3F4: Data Transmission

Handout 2: Channel models, Baseband transmission using PAM

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Modelling a channel

Channels are often modelled as *linear time-invariant* systems with additive noise:

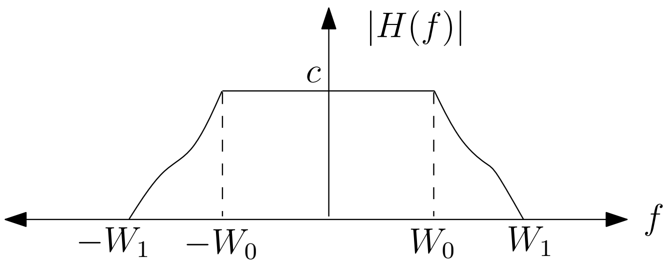
With such a model, channel output $y(t)$ generated from input $x(t)$ as

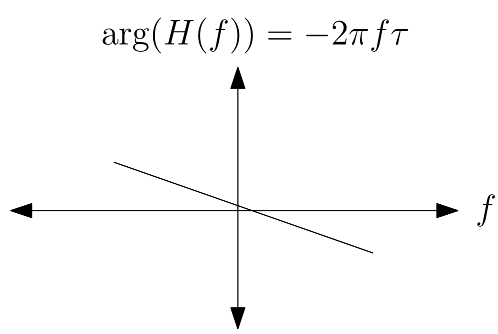
$$y(t) = h(t) \star x(t) + n(t)$$

In frequency domain:

$$Y(f) = H(f)X(f) + N(f)$$

For example, the frequency response of a telephone wire may look like:





Let us consider two situations with the above frequency response:

1. Suppose our input signal $x(t)$ is bandlimited to $[-W_0, W_0]$, where $|H(f)|$ is constant (say, equal to c). Then

$$\begin{aligned} Y(f) &= c\, e^{-j2\pi f\tau} X(f) + N(f) \\ \Rightarrow \quad y(t) &= c\, x(t-\tau) + n(t) \end{aligned}$$

   We can compensate for the constant channel gain c, and the constant delay τ at the receiver. So the channel is effectively:

$$y(t) = x(t) + n(t)$$

   Such a channel model – with only additive noise – is a good one for channels such as telephone cables carrying only voice.

2. Suppose our input signal $x(t)$ now has larger bandwidth spanning $[-W_1, W_1]$. Then the additive noise model is no longer adequate, so we need to understand how the channel *filters* the input signal:

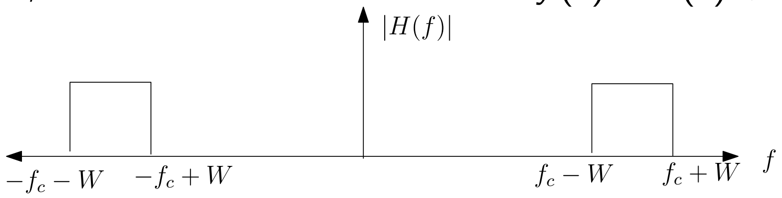
$$Y(f) = H(f)X(f) + N(f) \quad \text{or} \quad y(t) = \int h(u)x(t-u)du + n(t).$$

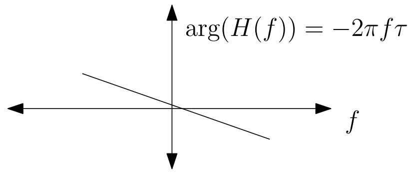
   Such a model is relevant when the telephone wire is used for DSL broadband communication.

Passband channel models

Here the signal is restricted to have frequency components in the band $[f_c + W, f_c - W]$, where f_c is a carrier frequency (typically $f_c \gg W$). As in baseband, there could be two scenarios in a passband channel.

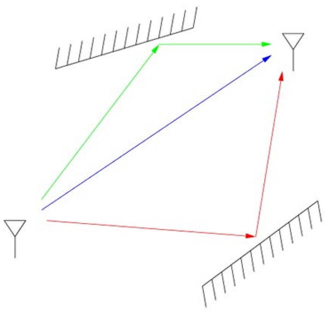
If $|H(f)|$ is constant throughout the band and the delay is also a constant τ, then an additive noise model $y(t) = x(t) + n(t)$ can be used.

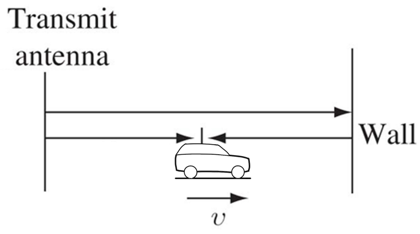




Such a model is suitable for applications such as communication with a geostationary satellite, where there is a single path from Tx to Rx.

However, in mobile wireless communication, there are typically multiple signal paths from Tx to Rx.





From "Fundamentals of Wireless Communication", Tse and Viswanath, CUP 2005

For example, if there are L signal paths with delays $\tau_1, \dots, \tau_L$ and attenuations $\alpha_1, \dots, \alpha_L$, the received signal is

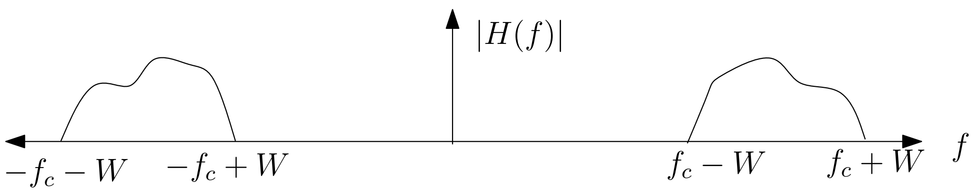
$$\begin{aligned} y(t) &= \alpha_1 x(t-\tau_1) + \alpha_2 x(t-\tau_2) + \ldots + \alpha_L x(t-\tau_L) + n(t) \\ &= x(t) \star h(t) \; + \; n(t), \end{aligned}$$

where $h(t) = \alpha_1 \delta(t - t_1) \, + \, \alpha_2 \delta(t - t_2) \, + \ldots + \, \alpha_L \delta(t - t_L)$.

In general

$$y(t) = \int h(u) x(t-u) du + n(t) \tag{1}$$

where $h(u)$ is the channel gain/attenuation along path with delay u.



Hence in such situations, we cannot use a simple additive noise model. In fact, even the LTI model in (1) is simplified because the path delays change with time if the Tx and/or Rx are mobile.

Noise model

$$y(t) = h(t) \star x(t) + n(t)$$

$n(t)$ is thermal noise at the Rx:

- Thermal noise is the noise generated by the thermal agitation of electrons inside an electrical conductor
- Happens regardless of the applied voltage
- All receivers (WiFi, mobile phone, AM, FM,...) generate thermal noise

We will model $n(t)$ as a Gaussian noise process.
If we operate in a frequency band where $H(f)$ is flat, then the effective channel

$$y(t) = x(t) + n(t)$$

is called an *additive Gaussian noise* channel

Baseband channel with additive Gaussian noise

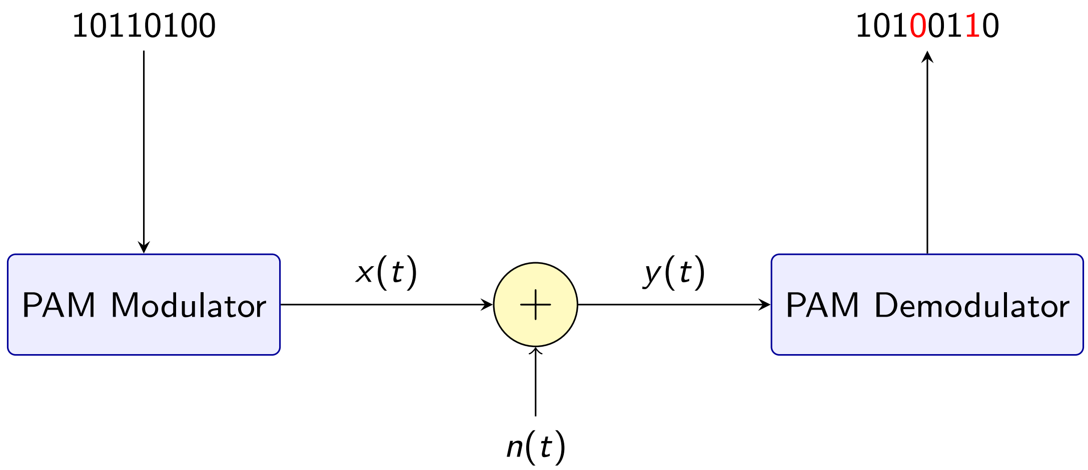
We will focus on understanding communication over a baseband channel whose frequency response is flat in the band $[-W, W]$ and zero outside.

- Recall from slides 3/4 that a constant attentuation factor c and a delay τ can be compensated for, so that the channel is effectively:

$$y(t) = x(t) + n(t)$$

- $x(t)$ has to be bandlimited to $[-W, W]$, as any frequency components outside this band will be cut off by the channel
- The most common modulation scheme for this channel is Pulse Amplitude Modulation (PAM), which you are already familiar with from 1B Paper 6 Communications

Here we will revisit PAM and study the underlying concepts in greater depth, as these form the foundation for more sophisticated modulation schemes



PAM Modulator has two parts:

1. Map input bits to symbols in a *constellation* $\mathcal{C}$
2. Use shifts of a baseband *pulse* to map the information symbols to a waveform $x(t)$

The Symbol Constellation

The set of values the bits are mapped to is called the *constellation*.
E.g., 4-ary constellation could be of the form $\{-3A, A, A, 3A\}$, for some real number $A > 0$

$$\text{Binary PAM}: \quad 0 \to -A, \quad 1 \to A$$
$$\text{4-ary PAM}: \quad 00 \to -3A, \quad 01 \to -A, \quad 10 \to A, \quad 11 \to 3A$$

Once we fix a constellation, a sequence of bits can be uniquely mapped to constellation symbols. E.g., with constellation $\{-A, A\}$

$$0\ 1\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 0 \longrightarrow -A,\ A, -A,\ A,\ A,\ A, -A, -A,\ A, -A$$

With constellation $\{-3A, -A, A, 3A\}$, the same sequence of bits is mapped as

$$\underline{01}\ \underline{01}\ \underline{11}\ \underline{00}\ \underline{10} \quad \longrightarrow \quad -A,\ -A,\ 3A,\ -3A,\ A$$

In a constellation with M symbols, each symbol represents $\log_2 M$ bits

The Pulse Waveform

The other component of Pulse Amplitude Modulation is a unit-energy *baseband* waveform denoted $p(t)$, called the *pulse* waveform.

A sequence of constellation symbols $X_0, X_1, X_2, \ldots$ is used to generate a *baseband* signal as follows

$$x(t) = \sum_k X_k p(t - kT)$$

- T is called the *symbol* time of the pulse
- Shift of the pulse by kT carries the symbol X_k, for $k = 0, 1, \ldots$

Thus we have the following steps to map bits to a baseband signal $x(t)$:

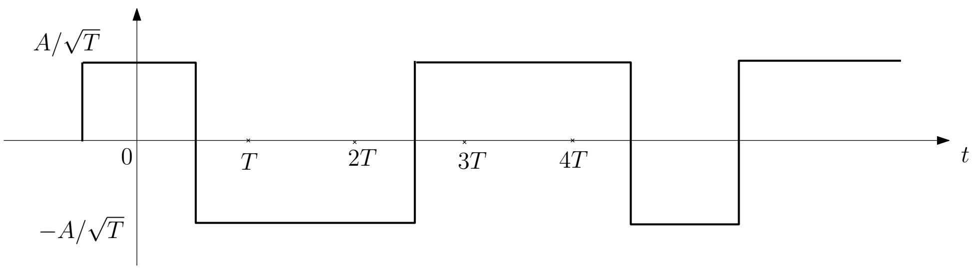
$$\ldots 0\ 1\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 0 \ldots \longrightarrow X_0, X_1, X_2, \ldots \longrightarrow \sum_k X_k p(t - kT)$$

Rate of Transmission

The modulated baseband signal is $x(t) = \sum_k X_k p(t - kT)$.
For example, with a rect pulse shape

$$p(t) = \frac{1}{\sqrt{T}} \text{rect}\left(\frac{t}{T}\right) = \begin{cases} \frac{1}{\sqrt{T}} & \text{for } t \in \left[\frac{-T}{2}, \frac{T}{2}\right] \\ 0 & \text{otherwise} \end{cases}$$

and $X_k \in \{+A, -A\}$, $x(t)$ looks like



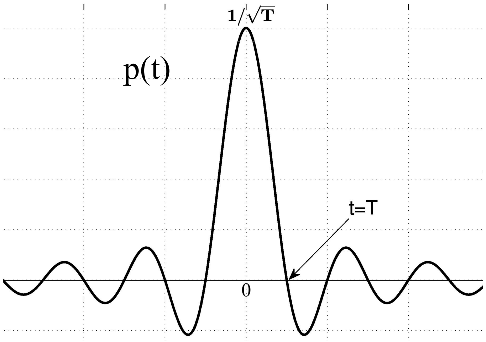
Every T seconds, a new symbol is introduced by shifting the pulse and modulating its amplitude with the symbol.

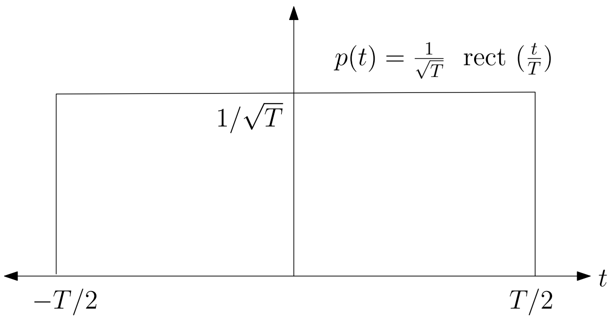
The *transmission rate* is $\frac{1}{T}$ **symbols/sec** or $\frac{\log_2 M}{T}$ **bits/second**

How do we choose the pulse $p(t)$?

E.g., are the sinc or rect functions good choices?

$$p(t) = \frac{1}{\sqrt{T}} \operatorname{sinc}\left(\frac{\pi t}{T}\right) \quad \text{or} \quad p(t) = \begin{cases} \frac{1}{\sqrt{T}} & \text{for } t \in \left[\frac{-T}{2}, \frac{T}{2}\right] \\ 0 & \text{otherwise} \end{cases}$$





(We use rect(t/T) to denote the function that equals 1 from $[-\frac{T}{2}, \frac{T}{2}]$ and zero elsewhere)

We will now address the question of how to choose a good pulse $p(t)$, and see that neither the sinc nor the rect are practical choices.

But a rect pulse is sometimes useful to visualize a PAM waveform.

Desirable Properties of the Pulse Shape $p(t)$

We would like $p(t)$ is chosen to satisfy the following criteria:

1. We want $p(t)$ to decay quickly in time:
   since symbol X_k is carried by $p(t - kT)$, its effect should not start much before $t = kT$ or last much beyond $t = (k+1)T$
2. We want $p(t)$ to be band-limited to $[-W, W]$.
3. The retrieval of the information sequence from the *noisy* received waveform $x(t) + n(t)$ should be simple and relatively reliable.
   In the absence of noise, the symbols $\{X_k\}_{k \in \mathbb{Z}}$ should be recovered perfectly at the receiver.

Let us compare the sinc and rect pulses with respect to the first two criteria . . .

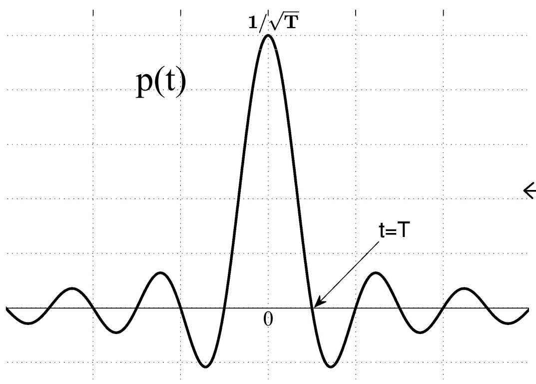
Time Decay vs. Bandwidth Trade-off

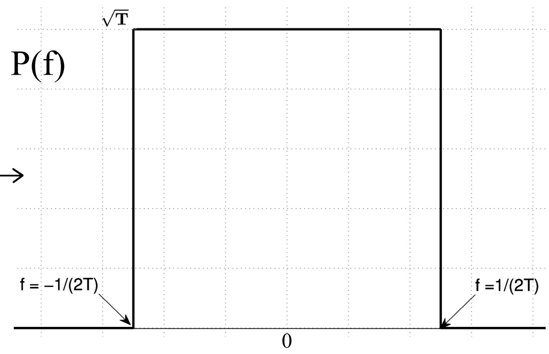
The first two objectives say that we want $p(t)$ to:

1. Decay quickly in time
2. Be approximately band-limited

But ... faster decay in time $\Leftrightarrow$ larger bandwidth

Consider the pulse $p(t) = \frac{1}{\sqrt{T}} \text{sinc}\left(\frac{\pi t}{T}\right)$

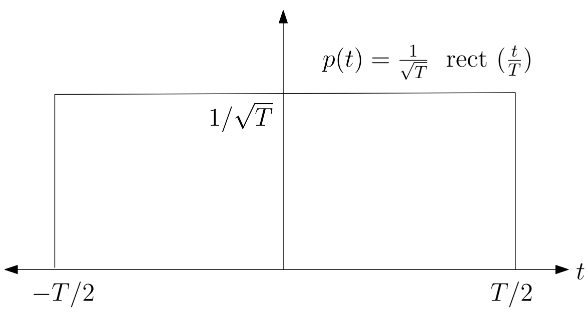


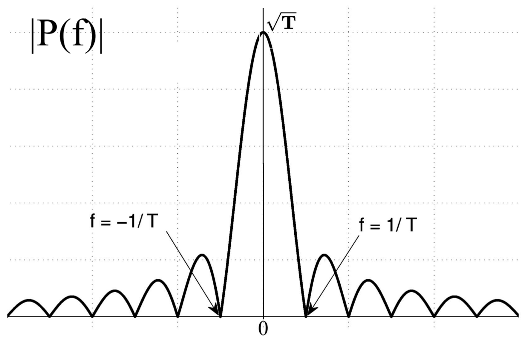


The sinc is perfectly band-limited to $W = \frac{1}{2T}$

But decays too slowly in time $|p(t)| \sim \frac{1}{|t|}$

Next consider the rectangular pulse $p(t) = \begin{cases} \frac{1}{\sqrt{T}} & \text{for } \left[\frac{-T}{2}, \frac{T}{2}\right] \\ 0 & \text{otherwise} \end{cases}$





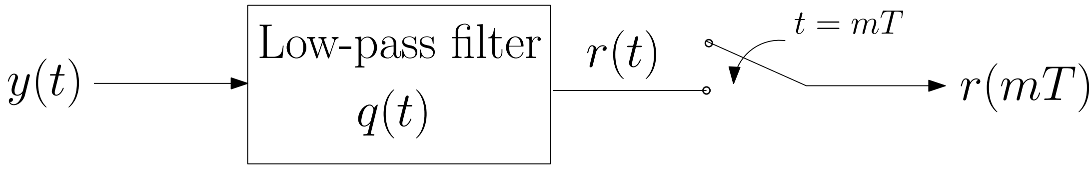
This pulse is perfectly time-limited to the symbol interval $[0, T)$. But ...

- Decays too slowly in freq. $|P(f)| \sim \frac{1}{|f|}$
- Main-lobe bandwidth $= \frac{1}{T}$

Neither of these is used in practice as we need a pulse that decays reasonably fast in both time and frequency.

To design a suitable pulse, let us start with the third criterion: easy retrieval of the symbols $\{X_k\}$ from the waveform $x(t)$

Demodulator structure



Consider the above demodulator structure, which:

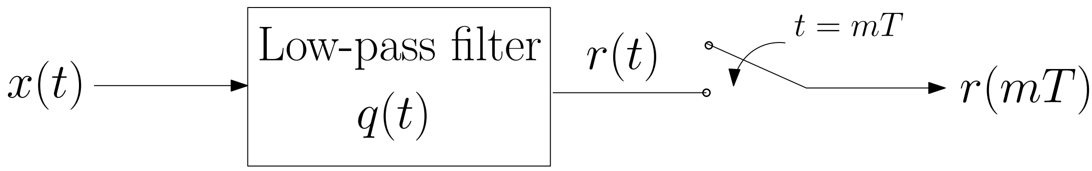
1. Low-pass filters the received signal using a filter with impulse response $q(t)$
2. Then samples the filter output at integer multiples of time T — this sampling operation is called *slicing*

Such a demodulator is simple to implement, and takes advantage of the fact that the signal is low-pass and filters out-of-band noise.

Key Q: How should we choose the filter impulse response $q(t)$ so that in the absence of noise, i.e., when $y(t) = x(t) = \sum_k X_k p(t - kT)$, the sampled output recovers the information symbols?

That is, we want:

$$r(mT) = X_m$$



When there is no noise, the filter output is

$$\begin{aligned} r(t) &= x(t) \star q(t) \\ &= \int_{-\infty}^{\infty} q(u) x(t-u)\, du \\ &= \sum_k X_k \int_{-\infty}^{\infty} q(u) p(t - kT - u)\, du \\ &= \sum_k X_k g(t - kT) \end{aligned} \tag{2}$$

where

$$g(t) = q(t) \star p(t) = \int_{-\infty}^{\infty} q(u) p(t-u)\, du.$$

- $g(t)$ can be considered the "effective" pulse at the receiver
- $p(t)$ is sometimes called *transmit filter*, $q(t)$ the *receive filter*, and $g(t)$ the *overall filter*

From (2), we see that

$$r(mT) = \sum_k X_k g((m-k)T).$$

Therefore, if we want $r(mT) = X_m$ for all integers m, then $g(t)$ should satisfy:

$$g(mT) = \begin{cases} 1, & m = 0 \\ 0, & m = \ldots, -2, -1, 1, 2, 1, \ldots \end{cases} \tag{3}$$

If $g(t)$ does not satisfy (3), we have **inter-symbol interference** (ISI):

$$r(mT) = X_m + \underbrace{\text{contributions from } \{X_k\}_{k \neq m}}_{\text{ISI}}$$

The condition (3) is equivalent to the following condition in the frequency domain, called the *Nyquist pulse shaping* criterion.

Nyquist Pulse Criterion

Let $G(f)$ denote the Fourier transform of the effective pulse $g(t)$. Then the demodulator will have no ISI (i.e., Eq. (3) will be satisfied) if and only if

$$\sum_{n=-\infty}^{\infty} G\left(f - \frac{n}{T}\right) = T.$$

Proof: Multiplying $g(t)$ with an impulse train, we obtain:

$$g(t)\left[\sum_{m=-\infty}^{\infty} \delta(t - mT)\right] = \sum_m g(mT)\delta(t - mT) \tag{4}$$

Now, if the condition in Eq. (3) for no ISI is satisfied, then $g(0) = 1$, and $g(mT) = 0$ for $m \neq 0$. Using this in the above, we get

$$g(t)\left[\sum_{m=-\infty}^{\infty} \delta(t - mT)\right] = \delta(t). \tag{5}$$

Taking Fourier transforms on both sides, we have

$$G(f) \star \mathcal{F}\left[\sum_{m=-\infty}^{\infty} \delta(t - mT)\right] = 1. \tag{6}$$

Proof (contd.)

The proof is then completed using the following fact (shown later):

$$\mathcal{F}\left[\sum_{m=-\infty}^{\infty}\delta(t-mT)\right]=\frac{1}{T}\sum_{n=-\infty}^{\infty}\delta\left(f-\frac{n}{T}\right). \tag{7}$$

Using (7) in (6), we obtain

$$G(f)\star\sum_{n=-\infty}^{\infty}\delta\left(f-\frac{n}{T}\right)=T, \tag{8}$$

which simplifies to the desired condition $\sum_n G\left(f-\frac{n}{T}\right)=T$.

Conversely, if $\sum_k G\left(f-\frac{n}{T}\right)=T$ is satisfied, then (8) holds, and we work backwards through the steps to see that

$$\sum_m g(mT)\delta(t-mT)=\delta(t),$$

or, $g(0)=1$ and $g(mT)=0$ for $m\neq 0$.
This completes the proof once we show the fact in Eq. (7).

To prove (7), we note that the $\sum_{m=-\infty}^{\infty}\delta(t-mT)$ is periodic with period T. So it can be expressed in terms of its Fourier series:

$$\sum_{m=-\infty}^{\infty}\delta(t-mT)=\sum_n c_n e^{jn\frac{2\pi}{T}t},$$

where the Fourier coefficients are

$$c_n=\frac{1}{T}\int_{-T/2}^{T/2}\left[\sum_m\delta(t-mT)\right]e^{-j\frac{2\pi n}{T}t}dt=\frac{1}{T}$$

Therefore

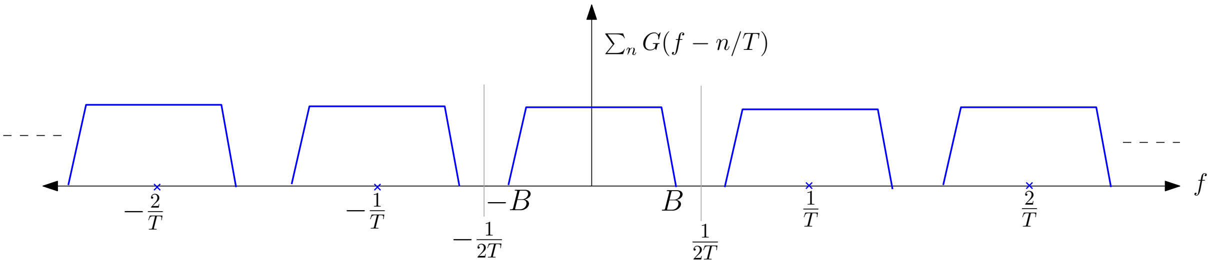
$$\mathcal{F}\left[\sum_{m=-\infty}^{\infty}\delta(t-mT)\right]=\mathcal{F}\left[\sum_n\frac{1}{T}e^{jn\frac{2\pi}{T}t}\right]=\sum_n\frac{1}{T}\delta\left(f-\frac{n}{T}\right),$$

as claimed in (7). □

Implications of Nyquist pulse criterion

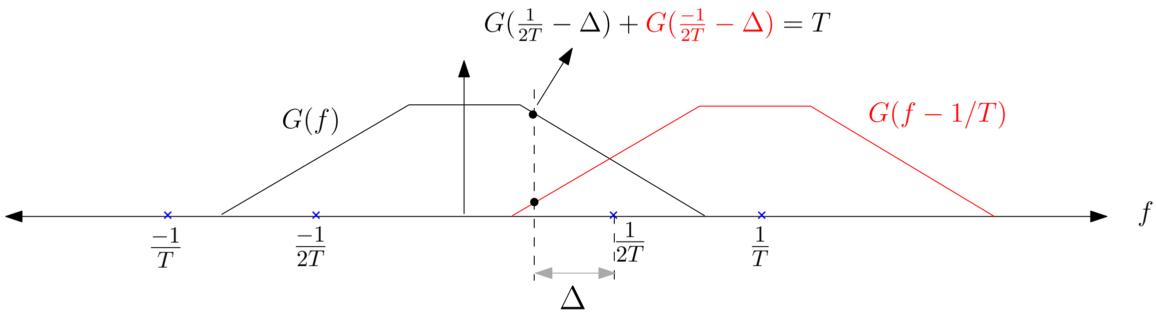
The Nyquist pulse criterion implies that in order to have no ISI, $G(f)$ must have bandwidth at least $1/(2T)$.

As shown below, if $G(f)$ were band-limited to $[-B, B]$, for $B < \frac{1}{2T}$, then $\sum_n G(f - n/T)$ has 'gaps' and thus cannot be uniformly equal to T.



- We are most interested in the case where the pulse bandwidth B lies in $(\frac{1}{2T}, \frac{1}{T}]$
- If B is much larger than the symbol rate $\frac{1}{T}$, then we are wasting a lot of bandwidth by signalling too slowly
- The sinc and the rect pulses on slides 15/16 both satisfy the criterion (most easily checked by the time domain criterion in Eq. (3))
- But the Nyquist criterion allows us to be much more flexible in our choice for $g(t)$

For example, we could choose:



If the pulse bandwidth B lies in $(\frac{1}{2T}, \frac{1}{T}]$, then, for all $\Delta \in [0, \frac{1}{2T})$ we need

$$G\left(\frac{1}{2T} - \Delta\right) + G\left(-\frac{1}{2T} - \Delta\right) = T.$$

Furthermore, if $G(f)$ is real and even (i.e., $g(t)$ is real and even), then $G(f) = G(-f)$, and the condition becomes:

$$G\left(\frac{1}{2T} - \Delta\right) + G\left(\frac{1}{2T} + \Delta\right) = T, \qquad \text{for } \Delta \in \left[0, \frac{1}{2T}\right].$$

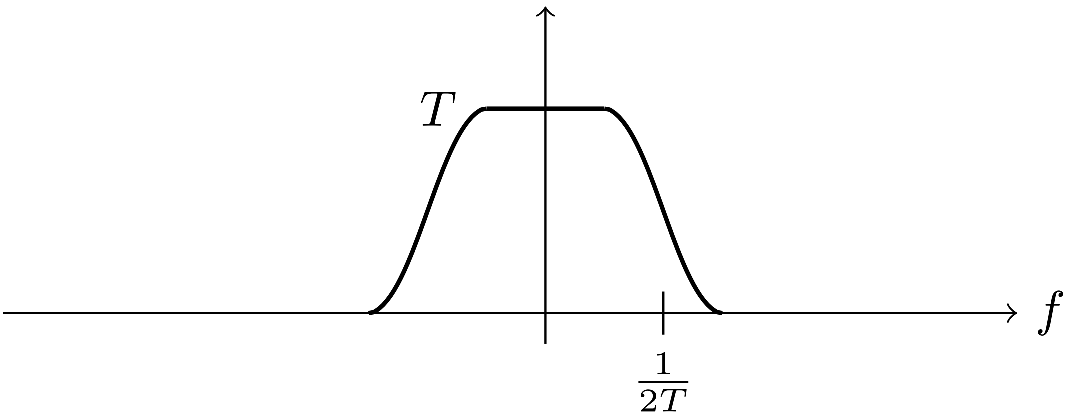
This condition is called *band-edge symmetry*

The raised-cosine family

In practice, the effective pulse $g(t)$ is often chosen to have a *raised cosine* transform.

The raised cosine frequency function, defined for any *rolloff factor* $\beta \in [0, 1]$, is defined by:

$$G_\beta(f) = \begin{cases} T, & 0 \leq |f| \leq \frac{1-\beta}{2T} \\ T \cos^2\left[\frac{\pi T}{2\beta}\left(|f| - \frac{(1-\beta)}{2T}\right)\right], & \frac{1-\beta}{2T} \leq |f| \leq \frac{1+\beta}{2T} \\ 0, & |f| \geq \frac{1+\beta}{2T} \end{cases}$$



$G_\beta(f)$ for $\beta = \frac{1}{2}$

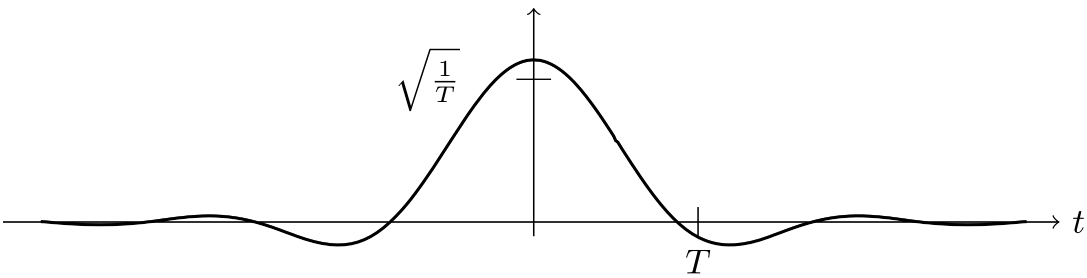
The root raised-cosine function

Recall that $G(f) = P(f)Q(f)$, where P, Q are the transmit and receive filters, respectively. We can therefore choose

$$P(f) = Q(f) = \sqrt{G_\beta(f)},$$

where $\sqrt{G_\beta(f)}$ is called the *root raised cosine* frequency function. The transmitted pulse $p(t)$ is therefore obtained as the inverse Fourier transform of $\sqrt{G_\beta(f)}$:

$$p(t) = \frac{4\beta}{\pi\sqrt{T}}\left[\frac{\cos\left[(1+\beta)\pi\frac{t}{T}\right] + \frac{(1-\beta)\pi}{4\beta}\text{sinc}\left[(1-\beta)\pi\frac{t}{T}\right]}{1 - \left(4\beta\frac{t}{T}\right)^2}\right]$$



$p(t)$ for $\beta = \frac{1}{2}$

Some properties of the root raised cosine pulse $p(t)$:

- For $\beta = 0$, $p(t)$ becomes the sinc pulse $\frac{1}{\sqrt{T}}\text{sinc}(\pi t/T)$ which decays as $\sim 1/t$
- For $0 < \beta \leq 1$, $p(t)$ decays as $\sim 1/t^2$, faster than sinc.
- The impulse response of the *overall* filter is $g(t) = p(t) \star p(t)$, the inverse Fourier transform of the raised cosine frequency function:

$$g(t) = \frac{1}{T}\text{sinc}\left(\frac{\pi t}{T}\right) \frac{\cos(\pi \beta t/T)}{1 - (2\beta t/T)^2}.$$

- For $\beta = 0$, $g(t)$ is a sinc, and hence decays as $\sim 1/t$. For $\beta \in (0, 1]$, the overall pulse $g(t)$ decays as $1/t^3$, which is much faster than sinc.
- As β increases from 0 to 1, pulse $p(t)$ decays faster, but bandwidth increases as well, from $\frac{1}{2T}$ to $\frac{1}{T}$.
- $p(t)$ and $g(t)$ are both real-valued, even, and have infinite support. $p(t)$ has to be truncated and delayed to make it causal.
- The faster the decay in t, the less the deviation from the Nyquist pulse criterion due to truncation.

Signal Space interpretation of Nyquist pulse criterion

- Let $G(f)$ be any real, non-negative function that satisfies the Nyquist criterion: $\sum_n G(f - n/T) = T$.
- Let the transmit and receive filters satisfy

$$|P(f)| = |Q(f)| = \sqrt{G(f)}.$$

Then for $P(f)Q(f) = G(f)$, we need

$$\arg(Q(f)) = -\arg(P(f)) \quad \Rightarrow \quad Q(f) = P^*(f).$$

As $p(t)$ is real, $P^*(f) = P(-f)$. Hence $Q(f) = P(-f) \Rightarrow q(t) = p(-t)$. With this choice,

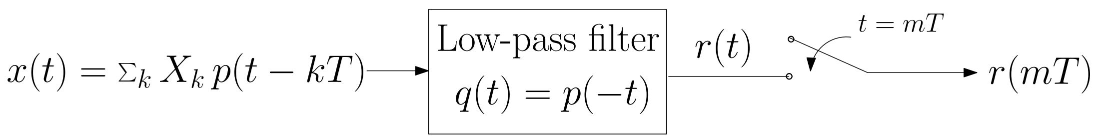
$$g(t) = p(t) \star p(-t) = \int_{-\infty}^{\infty} p(u)p(u-t)du. \tag{9}$$

Furthermore, $g(t)$ satisfies Nyquist pulse criterion. Therefore, from (9)

$$g(nT) = \int_{-\infty}^{\infty} p(u)p(u-nT)du = \begin{cases} 1, & n = 0 \\ 0, & n = \pm 1, \pm 2, \ldots \end{cases} \tag{10}$$

> Eq. (10) implies that $p(t)$ is orthonormal to each of its shifts by an integer multiple of T, i.e., $\{p(t-nT)\}_{n\in\mathbb{Z}}$ is an orthonormal set of functions.

Revisiting the demodulator for this case:



$$r(t) = \left[\sum_k X_k\, p(t-kT)\right] \star p(-t) = \sum_k X_k \int_{-\infty}^{\infty} p(u-kT)p(u-t)du$$

Therefore, for any integer m:

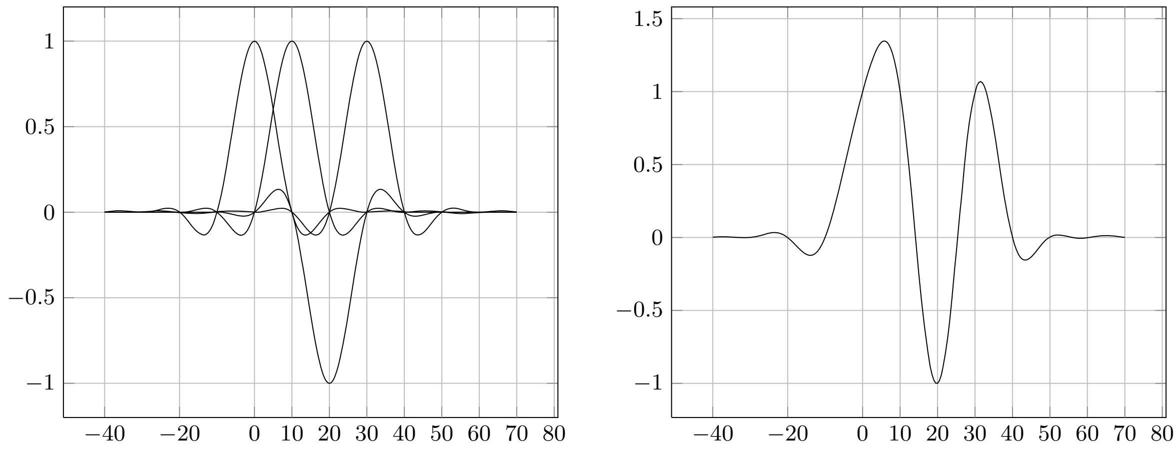
$$r(mT) = \sum_k X_k \int_{-\infty}^{\infty} p(u-kT)p(u-mT)du \overset{(i)}{=} \sum_k X_k \mathbf{1}\{k=m\} = X_m$$

where equality (i) is due to the orthonormality property (10). Thus:

- At time mT, the demodulator – also known as *matched filter* – computes the projection (inner product) of the received signal with the function $p(t-mT)$.
- The orthonormality property ensures that X_m is the only information symbol picked out at time mT.

Consider the (noiseless) rx filter output $r(t) = \sum_k X_k g(t - kT)$, with:

- Symbols $X_k \in \{\pm 1\}$, $T = 10$, and
- $g(t)$ is the raised-cosine function. That is, transmit pulse $p(t)$ is root raised cosine, and the matched filter is $p(-t)$.



(a) (b)

Figure from *Principles of Digital Communication* by B. Rimoldi, CUP 2016.

Panel (a) shows the individual waveforms $X_k g(t - kT)$, for $k = 0, 1, 2, 3$ with the symbols $\{1, 1, -1, 1\}$. Panel (b) shows the combined signal $r(t)$.

In the figure, note that $T = 10$ and the samples $\{r(kT)\}$ for $k = 0, 1, 2, 3$ are exactly equal to $1, 1, -1, 1$, respectively.

That is, there is no *inter-symbol interference* (ISI). This is because $g(t)$ satisfies the Nyquist pulse criterion.

There are a couple of factors that can introduce ISI:

1. One is if $g(t)$ does not exactly satisfy Nyquist pulse criterion. For example, this could be caused due to truncation of the root cosine pulses.
2. If the sampling times are not exactly integer multiples of kT, say, we sample at $\{kT + \Delta\}$. For example, sampling $r(t)$ at $t = \Delta$ gives $X_0 g(\Delta) + X_1 g(T + \Delta) + X_2 g(2T + \Delta) + \dots$.

Eye Diagrams

The *eye diagram* is a technique that allows us to visualise whether there is ISI, and examine the effect of imprecise sampling times.

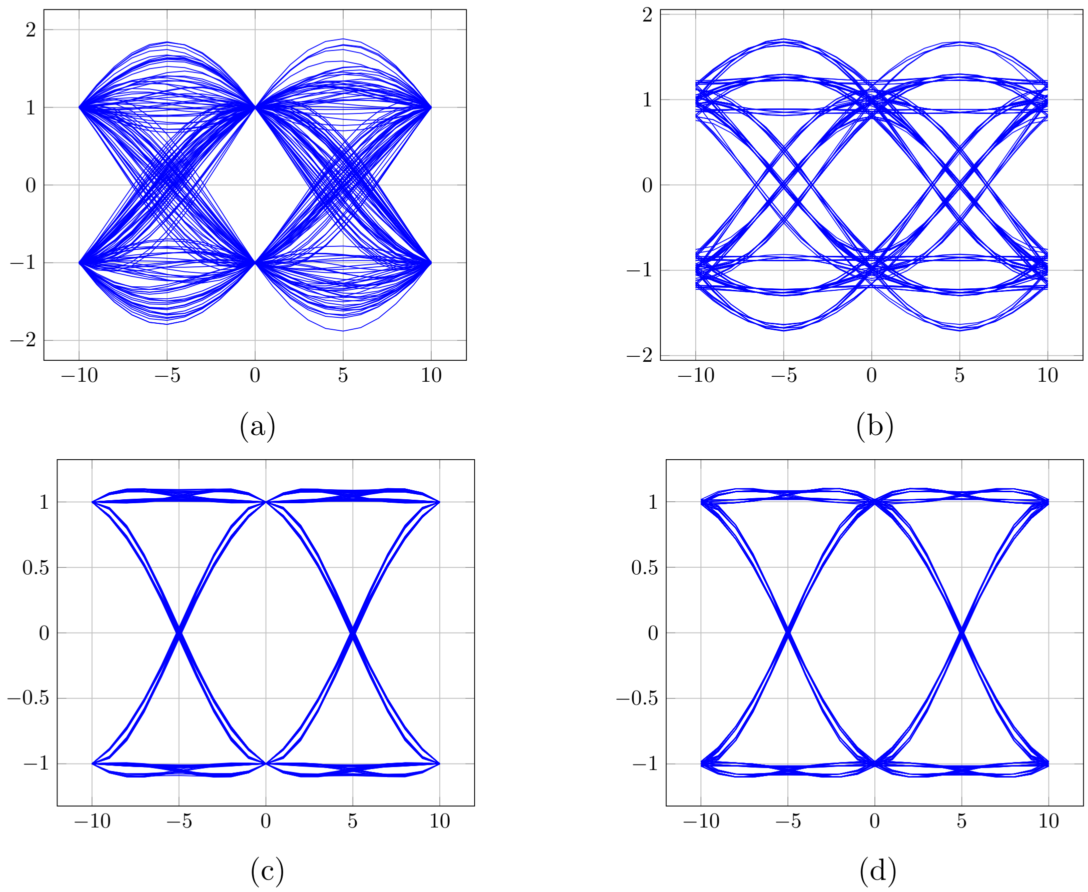
To construct the eye diagram:

- Generate a plot of $r(t)$ using a random selection of symbols $\{X_k\}$ from the constellation.
- Superimpose the plot from $[T, 3T]$, $[3T, 5T]$, ... on top of the plot from $[-T, T]$.
- Repeat many times by generating many different traces of $r(t)$ with random symbols $\{X_k\}$.

Essentially, the eye diagram is the superposition of traces of the form $r(t - kT)$ for $t \in [-T, T]$ and $k \in \mathbb{Z}$. (The interval $[-T, T]$ can be replaced by any interval that is an integer multiple of T).

Below we plot eye diagrams of $r(t) = X_k g(t - kT)$ for $X_k \in \{\pm 1\}$, with $T = 10$ and $g(t)$ from the raised cosine family.

- Roll-off factor is $\beta = 0.25$ for the top figures, and $\beta = 0.9$ for bottom ones.
- The pulse has been truncated to length $20T$ for figures on the left, and to length $4T$ for those on the right.



(Figure from *Principles of Digital Communication* by B. Rimoldi, CUP 2016.)

In the eye-diagrams on the previous page:

- The eye-diagram in (b) shows presence of ISI, as this has a small roll-off *and* a more truncated pulse.
- Also note that the eye in (c) is wider than in (a). A wider eye implies the system is more tolerant to small variations (jitter) in sampling time.
- The wider eye is characteristic of larger β: recall that as β increases, the pulse decays faster as a function of $|t|$.
- Hence a pulse with larger β can be truncated to shorter length, at the price of higher bandwidth.

The eye-diagram is popular because it can be obtained easily by looking at the matched filter output with an oscilloscope triggered by a clock that produces the sampling time.

The eye diagram is very informative even if the channel has attenuated the signal and/or has added noise. You will explore this in the 3F4 lab.

Summary

- A PAM signal $x(t) = \sum_k X_k p(t - kT)$ has rate $\frac{1}{T}$ symbols/sec. If symbols are drawn equally likely from an M-point constellation, tranmission rate $= \frac{\log_2 M}{T}$ bits/sec.
- Choosing the pulse $p(t)$ involves three considerations: should be bandlimited, decay quickly in time, and should facilitate efficient decoding
- Demodulator implemented as a low-pass filter + slicer.
- For no ISI at the output of demod, the overall filter $g(t) = p(t) \star q(t)$ has to satisfy Nyquist pulse criterion:

$$g(0) = 1, \quad \text{and } g(nT) = 0 \text{ for } n \neq 0, \quad \Leftrightarrow \quad \sum_n G\left(f - \frac{n}{T}\right) = T.$$

- A widely used practical choice is $p(t) =$ root raised-cosine
- If we choose $q(t) = p(-t)$, then:
  – Nyquist pulse criterion is equivalent to $p(t)$ being orthogonal to any shift of itself by integer multiple of T, i.e., $\{p(t - nT\}_{n \in \mathbb{Z}}$ forms an orthonormal basis.
  – The demod output at time nT is the projection of the received signal along the basis function $p(t - nT)$.
  (More on this in later lectures, and in Q.8 of Examples Paper 1.)