VAST

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```
library(tinyVAST)
library(fmesher)
set.seed(101)
```

`tinyVAST` is an R package for fitting vector autoregressive spatio-temporal (VAST) models. We here explore the capacity to specify the vector-autoregressive spatio-temporal component.

Spatio-temporal autoregressive model

We first explore the ability to specify a first-order autoregressive spatio-temporal process:

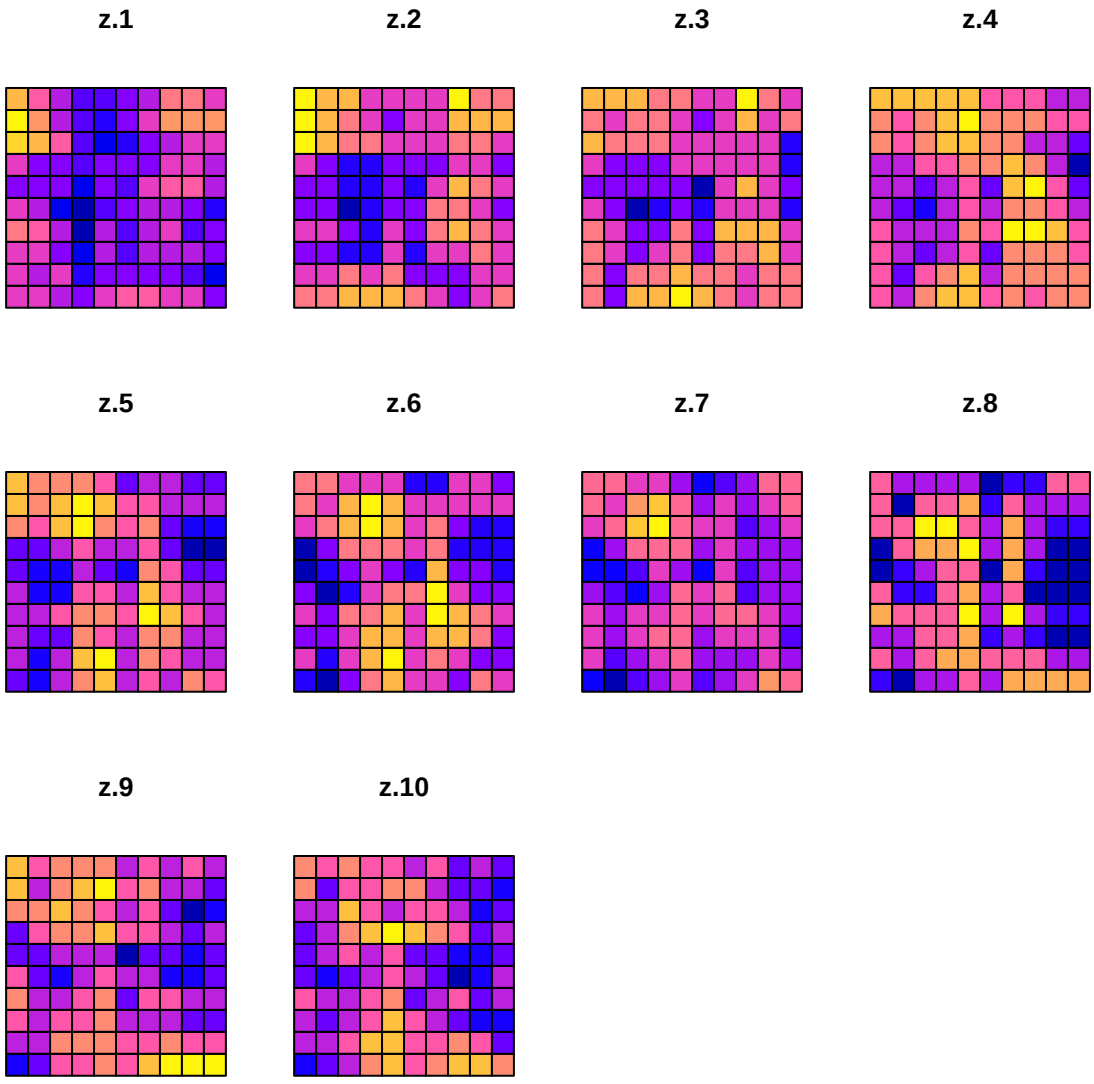
```
# Simulate settings
theta_xy = 0.4
n_x = n_y = 10
n_t = 10
rho = 0.8
spatial_sd = 0.5

# Simulate GMRFs
R = exp(-theta_xy * abs(outer(1:n_x, 1:n_y, FUN="-")) )
d = mvtnorm::rmvnorm(n_t, sigma=spatial_sd^2*kronecker(R,R) )

# Project through time and add mean
for( t in seq_len(n_t) ){
  if(t>1) d[t,] = rho*d[t-1,] + d[t,]
}
d = d + 2

# Shape into longform data-frame and add error
Data = data.frame( expand.grid(time=1:n_t, x=1:n_x, y=1:n_y), "var"="logn", z=as.vector(d))
Data$n = Data$z + rnorm(nrow(Data), sd=0.2)

# make mesh
mesh = fm_mesh_2d( Data[,c('x','y')] )

# fit model
out = fit( sem = "logn -> logn, 1, rho",
           data = Data,
           formula = n ~ 0 + factor(time),
           spatial_graph = mesh,
           quiet = TRUE )
out
```

```
#> $call
#> fit(data = Data, formula = n ~ 0 + factor(time), sem = "logn -> logn, 1, rho",
#>     spatial_graph = mesh, quiet = TRUE)
#>
#> $opt
#> $opt$par
#>   log_kappa       alpha       alpha       alpha       alpha       alpha       alpha       alpha
#> -0.02425752  1.86984224  1.83580262  1.87501954  1.85153103  1.52537711  1.71661702  1.58766220  1.3
#>      beta_z      beta_z   log_sigma
#>  0.81657948  0.38747642 -1.60062550
#>
#> $opt$objective
#> [1] 562.4018
#>
#> $opt$convergence
#> [1] 0
#>
#> $opt$iterations
#> [1] 53
#>
#> $opt$evaluations
#> function gradient
#>       76       54
#>
#> $opt$message
#> [1] "relative convergence (4)"
#>
#>
#> $sdrep
#> sdreport(.) result
#>              Estimate Std. Error
#> log_kappa -0.02425752 0.08572262
#> alpha      1.86984224 0.13882807
#> alpha      1.83580262 0.17846892
#> alpha      1.87501954 0.20059433
#> alpha      1.85153103 0.21408047
#> alpha      1.52537711 0.22261967
#> alpha      1.71661702 0.22813627
#> alpha      1.58766220 0.23174136
#> alpha      1.38155929 0.23411458
#> alpha      1.58792086 0.23568398
#> alpha      1.88280383 0.23672483
#> beta_z     0.81657948 0.03015651
#> beta_z     0.38747642 0.01720812
#> log_sigma -1.60062550 0.07505558
#> Maximum gradient component: 0.0007494347
#>
#> $run_time
#> Time difference of 5.164159 secs
```

The estimated values for `beta_z` then correspond to the simulated value for `rho` and `spatial_sd`.

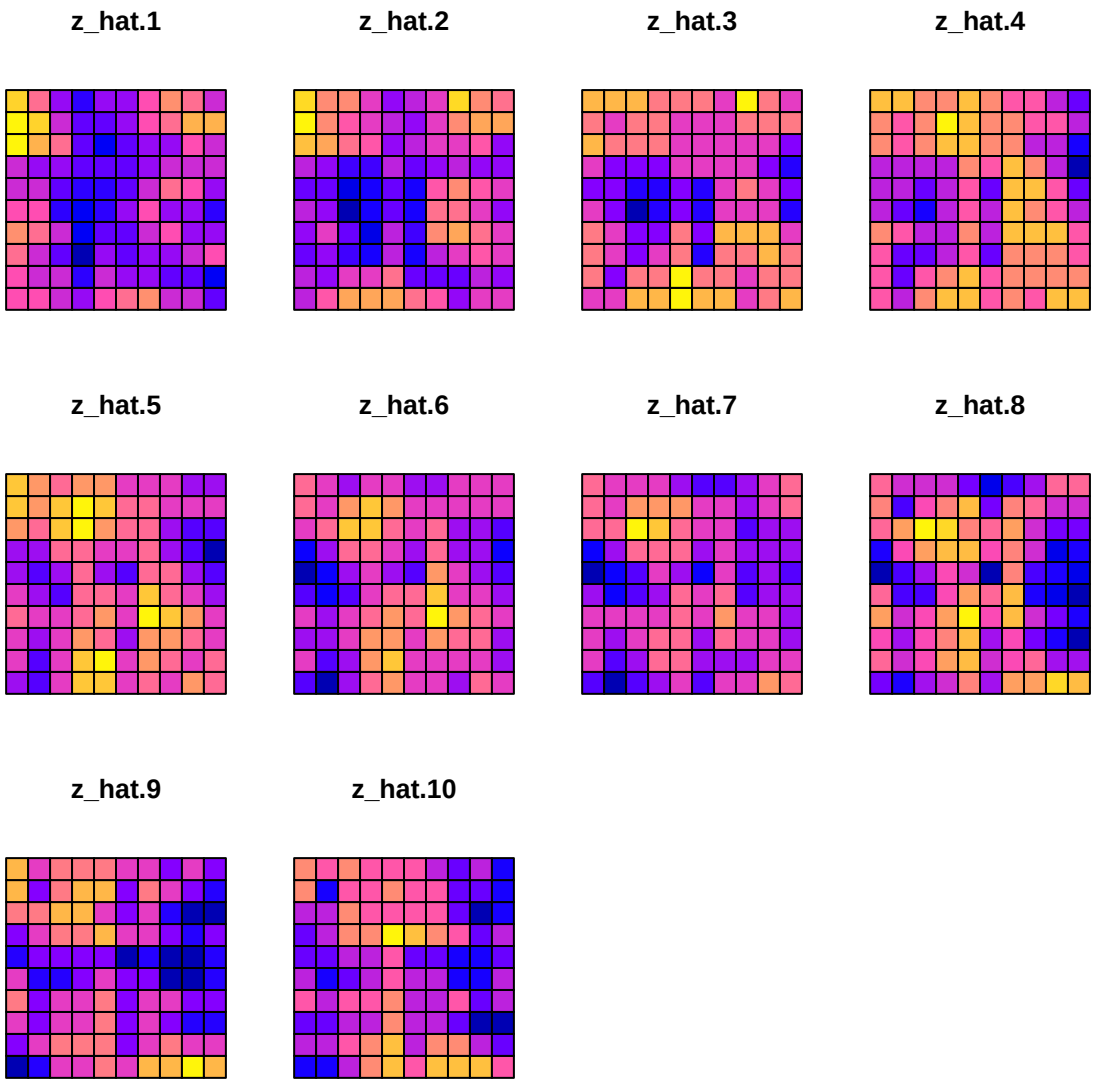
We can compare the true densities:

```
library(sf)
#> Warning: package 'sf' was built under R version 4.3.1
data_wide = reshape( Data[,c('x','y','time','z')],
                     direction = "wide", idvar = c('x','y'), timevar = "time")
sf_data = st_as_sf( data_wide, coords=c("x","y"))
sf_grid = sf::st_make_grid( sf_data )
sf_plot = st_sf(sf_grid, st_drop_geometry(sf_data) )
plot(sf_plot, max.plot=n_t )
```



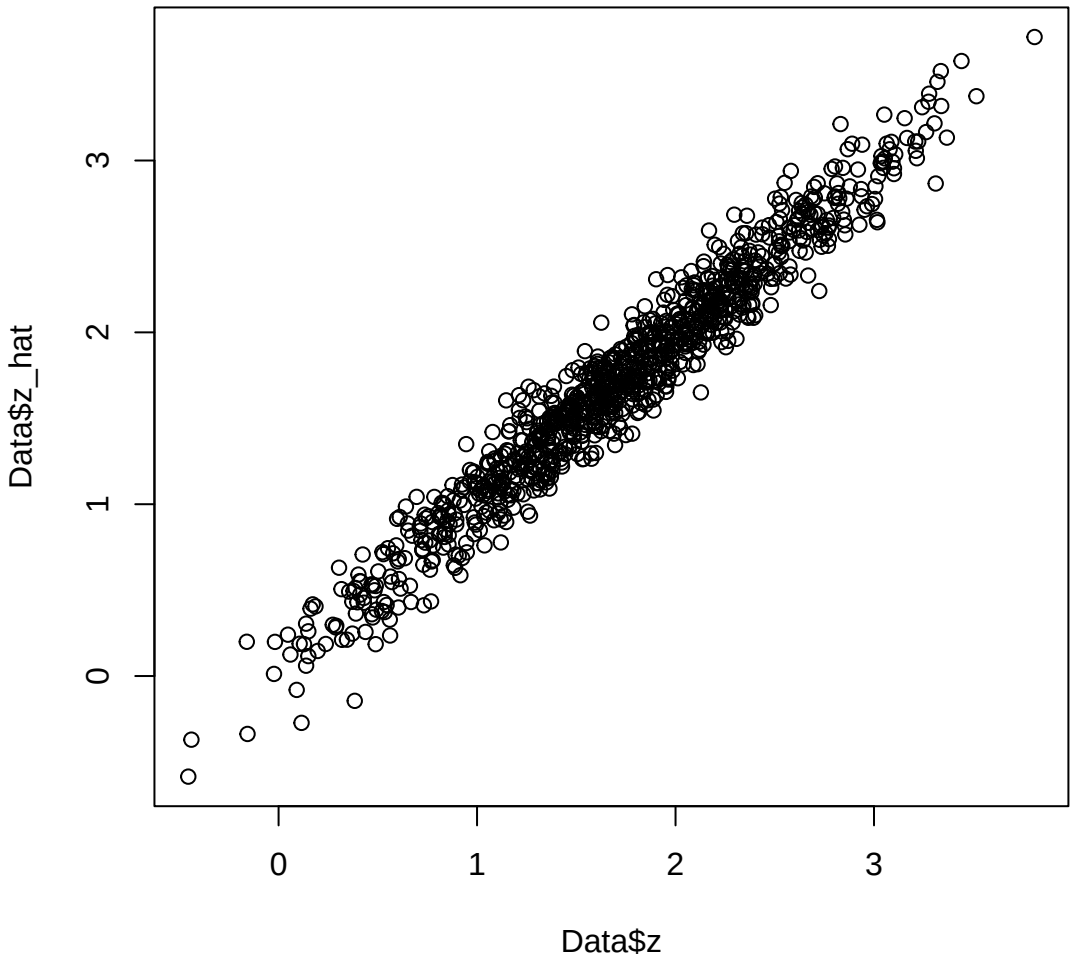
with the estimated densities:

```
Data$z_hat = predict(out)
data_wide = reshape( Data[,c('x','y','time','z_hat')],
                     direction = "wide", idvar = c('x','y'), timevar = "time")
sf_data = st_as_sf( data_wide, coords=c("x","y"))
sf_plot = st_sf(sf_grid, st_drop_geometry(sf_data) )
plot(sf_plot, max.plot=n_t )
```



where a scatterplot shows that they are highly correlated:

```
plot( x=Data$z, y=Data$z_hat )
```



Next, we compare this against the current version of VAST

```
library(INLA)
#> Warning: package 'INLA' was built under R version 4.3.1
#> Warning: package 'sp' was built under R version 4.3.1
library(VAST)
#> Warning: package 'units' was built under R version 4.3.1
#> Warning: package 'marginaleffects' was built under R version 4.3.1
settings = make_settings( purpose="index3",
                          n_x = n_x*n_y,
                          Region = "Other",
                          bias.correct = FALSE,
                          use_anisotropy = FALSE )
settings$FieldConfig['Epsilon','Component_2'] = 0
settings$FieldConfig['Omega',] = 0
settings$RhoConfig['Epsilon1'] = 4
```

```
myVAST = fit_model( settings=settings,
                Lat_i = Data[,'y'],
                Lon_i = Data[,'x'],
                t_i = Data[,'time'],
                b_i = exp(Data[,'n']),
                a_i = rep(1,nrow(Data)),
                observations_LL = cbind(Lat=Data[,'y'],Lon=Data[,'x']),
                grid_dim_km = c(100,100),
                ObsModel = c(1,4),
                newtonsteps = 0,
                loopnum = 0,
                control = list(eval.max=100, iter.max=100, trace=0) )
#> Warning: The `returnclass` argument of `ne_download()` sp as of rnaturalearth 1.0.0.
#> i Please use `sf` objects with {rnaturalearth}, support for Spatial objects (sp) will be removed in
#> i The deprecated feature was likely used in the FishStatsUtils package.
#>   Please report the issue to the authors.
#> This warning is displayed once every 8 hours.
#> Call `lifecycle::last_lifecycle_warnings()` to see where this warning was generated.
#> Warning in FishStatsUtils:::inla.barrier.fem.copy(mesh = anisotropic_mesh, : Please install the `INL
#>  which contains an implementation that runs faster!
#>              Component_1 Component_2
#> Omega                 -1          -1
#> Epsilon               -2          -1
#> Beta                  -2          -2
#> Epsilon_time          -3          -3
#> Eta1 Eta2
#>   -1   -1
#>    Coefficient_name Number_of_coefficients   Type
#> 1          beta1_ft                    10  Fixed
#> 2    Epsilon_rho1_f                     1  Fixed
#> 3      L_epsilon1_z                     1  Fixed
#> 4         logkappa1                     1  Fixed
#> 5         logSigmaM                     1  Fixed
#> 6 Epsiloninput1_sff                  1360 Random
#>   0:     4894.2283:  20.0000  20.0000  20.0000  20.0000  20.0000  20.0000  20.0000  20.0000  20.0000
#>   1:     4438.1813:  19.9995  19.9995  19.9995  19.9995  19.9995  19.9995  19.9995  19.9994  19.9995
#>   2:     4425.9216:  19.9871  19.9864  19.9871  19.9869  19.9812  19.9849  19.9824  19.9787  19.9820
#>   3:     4410.5566:  19.5801  19.5608  19.5799  19.5742  19.4176  19.5182  19.4501  19.3473  19.4398
#>   4:     4259.7142:  14.6347  14.4725  14.6561  14.6065  13.1418  14.0869  13.4506  12.4874  13.3478
#>   5:     4184.8409:  12.6473  12.4300  12.6777  12.6109  10.6372  11.9104  11.0528  9.75485  10.9142
#>   6:     3883.8869:  8.51254  8.23137  8.51344  8.39364  5.91481  7.45055  6.36317  4.74708  6.19091
#> Warning in nlminb(start = startpar, objective = fn, gradient = gr, control = nlminb.control, : NA/Na
#>   7:     3429.2064:  4.31122  3.99694  4.30956  4.13994  1.25063  2.97992  1.70061 -0.171716  1.5001
#> Warning in nlminb(start = startpar, objective = fn, gradient = gr, control = nlminb.control, : NA/Na
#>   8:     3181.2403:  2.61824  2.33640  2.63385  2.40572 -0.513021  1.13494 -0.177412 -2.05427 -0.343
#>   9:     2752.0904:  2.60860  2.34357  2.62177  2.36867 -0.438463  1.09902 -0.173796 -1.98127 -0.314
#>  10:     2692.9981:  2.58078  2.35240  2.60481  2.30909 -0.330328  1.03964 -0.173797 -1.88290 -0.279
#>  11:     2524.1742:  2.38827  2.19480  2.42028  2.07078 -0.397943 0.790873 -0.364955 -1.97958 -0.428
#>  12:     2484.1287:  2.38740  2.19524  2.41958  2.06840 -0.393764 0.788209 -0.365340 -1.97588 -0.427
#>  13:     2475.8920:  2.38549  2.19625  2.41803  2.06305 -0.384490 0.782159 -0.366342 -1.96778 -0.424
#>  14:     2473.3779:  2.42816  2.24524  2.46366  2.10372 -0.323073 0.821311 -0.320157 -1.90742 -0.372
#>  15:     2466.5210:  2.47704  2.33184  2.52336  2.10678 -0.157839 0.804363 -0.285000 -1.77883 -0.289
#>  16:     2465.0350:  2.52992  2.42097  2.58177  2.12244 -0.000242209 0.794713 -0.251257 -1.64502 -0.
```

```
#>  17:     2457.4230:  2.63922  2.72944  2.67644  1.98309 0.670643 0.443700 -0.372721 -1.11277 0.03439
#>  18:     2451.9494:  2.79709  3.05871  2.83630  2.05927  1.25362 0.380820 -0.255511 -0.470920 0.4149
#>  19:     2445.8974:  2.56599  2.91929  2.59635  2.15139  1.15237 0.443918 -0.0960517 0.115605 0.6342
#>  20:     2444.2949:  2.56505  2.91648  2.59612  2.15053  1.15140 0.445779 -0.0928384 0.115063 0.6328
#>  21:     2443.6387:  2.55895  2.90409  2.59251  2.14266  1.14890 0.447399 -0.0835520 0.114173 0.6257
#>  22:     2442.7980:  2.54661  2.88288  2.58347  2.12596  1.14561 0.444389 -0.0726652 0.113963 0.6125
#>  23:     2442.1636:  2.54153  2.87158  2.58035  2.12055  1.14214 0.448763 -0.0627556 0.112418 0.6063
#>  24:     2441.3384:  2.52861  2.84968  2.56967  2.10314  1.13857 0.446703 -0.0518165 0.111325 0.5922
#>  25:     2440.7351:  2.52504  2.84002  2.56782  2.10073  1.13450 0.454232 -0.0408404 0.109338 0.5875
#>  26:     2439.9154:  2.51202  2.81755  2.55622  2.08290  1.13096 0.453936 -0.0289248 0.107046 0.5727
#>  27:     2439.3645:  2.50735  2.80561  2.55335  2.08005  1.12582 0.463669 -0.0154226 0.104759 0.5670
#>  28:     2438.5410:  2.49420  2.78289  2.54081  2.06213  1.12227 0.464776 -0.00308151 0.101762 0.552
#>  29:     2437.9948:  2.48815  2.76828  2.53652  2.05880  1.11589 0.477212 0.0133159 0.0992179 0.5452
#>  30:     2437.1793:  2.47473  2.74538  2.52289  2.04087  1.11242 0.479625 0.0259217 0.0959573 0.5303
#>  31:     2436.7024:  2.46759  2.72914  2.51716  2.03733  1.10527 0.494030 0.0439490 0.0934880 0.5231
#>  32:     2435.8541:  2.45398  2.70632  2.50250  2.01955  1.10221 0.497747 0.0567641 0.0903360 0.5086
#>  33:     2435.3782:  2.44576  2.68865  2.49520  2.01581  1.09450 0.513951 0.0760981 0.0881433 0.5013
#>  34:     2434.5439:  2.43172  2.66580  2.47923  1.99817  1.09168 0.518539 0.0887000 0.0855052 0.4874
#>  35:     2434.1089:  2.42259  2.64719  2.47046  1.99428  1.08377 0.536030 0.108716 0.0837924 0.4802
#>  36:     2433.2524:  2.40827  2.62453  2.45335  1.97693  1.08131 0.541218 0.120860 0.0819129 0.4672
#>  37:     2432.8130:  2.39831  2.60512  2.44313  1.97284  1.07341 0.559741 0.141330 0.0807968 0.4605
#>  38:     2431.9676:  2.38364  2.58248  2.42489  1.95574  1.07121 0.565317 0.152976 0.0798196 0.4484
#>  39:     2431.5464:  2.37281  2.56234  2.41307  1.95097  1.06371 0.584149 0.173385 0.0793822 0.4420
#>  40:     2430.6918:  2.35793  2.53981  2.39394  1.93418  1.06186 0.590020 0.184578 0.0793688 0.4310
#>  41:     2430.2783:  2.34661  2.51930  2.38102  1.92923  1.05473 0.609285 0.204952 0.0796499 0.4253
#>  42:     2429.4233:  2.33157  2.49681  2.36111  1.91273  1.05319 0.615365 0.215770 0.0806210 0.4154
#>  43:     2429.0040:  2.31945  2.47560  2.34661  1.90677  1.04676 0.634304 0.235717 0.0816798 0.4101
#>  44:     2428.1586:  2.30428  2.45305  2.32602  1.89050  1.04559 0.640655 0.246378 0.0836546 0.4013
#>  45:     2427.7273:  2.29145  2.43120  2.30999  1.88341  1.03997 0.658935 0.265676 0.0855313 0.3965
#>  46:     2426.8986:  2.27620  2.40855  2.28878  1.86736  1.03918 0.665604 0.276324 0.0885506 0.3888
#>  47:     2426.4512:  2.26268  2.38603  2.27118  1.85898  1.03440 0.682898 0.294778 0.0913029 0.3845
#>  48:     2425.6438:  2.24744  2.36322  2.24942  1.84316  1.03413 0.690119 0.305732 0.0954615 0.3782
#>  49:     2425.1785:  2.23344  2.34023  2.23050  1.83367  1.03022 0.706473 0.323417 0.0991740 0.3745
#>  50:     2424.3960:  2.21829  2.31725  2.20826  1.81815  1.03056 0.714501 0.334996 0.104639 0.369846
#>  51:     2423.9112:  2.20397  2.29389  2.18819  1.80768  1.02761 0.729978 0.352053 0.109516 0.367195
#>  52:     2423.1555:  2.18908  2.27082  2.16567  1.79265  1.02883 0.739352 0.364855 0.116626 0.364576
#>  53:     2422.6498:  2.17477  2.24741  2.14489  1.78164  1.02712 0.754543 0.381898 0.123092 0.363607
#>  54:     2416.9743:  1.80216  1.65897  1.55225  1.38986  1.12153  1.01591 0.742850 0.383206 0.378278
#>  55:     2413.9521:  1.83467  1.63004  1.55186  1.44221  1.20890  1.25707 0.974291 0.499839 0.572938
#>  56:     2410.5992:  1.75624  1.64408  1.60392  1.57803  1.21778  1.36744  1.13168 0.825915 0.879241
#>  57:     2410.3696:  1.75861  1.64413  1.60611  1.57788  1.21645  1.36595  1.13255 0.823480 0.880902
#>  58:     2410.2605:  1.76204  1.64438  1.60929  1.57769  1.21447  1.36376  1.13363 0.820051 0.883390
#>  59:     2410.0962:  1.76471  1.64517  1.61297  1.57916  1.21553  1.36490  1.13902 0.822284 0.889381
#>  60:     2409.9854:  1.76854  1.64623  1.61685  1.57964  1.21410  1.36356  1.14130 0.820445 0.893727
#>  61:     2409.8548:  1.77101  1.64782  1.62018  1.58150  1.21571  1.36580  1.14659 0.823830 0.899884
#>  62:     2409.7557:  1.77445  1.64980  1.62399  1.58289  1.21573  1.36646  1.15010 0.824862 0.905651
#>  63:     2409.6493:  1.77680  1.65197  1.62707  1.58504  1.21774  1.36948  1.15515 0.828971 0.911899
#>  64:     2409.5606:  1.77991  1.65445  1.63062  1.58685  1.21849  1.37112  1.15880 0.831275 0.918068
#>  65:     2409.4675:  1.78214  1.65689  1.63355  1.58922  1.22079  1.37457  1.16363 0.835783 0.924405
#>  66:     2409.3868:  1.78499  1.65958  1.63695  1.59131  1.22203  1.37674  1.16731 0.838820 0.930845
#>  67:     2409.3027:  1.78711  1.66212  1.63982  1.59385  1.22457  1.38042  1.17206 0.843589 0.937310
#>  68:     2409.2283:  1.78975  1.66488  1.64316  1.59616  1.22617  1.38289  1.17579 0.847123 0.943969
#>  69:     2409.1510:  1.79176  1.66742  1.64603  1.59884  1.22891  1.38670  1.18052 0.852093 0.950556
```

```
#>  70:    2409.0816:  1.79423  1.67019  1.64933  1.60133  1.23081  1.38937  1.18432 0.856003 0.957380
#>  71:    2409.0097:  1.79613  1.67271  1.65221  1.60414  1.23372  1.39325  1.18907 0.861140 0.964061
#>  72:    2408.9443:  1.79845  1.67545  1.65551  1.60677  1.23585  1.39607  1.19295 0.865349 0.971003
#>  73:    2408.8768:  1.80026  1.67794  1.65840  1.60968  1.23889  1.40001  1.19773 0.870629 0.977750
#>  74:    2408.8149:  1.80244  1.68066  1.66169  1.61243  1.24122  1.40294  1.20168 0.875085 0.984772
#>  75:    2408.7510:  1.80416  1.68311  1.66459  1.61543  1.24438  1.40693  1.20649 0.880490 0.991564
#>  76:    2408.6921:  1.80622  1.68581  1.66787  1.61828  1.24687  1.40996  1.21052 0.885153 0.998643
#>  77:    2408.6313:  1.80787  1.68822  1.67079  1.62135  1.25013  1.41400  1.21537 0.890665  1.00547
#>  78:    2408.5749:  1.80981  1.69089  1.67405  1.62428  1.25276  1.41712  1.21946 0.895504  1.01258
#>  79:    2408.5169:  1.81139  1.69327  1.67697  1.62741  1.25611  1.42120  1.22435 0.901111  1.01942
#>  80:    2408.4628:  1.81324  1.69590  1.68023  1.63041  1.25887  1.42441  1.22850 0.906101  1.02657
#>  81:    2408.4072:  1.81476  1.69825  1.68315  1.63360  1.26229  1.42854  1.23343 0.911790  1.03342
#>  82:    2408.3553:  1.81652  1.70085  1.68638  1.63665  1.26517  1.43184  1.23766 0.916913  1.04058
#>  83:    2408.3020:  1.81798  1.70317  1.68930  1.63988  1.26866  1.43602  1.24263 0.922675  1.04744
#>  84:    2408.2520:  1.81966  1.70573  1.69251  1.64299  1.27163  1.43939  1.24692 0.927915  1.05461
#>  85:    2408.2007:  1.82106  1.70802  1.69542  1.64625  1.27519  1.44362  1.25193 0.933742  1.06148
#>  86:    2408.1525:  1.82268  1.71054  1.69861  1.64940  1.27825  1.44708  1.25629 0.939084  1.06865
#>  87:    2408.1032:  1.82403  1.71280  1.70151  1.65269  1.28186  1.45135  1.26134 0.944970  1.07552
#>  88:    2408.0567:  1.82558  1.71528  1.70467  1.65587  1.28501  1.45488  1.26576 0.950404  1.08269
#>  89:    2408.0092:  1.82688  1.71750  1.70755  1.65919  1.28868  1.45921  1.27085 0.956344  1.08957
#>  90:    2407.9644:  1.82836  1.71994  1.71069  1.66240  1.29190  1.46281  1.27534 0.961861  1.09674
#>  91:    2407.9185:  1.82961  1.72213  1.71355  1.66573  1.29561  1.46718  1.28047 0.967849  1.10361
#>  92:    2407.8753:  1.83104  1.72452  1.71666  1.66896  1.29890  1.47086  1.28501 0.973441  1.11078
#>  93:    2407.8311:  1.83224  1.72667  1.71949  1.67230  1.30265  1.47527  1.29018 0.979475  1.11766
#>  94:    2407.7893:  1.83361  1.72902  1.72256  1.67554  1.30600  1.47901  1.29478 0.985136  1.12483
#>  95:    2407.7467:  1.83477  1.73112  1.72537  1.67890  1.30979  1.48346  1.29998 0.991213  1.13171
#>  96:    2407.7063:  1.83609  1.73342  1.72840  1.68214  1.31319  1.48727  1.30464 0.996937  1.13888
#>  97:    2407.6652:  1.83720  1.73548  1.73117  1.68549  1.31701  1.49175  1.30988  1.00305  1.14577
#>  98:    2407.6261:  1.83846  1.73773  1.73416  1.68874  1.32045  1.49561  1.31459  1.00884  1.15294
#>  99:    2407.5865:  1.83952  1.73974  1.73690  1.69208  1.32430  1.50013  1.31985  1.01499  1.15984
#> 100:    2407.5487:  1.84074  1.74195  1.73984  1.69533  1.32778  1.50404  1.32461  1.02083  1.16701
#> 101:    2407.5104:  1.84176  1.74391  1.74253  1.69865  1.33164  1.50857  1.32990  1.02703  1.17393
#> 102:    2407.4739:  1.84292  1.74606  1.74543  1.70188  1.33516  1.51253  1.33470  1.03292  1.18112
#> 103:    2407.4368:  1.84390  1.74797  1.74807  1.70518  1.33903  1.51708  1.34002  1.03915  1.18805
#> 104:    2407.4015:  1.84502  1.75006  1.75091  1.70838  1.34256  1.52106  1.34485  1.04509  1.19525
#> 105:    2407.3657:  1.84595  1.75193  1.75351  1.71165  1.34643  1.52562  1.35019  1.05136  1.20221
#> 106:    2407.3314:  1.84703  1.75397  1.75630  1.71483  1.34997  1.52962  1.35505  1.05734  1.20943
#> 107:    2407.2968:  1.84792  1.75579  1.75884  1.71806  1.35384  1.53417  1.36039  1.06365  1.21642
#> 108:    2407.2635:  1.84897  1.75779  1.76158  1.72120  1.35737  1.53817  1.36527  1.06966  1.22367
#> 109:    2407.2299:  1.84983  1.75957  1.76407  1.72439  1.36123  1.54272  1.37061  1.07601  1.23070
#> 110:    2407.1976:  1.85086  1.76154  1.76677  1.72749  1.36475  1.54670  1.37550  1.08205  1.23798
#> 111:    2407.1650:  1.85170  1.76329  1.76922  1.73064  1.36859  1.55122  1.38083  1.08843  1.24505
#> 112:    2407.1336:  1.85272  1.76524  1.77189  1.73370  1.37208  1.55517  1.38570  1.09449  1.25237
#> 113:    2407.1017:  1.85355  1.76698  1.77432  1.73681  1.37590  1.55966  1.39102  1.10088  1.25947
#> 114:    2407.0711:  1.85459  1.76894  1.77699  1.73985  1.37936  1.56356  1.39586  1.10694  1.26683
#> 115:    2407.0401:  1.85544  1.77070  1.77943  1.74294  1.38316  1.56800  1.40115  1.11334  1.27396
#> 116:    2407.0101:  1.85651  1.77270  1.78213  1.74597  1.38658  1.57184  1.40594  1.11936  1.28136
#> 117:    2406.9798:  1.85740  1.77452  1.78460  1.74907  1.39035  1.57623  1.41119  1.12575  1.28851
#> 118:    2406.9505:  1.85855  1.77660  1.78739  1.75212  1.39372  1.57998  1.41590  1.13171  1.29593
#> 119:    2406.5926:  1.87626  1.81852  1.84483  1.83551  1.50456  1.71286  1.56765  1.30924  1.47333
#> 120:    2406.5687:  1.86111  1.80902  1.84847  1.83163  1.49772  1.69550  1.55719  1.29873  1.48966
#> 121:    2406.4527:  1.86820  1.79611  1.83532  1.82193  1.48461  1.67822  1.53650  1.29643  1.49251
#> 122:    2406.4334:  1.88364  1.83032  1.83555  1.81021  1.47608  1.67715  1.55159  1.28854  1.50931
```

```
#> 123:     2406.4113:  1.89525  1.83715  1.84756  1.81080  1.47304  1.68099  1.55100  1.29767  1.50550
#> 124:     2406.3846:  1.89454  1.84096  1.86088  1.82100  1.48031  1.68445  1.55917  1.30247  1.50745
#> 125:     2406.3670:  1.89541  1.84300  1.86202  1.83788  1.49141  1.68867  1.55906  1.30151  1.51100
#> 126:     2406.3635:  1.89346  1.84326  1.87075  1.84263  1.50011  1.70094  1.55330  1.30020  1.52176
#> 127:     2406.3424:  1.89179  1.84892  1.88073  1.84941  1.50245  1.69834  1.56362  1.31017  1.52339
#> 128:     2406.3406:  1.89273  1.84873  1.88075  1.84995  1.50185  1.69835  1.56309  1.31078  1.52340
#> 129:     2406.3373:  1.89400  1.84880  1.88069  1.85067  1.50169  1.69803  1.56327  1.31155  1.52381
#> 130:     2406.3352:  1.89537  1.84890  1.88075  1.85138  1.50142  1.69777  1.56337  1.31247  1.52436
#> 131:     2406.3331:  1.89664  1.84937  1.88079  1.85178  1.50118  1.69762  1.56356  1.31347  1.52497
#> 132:     2406.3313:  1.89755  1.85012  1.88096  1.85169  1.50075  1.69779  1.56348  1.31447  1.52558
#> 133:     2406.3297:  1.89827  1.85028  1.88159  1.85164  1.50051  1.69797  1.56379  1.31544  1.52675
#> 134:     2406.3283:  1.89886  1.85035  1.88193  1.85210  1.50051  1.69806  1.56417  1.31636  1.52794
#> 135:     2406.3269:  1.89890  1.85125  1.88153  1.85252  1.50047  1.69846  1.56386  1.31698  1.52823
#> 136:     2406.3257:  1.89951  1.85106  1.88239  1.85301  1.50062  1.69861  1.56431  1.31777  1.52949
#> 137:     2406.3249:  1.90010  1.85082  1.88330  1.85350  1.50090  1.69875  1.56493  1.31855  1.53077
#> 138:     2406.3236:  1.90032  1.85148  1.88382  1.85389  1.50127  1.69937  1.56522  1.31940  1.53164
#> 139:     2406.3225:  1.90045  1.85224  1.88401  1.85449  1.50156  1.69995  1.56531  1.32017  1.53233
#> 140:     2406.3215:  1.90089  1.85220  1.88453  1.85532  1.50208  1.70017  1.56601  1.32096  1.53352
#> 141:     2406.3207:  1.90133  1.85234  1.88525  1.85587  1.50231  1.70049  1.56649  1.32182  1.53468
#> 142:     2406.3196:  1.90144  1.85291  1.88597  1.85602  1.50268  1.70129  1.56692  1.32267  1.53563
#> 143:     2406.3188:  1.90181  1.85342  1.88615  1.85685  1.50290  1.70153  1.56719  1.32347  1.53648
#> 144:     2406.3178:  1.90216  1.85358  1.88648  1.85776  1.50358  1.70185  1.56794  1.32427  1.53753
#> 145:     2406.3148:  1.90303  1.85505  1.89117  1.85916  1.50636  1.70593  1.57171  1.32915  1.54393
#> 146:     2406.3075:  1.90591  1.85974  1.89509  1.86788  1.51421  1.71293  1.58044  1.33958  1.55543
#> 147:     2406.3060:  1.90422  1.86154  1.89492  1.86777  1.51434  1.71675  1.58198  1.34261  1.55822
#> 148:     2406.3041:  1.90541  1.86110  1.89548  1.86996  1.51583  1.71825  1.58586  1.34631  1.56277
#> 149:     2406.3025:  1.90848  1.86366  1.89801  1.87059  1.51816  1.72033  1.58909  1.35030  1.56559
#> 150:     2406.3007:  1.90619  1.86223  1.90056  1.87464  1.52265  1.72422  1.59433  1.35608  1.57448
#> 151:     2406.2993:  1.90783  1.86719  1.90588  1.88064  1.52966  1.73039  1.59939  1.36256  1.58032
#> 152:     2406.2984:  1.91072  1.86963  1.90878  1.88500  1.53166  1.73632  1.60776  1.37042  1.58790
#> 153:     2406.2980:  1.91045  1.86881  1.90668  1.88218  1.53016  1.73500  1.60752  1.37193  1.59037
#> 154:     2406.2979:  1.91161  1.87018  1.90857  1.88441  1.53364  1.73715  1.60891  1.37253  1.59141
#> 155:     2406.2978:  1.91161  1.87029  1.90895  1.88450  1.53360  1.73765  1.60937  1.37275  1.59156
#> 156:     2406.2978:  1.91139  1.87041  1.90889  1.88450  1.53359  1.73720  1.60914  1.37333  1.59207
#> 157:     2406.2978:  1.91129  1.87065  1.90922  1.88470  1.53361  1.73758  1.60951  1.37383  1.59242
#> 158:     2406.2978:  1.91125  1.87030  1.90888  1.88416  1.53319  1.73730  1.60944  1.37375  1.59254
#> 159:     2406.2978:  1.91129  1.87034  1.90884  1.88428  1.53326  1.73725  1.60933  1.37358  1.59242
#> 160:     2406.2978:  1.91128  1.87036  1.90889  1.88431  1.53331  1.73731  1.60938  1.37365  1.59242
myVAST
#> fit_model(.) result
#> $par
#>       beta1_ft       beta1_ft       beta1_ft       beta1_ft       beta1_ft       beta1_ft       beta
#>      1.9112804      1.8703625      1.9088857      1.8843138      1.5333074      1.7373072      1.609
#>       beta1_ft   L_epsilon1_z      logkappa1 Epsilon_rho1_f      logSigmaM
#>      1.9129480     -0.4256334     -4.5141119      0.9396206     -1.1819730
#>
#> $objective
#> [1] 2406.298
#>
#> $iterations
#> [1] 5
#>
#> $evaluations
```

```
#> function gradient
#>        9        6
#>
#> $time_for_MLE
#> Time difference of 0.8580542 secs
#>
#> $max_gradient
#> [1] 0.0009568968
#>
#> $Convergence_check
#> [1] "The model is likely not converged"
#>
#> $number_of_coefficients
#>  Total  Fixed Random
#>   1374     14   1360
#>
#> $AIC
#> [1] 4840.596
#>
#> $diagnostics
#>              Param starting_value     Lower        MLE     Upper final_gradient
#> 1        beta1_ft      1.9112762      -Inf  1.9112804       Inf  -1.601250e-04
#> 2        beta1_ft      1.8703611      -Inf  1.8703625       Inf  -1.014894e-04
#> 3        beta1_ft      1.9088872      -Inf  1.9088857       Inf   2.265181e-05
#> 4        beta1_ft      1.8843134      -Inf  1.8843138       Inf   2.843921e-05
#> 5        beta1_ft      1.5333092      -Inf  1.5333074       Inf   9.096030e-05
#> 6        beta1_ft      1.7373075      -Inf  1.7373072       Inf   8.865338e-05
#> 7        beta1_ft      1.6093756      -Inf  1.6093746       Inf   1.162486e-04
#> 8        beta1_ft      1.3736500      -Inf  1.3736440       Inf   1.717689e-04
#> 9        beta1_ft      1.5924244      -Inf  1.5924303       Inf  -1.646446e-04
#> 10       beta1_ft      1.9129472      -Inf  1.9129480       Inf  -1.281386e-04
#> 11   L_epsilon1_z     -0.4256342      -Inf -0.4256334       Inf   5.249151e-04
#> 12      logkappa1     -4.5141158 -6.214608 -4.5141119 -3.565449  -1.934040e-04
#> 13 Epsilon_rho1_f      0.9396210 -0.990000  0.9396206  0.990000  -9.568968e-04
#> 14      logSigmaM     -1.1819727      -Inf -1.1819730 10.000000  -4.259654e-05
#>
#> $SD
#> sdreport(.) result
#>                 Estimate Std. Error
#> beta1_ft        1.9112804 0.12533621
#> beta1_ft        1.8703625 0.16951598
#> beta1_ft        1.9088857 0.20052381
#> beta1_ft        1.8843138 0.22442668
#> beta1_ft        1.5333074 0.24390978
#> beta1_ft        1.7373072 0.25938468
#> beta1_ft        1.6093746 0.27247405
#> beta1_ft        1.3736440 0.28383066
#> beta1_ft        1.5924303 0.29303093
#> beta1_ft        1.9129480 0.30089411
#> L_epsilon1_z   -0.4256334 0.02168763
#> logkappa1      -4.5141119 0.10217800
#> Epsilon_rho1_f  0.9396206 0.02503805
#> logSigmaM      -1.1819730 0.03921188
```

```
#> Maximum gradient component: 0.0009568968
#>
#> $time_for_sdreport
#> Time difference of 3.663825 secs
#>
#> $time_for_run
#> Time difference of 34.44474 secs
```

Or with sdmTMB

```
library(INLA)
library(sdmTMB)
#> Warning: package 'sdmTMB' was built under R version 4.3.1
mesh = make_mesh(Data, c("x","y"), n_knots=n_x*n_y )

start_time = Sys.time()
mysdmTMB = sdmTMB(
  formula = n ~ factor(time),
  data = Data,
  mesh = mesh,
  spatial = "off",
  spatiotemporal = "ar1",
  time = "time"
)
Sys.time() - start_time
#> Time difference of 8.257607 secs
```

Bivariate spatio-temporal autoregressive model

We next highlight how to specify a bivariate spatio-temporal model with a cross-laggged (vector autoregressive) interaction.

```
# Simulate settings
theta_xy = 0.2
n_x = n_y = 10
n_t = 20
B = rbind( c( 0.5, -0.25),
           c(-0.1,  0.50) )

# Simulate GMRFs
R = exp(-theta_xy * abs(outer(1:n_x, 1:n_y, FUN="-")) )
d1 = mvtnorm::rmvnorm(n_t, sigma=0.2*kronecker(R,R) )
d2 = mvtnorm::rmvnorm(n_t, sigma=0.2*kronecker(R,R) )
d = abind::abind( d1, d2, along=3 )

# Project through time and add mean
for( t in seq_len(n_t) ){
  if(t>1) d[t,,] = t(B%*%t(d[t-1,,])) + d[t,,]
}
d[,,1] = d[,,1] + 2
d[,,2] = d[,,2] + 3
```

```r
# Shape into longform data-frame and add error
Data = data.frame( expand.grid(time=1:n_t, x=1:n_x, y=1:n_y, "var"=c("d1","d2")), z=as.vector(d))
Data$n = Data$z + rnorm(nrow(Data), sd=0.2)

# make mesh
mesh = fm_mesh_2d( Data[,c('x','y')] )

# Define sem
sem = "
  d1 -> d1, 1, b11
  d2 -> d2, 1, b22
  d2 -> d1, 1, b21
  d1 -> d2, 1, b12
  d1 <-> d1, 0, var1
  d2 <-> d2, 0, var1
"

# fit model
out = fit( sem = sem,
           data = Data,
           formula = n ~ 0 + var,
           spatial_graph = mesh,
           quiet = TRUE )
#> Warning in Ops.factor(variables, 0): '>' not meaningful for factors
out
#> $call
#> fit(data = Data, formula = n ~ 0 + var, sem = sem, spatial_graph = mesh,
#>     quiet = TRUE)
#>
#> $opt
#> $opt$par
#>   log_kappa       alpha       alpha      beta_z      beta_z      beta_z      beta_z      beta_z   lo
#> -0.37119583  1.93011294  2.92522186  0.40288545  0.41412626 -0.19531475 -0.08333714  0.34839338 -1.7
#>
#> $opt$objective
#> [1] 1232.655
#>
#> $opt$convergence
#> [1] 0
#>
#> $opt$iterations
#> [1] 54
#>
#> $opt$evaluations
#> function gradient
#>       76       55
#>
#> $opt$message
#> [1] "relative convergence (4)"
#>
#>
#> $sdrep
#> sdreport(.) result
```

```
#>              Estimate  Std. Error
#> log_kappa -0.37119583 0.046417958
#> alpha      1.93011294 0.067580368
#> alpha      2.92522186 0.066140302
#> beta_z     0.40288545 0.031927107
#> beta_z     0.41412626 0.031447063
#> beta_z    -0.19531475 0.031078105
#> beta_z    -0.08333714 0.029533568
#> beta_z     0.34839338 0.009009959
#> log_sigma -1.72787247 0.040607051
#> Maximum gradient component: 0.01482948
#>
#> $run_time
#> Time difference of 2.021682 mins
```

The values for `beta_z` again correspond to the specified value for interaction-matrix `B`