# Dynamic structural equation model

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```r
library(tinyVAST)
set.seed(101)
```

tinyVAST includes features to fit a dynamic structural equation model

```r
data(isle_royale, package="dsem")

# Convert to long-form
data = expand.grid( "time"=isle_royale[,1], "var"=colnames(isle_royale[,2:3]) )
data$logn = unlist(log(isle_royale[2:3]))

# Define cross-lagged DSEM
dsem = "
  # Link, lag, param_name
  wolves -> wolves, 1, arW
  moose -> wolves, 1, MtoW
  wolves -> moose, 1, WtoM
  moose -> moose, 1, arM
  wolves -> moose, 0, corr
"

# fit model
mytiny = fit( dsem = dsem,
              data = data,
              times = isle_royale[,1],
              variables = colnames(isle_royale[,2:3]),
              estimate_delta0 = FALSE,
              formula = logn ~ 0 + var,
              quiet = TRUE )
#>    0:    391.97944:  0.00000   0.00000 0.0100000 0.0100000 0.0100000 0.0100000 0.0100000  1.00000  1.
#>    1:    293.47883: 0.0432878 0.108614 0.137076 0.295137 0.299864 0.700110 0.304606 0.920827  1.4844
#>    2:    272.35246: 0.0702137 0.104050 0.249882 0.504281 0.312730 0.752622 0.324146 0.124328 0.94800
#>    3:    267.11302: 0.0331173 0.130867 0.0531818 0.164741 0.302841 0.735444 0.313854 0.237609 0.9178
#>    4:    222.17865: 0.0589472 0.127187 0.106932 0.347293 0.304470 0.742269 0.315607 0.311164 0.89946
#>    5:    208.04762: 0.0717159 0.124302 0.148031 0.408069 0.302962 0.740648 0.315093 0.163017 0.78815
#>    6:    194.08329: 0.0769018 0.143464 0.104822 0.324971 0.309062 0.758708 0.321504 0.138585 0.61835
#>    7:    177.09291: 0.101268 0.155892 0.125659 0.368199 0.322378 0.788853 0.334868 0.128494 0.435963
#>    8:    165.12572: 0.135840 0.177628 0.141177 0.338710 0.292867 0.713473 0.307171 0.112514 0.286754
#>    9:    161.67079: 0.138094 0.178119 0.152983 0.363757 0.301670 0.736209 0.316790 0.117254 0.281971
#>   10:    160.40593: 0.137281 0.181960 0.147957 0.337955 0.302699 0.741241 0.320134 0.122707 0.258864
#>   11:    158.99202: 0.142354 0.185057 0.163540 0.353935 0.304947 0.751270 0.326251 0.128201 0.239372
#>   12:    157.39856: 0.146652 0.190642 0.170507 0.338697 0.297404 0.741771 0.325368 0.134777 0.229954
#>   13:    148.34749: 0.218344 0.283264 0.404687 0.271365 0.202863 0.732150 0.395346 0.242010 0.192628
#>   14:    146.52996: 0.208329 0.292242 0.391447 0.224459 0.216367 0.799418 0.419694 0.190607 0.233675
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#>   15:    140.08045: 0.225738 0.313954 0.417170 0.247747 0.147623 0.764066 0.394473 0.180619 0.278054
#>   16:    134.20758: 0.240366 0.343996 0.446285 0.214776 0.0948538 0.799166 0.395334 0.169305 0.22237
#>   17:    131.16629: 0.250863 0.371955 0.498005 0.219067 0.0393347 0.828875 0.396977 0.175618 0.20655
#>   18:    127.66669: 0.263105 0.399929 0.533055 0.183490 -0.0190056 0.852045 0.399414 0.182196 0.2059
#>   19:    124.11699: 0.277800 0.446152 0.562137 0.190934 -0.0947508 0.873181 0.403304 0.155230 0.1954
#>   20:    118.40042: 0.312852 0.530145 0.627937 0.152709 -0.219049 0.927603 0.439546 0.168529 0.195308
#>   21:    117.92783: 0.338796 0.654110 0.677227 0.123690 -0.344532 0.906275 0.510641 0.159569 0.145858
#>   22:    110.15216: 0.356648 0.717696 0.700543 0.128534 -0.372120 0.930679 0.575301 0.146911 0.17024
#>   23:    107.54614: 0.375368 0.787620 0.717874 0.118302 -0.419856 0.911938 0.623342 0.139189 0.17066
#>   24:    103.19533: 0.424791 0.935402 0.780524 0.105800 -0.534974 0.922482 0.722012 0.158522 0.15011
#>   25:    101.68558: 0.492288  1.12347 0.747017 0.0785286 -0.657683 0.959822 0.810607 0.132680 0.1421
#>   26:    94.244117: 0.517084  1.34150 0.752566 0.119320 -0.785405 0.985749 0.831330 0.129256 0.13787
#>   27:    90.226592: 0.543544  1.60413 0.761883 0.101762 -0.802545 0.967723 0.891040 0.141854 0.125083
#>   28:    86.796561: 0.545694  1.87058 0.834900 0.0765051 -0.795957 0.969397 0.872722 0.120890 0.1364
#>   29:    83.245193: 0.598193  2.12200 0.841094 0.0773191 -0.830677  1.00824 0.820285 0.130225 0.1171
#>   30:    80.566600: 0.669509  2.36722 0.825623 0.0860949 -0.788875  1.00097 0.830899 0.131947 0.1184
#>   31:    76.468880: 0.717368  2.59846 0.800943 0.0967272 -0.722303 0.993675 0.741238 0.129476 0.1206
#>   32:    31.240246:  1.63782  5.40279 0.996746 0.0483751 -0.0279226 0.969621 0.0417951 0.116574 0.05
#>   33:    25.233335:  1.82195  5.96514  1.00764 0.0439629 0.0980274 0.984701 -0.0864806 0.113885 0.06
#>   34:    22.925166:  1.87341  6.12170  1.00913 0.0437779 0.128359 0.985682 -0.122445 0.113411 0.0458
#>   35:    21.536129:  1.92728  6.28088  1.01207 0.0444219 0.150916 0.987403 -0.151129 0.111700 0.0589
#>   36:    18.135935:  1.90721  6.18436 0.993389 0.0516422 0.0477960 0.990004 -0.0487407 0.109509 0.05
#>   37:    17.964235:  1.90764  6.18469 0.991105 0.0517743 0.0465553 0.990836 -0.0489936 0.111559 0.04
#>   38:    17.863867:  1.90810  6.18506 0.988817 0.0519411 0.0453085 0.991889 -0.0491077 0.112349 0.05
#>   39:    17.704288:  1.91086  6.19205 0.987425 0.0521669 0.0453187 0.992522 -0.0498778 0.112357 0.050
#>   40:    17.531223:  1.91661  6.20682 0.985240 0.0526118 0.0456340 0.993693 -0.0514661 0.112206 0.05
#>   41:    17.292444:  1.92793  6.23688 0.984374 0.0532120 0.0481722 0.994205 -0.0548554 0.111616 0.04
#>   42:    17.023175:  1.95152  6.29653 0.983620 0.0545119 0.0540375 0.994422 -0.0615471 0.110578 0.05
#>   43:    16.531427:  2.02525  6.39041 0.969635 0.0632538 0.0707887 0.988609 -0.0686677 0.109142 0.04
#>   44:    15.579062:  2.05907  6.26549 0.941086 0.0706254 0.0586445 0.989815 -0.0554868 0.112170 0.05
#>   45:    14.244291:  2.18658  6.26208 0.941913 0.0652179 0.0425333  1.00828 -0.0776921 0.104598 0.04
#>   46:    12.385984:  2.42401  6.09663 0.915920 0.0631887 0.0187646  1.01631 -0.0555112 0.104999 0.04
#>   47:    12.303620:  2.42423  6.09688 0.915371 0.0633145 0.0180130  1.01644 -0.0557555 0.105042 0.05
#>   48:    12.244880:  2.42688  6.09735 0.914884 0.0633578 0.0173414  1.01661 -0.0559352 0.105023 0.050
#>   49:    12.166238:  2.43217  6.09860 0.913194 0.0636205 0.0151437  1.01714 -0.0564914 0.105004 0.05
#>   50:    10.602009:  2.77919  6.13695 0.917823 0.0525166 0.00618072  1.02615 -0.0588759 0.0991222 0.0
#>   51:    9.7728681:  2.80148  6.25968 0.886404 0.0765020 -0.0324231  1.02386 -0.0241585 0.110542 0.0
#>   52:    9.2175357:  2.82142  6.37040 0.857442 0.0986281 -0.00358397  1.01073 -0.0460465 0.105609 0.0
#>   53:    8.9799768:  2.91923  6.38492 0.891825 0.0758988 -0.0797138  1.00350 0.0117300 0.0998864 0.03
#>   54:    8.9111399:  3.05367  6.32413 0.872155 0.0763884 -0.0663305  1.01025 -0.0331932 0.100953 0.08
#>   55:    7.8821121:  3.04259  6.39419 0.882485 0.0726891 -0.0484305 0.990200 -0.0493881 0.101782 0.0
#>   56:    7.2091894:  3.02524  6.36991 0.889071 0.0689770 -0.0604855 0.956270 -0.0348989 0.106260 0.0
#>   57:    6.5691096:  3.25947  6.46905 0.928805 0.0387380 -0.0808593 0.912876 -0.0258049 0.108132 0.0
#>   58:    6.4660455:  3.38437  6.49690 0.897624 0.0438598 -0.131594 0.859130 0.00178143 0.102779 0.04
#>   59:    6.0100656:  3.28029  6.40519 0.891682 0.0393210 -0.115249 0.874988 -0.0140715 0.104396 0.04
#>   60:    5.9691877:  3.33959  6.44124 0.912478 0.0245097 -0.110049 0.868105 -0.0258715 0.107989 0.04
#>   61:    5.8638039:  3.32800  6.43937 0.902296 0.0286487 -0.115492 0.866112 -0.0162057 0.106189 0.04
#>   62:    5.8533019:  3.32976  6.44091 0.899618 0.0265486 -0.117717 0.863356 -0.0154431 0.105994 0.04
#>   63:    5.8370056:  3.32787  6.44472 0.895029 0.0206084 -0.119828 0.860225 -0.0148022 0.105933 0.04
#>   64:    5.8252995:  3.32271  6.44654 0.891254 0.0147648 -0.119909 0.859887 -0.0150141 0.106006 0.04
#>   65:    5.8150381:  3.31758  6.44519 0.887778 0.00735763 -0.118609 0.861661 -0.0157496 0.106180 0.0
#>   66:    5.8100662:  3.31828  6.44076 0.886687 0.00384752 -0.117395 0.863794 -0.0162586 0.106306 0.0
#>   67:    5.8057883:  3.32266  6.43354 0.886856 0.00181174 -0.116424 0.865862 -0.0166882 0.106408 0.0
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#>  68:     5.8024493:   3.32867   6.42732 0.887607 0.00182408 -0.116076 0.866965 -0.0167794 0.106467 0.0
#>  69:     5.7993972:   3.33330   6.42375 0.889068 0.00316629 -0.116237 0.866906 -0.0167492 0.106488 0.0
#>  70:     5.7943944:   3.33802   6.42369 0.891133 0.00717801 -0.117155 0.865590 -0.0161952 0.106502 0.0
#>  71:     5.7900941:   3.33766   6.42739 0.893455 0.0109708 -0.117899 0.863834 -0.0159520 0.106501 0.04
#>  72:     5.7858240:   3.33184   6.43810 0.894578 0.0160509 -0.119027 0.861469 -0.0151691 0.106503 0.04
#>  73:     5.7844424:   3.32566   6.44341 0.894845 0.0170911 -0.118886 0.860921 -0.0153334 0.106479 0.04
#>  74:     5.7838579:   3.32283   6.44524 0.893977 0.0164838 -0.118851 0.861025 -0.0152520 0.106474 0.04
#>  75:     5.7833868:   3.32183   6.44511 0.893163 0.0152845 -0.118634 0.861393 -0.0153439 0.106475 0.04
#>  76:     5.7828440:   3.32239   6.44348 0.892416 0.0136958 -0.118496 0.861850 -0.0154171 0.106481 0.04
#>  77:     5.7824848:   3.32377   6.44211 0.892320 0.0132257 -0.118456 0.861984 -0.0154768 0.106487 0.04
#>  78:     5.7821805:   3.32533   6.44112 0.892692 0.0135189 -0.118596 0.861874 -0.0154256 0.106489 0.04
#>  79:     5.7820566:   3.32581   6.44117 0.893024 0.0140744 -0.118646 0.861715 -0.0154195 0.106489 0.04
#>  80:     5.7819972:   3.32571   6.44145 0.893196 0.0143785 -0.118710 0.861632 -0.0153694 0.106488 0.04
#>  81:     5.7819626:   3.32546   6.44167 0.893188 0.0144455 -0.118673 0.861630 -0.0153922 0.106488 0.04
#>  82:     5.7819423:   3.32527   6.44173 0.893127 0.0143480 -0.118674 0.861662 -0.0153788 0.106488 0.04
#>  83:     5.7819315:   3.32520   6.44172 0.893052 0.0142445 -0.118640 0.861693 -0.0154024 0.106488 0.04
#>  84:     5.7819259:   3.32520   6.44168 0.893020 0.0141751 -0.118647 0.861708 -0.0153948 0.106488 0.04
#>  85:     5.7819228:   3.32522   6.44165 0.893012 0.0141639 -0.118640 0.861708 -0.0154028 0.106488 0.04
#>  86:     5.7819210:   3.32525   6.44164 0.893025 0.0141777 -0.118649 0.861702 -0.0153965 0.106488 0.04
#>  87:     5.7819198:   3.32527   6.44165 0.893040 0.0142054 -0.118649 0.861695 -0.0153985 0.106488 0.04
#>  88:     5.7819193:   3.32527   6.44165 0.893049 0.0142175 -0.118653 0.861692 -0.0153949 0.106488 0.04
#>  89:     5.7819191:   3.32527   6.44166 0.893049 0.0142193 -0.118651 0.861692 -0.0153967 0.106488 0.04
#>  90:     5.7819189:   3.32526   6.44166 0.893046 0.0142142 -0.118651 0.861694 -0.0153960 0.106488 0.04
#>  91:     5.7819189:   3.32526   6.44165 0.893043 0.0142096 -0.118650 0.861695 -0.0153970 0.106488 0.04
#>  92:     5.7819188:   3.32526   6.44165 0.893041 0.0142071 -0.118650 0.861696 -0.0153967 0.106488 0.04
#>  93:     5.7819188:   3.32526   6.44165 0.893041 0.0142071 -0.118650 0.861695 -0.0153969 0.106488 0.04
#>  94:     5.7819188:   3.32526   6.44165 0.893042 0.0142081 -0.118650 0.861695 -0.0153967 0.106488 0.04
#>  95:     5.7819188:   3.32526   6.44165 0.893043 0.0142091 -0.118650 0.861695 -0.0153967 0.106488 0.04
#>  96:     5.7819188:   3.32526   6.44165 0.893043 0.0142096 -0.118650 0.861695 -0.0153966 0.106488 0.04
#>  97:     5.7819188:   3.32526   6.44165 0.893043 0.0142095 -0.118650 0.861695 -0.0153967 0.106488 0.04
#>  98:     5.7819188:   3.32526   6.44165 0.893043 0.0142093 -0.118650 0.861695 -0.0153966 0.106488 0.04
mytiny
#> $call
#> fit(data = data, formula = logn ~ 0 + var, dsem = dsem, estimate_delta0 = FALSE,
#>     times = isle_royale[, 1], variables = colnames(isle_royale[,
#>         2:3]), quiet = TRUE)
#>
#> $opt
#> $opt$par
#>      alpha_j      alpha_j       beta_z       beta_z       beta_z       beta_z       beta_z        bet
#>   3.32526215   6.44165380   0.89304284   0.01420930  -0.11865012   0.86169491  -0.01539665   0.106488
#>
#> $opt$objective
#> [1] 5.781919
#>
#> $opt$convergence
#> [1] 0
#>
#> $opt$iterations
#> [1] 98
#>
#> $opt$evaluations
#> function gradient
```

```
#>      125      99
#>
#> $opt$message
#> [1] "relative convergence (4)"
#>
#>
#> $sdrep
#> sdreport(.) result
#>                Estimate    Std. Error
#> alpha_j      3.32526215 2.483419e-01
#> alpha_j      6.44165380 2.116017e-01
#> beta_z       0.89304284 8.420604e-02
#> beta_z       0.01420930 1.279148e-01
#> beta_z      -0.11865012 6.477637e-02
#> beta_z       0.86169491 7.080254e-02
#> beta_z      -0.01539665 6.067738e-02
#> beta_z       0.10648808 9.881848e-03
#> beta_z       0.04810383 4.460377e-03
#> log_sigma  -12.52406863 1.829804e+04
#> Maximum gradient component: 1.288584e-05
#>
#> $run_time
#> Time difference of 0.269253 secs
```

We can then compare this with package dsem

```
library(dsem)

# Keep in wide-form
dsem_data = ts( log(isle_royale[,2:3]), start=1959)
family = c("normal","normal")

# initial first without delta0 (to improve starting values)
mydsem = dsem::dsem( sem = dsem,
            tsdata = dsem_data,
            estimate_delta0 = FALSE,
            quiet = TRUE,
            getsd = FALSE,
            family = family )
mydsem
#> $par
#>       beta_z        beta_z        beta_z        beta_z        beta_z        beta_z        beta_z
#>   0.895834720   0.007358847  -0.109332511   0.875012562  -0.017355229   0.378795847  -0.172873038  -1.
#>
#> $objective
#> [1] 7.739638
#>
#> $iterations
#> [1] 79
#>
#> $evaluations
#> function gradient
#>       96       80
```

```
#> 
#> $time_for_MLE
#> Time difference of 0.07714605 secs
#> 
#> $max_gradient
#> [1] 7.714655e-07
#> 
#> $Convergence_check
#> [1] "There is no evidence that the model is not converged"
#> 
#> $number_of_coefficients
#>   Total  Fixed Random
#>     133      9    124
#> 
#> $AIC
#> [1] 33.47928
#> 
#> $diagnostics
#>        Param starting_value Lower          MLE Upper final_gradient
#> 1     beta_z           0.01  -Inf   0.895834720   Inf   4.785205e-09
#> 2     beta_z           0.01  -Inf   0.007358847   Inf  -5.078683e-09
#> 3     beta_z           0.01  -Inf  -0.109332511   Inf  -2.031211e-08
#> 4     beta_z           0.01  -Inf   0.875012562   Inf  -5.821149e-08
#> 5     beta_z           0.01  -Inf  -0.017355229   Inf  -5.373382e-09
#> 6     beta_z           1.00  -Inf   0.378795847   Inf   2.119351e-09
#> 7     beta_z           1.00  -Inf  -0.172873038   Inf  -7.714655e-07
#> 8 lnsigma_j            0.00  -Inf -15.799262455   Inf   1.628788e-12
#> 9 lnsigma_j            0.00  -Inf -11.977331517   Inf   2.141499e-09
#> 
#> $time_for_run
#> Time difference of 0.07876801 secs
```