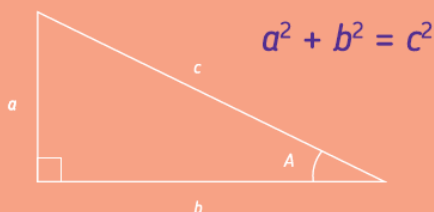# GCSE Maths: Formulae you'll need to know

As provided by AQA, these are the formulae required for the new GCSE

*These will **not** be given in the exam, so you will need to recall as well as use these formulae.*

Pythagoras' theorem

In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$

Trigonometry formulae

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:
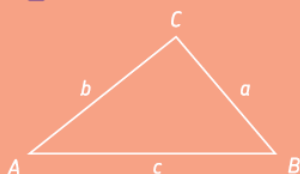
$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$

In any triangle ABC where a, b and c are the length of the sides:

$$\text{sine rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{cosine rule: } a^2 = b^2 + c^2 - 2bc \cos A$$
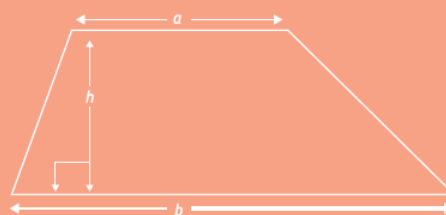
$$\text{Area} = \frac{1}{2} ab \sin C$$

Circumference and area of a circle

Where r is the radius and d is the diameter:

$$\text{Circumference of a circle} = 2\pi r = \pi d$$

$$\text{Area of a circle} = \pi r^2$$

Perimeter, area, surface area and volume formulae

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

$$\text{Area of a trapezium} = \frac{1}{2}(a + b)\,h$$

Volume of a prism = area of cross section x length

Compound interest

Where P is the principal amount, r is the interest rate over a given period and n is the number of times that the interest is compounded:

$$\text{Total accrued} = P\left(1 + \frac{r}{100}\right)^n$$

The quadratic formula

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Probability

Where $P(A)$ is the probability of outcome A and $P(B)$ is the probability of outcome B:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A \text{ given } B)\, P(B)$$

Because learning a rule or formula is easier when it is fully understood, the pages that follow give extra details for all of the formulae, including where they come from, when and how to use them, and some bonus info designed to provide a little background or prepare you for future studies.

Remember that a memorised formula is only useful to you if you recognise the type of question that will require it, and understand it well enough to apply it.

Pythagoras' theorem

In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$
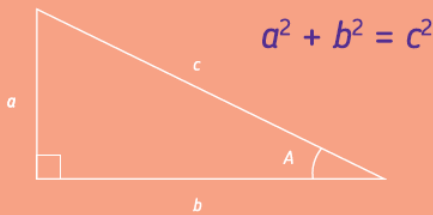
Bonus info:

Pythagoras' theorem works both ways, so if the three side lengths satisfy the formula, the triangle must be right-angled.

Some right-angled triangles have all sides whole number lengths. Sets of numbers like this are called Pythagorean Triples. The first two are: 3, 4, 5 and 5, 12, 13. Create your own by picking a starting number $n > 1$ and using: $a = n^2 - 1$ $b = 2n$ $c = n^2 + 1$. Can you see why only even values of n give primitive triples (no common factors)?

When to use it:

When you know the length of two sides of a right-angled triangle and need the length of the third side.

How to use it:

Label the sides a, b and c, taking care to label the longest side c, then substitute in.

What to watch out for:

You can't square root two added terms separately, so $\sqrt{3^2 + 4^2}$ is not $3 + 4$.

Where it comes from:

Rearrange these identical right-angled triangles to show that the leftover areas are the same for any right-angled triangle:

Trigonometry formulae

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$

Alternative forms:

You may find it easier to recall them as:

$$\sin \theta = \frac{opp}{hyp} \quad \cos \theta = \frac{adj}{hyp} \quad \tan \theta = \frac{opp}{adj}$$
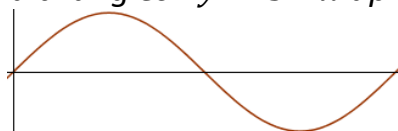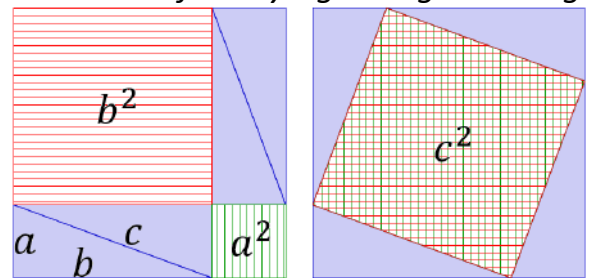
Often remembered using: "SohCahToa"

Bonus info:

Trigonometry was initially created to help map-makers. By sighting points with a known elevation (height) and measuring the angle, if you know the distance away you can calculate the height of your own hill. The trig 'ratios' give a ratio of two sides:

$$\theta \quad tan(\theta) = \frac{2}{5}$$
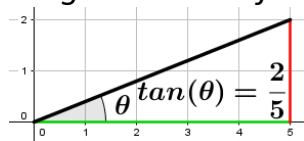
Here, $\tan \theta$ gives the height compared to the width, which is the same as gradient. Trig functions work for any angle, and go far beyond triangles. $y = \sin x$ up to $360°$:

When to use it:

In right-angled triangles, when you know a side and an angle and need another side, or you know two sides and need an angle.

How to use it:

Label the sides of your right-angled triangle (hypotenuse: longest, opposite: opposite the angle, adjacent: next to the angle), choose the right rule, substitute your values and rearrange. To find an angle you'll need the inverses $\sin^{-1}$, etc.

Where it comes from:

All right-angled triangles with the same angle are similar (same shape if not size), the ratio of side lengths is the same. The ratio changes when the angle changes, so we need to 'look it up' on a calculator for each different angle.

When your calculator works out $\sin x$, it uses $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \dfrac{x^9}{9!} \ldots$ *

(the further you go, the better the answer) The red graph shows this approximation:
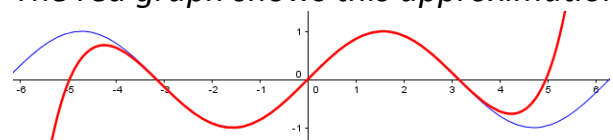
*also, the angle is measured in radians, not degrees!

In any triangle ABC where a, b and c are the length of the sides:

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
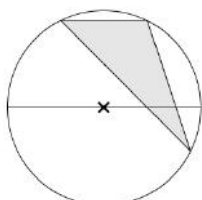
Alternative forms:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Easier for finding unknown angles, but generally quoted the other way because…

Bonus info:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = d$$

Each fraction is also equal to the diameter of the triangle's circumcircle:

When to use it:

For any triangle (not just right-angled) where you know the size of a side and the opposite angle, and one other thing.

How to use it:

Label the triangle so that side length a is opposite angle A, etc, then substitute in the three parts you know or need.

Where it comes from:

Using the area formula $\frac{1}{2}ab\sin C$ from different corners ($\frac{1}{2}ac\sin B$, etc) and setting them equal to one another. Eg:

$$\frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A \Rightarrow a\sin C = c\sin A$$

cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Alternative forms:

$\cos A = \dfrac{b^2+c^2-a^2}{2bc}$ *is a more convenient rearrangement for finding a missing angle from three known side lengths.*

Where it comes from:

Dropping a line perpendicularly to the base of a triangle to form right angled triangles, using right-angled trig to find the height, and substituting into Pythagoras' theorem.

Bonus info:

The cosine rule can be thought of as a stronger version of Pythagoras' theorem for finding a third side which works for any angle. Substituting in $A = 90°$ simplifies to $a^2 = b^2 + c^2$ (which, since in this case a is the hypotenuse, is just Pythagoras).

When to use it:

For any triangle (not just right-angled) where you know two sides and the angle in between (to find the third side) or you know three sides (to find an angle).

How to use it:

Label the triangle as for Sine Rule, but ensure that the missing side is labelled a (or the missing angle is labelled A) then substitute into the formula and rearrange.

What to watch out for:

*After substituting you may end up with something like $12 = 16 - 10\cos A$. This does **not** simplify to $12 = 6\cos A$. It should become $-4 = -10\cos A$, then $0.4 = \cos A$. Treat number parts and $\cos A$ parts separately, just as with x in $12 = 16 - 10x$, which becomes $x = 0.4$.*

Area = $\dfrac{1}{2} ab \sin C$
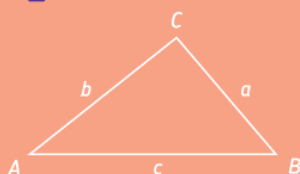
Alternative forms:

$$Area = \frac{1}{2}bc\sin A \qquad Area = \frac{1}{2}ac\sin B$$

Where it comes from:

Dropping a line perpendicularly to the base of a triangle to form right-angled triangles, using right-angled trigonometry to find the height, then applying $Area = \frac{1}{2}bh$.

When to use it:

For any triangle (not just right-angled) where you know two sides and the angle in between, to calculate area without needing to know the height of the triangle.
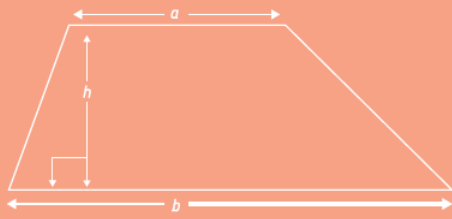
How to use it:

Label the triangle as for Sine Rule and Cosine Rule, but ensure the angle you are using is labelled C, and the sides a and b. Substitute directly into the formula.

Bonus info:

This formula allows us to find a rule for the area of a regular n-sided polygon with 'radius' r: $A = r^2 \times \frac{n}{2}\sin\left(\frac{360}{n}\right)$

Perimeter, area, surface area and volume formulae

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

Area of a trapezium $= \dfrac{1}{2}(a + b)\,h$

Alternative forms:
A more memorable form of this formula is:
$$Average\ Width \times Height$$

Bonus info:
The general form of the formula given above actually applies to a square, rectangle, parallelogram and rhombus (it's just that the width is constant for those).

For a kite with given diagonals: $A = \dfrac{1}{2}d_1 d_2$

When to use it:
When you know the length of the two parallel sides of a trapezium and the vertical height.
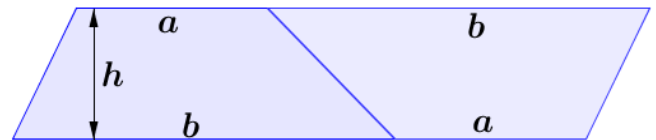
How to use it:
Find the average of the base width and top width (add together and divide by 2, or, for simpler numbers, go halfway between), then multiply by the vertical height.

What to watch out for:
Take care to use the vertical height, not the slanting length of either of the two sides – unless one side is actually vertical, these will both be longer than the height.

Where it comes from:
Putting two trapezia next to each other always creates a parallelogram with area $(a + b)h$, so just halve this.

Volume of a prism = area of cross section x length

Alternative forms:
Since a cuboid is a rectangular prism, the formula becomes $(h \times w) \times l$. Since a cylinder is a circular prism, we get $\pi r^2 h$.

Where it comes from:
Volume is the 3D space inside a shape. Area is 2D space, so the cross-sectional area is the number of squares that would fit on the front. This is also the number of cubes in a single layer. We multiply by the length because this gives the number of layers.

Bonus info:
A pyramid takes up exactly $\dfrac{1}{3}$ the space of the prism that contains it (ie, same base and height). So a cone has volume $\dfrac{1}{3}\pi r^2 h$.

When to use it:
When you have a prism (constant cross-section all the way through) where you know (or can calculate) the area of the front/top (cross-section) and you know the length/height.

How to use it:
Work out the area of the cross-section and multiply by the length, or working backwards, you can divide the volume by the length to find the cross-sectional area.

What to watch out for:
Take care to accurately find the area of the cross-section. Eg, for a triangular prism use $\dfrac{1}{2}(base \times height)$. Recall that the area will be measured in cm^2 or m^2 while the volume will be in cm^3 or m^3.

Circumference and area of a circle

Where r is the radius and d is the diameter:

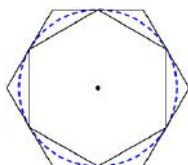### Circumference of a circle = $2\pi r = \pi d$

Alternative forms:
$$C = \pi d \qquad C = 2\pi r$$
This first one is the easiest to remember – just multiply the diamter by π.

Where it comes from:
Sandwiching a circle between two polygons lets us approximate π. The circumference must be more than the perimeter of the inner polygon but less than the outer polygon.

Bonus info:
Long before computers found π to 13 trillion decimal places, Archimedes used this trick (with 96-sides) to show that $\frac{223}{71} < \pi < \frac{22}{7}$. Averaging gives $\pi \approx 3.142$. Not bad.
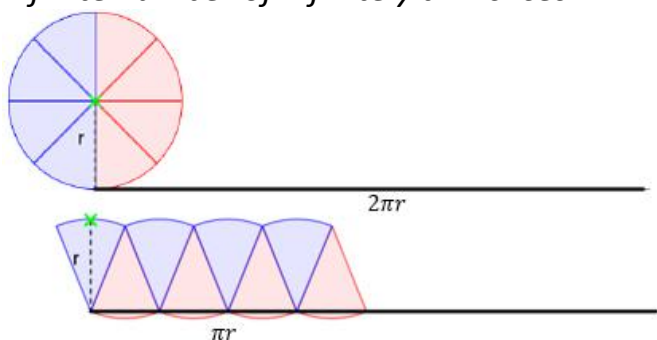
When to use it:
To find the distance around a circle when you know the radius or the diameter, or vice versa.

How to use it:
To find the circumference, multiply the diameter by π. If you have the radius, double it to get the diameter, then $\times \pi$.

You can extend this rule to work for arc length (a piece of the circumference) by multiplying by the fraction $\frac{\theta}{360}$ for angle θ: $\quad l = \frac{\theta}{360}(2\pi r)$

What to watch out for:
Circumference is the perimeter of a circle. Because it's a distance, the units are the same length units as the radius or diameter (eg cm or m). Most common errors involve mixing this up with area.

Area of a circle = πr^2

Alternative forms:
$$A = \pi r^2$$

Where it comes from:
The precise proof involves calculus, which involves adding together the area of an infinite number of infinitely thin slices:
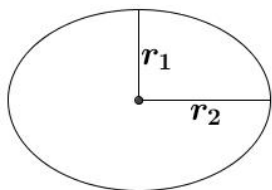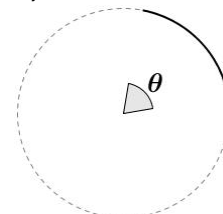
The thinner the slices, the closer this second rearrangement gets to a rectangle of height r and width πr (half the circumference).

Bonus info:
An ellipse has two different radii (major and minor), but the area formula looks similar:

$$A = \pi r_1 r_2$$

When to use it:
To find the area of a circle when you know the radius, or by reversing the process, to calculate the radius from a given area.

How to use it:
First square the radius, then multiply by π. If you only have the diameter, just halve it to get the radius. For more complicated questions (eg find the radius from a given area), substitute values into the formula and rearrange.

You can extend this rule to work for sector area (a piece of the area) by multiplying by the fraction $\frac{\theta}{360}$ for angle θ:
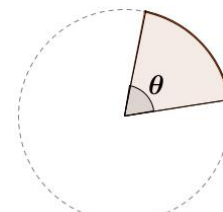
$$A = \frac{\theta}{360}(\pi r^2)$$

What to watch out for:
Remember to use πr^2, not $(\pi r)^2$. That is, square the radius, then multiply by π. Your scientific calculator will follow the order of operations rules automatically, so just type in $\pi \times r^2$. If in doubt, estimate (eg a square with the same width as the diameter should be a bit bigger.)

Compound interest

Where P is the principal amount, r is the interest rate over a given period and n is the number of times that the interest is compounded:

$$\text{Total accrued} = P\left(1 + \frac{r}{100}\right)^n$$

When to use it:
When you are saving (or borrowing) an initial amount, and being paid (or paying) interest annually at a known rate.

How to use it:
First find the appropriate percentage multiplier (eg 1.025 for 2.5% interest), then multiply the 'principal' (original amount) by this the right number of times. Use the power button to simplify the process (eg $\times 1.025^{20}$ for 20 years).

Alternative forms:
Since r is given as a percentage, $1 + \frac{r}{100}$ is simply the multiplier for an $r\%$ increase. Call this m and the formula is:
$$Total = Initial \times m^n$$

Where it comes from:
Compound interest means interest each year is calculated based on the total in the account (including interest paid from any previous years). Multiplying by 1.04 does a 4% increase, so multiplying by 1.04^5 does a 4% increase five times (for 5 years).

Bonus info:
Percentage multipliers are not only useful for efficient interest calculations, but also for reversing a percentage change.
To reverse a 4% increase, divide by 1.04.
To reverse 5 years of it, divide by 1.04^5.

What to watch out for:
Read questions carefully so you know whether you are being asked for the final amount or the interest earned (in this case, do exactly the same, but subtract the initial amount right at the end). If you need to reverse a percentage increase or decrease, remember to divide by the multiplier (eg $\div 1.025$ to reverse a 2.5% increase, or $\div 0.8$ to reverse a 20% decrease). Do not confuse the reverse of an increase with a decrease - convince yourself by increasing £100 by 50% then decreasing by 50% (you get £75, not £100).

The quadratic formula

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

When to use it:
When solving a quadratic equation written in the form $ax^2 + bx + c = 0$. If it is possible to factorise, this is usually quicker, but the formula, just like completing the square, will always work (provided there are solutions to be found). If a question asks for answers to $3\,s.f.$ that's usually a good indication you'll need the formula.

Alternative forms:
$$x = \frac{-b \pm \sqrt{\Delta}}{2a} \quad where \quad \Delta = b^2 - 4ac$$

Bonus info:
$b^2 - 4ac$ *is called the discriminant. If $-ve$, you can't square-root it, so there are no solutions (ie the curve never crosses the x-axis), if $+ve$ there are two distinct (different) solutions, and if 0, exactly one.*

Where it comes from:
Completing the square with $ax^2 + bx + c$ gives $a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right]$ which, if $= 0$, rearranges to give the quadratic formula.

How to use it:
If necessary, rearrange the quadratic to $ax^2 + bx + c = 0$, and identify the values of a, b and c. Substitute into the formula and calculate (usually with a calculator).

What to watch out for:
Take care to write the quadratic in the correct format, and watch out for negative signs. Often $x^2 - 4x + 2 = 5$ is interpreted wrongly as having $b = 4$ (forgetting the $-$ sign), or $c = 2$ (failing to rearrange fully to ensure $= 0$ at the end).

Probability

Where $P(A)$ is the probability of outcome A and $P(B)$ is the probability of outcome B:

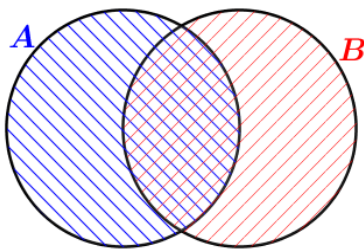$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Alternative forms:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

This is the same, but using formal notation.

Where it comes from:

When events are not mutually exclusive, they may both occur together. To find the chance of being in one or other category, you can add up both, but then you must subtract the overlap which has been counted twice:

Bonus info:

This is a very commonly misunderstood result. A student with a 50% chance of passing Maths and a 70% chance of passing English doesn't have a 120% chance of passing either, because they may pass both.
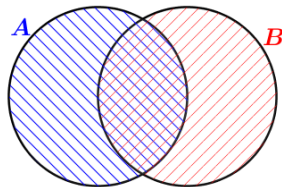
When to use it:

When finding the probability of either one or both events happening when you know the chance of each and the chance of both, or to find the probability of both when you know each and the chance of either.

How to use it:

You can directly apply the formula, or sketch a Venn diagram to ensure you take into account the key concept: unless events are mutually exclusive, they may both occur, so don't double-count.

What to watch out for:

In maths, the word 'or' is generally inclusive (ie "Win with a 3 on the dice roll or a tail on the coin toss" implies the unstated "...or both"). Outside maths 'or' is often an 'exclusive or' ("Tea or coffee?" is offering you one or the other but not both). If in doubt, sketch the Venn diagram to get it clear in your head which bits you want, and check that any probabilities you find are sensible (ie extra conditions like 'and' make success less likely, extra ways to win like 'or' make success more likely, and $0 \le p \le 1$ always.

$$P(A \text{ and } B) = P(A \text{ given } B) \, P(B)$$

Alternative forms:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Where it comes from:

If you know that B happens, just look within region B. The chance of A given B

is the size of the overlap relative to B.

Bonus info:

A medical test with 0% false negative rate (the sick are told they're sick) and 1% false positive rate (healthy told they're sick) may seem pretty good, but if you are told you're sick, conditional probability shows that your chance of actually being sick, if the illness is rare (say 1 in 1000) are only around 10%.

When to use it:

To find the probability of one event occurring given that another event has occurred (events may not be independent, so they could affect one another).

How to use it:

The chance of A given that B happens is the chance of both out of (compared to) the chance of B. Divide the size of the overlap of the Venn diagram by the size of the category you know you are already in.

What to watch out for:

The probability of A given B is not usually the same as the probability of B given A. For example, it may be true that most astronauts are American, but it's not true that most Americans are astronauts.
As with all probability problems, check that your probability is between 0 and 1.