Función de Onda

miércoles, 9 de agosto de 2023 9:07 p. m.

Patrón de Interferencia

$\Psi(x,t) = A e^{i(\vec{n}\cdot\vec{x} - \omega t)} \to E(x,t)$

$\vec{E}(x,t) = \vec{E}_1(x,t) + \vec{E}_2(x,t)$. Intensidad $\to I \propto |\vec{E}|^2 = |\vec{E}_1|^2 + |\vec{E}_2|^2 + 2\vec{E}_1\cdot\vec{E}_2$

Para Complejos $(E_1+E_2)(E_1^*+E_2^*) = |E_1|^2 + |E_2|^2 + E_1E_2^* + E_1^*E_2$

$\to 2\,Re(E_1 E_2^*) =$

Función de onda

Ecuación de Schrödinger Hipótesis $\rho(\vec{x},t)\, d^3x = |\Psi(x,t)|^2 d^3x$

$\to$ Densidad de probabilidad

• Ecuación de movimiento

1. $\frac{d^2x}{dt^2} = \frac{F}{m} \to x \xrightarrow{\iint} x(t)$

$\Psi(x,t_0) \to \Psi(x,t)$ Ecuación Dif. de primer orden para t

2. Lineal $L\Psi = 0$ 3. $\int |\Psi(x,t)|^2 d^3x = 1$

4. $\Psi(x,t) = A e^{i(k\cdot x - \omega t)} = A e^{\frac{i}{\hbar}(\vec{p}\cdot\vec{x} - \frac{p^2}{2m}t)}$ $\frac{\partial\Psi}{\partial t} = L\Psi + c$

$\Psi(x,t) = A e^{\frac{i}{\hbar}(\vec{p}\cdot\vec{x} - \frac{p^2}{2m}t)}$

$\frac{\partial\Psi}{\partial t} = \frac{-i p^2}{2\hbar m}\Psi$. $\nabla\Psi = \frac{i}{\hbar}\vec{p}\,\Psi \to \nabla^2\Psi = -\frac{p^2}{\hbar^2}\Psi$

$\frac{p^2}{\hbar^2}\Psi = -\nabla^2\Psi \to \frac{\partial\Psi}{\partial t} = -\frac{i\hbar\, p^2}{2m\hbar^2}\Psi = \frac{i\hbar}{2m}\nabla\Psi = \frac{-\hbar}{2mi}\nabla\Psi$

$\hbar i\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi$ Ec. de Schrödinger partícula libre.

Para una solución general $\to \Psi(x,t) = \sum_{k=1}^{n} \underbrace{A(p_k)}_{\text{Función de peso}} e^{i(\vec{p}_k\cdot\vec{x} - \frac{p_k^2}{2m}t)}$

Normalización de ec. De onda

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Paso al continuo

$$\varphi(p) = A e^{\left(\frac{-(\vec{p}-\vec{p}_0)^2 \, \delta^2}{\hbar^2}\right)}$$

$$\Psi(x,t) = \int \frac{d^3p}{(2\pi\hbar)^3} \underbrace{\varphi(p)}_{} e^{\frac{i}{\hbar}\left(\vec{p}\cdot\vec{x} - \frac{p^2}{2m}t\right)}$$

Función de peso → Dist. Gaussiana

$$\Psi(x,t) = \int \frac{d^3p}{(2\pi\hbar)^3} A e^{\underbrace{\frac{i}{\hbar}\left(p\cdot x - \frac{p^2}{2m}t\right) - \frac{1}{\hbar^2}\left(p^2 + p_0^2 - 2\vec{p}\cdot\vec{p}_0\right)\delta^2}} \Rightarrow 1 \text{ dimensión } \int \frac{dp}{2\pi\hbar} A e^{(-ap^2 + bp + c)}$$

$$-ap^2 + bp + c = -\left(\frac{\delta^2}{\hbar^2} + \frac{it}{2m\hbar}\right)p^2 + \left(\frac{p_0\delta^2}{\hbar^2} + i\frac{x}{2\hbar}\right)p + \left(-\frac{\delta^2 p_0^2}{\hbar^2}\right)$$

$$\tilde{F}(\omega) = \int \frac{dt}{\sqrt{2\pi}} e^{-i\omega t} F(t)$$

$$\Psi(x,t) = \int \frac{dp}{(2\pi\hbar)} A e^{\frac{i}{\hbar}\left(p\cdot x - \frac{p^2}{2m}t\right) - \frac{1}{\hbar^2}\left(p^2 + p_0^2 - 2\vec{p}\cdot\vec{p}_0\right)} \Rightarrow \int \frac{dp}{2\pi\hbar} A e^{\left(-a\left(p - \frac{b}{a}\right)^2 + \frac{b^2}{a} - c\right)}$$

$$\Psi(x,t) = \frac{A}{2\pi\hbar}\sqrt{\frac{\pi}{a}}\, e^{\left(\frac{b^2}{a} - c\right)} \rightarrow \text{Módulo}$$

$$|\Psi(x,t)|^2 = \frac{A^2 \pi}{(2\pi\hbar)^2 |a|} \left| e^{\left(\frac{b^2 - ca}{a}\right)} \right|^2$$

$$|e^w|^2 = e^w \cdot e^{w^*} = e^{w + w^*} = e^{2Re(w)} = e^{2Re\left(\frac{b^2 - ca}{a}\right)} = e^{2\frac{-(x-vt)^2}{4(\delta^2+\Delta^2)}} = e^{\frac{-(x-vt)^2}{2(\delta^2+\Delta^2)}}$$

$$\frac{-x^2}{4\hbar^2} + i\left(\frac{\delta^2 p_0}{\hbar^3}\left(x - \frac{v_0 t}{2}\right)\right)$$

$$|\Psi(x,t)|^2 = \frac{A^2 \pi}{(2\pi\hbar)^2 |a|} e^{\frac{(x-vt)^2}{2(\delta^2+\Delta^2)}}$$

$$\frac{(x - vt)^2}{4(\delta^2+\Delta^2)}$$

$$\sqrt{\alpha}$$

$$\int dx |\Psi(x,t)|^2 = 1 = \frac{A^2}{4\pi\hbar^2 |a|} \int e^{\frac{-(x-vt)^2}{2\delta^2(1+\delta^2)}} dx$$

$$= \frac{A^2}{4\pi\hbar^2|a|}\sqrt{\frac{\pi}{\frac{1}{2\delta^2(1+\delta^2)}}} = \frac{A^2}{4\pi}$$

$$= b^2 - ca$$

$$= \frac{A^2\, 2m\hbar^2}{4\pi\hbar^2 \sqrt{4\delta^2 \hbar^2 + t^2\hbar^2}} \cdot \left(\sqrt{\pi\left(2\delta^2 + \frac{2\delta^2 + t^2\hbar^2}{4\delta^4 m^2}\right)}\right)$$

$$\left(\frac{\hbar\, it}{2m}\right) \quad 2Re\left(\frac{(b^2 - ca)}{a}\frac{a^*}{a^*}\right) = \frac{2Re\left((b^2 - ca)\, a^*\right)}{a}$$

$$\int_{-\infty}^{\infty} e^{-x^2} dx = I \Rightarrow I^2 = \left(\int_{-\infty}^{\infty} e^{-x^2} dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2} dy\right)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)} dx\, dy = \int_0^{2\pi}\int_0^{\infty} e^{-r^2} r\, dr\, d\theta$$

$$u = r^2 \qquad du = 2r\,dr \qquad = 2\pi \int_0^\infty e^{-u} \frac{du}{2} = \pi$$

$$\frac{A^2 m}{2\sqrt{\pi}\,\hbar}\sqrt{\frac{\pi\left(4\delta^6 m^2 + 2\delta^2 + t^2\hbar^2\right)}{4\delta^6 m^4 + \delta^4 t^2 \hbar^2 m^2}}$$

$$|a|^2 = \frac{\delta^4}{\hbar^4} + \frac{t^2}{4m^2\hbar^2} \Rightarrow |a| = \sqrt{\frac{\delta^4}{\hbar^4} + \frac{t^2}{4m^2\hbar^2}}$$

$$|a| = \frac{\sqrt{4\delta^4 m^2 + t^2\hbar^2}}{2m\hbar^2}$$

$$\frac{A^2 \sqrt{2\pi}\,\delta}{2\pi\hbar \cdot \sqrt{4\delta^4 m^2 + t^2\hbar^2}} \ldots = 1 \Rightarrow A^2 = \sqrt{8\pi\delta^2}$$

$$A = \sqrt{8\pi\delta^2}$$

$$I = \int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

$$|a| = \sqrt{4\delta^2 m^2 + t^2}$$

$$\int e^{\frac{-(x-vt)^2}{2(\delta^2+\Delta^2)}} dx = \int_{-\infty}^{\infty} e^{\frac{-(x-vt)^2}{2(\delta^2+\Delta^2)}} dx$$

$$u = (x - vt) \qquad du = dx \qquad \int_{-\infty}^{\infty} e^{-u^2/2(\delta^2+\Delta^2)} du =$$

$$t = \frac{u}{\sqrt{2\delta^2+\Delta^2}} \qquad dt\sqrt{2\delta^2+\Delta^2} = du \qquad = \alpha\int_{-\infty}^{\infty} e^{-t^2} dt = \sqrt{\frac{\pi}{2\delta^2+\Delta^2}}$$

$$\left(\frac{\delta^2 p_0}{\hbar^2} + i\frac{x}{2\hbar}\right)^2 - \frac{\delta^2 p_0^2}{\hbar^2}\left(\frac{\delta^2}{\hbar^2} + \frac{it}{2m\hbar}\right)$$

$$= \cancel{\frac{\delta^4 p_0^2}{\hbar^4}} + i\frac{\delta^2 p_0 x}{\hbar^3} - \frac{x^2}{4\hbar^2} - \cancel{\frac{\delta^4 p_0^2}{\hbar^4}} - \frac{i\,\delta^2 p_0^2 t}{2m\hbar^3}$$

$$= \frac{-x^2}{4\hbar^2} + i\left(\frac{\delta^2 p_0}{\hbar^3}\left(x - \frac{p}{2m}t\right)\right) =$$

$$(b^2 - ca)\, a^* = \left(\left(\frac{-x^2}{4\hbar^2}\right) + i\left(\frac{\delta^2 p_0}{\hbar^3}\left(x - \frac{v_0 t}{2}\right)\right)\right).$$

$$= \frac{-\delta^2 x^2}{4\hbar^4} + \frac{ixt}{4m\hbar^3} + \frac{i\,\delta^4}{\hbar^5} p_0\left(x - \frac{v_0 t}{2}\right) + \frac{\delta^2 p_0 t}{4m^2\hbar^4}\left(x - \frac{v_0 t}{2}\right)$$

$$\frac{Re\left((b^2 - ca)\, a^*\right)}{|a|^2} = \frac{\frac{\delta^2 p_0 t}{2m\hbar^4}\left(x - \frac{v_0 t}{2}\right) - \frac{\delta^2 x^2}{4\hbar^4}}{\ldots} = \frac{\frac{\delta^2}{\hbar^4}\left(\frac{p_0 t}{2m}\left(x - v \ldots\right)\right)}{\frac{\delta^2}{\hbar^4}\left(\delta^2 + \frac{t^2}{4m^2} \ldots\right)}$$

$$\Delta = \Delta(t) = \frac{t\hbar}{2m\delta^2}$$

$$= \frac{vt}{2}x - \frac{v^2 t^2}{4} - \frac{x^2}{4} = \frac{1}{4}\left(x^2 + 2vtx - (vt)^2\right)$$

Condición de normalización

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$$\int |\Psi(x,t)|^2 dx = \int \left| \frac{A}{2\pi\hbar|a|} e^{-\frac{(x-vt)^2}{2d^2(1+\Delta^2)}} \right| = \frac{A^2}{\underbrace{4\pi^2\hbar^2|a|}_{0}} \cdot \sqrt{2\pi d^2(1+\Delta^2)} = A^2\sqrt{\frac{2\pi d^2 \cancel{(1+\Delta^2)}}{16\pi^4 d^4 \cancel{(1+\Delta^2)}}} = A^2\sqrt{\frac{1}{8\pi d^2}} = 1$$

$$\boxed{A^2 = \sqrt{8\pi}\, d}$$

$$4\pi\hbar^2\sqrt{\frac{d^4}{\hbar^4}+\frac{t^2}{4\hbar^2 m^2}} = 4\pi\hbar^2\sqrt{\frac{4d^4\hbar^2m^2+t^2\hbar^2}{4\hbar^6m^2}}$$

$$= \frac{4\pi^2\cancel{\hbar^2}}{2\hbar^{\cancel{3}}m} = \frac{2\pi}{\hbar m}\sqrt{4d^4\hbar^2m^2(1+\Delta^2)}$$

$$= \frac{4\pi^2 d^2 \cancel{\hbar m}}{\cancel{\hbar m}}\sqrt{1+\Delta^2}$$

$$= 4\pi^2 d^2\sqrt{1+\Delta^2}$$