CS2214 - Assignment 1

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Question 1

Part (i)

We show that the proposition $s = (p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge r)$ satisfies the given requirement.

p	q	r	$p \wedge q \wedge \neg r$	$\neg p \wedge q \wedge r$	$(p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge r)$	s
T	T	T	F	F	F	F
T	T	F	T	F	T	T
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	T	T
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Table 1: Truth Table for $(p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge r)$

Part (ii)

We show that the proposition $(p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r)$ satisfies the given requirements:

p	q	r	$p \wedge \neg q \wedge \neg r$	$\neg p \wedge q \wedge \neg r$	$\neg p \wedge \neg q \wedge r$	$(p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r)$	s
T	T	T	F	F	F	F	F
T	T	F	F	F	F	F	F
T	F	T	F	F	F	F	F
T	F	F	T	F	F	T	T
F	T	T	F	F	F	F	F
F	T	F	F	T	F	T	T
F	F	T	F	F	T	T	T
F	F	F	F	F	F	F	F

Table 2: Truth Table for $(p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r)$

Question 2

Part (i)

We construct a truth table for this proposition. Since all of its rows show T, we conclude it is a tautaology.

p	q	r	$p \wedge q \wedge r$	$p \wedge q$	$(p \wedge q) \vee r$	$(p \wedge q \wedge r) \to ((p \wedge q) \vee r)$
T	T	T	T	T	T	T
T	T	F	F	T	T	T
T	F	T	F	F	T	T
T	F	F	F	F	F	T
F	T	T	F	F	T	T
F	T	F	F	F	F	T
F	F	T	F	F	T	T
F	F	F	F	F	F	T

Table 3: Truth Table for $(p \wedge q \wedge r) \to ((p \wedge q) \vee r)$

Part (ii)

This proposition is not a tautology, but it is satisfiable as seen in the table below.

p	$\neg p$	$p \to \neg p$
T	F	F
F	T	T

Table 4: Truth Table for $p \to \neg p$

Part (iii)

This proposition is not a tautology, but it is satisfiable as seen in the table below.

p	q	$p \to q$	$(p \to q) \wedge q$	$((p \to q) \wedge q) \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Table 5: Truth Table for $((p \to q) \wedge q) \to p$

Question 3

Let the following propositions be labelled as follows: f = "Gail scored a 100 in the final." a = "Gail scored 80% in the assignments." c = "Gail will get an A in the course."

As such, we can write (i) and (ii) using the labels above as:

- (i): $(f \wedge a) \rightarrow c$
- (ii): $f \rightarrow (\neg a \vee c)$

Note if (i) and (ii) were equivalent, one can arrive from one to the other. We claim that in fact both are equivalent to each other.

Proof.

$$(f \wedge a) \rightarrow c$$

$$\equiv \neg\neg((f \wedge a) \rightarrow c) \text{ (Double Negation Law)}$$

$$\equiv \neg((f \wedge a) \wedge \neg c) \text{ (Negation of Implication)}$$

$$\equiv \neg(f \wedge (a \wedge \neg c)) \text{ (Associativity of AND)}$$

$$\equiv (\neg f \vee (\neg a \vee c)) \text{ (DeMorgan's Law)}$$

$$\equiv f \rightarrow (\neg a \vee c) \text{ (Implication Equivalence*)}$$

□

This proof is bi-directional, so it proves the equivalence both (i) and (ii) to each other, as claimed. Finally, the last (starred) justification refers to the following:

$$(p \rightarrow q) \equiv (\neg p \vee q)$$

Question 4

Let the following predicates be labelled as follows, in which the domain $U = \{x : x \text{ is a family member of a large family}\}$.

- $T(x) =$ "x eats Tuna".
- $M(x) =$ "x eats Salmon".
- $S(x) =$ "x eats Shrimp".

The argument provided in notation form goes as follows:

A. $\forall x \in U, T(x) \to M(x)$
B. $\exists x \in U, S(x)$
C. $\forall x \in U, \neg(M(x) \wedge S(x))$

$\therefore \exists x \in U, \neg T(x)$
We claim the argument above is valid, and we shall prove it using rules of inference. Before that, however, we shall re-write the premises A and C so that we have:
A'. $\forall x \in U, \neg T(x) \vee M(x)$ (Implication Equivalence Identity)
C'. $\forall x \in U, \neg M(x) \vee \neg S(x)$ (DeMorgan's Law)
Now we proceed to the proof.

Proof.
1. $\forall x \in U, \neg M(x) \vee \neg S(x)$ (Premise C')
2. $\exists x \in U, S(x)$ (Premise B)
3. $(\forall x \in U, \neg M(x) \vee \neg S(x)) \wedge (\exists x \in U, S(x))$ (Conjunction from 1 and 2)
4. $(\exists c \in U, \neg M(c) \vee \neg S(c)) \wedge (\exists c \in U, S(c))$ (Universal Instantation and Existential Instantiation for a particular c from 3)
5. $\exists c \in U, (\neg M(c) \wedge S(c)) \vee (\neg S(c) \wedge S(c))$ ($\wedge$ Distributivity from 4)
6. $\exists c \in U, (\neg M(c) \wedge S(c)) \vee F$ (Domination Law from 5)
7. $\exists c \in U, \neg M(c) \wedge S(c)$ (Domination Law from 6)
8. $\exists c \in U, \neg M(c)$ (Simplification from 7)
9. $\forall x \in U, T(x) \to M(x)$ (Premise A)
10. $\exists c \in U, T(c) \to M(c)$ (Universal Instantiation for a particular c from 9)
11. $\exists c \in U, \neg T(c)$ (Modus Tollens from 8 and 10)
12. $\exists x \in U, \neg T(x)$ (Existential Generalization from 11)

$\square$