Module 3A1: Fluid Mechanics I

INCOMPRESSIBLE FLOW DATA CARD

Continuity equation $\nabla \cdot \boldsymbol{u} = 0$

Momentum equation (inviscid) $\rho \dfrac{D\boldsymbol{u}}{Dt} = -\nabla p + \rho \boldsymbol{g}$

D/Dt denotes the material derivative, $\partial/\partial t + \boldsymbol{u} \cdot \nabla$

Vorticity $\boldsymbol{\omega} = \text{curl } \boldsymbol{u}$

Vorticity equation (inviscid) $\dfrac{D\omega}{Dt} = \boldsymbol{\omega} \cdot \nabla \boldsymbol{u}$

Kelvin's circulation theorem (inviscid) $\dfrac{D\Gamma}{Dt} = 0, \;\; \Gamma = \oint \boldsymbol{u} \cdot \boldsymbol{dl} = \int \boldsymbol{\omega} \cdot \boldsymbol{dS}$

For an irrotational flow

velocity potential ϕ $\boldsymbol{u} = \nabla\phi$ and $\nabla^2 \phi = 0$

Bernoulli's equation for inviscid flow: $\dfrac{p}{\rho} + \tfrac{1}{2}|\boldsymbol{u}|^2 + gz + \dfrac{\partial \phi}{\partial t} = \text{constant}$ throughout flow field

TWO-DIMENSIONAL FLOW

Streamfunction ψ

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}$$

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \qquad u_\theta = -\frac{\partial \psi}{\partial r}$$

Lift force Lift / unit length $= \rho U(-\Gamma)$

For an irrotational flow

complex potential *F(z)* $F(z) = \phi + i\psi$ is a function of $z = x + iy$

$$\frac{dF}{dz} = u - iv$$

TWO-DIMENSIONAL FLOW (continued)

Summary of simple 2 - D flow fields

	ϕ	ψ	$F(z)$	$\boldsymbol{u}$
Uniform flow (x - wise)	Ux	Uy	Uz	$u = U,\ v = 0$
Source at origin	$\frac{m}{2\pi}\ln r$	$\frac{m}{2\pi}\theta$	$\frac{m}{2\pi}\ln z$	$u_r = \frac{m}{2\pi r},\ u_\theta = 0$
Doublet (x - wise) at origin	$-\frac{\mu\cos\theta}{2\pi r}$	$\frac{\mu\sin\theta}{2\pi r}$	$-\frac{\mu}{2\pi z}$	$u_r = \frac{\mu\cos\theta}{2\pi r^2},\ u_\theta = \frac{\mu\sin\theta}{2\pi r^2}$
Vortex at origin	$\frac{\Gamma}{2\pi}\theta$	$-\frac{\Gamma}{2\pi}\ln r$	$-\frac{i\Gamma}{2\pi}\ln z$	$u_r = 0,\ u_\theta = \frac{\Gamma}{2\pi r}$

THREE-DIMENSIONAL FLOW

Summary of simple 3 - D flow fields

	ϕ	$\boldsymbol{u}$
Source at origin	$-\frac{m}{4\pi r}$	$u_r = \frac{m}{4\pi r^2},\quad u_\theta = 0,\quad u_\psi = 0$
Doublet at origin (with θ the angle from the doublet axis)	$-\frac{\mu\cos\theta}{4\pi r^2}$	$u_r = \frac{\mu\cos\theta}{2\pi r^3},\quad u_\theta = \frac{\mu\sin\theta}{4\pi r^3},\quad u_\psi = 0$

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