Impact Evaluation Methods

Topic 8: Difference-in-Differences

Alex Alekseev

June 28, 2024

University of Regensburg, Department of Economics

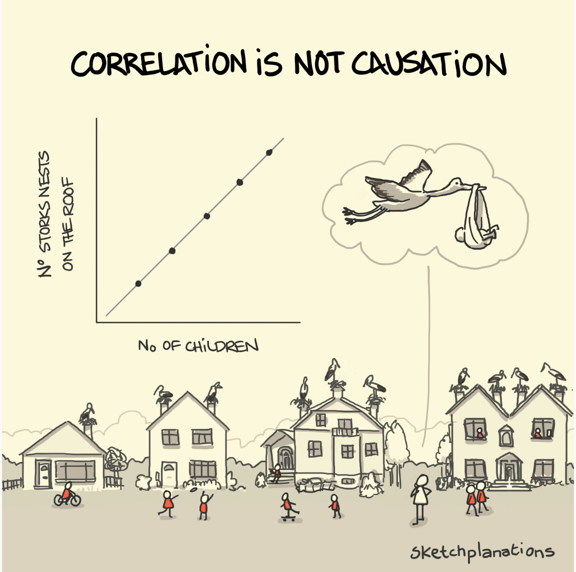


Figure 1: Source: Sketchplanations.com

Previously on *Impact Evaluation Methods*...

- Instrumental variables
- Heterogeneous treatment effects
- LATE

History and background

First DD

- One of the first recorded examples the DD design to answer a causal question is John Snow's work on the origins of cholera
- He challenged the received knowledge at that time that cholera was spread through air
- Showed instead that cholera was spread by fecally-contaminated water in his 1855 work titled "On the Mode of Communication of Cholera"

Cholera

- A bacterial disease of the small intestine with acute symptoms such as vomiting and diarrhea
- In the 19th century, it was usually fatal
- There were three main epidemics that hit London at that time, tens of thousands of people suffered and died from the disease
- Doctors could not help the victims

Miasma Theory

- The dominant theory about cholera transmission at that time was the **miasma theory**
- Diseases were spread by microscopic poisonous particles that infected people by floating through the air
- Treatments were designed to stop poisonous dirt from spreading through the air
- Methods like quarantining the sick were strangely ineffective at slowing down cholera

John Snow

- John Snow worked in London during these epidemics
- Originally he accepted the miasma theory
- He covered the sick with burlap bags, for instance, but the disease still spread
- He changed his mind and began look for a new explanation

Alternative Hypothesis

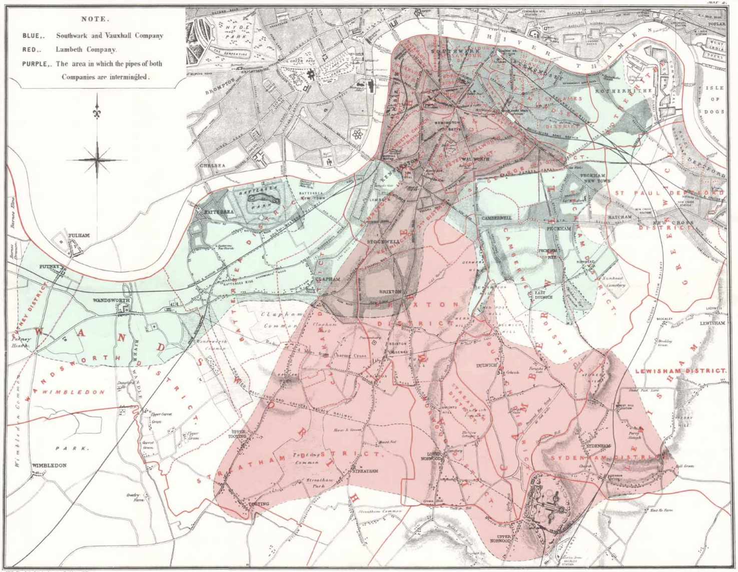
- John Snow came to believe that cholera spread by dirty drinking water
- He had a few ways of providing evidence, one of which is very similar to a modern-day **difference-in-differences** research design (see Coleman (2019) for a review)
- He used a **natural experiment** in London during which one of the water companies changed its water intake point

Natural Experiment

- London's water needs were served by a number of competing companies
- Water taken in from the parts of the Thames that were downstream of London contained everything that Londoners dumped in the river
- In 1849, the Lambeth water company had moved its intake pipes upstream higher up the Thames
- They did this to obtain cleaner water, but it had the added benefit of being too high up the Thames to be infected with cholera

Counterfactuals

- Snow realized that it had given him a natural experiment to test his hypothesis
- If his theory was right, then the Lambeth houses should have lower cholera death rates than some other set of households whose water was infected
- He found his explicit counterfactual in the Southwark and Vauxhall Waterworks Company that have not moved their intake point upstream and who served similar households


NOTE.
BLUE,. Southwark and Vauxhall Company
RED,. Lambeth Company.
PURPLE,. The area in which the pipes of both Companies are intermingled.
HYDE PARK
BROMPTON
CHELSEA
BATTERSEA
FULHAM
PUTNEY
WANDSWORTH
CLAPHAM
STOCKWELL
BRIXTON
CAMBERWELL
PECKHAM
ROTHERHITHE
ISLE OF DOGS
LEWISHAM
LEWISHAM DISTRICT.
WIMBLEDON
STREATHAM
TOOTING
SYDENHAM
UPPER NORWOOD
DULWICH
WALWORTH
SOUTHWARK
RIVER THAMES

Snow's DD Table

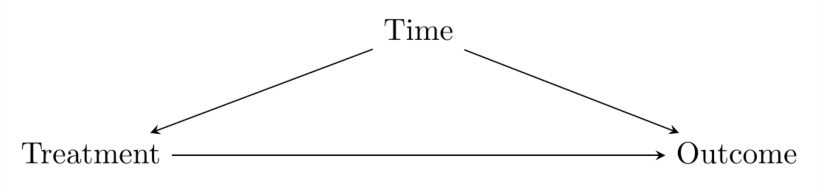
Snow collected the data on death rates (per 10,000 people in 1851), which are summarize in the table below.

Company	Death rate in 1849	Death rate in 1854
Southwark and Vauxhall	135	147
Lambeth	85	19

- Treatment effect: $19 - (85 + (147 - 135)) = -78$
- Alternatively: $19 - (147 - (135 - 85)) = -78$

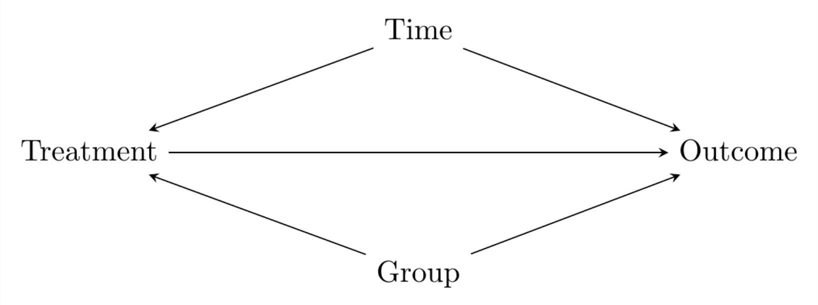
Causal diagram of DD

Time Back Door



- Identifying the effect of **Treatment** on **Outcome** requires us to close the back door that goes through **Time**
- But we can't do this entirely, because all of the variation in Treatment is explained by Time
- You're either in a pre-treatment time and untreated, or in a post-treatment time and treated.

Another Group



- DD design brings in another group that is never treated
- So now in the data we have both the group that receives treatment at a certain point, and another group that never receives treatment
- At first this seems counterintuitive

Isolating Variation

- Now that we have that untreated group, even though we've added a new back door, we can now close both back doors
- First, we isolate the within variation (across time) for both the treated group and untreated group
- Because we have isolated within variation, we are controlling for group differences and closing the back door through **Group**

Isolating Variation

- Now that we have that untreated group, even though we've added a new back door, we can now close both back doors
- First, we isolate the within variation (across time) for both the treated group and untreated group
- Because we have isolated within variation, we are controlling for group differences and closing the back door through **Group**
- Second, we compare the within variation in the treated group to the within variation in the untreated group
- Because the within variation in the untreated group is affected by time, doing this comparison controls for time differences and closes the back door through **Time**.

Two-by-two DD

Some Notation

- Let $\mathbb{E}[Y_{it}]$ be the expected outcome of group i at time t
- The group index i will take values of T (treated, the group that experienced treatment between the two dates) or U (untreated, the group that did not experience treatment)
- The time index t will take values of 0 (pre-treatment date) and 1 (post-treatment date)
- This gives us four expected outcomes in total that we can arrange in a 2x2 table

DD Table

	$t = 0$	$t = 1$	Difference
$i = U$	$\mathbb{E}[Y_{U0}]$	$\mathbb{E}[Y_{U1}]$	$\mathbb{E}[Y_{U1}] - \mathbb{E}[Y_{U0}]$
$i = T$	$\mathbb{E}[Y_{T0}]$	$\mathbb{E}[Y_{T1}]$	$\mathbb{E}[Y_{T1}] - \mathbb{E}[Y_{T0}]$
Difference	$\mathbb{E}[Y_{T0}] - \mathbb{E}[Y_{U0}]$	$\mathbb{E}[Y_{T1}] - \mathbb{E}[Y_{U1}]$	$\mathbb{E}[Y_{T1}] + \mathbb{E}[Y_{U0}] - \mathbb{E}[Y_{T0}] - \mathbb{E}[Y_{U1}]$

Simplify

- Suppose that the difference in expected outcomes at time $t = 0$ between the treated and untreated groups is

$$\lambda \equiv \mathbb{E}[Y_{T0}] - \mathbb{E}[Y_{U0}].$$

- Likewise, assume that the difference for the untreated group between the post- and pre-treatment dates is

$$\tau \equiv \mathbb{E}[Y_{U1}] - \mathbb{E}[Y_{U0}].$$

Treatment Effect

- Then we can find the post-treatment outcome for the treated group in one of the two ways
- First, we can assume that the difference in expected outcomes for the treated group between the dates is caused by the treatment (δ) and time (τ) effects

$$\mathbb{E}[Y_{T1}] = \mathbb{E}[Y_{T0}] + \delta + \tau.$$

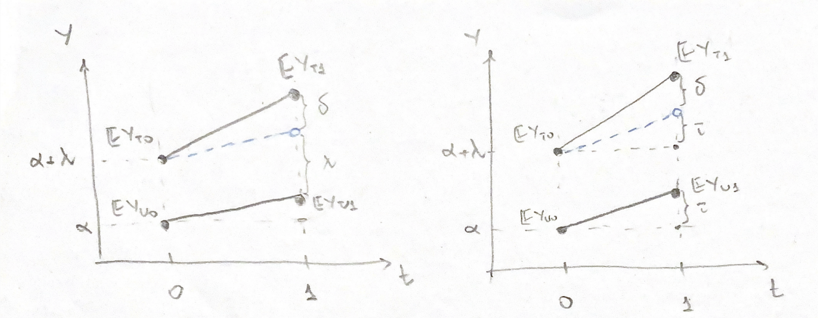
- Alternatively, we can assume that the difference in expected outcomes at the post-treatment date between the treated and untreated groups is caused by the treatment (δ) and group (λ) effects

$$\mathbb{E}[Y_{T1}] - \mathbb{E}[Y_{U1}] = \delta + \lambda.$$

Simplified DD Table

	$t = 0$	$t = 1$	Difference
$i = U$	$\mathbb{E}[Y_{U0}]$	$\mathbb{E}[Y_{U1}]$	τ
$i = T$	$\mathbb{E}[Y_{T0}]$	$\mathbb{E}[Y_{T1}]$	$\delta + \tau$
Difference	λ	$\delta + \lambda$	δ

Geometry of DD



DD in action

Card and Krueger (1994)

- A classic example of DD in labor economics is Card and Krueger (1994)
- Studied the effect of a minimum wage increase on employment
- In theory, in a competitive labor market, increases in the minimum wage move us up a downward-sloping demand curve
- Higher minimum wage therefore reduces employment, perhaps hurting the very workers these policies were designed to help

What They Did

- On April 1, 1992, New Jersey raised the state minimum from \$4.25 to \$5.05
- Card and Krueger collected data on employment at fast food restaurants in New Jersey in February 1992 and again in November 1992.
- These restaurants (Burger King, Wendy's, and so on) are big minimum-wage employers
- Card and Krueger collected data from the same type of restaurants in eastern Pennsylvania
- The minimum wage in Pennsylvania stayed at \$4.25 throughout this period

DD Table

The table below shows the average full-time equivalent (FTE) employment at restaurants in Pennsylvania (PA, untreated) and New Jersey (NJ, treated) before (February, $t = 0$) and after (November, $t = 1$) a minimum wage increase in New Jersey.

	February	November	Difference
$i = PA$	23.33	21.17	-2.16
$i = NJ$	20.44	21.03	0.59
Difference	-2.89	-0.14	2.75

Two-way fixed effects

Regression

- In practice, the DD is estimated using a regression
- For a simple two-by-two case the DD estimated is equivalent to an interaction effect in the following model.

$$Y = \alpha + \lambda \mathbb{I}(i = T) + \tau \mathbb{I}(t = 1) + \delta \mathbb{I}(i = T)\mathbb{I}(t = 1) + \epsilon,$$

- You can easily verify that plugging all the four possible combinations of i and t will yield the four outcomes from the table
- The interaction effect tells us by how much bigger the group effect is when $t = 1$ relative to $t = 0$
- Equivalently, it tells us by how much bigger the time effect is when $i = T$ relative to $i = U$
- Both of these interpretations are identical to the DD effect we introduced earlier using differences in expected outcomes.

Many Groups and Dates

- The regression framework makes it easy to extend the model to include multiple dates or groups
- The model will take the following form, which is a generalization of the model above
$$Y = \alpha_g + \alpha_t + \delta D + \epsilon.$$
- This model is know as two-way fixed effects model
- The "two-way" part comes from the fact that we are including the so-called fixed effects for both groups (α_g) and time (α_t)
- These fixed effects allow different groups and different dates to have their own conditional means
- The D variable is an indicator equal to 1 if the observation is in the treated group after the treatment occurred.

Controls

- The model can also include additional controls
- However, notice that any control variables that vary over group but don't change over time are unnecessary
- The inclusion of time-varying controls imposes some statistical problems
- If you need to include covariates, it's often a good idea to show your results both with and without them.

Rollout Designs

- If you have a single treatment period, the regression can be an easy way to estimate difference-in-differences
- The downside of this regression approach for estimating DD is that it doesn't work very well for the so-called **rollout designs**
- In the rollout designs, the treatment is applied at different times to different groups
- Researchers used the two-way fixed effects for these cases for a long time, but it turns out to not work as expected

Treatment effect and assumptions

Treatment effect

- The graphical analysis we conducted already suggests an answer to a question of what kind of a treatment effect does DD give
- But let's look at it more formally

$$
\begin{aligned}
\mathbb{E}[Y_{T1}] + \mathbb{E}[Y_{U0}] - \mathbb{E}[Y_{T0}] - \mathbb{E}[Y_{U1}] &= \mathbb{E}[Y^1_{T1}] + \mathbb{E}[Y^0_{U0}] - \mathbb{E}[Y^0_{T0}] - \mathbb{E}[Y^0_{U1}] \\
&\quad + \mathbb{E}[Y^0_{T1}] - \mathbb{E}[Y^0_{T1}] \\
&= \underbrace{\mathbb{E}[Y^1_{T1}] - \mathbb{E}[Y^0_{T1}]}_{ATT} \\
&\quad + \underbrace{\mathbb{E}[Y^0_{T1}] - \mathbb{E}[Y^0_{T0}]}_{\tau_T} - \underbrace{(\mathbb{E}[Y^0_{U1}] - \mathbb{E}[Y^0_{U0}])}_{\tau_U}.
\end{aligned}
$$

ATT

- Therefore, the DD estimates an ATT...
- plus a term that is a difference in the time effects for the treated (τ_T) and untreated (τ_U) groups
- If the two time effects are not equal, the DD estimate is biased relative to the true ATT
- Hence, the main identifying assumption of DD is that $\tau_T = \tau_U$
- This assumption is called **parallel trends**

Alternative Formulation

- Notice that due to the symmetry of the design, we can rewrite the expression above as

$$\begin{aligned}\mathbb{E}[Y_{T1}] + \mathbb{E}[Y_{U0}] - \mathbb{E}[Y_{T0}] - \mathbb{E}[Y_{U1}] = & \underbrace{\mathbb{E}[Y^1_{T1}] - \mathbb{E}[Y^0_{T1}]}_{ATT} \\ & + \underbrace{\mathbb{E}[Y^0_{T1}] - \mathbb{E}[Y^0_{U1}]}_{\lambda_1} - \underbrace{(\mathbb{E}[Y^0_{T0}] - \mathbb{E}[Y^0_{U0}])}_{\lambda_0}.\end{aligned}$$

- Another way to state the parallel trends assumption is then to assume that the group effects do not change over time, $\lambda_1 = \lambda_0$

Parallel Trends

- The parallel trends assumption is what allows us to difference out the time and group effects in the table to get δ
- It also allows us to construct the counterfactual in the graph
- Without this assumption, DD does not identify any meaningful effect
- Notice that the parallel trends assumption involves the counterfactual $\mathbb{E}[Y^0_{T1}]$
- We never observe this value, hence the identifying assumption of DD is inherently untestable
- We can, however, use several diagnostic tests: the test of prior trends and the placebo test.

Prior Trends

- Look at whether the treated and untreated groups already had differing trends in the lead-up to the treatment date
- First, graph the average outcomes over time in the pre-treatment period and see if they look different
- Second, perform a statistical test to see if the trends are different, and if so, how much different
- The simplest form of this uses the regression model

$$Y = \alpha_g + \beta_1 t + \beta_2 t \times Group + \epsilon$$

estimated using only data from before the treatment period, where the interaction term allows the time trend to be different for each group.

What If It Fails?

- When failing a prior trends test, some researchers will see this as a reason to add "controls for trends" by including the **Time** variable in the model directly
- However, this can have the unfortunate effect of controlling away some of the actual treatment effect, especially for treatments with effects that get stronger or weaker over time

Placebo

1. Use only the data that came before the treatment went into effect
2. Pick a fake treatment period.
3. Estimate the same DD model you were planning to use, but create the **Treated** variable as equal to 1 you're in the treated group and after the fake treatment date you picked
4. If you find an "effect" for that treatment date where there really shouldn't be one, that's evidence that there's something wrong with your design, which may imply a violation of parallel trends.

Alternative Placebo

- Another way to do this if you have multiple untreated groups is to use all of the data, but drop the data from the treated groups
- Then, assign different untreated groups to be fake treated groups, and estimate the DD effect for them
- This approach is less common since it doesn't address parallel trends quite as directly
- This is a very common placebo test for the synthetic control method

Next Time on *Impact Evaluation Methods*...

Your presentations