3F4: Data Transmission (Lent 2019)
Examples Paper 3
Channel Coding & Network Algorithms

1. (A simple binary code) Consider a binary code with the following codebook:

$$B = \{\,(00100),\,(10010),\,(01001),\,(11111)\}$$

   a) What is the minimum distance of this code? What is the code rate?
   b) Is this code linear? (prove your answer)
   c) Decode using the minimum distance rule if the received vector is: i) (00000); ii) (01101).

2. (Minimum-distance decoding) A codeword from an (n, k) binary block code with codebook B is transmitted over a binary symmetric channel with crossover probability $p < \frac{1}{2}$. The maximum likelihood decoder is defined as

$$\hat{\underline{x}} = \arg\max_{\underline{x} \in B} \Pr(\underline{Y} \mid \underline{x}), \tag{1}$$

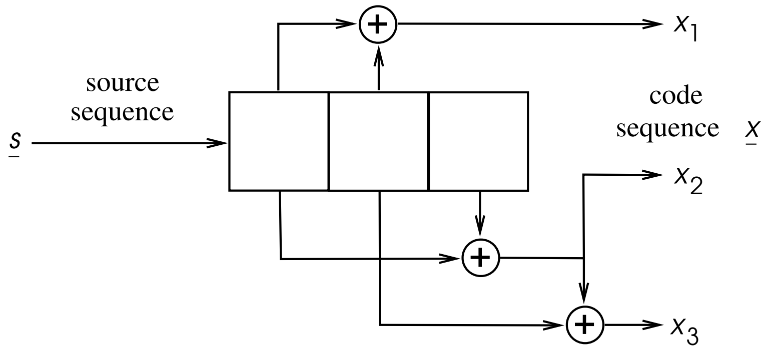
i.e., it determines the codeword in B that maximises the likelihood of receiving $\underline{Y}$. Prove that the maximum-likelihood decoder in (1) is equivalent to the minimum-distance encoder given by:

$$\hat{\underline{x}} = \arg\min_{\underline{x} \in B} d(\underline{x}, \underline{Y}),$$

where $d(\underline{x}, \underline{Y})$ denotes the Hamming distance between $\underline{x}$ and $\underline{Y}$.

Hint: First show that $\Pr(\underline{Y} \mid \underline{x}) = p^{d(\underline{x},\underline{Y})}(1-p)^{n-d(\underline{x},\underline{Y})}$.

3. (A very simple convolutional code) The block diagram of a rate 1/3 convolutional code is shown in the figure below.



   i) Draw the state diagram of the code.
   ii) Suppose the code is used on a binary symmetric channel, and the received sequence is $\underline{Y} = (111, 111, 111, 111, 111, 111)$. Using the Viterbi algorithm, determine the decoded sequence $\hat{\underline{x}}$, and the corresponding input sequence $\hat{\underline{s}}$.
   iii) Repeat part ii) when the received sequence is $\underline{Y} = (101, 111, 011, 111)$.
   iv) Draw the modified state diagram and the extended state diagram of the code.
   v) Determine the first **three** terms of the extended transfer function of this code (namely, up to and including all terms of order D^9), and interpret them.

4. Past exam questions on convolutional codes: 2017 qn. 2(d), 2016 qn. 2(a), and 2015 qn. 2.

5. (Window sizes and throughput in TCP-Reno) Consider a single TCP-Reno connection over a link with bandwidth 15 Mbps. Assume each packet has a size of 1500 bytes and the round trip time RTT = 80 ms (so that each transmission takes place over 40 ms, and the 'ack' takes another 40 ms to reach the transmitter). Further, assume packet delays/losses occur when the transmission rate exceeds the link capacity, but no time-outs occur during transmissions.

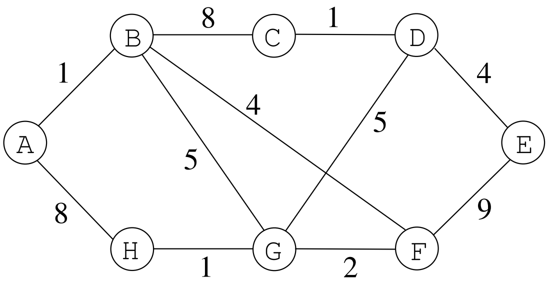
(a) What is the maximum possible window size (in terms of packets) for this TCP connection? [*Note.* In reality, most versions of TCP-Reno set a maximum window size beyond which the window size cannot increase. In this problem, we assume that such an upper limit does not exist, but rather we are interested in computing the maximum window size limit that is naturally imposed by the available bandwidth and the RTT.]

(b) Suppose that *ssthresh* is set to 20. How many packets were transmitted during the slow-start phase?

(c) In the long-run, what is the average throughput, in bits/second, over a congestion avoidance phase, from the time it starts until the first time a delay/loss is detected?

6. (Dijkstra's algorithm)

   Consider the network shown below. Use a table similar to the one used in the example in class, to illustrate the computation process of Dijkstra's algorithm at node A. Specifically, compute the minimum-cost paths, together with the associated costs, from A to all other nodes in the network. [Note. Here costs are symmetric, $c_{ij} = c_{ji}$, for all links (i, j) shown.]



7. (Bellman-Ford algorithm) Consider the same network as in the previous question. Use the Bellman-Ford algorithm to find the minimum-cost paths from all nodes to node A, and to compute the associated minimum costs ω^*_{iA} for all $i = B, C, D, E, F, G$ and H.

   Compare with the results of the previous problem.

SELECTED ANSWERS:

Problem 1. 3, 2/5 bits/trans, no, (00100), (01001)
Problem 2.ii. $\hat{\underline{x}} = (111, 101, 011, 111, 101, 011)$, $\hat{\underline{s}} = 100100$
Problem 2.iii. $\hat{\underline{x}} = (111, 101, 011, 111)$, $\hat{\underline{s}} = 1001$
Problem 5. 50, 31, 5.625 Mbps
Problem 6. $A \to B$ with cost 1, $A \to B \to C$ with cost 9, $A \to B \to C \to D$ with cost 10,
$A \to B \to F \to E$ with cost 14, $A \to B \to F$ with cost 5, $A \to B \to G$ with cost 6,
$A \to B \to G \to H$ with cost 7
Problem 7. Same as problem 6, in reverse order