Normalización de ec. De onda

viernes, 11 de agosto de 2023 8:21 a. m.

Paso al continuo

$$\varphi(p) = A e^{\left(\frac{-(\vec{p}-\vec{p}_0)^2 \partial^2}{\hbar^2}\right)}$$

$$\Psi(x,t) = \int \frac{d^3p}{(2\pi\hbar)^3}\, \underset{\sim}{\varphi(p)}\, e^{\frac{i}{\hbar}\left(\vec{p}\cdot\vec{x} - \frac{p^2}{2m}t\right)}$$

Función de peso → Dist. Gaussiana

$$\Psi(x,t) = \int \frac{d^3p}{(2\pi\hbar)^3} A e^{\underbrace{\frac{i}{\hbar}\left(p\cdot x - \frac{p^2}{2m}t\right) - \frac{1}{\hbar^2}\left(p^2 + p_0^2 - 2\vec{p}\cdot\vec{p}_0\right)\partial^2}} \Rightarrow 1 \text{ dimensión } \int \frac{dp}{2\pi\hbar} A e^{(-ap^2+bp+c)}$$

$$-ap^2 + bp + c = -\left(\frac{\partial^2}{\hbar^2} + \frac{it}{2m\hbar}\right)p^2 + \left(\frac{p_0\partial^2}{\hbar^2} + i\frac{x}{2\hbar}\right)p + \left(-\frac{\partial^2 p_0^2}{\hbar^2}\right)$$

$$\tilde{F}(\omega) = \int \frac{dt}{\sqrt{2\pi}} e^{-i\omega t} F(t)$$

$$\Psi(x,t) = \int \frac{dp}{(2\pi\hbar)} A e^{\frac{i}{\hbar}\left(p\cdot x - \frac{p^2}{2m}t\right) - \frac{1}{\hbar^2}\left(p^2 + p_0^2 - 2\vec{p}\cdot\vec{p}_0\right)} \Rightarrow \int \frac{dp}{2\pi\hbar} A e^{\left(-a\left(p - \frac{b}{a}\right)^2 + \frac{b^2}{a} - c\right)}$$

$$\Psi(x,t) = \frac{A}{2\pi\hbar}\sqrt{\frac{\pi}{a}}\, e^{\left(\frac{b^2}{a} - c\right)} \to \text{Módulo}$$

$$|\Psi(x,t)|^2 = \frac{A^2\pi}{(2\pi\hbar)^2|a|}\left|e^{\left(\frac{b^2-ca}{a}\right)}\right|^2$$

$$|e^w|^2 = e^w \cdot e^{w^*} = e^{w+w^*} = e^{2Re(w)} = e^{2Re\left(\frac{b^2-ca}{a}\right)} = e^{\frac{-(x-vt)^2}{4(\partial^2+\Delta^2)}} = e^{\frac{-(x-vt)^2}{2(\partial^2+\Delta^2)}}$$

$$\frac{-x^2}{4\hbar^2} + i\left(\frac{\partial^2 p_0}{\hbar^3}\left(x - \frac{v_0 t}{2}\right)\right)$$

$$|\Psi(x,t)|^2 = \frac{A^2\pi}{(2\pi\hbar)^2|a|} e^{\frac{(x-vt)^2}{2(\partial^2+\Delta^2)}}$$

$$\frac{x-vt)^2}{4(\partial^2+\Delta^2}$$

$$= b^2 - ca$$

$$\int dx\, |\Psi(x,t)|^2 = 1 = \frac{A^2}{4\pi\hbar^2|a|}\int e^{\frac{-(x-vt)^2}{2\partial^2(1+\delta^2)}} dx$$

$$= \frac{A^2}{4\pi\hbar^2|a|}\sqrt{\frac{\pi}{\frac{1}{2\partial^2(1+\delta^2)}}} = \frac{A^2}{4\pi}$$

$$= \frac{A^2\, 2m\hbar^2}{4\pi\hbar^2\sqrt{4\partial^2\hbar^2 + t^2\hbar^2}} \cdot \left(\sqrt{\frac{\pi\left(2\partial^2 + 2\partial^2 + t^2\hbar^2\right)}{4\partial^4 m^2}}\right)$$

$$2Re\left(\frac{(b^2-ca)}{a}\frac{a^*}{a^*}\right) = \frac{2Re\left((b^2-ca)a^*\right)}{a}$$

$$\int_{-\infty}^{\infty} e^{-x^2} dx = I \Rightarrow I^2 = \left(\int_{-\infty}^{\infty} e^{-x^2} dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2} dy\right)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)} dx\, dy = \int_0^{2\pi}\int_0^{\infty} e^{-r^2} r\, dr\, d\theta$$

$u = r^2$, $du = 2r\,dr$ $= \pi$

$$I = \int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

$$|a| = \sqrt{\frac{\partial^4}{\hbar^4} + \frac{t^2}{4m^2\hbar^2}}$$

$$\frac{A^2 m}{2\sqrt{\pi}} \sqrt{\frac{\pi\left(4\partial^6 m^2 + 2\partial^4 t^2\hbar^2\right)}{4\partial^6 m^4 + \partial^4 t^2\hbar^2 m^2}}$$

$$\frac{A^2}{\sqrt{8\pi\partial^2}} = 1 \Rightarrow A^2 = \sqrt{8\pi\partial^2}$$

$$A = \sqrt{8\pi\partial^2}$$

$$\int e^{\frac{-(x-vt)^2}{2(\partial^2+\Delta^2)}} dx = \int_{-\infty}^{\infty} e^{\frac{-(x-vt)^2}{2(\partial^2+\Delta^2)}} dx$$

$u = (x-vt)$, $du = dx$

$$\int_{-\infty}^{\infty} e^{-u^2/2(\partial^2+\Delta^2)} du =$$

$t = \frac{u}{\sqrt{2\partial^2+\Delta^2}}$, $dt\sqrt{2\partial^2+\Delta^2} = du$

$$= \alpha\int_{-\infty}^{\infty} e^{-t^2} dt = \sqrt{\frac{\pi}{2\partial^2+\Delta^2}}$$

$$\left(\frac{\partial^2 p_0}{\hbar^2} + i\frac{x}{2\hbar}\right)^2 - \frac{\partial^2 p_0^2}{\hbar^2}\left(\frac{\partial^2}{\hbar^2} + \frac{it}{2m\hbar}\right)$$

$$= \frac{\partial^4 p_0^2}{\hbar^4} + i\frac{\partial^2 p_0 x}{\hbar^3} - \frac{x^2}{4\hbar^2} - \frac{\partial^4 p_0^2}{\hbar^4} - \frac{i\,\partial^2 p_0^2 t}{2m\hbar^3}$$

$$= \frac{-x^2}{4\hbar^2} + i\left(\frac{\partial^2 p_0}{\hbar^3}\left(x - \frac{p}{2m}t\right)\right) =$$

$$(b^2 - ca)a^* = \left(\left(\frac{-x^2}{4\hbar^2}\right) + i\left(\frac{\partial^2 p_0}{\hbar^3}\left(x - \frac{v_0 t}{2}\right)\right)\right).$$

$$= \frac{-\partial^2 x^2}{4\hbar^4} + \frac{ixt}{4m\hbar^3} + \frac{i\,\partial^4}{\hbar^5}p_0\left(x - \frac{v_0 t}{2}\right) + \frac{\partial^2 p_0 t}{2m\hbar^4}\left(x - \frac{v_0 t}{2}\right)$$

$$\frac{Re\left((b^2-ca)a^*\right)}{|a|^2} = \frac{\frac{\partial^2 p_0 t}{2m\hbar^4}\left(x - \frac{v_0 t}{2}\right) - \frac{\partial^2 x^2}{4\hbar^4}}{\frac{\partial^2}{\hbar^4}\left(\partial^2 + \frac{t^2}{4m^2}\right)}$$

$$\Delta = \Delta(t) = \frac{t\hbar}{2m\partial^2}$$

$$= \frac{vt}{2}x - \frac{v^2t^2}{4} - \frac{x^2}{4} = \frac{1}{4}\left(x^2 + 2vtx - (vt)^2\right)$$