Dynamic structural equation models

James T. Thorson

```
library(tinyVAST)
set.seed(101)
options("tinyVAST.verbose" = FALSE)
```

`tinyVAST` includes features to fit a dynamic structural equation model. We here show this using a bivariate vector autoregressive model for wolf and moose abundance on Isle Royale.

```
data(isle_royale, package="dsem")

# Convert to long-form
data = expand.grid( "time"=isle_royale[,1], "var"=colnames(isle_royale[,2:3]) )
data$logn = unlist(log(isle_royale[2:3]))

# Define cross-lagged DSEM
dsem = "
  # Link, lag, param_name
  wolves -> wolves, 1, arW
  moose -> wolves, 1, MtoW
  wolves -> moose, 1, WtoM
  moose -> moose, 1, arM
  wolves -> moose, 0, corr
"

# fit model
mytiny = tinyVAST( dsem = dsem,
                   data = data,
                   times = isle_royale[,1],
                   variables = colnames(isle_royale[,2:3]),
                   formula = logn ~ 0 + var )
#> Warning in nlminb(start = opt$par, objective = obj$fn, gradient = obj$gr, : NA/NaN function evaluati
mytiny
#> $call
#> tinyVAST(formula = logn ~ 0 + var, data = data, dsem = dsem,
#>     times = isle_royale[, 1], variables = colnames(isle_royale[,
#>         2:3]))
#>
#> $opt
#> $opt$par
#>      alpha_j      alpha_j       beta_z       beta_z       beta_z       beta_z       beta_z       bet
#>   3.32526212   6.44165421   0.89304301   0.01420970  -0.11865018   0.86169482  -0.01539658   0.37749
#>
#> $opt$objective
#> [1] 5.781919
#>
```

```
#> $opt$convergence
#> [1] 0
#>
#> $opt$iterations
#> [1] 93
#>
#> $opt$evaluations
#> function gradient
#>      116       94
#>
#> $opt$message
#> [1] "relative convergence (4)"
#>
#>
#> $sdrep
#> sdreport(.) result
#>               Estimate   Std. Error
#> alpha_j     3.32526212 2.483496e-01
#> alpha_j     6.44165421 2.116034e-01
#> beta_z      0.89304301 8.420632e-02
#> beta_z      0.01420970 1.279152e-01
#> beta_z     -0.11865018 6.477638e-02
#> beta_z      0.86169482 7.080269e-02
#> beta_z     -0.01539658 6.067738e-02
#> beta_z      0.37749042 3.503686e-02
#> beta_z      0.17052360 1.582618e-02
#> log_sigma -12.56122233 1.909999e+04
#> Maximum gradient component: 9.935109e-05
#>
#> $run_time
#> Time difference of 0.3094311 secs
```

And we can specifically inspect the estimated interaction matrix:

	wolves	moose
wolves	0.893	-0.119
moose	0.014	0.862

We can then compare this with package `dsem`

```
library(dsem)

# Keep in wide-form
dsem_data = ts( log(isle_royale[,2:3]), start=1959)
family = c("normal","normal")

# initial first without delta0 (to improve starting values)
mydsem = dsem::dsem( sem = dsem,
             tsdata = dsem_data,
             getsd = FALSE,
             family = family )
#>   Coefficient_name Number_of_coefficients   Type
```

```
#> 1           beta_z                     7  Fixed
#> 2       lnsigma_j                      2  Fixed
#> 3            mu_j                      2 Random
#> 4            x_tj                    122 Random
#>   0:    165.31877: 0.0100000 0.0100000 0.0100000 0.0100000 0.0100000  1.00000  1.00000  0.00000  0.
#>   1:    112.33035: 0.129461 -0.0100018 -0.0340476 0.0574367 -0.0233558 0.549854 0.464639 -0.451120
#>   2:    85.669125: 0.318452 -0.0454305 -0.116662 0.133698 -0.0865613 0.357200 0.0120025 -0.662996 -
#>   3:    85.307369: 0.701552 -0.125636 -0.398766 0.204560 -0.318556 0.689880 0.0121052 -0.841226 -1.
#>   4:    66.438071: 0.698006 -0.135288 -0.244615 0.135383 -0.139501 0.468339 0.0110866 -0.827798 -1.
#>   5:    62.293163: 0.778498 -0.169673 -0.305856 0.202248 -0.141339 0.190316 0.0128312 -0.953311 -1.
#>   6:    61.911130:  1.00275 -0.243283 -0.341661 0.283020 -0.126047 0.342016 -0.133617 -0.947107 -1.
#>   7:    54.312553: 0.897268 -0.180482 -0.285442 0.284763 -0.0499491 0.315894 -0.100907 -0.992907 -1
#>   8:    50.901932: 0.896714 -0.163882 -0.328025 0.376529 -0.0528561 0.251628 -0.206529 -1.04007 -1.
#>   9:    45.631636: 0.910370 -0.119261 -0.306726 0.455182 -0.00246740 0.285953 -0.134579 -1.12118 -1
#>  10:    41.410364: 0.924938 -0.0874709 -0.265096 0.541670 0.0604321 0.288357 -0.236770 -1.14234 -1.
#>  11:    34.474412: 0.940385 -0.0446181 -0.270721 0.630373 0.0667309 0.283989 -0.138549 -1.20299 -1.
#>  12:    31.285695: 0.995733 0.00402205 -0.323513 0.867515 0.0526206 0.247826 -0.185457 -1.37155 -1.
#>  13:    24.660710: 0.976833 0.0222760 -0.236833 0.837048 0.131732 0.253163 -0.152866 -1.37362 -1.70
#>  14:    22.596533: 0.988173 0.0641420 -0.261280 0.894338 0.0891838 0.292494 -0.133802 -1.39091 -1.7
#>  15:    20.054733:  1.00157 0.0852109 -0.200443 0.905897 0.108574 0.290203 -0.148714 -1.42578 -1.89
#>  16:    19.006945: 0.985125 0.0588261 -0.200414 0.818967 0.0459431 0.262495 -0.112928 -1.45661 -1.9
#>  17:    18.538513: 0.989469 0.0601250 -0.192612 0.834540 0.0480598 0.275136 -0.168131 -1.45833 -1.9
#>  18:    16.705161: 0.991200 0.0630879 -0.184299 0.855909 0.0390567 0.279812 -0.137396 -1.47558 -1.9
#>  19:    16.432569: 0.988740 0.0642804 -0.171069 0.856440 0.0438731 0.284289 -0.133181 -1.47930 -2.0
#>  20:    16.210216: 0.987846 0.0653563 -0.175606 0.862110 0.0309054 0.288148 -0.139017 -1.48320 -2.0
#>  21:    15.567351: 0.987501 0.0653313 -0.155568 0.862639 0.0295660 0.289885 -0.132406 -1.49494 -2.0
#>  22:    14.482815: 0.991419 0.0632132 -0.142092 0.871917 -0.000730430 0.293296 -0.139951 -1.52254 -
#>  23:    13.043707: 0.964947 0.0489707 -0.0903498 0.862248 -0.0292181 0.328138 -0.128538 -1.57566 -2
#>  24:    12.671059: 0.991744 0.0157822 -0.0912287 0.944517 -0.00572376 0.303002 -0.143741 -1.67040 -
#>  25:    11.757880: 0.956584 0.0334033 -0.102907 0.912554 -0.0343353 0.317914 -0.149452 -1.67433 -2.
#>  26:    11.478590: 0.953043 0.0338680 -0.0870746 0.903511 -0.0214264 0.317087 -0.140338 -1.67661 -2
#>  27:    11.267569: 0.955132 0.0313884 -0.0972948 0.895857 -0.0282228 0.316102 -0.144821 -1.69367 -2
#>  28:    11.139343: 0.952142 0.0309275 -0.0966821 0.890300 -0.0222160 0.321394 -0.141618 -1.71280 -2
#>  29:    10.965962: 0.947761 0.0317989 -0.110679 0.893537 -0.0141059 0.336069 -0.147483 -1.75304 -2.
#>  30:    10.718902: 0.954170 0.0305499 -0.108192 0.877771 -0.0212019 0.325878 -0.144096 -1.79190 -2.
#>  31:    10.670802: 0.951043 0.0318156 -0.103643 0.879523 -0.0192416 0.328606 -0.148620 -1.79401 -2.
#>  32:    10.622457: 0.948401 0.0322213 -0.104131 0.882543 -0.0235029 0.330187 -0.143278 -1.79620 -2.
#>  33:    10.515389: 0.941865 0.0270053 -0.0950321 0.886853 -0.0273500 0.333119 -0.147991 -1.80312 -2
#>  34:    9.8447364: 0.956296 0.00708773 -0.157635 0.862217 0.0301384 0.348981 -0.159733 -1.98655 -2.
#>  35:    9.2725608: 0.959064 0.0456067 -0.0810065 0.890599 -0.0300462 0.377215 -0.146430 -2.20747 -2
#>  36:    9.2637046: 0.943309 0.0326776 -0.0728563 0.895722 -0.0649503 0.369145 -0.173980 -2.23393 -2
#>  37:    8.6733728: 0.923792 0.0125871 -0.0740055 0.895119 -0.0503543 0.364927 -0.160897 -2.22080 -2
#>  38:    8.6474541: 0.917424 0.0285134 -0.0846766 0.890027 -0.0263341 0.355137 -0.160795 -2.20883 -2
#>  39:    8.5523724: 0.897160 0.00368012 -0.0976490 0.893009 -0.0284047 0.358842 -0.162673 -2.21956 -
#>  40:    8.5026763: 0.903791 0.0124612 -0.0940695 0.889858 -0.0306526 0.360592 -0.159439 -2.25677 -2
#>  41:    8.4673789: 0.899871 0.000379081 -0.0922837 0.888264 -0.0289681 0.362893 -0.161847 -2.29368
#>  42:    8.4276995: 0.910125 0.0179802 -0.0954518 0.886807 -0.0295968 0.362855 -0.163899 -2.32371 -2
#>  43:    8.3774166: 0.907003 0.0158851 -0.0957090 0.886929 -0.0294851 0.362471 -0.160052 -2.35525 -3
#>  44:    8.2517913: 0.894444 0.0196119 -0.102589 0.886285 -0.0238675 0.369818 -0.162553 -2.48249 -3.
#>  45:    8.1417047: 0.890241 0.00809558 -0.0962901 0.887871 -0.0307037 0.363810 -0.167356 -2.59958 -
#>  46:    8.0297357: 0.899186 0.00492759 -0.0990372 0.885101 -0.0252363 0.371720 -0.168080 -2.74789 -
#>  47:    7.8887318: 0.899877 0.0122204 -0.115611 0.877326 -0.00818792 0.375758 -0.168825 -3.11090 -3
#>  48:    7.8560062: 0.895740 0.00172195 -0.111814 0.875747 -0.0167326 0.381283 -0.168450 -3.27775 -3
```

```
#>  49:     7.8203927: 0.895502 0.00417037 -0.107001 0.877540 -0.0192068 0.380513 -0.169697 -3.41700 -3
#>  50:     7.7805608: 0.898054 0.00720056 -0.108188 0.878166 -0.0184696 0.377057 -0.171573 -3.73591 -3
#>  51:     7.7625544: 0.894914 0.00752110 -0.107634 0.875319 -0.0182304 0.378382 -0.172185 -4.06224 -4
#>  52:     7.7516648: 0.895507 0.00692828 -0.108193 0.876279 -0.0178507 0.378778 -0.172056 -4.38551 -4
#>  53:     7.7465960: 0.895706 0.00715436 -0.109348 0.875415 -0.0173041 0.378938 -0.172603 -4.70713 -4
#>  54:     7.7435380: 0.896065 0.00733544 -0.109208 0.875009 -0.0174847 0.378686 -0.172518 -5.03033 -4
#>  55:     7.7419356: 0.895756 0.00721843 -0.109230 0.875279 -0.0173935 0.378835 -0.172771 -5.35189 -4
#>  56:     7.7409834: 0.895818 0.00733075 -0.109243 0.875119 -0.0174058 0.378784 -0.172781 -5.67464 -5
#>  57:     7.7403244: 0.895810 0.00733114 -0.109290 0.875085 -0.0173747 0.378799 -0.172835 -6.10059 -5
#>  58:     7.7400049: 0.895829 0.00734739 -0.109309 0.875045 -0.0173676 0.378795 -0.172847 -6.50614 -5
#>  59:     7.7398377: 0.895831 0.00735152 -0.109322 0.875030 -0.0173604 0.378797 -0.172861 -6.91166 -6
#>  60:     7.7397481: 0.895833 0.00735505 -0.109326 0.875022 -0.0173586 0.378796 -0.172866 -7.31721 -6
#>  61:     7.7396995: 0.895834 0.00735697 -0.109329 0.875017 -0.0173568 0.378796 -0.172870 -7.72276 -6
#>  62:     7.7396727: 0.895834 0.00735782 -0.109331 0.875015 -0.0173562 0.378796 -0.172871 -8.12832 -6
#>  63:     7.7396553: 0.895835 0.00735847 -0.109332 0.875014 -0.0173556 0.378796 -0.172872 -8.63142 -7
#>  64:     7.7396470: 0.895835 0.00735864 -0.109332 0.875013 -0.0173554 0.378796 -0.172873 -9.10928 -7
#>  65:     7.7396428: 0.895835 0.00735878 -0.109332 0.875013 -0.0173553 0.378796 -0.172873 -9.58713 -7
#>  66:     7.7396406: 0.895835 0.00735881 -0.109332 0.875013 -0.0173553 0.378796 -0.172873 -10.0650 -8
#>  67:     7.7396395: 0.895835 0.00735884 -0.109332 0.875013 -0.0173552 0.378796 -0.172873 -10.5428 -8
#>  68:     7.7396389: 0.895835 0.00735884 -0.109332 0.875013 -0.0173552 0.378796 -0.172873 -11.0207 -8
#>  69:     7.7396386: 0.895835 0.00735885 -0.109333 0.875013 -0.0173552 0.378796 -0.172873 -11.4986 -9
#>  70:     7.7396384: 0.895835 0.00735885 -0.109333 0.875013 -0.0173552 0.378796 -0.172873 -11.9764 -9
#>  71:     7.7396383: 0.895835 0.00735885 -0.109333 0.875013 -0.0173552 0.378796 -0.172873 -12.4543 -9
#>  72:     7.7396383: 0.895835 0.00735885 -0.109333 0.875013 -0.0173552 0.378796 -0.172873 -12.9321 -1
#>  73:     7.7396383: 0.895835 0.00735885 -0.109333 0.875013 -0.0173552 0.378796 -0.172873 -13.4100 -1
#>  74:     7.7396383: 0.895835 0.00735885 -0.109333 0.875013 -0.0173552 0.378796 -0.172873 -13.8878 -1
#>  75:     7.7396382: 0.895835 0.00735885 -0.109333 0.875013 -0.0173552 0.378796 -0.172873 -14.3657 -1
#>  76:     7.7396382: 0.895835 0.00735885 -0.109333 0.875013 -0.0173552 0.378796 -0.172873 -14.8436 -1
#>  77:     7.7396382: 0.895835 0.00735885 -0.109333 0.875013 -0.0173552 0.378796 -0.172873 -15.3214 -1
#>  78:     7.7396382: 0.895835 0.00735885 -0.109333 0.875013 -0.0173552 0.378796 -0.172873 -15.7993 -1
#>   0:     7.7396382: 0.895835 0.00735885 -0.109333 0.875013 -0.0173552 0.378796 -0.172873 -15.7993 -1
#>   1:     7.7396382: 0.895835 0.00735885 -0.109333 0.875013 -0.0173552 0.378796 -0.172873 -15.7993 -1
mydsem
#> $par
#>        beta_z        beta_z        beta_z        beta_z        beta_z        beta_z        beta_z
#>   0.895834720   0.007358847  -0.109332511   0.875012562  -0.017355229   0.378795847  -0.172873038 -1
#>
#> $objective
#> [1] 7.739638
#>
#> $iterations
#> [1] 79
#>
#> $evaluations
#> function gradient
#>       96       80
#>
#> $time_for_MLE
#> Time difference of 0.06932712 secs
#>
#> $max_gradient
#> [1] 7.714655e-07
#>
```

```
#> $Convergence_check
#> [1] "There is no evidence that the model is not converged"
#>
#> $number_of_coefficients
#>  Total  Fixed Random
#>    133      9    124
#>
#> $AIC
#> [1] 33.47928
#>
#> $diagnostics
#>       Param starting_value Lower           MLE Upper final_gradient
#> 1    beta_z           0.01  -Inf   0.895834720   Inf   4.785205e-09
#> 2    beta_z           0.01  -Inf   0.007358847   Inf  -5.078683e-09
#> 3    beta_z           0.01  -Inf  -0.109332511   Inf  -2.031211e-08
#> 4    beta_z           0.01  -Inf   0.875012562   Inf  -5.821149e-08
#> 5    beta_z           0.01  -Inf  -0.017355229   Inf  -5.373382e-09
#> 6    beta_z           1.00  -Inf   0.378795847   Inf   2.119351e-09
#> 7    beta_z           1.00  -Inf  -0.172873038   Inf  -7.714655e-07
#> 8 lnsigma_j           0.00  -Inf -15.799262455   Inf   1.628788e-12
#> 9 lnsigma_j           0.00  -Inf -11.977331517   Inf   2.141499e-09
#>
#> $time_for_run
#> Time difference of 0.07067394 secs
```

where we again inspect the estimated interaction matrix:

	wolves	moose
wolves	0.896	-0.109
moose	0.007	0.875