# 3F4: Data Transmission

## Handout 1: Introduction, Signal space concepts

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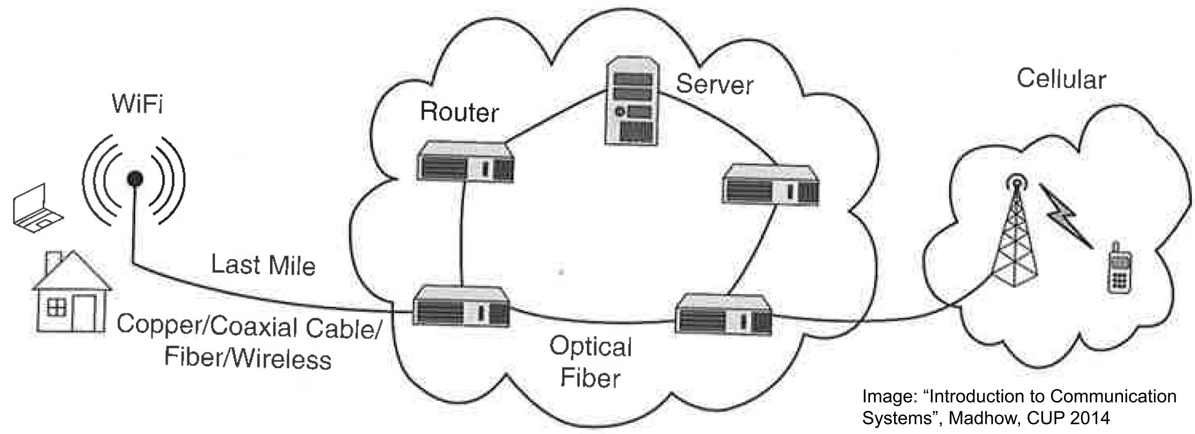


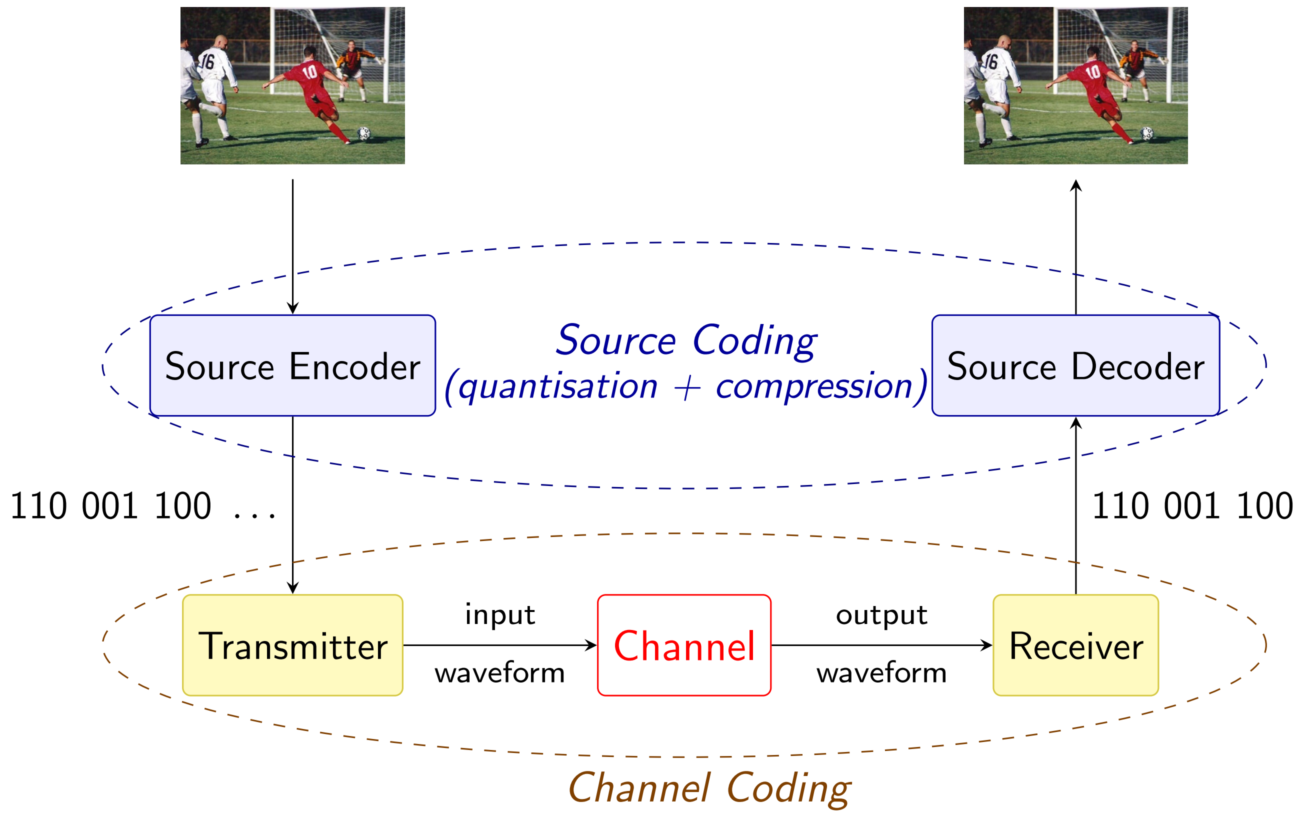

Image: "Introduction to Communication Systems", Madhow, CUP 2014

High-speed digital communication is now ubiquitous.

With the ever-increasing volumes of data being transmitted over networks, it is important to design efficient communication systems and algorithms that make optimal use of scarce resources such as power and network bandwidth.

This course is about the fundamental principles of designing such communication systems.

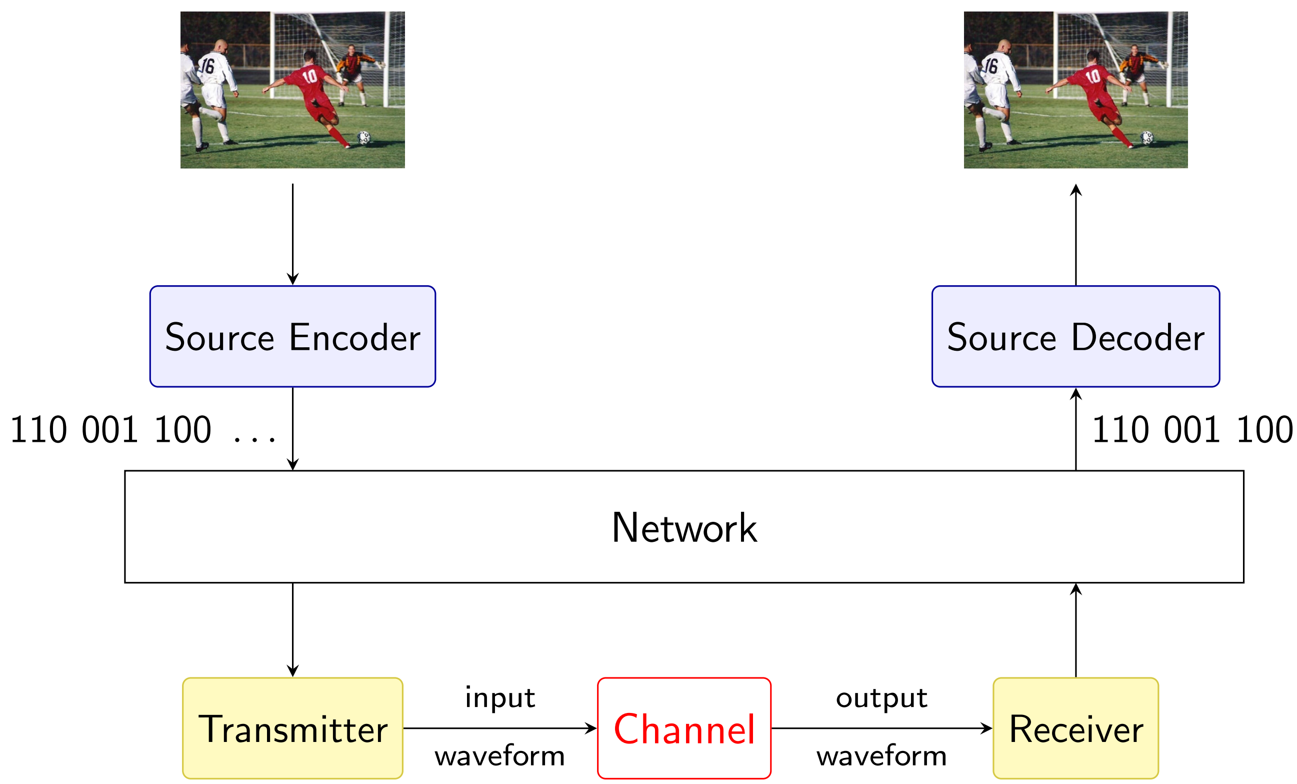
# An end-to-end communication system



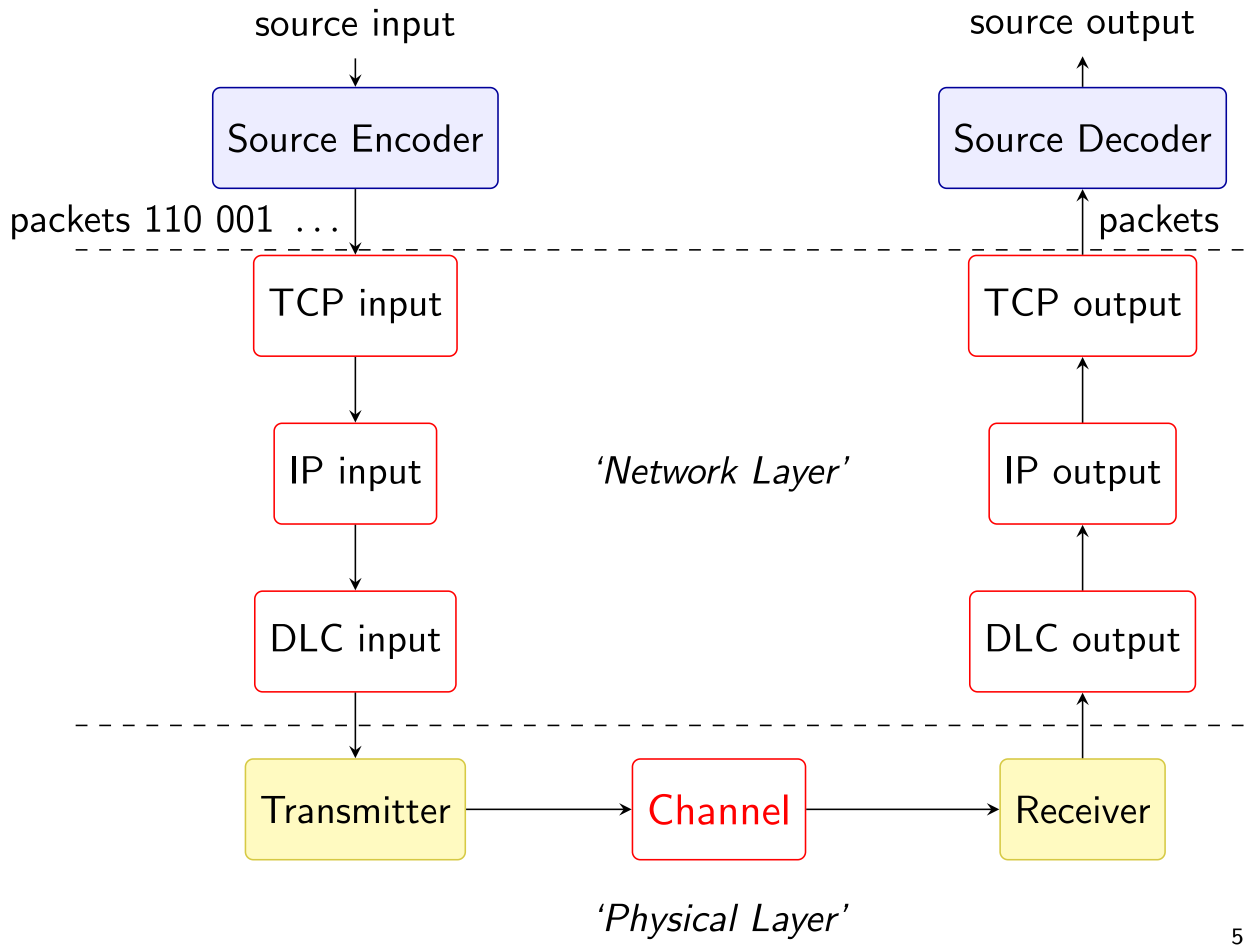

The picture above is highly oversimplified



In reality:



A more realistic (but still simplified) picture of a source-destination link is:






The 'network layer' is actually a combination of several layers:

1. There is a **TCP** (transport control protocol) module associated with each source-destination pair. TCP is responsible for end-to-end error recovery, and slowing down the transmission rate when the network becomes congested.
2. There is an **IP** (internet protocol) module associated with each node in the network. This is responsible for routing data packets through the network and to reduce congestion.
3. There is a **data link control** module associated with each channel. It accomplishes rate matching and an extra layer of error recovery on the channel.

Each layer adds some overhead: TCP and IP modules add headers, the DLC includes both header and trailer. What enters the physical layer is a sequence of frames, where each frame has the following structure:

| | | | | |
|---|---|---|---|---|
| DLC header | IP header | TCP header | SOURCE ENCODED PACKET | DLC trailer |

There are many more modules than the ones shown in the figure in slide 5, but we have chosen the key ones to illustrate the *layered* architecture of the network.

The key advantage of such a layered architecture is that each layer can be designed more or less independently.
For example, the following tasks can all be tackled independently of one another:

- Designing a good source encoder (quantisation + compression scheme),
- Designing good network layer algorithms (e.g., for routing and congestion control),
- Designing good transmission techniques for the physical layer,

**In this course:**

- Focus mostly on the physical layer (transmission techniques)
- The last three lectures will focus on network algorithms for routing and congestion control

Always keep in the mind the high-level picture on slide 5!



## Course Outline

Physical Layer (13L) + introduction to Network Layer (3L):

- Fundamentals of modulation & demodulation for baseband and passband channels (7L, Dr Venkataramanan)
- Advanced concepts in modulation (Equalization, OFDM) (3L, Dr Venkataramanan)
- Error-correction and convolutional coding (3L, Prof. Kontoyiannis)
- Network-layer algorithms for routing and congestion control (3L, Prof. Kontoyiannis)

Handouts, Examples Papers in Moodle

3F4 Lab (on baseband transmission) handled by Dr Jossy Sayir

Questions and active participation in lectures encouraged!

The lecture notes will be self-contained, but some useful references (available in the CUED library) are:

For physical layer (first 13 lectures):


- R. Gallager, *Principles of Digital Communication*, Cambridge University Press, 2008.
- B. Rimoldi, *Principles of Digital Communication: A Top-Down Approach*, Cambridge University Press, 2016.
- U. Madhow *Fundamentals of Digital Communication*, Cambridge University Press, 2008.

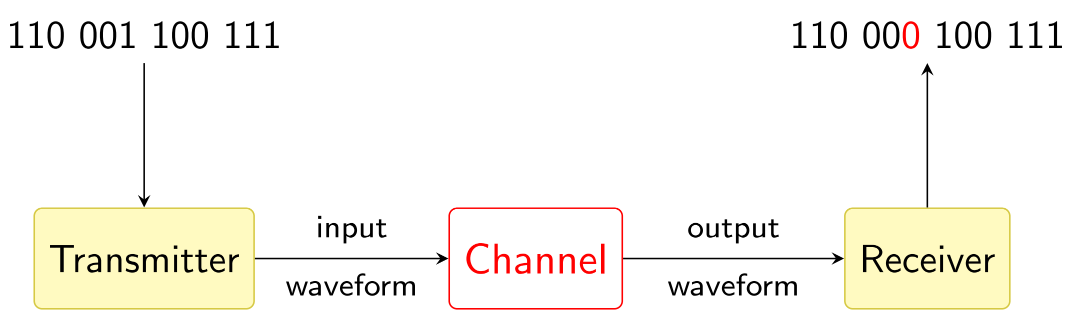

For network algorithms (last 3 lectures):


- R. Srikant and L. Ying *Communication Networks*, Cambridge University Press, 2014.


The syllabus for 3F4 has been significantly was significantly revised last year (2017-18), so many questions from previous Tripos papers **may not be relevant**. A list of relevant past Tripos questions will be posted on Moodle.

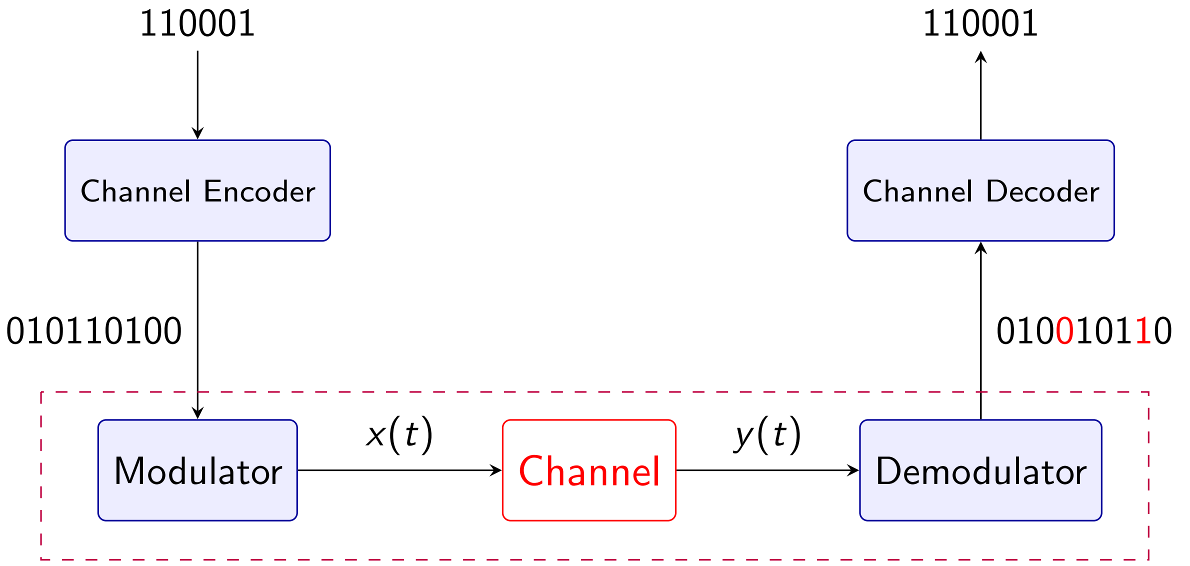


# The Physical Layer




Transmitter does two things:

1. *Coding*: Adding redundancy to the data bits to protect against noise
2. *Modulation*: Transforming the coded bits into waveforms




In the first 10 lectures our focus will be *within* the dashed box. We will study channel models, modulation and demodulation schemes

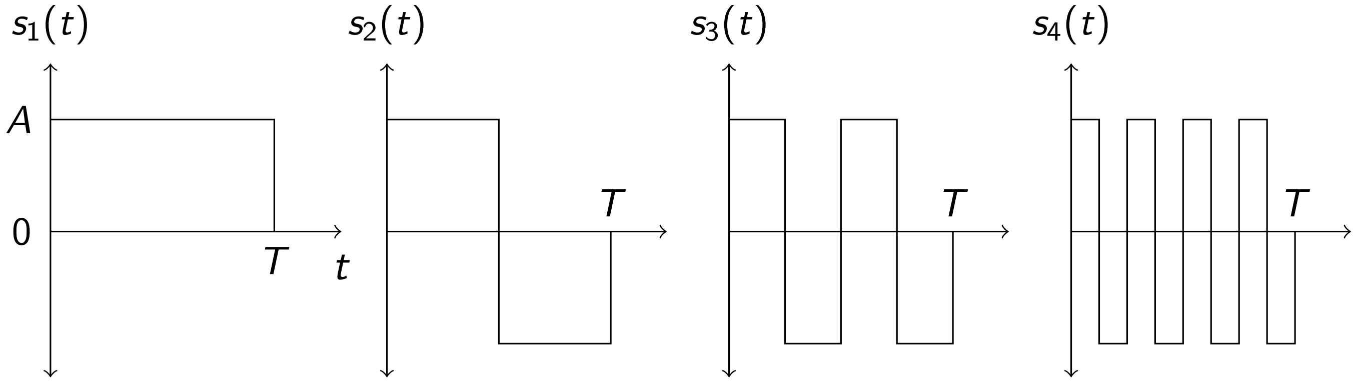
- Elegant application of Fourier theory, linear algebra, and probability to solve a key engineering problem
- We begin with some general linear algebra principles that will help understand mapping bits to continuous-time waveforms



## Mapping bits to waveforms

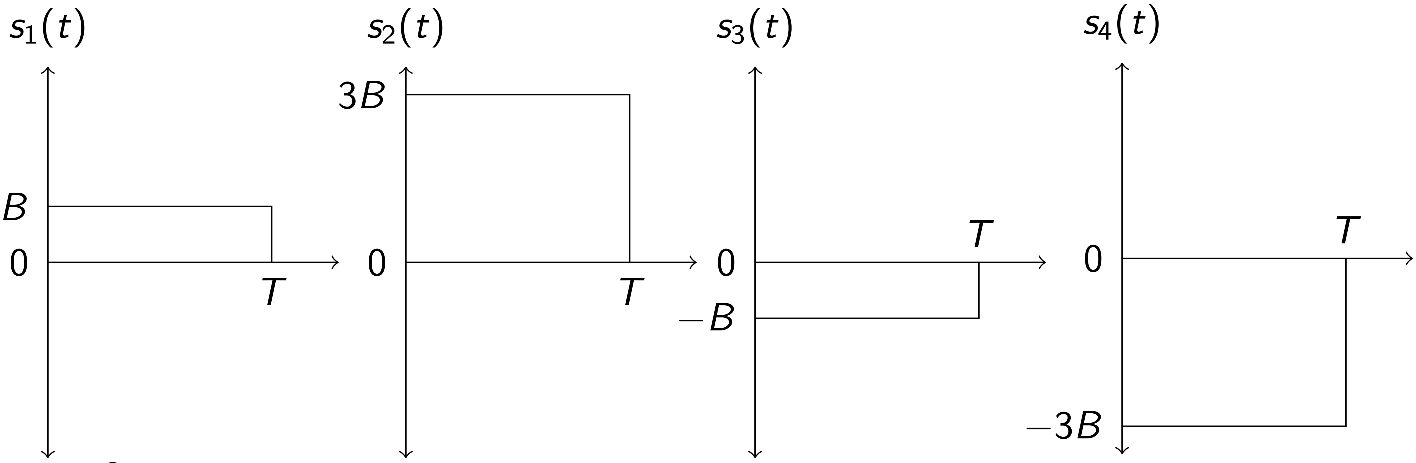
- Suppose we want to transmit $m$ bits in $T$ seconds using a waveform $x(t)$, $t \in [0, T)$.
- Note that each $m$-bit pattern indexes one of $M = 2^m$ messages
- We choose a collection of $M$ waveforms $\{s_1(t), \ldots, s_M(t)\}$. Map each $m$-bit sequence to one of these waveforms.

**Example**: $m = 2$ bits. One choice of $M = 4$ waveforms could be

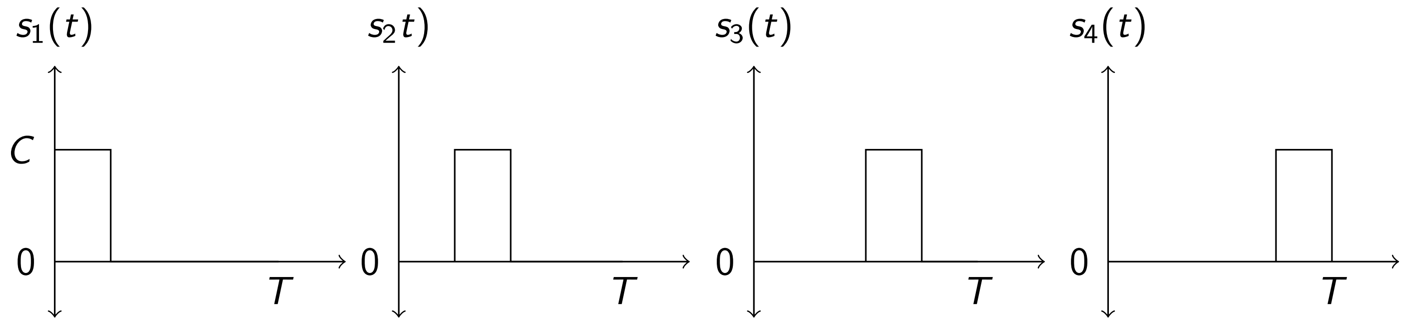



$00 \to s_1(t)$ $01 \to s_2(t)$ $10 \to s_3(t)$ $11 \to s_4(t)$

We could also choose something like:




OR






- We need a systematic way to represent finite-energy functions.
- We will see that by fixing a set of basis functions, signal waveforms can be represented as "vectors", which obey properties similar to vectors in Euclidean space $\mathbb{R}^n$.
- Let us first review what a vector space is ...

## Vector Spaces

A vector space $\mathcal{V}$ is a set of elements (called "vectors") that is **closed under addition and scalar multiplication**. The scalars are often chosen to be real or complex-valued, but they can be chosen over some other field as well.

That is, if $\underline{v}_1, \underline{v}_2 \in \mathcal{V}$, then $a\underline{v}_1 + b\underline{v}_2 \in \mathcal{V}$, for any scalars $a, b$.

You are already familiar with the Euclidean vector space $\mathbb{R}^k$.

A set of *linearly independent* vectors, say $\{\underline{v}_1, \underline{v}_2, \ldots, \underline{v}_k\}$, is called a **basis** of the vector space $\mathcal{V}$, if $v \in \mathcal{V}$ can be expressed as a linear combination of the form

$$\underline{v} = a_1\, \underline{v}_1 + a_2\, \underline{v}_2 + \ldots + a_k\, \underline{v}_k$$

for some scalars $a_1, \ldots, a_k$.
If the number of vectors in the basis is a finite number $k$, the vector space is said to have *dimension* $k$.
Q: Specify a basis for $\mathbb{R}^3$. Is the basis unique?



## Inner product, Orthogonality, Norm

You are familiar with the *inner-product* ("dot-product") in Euclidean space $\mathbb{R}^n$: if

$$\underline{v}_1 = x_1\underline{e}_1 + x_2\underline{e}_2 + \ldots + x_n\underline{e}_n,$$
$$\underline{v}_2 = y_1\underline{e}_1 + y_2\underline{e}_2 + \ldots + y_n\underline{e}_n,$$

then

$$\langle \underline{v}_1, \underline{v}_2 \rangle = x_1 y_1 + \ldots + x_n y_n.$$

(Here $\underline{e}_1, \ldots, \underline{e}_n$ are the unit vectors along the $n$ axes of $\mathbb{R}^n$)

- Recall that vectors $\underline{v}_1$ and $\underline{v}_2$ are *orthogonal* if $\langle \underline{v}_1, \underline{v}_2 \rangle = 0$. E.g. in $\mathbb{R}^n$, $\underline{e}_i, \underline{e}_j$ are orthogonal iff $i \neq j$.
- The **norm** ("length") of a vector $\underline{v}$ is defined as

$$\|\underline{v}\| = \sqrt{\langle \underline{v}, \underline{v} \rangle}$$

We will now extend these concepts to a vector space of *signals*. This vector space is called the *signal space*.

## The Signal Space

To understand communication over a continuous-time channel such as

$$Y(t) = X(t) + N(t),$$

it is useful to consider the vector-space of *finite-energy* signals.
Let $\mathcal{L}_2$ be the set of complex-valued signals (functions) $x(t)$ with finite energy, i.e.,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt < \infty$$

It can be shown that $\mathcal{L}_2$ is a vector space (set of finite-energy signals is closed under addition and scalar multiplication.)

- The inner product in this space can be defined as follows. For $x(.), y(.) \in \mathcal{L}_2$,
$$\langle x, y \rangle = \int_{-\infty}^{\infty} x(t) y^*(t) dt$$
- The norm of a signal is the square-root of its energy:
$$\|x\| = \sqrt{\langle x, x \rangle} = \left[ \int_{-\infty}^{\infty} |x(t)|^2 dt \right]^{1/2}$$



## Orthonormal Basis

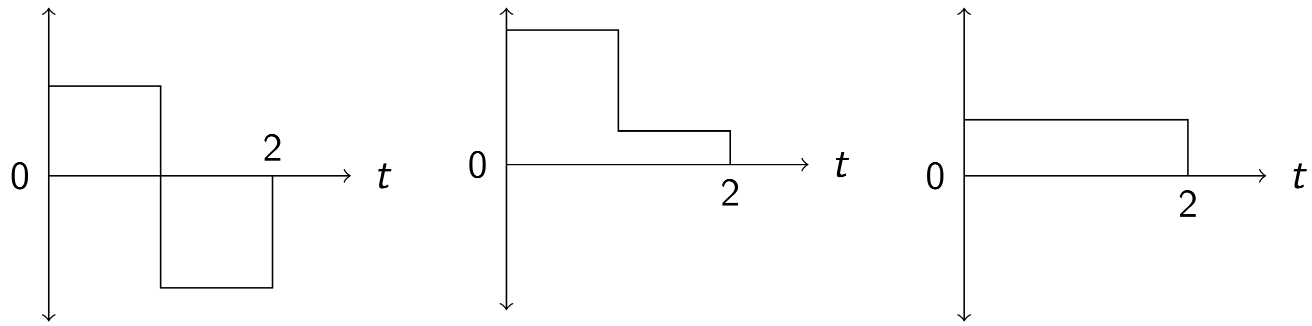
For any vector space $\mathcal{L} \subset \mathcal{L}_2$, the set of functions $\{f_i(.),\ i = 1, 2, \ldots\}$ is called an **orthonormal** basis for $\mathcal{L}$ if

1. Every $x(.) \in \mathcal{L}$ can be expressed as
$$x(t) = \sum_i x_i f_i(t),$$
for some *scalars* $x_1, x_2, \ldots$, and
2. The functions $\{f_i(.),\ i = 1, 2, \ldots\}$ are *orthonormal*, i.e.,
$$\langle f_\ell, f_m \rangle = \int f_\ell(t) f_m^*(t) dt = \begin{cases} 1 & \text{if } \ell = m \\ 0 & \text{if } \ell \neq m \end{cases}$$

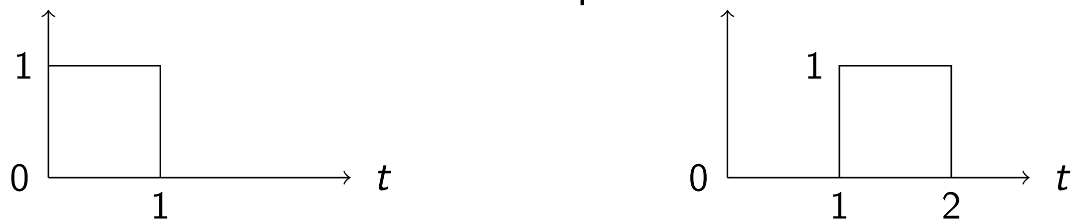
If these two conditions are satisfied, the orthonormal basis $\{f_i\}$ is said to *span* the vector space $\mathcal{L}$.
The number of elements (functions) in the basis is called the *dimension* of the vector space $\mathcal{L}$

# Example

Let $\mathcal{L}$ be the set of all functions $f(t)$ defined for $t \in [0, 2]$ that are piece-wise constant in the intervals $[0, 1]$ and $(1, 2]$.
E.g., some such functions are:




- This set of functions is a vector space (why?)
- An orthonormal basis for this space is:




What is the dimension of this vector space? Specify the projection coeffs. for the above functions with this basis. Is the basis unique?



Exercise: Find an orthonormal basis for the vector space spanned by each of the three sets of functions in pp. 12–13. Also specify the dimension of the space in each case.

# Gram-Schmidt procedure

- Given a set of functions, you can often you can find an orthonormal basis for them by careful inspection.
- But this may not always work, especially when there are many functions with no apparent structure.
- The Gram-Schmidt procedure gives a systematic way to find an orthonormal basis for a given set of functions (or vectors).



Given functions $\{x_1(t), x_2(t), \ldots, x_m(t)\}$, we find an orthonormal basis $\{f_1(t), f_2(t) \ldots\}$ as follows:

1. Let $f_1(t) = \frac{x_1(t)}{\|x_1(t)\|}$
2. Find the part $x_2$ orthogonal to $f_1$, and normalise. Let
$$g_2(t) = x_2(t) - \langle x_2, f_1 \rangle f_1(t).$$
Then, $f_2(t) = \frac{g_2(t)}{\|g_2(t)\|}$.
3. Find the part of $x_3$ orthogonal to $f_1, f_2$, and normalise. Let
$$g_3(t) = x_3(t) - \langle x_3, f_1 \rangle f_1(t) - \langle x_3, f_2 \rangle f_2(t).$$
Then, $f_3(t) = \frac{g_3(t)}{\|g_3(t)\|}$.

$\vdots$

If the dimension of the space is $k$, only $f_1, \ldots f_k$ will be non-zero.

Why do we care about an orthonormal basis for a given vector space $\mathcal{L}$? Consider any function $x(.) \in \mathcal{L}$, expressed in terms of an orthonormal basis $\{f_i(t)\}$ as

$$x(t) = \sum_i x_i f_i(t).$$

Each coefficient $x_j$ can be calculated as

$$x_j = \langle x, f_j \rangle = \int \Big( \sum_i x_i f_i(t) \Big) f_j^*(t) dt = \sum_i x_i \int f_i(t) f_j^*(t) dt = x_j$$

The coefficients $x_1, x_2, \ldots$ are called the *projection coefficients*, and $x_1 f_1(t),\ x_2 f_2(t) \ldots$ are the projections of the signal $x(t)$ along $f_1(t), f_2(t), \ldots$, respectively.

The inner product between $x(t)$ and $y(t)$ is

$$\langle x(t), y(t) \rangle = \int \Big( \sum_i x_i f_i(t) \Big) \Big( \sum_j y_j f_j(t) \Big)^* dt = \sum_i x_i y_i^*$$

The energy of $x(t)$ can therefore be written as

$$\int |x(t)|^2 dt = \langle x(t), x(t) \rangle = \sum_i |x_i|^2$$



Thus, if we fix an orthonormal basis for $\mathcal{L}$, we can treat functions in $\mathcal{L}$ just like vectors in Euclidean space, i.e.,

$$x(t) \leftrightarrow (x_1, x_2, \ldots x_k),$$

where $k$ is the number of elements in the orthonormal basis, i.e., the *dimension* of the signal space.

- We can use this vector representation to easily compute inner products between signals, energies etc.
- We have effectively converted continuous-time operations (integrals over $t$) into discrete-time operations (summations)!

Signal space is a convenient framework to analyse modulation and demodulation.