Dynamic structural equation model

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```
library(tinyVAST)
set.seed(101)
```

`tinyVAST` includes features to fit a dynamic structural equation model

```
data(isle_royale, package="dsem")

# Convert to long-form
data = expand.grid( "time"=isle_royale[,1], "var"=colnames(isle_royale[,2:3]) )
data$logn = unlist(log(isle_royale[2:3]))

# Define cross-lagged SEM
sem = "
  # Link, lag, param_name
  wolves -> wolves, 1, arW
  moose -> wolves, 1, MtoW
  wolves -> moose, 1, WtoM
  moose -> moose, 1, arM
  wolves -> moose, 0, corr
"

# fit model
mytiny = fit( sem = sem,
                data = data,
                times = isle_royale[,1],
                variables = colnames(isle_royale[,2:3]),
                estimate_delta0 = FALSE,
                formula = logn ~ 0 + var,
                quiet = TRUE )
#>   0:     391.97944:  0.00000  0.00000 0.0100000 0.0100000 0.0100000 0.0100000 0.0100000  1.00000  1.
#>   1:     293.47883: 0.0432878 0.108614 0.137076 0.295137 0.299864 0.700110 0.304606 0.920827  1.4844
#>   2:     272.35246: 0.0702137 0.104050 0.249882 0.504281 0.312730 0.752622 0.324146 0.124328 0.94800
#>   3:     267.11302: 0.0331173 0.130867 0.0531818 0.164741 0.302841 0.735444 0.313854 0.237609 0.9178
#>   4:     222.17865: 0.0589472 0.127187 0.106932 0.347293 0.304470 0.742269 0.315607 0.311164 0.89946
#>   5:     208.04762: 0.0717159 0.124302 0.148031 0.408069 0.302962 0.740648 0.315093 0.163017 0.78815
#>   6:     194.08329: 0.0769018 0.143464 0.104822 0.324971 0.309062 0.758708 0.321504 0.138585 0.61835
#>   7:     177.09291: 0.101268 0.155892 0.125659 0.368199 0.322378 0.788853 0.334868 0.128494 0.435963
#>   8:     165.12572: 0.135840 0.177628 0.141177 0.338710 0.292867 0.713473 0.307171 0.112514 0.286754
#>   9:     161.67079: 0.138094 0.178119 0.152983 0.363757 0.301670 0.736209 0.316790 0.117254 0.281971
#>  10:     160.40593: 0.137281 0.181960 0.147957 0.337955 0.302699 0.741241 0.320134 0.122707 0.258864
#>  11:     158.99202: 0.142354 0.185057 0.163540 0.353935 0.304947 0.751270 0.326251 0.128201 0.239372
#>  12:     157.39856: 0.146652 0.190642 0.170507 0.338697 0.297404 0.741771 0.325368 0.134777 0.229954
#>  13:     148.34749: 0.218344 0.283264 0.404687 0.271365 0.202863 0.732150 0.395346 0.242010 0.192628
#>  14:     146.52996: 0.208329 0.292242 0.391447 0.224459 0.216367 0.799418 0.419694 0.190607 0.233675
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#>  15:     140.08045: 0.225738 0.313954 0.417170 0.247747 0.147623 0.764066 0.394473 0.180619 0.278054
#>  16:     134.20758: 0.240366 0.343996 0.446285 0.214776 0.0948538 0.799166 0.395334 0.169305 0.22237
#>  17:     131.16629: 0.250863 0.371955 0.498005 0.219067 0.0393347 0.828875 0.396977 0.175618 0.20655
#>  18:     127.66669: 0.263105 0.399929 0.533055 0.183490 -0.0190056 0.852045 0.399414 0.182196 0.2059
#>  19:     124.11699: 0.277800 0.446152 0.562137 0.190934 -0.0947508 0.873181 0.403304 0.155230 0.1954
#>  20:     118.40042: 0.312852 0.530145 0.627937 0.152709 -0.219049 0.927603 0.439546 0.168529 0.19530
#>  21:     117.92783: 0.338796 0.654110 0.677227 0.123690 -0.344532 0.906275 0.510641 0.159569 0.14585
#>  22:     110.15216: 0.356648 0.717696 0.700543 0.128534 -0.372120 0.930679 0.575301 0.146911 0.17024
#>  23:     107.54614: 0.375368 0.787620 0.717874 0.118302 -0.419856 0.911938 0.623342 0.139189 0.17066
#>  24:     103.19533: 0.424791 0.935402 0.780524 0.105800 -0.534974 0.922482 0.722012 0.158522 0.15011
#>  25:     101.68558: 0.492288  1.12347 0.747017 0.0785286 -0.657683 0.959822 0.810607 0.132680 0.1421
#>  26:     94.244117: 0.517084  1.34150 0.752566 0.119320 -0.785405 0.985749 0.831330 0.129256 0.13787
#>  27:     90.226592: 0.543544  1.60413 0.761883 0.101762 -0.802545 0.967723 0.891040 0.141854 0.12508
#>  28:     86.796561: 0.545694  1.87058 0.834900 0.0765051 -0.795957 0.969397 0.872722 0.120890 0.1364
#>  29:     83.245193: 0.598193  2.12200 0.841094 0.0773191 -0.830677  1.00824 0.820285 0.130225 0.1171
#>  30:     80.566600: 0.669509  2.36722 0.825623 0.0860949 -0.788875  1.00097 0.830899 0.131947 0.1184
#>  31:     76.468880: 0.717368  2.59846 0.800943 0.0967272 -0.722303 0.993675 0.741238 0.129476 0.1206
#>  32:     31.240247:  1.63782  5.40279 0.996746 0.0483751 -0.0279226 0.969621 0.0417952 0.116574 0.05
#>  33:     25.233334:  1.82195  5.96514  1.00764 0.0439629 0.0980274 0.984701 -0.0864806 0.113885 0.06
#>  34:     22.925167:  1.87341  6.12170  1.00913 0.0437779 0.128359 0.985682 -0.122445 0.113411 0.0458
#>  35:     21.536129:  1.92728  6.28088  1.01207 0.0444219 0.150916 0.987403 -0.151129 0.111700 0.0589
#>  36:     18.135929:  1.90721  6.18436 0.993389 0.0516422 0.0477958 0.990004 -0.0487405 0.109509 0.05
#>  37:     17.964229:  1.90764  6.18469 0.991105 0.0517743 0.0465551 0.990836 -0.0489934 0.111559 0.04
#>  38:     17.863862:  1.90810  6.18506 0.988817 0.0519411 0.0453083 0.991889 -0.0491075 0.112349 0.05
#>  39:     17.704282:  1.91086  6.19205 0.987425 0.0521669 0.0453186 0.992522 -0.0498777 0.112357 0.05
#>  40:     17.531216:  1.91661  6.20682 0.985240 0.0526118 0.0456338 0.993693 -0.0514659 0.112206 0.05
#>  41:     17.292437:  1.92793  6.23688 0.984374 0.0532120 0.0481720 0.994205 -0.0548553 0.111616 0.04
#>  42:     17.023169:  1.95153  6.29653 0.983620 0.0545119 0.0540374 0.994422 -0.0615470 0.110578 0.05
#>  43:     16.531412:  2.02526  6.39041 0.969634 0.0632541 0.0707889 0.988609 -0.0686675 0.109142 0.04
#>  44:     15.579043:  2.05907  6.26549 0.941085 0.0706256 0.0586444 0.989815 -0.0554867 0.112170 0.05
#>  45:     14.244263:  2.18659  6.26209 0.941913 0.0652182 0.0425338  1.00828 -0.0776924 0.104598 0.04
#>  46:     12.385983:  2.42401  6.09663 0.915919 0.0631891 0.0187644  1.01631 -0.0555111 0.104999 0.04
#>  47:     12.303612:  2.42423  6.09687 0.915370 0.0633148 0.0180129  1.01644 -0.0557553 0.105042 0.05
#>  48:     12.244870:  2.42688  6.09735 0.914883 0.0633582 0.0173413  1.01661 -0.0559350 0.105023 0.05
#>  49:     12.166225:  2.43217  6.09860 0.913193 0.0636209 0.0151434  1.01714 -0.0564913 0.105004 0.05
#>  50:     10.601993:  2.77919  6.13694 0.917821 0.0525167 0.00617886  1.02615 -0.0588749 0.0991225 0.
#>  51:     9.7728403:  2.80149  6.25968 0.886403 0.0765018 -0.0324250  1.02386 -0.0241564 0.110541 0.0
#>  52:     9.2175173:  2.82143  6.37041 0.857444 0.0986268 -0.00358367  1.01073 -0.0460460 0.105609 0.
#>  53:     8.9799692:  2.91924  6.38492 0.891829 0.0758957 -0.0797135  1.00350 0.0117286 0.0998862 0.0
#>  54:     8.9112641:  3.05368  6.32412 0.872152 0.0763886 -0.0663287  1.01025 -0.0332002 0.100954 0.0
#>  55:     7.8821340:  3.04257  6.39419 0.882479 0.0726922 -0.0484307 0.990201 -0.0493890 0.101782 0.0
#>  56:     7.2091192:  3.02527  6.36993 0.889085 0.0689692 -0.0604854 0.956275 -0.0348962 0.106261 0.0
#>  57:     6.5690422:  3.25930  6.46898 0.928772 0.0387637 -0.0808484 0.912915 -0.0258039 0.108128 0.0
#>  58:     6.4647388:  3.38431  6.49691 0.897694 0.0438401 -0.131542 0.859167 0.00174981 0.102787 0.04
#>  59:     6.0094698:  3.28045  6.40528 0.891718 0.0393087 -0.115254 0.874962 -0.0140661 0.104397 0.04
#>  60:     5.9689302:  3.33960  6.44122 0.912456 0.0245201 -0.110066 0.868087 -0.0258628 0.107986 0.04
#>  61:     5.8638005:  3.32802  6.43937 0.902297 0.0286517 -0.115498 0.866102 -0.0162072 0.106189 0.04
#>  62:     5.8533065:  3.32976  6.44092 0.899621 0.0265515 -0.117718 0.863354 -0.0154414 0.105994 0.04
#>  63:     5.8369695:  3.32784  6.44474 0.895017 0.0205965 -0.119828 0.860224 -0.0147989 0.105933 0.04
#>  64:     5.8252522:  3.32267  6.44656 0.891234 0.0147373 -0.119907 0.859890 -0.0150121 0.106006 0.04
#>  65:     5.8150263:  3.31757  6.44519 0.887772 0.00734691 -0.118604 0.861668 -0.0157502 0.106180 0.0
#>  66:     5.8100538:  3.31829  6.44074 0.886684 0.00383683 -0.117391 0.863803 -0.0162601 0.106307 0.0
#>  67:     5.8057767:  3.32268  6.43350 0.886859 0.00180287 -0.116420 0.865870 -0.0166919 0.106409 0.0
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#>  68:     5.8024395:  3.32871  6.42729 0.887611 0.00182578 -0.116075 0.866969 -0.0167799 0.106468 0.0
#>  69:     5.7993840:  3.33334  6.42372 0.889081 0.00317715 -0.116239 0.866903 -0.0167500 0.106488 0.0
#>  70:     5.7943738:  3.33804  6.42370 0.891145 0.00720984 -0.117164 0.865577 -0.0161895 0.106502 0.0
#>  71:     5.7900674:  3.33765  6.42744 0.893475 0.0110153 -0.117909 0.863814 -0.0159467 0.106501 0.04
#>  72:     5.7858147:  3.33180  6.43815 0.894587 0.0160705 -0.119029 0.861459 -0.0151682 0.106503 0.04
#>  73:     5.7844406:  3.32563  6.44344 0.894833 0.0170892 -0.118885 0.860923 -0.0153313 0.106479 0.04
#>  74:     5.7838569:  3.32280  6.44525 0.893968 0.0164722 -0.118848 0.861030 -0.0152532 0.106474 0.04
#>  75:     5.7833857:  3.32182  6.44511 0.893161 0.0152762 -0.118631 0.861396 -0.0153461 0.106475 0.04
#>  76:     5.7828398:  3.32240  6.44346 0.892416 0.0136884 -0.118495 0.861852 -0.0154185 0.106481 0.04
#>  77:     5.7824798:  3.32379  6.44209 0.892323 0.0132240 -0.118456 0.861984 -0.0154778 0.106487 0.04
#>  78:     5.7821785:  3.32534  6.44112 0.892696 0.0135242 -0.118598 0.861872 -0.0154249 0.106489 0.04
#>  79:     5.7820558:  3.32581  6.44117 0.893027 0.0140786 -0.118646 0.861714 -0.0154191 0.106489 0.04
#>  80:     5.7819968:  3.32571  6.44145 0.893196 0.0143801 -0.118710 0.861632 -0.0153690 0.106488 0.04
#>  81:     5.7819624:  3.32546  6.44167 0.893187 0.0144452 -0.118672 0.861630 -0.0153922 0.106488 0.04
#>  82:     5.7819421:  3.32527  6.44173 0.893126 0.0143472 -0.118674 0.861662 -0.0153789 0.106488 0.04
#>  83:     5.7819314:  3.32519  6.44172 0.893051 0.0142438 -0.118640 0.861694 -0.0154025 0.106488 0.04
#>  84:     5.7819258:  3.32520  6.44168 0.893020 0.0141749 -0.118647 0.861708 -0.0153949 0.106488 0.04
#>  85:     5.7819227:  3.32522  6.44165 0.893012 0.0141639 -0.118640 0.861708 -0.0154027 0.106488 0.04
#>  86:     5.7819210:  3.32525  6.44164 0.893025 0.0141778 -0.118649 0.861702 -0.0153966 0.106488 0.04
#>  87:     5.7819198:  3.32527  6.44165 0.893040 0.0142055 -0.118649 0.861695 -0.0153984 0.106488 0.04
#>  88:     5.7819193:  3.32527  6.44165 0.893049 0.0142175 -0.118653 0.861692 -0.0153949 0.106488 0.04
#>  89:     5.7819191:  3.32527  6.44166 0.893049 0.0142193 -0.118651 0.861692 -0.0153967 0.106488 0.04
#>  90:     5.7819189:  3.32526  6.44166 0.893046 0.0142142 -0.118651 0.861694 -0.0153960 0.106488 0.04
#>  91:     5.7819189:  3.32526  6.44165 0.893043 0.0142096 -0.118650 0.861695 -0.0153970 0.106488 0.04
#>  92:     5.7819188:  3.32526  6.44165 0.893041 0.0142071 -0.118650 0.861696 -0.0153967 0.106488 0.04
#>  93:     5.7819188:  3.32526  6.44165 0.893041 0.0142071 -0.118650 0.861695 -0.0153969 0.106488 0.04
#>  94:     5.7819188:  3.32526  6.44165 0.893042 0.0142081 -0.118650 0.861695 -0.0153967 0.106488 0.04
#>  95:     5.7819188:  3.32526  6.44165 0.893043 0.0142091 -0.118650 0.861695 -0.0153967 0.106488 0.04
#>  96:     5.7819188:  3.32526  6.44165 0.893043 0.0142096 -0.118650 0.861695 -0.0153966 0.106488 0.04
#>  97:     5.7819188:  3.32526  6.44165 0.893043 0.0142095 -0.118650 0.861695 -0.0153967 0.106488 0.04
#>  98:     5.7819188:  3.32526  6.44165 0.893043 0.0142093 -0.118650 0.861695 -0.0153966 0.106488 0.04
mytiny
#> $call
#> fit(data = data, formula = logn ~ 0 + var, sem = sem, estimate_delta0 = FALSE,
#>     times = isle_royale[, 1], variables = colnames(isle_royale[,
#>         2:3]), quiet = TRUE)
#>
#> $opt
#> $opt$par
#>      alpha_j      alpha_j       beta_z       beta_z       beta_z       beta_z       beta_z       bet
#>   3.32526215   6.44165380   0.89304284   0.01420930  -0.11865012   0.86169491  -0.01539665   0.10648
#>
#> $opt$objective
#> [1] 5.781919
#>
#> $opt$convergence
#> [1] 0
#>
#> $opt$iterations
#> [1] 98
#>
#> $opt$evaluations
#> function gradient
```

```
#>      125       99
#>
#> $opt$message
#> [1] "relative convergence (4)"
#>
#>
#> $sdrep
#> sdreport(.) result
#>               Estimate   Std. Error
#> alpha_j     3.32526215 2.483419e-01
#> alpha_j     6.44165380 2.116017e-01
#> beta_z      0.89304284 8.420604e-02
#> beta_z      0.01420930 1.279148e-01
#> beta_z     -0.11865012 6.477637e-02
#> beta_z      0.86169491 7.080254e-02
#> beta_z     -0.01539665 6.067738e-02
#> beta_z      0.10648808 9.881848e-03
#> beta_z      0.04810383 4.460377e-03
#> log_sigma -12.52451040 1.805702e+04
#> Maximum gradient component: 1.345814e-05
#>
#> $run_time
#> Time difference of 0.3097789 secs
```

We can then compare this with package **dsem**

```
library(dsem)

# Keep in wide-form
dsem_data = ts( log(isle_royale[,2:3]), start=1959)
family = c("normal","normal")

# initial first without delta0 (to improve starting values)
mydsem = dsem( sem = sem,
               tsdata = dsem_data,
               estimate_delta0 = FALSE,
               quiet = TRUE,
               getsd = FALSE,
               family = family )
mydsem
#> $par
#>        beta_z        beta_z        beta_z        beta_z        beta_z        beta_z        beta_z
#>   0.895834720   0.007358847  -0.109332511   0.875012562  -0.017355229   0.378795847  -0.172873038 -1
#>
#> $objective
#> [1] 7.739638
#>
#> $iterations
#> [1] 79
#>
#> $evaluations
#> function gradient
#>       96       80
```

```
#>
#> $time_for_MLE
#> Time difference of 0.07090306 secs
#>
#> $max_gradient
#> [1] 7.714655e-07
#>
#> $Convergence_check
#> [1] "There is no evidence that the model is not converged"
#>
#> $number_of_coefficients
#>  Total  Fixed Random
#>    133      9    124
#>
#> $AIC
#> [1] 33.47928
#>
#> $diagnostics
#>       Param starting_value Lower           MLE Upper final_gradient
#> 1    beta_z           0.01  -Inf   0.895834720   Inf   4.785205e-09
#> 2    beta_z           0.01  -Inf   0.007358847   Inf  -5.078683e-09
#> 3    beta_z           0.01  -Inf  -0.109332511   Inf  -2.031211e-08
#> 4    beta_z           0.01  -Inf   0.875012562   Inf  -5.821149e-08
#> 5    beta_z           0.01  -Inf  -0.017355229   Inf  -5.373382e-09
#> 6    beta_z           1.00  -Inf   0.378795847   Inf   2.119351e-09
#> 7    beta_z           1.00  -Inf  -0.172873038   Inf  -7.714655e-07
#> 8 lnsigma_j           0.00  -Inf -15.799262455   Inf   1.628788e-12
#> 9 lnsigma_j           0.00  -Inf -11.977331517   Inf   2.141499e-09
#>
#> $time_for_run
#> Time difference of 0.07221317 secs
```