, i.e., the efficient type produces more.

- For a given type, production should take place if, at the respective first-best quantity, the achievable surplus is higher than the costs:

$$S(\underline{q}^*) - \underline{\theta}\underline{q}^* > 0 \quad \textit{and} \quad S(\overline{q}^*) - \overline{\theta}\overline{q}^* > 0,$$

- Hence, assuming that this condition is satisfied, the optimal contract is either $(\underline{q}^*, \underline{t}^* = \underline{\theta}\underline{q}^*)$ or $(\overline{q}^*, \overline{t}^* = \overline{\theta}\overline{q}^*)$. Otherwise, the optimal contract is $(0, 0)$ and production does not take place (**"shut-down"**).

4.2. The Revelation Principle: A Piece of Art

- In our two-type setup, we will only consider a relatively special form of interaction between P and A: P offers two different contracts and each agent type decides whether to accept one of the contracts or to reject. If both contracts meet certain conditions, each type chooses the contract that is designed for him.
- How restrictive is this? Are there other, possibly more complex mechanisms, which yield a better result for P?
- For example, P could possibly offer 20 instead of 2 different contracts among which every type can choose.
- However, the **revelation principle** (maybe the most important result in mechanism design; due to Myerson, 1979) states that when looking for an optimal contract, without loss of generality, one can restrict attention to incentive-compatible direct mechanisms (which in the present case will boil down to a menu of two contracts with certain properties).

Basic Idea

- A contract generally stipulates a non-cooperative game so that rational parties will play an (appropriately defined) equilibrium of this game (Bayesian, Perfect Bayesian).

- An allocation (quantities and transfers) is thus feasible under a given contract if it is an equilibrium outcome of the game stipulated by the contract.

- The revelation principle states that every contractually attainable allocation can also be attained through a mechanism where (i) the principal directly asks the agent about his private information (direct mechanism) and (ii) where the agent has an incentive to report truthfully (incentive-compatibility).

Definition (Direct Mechanism)

In a direct mechanism, the principal commits to offer the transfer $t(\widetilde{\theta})$ and the production level $q(\widetilde{\theta})$ for a given **message** $\widetilde{\theta} \in \Theta$ by the agent about his type. Formally, a direct mechanism is a mapping $g(\cdot)$ from the set of possible types Θ to the set of possible allocations A (i.e., a quantity q and a transfer t).

Note: While the type θ itself is unobservable (and hence not verifiable), the message $\widetilde{\theta}$ (which is not necessarily truthful) will be observable and verifiable.

Definition (Incentive-Compatible Direct Mechanisms)

A direct mechanism is incentive-compatible (that is, truthful) if it is individually rational for the agent to truthfully announce his type (for any type he may be).

- In the present example, for a direct mechanism to be truthful the following two incentive compatibility conditions for $\Theta = \{\underline{\theta}, \overline{\theta}\}$ need to be satisfied:

$$t(\underline{\theta}) - \underline{\theta} q(\underline{\theta}) \geq t(\overline{\theta}) - \underline{\theta} q(\overline{\theta}) \quad (6)$$

$$t(\overline{\theta}) - \overline{\theta} q(\overline{\theta}) \geq t(\underline{\theta}) - \overline{\theta} q(\underline{\theta}) \quad (7)$$

- That is, type $\underline{\theta}$ has to be better off by sending the message $\widetilde{\theta} = \underline{\theta}$ than by sending the message $\widetilde{\theta} = \overline{\theta}$, and analogously for type $\overline{\theta}$.
- In the following, we will illustrate why the revelation principle holds true.
- To do so, we introduce the (all-encompassing) general class of mechanisms where, conditionally on some message m received from the agent, a production level $\widetilde{q}(m)$ and a corresponding transfer $\widetilde{t}(m)$ are implemented, and where m is from some arbitrary message space M.

Definition (General Mechanism)

In general, a mechanism is a message space M and a mapping $\widetilde{g}(\cdot)$ from M to A, with $\widetilde{g}(m) = (\widetilde{t}(m), \widetilde{q}(m))$ for all $m \in M$.

- Given a mechanism M, an agent of type θ optimally chooses a message $m^*(\theta)$ that maximizes his expected utility, i.e., that satisfies:

$$\widetilde{t}(m^*(\theta)) - \theta\widetilde{q}(m^*(\theta)) \geq \widetilde{t}(\widetilde{m}) - \theta\widetilde{q}(\widetilde{m}) \text{ for all } \widetilde{m} \in M \qquad (8)$$

- The mechanism $(M, \widetilde{g}(\cdot))$ therefore induces an allocation rule $a(\theta)$, mapping the set of types into the set of allocations:

$$a(\theta) = \left(\widetilde{t}(m^*(\theta)), \widetilde{q}(m^*(\theta))\right).$$

Theorem (Revelation Principle)

Any allocation rule $a(\theta)$ obtained with an arbitrary mechanism $(M, \widetilde{g}(\cdot))$ can also be implemented with an incentive-compatible direct mechanism. Hence, when looking for optimal contracts, without loss of generality one can restrict attention to incentive-compatible direct mechanisms.

Proof

- Any mechanism $(M, \widetilde{g}(\cdot))$ induces an allocation rule $a(\theta) = (\widetilde{t}(m^*(\theta)), \widetilde{q}(m^*(\theta)))$.

- Construct a direct mechanism that replicates this allocation rule:

$$g(\theta) = (t(\theta), q(\theta)) \equiv (\widetilde{t}(m^*(\theta)), \widetilde{q}(m^*(\theta))) \text{ for all } \theta \tag{9}$$

- As under the allocation rule $(\widetilde{t}(m^*(\theta)), \widetilde{q}(m^*(\theta)))$, the agent optimally chooses $m^*(\theta)$ given θ, sending the message $\widetilde{\theta} = \theta$ must also be the agent's best reply in the direct mechanism (because it implements the same allocation).

- Let θ be the agent's true type and θ' some other arbitrary value belonging to Θ. Then, as $m^*(\cdot)$ is optimal for the agent, it has to hold that:

$$\widetilde{t}(m^*(\theta)) - \theta\widetilde{q}(m^*(\theta)) \geq \widetilde{t}(m^*(\theta')) - \theta\widetilde{q}(m^*(\theta')) \tag{10}$$

- And hence, the associated direct mechanism $g(\theta)$ satisfies:

$$t(\theta) - \theta q(\theta) \geq t(\theta') - \theta q(\theta') \tag{11}$$

- Intuition: the direct mechanism "does the equilibrium calculations" for the agent, such that for any type θ the constructed direct mechanism, in equilibrium, leads to the same allocation as the general mechanism $(M, \widetilde{g}(\cdot))$. **QED**

- Hence, the Revelation Principle says that for any arbitrary general mechanism there exist some incentive-compatible direct mechanism that leads to the same allocation.

- Put differently: Take some allocation as given. If there does not exist an incentive-compatible direct mechanism implementing this allocation, then there does not exist any mechanism that implements this allocation.

- Hence, from the Revelation Principle, we know that by restricting attention to incentive-compatible direct mechanisms one does not risk missing implementable allocations.

- As we will see, an important assumption underlying the revelation principle is the ability to commit to the initial contract (mechanism).