ASSIGNMENT-3
CS 2214: DISCRETE STRUCTURES FOR COMPUTING
DUE FEBRUARY 16 TH 2020, 11:55 PM

Instructions: Please submit a **single pdf file** to gradescope.

1. Consider the two systems of linear equations

$$x'' = ax' + by' \qquad\qquad x' = px + qy$$
$$y'' = cx' + dy' \qquad\qquad y' = rx + sy$$

We may rewrite the above equations as $\begin{pmatrix} x'' \\ y'' \end{pmatrix} = A \begin{pmatrix} x' \\ y' \end{pmatrix}$ and $\begin{pmatrix} x' \\ y' \end{pmatrix} = B \begin{pmatrix} x \\ y \end{pmatrix}$, where $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $B = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ are the coefficient matrices. Suppose C is a matrix such that $\begin{pmatrix} x'' \\ y'' \end{pmatrix} = C \begin{pmatrix} x \\ y \end{pmatrix}$. By substitution show that $C = AB$.

In summary, matrix multiplication appears naturally via substitution of one set of linear equations into another.

2. Find a closed form formula for

$$\sum_{k=0}^{n} (3k+1)^2 = 1^2 + 4^2 + \ldots + (3n+1)^2$$

Hint: Use the formula for $\sum_{k=1}^{n} k^2$ and $\sum_{k=1}^{n} k$ derived in class.

3. Prove that 13 divides $3^{n+1} + 4^{2n-1}$ for every positive integer n using induction.

4. Let $\{F_n\}_{n\geq 0}$ denote the Fibonacci sequence. Prove that, if

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

then

$$A^n = \begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix}$$

for $n \geq 1$ using induction.