A Flat-Response Single-Tuned I. F. Amplifier*

E. H. B. BARTELINK†, SENIOR MEMBER, I.R.E., J. KAHNKE‡, ASSOCIATE, I.R.E., AND R. L. WATTERS§, ASSOCIATE, I.R.E.

Summary—**An intermediate-frequency amplifier, providing double-tuned response using single-tuned circuits with negative feedback, is described. Particular attention is centered on the problems arising in the case where relatively narrow pass bands are wanted.**

General

IN THE COURSE of some special work on radar systems, the authors found it desirable to have "flat-topped" intermediate-frequency amplifiers, mainly because they will allow some deviation in transmitter and local-oscillator frequencies without affecting the amplifier gain. Such amplifiers should be useful in many other applications.

At the time work on this project was started, there were no amplifiers readily available which combined the desired "flat-top" response with the compactness required. During a visit to the Radiation Laboratory at the Massachusetts Institute of Technology, the authors learned from L. A. Turner about the use of negative feedback to obtain such a response from single-tuned circuits. The application at the Radiation Laboratory had been for rather wide pass bands, 10 Mc. and higher, while in the case under discussion a much narrower pass-band was needed.

The idea of using negative feedback to control the response curve has been known for some time. Wheeler[1] pointed out the effects of negative feedback on response. Feedback methods for the wide-band case were used by H. N. Beveridge and A. J. Ferguson and their co-workers in the National Research Council of Canada, and by E. Feenberg and W. W. Hansen in this country. These feedback applications contemplated "chain" feedback in which every stage was so equipped. In such cases, appreciable care has to be taken in the termination of the amplifier because reflected waves, similar to those in transmission lines and filters, can occur. Additional problems arise if gain control is desired, as is the case in most applications.

H. J. Lipkin, of the Radiation Laboratory, had proposed and used a different type of single-tuned feedback amplifier for the same wide pass bands (10 Mc. or more). In this amplifier, every stage of "feedback" amplification is separated from the next by a stage of "normal" amplification. This makes the amplifier unidirectional and eliminates all reflected-wave problems. In addition, gain control can easily be applied to the "normal" stages. This type of feedback amplifier, which is by far the more desirable for a number of applications, is discussed in this paper.

It was found that, if the same techniques are used for narrow-band amplifiers (4 Mc. or less) as for wide-band amplifiers (10 Mc. or more), some particular problems arise. Initially, a direct plate-to-plate feedback resistor was mounted in an existing intermediate-frequency amplifier as shown in Fig. 1. A very large amount of spurious feedback was encountered. While it proved possible to neutralize this spurious feedback and obtain the desired response curves, the resulting amplifier was highly unstable so far as the shape of the response curve was concerned. Part of this trouble was traced to the fact that decouplings, chiefly in screens and filaments, which are sufficient for the original purpose, are inadequate to produce stable response curves in the feedback amplifier. Even after these troubles were eliminated in a specially designed amplifier, it was observed that the direct plate-to-plate feedback method still resulted in excessive instability in the response curves.

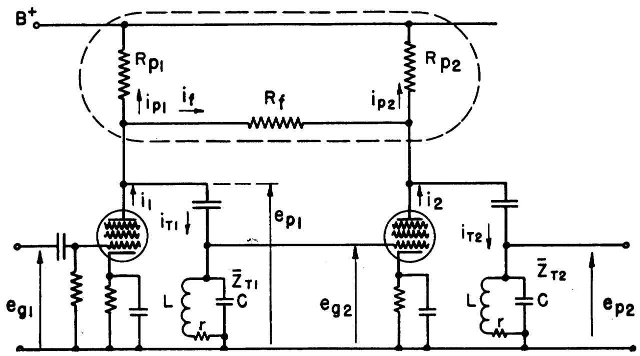

Fig. 1—Circuit diagram of the i.f. amplifier with high-impedance feedback.

To overcome this, a new method of applying the feedback was used which permits the use of low-impedance elements in the feedback circuit. The response curves of this arrangement proved to be highly stable. A new amplifier, designed to have the same physical dimensions as the existing single-tuned unit and in which particular care was given to decoupling and shielding, resulted in a very stable unit. The gain can be controlled by varying screen or control-grid voltages of the "straight" amplifier stages.

The following contains an analysis of the basic circuit showing the analogy of its response with that of the double-tuned intermediate-frequency stage, the analysis of the low-impedance feedback circuit, and a report on the development and behavior of the final unit.

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† General Telephone Service Corporation, New York 4, N. Y.
‡ Engineering Research Associates, Inc., St. Paul 4, Minn.
§ Research Laboratory, General Electric Company, Schenectady, N. Y.

[1] H. A. Wheeler, "Wide-band amplifiers for television," Proc. I.R.E., vol. 27, pp. 429–438; July, 1939.

Analysis of the Basic Circuit

The basic circuit is shown in Fig. 1. Simple reasoning shows that flat-topped and double-humped response curves may be expected from this circuit. The first approach is as follows: It is well known that any two coupled tuned circuits will produce "flat" and "double-humped" responses under suitable coupling conditions. The only exception is the case where the coupling is a pure unidirectional network like a tube. This case will have a response equal to the product of the single-tuned circuit responses. By inserting the feedback resistor R_F in Fig. 1, the bidirectional coupling and thus the ability to product flat and double-humped responses has been restored.

The other way of reasoning is this: For large values of $\mu\beta$ the standard negative feedback equation

$$e_p = \frac{\mu\beta}{1-\mu\beta}\cdot\frac{e_g}{\beta}$$

approaches $e_p = (1/\beta)e_g$. For a purely resistive feedback system this will give a gain which is independent of frequency. In the actual circuit of Fig. 1, however, the transfer constant of the feedback voltage is a maximum at the resonance frequency of the first plate circuit, i.e., more feedback exists at this frequency, and a dip in the response curve may be expected at this point.

Using the symbols appearing in Fig. 1, the following equations can be obtained:

$$i_1 = i_{p1} + i_{T1} + i_F \tag{1}$$

$$i_2 = i_{p2} + i_{T_2} - i_F \tag{2}$$

$$i_1 = g_{m1}e_{g1} \tag{3}$$

$$i_2 = g_{m2}e_{g2} \tag{4}$$

$$i_{p1}R_{p1} = i_{T1}Z_{T1} \tag{5}$$

$$i_{p2}R_{p2} = i_{T2}Z_{T2} \tag{6}$$

$$i_{p1}R_{p1} - i_{p2}R_{p2} = i_F R_F \tag{7}$$

$$e_{g2} = -\, i_{p1}R_{p1}. \tag{8}$$

These must be solved for the eight unknowns:

$$i_1,\ i_{p1},\ i_{T1},\ i_2,\ i_{p2},\ i_{T2},\ i_F,\ e_{g2}.$$

After some manipulation, (1) to (8) yield

$$g_{m1}e_{g1} = A_1 i_{p1} - B_{21}i_{p2} \tag{9}$$

$$0 = A_2 i_{p2} - B_{12}i_{p1} \tag{10}$$

where

$$A_1 = \left(1 + \frac{R_{p1}}{Z_{T_1}} + \frac{R_{p1}}{R_F}\right) \tag{11}$$

$$A_2 = \left(1 + \frac{R_{p2}}{Z_{T2}} + \frac{R_{p2}}{R_F}\right) \tag{12}$$

$$B_{21} = \frac{R_{p2}}{R_F} \tag{13}$$

$$B_{12} = \frac{R_{p1}}{R_F}(1 - g_{m2}R_F). \tag{14}$$

Combining the preceding with

$$e_{p2} = i_{p2}R_{p2}, \tag{15}$$

the gain G becomes

$$G = -\, g_{m1}R_{p2}\frac{B_{12}}{A_1A_2 - B_{12}B_{21}} = -\, g_{m1}R_{p2}\frac{B_{12}}{N} \tag{16}$$

where

$$N = A_1A_2 - B_{12}B_{21}. \tag{16a}$$

All the frequency-dependent terms are concentrated in N. Substitution of (11), (12), (13), and (14) in (16a) gives

$$N = \left(1 + \frac{R_{p1}}{R_F} + \frac{R_{p1}}{Z_{T1}}\right)\left(1 + \frac{R_{p2}}{R_F} + \frac{R_{p2}}{Z_{T2}}\right) + (G_{m2}R_F - 1)\frac{R_{p1}R_{p2}}{R_F{}^2}. \tag{17}$$

Introducing

$$\left\{\begin{aligned} Z_T &= \frac{(r + j\omega L)\dfrac{1}{i\omega c}}{r + j\left(\omega L - \dfrac{1}{\omega c}\right)} \\ \omega_c{}^2 &= \frac{1}{Lc} \\ \omega &= (1+\delta)\omega_0 \\ Q_T &= \frac{\omega_0 L}{r} = \frac{X_0}{r} \end{aligned}\right. \tag{18}$$

and assuming $\omega_{c1} = \omega_{c2} = \omega_c$, then

$$Z_T = \frac{X_0{}^2}{r}\cdot\frac{1 - j\dfrac{1}{Q_T(1+\delta)}}{1 + 2j\delta Q_T\dfrac{1+\delta/2}{1+\delta}}. \tag{19}$$

Equation (19) is exact and not restricted to the case where the pass band is a small fraction of the carrier. If Q_T is large, the complex term in the numerator can be neglected. For the subsequent calculations a new variable ϵ is introduced, defined by

$$\epsilon = \delta\frac{1+\delta/2}{1+\delta} = \frac{1}{2}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right). \tag{20}$$

For small frequency deviations, ϵ approaches δ. Substitution of (20) in (19) under the assumption $Q_T \ll 1$ results in

$$\frac{1}{Z_T} = \frac{1 + 2j\epsilon Q_T}{X_0 Q_T}. \tag{21}$$

Neglecting the complex term in the numerator of (19) results in a small shift in the resonance frequency as compared to the value $\omega_0 = 1/\sqrt{LC}$. Assuming $Q_{T1} = Q_{T2}$

$=Q_T$, resonance will be obtained at a deviation $\delta^1 = 1/2Q_T^2$ instead of $\delta=0$. This effect shifts the response curve but does not affect its shape.

Introducing the symbols

$$\left\{\begin{array}{l} \dfrac{X_{01}}{R_{p1}}=Q_{p1} \\[2ex] \dfrac{R_{p1}}{R_F}=\alpha_1 \\[2ex] (1+\alpha_1)Q_{p1}+\dfrac{1}{Q_{T1}}=p_1 \end{array}\right. \quad \left\{\begin{array}{l} \dfrac{X_{02}}{R_{p2}}=Q_{p2} \\[2ex] \dfrac{R_{p2}}{R_F}=\alpha_2 \\[2ex] (1+\alpha_2)Q_{p2}+\dfrac{1}{Q_{T2}}=p_2 \end{array}\right. \quad \left\{\begin{array}{l} G_F=g_{m2}R_F \\[2ex] Q_F{}^2=\dfrac{X_{01}X_{02}}{R_F{}^2} \end{array}\right. \tag{22}$$

$$K^2=(G_F-1)Q_F{}^2,$$

equation (17) simplifies to

$$\boxed{N = \frac{1}{Q_{p1}Q_{p2}}\left((p_1+2j\epsilon)(p_2+2j\epsilon)+K^2\right).} \tag{23}$$

This form is identical to the one for the double-tuned intermediate-frequency-transformer response. Thus the well-known criteria for critical coupling, attenuation, bandwidth, etc., can be applied directly to this case. Notably, "critical coupling" is obtained when

$$K^2 = K_0{}^2 = \frac{p_1{}^2+p_2{}^2}{2}\cdot \tag{24}$$

The bandwidth at the 1/2-power point is given by

$$\epsilon_{0.707} = 1/2\sqrt{p_1p_2+K^2}\sqrt{g+\sqrt{1+g^2}} \tag{24a}$$

where

$$g = \frac{K^2-K_0{}^2}{p_1p_2+K^2}\cdot$$

If $p_1=p_2=p$, (24) reduces to $K_0=p$, and (24a) to

$$\epsilon_{0.707} = 1/2\sqrt{(K^2-K_0{}^2)+\sqrt{2(K^4+K_0{}^4)}}. \tag{24b}$$

For $K\approx K_0$,

$$\epsilon_{0.707} = 0.707K_0 = 0.707p$$

$$= 0.707\left\{\left(1+\frac{R_p}{R_F}\right)\frac{X_0}{R_p}+\frac{r}{X_0}\right\}, \tag{24c}$$

and this same value holds in the region near critical coupling.

Equation (23) also determines the location ϵ_m of the peaks which occur at

$$\epsilon_m = 1/2\sqrt{K^2-K_0{}^2}. \tag{24d}$$

Introducing (23) in (16), the gain becomes:

$$G = \frac{g_{m1}R_{p1}\cdot g_{m2}R_{p2}\left(1-\dfrac{\alpha_1}{g_{m2}R_{p1}}\right)}{\dfrac{1}{Q_{p1}Q_{p2}}\left[(p_1+2j\epsilon)(p_2+2j\epsilon)+K^2\right]}\cdot \tag{25}$$

For the bandwidth in the particular region used, R_p is approximately 7000 ohms; R_F is approximately 15,000 ohms, resulting in $\alpha_1\approx 7/15$; $g_mR_p\approx 70$, which will give $\alpha_1/g_{m2}\,R_{p1}\approx 1/150$. It is clear that this term may be neglected. The gain at resonance is

$$G_0 = \frac{g_{m1}R_{p1}g_{m2}R_{p2}}{1+(\alpha_1+\alpha_2)+(G_F-1)\alpha_1\alpha_2}\cdot \tag{26}$$

In this case, where G_F is in the order of 150, (26) may be rewritten:

$$G_0 = \frac{g_{m1}R_{p1}g_{m2}R_{p2}}{1+\dfrac{R_{p1}+R_{p2}}{R_F}+g_{m2}R_F\dfrac{R_{p1}R_{p2}}{R_F{}^2}}\cdot \tag{27}$$

Low-Impedance Feedback Circuit

Experiments on the direct plate-to-plate feedback circuit, applied to the existing intermediate-frequency amplifier, showed that the values of R_F necessary to obtain the required bandpass, i.e., 1 to 5 Mc., were of the same order of magnitude as the impedance of the path through the stray capacitance in and around the feedback network. In particular, the original modification of this unit, constructed for 2.5-Mc. bandwidth, had a feedback resistor of 15,000 ohms. The capacitance across the resistor alone was of the order of ½ micromicrofarad or approximately 10,000 ohms at about 30 Mc., where the amplifier was operated.

The effects of the spurious capacitances in the feedback network were eliminated by converting from a high-voltage, low-current to a low-voltage, high-current system. This is done by inserting the feedback at points of lower potential, as shown in Fig. 2. The feedback resistor was reduced to 3000 ohms and the tap was located approximately in the middle of the plate load resistor. The bandwidth of 2.5 Mc. remained the same, while the gain increased by a factor of 1.6 over the gain of a similar single-tuned amplifier having the same bandwidth.

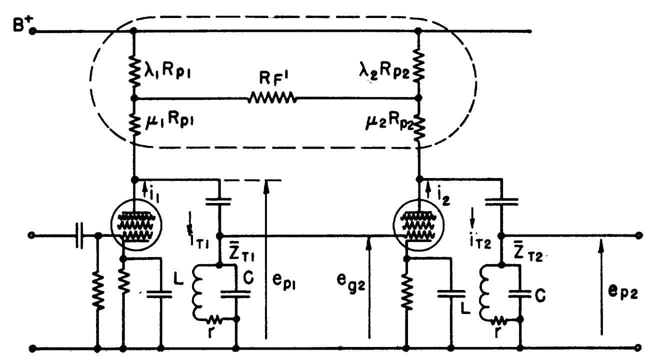

Fig. 2—Circuit diagram of the i.f. amplifier with low-impedance feedback.

Analyses of the New Circuit

As can be seen from comparison of Figs. 1 and 2, the two circuits are identical except for the part circled by the dotted line. This part forms a four-terminal network and can, therefore, be changed by pi-tee equivalent transformations. As the components are pure resistors, this equivalence is independent of frequency.

Fig. 3(a) to (c) shows the steps which will transform the new circuit into the old one.

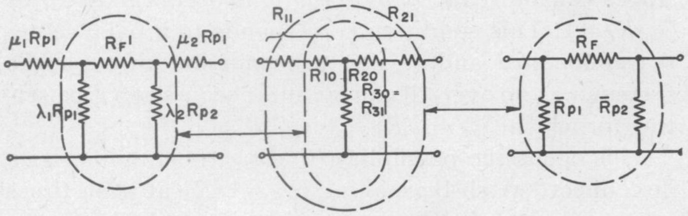

Fig. 3—Transformations to equivalent feedback network.

These transformations result in

$$\overline{R}_{p1} = \frac{1}{(R_L + R_{F'})} \cdot \frac{P}{R_{p2}(\mu_2 R_L + R_{F'})}$$

$$\overline{R}_{p2} = \frac{1}{R_L + R_{F'}} \cdot \frac{P}{R_{p1}(\mu_1 R_1 + R_{F'})}; \tag{28}$$

$$\overline{R}_F = \frac{1}{R_L + R_{F'}} \cdot \frac{P}{R_m^2}$$

where

$$R_L = \lambda_1 R_{p1} + \lambda_2 R_{p2}$$

$$R_m^2 = \lambda_1 R_{p1} \cdot \lambda_2 R_{p2}$$

$$\begin{aligned} P = {} & (R_{p1} R_{p2}(\mu_1 R_L + R_{F'}))(\mu_2 R_L + R_{F'}) \\ & + R_{p1}(\mu_1 R_L + R_{F'}) R_m^2 \\ & + R_{p2}(\mu_2 R_L + R_{F'}) R_m^2. \end{aligned} \tag{29}$$

Considerable simplification results if plate load resistors and tapping ratios are equal. In practice, this case will be the most common.

Introducing

$$\begin{aligned} \lambda_1 &= \lambda_2 = \lambda \\ \mu_1 &= \mu_2 = \mu \\ 1 - \lambda &= \mu \\ \alpha &= \frac{R_p}{R_{F'}}, \end{aligned} \tag{29a}$$

then (29) simplifies to

$$\overline{R}_p = R_p \frac{1 + 2\alpha\mu\lambda + 2\alpha\lambda^2}{(1 + 2\alpha\lambda)}$$

$$\overline{R}_F = R_F \frac{(1 + 2\alpha\mu\lambda)^2 \left(1 + \dfrac{2\alpha\lambda^2}{1 + 2\alpha\mu\lambda}\right)}{\lambda^2(1 + 2\lambda\alpha)}. \tag{30}$$

Under all conditions, $\mu\lambda < 1/4$, while usually $\alpha < 1/2$. In order to get appreciable reduction of the effect of spurious capacitances, it is necessary that $\lambda \leqq 1/2$. Under these conditions, $\begin{cases} 2\alpha\lambda < 1/2 \\ 2\alpha\mu\lambda < 1/4. \end{cases}$

In an actual example, the following values were used:

$$\begin{aligned} R_p &= 1750 \\ \lambda R_p &= 750 \\ R_{F'} &= 3000. \end{aligned}$$

Introducing these values,

$$\begin{aligned} \overline{R}_p &= R_p \\ \overline{R}_F &= 7 R_{F'}. \end{aligned} \tag{30a}$$

Thus the plate load has not been changed, but the effect of the feedback resistor equals that of one 7 times larger in the original scheme.

Tests of the amplifier with the low-impedance feedback circuit showed that the high-frequency side of the response curve peaked up considerably over the low-frequency side. The feedback network is an H type of structure. Each of the components in the structure has resistance plus a small inherent capacitance of its own, and if the RC products are not balanced around the circuit, the network will be frequency-dependent. Addition of a balancing capacitance from plate to plate would cure this, but it may introduce appreciable lead capacitances which are hard to control. Inspection of the circuit will show that the same result may be obtained by a capacitance inserted between the grid and the plate of the last tube of the pair. To balance capacitances all around the loop stably, a small variable capacitor of about 1.2-micromicrofarad maximum capacitance was added at this point. By adjusting this capacitor, it was possible to produce a double-humped response curve having equal

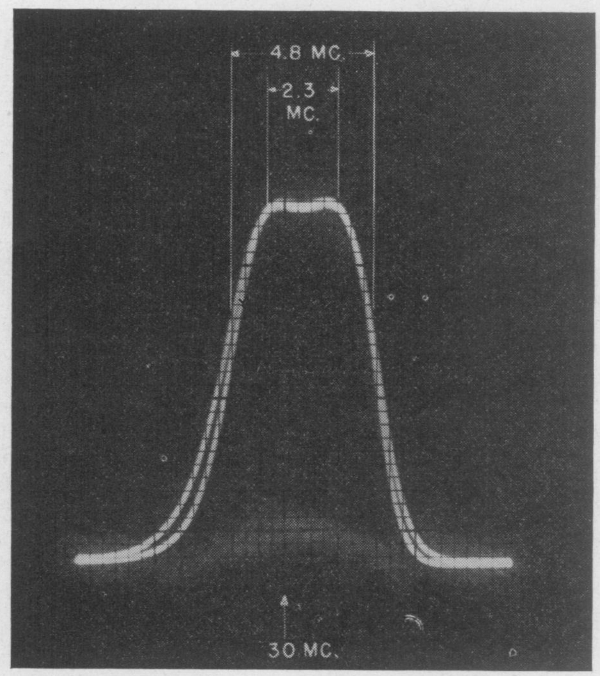

Fig. 4—Band-pass characteristics of the i.f. amplifier.

peaks. Reduction of the capacitance brought up the high-frequency side, while too much capacitance brought up the low-frequency side. Further increase of the response toward the low-frequency side produced oscillation. When the size of the capacitance is adjusted properly, it is possible to line the amplifier up at any frequency over the range of the tuning coils and preserve the double-hump response curve.

A typical response curve of one of the amplifiers is shown in Fig. 4. The bandwidth between the tops of the two slight peaks is 2.3 Mc. The bandwidth at 0.707 down is 4.8 Mc. The measured voltage gain to and including a diode detector (from r.m.s. to d.c.) was 20,000 for two feedback pairs. The gain of a pair of stages was approximately 200.

Adjustment Procedure

As can be seen (for instance, from (24b) and (22)), the bandwidth increases and decreases with the factor $K^2 = Q_F^2(G_{m2}R_F - 1)$. One can, therefore, reduce the bandwidth by reducing the gain of the second tube to where a sharp response is obtained. In this condition it is very easy to tune the different stages of the amplifier to the same frequency.

If the particular shape of the response curve is not important, this provides a means for changing the bandwidth of an amplifier by a d.c. control.

If it is desired to keep a flat response curve, the pass band of the amplifier can be changed over smaller ranges merely by changing the feedback resistors and readjusting the second-stage gain to critical coupling. Larger changes in bandwidth will, in general, call for a new set of loading resistors.

It will be seen that the degree of coupling between the stages can be reduced to zero. This occurs if $K=0$ or $G_{m2}R_F = 1$. This condition corresponds to a balance between forward and reverse transmission through the system, i.e., no over-all gain at all; it occurs at a gain setting for which $G_{m2} = 1/R_F$.

This opens the possibility of using the amplifier as a disconnect switch by making $G_{m2} = 1/R_F$ at such times as suppression of the output voltage is desired.

Another interesting effect results from these conditions. As the mutual conductance of the second tube is further reduced, the phase or "polarity" of the gain reverses, and the output amplitude begins to increase again. (The influence of small values of K on the term N in (23) may be neglected.) For the extreme case, $G_{m2} = 0$, the arrangement presents two tuned circuits in the plate of tube No. 1 coupled by resistor R_F and having an over-all gain at resonance of $g_{m1}R_{p1}(R_{p2}/R_F$. If two of such pairs are used, fed by voltages 90° apart, and if their outputs are fed into a common circuit (for instance, through two cathode followers), it is possible, by proper control of the two second screen voltages, to obtain a voltage which can be phase-shifted by any desired amount with respect to the phase of the input voltage.

The Radiation Resistance of an Antenna in an Infinite Array or Waveguide*

HAROLD A. WHEELER†, FELLOW, I.R.E.

Summary—**The electromagnetic field in front of an infinite flat array of antennas can be subdivided into wave channels, each including one of the antennas. Each channel behaves like a hypothetical waveguide similar to a transmission line made of two conductors in the form of parallel strips. A simple derivation then leads to the radiation resistance of each antenna and to some limitations on the antenna spacing. In the usual flat array of half-wave dipoles, each allotted a half-wave-square area, and backed by a plane reflector at a quarter-wave distance, the radiation resistance of each dipole is $480/\pi = 153$ ohms. In a finite array, this derivation is a fair approximation for all antennas except those too close to the edge. This derivation also verifies the known formula for the directive gain of a large flat array in terms of its area. The same viewpoint leads to the radiation resistance of an antenna in a rectangular waveguide, which has previously been derived by more complicated methods.**

I. Introduction

In THE SCIENCE of radio antennas, one of the most fundamental and useful concepts is the radiation resistance of a thin conductor of a certain length and configuration. The classic example is the half-wave dipole in free space, whose radiation resistance is 73.13 ohms. Its exact value was difficult of computation because it involved the spherical electromagnetic wave with all its complexities.

In combining several elementary antennas into a directive array, it has been customary to compute the self and mutual impedances associated with radiation, and to obtain therefrom the radiation resistance of each antenna with respect to its own current. With a greater number of antennas in an array, this procedure involves a number of components proportional to the square of the number of antennas. However, the interactions of the more distant elements usually becomes negligible for practical approximations.

This circumstance suggests the possibility of attacking the problem by assuming an array of infinite dimensions as an approximation to a finite array of a large number of elements. It devolves that many cases of the infinite array yield extremely simple solutions for the radiation resistance of the component antennas, and

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† Consulting Radio Physicist, Great Neck, L. I., N. Y.