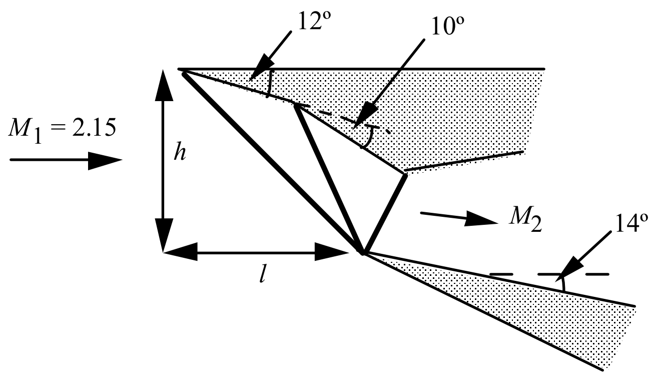
Paper 3A3: Fluid mechanics

TWO-DIMENSIONAL COMPRESSIBLE FLOW

Examples paper 2

Oblique shock waves

1 In the engine intakes of supersonic aircraft, the flow must be decelerated to subsonic speeds before it enters the compressor. The figure shows broadly how this is achieved on Concorde, with the flow passing through three oblique shocks.



(a) Calculate M_2, and the distance l.

(b) Find the fractional loss in stagnation pressure, $(p_{01} - p_{02})/p_{01}$ and compare it with that for a normal shock with the same inlet Mach number. How does a stagnation pressure loss manifest itself in terms of aircraft performance?

2 (a) Starting with the formula for density ratio across an oblique shock, show that, as the upstream Mach number becomes very large, the density ratio tends to $(\gamma+1)/(\gamma-1)$. Deduce the limiting value of the ratio of normal velocity upstream to that downstream and hence, derive the relation between deflection angle θ and shock angle β for very high Mach number oblique shock waves:

$$\tan\theta = \tan(\beta - (\beta - \theta)) = \frac{\tan\beta - \tan(\beta-\theta)}{1 + \tan\beta\tan(\beta-\theta)}$$

$$= \frac{2\tan\beta}{\gamma + 1 + (\gamma-1)\tan^2\beta}$$

Deduce the value of β for which $\theta = \theta_{max}$ and find θ_{max}.

*(b) Show that the energy equation reduces to

$$u_1^2 = \frac{2c_2^2}{\gamma - 1} + u_2^2$$

where subscripts 1 and 2 refer to conditions upstream and downstream of the shock. Show, further, that the flow is sonic downstream of a very strong oblique shock wave when $\theta = \theta_{max}$.

3 An oblique shock wave, with an upstream Mach number of 2 is generated by turning the flow through an angle θ. The shock wave impinges on a nearby wall which is parallel to the flow upstream. Determine the maximum value of θ for simple reflection at the wall. Sketch the flow when θ exceeds this value.

4 For an oblique shock inclined at angle β to the flow direction and giving flow deflection θ, show that

$$\frac{V_2}{V_1} = \frac{\cos\beta}{\cos(\beta - \theta)}$$

where V_1, V_2 are the upstream and downstream flow speeds respectively. Show further that, for a weak oblique shock ($\beta = \mu + \varepsilon$, $\varepsilon << 1$, $\theta << 1$),

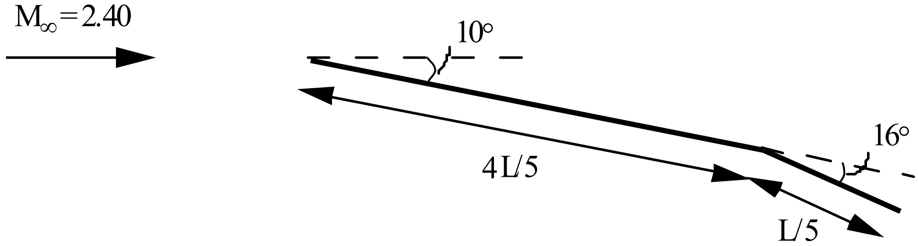
$$\frac{V_2 - V_1}{V_1} = -\frac{\theta}{\sqrt{M^2 - 1}}$$

to leading order in θ, ε.

As the shock strength becomes infinitesimal, we can replace V_2–V_1, by dV, θ by $d\theta$, and obtain the same relationship as applies across $+\mu$ waves: $d\nu + d\theta = 0$. What does this imply about this limiting form of shock wave? Hence explain why, although shock waves must be compressive, $+\mu$ waves can be compressive or expansive.

Shock-Expansion Theory

5 On re-entry to the earth's atmosphere, the space shuttle experiences a significant nose-up pitching moment, which is countered by a flap at the rear of the wing. A similar, two-dimensional geometry is shown.



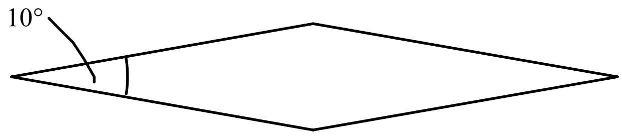
(a) The pitching moment coefficient is defined by

$$C_m = \frac{\text{moment about leading edge}}{\frac{1}{2}\rho_\infty U_\infty^2 L^2},$$

where ρ_∞ and U_∞ are the free-stream speed and density respectively. For the upstream Mach number and angle of attack shown, use shock-expansion theory to calculate the surface pressures in terms of p_∞, the free-stream pressure. Hence obtain C_m.

(b) Flaps on subsonic aircraft are progressively cambered, rather than set at an abrupt angle to the wing. Why is this?

6 The two-dimensional, symmetrical, double-wedge aerofoil shown is placed at an incidence of 10° in an airstream of Mach number 2.5. Using shock-expansion theory, determine the lift and drag coefficients at this incidence, and compare them with the values obtained from linearised theory.



Answers

NB: Values obtained using Houghton and Brock, CUED Library TL.114, where appropriate.

1 (a) 0.95, 1.25h (b) 0.053, compared with 0.348 for single normal shock

2 $\tan\theta_{max} = \dfrac{1}{\sqrt{\gamma^2 - 1}}$ when $\tan\beta = \sqrt{\dfrac{\gamma+1}{\gamma-1}}$

3 12.9°

5 (a) 0.283

6 0.315, 0.070 (shock-expansion theory)

0.305, 0.066 (linearised theory)

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