Engineering Tripos Part IIA | THIRD YEAR

Module 3F1 – Signals and Systems

Examples Paper 3F1/2

9. Compute the frequency responses of the FIR filters with the following impulse responses, exploiting symmetry to express each frequency response as the product of a pure delay term and a frequency-dependent gain. State what type of filter (eg. highpass, lowpass, etc.) each is

   (a) 1, 2, 1
   (b) -1, 2, -1
   (c) -1, 0, 2, 0, -1
   (d) 1, 2, 2, 1

10. Show that the filter $y(n) = ay(n-1) - ax(n) + x(n-1)$ is all-pass.

11. We want to design a FIR bandpass filter having a duration $M = 201$. $H_d\left(e^{j\theta}\right)$ represents the ideal characteristic of the noncausal bandpass filter defined by

$$H_d\left(e^{j\theta}\right) = \begin{cases} 1 & \text{if } 0.4\pi < |\theta| < 0.5\pi \\ 0 & \text{otherwise} \end{cases}$$

   (a) Determine the impulse response $h_d(n)$ corresponding to $H_d\left(e^{j\theta}\right)$.
   (b) Explain how you would use the Hamming window

$$w(n) = 0.54 + 0.46\cos\left(\frac{2\pi n}{M-1}\right), \quad -\frac{M-1}{2} \le n \le \frac{M-1}{2}$$

   to design a FIR bandpass filter with impulse response $h(n)$ for $0 \le n \le 200$.
   (c) Explain the advantages of using Hamming window compared to a rectangular window.

12. Find the forward difference $s = \dfrac{z-1}{T}$, backward difference $s = \dfrac{z-1}{zT}$ and Tustin transformation $s = \dfrac{2}{T}\dfrac{z-1}{z+1}$ of the analog low-pass filter

$$G(s) = \frac{a}{s+a}$$

   ($a > 0$) assuming a sampling period of T seconds. What conditions must aT satisfy for these digital filters to be stable? For what range of values of aT would these filters be reasonable approximations of $G(s)$?

13. For an audio system with sampling rate 44.1 kHz, a bandpass filter is required with 3dB corner frequencies at 6.5084 kHz and 7.5861 kHz. An analogue lowpass filter with the transfer function $H(s) = \frac{1}{s+1}$ has a 3dB corner frequency of 1 rad/sec. Using the lowpass to bandpass transformation together with the bilinear transform

$$s \to \frac{1 - z^{-1}}{1 + z^{-1}}$$

design the required digital filter. Calculate the poles and zeros of the digital filter and show it gives the desired bandpass response.

14. A 4th order analogue Butterworth lowpass filter with cutoff frequency 1 rad/sec has poles at $s = -0.3827 \pm j0.9239$ and $-0.9239 \pm j0.3827$. Design a 4th order lowpass digital filter with sampling rate 8kHz, unit DC gain, and cutoff frequency 1kHz using the bilinear transform

$$s \to \frac{1 - z^{-1}}{1 + z^{-1}}\ .$$

15. A motor driving a rotating inertia has transfer function $1/s(s+1)$ from the motor current input to the position output. The output is sampled with period T, and the current input is held constant between sampling points. Show that the equivalent discrete-time system, from the sequence of current inputs to the sampled outputs, has the z-plane transfer function,

$$G(z) = \frac{(e^{-T} - 1 + T)z^{-1} + [1 - (1+T)e^{-T}]z^{-2}}{(1 - z^{-1})(1 - e^{-T}z^{-1})}.$$

16. Consider a continuous time system with transfer function $G(s)$ connected as shown in Figure 1. The output of the first-order hold DAC is the linear extrapolation through the last two discrete inputs. Explain why the discrete system taking $\{u(kT)\}$ to $\{y(kT)\}$ has a z-transfer function. Show that the transfer function is given by the expression:

$$H(z) = \frac{(z-1)^2}{Tz^2} \mathcal{Z} \left(\mathcal{L}^{-1} \left(G(s) \frac{Ts+1}{s^2} \right) \bigg|_{t=kT} \right).$$

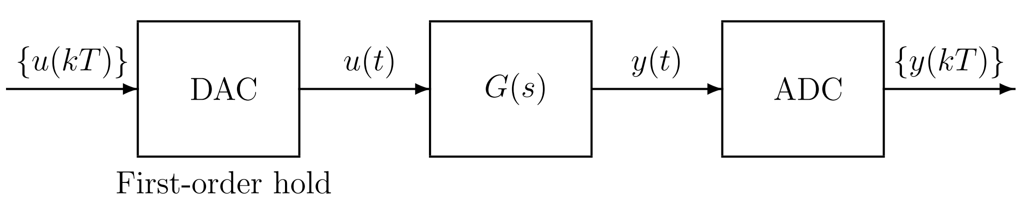

Figure 1: Block diagram for question 10

17. The plots of $G(e^{j\theta})$ for θ ranging from 0^+ to π are shown in Figures 2-4 (in random order) for the following transfer functions:

$$\text{(i)}\frac{1}{z^2(z-1)}, \text{(ii)}\frac{4z-2}{3(z-1)^2}, \text{(iii)}\frac{4}{(z-1)^3}.$$

Sketch the complete Nyquist diagrams for each transfer function and use the Nyquist criterion to determine for what values of gain (if any) closed loop stability will be achieved when constant gain negative feedback is connected around them.

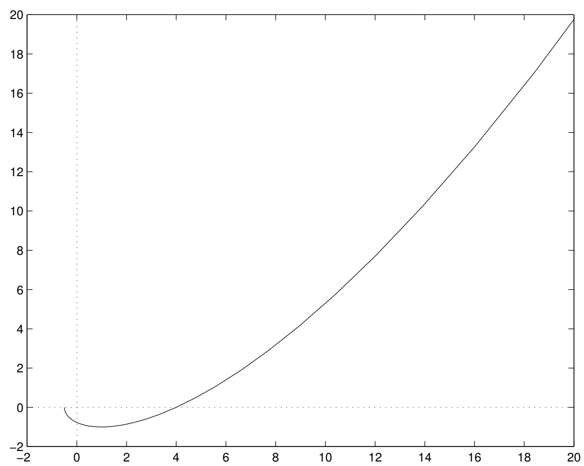

Figure 2: Plot of $G(e^{j\theta})$.

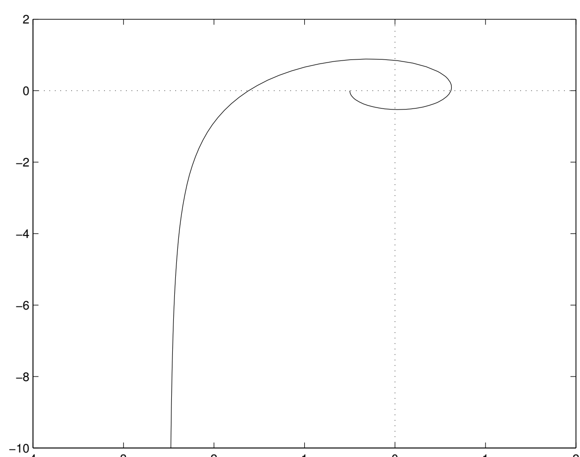

Figure 3: Plot of $G(e^{j\theta})$.

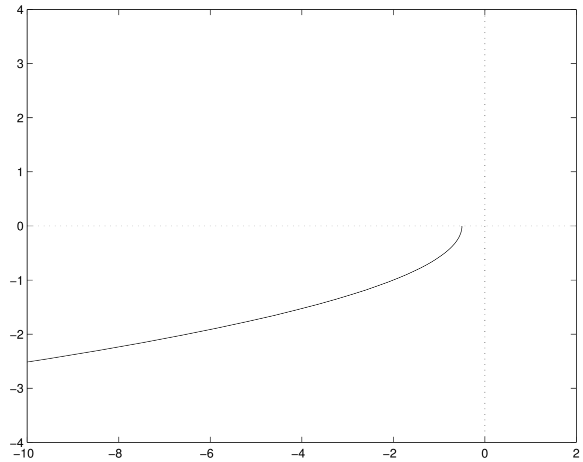

Figure 4: Plot of $G(e^{j\theta})$.

18. Show the following properties of the DFT of an N-point data sequence $\{x_n\}$.

    (a) Periodic spectrum, i.e. $X_{k+N} = X_k$.

    (b) Periodic data, i.e. $x_{n+N} = x_n$ (using the inverse DFT).

    (c) Conjugate symmetry for real-valued $\{x_n\}$, i.e. $X_k = X^*_{N-k}$.

    (d) Let X_k be the N-point DFT of the sequence x_n, $0 \leq n \leq N-1$. Define a new sequence $y_n = X_n$ and compute its DFT Y_k. Show that $Y_k = Nx_{-k} = Nx_{N-k}$.

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