Impact Evaluation Methods

Topic 3: Causal Diagrams

Alex Alekseev

May 15, 2025

University of Regensburg, Department of Economics

Previously on *Impact Evaluation Methods*...

- Potential outcomes framework (aka counterfactual model)
- Treatment effects: ATE, ATT, ATU
- Naive treatment effect and bias
- Randomization
- Regression interpretation
- SUTVA

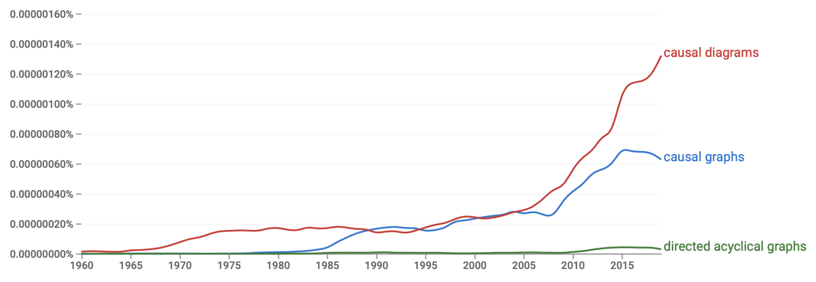
Confounders and Colliders

Introduction

Background

- A graphical approach to causal modeling is not new
- Sewall Wright (one of the fathers of modern genetics) invented the path analysis
- Adapted to economic modeling by Wright's **father** Philip Wright
- Largely ignored by economists
- 2000 book **Causality: Models, Reasoning, and Inference** by computer scientist and Turing Award winner Judea Pearl
- Immensely helpful for designing a credible identification strategy

Causal diagrams grow in popularity



Causal Diagrams vs. Potential Outcomes Framework

- Pearl's work provides a language and a framework for thinking about causality
- Pearl proves that the fundamental concepts underlying the potential outcome framework and his perspective are equivalent
- We suppress potential outcome random variables and use only observed outcome variables
- Graphs nonetheless provide a direct and powerful way of thinking about causality and the identification strategies

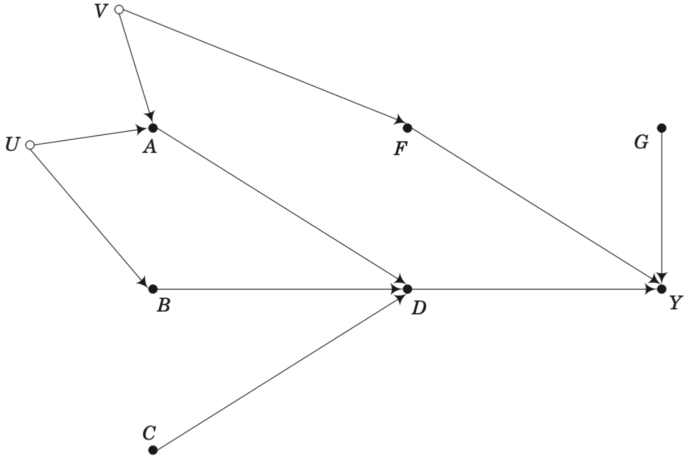
Three Strategies

- Pearl shows that there are three basic strategies for identifying a causal effect
- We will briefly preview all three and talk more specifically about the first one
- The other two will be covered in later lectures
- The strategies are
  - Conditioning on variables that block all back-door paths
  - Conditioning on variables that allow for estimation by a mechanism
  - Using an instrumental variable that is an exogenous shock to to the cause

Basics of DAGs

Do you like dags?

An Example of a DAG



One Way Only

- A DAG does not permit a representation of simultaneous causation
- Only directed edges are permissible, and direct causation can run in only one direction, as in $X \rightarrow Y$
- The arrows do not tell anything about the size or the shape of the effect
- $X \rightarrow Y$ simply means that X causes Y, without specifying whether the effect is, e.g., linear or quadratic.

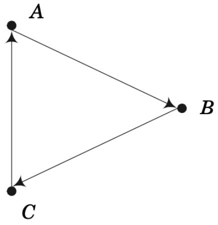
A Path

Definition

A path is any any sequence of edges pointing in any direction that connects one variable to another. Notice that a path does not have to follow the direction of arrows. A **path**, in other words, means an **undirected** path.

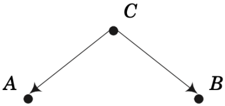
No Cycles

- A causal diagram is an acyclic graph
- No directed paths emanating from a causal variable also terminate at the same causal variable
- You cannot, e.g., have graphs like this one:

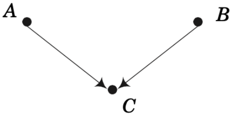


Three Basic Patterns

(a) Mediation



(b) Mutual dependence

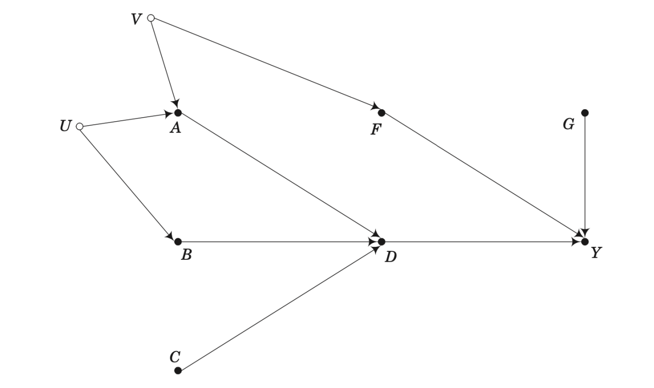


(c) Mutual causation

Colliders

- Does not generate an unconditional association between the variables that cause the collider variable
- If nothing is known about the value that C takes on, then knowing the value that A takes on yields no information about the value that B takes on
- The collider variable C “blocks” the possible causal effects of A and B on each other
- The incautious handling of colliders can create conditional dependence that can sabotage a causal analysis

Back to our DAG



Given the structure of causal relationships represented in the graph, which variables must we observe and then use in our analysis to estimate the causal effect of D on Y?

Probability Distributions

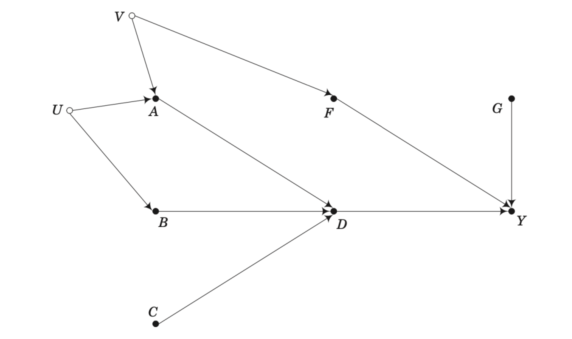
- The causal variable D has a probability distribution
- The causal effects emanating from the variables A, B, and C are explicitly represented in the graph
- The relative sizes of these effects are not represented
- Other causes of D that are unrelated to A, B, and C are left implicit
- The outcome variable, Y, is caused by F , G, and D, but there are other implicit causes that are unrelated to F, G, and D that give Y its probability distribution

The Three Strategies

The Three Strategies

- One can **condition** on variables (with procedures such as stratification, matching, weighting, or regression) that block all back-door paths from the causal variable to the outcome variable.
- One can use exogenous variation in an appropriate **instrumental variable** to isolate covariation in the causal and outcome variables.
- One can establish an isolated and exhaustive **mechanism** that relates the causal variable to the outcome variable and then calculate the causal effect as it propagates through the mechanism.

Conditioning



Two back-door paths:

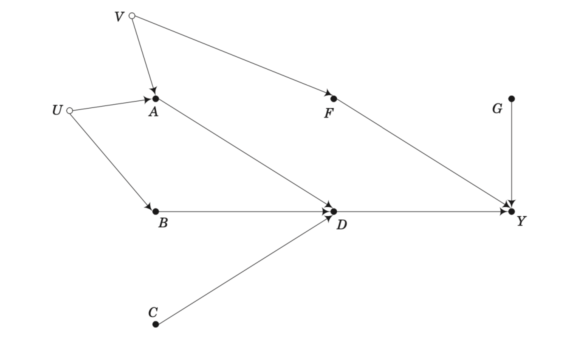
- $D \leftarrow A \leftarrow V \rightarrow F \rightarrow Y$
- $D \leftarrow B \leftarrow U \rightarrow A \leftarrow V \rightarrow F \rightarrow Y$

Back-Door Paths

Definition

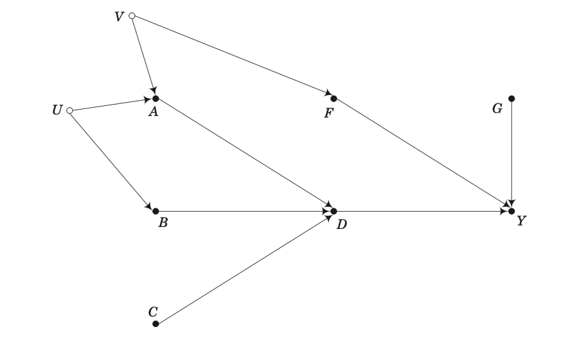
A back-door path is a path between any causally ordered sequence of two variables that includes a directed edge $\rightarrow$ that points to the first variable.

Blocking Paths



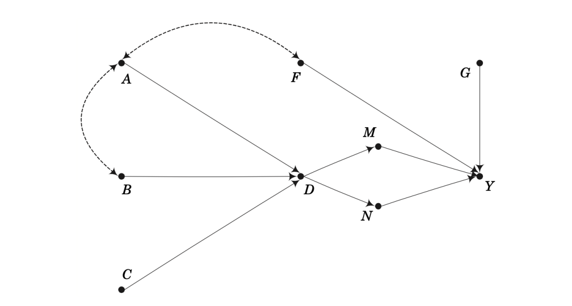
- Both of the back-door paths can be blocked by conditioning on A and B **or** by conditioning on F
- One cannot identify the causal effect of D on Y by conditioning only on A

Instrumental Variables



- Use a localized exogenous shock to both the causal variable and the outcome variable
- C is a **valid** instrument for D
- It causes D but does not have an effect on Y except though its effect on D
- For this estimation strategy, A, B, F, G do not need to be observed.

Front-Door Method



- The mediating variables M and N completely account for the causal effect of D on Y
- M and N are not determined by anything other than D
- The causal effect of D on Y can be calculated by estimation of the causal effect of D on M and N and then the causal effects of M and N on Y

Discussion

Ideal World

- In an ideal scenario, all three strategies could be used and all three would generate equivalent estimates
- If a causal effect estimate generated by conditioning is similar to a causal effect estimate generated by a valid instrumental variable estimator, then each estimate is bolstered
- If a front door method then generates a third equivalent estimate, all three causal effect estimates would be even more convincing
- An elaborated explanation of how the causal effect comes about is also available
- But... It is rare that one can specify causes as cleanly as in the causal diagrams in these figures

Still Useful

- Often more than one way to estimate a causal effect
- "Control for all other causes of the outcome variable" can be poor guide for practice
- The strategies are not well suited to **discovering** the causes of outcomes and then comprehensively estimating the relative effects of all alternative causes
- The methods are not irrelevant to a broader question, but they are designed to answer simpler subordinate questions

Having a Theory is Important

- Suppose we estimated the effect of D on Y by observing only A, B, D, Y and then conditioning on A and B
- We found that D had a small effect on Y
- We would then want to observe both F and G
- If we did not have a theory that suggested that F and G have causal effects on Y, determining that D has a small effect on Y would not help us

Where Do DAGs Come From?

- A causal diagram is a theoretical representation of the state-of-the-art knowledge about the phenomena you are studying
- A causal diagram should describe all causal relationships relevant to the causal effect of interest
- Drawing a diagram requires you to make the explicit commitment to a causal paths that might exist and the complete commitment to the lack of causal paths
- Drawing a causal diagram requires you to make choices about which arrows to include and which arrows to exclude
- Each choice is an explicit assumption that you are making and which you should be able to defend.

Poorly Specified Theories

- A well-specified theory is needed to justify assumptions about underlying causal relationships
- Suppose a theory is poorly specified, or divergent theories exist that support alternative assumptions
- Alternative causal effect estimates may be considered valid conditional on the validity of alternative maintained assumptions

Conditioning on Observables

Basic Strategy

- One of the most basic strategies is to analyze a relationship within groups defined by one or more variables
- This approach is called subgroup analysis, subclassification, stratification, or tabular decomposition
- The motivation is to analyze the data after conditioning on membership in groups identified by values of a variable that is thought to be related to both the causal variable and the outcome variable.

How It Works

- Suppose we are interested in the causal effect of A on B but are worried that C causes both
- If analysis is carried out for a group in which all individuals have a particular value for the variable C, then the variable C is constant within the group and cannot therefore be associated with A or B
- In practice, we will often use other conditioning methods, such as regression.

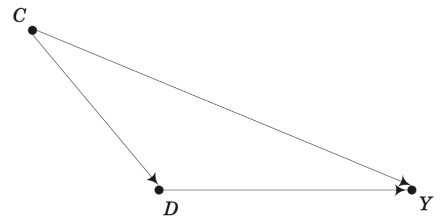
Colliders

- Conditioning on a collider adds complications by creating new associations
- Consider a simple model in which $Y = A + B$
- Suppose that A and B are independent in the population.
- Now if we fix Y at a level $\bar{Y}$, we get $\bar{Y} = A + B$
- What does it do to A and B? It creates a deterministic and negative relationship between them! $A = \bar{Y} - B$

The Back-Door Criterion

A Typical Concern

- The causal variable D and the outcome variable Y are mutually dependent on a common third variable C



Confounders

- The total association between D and Y represents
  - the genuine causal effect of D on Y
  - the common dependence of D and Y on C
- The causal effect of D on Y is confounded by C, or C is a **confounder**
- The causal effect of D on Y can be identified by conditioning on C.

Back-Door Paths

- Causal diagrams characterize the confounding using the language of **back-door paths**
- In our graph, there are two paths that connect D and Y: $D \to Y$ and $D \leftarrow C \to Y$
- The path $D \leftarrow C \to Y$ is a back-door path because it includes a directed edge pointing to D
- The path $D \to Y$ is not a back-door path because it does not include a directed edge pointing to D.

The Problem with Back-Door Paths

- They may contribute to the association between D and Y
- The observed association between D and Y may not consistently estimate the causal effect of D on Y
- The observed association between D and Y does not identify the causal effect because the total association is a composite of the true causal effect $D \to Y$ and the back-door path $D \leftarrow C \to Y$

Back-Door Criterion

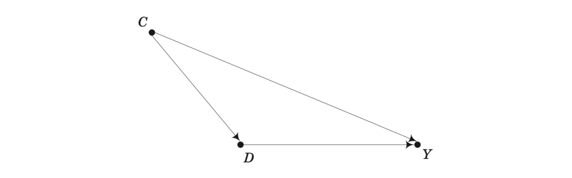
- The causal effect is identified by conditioning on a set of variables Z if and only if all back-door paths between the causal variable and the outcome variable are blocked after conditioning on Z
- All back-door paths are blocked by Z if and only if each back-door path
  1. contains a chain of mediation $A \rightarrow C \rightarrow B$, where the middle variable C is in Z, or
  2. contains a fork of mutual dependence $A \leftarrow C \rightarrow B$, where the middle variable C is in Z, or
  3. contains an inverted fork of mutual causation $A \rightarrow C \leftarrow B$, where the middle variable C and all of C's descendants are not in Z.

Important Note

...or...

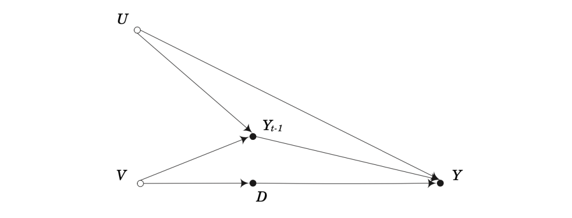
Because the "or" in the back-door criterion is inclusive, one can condition on colliders and still satisfy the back-door criterion if the back-door paths along which the colliders lie are otherwise blocked because Z satisfies condition 1 or condition 2 with respect to another variable on the same back-door path.

Back to Our Example



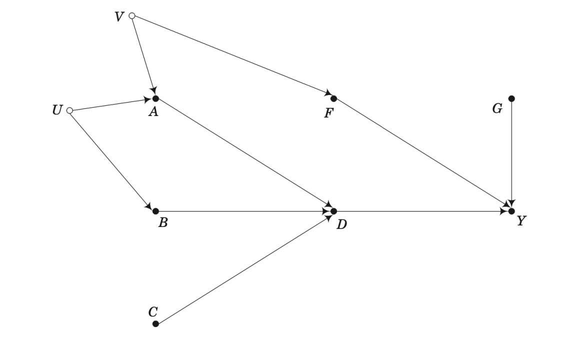
- There is a single back-door path: a fork of mutual dependence where C causes both D and Y
- Conditioning on C blocks $D \leftarrow C \rightarrow Y$ because C is the middle variable in a fork of mutual dependence
- Conditioning on C satisfies the back-door criterion, and identifies the causal effect of D on Y.

Another Example



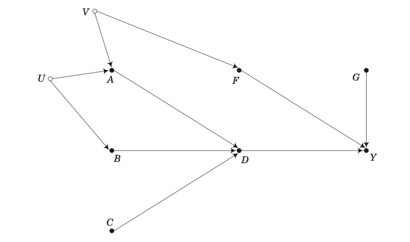
- There are two back-door paths: $D \leftarrow V \rightarrow Y_{t-1} \rightarrow Y$ and $D \leftarrow V \rightarrow Y_{t-1} \leftarrow U \rightarrow Y$.
- Y_{t-1} does not satisfy the back-door criterion
- It blocks the first back-door path $D \leftarrow V \rightarrow Y_{t-1} \rightarrow Y$
- For the second path $D \leftarrow V \rightarrow Y_{t-1} \leftarrow U \rightarrow Y$, Y_{t-1} is a collider
- After conditioning on Y_{t-1}, at least one back-door path will remain unblocked

Our Old Example



- $D \leftarrow A \leftarrow V \rightarrow F \rightarrow Y$
- $D \leftarrow B \leftarrow U \rightarrow A \leftarrow V \rightarrow F \rightarrow Y$

Conditioning Strategies



- F satisfies the back-door criterion and conditioning on it identifies the causal effect
- A alone does not satisfy the back-door criterion. Conditioning on A would unblock the second back-door path.
- A and B together satisfy the back-door criterion, and conditioning on them together identifies the causal effect of D on Y.

Takeaway

- Conditioning on variables that lie along back-door paths can be an effective strategy to identify a causal effect
- If all back-door paths between the causal variable and the outcome variable are blocked ...
- ...then back-door paths do not contribute to the association between the causal variable and the outcome variable
- The remaining association between the causal variable and outcome variable identifies the causal effect
- Conditioning on a collider variable has the opposite effect

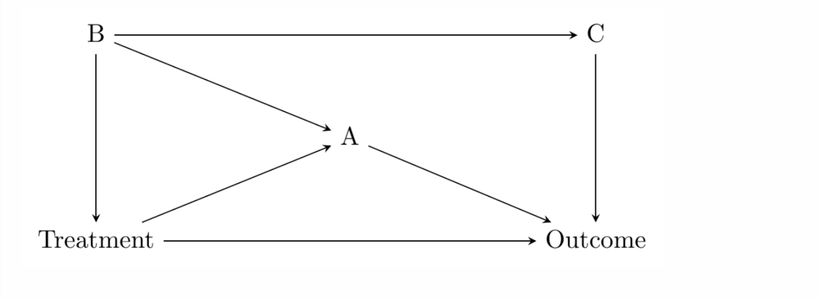
Generalization

- The back-door criterion is a generalization of various previous solutions for how to solve the omitted-variable bias problem
- The back-door criterion shows that researchers do not need to condition on all omitted direct causes of an outcome variable
- Researchers need to condition on only a minimally sufficient set of variables that renders all back-door paths blocked
- Just write down each back-door path and then determine whether or not each endogenous variable is a collider along any of these back-door paths

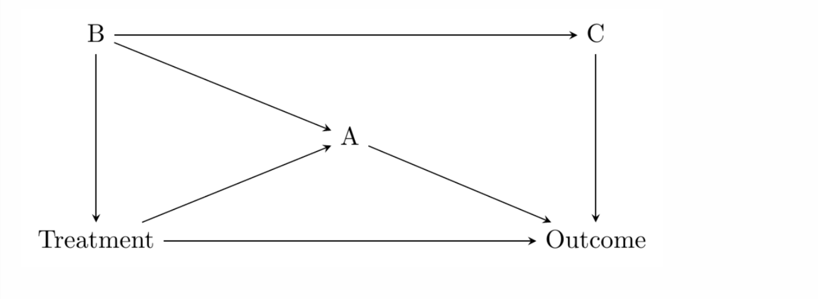
How To Find Back-Door Paths

1. Start at the treatment variable
2. Follow one of the arrows coming in or out of the treatment variable to find another variable
3. Then, follow one of the arrows coming in or out of *that* variable
4. Keep repeating step 3 until you either come to a variable you've already visited (a loop) or find the outcome variable (a path, write it down)
5. Every time you either find a path or a loop, back up one and try a different arrow in/out until you have tried them all. Then, back up again and try all *those* arrows
6. Once you've tried all the ways out of the treatment variable and all the eventual paths, you've got all the paths!

Example



Example



- $Treatment \rightarrow Outcome$
- $Treatment \rightarrow A \rightarrow Outcome$
- $Treatment \rightarrow A \leftarrow B \rightarrow C \rightarrow Outcome$
- $Treatment \leftarrow B \rightarrow A \rightarrow Outcome$
- $Treatment \leftarrow B \rightarrow C \rightarrow Outcome$

Homework

Develop all possible conditioning strategies to identify the effect of Treatment on Outcome

Next Time on Impact Evaluation Methods...

Regression