Impact Evaluation Methods

Topic 7: Instrumental Variables, Part 2

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Previously on *Impact Evaluation Methods*...

- Instrumental variables
- History
- Homogeneous treatment effects
- Binary treatment and instrument
- Non-binary treatment and instrument
- Assumptions and weaknesses
- IV in action

Heterogeneous treatment effects

Heterogeneous TE and LATE

- The assumption of homogeneous treatment effects is rather strong
- If we allow treatment effects to be heterogeneous, then what kind of an average treatment effect would we estimate using IV?
- We will introduce a new treatment effect parameter: the local average treatment effect (LATE)
- The following discussion uses the ideas developed in Imbens and Angrist (1994)

Potential Treatment Assignment

- Imbens and Angrist (1994) classify individuals into
  - those who respond positively to an instrument (compliers)
  - those who remain unaffected by an instrument (always takers and never takers)
  - those who rebel against an instrument (defiers)
- We can define potential treatment assignment variables, $D^{Z=z}$, for each state z of the instrument Z
- When D and Z are binary, there are four possible groups of individuals in the population.

Classification of Individuals by Compliance

	$D^1 = 0$	$D^1 = 1$
$D^0 = 0$	Never takers ($\tilde{C} = n$)	Compliers ($\tilde{C} = c$)
$D^0 = 1$	Defiers ($\tilde{C} = d$)	Always takers ($\tilde{C} = a$)

Here, variable $\tilde{C}$ represents compliance status and takes one of the four possible values.

Observed Treatment Assignment

- The observed treatment assignment (D) can now be represented using its own **switching equation**:

$$D = D^0 + (D^1 - D^0)Z.$$

- The difference $D^1 - D^0$ is the individual-level causal effect of the instrument on D
- This difference equals 1 for compliers, -1 for defiers, and 0 for always takers and never takers

LATE Assumptions

- We need three assumptions to characterize the treatment effect we would obtain using the Wald estimator

1. Independence: $(Y^1, Y^0, D^1, D^0) \perp Z$
2. Relevance: $D^1 - D^0 \neq 0$ for all i
3. Monotonicity: $D^1 - D^0 \geqslant 0$ for all i or $D^1 - D^0 \leqslant 0$ for all i

Independence

- The **independence** assumption states that the potential outcomes and potential treatment assignments (but not the observed ones) are independent of the instrument
- It implies the validity assumption (or exclusion restriction)...
- ...and that the reduced form gives us the causal effect of Z on Y
- It also implies that the first stage can be used to estimate the causal effect of Z on D:

$$\begin{aligned} \mathbb{E}[D \mid Z = 1] - \mathbb{E}[D \mid Z = 0] &= \mathbb{E}[D^1 \mid Z = 1] - \mathbb{E}[D^0 \mid Z = 0] \\ &= \mathbb{E}[D^1 - D^0] \end{aligned}$$

Relevance and Monotonicity

- The **relevance** assumption (nonzero effect of Z on D) means that the instrument must cause treatment assignment for at least some individuals
- There must be at least some compliers or some defiers in the population of interest
- The **monotonicity** assumption further specifies that the effect of Z on D must be either weakly positive or weakly negative for all individuals i
- There may be either defiers or compliers in the population but not both

LATE Theorem

- If the assumptions hold, then an instrument identifies a local average treatment effect (LATE): the average causal effect of the treatment for the subset of the population whose treatment selection is induced by the instrument
- In particular, if there are no defiers, then

$$LATE = \frac{\mathbb{E}[Y \mid Z = 1] - \mathbb{E}[Y \mid Z = 0]}{\mathbb{E}[D \mid Z = 1] - \mathbb{E}[D \mid Z = 0]} = \mathbb{E}[Y^1 - Y^0 \mid \tilde{C} = c]$$

Math time: Proof of the LATE formula

- Consider the numerator. By independence, we have

$$\mathbb{E}[Y \mid Z=1] = \mathbb{E}[Y^0 + D^1(Y^1 - Y^0)]$$
$$\mathbb{E}[Y \mid Z=0] = \mathbb{E}[Y^0 + D^0(Y^1 - Y^0)]$$

- Taking the difference between the two, we get

$$\begin{aligned}
&\mathbb{E}[Y \mid Z=1] - \mathbb{E}[Y \mid Z=0] \\
&\qquad = \mathbb{E}[Y^0 + D^1(Y^1 - Y^0)] - \mathbb{E}[Y^0 + D^0(Y^1 - Y^0)] \\
&\qquad = \mathbb{E}[(Y^1 - Y^0)(D^1 - D^0)] \\
&\qquad = \mathbb{E}[(Y^1 - Y^0)(D^1 - D^0) \mid D^1 > D^0]\mathbb{P}(D^1 > D^0) \\
&\qquad + \mathbb{E}[(Y^1 - Y^0)(D^1 - D^0) \mid D^1 = D^0]\mathbb{P}(D^1 = D^0) \\
&\qquad = \mathbb{E}[Y^1 - Y^0 \mid D^1 > D^0]\mathbb{P}(D^1 > D^0)
\end{aligned}$$

Math time: Proof of the LATE formula

- Now consider the denominator. Again, by independence we have

$$
\begin{aligned}
\mathbb{E}[D \mid Z = 1] &= \mathbb{E}[D^1] \\
\mathbb{E}[D \mid Z = 0] &= \mathbb{E}[D^0].
\end{aligned}
$$

- Taking the difference, we get

$$
\begin{aligned}
\mathbb{E}[D \mid Z = 1] - \mathbb{E}[D \mid Z = 0] &= \mathbb{E}[D^1 - D^0] \\
&= \mathbb{E}[D^1 - D^0 \mid D^1 > D^0]\mathbb{P}(D^1 > D^0) \\
&+ \mathbb{E}[D^1 - D^0 \mid D^1 = D^0]\mathbb{P}(D^1 = D^0) \\
&= \mathbb{P}(D^1 > D^0)
\end{aligned}
$$

Math time: Proof of the LATE formula

- Dividing the two parts, we get

$$\frac{\mathbb{E}[Y \mid Z = 1] - \mathbb{E}[Y \mid Z = 0]}{\mathbb{E}[D \mid Z = 1] - \mathbb{E}[D \mid Z = 0]} = \mathbb{E}[Y^1 - Y^0 \mid D^1 > D^0].$$

- The case when $D^1 > D^0$ is only possible when $D^1 = 1$ and $D^1 = 0$, in other words, when $\tilde{C} = c$ (compliers).

LATE Interpretation

- The difference in the average value of Y, when examined across Z, is not a function of the outcomes of always takers and never takers
- Defiers and compliers contribute **all the variation** that generates the IV estimate because only their behavior is responsive to the instrument
- If compliers are present but defiers are not, then the causal estimate is interpretable as the average causal effect for compliers
- If defiers are present but compliers are not, then the causal estimate is interpretable as the average causal effect for defiers
- If both compliers and defiers are present, then the estimate generated by the ratio **does not have a well-defined causal interpretation**

Counting Compliers

- Using our assumptions, we can actually count all the four groups of individuals
- The proof of the LATE theorem, in particular, shows that the probability of being a complier is

$$\begin{aligned}\mathbb{P}(\tilde{C} = c) &= \mathbb{P}(D^1 > D^0) = \mathbb{E}[D \mid Z = 1] - \mathbb{E}[D \mid Z = 0] \\ &= \mathbb{P}(D = 1 \mid Z = 1) - \mathbb{P}(D = 1 \mid Z = 0),\end{aligned}$$

which is simply the first stage

Counting Always Takers

- To find the probabilities of being an always taker or a never taker, we can use the following observations

$$
\begin{aligned}
\mathbb{P}(D = 1 \mid Z = 0) &= \mathbb{P}(D^0 = 1, D^1 = 1 \mid Z = 0) \\
&\quad + \mathbb{P}(D^0 = 1, D^1 = 0 \mid Z = 0) \\
&= \mathbb{P}(\tilde{C} = a) + \mathbb{P}(\tilde{C} = d) \\
&= \mathbb{P}(\tilde{C} = a),
\end{aligned}
$$

  where we use independence and the assumption of no defiers.

Counting Never Takers

- Similarly, we can find that

$$\begin{aligned}\mathbb{P}(D = 0 \mid Z = 1) &= \mathbb{P}(D^0 = 0, D^1 = 0 \mid Z = 1) \\ &\quad + \mathbb{P}(D^0 = 1, D^1 = 0 \mid Z = 1) \\ &= \mathbb{P}(\tilde{C} = n) + \mathbb{P}(\tilde{C} = d) \\ &= \mathbb{P}(\tilde{C} = n).\end{aligned}$$

Table of Compliance

- Depending on the values of the treatment status and the instrument, we can construct the following table, which shows where different individuals are located based on their compliance status

	$D = 0$	$D = 1$
$Z = 0$	never takers + compliers	always takers
$Z = 1$	never takers	always takers + compliers

Counting in Cells

- For example, the joint probability of being a never taker and being in the top left cell is

$$\mathbb{P}(\tilde{C} = n, D = 0, Z = 0) = \mathbb{P}(\tilde{C} = n, Z = 0) = \mathbb{P}(\tilde{C} = n)\mathbb{P}(Z = 0)$$

- Similarly, we can find the probability of being an always taker and being in the bottom right cell.

$$\mathbb{P}(\tilde{C} = a, D = 1, Z = 1) = \mathbb{P}(\tilde{C} = a, Z = 1) = \mathbb{P}(\tilde{C} = a)\mathbb{P}(Z = 1)$$

- The probabilities of being a complier in each of the diagonal cells are

$$\mathbb{P}(\tilde{C} = c, D = 0, Z = 0) = \mathbb{P}(\tilde{C} = c)\mathbb{P}(Z = 0)$$
$$\mathbb{P}(\tilde{C} = c, D = 1, Z = 1) = \mathbb{P}(\tilde{C} = c)\mathbb{P}(Z = 1)$$

Math time: LATE redefined

- Consider the observed outcomes in the diagonal cells
- For example, in the top left cell, the average outcome would be the weighted average of outcomes for never takers and compliers

$$\begin{aligned}\mathbb{E}[Y \mid D=0, Z=0] = {} & \mathbb{E}[Y \mid D=0, Z=0, \tilde{C}=n]\mathbb{P}(\tilde{C}=n \mid D=0, Z=0) \\ & + \mathbb{E}[Y \mid D=0, Z=0, \tilde{C}=c]\mathbb{P}(\tilde{C}=c \mid D=0, Z=0).\end{aligned}$$

- Notice that the conditional expectations on the RHS can be rewritten as

$$\begin{aligned}\mathbb{E}[Y \mid D=0, Z=0, \tilde{C}=n] &= \mathbb{E}[Y^0 + (Y^1 - Y^0)D \mid D=0, Z=0, \tilde{C}=n] \\ &= \mathbb{E}[Y^0 \mid \tilde{C}=n], \\ \mathbb{E}[Y \mid D=0, Z=0, \tilde{C}=c] &= \mathbb{E}[Y^0 + (Y^1 - Y^0)D \mid D=0, Z=0, \tilde{C}=c] \\ &= \mathbb{E}[Y^0 \mid \tilde{C}=c]\end{aligned}$$

Math time: LATE redefined

- The conditional probabilities are given by the Bayes' rule:

$$
\begin{aligned}
\mathbb{P}(\tilde{C} = n \mid D = 0, Z = 0) &= \frac{\mathbb{P}(\tilde{C} = n, D = 0, Z = 0)}{\mathbb{P}(D = 0, Z = 0)} \\
&= \frac{\mathbb{P}(\tilde{C} = n)\mathbb{P}(Z = 0)}{\mathbb{P}(D = 0 \mid Z = 0)\mathbb{P}(Z = 0)} \\
&= \frac{\mathbb{P}(\tilde{C} = n)}{\mathbb{P}(D = 0 \mid Z = 0)}, \\
\mathbb{P}(\tilde{C} = c \mid D = 0, Z = 0) &= \frac{\mathbb{P}(\tilde{C} = c)}{\mathbb{P}(D = 0 \mid Z = 0)}.
\end{aligned}
$$

Math time: LATE redefined

- Hence, the control-state outcome for compliers is

$$\mathbb{E}[Y^0 \mid \tilde{C} = c] = \frac{\mathbb{E}[Y \mid D = 0, Z = 0]\mathbb{P}(D = 0 \mid Z = 0) - \mathbb{E}[Y^0 \mid \tilde{C} = n]\mathbb{P}(\tilde{C} = n)}{\mathbb{P}(\tilde{C} = c)}.$$

- Following similar steps, we can find the treatment-state outcome for compliers

$$\mathbb{E}[Y^1 \mid \tilde{C} = c] = \frac{\mathbb{E}[Y \mid D = 1, Z = 1]\mathbb{P}(D = 1 \mid Z = 1) - \mathbb{E}[Y^1 \mid \tilde{C} = a]\mathbb{P}(\tilde{C} = a)}{\mathbb{P}(\tilde{C} = c)}.$$

- The difference between the two will give the average treatment effect for compliers.

Math time: Private schools and vouchers, part 2

- Recall our example. We will assume that there are no defiers. First, let's find the proportions of the remaining three groups.

$$\mathbb{P}(\tilde{C} = a) = \mathbb{P}(D = 1 \mid Z = 0) = 1/9(0.111)$$
$$\mathbb{P}(\tilde{C} = n) = \mathbb{P}(D = 0 \mid Z = 1) = 4/5(0.8).$$

- The proportion of compliers can be found as either one minus the previous two proportions or using the first stage:

$$\mathbb{P}(\tilde{C} = c) = \mathbb{P}(D = 1 \mid Z = 1) - \mathbb{P}(D = 1 \mid Z = 0) = 1/5 - 1/9 = 4/45(0.0889).$$

Math time: Private schools and vouchers, part 2

- Now let's find the proportions of individuals in each of the four cells defined by Z and D.

$$\mathbb{P}(\tilde{C} = c, D = 0, Z = 0) = \mathbb{P}(\tilde{C} = c)\mathbb{P}(Z = 0) = 4/45 \times 9/10 = 0.08$$
$$\mathbb{P}(\tilde{C} = c, D = 1, Z = 1) = \mathbb{P}(\tilde{C} = c)\mathbb{P}(Z = 1) = 4/45 \times 0.1 \approx 0.0089$$
$$\mathbb{P}(\tilde{C} = n, D = 0, Z = 0) = \mathbb{P}(\tilde{C} = n)\mathbb{P}(Z = 0) = 4/5 \times 0.9 = 0.72$$
$$\mathbb{P}(\tilde{C} = a, D = 1, Z = 1) = \mathbb{P}(\tilde{C} = a)\mathbb{P}(Z = 1) = 1/9 \times 0.1 \approx 0.0111$$

	$D = 0$	$D = 1$
$Z = 0$	nt (0.72) + compliers (0.08)	at (0.1)
$Z = 1$	nt (0.08)	at (0.0111) + compliers (0.0089)

Math time: Private schools and vouchers, part 2

- Finally, the outcomes for compliers are

$$\mathbb{E}[Y^1 \mid \tilde{C} = c] = \frac{\mathbb{E}[Y \mid D = 1, Z = 1]\mathbb{P}(D = 1 \mid Z = 1) - \mathbb{E}[Y^1 \mid \tilde{C} = a]\mathbb{P}(\tilde{C} = a)}{\mathbb{P}(\tilde{C} = c)}$$

$$= \frac{58 \times 1/5 - 60 \times 1/9}{4/45} = 55.5$$

$$\mathbb{E}[Y^0 \mid \tilde{C} = c] = \frac{\mathbb{E}[Y \mid D = 0, Z = 0]\mathbb{P}(D = 0 \mid Z = 0) - \mathbb{E}[Y^0 \mid \tilde{C} = n]\mathbb{P}(\tilde{C} = n)}{\mathbb{P}(\tilde{C} = c)}$$

$$= \frac{50 \times 8/9 - 50 \times 4/5}{4/45} = 50$$

- The average treatment effect for compliers is then 5.5
- No information about the effect of private schooling for the always takers or the never takers
- No way to estimate the counterfactuals for them

Math time: Private schools and vouchers, part 2

	$D = 0$	$D = 1$
$Z = 0$	50	60
$Z = 1$	50	58

	$D = 0$	$D = 1$
$Z = 0$	nt (0.72) + compliers (0.08)	at (0.1)
$Z = 1$	nt (0.08)	at (0.0111) + compliers (0.0089)

Discussion of LATE

Different IVs, Different LATEs

- LATE estimators depend on the instrument under consideration
- **Different instruments** define **different average treatment effects** for the same group of treated individuals
- The meanings of the labels for $\tilde{C}$ depend on the instrument, such that some individuals can be never takers for one instrument and compliers for another
- This also means that different IVs will in general produce different estimated causal effects

Is LATE Interesting?

- In our example, the IV estimate **does not provide any information** about the average effect for the always or never takers
- The IV estimate is an estimate of a narrowly defined average effect only among those induced to take the treatment by the voucher policy intervention
- However, this is precisely what should be of interest for policy evaluation purposes
- If the policy question is "What is the effect of vouchers on school performance?" then they presumably care most about the average effect for compliers

Many Instruments

- If more than one instrument is available, the traditional econometric literature suggests that they should all be used to **overidentify** the model
- Overidentified models generate a **mixture-of-LATEs** problem
- Since each instrument defines a LATE for a different group of individuals, the estimated causal effect would be averaged across these different groups
- Even though one would probably be more interested in each group separately

LATE with one-sided non-compliance

LATE, ATT, ATU

- In general, LATE **is not identical** to either ATT or ATU
- However, in a special case of one-sided non-compliance LATE coincides with ATT
- In many **randomized trials**, participation is voluntary among those randomly assigned to receive treatment
- On the other hand, no one in the control group has access to the experimental intervention
- This means that $\mathbb{P}(D = 1 \mid Z = 0) = 0$

Compliance Problem

- Since the group that receives (i.e., complies with) the assigned treatment is a self-selected subset of those offered treatment, a comparison between those actually treated and the control group is misleading
- The selection bias in this case is almost always **positive**: those who take their medicine in a randomized trial tend to be healthier
- IV using the randomly assigned treatment intended as an instrumental variable for treatment received solves this sort of compliance problem
- Moreover, LATE is the effect of treatment on the treated.

LATE and ATT

- Formally, let the assumption of the LATE theorem hold and let $\mathbb{P}(D = 1 \mid Z = 0) = 0$. Then

$$\frac{\mathbb{E}[Y \mid Z = 1] - \mathbb{E}[Y \mid Z = 0]}{\mathbb{E}[D \mid Z = 1]} = \mathbb{E}[Y^1 - Y^0 \mid D = 1]$$

- In other words, the average treatment effect on the treated is equal to the **intention-to-treat** (ITT) divided by compliance

Math time: Proof of the LATE and ATT formula

- First, consider

$$\mathbb{E}[Y \mid Z = 1] = \mathbb{E}[Y^0 + (Y^1 - Y^0)D \mid Z = 1].$$

- Since $\mathbb{P}(D = 1 \mid Z = 0) = 0$ (or equivalently, $\mathbb{P}(D = 0 \mid Z = 0) = 1$), we have that

$$\mathbb{E}[Y \mid Z = 0] = \mathbb{E}[Y^0]$$

- Taking the difference between the two, we obtain

$$\begin{aligned}
\mathbb{E}[Y \mid Z = 1] - \mathbb{E}[Y \mid Z = 0] &= \mathbb{E}[(Y^1 - Y^0)D \mid Z = 1] \\
&= \mathbb{E}[(Y^1 - Y^0)D \mid D = 1, Z = 1]\mathbb{P}(D = 1 \mid Z = 1) \\
&+ \mathbb{E}[(Y^1 - Y^0)D \mid D = 0, Z = 1]\mathbb{P}(D = 0 \mid Z = 1) \\
&= \mathbb{E}[Y^1 - Y^0 \mid D = 1]\mathbb{P}(D = 1 \mid Z = 1)
\end{aligned}$$

Math time: Proof of the LATE and ATT formula

- We know that

$$\mathbb{E}[D \mid Z = 1] = \mathbb{P}(D = 1 \mid Z = 1).$$

- The result obtains after dividing both parts.
- In other words, the average treatment effect on the treated is equal to the intention-to-treat (ITT) divided by compliance

Next Time on *Impact Evaluation Methods*...

Regression discontinuity