Función de Onda

miércoles, 9 de agosto de 2023 9:07 p. m.

Patrón de Interferencia

$$\Psi_{(x,t)} = A e^{i(\vec{n}\cdot\vec{x} - \omega t)} \rightarrow E(x,t)$$

$\vec{E}(x,t) = \vec{E}_1(x,t) + \vec{E}_2(x,t)$. Intensidad $\rightarrow I \propto |\vec{E}|^2 = |\vec{E}_1|^2 + |\vec{E}_2|^2 + 2\vec{E}_1\cdot\vec{E}_2$

Para Complejos $(E_1+E_2)(E_1^* + E_2^*) = |E_1|^2 + |E_2|^2 + E_1E_2^* + E_1^*E_2$

$\rightarrow 2\,Re(E_1 \cdot E_2^*) =$

Función de onda

Ecuación de ~~~~nrödinger Hipótesis $\rho(\vec{x},t)\, d^3x = |\Psi(x,t)|^2 d^3x$

$\rightarrow$ Densidad de probabilidad

• Ecuación de movimiento

1. $\dfrac{d^2x}{dt^2} = \dfrac{F}{m} \rightarrow x \xrightarrow{\iint} x(t)$

$\Psi(x,t_0) \rightarrow \Psi(x,t)$ Ecuación Dif. de primer orden para t

2. Lineal $L\Psi = 0$ 3. $\int |\Psi(x,t)|^2 d^3x = 1$

4. $\Psi(x,t) = A e^{i(k\cdot x - \omega t)} = A e^{\frac{i}{\hbar}(\vec{p}\cdot\vec{x} - \frac{p^2}{2m}t)}$ $\dfrac{\partial\Psi}{\partial t} = L\Psi + c$

$$\Psi(x,t) = A e^{\frac{i}{\hbar}(\vec{p}\cdot\vec{x} - \frac{p^2}{2m}t)}$$

$$\frac{\partial\Psi}{\partial t} = \frac{-ip^2}{2\hbar m}\Psi. \quad \nabla\Psi = \frac{i}{\hbar}\vec{p}\,\Psi \rightarrow \nabla^2\Psi = -\frac{p^2}{\hbar^2}\Psi$$

$$\frac{p^2}{\hbar^2}\Psi = -\nabla^2\Psi \rightarrow \frac{\partial\Psi}{\partial t} = -\frac{i\hbar\, p^2}{2m\hbar^2}\Psi = \frac{i\hbar}{2m}\nabla\Psi = \frac{-\hbar}{2mi}\nabla\Psi$$

$$\hbar i \frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi$$

Ec. de Schrödinger partícula libre.

Para una solución general $\rightarrow \Psi(x,t) = \sum_{l=1}^{n} \underbrace{A(p_l)}_{\text{Función de peso}}\, e^{i(\vec{p}_l\cdot\vec{x} - \frac{p_l^2}{2m}t)}$

Normalización de ec. De onda

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Paso al continuo

$$\Psi(x,t) = \int \frac{d^3p}{(2\pi\hbar)^3}\, \underbrace{\varphi(p)}_{\text{Función de peso} \to \text{Dist. Gaussiana}}\, e^{\frac{i}{\hbar}\left(\vec{p}\cdot\vec{x} - \frac{p^2}{2m}t\right)}$$

$$\varphi(p) = A\, e^{\left(-\frac{(\vec{p}-\vec{p}_0)^2 d^2}{\hbar^2}\right)}$$

$$\Psi(x,t) = \int \frac{d^3p}{(2\pi\hbar)^3} A e^{\frac{i}{\hbar}\left(p\cdot x - \frac{p^2}{2m}t\right) - \frac{1}{\hbar^2}\left(p^2+p_0^2-2\vec{p}\cdot\vec{p}_0\right)d^2} \Rightarrow 1 \text{ dimensión } \int \frac{dp}{2\pi\hbar} A e^{(-ap^2+bp+c)}$$

$$-ap^2+bp+c = -\left(\frac{d^2}{\hbar^2}+\frac{it}{2m\hbar}\right)p^2 + \left(\frac{p_0 d^2}{\hbar^2} + i\frac{x}{2\hbar}\right)p + \left(-\frac{d^2 p_0^2}{\hbar^2}\right)$$

$$\tilde{F}(\omega) = \int \frac{dt}{\sqrt{2\pi}}\, e^{-i\omega t} f(t)$$

$$\Psi(x,t) = \int \frac{dp}{(2\pi\hbar)} A e^{\frac{i}{\hbar}\left(p\cdot x - \frac{p^2}{2m}t\right) - \frac{1}{\hbar^2}\left(p^2+p_0^2-2\vec{p}\cdot\vec{p}_0\right)} \Rightarrow \int \frac{dp}{2\pi\hbar} A e^{\left(-a\left(p-\frac{b}{2a}\right)^2 + \frac{b^2}{a} - c\right)}$$

$$\Psi(x,t) = \frac{A}{2\pi\hbar}\sqrt{\frac{\pi}{a}}\, e^{\left(\frac{b^2}{a}-c\right)} \xrightarrow{\text{Módulo}} |\Psi(x,t)|^2 = \frac{A^2\pi}{(2\pi\hbar)^2|a|}\left|e^{\left(\frac{b^2-ca}{a}\right)}\right|^2$$

$$|e^w|^2 = e^w \cdot e^{w^*} = e^{w+w^*} = e^{2\mathrm{Re}(w)} = e^{2\mathrm{Re}\left(\frac{b^2-ca}{a}\right)} = e^{2\frac{-(x-vt)^2}{4(d^2+\Delta^2)}} = e^{\frac{-(x-vt)^2}{2(d^2+\Delta^2)}}$$

$$|\Psi(x,t)|^2 = \frac{A^2\pi}{(2\pi\hbar)^2|a|}\, e^{\frac{(x-vt)^2}{2(d^2+\Delta^2)}}$$

$$2\mathrm{Re}\left(\frac{(b^2-ca)}{a}\frac{a^*}{a^*}\right) = 2\mathrm{Re}\left(\frac{(b^2-ca)a^*}{a}\right)$$

$$\int_{-\infty}^{\infty} e^{-x^2}dx \Rightarrow I^2 = \left(\int_{-\infty}^{\infty} e^{-x^2}dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2}dy\right) = \dots = \pi \Rightarrow I = \sqrt{\pi}$$

$$|a|^2 = \dots \qquad |a| = \sqrt{\frac{\dots}{2m\hbar^3}} \qquad |a| = \frac{\sqrt{\dots}}{2m\hbar^2}$$

$$\int e^{-\frac{(x-vt)^2}{2(d^2+\Delta^2)}}dx = \int_{-\infty}^{\infty} e^{-\frac{(x-vt)^2}{2(d^2+\Delta^2)}}dx \qquad u = (x-vt),\ du = dx \qquad = \int_{-\infty}^{\infty} e^{-u^2/2(d^2+\Delta^2)}du$$

$$t = \frac{u}{\sqrt{2d^2+\Delta^2}} \qquad dt\sqrt{2d^2+\Delta^2} = du \qquad = \alpha\int_{-\infty}^{\infty} e^{-t^2}dt = \sqrt{\frac{\pi}{2d^2+\Delta^2}}$$

$$\int dx\,|\Psi(x,t)|^2 = 1 = \frac{A^2}{4\pi\hbar^2|a|}\int e^{\frac{-(x-vt)^2}{2d^2(1+\Delta^2)}}dx$$

$$= \frac{A^2}{4\pi\hbar^2|a|}\sqrt{\frac{\pi}{\frac{1}{2d^2(1+\Delta^2)}}} = \frac{A^2}{4\pi}$$

$$= \frac{A^2\, 2m\hbar^2}{4\pi\hbar^2\sqrt{4d^4m^2+t^2\hbar^2}}\cdot\left(\sqrt{\pi\left(2d^2+\frac{2d^2t^2\hbar^2}{4d^4m^2}\right)}\right)$$

$$= \frac{A^2 m}{2\sqrt{\pi}}\sqrt{\frac{8d^6m^2+2d^2t^2\hbar^2}{16d^6m^4+4d^4t^2\hbar^2m^2}}$$

$$= \frac{A^2 m}{2\sqrt{\pi}}\cdot\left(\sqrt{\frac{2(4d^6m^2+2d^2t^2\hbar^2)}{4(4d^6m^4+d^4t^2\hbar^2m^2)}}\right)$$

$$= \frac{A^2}{2\sqrt{2\pi}}\left(\frac{4d^6m^2+2d^2t^2\hbar^2}{4d^6m^2+d^4t^2\hbar^2}\right)$$

$$= \frac{A^2}{\sqrt{8\pi d^2}} = 1 \Rightarrow A^2 = \sqrt{8\pi d^2}$$

$$A = \sqrt[4]{8\pi d^2}$$

$$= b^2 - ca$$

$$= \dots = -\frac{x^2}{4\hbar^2} + i\left(\frac{d^2p_0}{\hbar^3}\left(x - \frac{v_0t}{2}\right)\right)$$

$$\mathrm{Re}\left(\frac{(b^2-ca)a^*}{|a|^2}\right) = \dots$$

$$\Delta(t) = \dots$$

$$= \dots = \frac{(x-vt)^2}{4(d^2+\Delta^2)}$$

Condición de normalización

lunes, 14 de agosto de 2023 2:25 p. m.

$$\int |\Psi(x,t)|^2 dx = \int \left| \frac{A}{2\pi\hbar|a|} e^{\frac{-(x-vt)^2}{2\delta^2(1+\Delta^2)}} \right| = \frac{A^2}{\underbrace{4\pi^2\hbar^2|a|}_{0}} \cdot \sqrt{2\pi\delta^2(1+\Delta^2)} = A^2 \sqrt{\frac{2\pi\delta^2\cancel{(1+\Delta^2)}}{16\pi^4\delta^4\cancel{(1+\Delta^2)}}} = A^2\sqrt{\frac{1}{8\pi\delta^2}} = 1$$

$$\boxed{A^2 = \sqrt{8\pi}\,\delta}$$

$$4\pi\hbar^2\sqrt{\frac{\delta^4}{\hbar^4} + \frac{t^2}{4\hbar^2 m^2}} = 4\pi\hbar^2\sqrt{\frac{4\delta^4\hbar^2 m^2 + t^2\hbar^2}{4\hbar^6 m^2}}$$

$$= \frac{4\pi^2\cancel{\hbar^2}}{2\hbar^{\cancel{3}} m} = \frac{2\pi}{\hbar m}\sqrt{4\delta^4\hbar^2 m^2(1+\Delta^2)}$$

$$= \frac{4\pi^2\delta^2\cancel{\hbar m}}{\cancel{\hbar m}}\sqrt{1+\Delta^2}$$

$$= 4\pi^2\delta^2\sqrt{1+\Delta^2}$$