1 A pair of vortices with the *same* circulation, Γ, are located initially as shown in Fig. 1 in an otherwise still fluid.

(a) At this instant of time write down the complex potential for this arrangement. [20%]

(b) Find any stagnation points in the flow and sketch the pattern at this instant. [20%]

(i) This flow is unsteady. Explain (briefly) why. [10%]

(ii) The pair rotates about the origin. Find the angular velocity of this rotation, $d\beta/dt$, where β is the angle of a line joining one vortex to the origin with the x-axis. [10%]

(c) A sink of strength m is now added at the origin causing the vortices to spiral inwards. Find the rate at which a, (the distance of either vortex from the origin), changes, i.e, find da/dt. [20%]

(d) The vortices spiral towards the origin. Find the equation of this spiral trajectory, i.e. the trajectory may be written as $a = f(\beta)$ where f is some function. Find the function $f(\beta)$. [20%]

Note: f depends also on m, Γ.

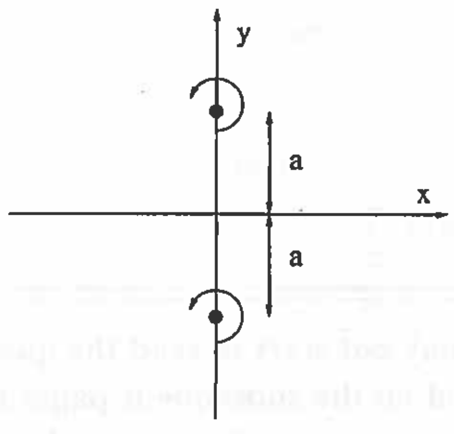

Fig. 1

2 A circular cylinder of radius a in a uniform flow from left to right is to be modelled by a doublet in a uniform flow of speed U.

(a) Write down the complex potential for this flow and find the appropriate strength of the doublet used to model the flow. [20%]

(b) Show that the complex potential you derived satisfies the correct boundary conditions and note the position of the stagnation points. [20%]

(c) A source of strength $2\pi aU$ is added to this flow at $z = -a$ and a sink of strength $-2\pi aU$ at $z = +a$.

(i) How many stagnation points are there now? Indicate roughly where they will be. [20%]

(ii) Find all the stagnation points. Note that the symmetry of the problem will simplify the mathematics. [20%]

(iii) Sketch the flow pattern. [20%]