CS3340B - Sample Midterm Q1a

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(1)

$$n \in \Theta\left(n + \frac{n}{\log_2 n}\right)$$

Proof.

- $n \in O\left(n + \frac{n}{\log_2 n}\right)$

  $$n > \log_2 n \quad \forall n > 1 \implies \frac{n}{\log_2 n} > 1 \implies n + \frac{n}{\log_2 n} > n \quad \forall n > 1$$
  $$\therefore n \leq n + \frac{n}{\log_2 n} \quad \forall n \geq 2$$

  Notice we pick n_0 to be 2 and not 1 because we require $n \geq n_0$. It is important to pay attention that $n > 1$ to prevent division by zero.

- $n \in \Omega\left(n + \frac{n}{\log_2 n}\right)$

  $$\log_2 n \geq 1 \quad \forall n \geq 2 \implies n + n = 2n \geq n + \frac{n}{\log_2 n}$$
  $$\therefore n \geq \frac{1}{2}\left(n + \frac{n}{\log_2 n}\right) \quad \forall n \geq 2$$

□

(2)

$$n + \sqrt{n} \in \Theta(n)$$

Proof.

- $n + \sqrt{n} \in O(n)$

  $$n \geq \sqrt{n} \quad \forall n \geq 0 \implies n + \sqrt{n} \leq n + n = 2n$$
  $$\therefore n + \sqrt{n} \leq 2n \quad \forall n \geq 0$$

- $n + \sqrt{n} \in \Omega(n)$

  $$\sqrt{n} \geq 0 \quad \forall n \geq 0 \implies n + \sqrt{n} \geq n$$
  $$\therefore n + \sqrt{n} \geq n \quad \forall n \geq 0$$

□

(3)

$$n \log_2 n^2 \in O\left(n^{\log_2 n}\right)$$

Proof.

$$\begin{aligned}
n \log_2 n^2 &= 2n \log_2 n \\
&\leq 2n \cdot n \\
&= 2n^2 \\
&\leq 2n^2 n^{\log_2 (n) - 2} \\
&= 2n^{\log_2 n} \quad \forall n \geq 4
\end{aligned}$$

□

(4)

$$\left(\frac{2}{3}\right)^n \in O(n)$$

Proof.

$$\frac{2}{3} < 1 \Longrightarrow \left(\frac{2}{3}\right)^n < 1^n = 1 \leq n \quad \forall n \geq 1$$

□

(5)

$$2^n \in \Omega\left(\frac{3}{2}^n\right)$$

Proof.

$$2 = \frac{4}{2} > \frac{3}{2} \Longrightarrow 2^n \geq \left(\frac{3}{2}\right)^n \quad \forall n \geq 0$$

□