FINAL EXAM
CS 2214B: DISCRETE STRUCTURES FOR COMPUTING
DUE APRIL 23RD, 2020, 02:00 PM

Instructions: Please submit a **single pdf file** to gradescope. Each question is worth 20 points.

1. Let n be an odd number and X be a set with n elements. Find the no. of subsets of X which has even no. of elements. The answer should be a number depending only on n. (For example, when $n = 5$, you need to find the no.of subsets of X which has either 0 elements or 2 elements or 4 elements)
2. Let $p_1, \dots, p_{n+1}$ be n points inside a circle C such that none of them are the center of C. Let $l_1, \dots, l_{n+1}$ be the lines (radii) connecting the center and $p_1, \dots, p_{n+1}$ respectively. Prove that there are two distinct lines l_i and l_j such that the (smaller) angle between them is at most $\frac{2\pi}{n}$.
3. Let S be a finite set of propositions. We define a relation R on S as follows: we say a proposition $p \in S$ is related to a proposition $q \in S$ by R iff $p \wedge q = T$. Is R an equivalence relation? Explain why or why not.
4. Prove that $\binom{n}{k}$ is divisible by n for all $1 \leq k \leq n - 1$, when n is a prime number. Give an example of a positive integer m such that m does not divide $\binom{m}{k}$ for some $1 \leq k \leq m - 1$.
5. A set of propositions $\{p_n : n \in \mathbb{N}\}$ are defined inductively as follows:
   i. $p_0 = T$ and $p_1 = T$.
   ii. $p_{n+1} = (p_n \rightarrow p_{n-1})$.

   Prove that $p_n = T$ for all $n \geq 2$ using induction.