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4F7-STATISTICAL SIGNAL ANALYSIS

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EXAMPLES PAPER

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Question 1: Consider the following hidden Markov model,

$$(0.1) \quad \begin{aligned} X_{k+1} &= aX_k + bW_{k+1}, \\ Y_k &= cX_k + dV_k, \quad k = 0, 1, \dots \end{aligned}$$

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where $\{V_k\}$ and $\{W_k\}$ are independent and identically distributed

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$\mathcal{N}(0, 1)$ and X_0 is $\mathcal{N}(0, b^2)$. Give the expressions for the tran-

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sition probability density function $f(x_k, x_{k+1})$ and the observa-

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tion probability density function $g(x_k, y_k)$.

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Question 2: Henceforth, let $a = 0$ and $c = 1$. Find the expres-

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sion for $p(x_0, \dots, x_n \mid y_0, \dots, y_n)$. What happens as $d \rightarrow 0$?

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Question 3: Construct a self-normalizing importance sampling

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estimate of $p(x_0, \dots, x_n \mid y_0, \dots, y_n)$ and an importance sam-

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pling estimate of $p(y_0, \dots, y_n)$. Show that the estimate of $p(y_0, \dots, y_n)$

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is unbiased.

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Question 4: Find the variance σ^2/N of the self-normalizing im-

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portance sampling of $p(x_0, \dots, x_n \mid y_0, \dots, y_n)$. Find the vari-

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ance σ_0^2/N of the estimate that uses N independent samples

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from $p(x_0, \dots, x_n \mid y_0, \dots, y_n)$.

18 **Question 5:** Find the number of samples N_1 such that $\sigma^2/N_1 =$
19 σ_0^2/N . Discuss what happens to the ratio N_1/N_0 as $d \rightarrow 0$?

20 **Question 6:** Construct importance sampling estimates of $p(y_0), \dots, p(y_n)$
21 and calculate the variance of the estimate of $p(y_k)$.

22 **Question 7:** Show that the product of the importance sampling
23 estimates of $p(y_0), \dots, p(y_n)$ is also an unbiased estimate of
24 $p(y_0, \dots, y_n)$. Compare the variance of this new estimate with
25 that of the importance sampling estimate of $p(y_0, \dots, y_n)$ from
26 Question 3.

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