Instructions: Please submit a **single pdf file** to gradescope. Each question is worth 25 points.

1. Let X be a finite set of cardinality n and $0 \leq r \leq n$. Provide a bijection between the sets A and B defined below. Prove that the function you provide is in fact a bijection.
   $A =$ Set of all subsets of X of size r.
   $B =$ Set of all strings of length n with exactly r 1's.
   For example, when $X = \{a_1, a_2, a_3\}$ and $r = 2$, we have $A = \{\{a_1, a_2\}, \{a_1, a_3\}, \{a_2, a_3\}\}$ and $B = \{110, 101, 011\}$.
   Hint: Give an inverse for your function instead of proving that it is $1 - 1$ or onto.

2. Consider a square having sides of length 2 and let $p_1, \ldots p_5$ be five distinct points in the interior of the square. Prove that there are at least two distinct points p_i and p_j such that the distance between them is at most $\sqrt{2}$. Hint: What are the pigeons and what are the holes?

3. Find the number of strings of length 7 with at most three 0's.

4. Find the coefficient of x^{94} in $(x + \frac{1}{x^2})^{100}$.