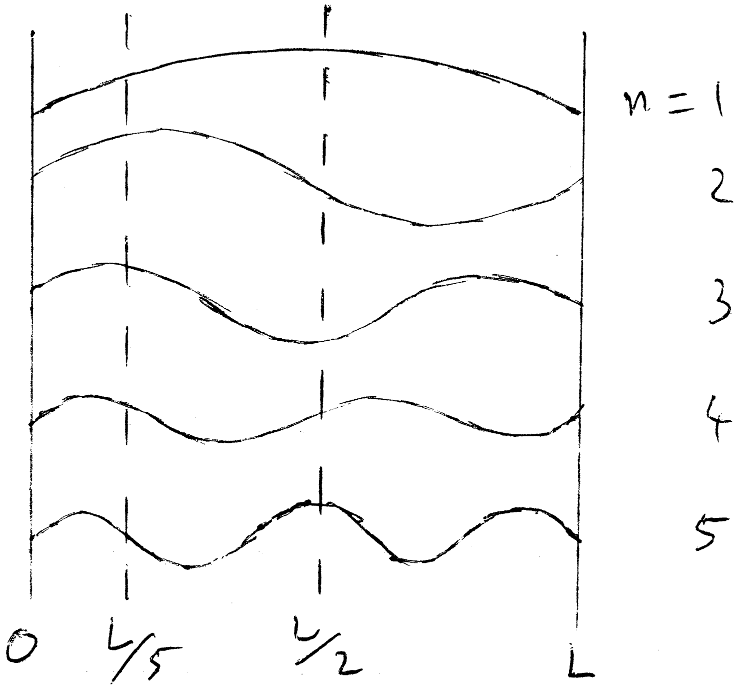
Part IIA Module 3C6 Sheet 3 Solutions

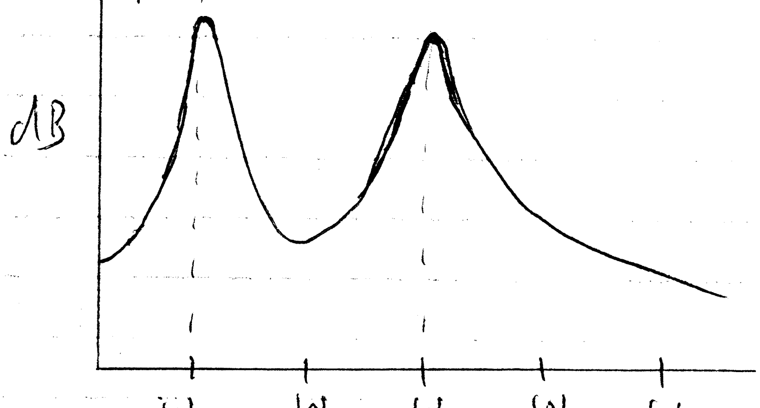
1. (a)



Modes 2, 4, 6 ... have nodes at $L/2$
Modes 5, 10, .. have nodes at $L/5$
None of these will appear in the transfer function from $L/2$ to $L/5$.
So we only see modes 1, 3 in this frequency range.

$$u_n(L/2) \times u_n(L/5) \text{ is } \begin{matrix} >0 & \text{for } n=1 \\ <0 & \text{for } n=3 \end{matrix}$$

So no antiresonance in between.
So transfer function is



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(b)(i) Top data shows peaks at equal intervals (except where missing). Bottom data shows peaks close together at low frequencies, getting wider apart at higher frequencies.
So a plausible guess is that the top data is a stretched string (notice that it is tuned to A 440 Hz) and the bottom data is a bending beam.

(ii) Top data:
Top curve shows peaks 1, 2, 3, 4, 6, 7, 8, 9, 11 ... with 5, 10 being small. It also shows an antiresonance between every pair of peaks.
Suggests driving-point response at $L/5$.

Middle curve has the same set of peaks but <u>no</u> antiresonances. So sign reverses every time.
Suggests transfer function from $L/5$ to $4L/5$.

Bottom curve shows peaks 1, 3, 7, 9, 11 as strong, others being small or absent.
This is the pattern found in part (a), so this is the transfer function from $L/2$ to $L/5$.

Bottom data:
Top curve shows all peaks strong, and antiresonances in every gap. Suggest driving point response <u>near</u> <u>one</u> <u>end</u>.
Middle curve shows all peaks but no antiresonances. Suggests transfer function from one end to the other.
Bottom curve has peaks 2, 4, 6, at reduced height. Suggests driving point <u>near</u> the middle of a symmetrical beam.

1 cont.

(iii) For a stretched string there isn't much choice about boundary conditions (but the measured string is on a violin, and rigid-body motion of the whole shows as a mode at $\omega = 0$)
For the beam there is a choice.
We have already guessed that the beam is symmetric, i.e. same boundary conditions at both ends.
If it was pinned-pinned, frequencies would be in the ratio $1:4:9:16 \ldots$
If it was free-free or clamped-clamped, frequencies would be approximately in the ratios
$$4\cdot73^2 : 7\cdot85^2 : 11\cdot0^2 : 14\cdot14^2 : 17\cdot28^2$$
(from values of "alpha" in lecture notes, p19)
$$\rightarrow \quad 1 : 2\cdot75 : 5\cdot50 : 9\cdot25 : 13\cdot75$$

Measuring from the graph gives ratios
$$1 : 2\cdot75 : 5\cdot41 : 8\cdot94 : 13\cdot35$$
So a very convincing match to free-free or clamped-clamped, definitely not simply supported. In fact, it is a free-free beam, so all modes have large amplitudes at the ends, thus making sense of the top curve.

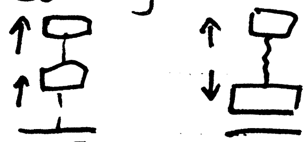
(iv) Nothing very odd in the string data, but the beam data shows small peaks at around 750 Hz, 1500 Hz. These are obviously not part of the sequence of bending modes. They are the first two torsional modes of the beam, which was a flat strip of steel excited near it centre-line, but probably not exactly on the centre-line.

2. (a) $V = \frac{1}{2}k(x-y)^2 + \frac{1}{2}2k\,y^2$

$\rightarrow K = k\begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$

$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}\cdot 2m\dot{y}^2$

$\rightarrow M = m\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

Modes are roughly 

Try (i) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

(i) gives $\frac{m\omega^2}{k} \simeq \frac{2}{3}, \frac{6}{3}$ i.e. 0.667, 2

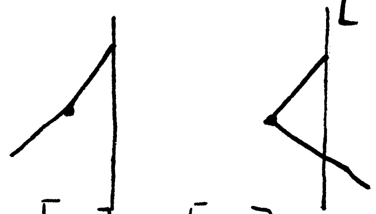
(ii) gives $\frac{m}{k}\omega^2 \simeq \frac{3}{6}, \frac{11}{6}$ i.e. 0.5, 1.833

Of these $\begin{cases} 0.5 \text{ is lower, so better} \\ 2 \text{ is higher, so better} \end{cases}$ so these are best guesses.

In fact, if you work out the exact solutions you will find that $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ are the correct modes shapes, so these are in fact the correct answers for ω^2. See plot on next page.

(b) From sheet 4: $K = mga\begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}$, $M = \frac{ma^2}{3}\begin{bmatrix} 32 & 10 \\ 10 & 4 \end{bmatrix}$

Modes:



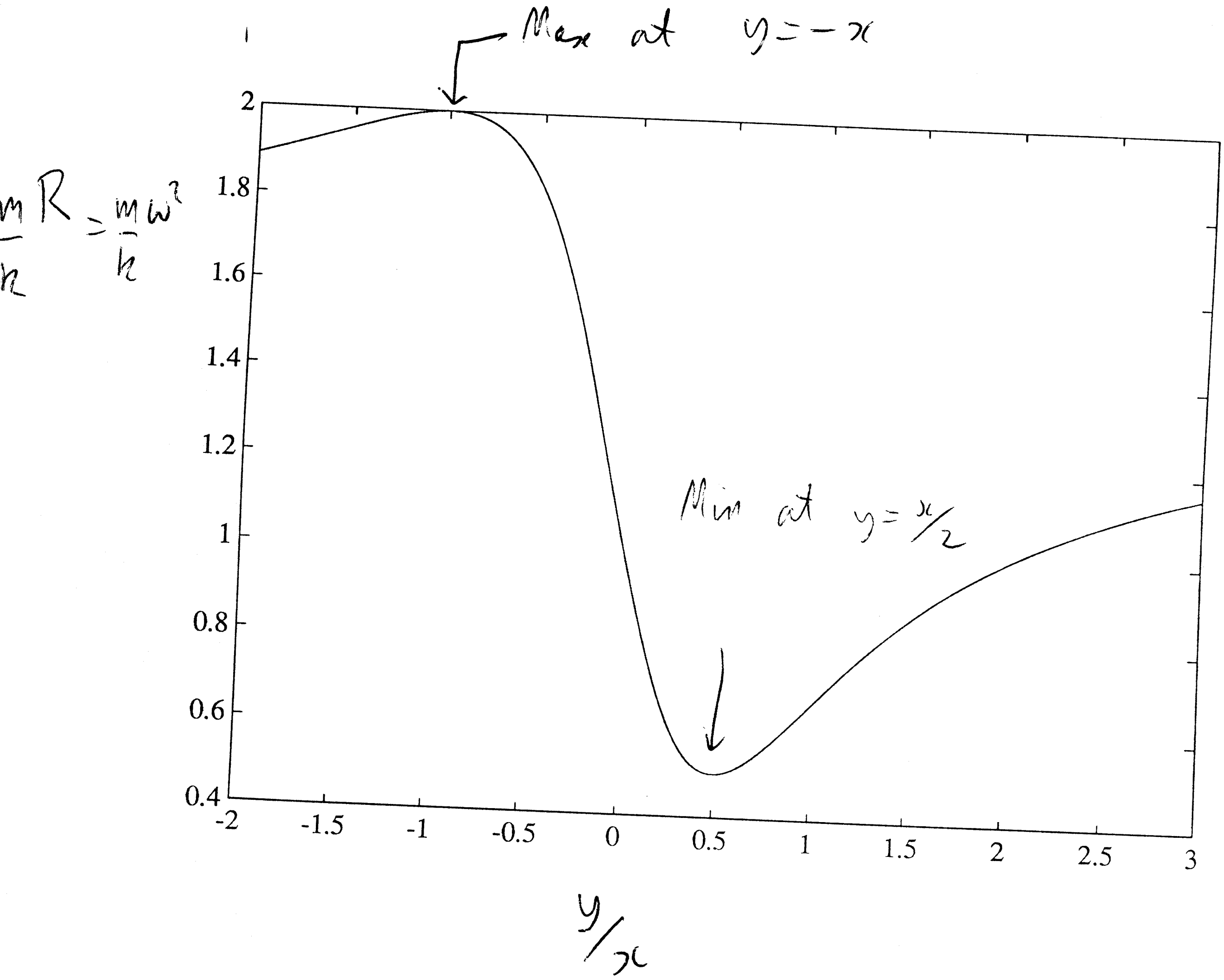
So try (i) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

(i) gives $\omega^2\frac{a}{3g} \simeq \frac{7}{56}, \frac{4}{8}$

(ii) gives $\omega^2\frac{a}{3g} \simeq \frac{52}{424}, \frac{7}{8}$

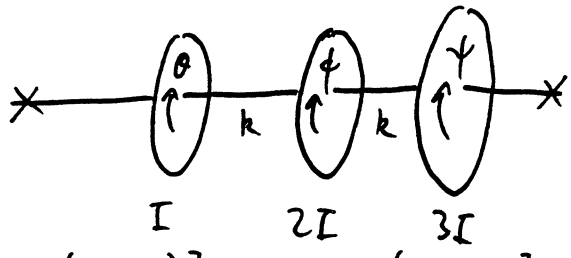
(ii) is lower, ∴ better (ii) is higher, ∴ better

[NB exact answers from sheet 4, are $\omega^2\frac{a}{3g} = \cdot 103, 1\cdot 04$]



Rayleigh quotient R for Q2 part 1

3.



$V = \frac{1}{2}k(\theta-\phi)^2 + \frac{1}{2}k(\phi-\psi)^2$

so $K = k\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

$T = \frac{1}{2}I\dot\theta^2 + \frac{1}{2}\cdot 2I\dot\phi^2 + \frac{1}{2}\cdot 3I\dot\psi^2$

so $M = I\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Rigid body mode is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, so if $\begin{bmatrix} \theta \\ \phi \\ \psi \end{bmatrix}$ is

orthogonal to this, need $\theta + 2\phi + 3\psi = 0$. ①

So guess some plausible mode shapes satisfying ①:

try $\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -5 \\ 4 \\ -1 \end{bmatrix}$

These give $\frac{\omega^2 I}{k} \simeq \frac{4}{6}, \frac{106}{60}$, i.e. 0.67, 1.77

For exact results: use mode $\underline{y} = \begin{bmatrix} -2a-3b \\ a \\ b \end{bmatrix}$ to satisfy ①.

Now $K\underline{y} = \omega^2 M\underline{y}$ gives

$$\begin{cases} k(2a+3b) + 2ka - kb = 2I\omega^2 a \\ -ka + kb = 3I\omega^2 b \end{cases}$$

So $\left(1 - 3\frac{I\omega^2}{k}\right) b = a$, and hence

$$(4k - 2I\omega^2)\left(1 - 3\frac{I\omega^2}{k}\right) + 2k = 0$$

or $(2-\lambda)(1-3\lambda) + 1 = 0$ with $\lambda = \frac{I\omega^2}{k}$

So $\lambda = \dfrac{7 \pm \sqrt{49-36}}{6} = \dfrac{7 \pm \sqrt{13}}{6}$

i.e. $w^2 = \dfrac{k}{6I}\left(7 \pm \sqrt{13}\right) = \dfrac{k}{I} \times \begin{cases} 0.566 \\ 1.768 \end{cases}$

So our guess for the higher mode is very accurate, the guess for the lower mode close but not quite so good.

4. $V = \frac{1}{2} k x^2 + \frac{1}{2} k y^2 + \frac{1}{2} s (x-y)^2$

so $K = \begin{bmatrix} k+s & -s \\ -s & k+s \end{bmatrix}$

$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2}(m+\delta)\dot{y}^2$

So $M = m\begin{bmatrix} 1 & 0 \\ 0 & 1+\delta \end{bmatrix}$.

When $\delta = 0$, the system is symmetric and the modes are $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

So the approximate frequencies when δ is small are

$$w^2 \simeq \frac{2k}{2m+\delta m}, \quad \frac{2k+4s}{2m+\delta m}$$

First line of eigenvector equation is $(k+s)x - sy = mw^2 x$

so $\dfrac{y}{x} = \dfrac{k+s-mw^2}{s} \simeq \begin{cases} \dfrac{k}{s} + 1 - \dfrac{2k}{s(2+\delta)} \simeq \dfrac{k}{s} + 1 - \dfrac{2k}{2s}\left(1 - \frac{\delta}{2}\right) \\ \quad = 1 + \dfrac{k\delta}{2s} \\ \dfrac{k}{s} + 1 - \dfrac{2(k+2s)}{s(2+\delta)} \simeq \dfrac{k}{s} + 1 - \dfrac{(k+2s)}{s}\left(1 - \frac{\delta}{2}\right) \\ \quad = -1 + \dfrac{\delta}{2s}(k+2s) \end{cases}$

For orthogonality, $m x_1 x_2 + m(1+\delta) y_1 y_2 = 0$

i.e. $\dfrac{x_1}{y_1} \cdot \dfrac{x_2}{y_2} = -(1+\delta)$, which works with these values.

5 (i) The d.e. for a free bar is $EA\,y'' - m\ddot{y} = 0$ (EP 5, Q6)

and a solution to this is $y = U_j(z)\,e^{i\omega_j t}$ where $U_j(z)$ is the j^{th} mode shape & ω_j is the j^{th} modal frequency ($m = \rho A$)

So $EA\,U_j'' + m\omega_j^2 = 0$ ①

Forced vibration, $-EA\,y'' + m\ddot{y} = f(z,t)$ and the modal approach says $y(z,t) = \sum_{j=1}^{\infty} U_j(z)\,q_j(t)$ $\therefore \sum_{j=1}^{\infty}\left(-EA\,U_j''q_j + mU_j\ddot{q}_j\right) = f(z,t)$

Harmonic forcing $\therefore f(z,t) = F(z)\,e^{i\omega t}$, response $q_j(t) = Q_j(\omega)\,e^{i\omega t}$

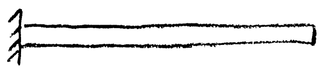
$\therefore \sum_{j=1}^{\infty}\left(m\omega_j^2 - m\omega^2\right)U_j(z)\,Q_j(\omega) = F(z)$ (using ①)

and multiply by $U_k(z)$ & integrate $\int_0^L dz$ noting orthogonality

$\therefore (\omega_j^2 - \omega^2)\,Q_j(\omega) = \int_0^L U_j(z)\,F(z)\,dz$ and for a point force at $z = s$ then $F(z) = F\,\delta(z-s)$ so RHS $= F\,U_j(z=s)$

So $Q_j(\omega) = \dfrac{F\,U_j(s)}{\omega_j^2 - \omega^2}$ and now sum over modes

$\therefore H(s,z,\omega) = \dfrac{Y(z,\omega)}{F} = \sum_{j=1}^{\infty}\dfrac{U_j(z)\,U_j(s)}{\omega_j^2 - \omega^2}$ where $y(z,t) = Y(z,\omega)\,e^{i\omega t}$

Now mode shapes for  are found from boundary conditions given $U_j(z) = A\cos\alpha z + B\sin\alpha z$ $\alpha = \sqrt{\dfrac{m\omega_j^2}{EA}}$

At $z = 0$, $U_j(z) = 0$ (zero displacement) $\therefore A = 0$

At $z = L$, $U_j'(z) = 0$ (zero strain) $\therefore B\cos\alpha L = 0$ $\therefore \alpha L = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

$\therefore \underline{\omega_j = (\alpha L)\sqrt{\dfrac{EA}{mL^2}}}$ and $U_j(z) = \sqrt{\dfrac{2}{mL}}\sin\alpha z$

(which is normalized to satisfy $m\int_0^L U_j(z)\,U_j(z)\,dz = 1$)

At $z = L$, $U_j(z) = \sqrt{\dfrac{2}{mL}}$ for all modes

so $\underline{H(s=L, z=L, \omega) = \dfrac{2}{mL}\sum_{j=1}^{\infty}\dfrac{1}{\omega_j^2 - \omega^2}}$ with $m = \rho A$

(ii) Use a different boundary condition at $z = L$ for the applied force

$EA\,y'' - m\ddot{y} = 0$ and put $y = Y(z)\,e^{i\omega t}$ $\therefore Y'' + \alpha^2 Y = 0$ ($\alpha^2 = \dfrac{m\omega^2}{EA}$)

$Y = A\cos\alpha z + B\sin\alpha z$, $Y = 0$ at $z = 0$ $\therefore A = 0$

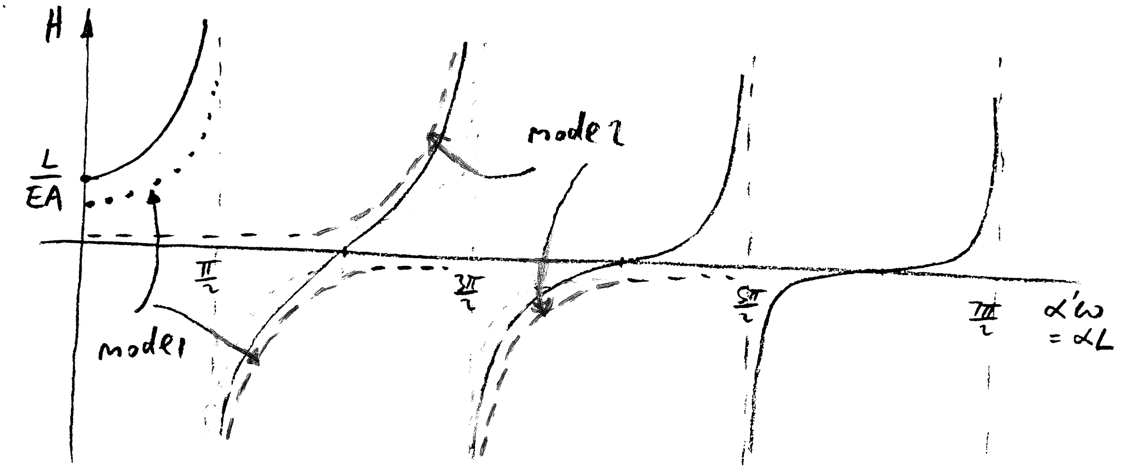
$EA\,Y' = F$ at $z = L$ $\therefore EAB\alpha\cos\alpha L = F$ $\therefore B = \dfrac{F}{EA\alpha\cos\alpha L}$

$\therefore Y = \dfrac{F}{EA\alpha}\dfrac{\sin\alpha z}{\cos\alpha L}$ and at $z = L$, $H(L,L,\omega) = \dfrac{Y(\omega)}{F} = \dfrac{L\tan\alpha L}{EA\alpha L}$

but $\alpha = \sqrt{\dfrac{\rho}{E}}\,\omega = \sqrt{\dfrac{\rho}{E}}\,L\,\dfrac{\omega}{L} = \alpha'\dfrac{\omega}{L}$ $\therefore \alpha'\omega = \alpha L$

$$\therefore H(L,L,\omega) = \frac{L}{EA}\,\frac{\tan\alpha'\omega}{\alpha'\omega}$$

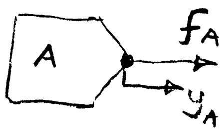
$\alpha' = \sqrt{\dfrac{\rho}{E}}\,L$

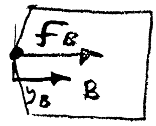


$\frac{L}{EA} \frac{\tan \alpha'\omega}{\alpha'\omega}$

You can see how, in principle, the modes will add up to give the same answer.

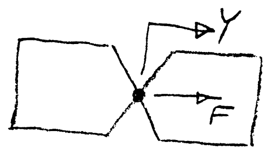
(iii) Joining subsystems A & B with FRF's $H_A(\omega)$ & $H_B(\omega)$





so $Y_A(\omega) = H_A(\omega) F_A$ and $Y_B(\omega) = H_B(\omega) F_B$

but $Y_A = Y_B = Y$ and $F_A + F_B = F$

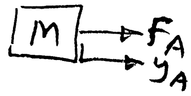


$\therefore \quad Y\left(\frac{1}{H_A} + \frac{1}{H_B}\right) = F \qquad \therefore \quad H_{AB}(\omega) = \frac{Y}{F} = \frac{1}{\frac{1}{H_A} + \frac{1}{H_B}}$

$= \frac{H_A H_B}{H_A + H_B}$

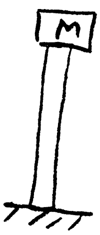
This will have resonances when $H_A + H_B = 0$

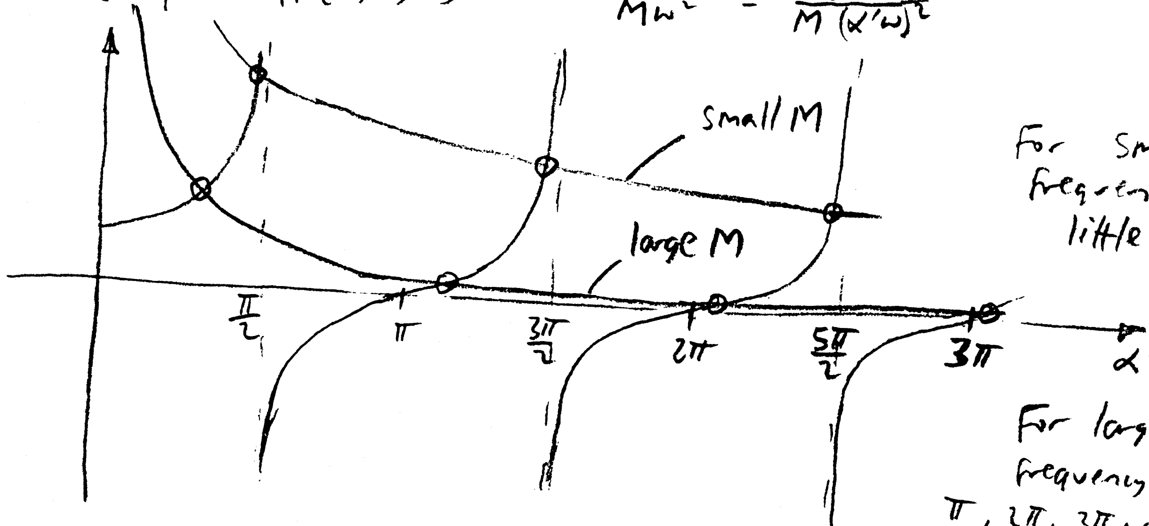
Now take as system A a mass M for which $f_A = M \ddot{y}_A \quad \therefore \quad H_A = \frac{-1}{M\omega^2}$



and for system B use $H(L, L, \omega)$ as above so $\frac{-1}{M\omega^2} + H(L,L,\omega) = 0$ at resonances

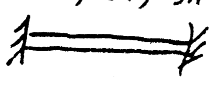
(iv) For natural frequencies of the joined system solve for $H(L,L,\omega) = \frac{1}{M\omega^2} = \frac{\alpha'^2}{M(\alpha'\omega)^2}$



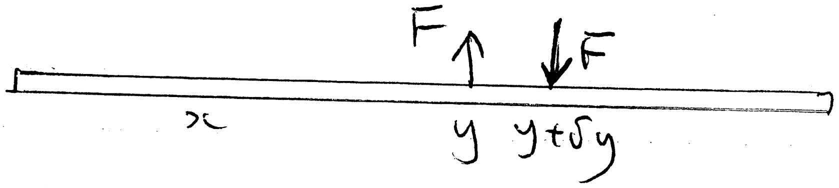


For small M, low frequency modes are little changed.

For large M, high frequency modes follow the $\pi, 2\pi, 3\pi \ldots$ sequence for



6. (i)



Moment $M = F\delta_y$

Response at x due to pair of forces is

$$-F\sum_j \frac{u_j(x)\,u_j(y)}{-\omega^2+\omega_j^2} + F\sum_j \frac{u_j(x)\,u_j(y+\delta_y)}{-\omega^2+\omega_j^2}$$

$$= M\sum_j \frac{u_j(x)\left[u_j(y+\delta_y)-u_j(y)\right]/\delta_y}{-\omega^2+\omega_j^2}$$

Now let $\delta_y \to 0$, $F\to\infty$ keeping M constant.

$$\text{Response} \rightarrow M\sum_j \frac{u_j(x)\;u_j'(y)}{-\omega^2+\omega_j^2}$$

To get transfer function, divide by M (the input).

If the required output variable is the rotation θ, i.e. the slope at x, then obviously just need to take $\frac{d}{dx}$ $\rightarrow$ $H_{\theta,M} = \sum \frac{u_j'(x)\;u_j'(y)}{-\omega^2+\omega_j^2}$

and $$H_{\theta F} = \sum \frac{u_j'(x)\;u_j(y)}{-\omega^2+\omega_j^2}$$

$H_{\theta F} = H_{YM}$ by reciprocity: moment M is the generalised force corresponding to rotation θ, just as F is the generalised force corresponding to displacement.

6 cont

(ii) At an end of a beam, the displacement and rotation are related to the force and bending moment via a 2×2 matrix of transfer functions:

For beam A, the 2×2 matrix is $[H_A] = \begin{bmatrix} H_{YF} & H_{YM} \\ H_{\theta F} & H_{\theta M} \end{bmatrix}$

For beam B, paying careful attention to signs, the corresponding matrix is $[H_B] = \begin{bmatrix} H_{YF} & -H_{YM} \\ -H_{\theta F} & H_{\theta M} \end{bmatrix}$

Now join the two beams. The displacement and rotation must be equal: $\begin{bmatrix} Y_A \\ \theta_A \end{bmatrix} = \begin{bmatrix} Y_B \\ \theta_B \end{bmatrix} = \begin{bmatrix} Y \\ \theta \end{bmatrix}$ say.

If a force and/or moment are applied at the junction point, these are simply the sums of the forces + moments acting on the two separate beams:

$$\begin{bmatrix} F \\ M \end{bmatrix} = \begin{bmatrix} F_A \\ M_A \end{bmatrix} + \begin{bmatrix} F_B \\ M_B \end{bmatrix}$$

$$= [H_A]^{-1} \begin{bmatrix} Y \\ \theta \end{bmatrix} + [H_B]^{-1} \begin{bmatrix} Y \\ \theta \end{bmatrix}$$

∴ the 2×2 transfer function matrix $[\bar{H}]$ at the join satisfies

$$[\bar{H}]^{-1} = [H_A]^{-1} + [H_B]^{-1} = \frac{1}{\Delta} \begin{bmatrix} 2H_{\theta M} & 0 \\ 0 & 2H_{YH} \end{bmatrix}, \ \Delta \text{ as given.}$$

Use the inversion formula again to get $\bar{H}_{YF} = \dfrac{\Delta}{2H_{\theta M}}$, $\bar{H}_{\theta M} = \dfrac{\Delta}{2H_{YF}}$

7. (i)

Assume a mode shape $Y(z) = \sin\frac{\pi}{L}$

We have max Strain energy in bending

$$V_{max} = \int_0^L \frac{1}{2} EI \left(\frac{d^2Y}{dz^2}\right)^2 dz$$

and $\frac{d^2Y}{dz^2} = -\left(\frac{\pi}{L}\right)^2 \sin\frac{\pi z}{L}$ $\quad \therefore \quad V_{max} = \frac{1}{2} EI \left(\frac{\pi}{L}\right)^4 \int_0^L \sin^2\frac{\pi z}{L}\, dz = \frac{1}{2} EI \left(\frac{\pi}{L}\right)^4 \frac{L}{2}$

The maximum beam kinetic energy is

$$T_{max} = \int_0^L \frac{1}{2} m \dot{y}_{max}^2 dz$$

$$= \omega^2 \int_0^L \frac{1}{2} m Y^2 dz$$

$$= \omega^2 \frac{1}{2} m \frac{L}{2}$$

to which must be added the KE of the mass M which is $\frac{1}{2} M v_{max}^2$

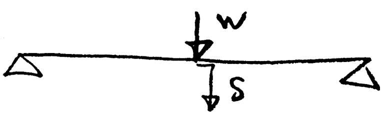
and $v_{max} = \omega y_{max} = \omega$

Equate T_{max} and V_{max} $\quad \therefore \quad \omega^2 = \dfrac{EI\left(\frac{\pi}{L}\right)^4 \frac{L}{2}}{m\frac{L}{2} + M}$

$$\therefore \quad \omega = \left(\frac{\pi}{L}\right)^2 \sqrt{\frac{EI}{m + \frac{2M}{L}}}$$

(ii) For beam mass $m = 0$, $\omega = \left(\frac{\pi}{L}\right)^2 \sqrt{\frac{EIL}{2M}} = \sqrt{\frac{48.7 EI}{ML^3}}$

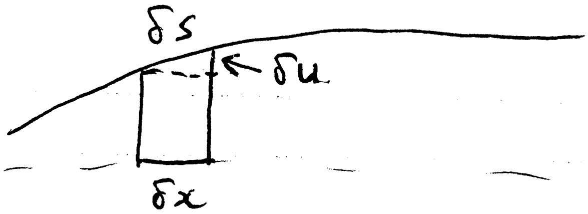
The true answer can be got using 1A structures data book



$\delta = \frac{WL^3}{48EI}$ $\quad \therefore$ Stiffness $k = \frac{W}{\delta} = \frac{48EI}{L^3}$

Frequency is then $\quad \omega = \sqrt{\frac{k}{M}} = \sqrt{\frac{48EI}{ML^3}}$ $\quad$ - very good!

8. (i)



$$\delta s = \sqrt{\delta x^2 + \delta u^2} = \delta x\left[1 + \left(\frac{du}{dx}\right)^2\right]^{1/2}$$

$$\simeq \delta x\left[1 + \frac{1}{2}\left(\frac{du}{dx}\right)^2\right] \quad \text{(binomial)}$$

So increase in length $\simeq \dfrac{\delta x}{2}\left(\dfrac{du}{dx}\right)^2$

(ii) Work done by this stretch against the pre-existing tension P

is $\delta W \simeq P\,\dfrac{\delta x}{2}\left(\dfrac{du}{dx}\right)^2$

$\therefore$ Potential energy $V = \dfrac{P}{2}\displaystyle\int_0^L \left(\frac{du}{dx}\right)^2 dx$

~~(iii) Kinetic energy~~ $\cancel{T = \frac{1}{2} m \int \left(\frac{\partial w}{\partial t}\right)^2 dx}$

~~so for~~ $\cancel{w(x,t) = u(x)\, e^{i\omega t}}$

~~maximum~~ potential energy is $\dfrac{1}{2} m\omega^2 \displaystyle\int_0^L u^2\,dx$

(iii) Try $u(x) = \sin\dfrac{\pi x}{L}$

Then $V = \dfrac{1}{2} P \displaystyle\int_0^L \frac{\pi^2}{L^2}\sin^2\frac{\pi x}{L}\,dx = \frac{P\pi^2}{4L}$

$T = \dfrac{1}{2} m\omega^2 \displaystyle\int_0^L \sin^2\frac{\pi x}{L}\,dx = \frac{mL\omega^2}{4}$

8 cont

So Rayleigh quotient:

$$\omega^2 \simeq \frac{P\pi^2/4L}{mL/4} = \frac{P}{m}\frac{\pi^2}{L^2}$$

ie $$\omega \simeq \frac{\pi}{L}\sqrt{\frac{P}{m}}$$

This is the exact answer, because the correct mode shape has been used.

(iv) Try $u(x) = \frac{x}{L}\left(1-\frac{x}{L}\right) \rightarrow u' = \frac{1}{L} - \frac{2x}{L^2}$

So $$V = \frac{1}{2}\frac{P}{L^2}\int_0^L \left(1-\frac{2x}{L}\right)^2 dx$$

$$= \frac{1}{2}\frac{P}{L^2}\left[x - \frac{4}{L}\frac{x^2}{2} + \frac{4}{L^2}\frac{x^3}{3}\right]_0^L$$

$$= \frac{1}{6}\frac{P}{L}$$

and $$T = \frac{1}{2}m\omega^2\int_0^L \frac{x^2}{L^2}\left(1-\frac{x}{L}\right)^2 dx$$

$$= \frac{1}{2}\frac{m\omega^2}{L^2}\left[\frac{x^3}{3} - \frac{2}{L}\frac{x^4}{4} + \frac{1}{L^2}\frac{x^5}{5}\right]_0^L$$

$$= \frac{1}{2}m\omega^2 L\left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right) = \frac{1}{60}mL\omega^2$$

So Rayleigh quotient: $\omega^2 \simeq \frac{1/6 \; P/L}{1/60 \; mL} = \frac{10}{L^2}\frac{P}{m}$

$\therefore \omega \simeq \frac{\sqrt{10}}{L}\sqrt{\frac{P}{m}}$, very close to exact result from (iv).