Intermediate-Frequency Gain Stabilization with Inverse Feedback*

G. FRANKLIN MONTGOMERY†, MEMBER, IRE

Summary—**Increased gain stability and gain-bandwidth product result from the use of inverse feedback in an intermediate-frequency amplifier. Improvement in gain stability is related to the number of cascaded stages, the stage gain, and the magnitude of the feedback. A circuit is described which uses feedback over a pair of cascaded stages. Generalized selectivity curves for this feedback couple are shown, and the design procedure is outlined. A description of an experimental amplifier concludes the paper.**

Introduction

INCREASED GAIN STABILITY and gain-bandwidth product result from the use of inverse feedback in an intermediate-frequency amplifier. In addition, the response curve of the amplifier may be designed to have a flatter top and steeper skirts than the curve for an amplifier of cascaded, synchronous, single-tuned stages without feedback. The improvement in flatness and gain-bandwidth product has been described by previous investigators for the case of feedback over a single stage.[1,2] This paper analyzes a method using feedback over pairs of stages and presents experimental confirmation of the design procedure.

Formulas introduced in the text are derived in detail in the appendix, where a complete list of symbol definitions appears.

Stability

Consider an amplifier of n identical cascaded stages, each stage having a voltage gain of A_1. The A_1 of each stage may be expected to vary with changes in tube transconductance due to power supply variations and tube aging, and with variations in constants of the interstage coupling networks. In the worst possible case of gain instability, a particular variation in A_1 will occur simultaneously in all stages. We shall examine the effect of such a simultaneous A_1 variation on the over-all gain A_n by forming the ratio of the percentage variation in A_n to the corresponding percentage variation in A_1. A convenient measure of the instability is the limit of this ratio as either of the variations approaches zero. This limit is called the instability factor

$$I = \lim_{\Delta A_1 \to 0} \left[\frac{\dfrac{\Delta A_n}{A_n}}{\dfrac{\Delta A_1}{A_1}} \right] = \left[\frac{A_1}{A_n} \right] \frac{dA_n}{dA_1}$$

and should be as small as possible for good gain stability.

If inverse feedback is applied over groups of m stages, and β is the voltage gain of the feedback network, then the over-all gain of the amplifier is[3]

$$A_n = \left[\frac{A_1{}^m}{1 + \beta A_1{}^m} \right]^{n/m}.$$

Differentiation of this expression leads to

$$I = n \left[\frac{A_n{}^{1/n}}{A_1} \right]^m \tag{1}$$

which is the general expression for I.

In the particular case where A_n is to equal a constant assigned value A_{ns}, we can find the number of stages n which results in the greatest gain stability. That is, if the over-all gain is held constant by increasing n and β simultaneously, the gain stability increases to a maximum (minimum I) and then decreases. For the maximum stability condition

$$n = m \ln A_{ns}$$

$$\beta = \frac{1}{e} - \frac{1}{A_1{}^m}$$

$$I = \frac{ne}{A_1{}^m}$$

where $e = 2.718$, and ln indicates the logarithm to the base e.

For a constant A_1, maximum stability is achieved at the cost of stage gain, since, for this condition

$$A_{ns} = e^{n/m}$$

and each m group has a gain of e. In practice, this degree of gain reduction is much larger than is necessary for good stability, the usual gain per stage being several times this value. If, for example, the number of stages n is $1/k$ times the number required for maximum stability, then I will be greater by the factor e^{k-1}/k. This factor is only 1.36 for $k=2$ and 2.46 for $k=3$, so that n may be considerably reduced without serious reduction of stability.

Notice from (1) that under any condition of feedback, if A_n and n are both held constant, it is more advantageous to increase feedback by increasing m rather than β, since $(A_n{}^{1/n}/A_1) < 1$. Because of difficulty in designing the β network, however, m will not usually be greater than three.

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† Central Radio Propagation Laboratory, National Bureau of Standards, Washington, D. C.

1 S. N. Van Voorhis, "Microwave Receivers," McGraw-Hill Book Co., New York, N. Y., pp. 88–91 and 175–178; 1948.

2 E. H. B. Bartelink, J. Kahnke, and R. L. Watters, "A flat-response single-tuned if amplifier," Proc. I.R.E., vol. 36, p. 474; April, 1948.

3 F. E. Terman, "Radio Engineering," McGraw-Hill Book Co., New York, N. Y., p. 248; 1937.

It should not be inferred that the use of feedback inevitably results in reduced gain per stage. Feedback increases the gain-bandwidth product by a factor dependent upon the magnitude of feedback and the β circuit characteristics. If the interstage coupling networks can be properly modified, feedback will produce in a given number of stages a gain higher than that of a single-tuned zero-feedback amplifier while maintaining the same bandwidth and improving the gain stability. The price which must be paid for this improved operation is the use of coupling networks of higher Q than would be required ordinarily. The amount of feedback which can be used in practice for a given bandwidth is thus limited by the obtainable circuit Q's, and this limitation is such that feedback reduces the gain per stage for bandwidths which are a small fraction of the center frequency. Relatively wide-band amplifiers do not suffer this limitation.

For a single-tuned stage with zero feedback, the gain-bandwidth product is

$$\Pi_o = \frac{g_m}{2\pi C_1}.$$

For the feedback couple to be described, the gain-bandwidth product, per stage, is

$$\Pi_B \approx 1.7\Pi_o \tag{2}$$

in the more useful range of feedback values.

Particular Method

Fig. 1 is the schematic diagram of a cascaded pair of radio-frequency stages with inverse feedback. Power supply connections are omitted for simplicity. Symbols necessary to the design of this feedback couple are defined in the following list:

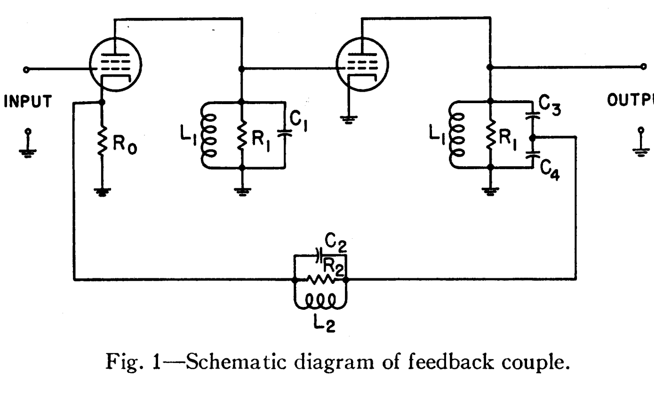

Fig. 1—Schematic diagram of feedback couple.

a = normalized voltage gain of feedback couple
A_0 = center-frequency voltage gain of couple with zero feedback
A_{c0} = center-frequency voltage gain of feedback couple
B = feedback factor
C_1 = plate-load capacitance in farads
C_2 = feedback capacitance in farads
C_3 = divider capacitance in farads
C_4 = divider capacitance in farads
f_0 = center frequency in cycles per second
Δf = bandwidth in cycles per second
g_m = tube transconductance in mhos
N = step-down ratio of tuned output circuit
$\omega_0 = 2\pi f_0$ in radians per second
P = design parameter
$Q_1 = Q$ of tuned circuit consisting of shunt-connected L_1, C_1, R_1
$Q_2 = Q$ of tuned circuit consisting of shunt-connected L_2, C_2, R_2
R_0 = cathode-bias resistance in ohms
R_1 = effective plate-load resistance in ohms
R_2 = effective feedback resistance in ohms.

Normalized gain curves for several values of feedback are given in Fig. 2, where a is plotted against $Q_1(\Delta f/f_0)$.

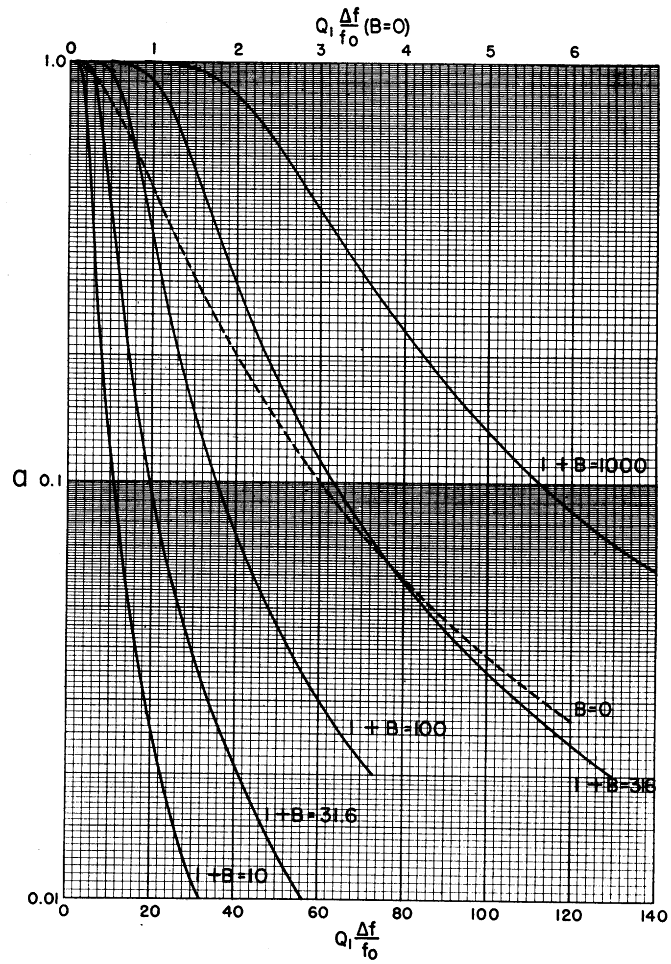

Fig. 2—Normalized gain for smooth response.

The curve labeled $B=0$, plotted to a different horizontal scale, is the normalized response of the couple with zero feedback. Note that the curves for appreciable feedback are more nearly rectangular than the curve for zero feedback.

The design procedure, using formulas developed in the Appendix and information given in the curves, is as follows:

Given Δf, f_0, g_m, and A_{c0}

A. Choose value of $1+B$. In Fig. 2 or 3, find $Q_1(\Delta f/f_0)$ for $a=1/\sqrt{2}$.

B. Calculate Q_1. If Q_1 is impractically large, choose a smaller value of $1+B$.

C. Calculate

$$C_1 = \frac{g_m Q_1}{\omega_0 \sqrt{A_{c0}}(1+B)}.$$

D. Calculate

$$R_1 = \frac{Q_1}{\omega_0 C_1}.$$

E. In Fig. 4, find P and A_0/N.

F. Calculate $A_0=(1+B)A_{c0}$, and N.

G. Choose R_0 from tube data.

H. Calculate

$$R_2 = \left[\frac{A_0}{NB} - 1\right] R_0.$$

I. Verify $N^2R_2 \gg R_1$. If this is not true, choose a smaller $1+B$ and redesign.

J. Calculate

$$Q_2 = \frac{A_0 P}{NB} Q_1.$$

K. Calculate

$$C_2 = \frac{Q_2}{\omega_0 R_2}.$$

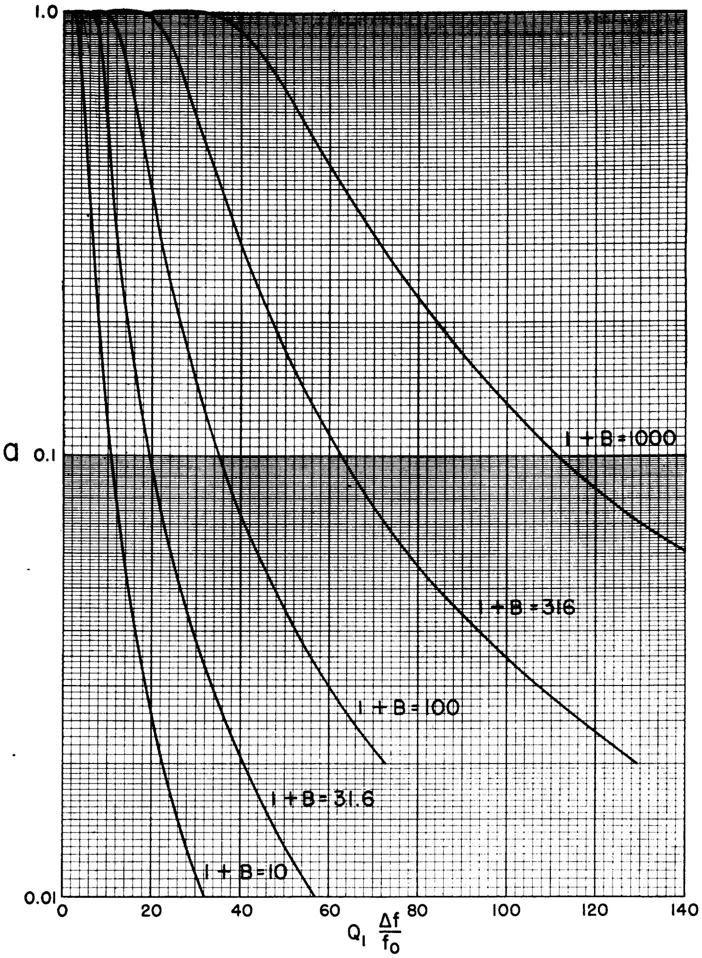

Fig. 3—Normalized gain for peaked response.

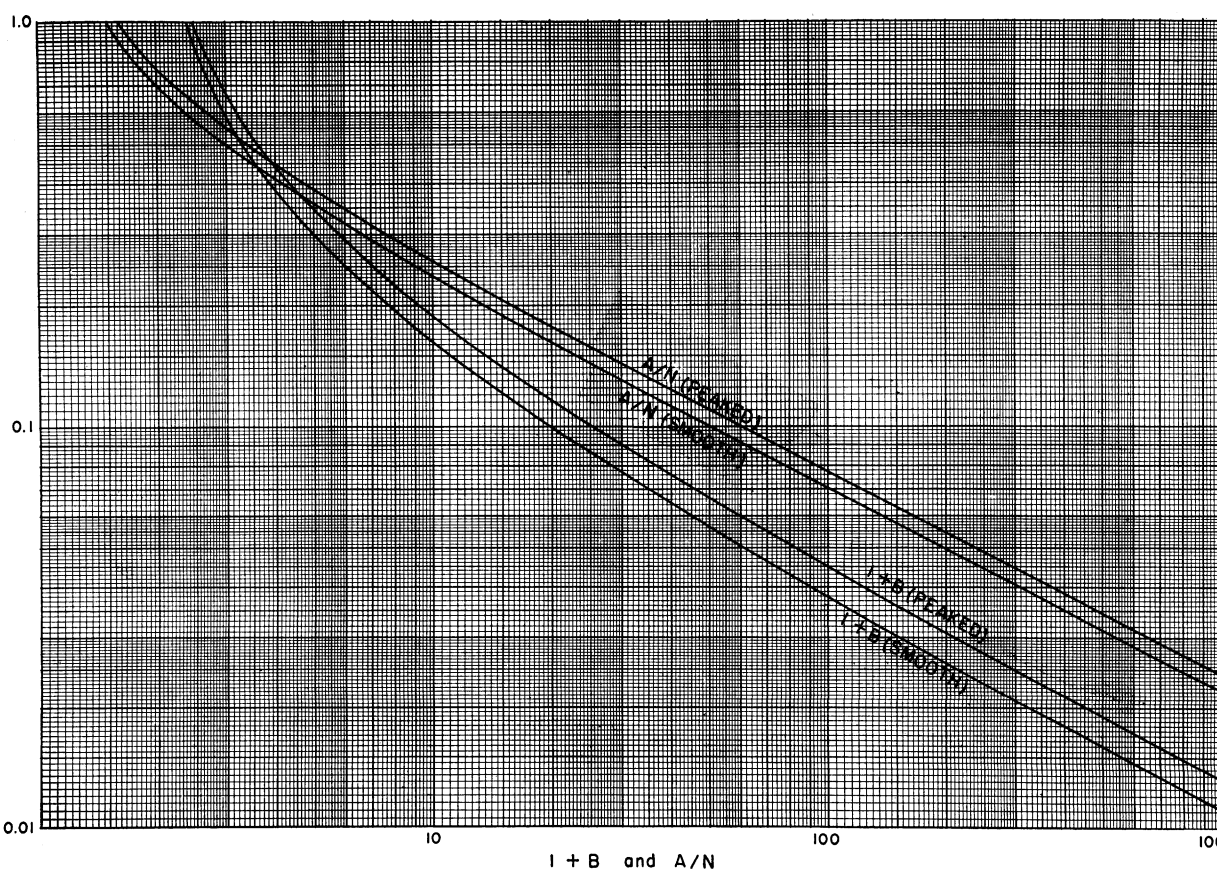

Fig. 4—Design factor chart.

L. Calculate

$$C_3 = \frac{NC_1}{N-1}, \qquad C_4 = NC_1.$$

Generally, the feedback factor B should be chosen as large as possible in order to obtain maximum stability. The improvement in center-frequency gain stability over a zero-feedback amplifier can be calculated directly from (1).

For example, if $n=4$, $m=2$, $A_n=10^4$, $A_1=10^2$, $I=4(0.1)^2=0.04$; a two-stage zero-feedback amplifier with equal gain has $I=2$. Consequently, an amplifier consisting of two feedback couples with the given constants has one-fiftieth the gain variation of a similar two-stage amplifier with no feedback.

In the appendix, it is shown that the design may be proportioned to give peaked responses at the extremes of the pass band. Curves for a slightly peaked response are given in Fig. 3.

Experimental Results

The electrical arrangement represented by Fig. 1 must be duplicated in practice as closely as possible if results are to match the predictions of the design. Several practices, noted as critical during the course of the experimental work, should be followed:

1. Ensure adequate by-passing of the "ground" ends of the tuned plate circuits. This is especially important in the tapped output circuit. The design formulas are based on an output impedance at the tap which is usually a few ohms, and it does not take much reactance in the by-pass capacitor to modify the output impedance considerably.
2. By-pass the screen grid of the first stage directly to the cathode. The formulas for R_0 and C_1 will not be correct if the screen is by-passed to ground.
3. Install power lead decoupling as in an ordinary zero-feedback amplifier. Because of the greater gain-bandwidth product, the gain per stage will be even larger than for a zero-feedback amplifier. A small amount of regeneration may work mischief with a carefully calculated design.

Tuning the amplifier is greatly facilitated by providing a switch to break the feedback line to the cathode of the first stage. The amplifier plate circuits are peaked in the normal fashion. The feedback is then switched in, and the feedback tuned circuit is adjusted for maximum response at the center frequency. If a slight asymmetry of the response develops, it is usually possible to minimize it by detuning the feedback circuit. A large asymmetry indicates a design error, regeneration, or inadequate by-passing. A useful final adjustment is the value of R_0. A bandwidth which is too large can be decreased by decreasing R_0. If the bandwidth is too small, R_0 should be increased.

In Fig. 5 is shown the gain characteristic of a feedback couple using two 6SK7 tubes with the following design values: $\Delta f=30$ kc, $f_0=450$ kc, $g_m=2{,}000$ micromhos, and $A_{c0}=100$. The normalized gain curve for $1+B=10$, taken from Fig. 2, is superimposed for comparison.

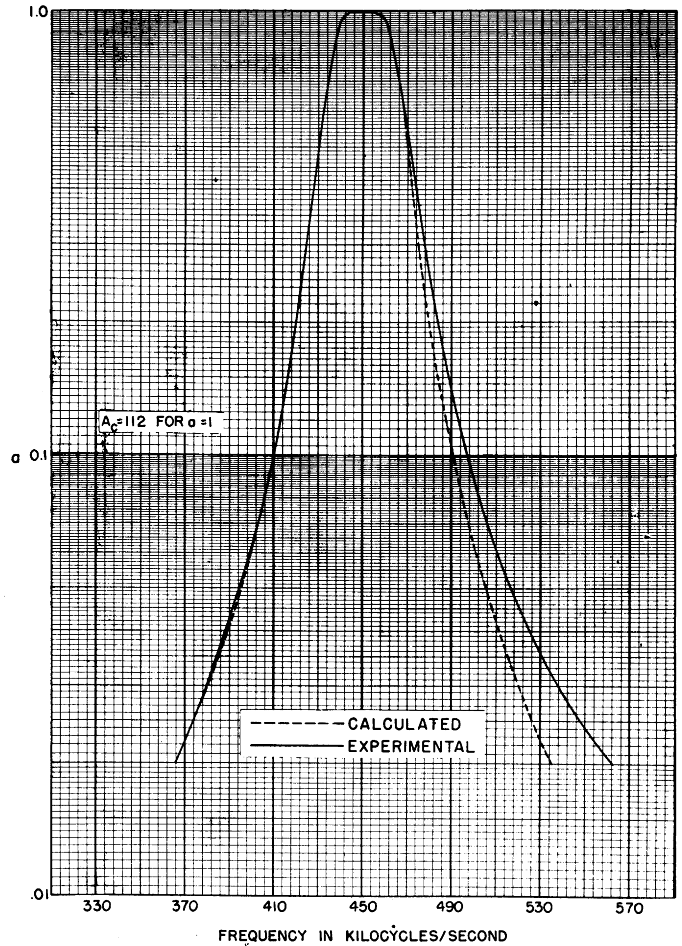

Fig. 5—Response of experimental amplifier.

The over-all gain and bandwidth agree well with the given values. The skirts are not quite as narrow as the normalized gain curve predicts. The lack of agreement is due to approximations that were made in the analysis. In particular, it was assumed that $N^2R_2 \gg R_1$, and that

$$\left|\frac{A}{N}\right| \gg |g_m Z_2|.$$

The latter inequality will not hold as well for frequencies far removed from resonance as for the center frequency. To the extent that the approximations are not achieved in practice, the skirts may be expected to deviate from the calculated values by small amounts.

The normalized gain curves plotted in Figs. 2 and 3 are also approximate in that the quantity u is considered equal to $\Delta f/f_0$, as explained in the appendix. This approximation was chosen because it allows the normalized response curves to be plotted as symmetrical characteristics, facilitating reading of bandwidth values. The approximation fails for bandwidths which are a large fraction of the center frequency, and in such cases it is advisable to plot curves with u equal to its exact value.

In Fig. 6 is shown the variation in gain of the experimental couple with plate supply voltage. A similar curve is shown for the same amplifier with zero feedback.

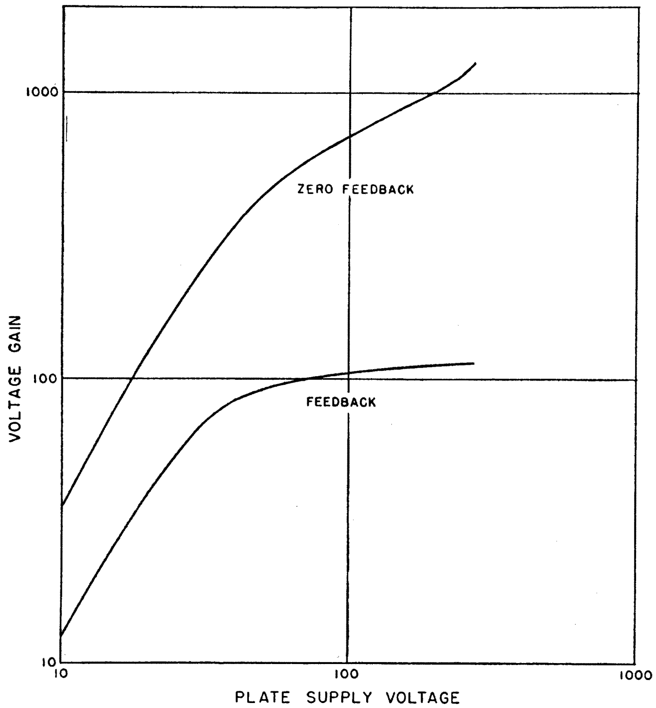

Fig. 6—Gain variation with supply voltage.

Appendix

Complete List of Symbols

a = normalized voltage gain of feedback couple
A = complex voltage gain of couple with zero feedback
A_0 = center-frequency voltage gain of couple with zero feedback
A_1 = voltage gain of single stage
A_c = complex voltage gain of feedback couple
A_{c0} = center-frequency voltage gain of feedback couple
A_n = voltage gain of cascade amplifier
A_{ns} = voltage gain of cascade amplifier with maximum stability
B = feedback factor
β = voltage gain of feedback network
C_1 = plate-load capacitance in farads
C_2 = feedback capacitance in farads
C_3 = divider capacitance in farads
C_4 = divider capacitance in farads
e = natural logarithmic base, 2.718
f = frequency in cycles per second
f_0 = center frequency in cycles per second
Δf = bandwidth in cycles per second
g_m = tube transconductance in mhos
I = instability factor
k = reduction factor
L_1 = plate load inductance in henries
L_2 = feedback inductance in henries
m = number of stages in each feedback loop
n = total number of stages
N = step-down ratio of tuned output circuit
ω = $2\pi f$ in radians per second
ω_0 = $2\pi f_0$
$\Delta\omega$ = $2\pi\Delta f$
P, P_1, P_2, P_3 = design parameters
Π_0 = single-tuned gain-bandwidth product
Π_B = feedback couple gain-bandwidth product
Q_1 = Q of tuned circuit consisting of shunt-connected L_1, C_1, R_1
Q_2 = Q of tuned circuit consisting of shunt-connected L_2, C_2, R_2
R_0 = cathode-bias resistance in ohms
R_1 = effective plate load resistance in ohms
R_2 = effective feedback resistance in ohms
σ = root of gain-bandwidth equation

$$u = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}$$

x = $Q_1 u$
x_c = value of x at $a = 1/2$
$\boldsymbol{Z}$ = parallel-tuned circuit impedance in ohms
$\boldsymbol{Z}_2$ = feedback-tuned circuit impedance in ohms.

Feedback Couple Analysis

The complex impedance of a two-pole formed by shunt-connected inductance L, capacitance C, and resistance R is

$$\boldsymbol{Z} = \frac{R}{1 + jQu}$$

where

$$Q = \frac{R}{\omega_0 L},$$

and

$$u = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \approx \frac{\Delta\omega}{\omega_0}.$$

In the circuit of Fig. 1

$$A_c = \frac{A}{1 + \left[\frac{A}{N} + g_m \boldsymbol{Z}_2\right]\left[\frac{R_0}{R_0 + \boldsymbol{Z}_2}\right]}$$

where it is assumed that $N^2 R_2 \gg R_1$ and each plate resistance is much larger than its corresponding load impedance, so that

$$A = \frac{A_0}{(1 + jQ_1 u)^2}.$$

If it is also assumed that

$$\left|\frac{A}{N}\right| \gg |g_m \boldsymbol{Z}_2|,$$

then

$$A_c = \frac{A}{1 + \frac{A}{N}\left[\frac{R_0}{R_0 + \boldsymbol{Z}_2}\right]}$$

which then becomes

$$A_c = \frac{A_0(1+jPx)}{1+B-(1+2P)x^2+j\left[2+\frac{A_0P}{N}+P(1-x^2)\right]x}$$

where

$$B = \frac{A_0}{N\left[1 + \frac{R_2}{R_0}\right]}$$

$$P = \frac{NBQ_2}{A_0Q_1}$$

and $x = Q_1u$.

If we let

$$P_1 = \left[\frac{2 + (1 + A_0/N)P}{1 + B}\right]^2 - 2\left[\frac{1 + 2P}{1 + B}\right]$$

$$P_2 = \frac{1 + 2(1 - A_0/N)P^2}{(1 + B)^2}$$

and

$$P_3 = \frac{P^2}{(1 + B)^2},$$

then the normalized gain

$$a = \left| A_c \right| \left[\frac{1 + B}{A_0}\right]$$

becomes

$$a = \left[\frac{1 + P^2x^2}{1 + P_1x^2 + P_2x^4 + P_3x^6}\right]^{1/2}.$$

The relative values of the coefficients in this equation determine the shape of the normalized response.

It can be seen by inspection that, for maximum flatness with no inflection,

$$P_2 = 0, \qquad P_1 = P^2.$$

These two conditions give

$$\frac{A_0}{N} = 1 + \frac{1}{2P^2}$$

and

$$1 + B = \frac{1 + 2P}{P^2}\left[\sqrt{1 + (\tfrac{1}{2} + P)^2} - 1\right]. \qquad (3)$$

Plots of these two functions are given in Fig. 4. The equation for a now becomes

$$a = \left[\frac{1}{1 + \frac{P^2x^6}{(1 + B)^2(1 + P^2x^2)}}\right]^{1/2}. \qquad (4)$$

Plots of this function are given in Fig. 2.

It should be noted that responses with almost any degree of peaking are available with different choices of the coefficients P_1, P_2, and P_3. For any particular set of these coefficients, it is necessary to derive new expressions for A_0/N, $1+B$, and a in terms of P. Responses may be obtained which give three peaks to the usual selectivity curve, or two inflections in the equation for a. A slightly peaked response with one inflection is obtained by setting

$$P_2 = -1/[5(1 + B)^2], \qquad P_1 = P^2.$$

The corresponding functions are plotted in Figs. 3 and 4.

Gain-Bandwidth Product

It can be shown that

$$\Pi_B = \Pi_0 \frac{x_c}{\sqrt{1 + B}}$$

where Π_0 is the gain-bandwidth product for a single-tuned stage, Π_B is the product for the feedback couple, and x_c is the value of x at $a=1/2$. The ratio $x_c/\sqrt{1+B}$ is approximately 1.7 over the useful range of $1+B$ in Fig. 2, and therefore

$$\Pi_B \approx 1.7\Pi_0. \qquad (2)$$

It is interesting to examine the limiting value of Π_B as B and A_0 become infinite. Substitution of $a=1/2$, $x=x_c$ in (4) yields

$$\sigma^3 = 3\left[\sigma + \frac{1}{P^2(1 + B)}\right]$$

where

$$\sigma = \frac{x_c^2}{1 + B}.$$

From (3), we note that

$$\lim_{B\to\infty} P^2(1 + B) = \sqrt{5/4} - 1.$$

An approximate solution for the real root gives

$$\lim_{B\to\infty} \sigma \approx 3.279$$

and therefore

$$\lim_{B\to\infty} \Pi_B \approx 1.811\Pi_0.$$