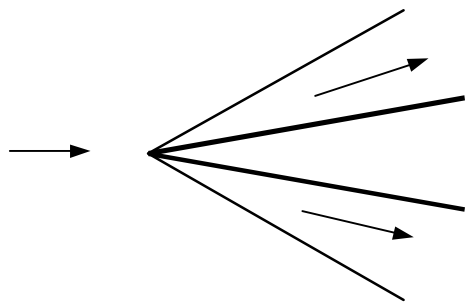
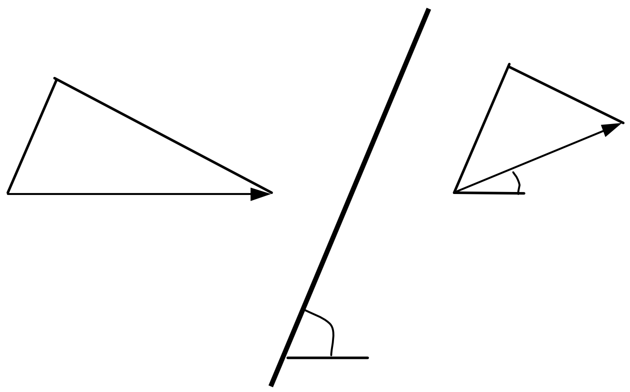
OBLIQUE SHOCK WAVES



In most practical situations, shocks are not in fact normal to the flow direction and usually involve some turning of the flow.

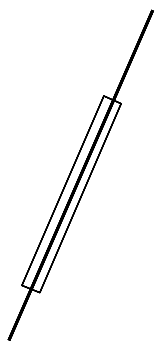


Consider a shock wave at angle β to the incoming flow, with velocity V decomposed into components (u,v) normal and parallel to the shock.

A control volume analysis applied to the flow through the oblique shock shown exactly parallels that for a normal shock.

The *differences* are that

(i) now have *tangential* (i.e. parallel to the shock) momentum equation



Since

$\Rightarrow$ (3.1)

(ii) the continuity and momentum equations only involve u_1 and u_2 (instead of V_1 and V_2)

$\Rightarrow$ (3.2)

$\Rightarrow$ (3.3)

(iii) the energy equation

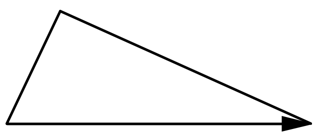
$$\Rightarrow \qquad (3.4)$$

Equations (3.1) – (3.4) are solved in the same manner as the corresponding ones for a normal shock, energy × continuity ÷ momentum²

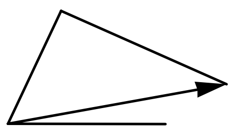
$$\frac{\left(1+\frac{\gamma-1}{2}M_1^2 \sin^2\beta\right)M_1^2 \sin^2\beta}{\left(1+\gamma M_1^2 \sin^2\beta\right)^2} = \frac{\left(1+\frac{\gamma-1}{2}M_2^2 \sin^2(\beta-\theta)\right)M_2^2 \sin^2(\beta-\theta)}{\left(1+\gamma M_2^2 \sin^2(\beta-\theta)\right)^2} \qquad (3.5)$$

leading to a quadratic equation for

Another View



Imagine an observer moving parallel to the shock at speed v (the component of *V* parallel to the shock). Since $v_1 = v_2$, the observer will see the flow as a <u>normal</u> shock with upstream Mach number $M_1 \sin\beta$.



The moving observer sees the same value for every **static** quantity as does a stationary one. Thus all relationships between **static** flow quantities (but *not* stagnation ones) are identical with those for a normal shock if we replace

M_1 by $M_{\perp 1} =$ and M_2 by $M_{\perp 2} =$

Thus, for example,

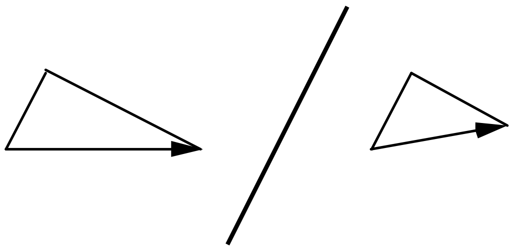
(3.6)

(3.7)

Remember that the moving observer will see *different* values of stagnation quantities to an observer at rest (if a fluid particle is brought to rest relative to the moving observer it will have speed v relative to an observer at rest). Care is thus necessary in framing relationships between upstream and downstream stagnation quantities.

Flow Turning

The final quantity of interest is the angle θ through which the flow has been turned.



Continuity gives

(3.8)

and the density ratio across a shock is given by equation the normal shock relationship with M_1 replaced by $M_1 \sin\beta$

$$\frac{\rho_2}{\rho_1} = \qquad (3.9)$$

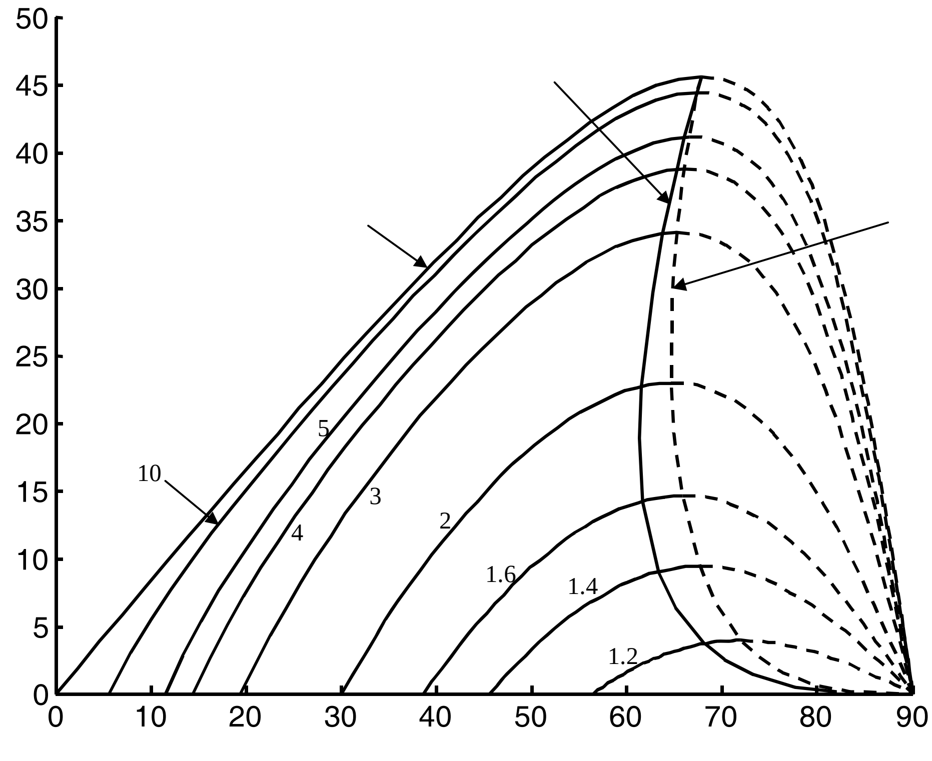
Equation (3.8) can be solved for $\tan(\beta-\theta)$ which is then substituted into the identity

$$\tan\theta =$$

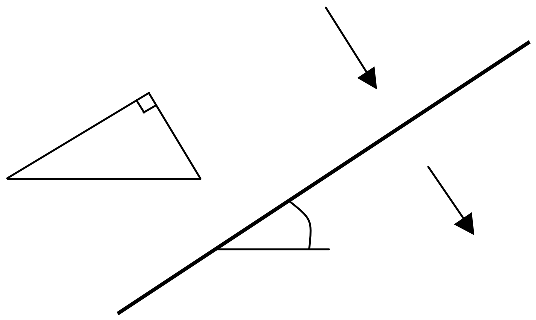
to give

(3.10)

The figure shows a plot of the deflection against the shock wave angle for various values of M_1.



Shock Angle β



Consequences

1) For each M_1, β must lie between $90°$ (normal shock) and $\beta = \sin^{-1}\dfrac{1}{M_1}$, corresponding to $M_{\perp 1} = M_1 \sin\beta = 1$. (i.e. is the Mach Angle μ).

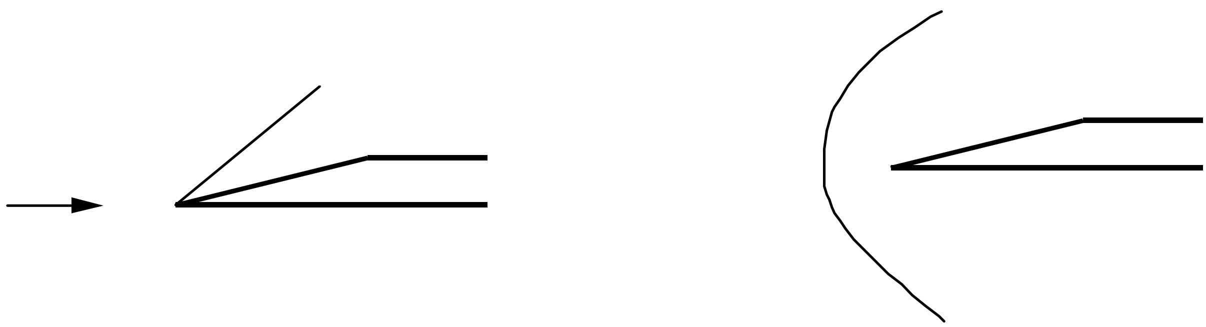
2) For each M_1 there is a *maximum possible deflection* $\theta = \theta_{\max}$, (which increases as M_1 increases.)

3) For each value of $\theta < \theta_{\max}$ there are *two* possible values of the shock angle β. In practice, it is very rare to find the stronger branch (higher value of $M_\perp = M_1 \sin\beta$). Nearly always find the part of the curve shown solid (other than, of course, normal shocks).

4) The Mach number for the perpendicular velocity downstream of the shock is *always subsonic* i.e. $M_2 \sin(\beta - \theta) < 1$. But M_2 is nearly always greater than unity (apart from a small region near $\theta = \theta_{\max}$)

5) Shocks which correspond to the part of the curve to the left of $\theta = \theta_{\max}$ are referred to as 'weak' shocks and those corresponding to the right as 'strong' shocks.

6) The wedge angle determines the flow turning, θ, which then determines β.
If $\theta > \theta_{\max}$ for this value of M_1, then the shock becomes normal and 'stands off'



7) It can be shown that the entropy rise across a normal shock is given by

$$\frac{\Delta s}{R} = \ln\frac{p_{01}}{p_{02}} \text{ and } \quad \frac{p_{01}}{p_{02}} = 1 + O(M_1 - 1)^3 \qquad \text{as } M_1 \to 1.$$

For oblique shocks, we have

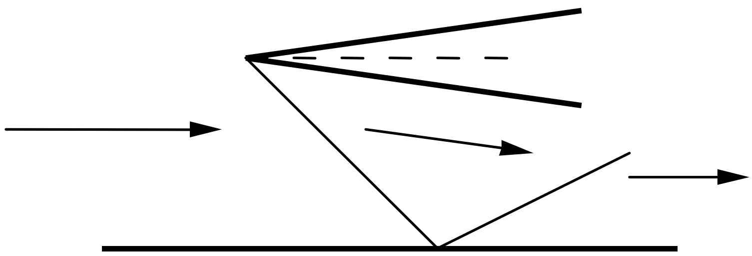
$$\frac{\Delta s}{R} = O(M_1 \sin\beta - 1)^3 \quad \text{as } M_1 \sin\beta \to 1.$$

E.g. with

Thus oblique shocks are often nearly isentropic even for relatively high Mach numbers.

SHOCK WAVE REFLECTION

Regular Reflection



When an oblique shock meets a solid boundary a reflected wave forms, exactly as for characteristics, since downstream of the shock the flow is aligned with the wedge direction, and it must be turned back to the original direction at the wall.

The turning of the flow back to the original direction requires another shock.

E.g. M_1 = and θ =

From the CUED gas flow tables:

Oblique Shock Tables ($\gamma = 1.4$)

M_1	θ	β	$\frac{p_2}{p_1}$	$\frac{\rho_2}{\rho_1}$	$\frac{T_2}{T_1}$	M_2	$\frac{p_{02}}{p_{01}}$
2.50	2.000	25.050	1.1405	1.0984	1.0384	2.4155	0.99977
	4.000	26.609	1.2961	1.2029	1.0775	2.3326	0.99822
	6.000	28.259	1.4679	1.3133	1.1177	2.2505	0.99427
	8.000	30.005	1.6568	1.4289	1.1595	2.1685	0.98703
	10.000	31.851	1.8639	1.5493	1.2031	2.0859	0.97589
	12.000	33.802	2.0900	1.6737	1.2488	2.0022	0.96046
	14.000	35.866	2.3364	1.8015	1.2969	1.9169	0.94057
	16.000	38.057	2.6042	1.9322	1.3478	1.8295	0.91625
	18.000	40.389	2.8949	2.0652	1.4018	1.7394	0.88767

$\Rightarrow$ M_2 = (and β =)

Thus for second shock M = and θ =

2.15	2.000	29.293	1.1243	1.0872	1.0341	2.0749	0.99984
	4.000	30.960	1.2606	1.1794	1.0688	2.0008	0.99874
	6.000	32.725	1.4094	1.2763	1.1043	1.9271	0.99590
	8.000	34.596	1.5719	1.3777	1.1410	1.8529	0.99065
2.20	2.000	28.592	1.1266	1.0888	1.0347	2.1237	0.99983
	4.000	30.238	1.2654	1.1826	1.0700	2.0485	0.99867
	6.000	31.981	1.4173	1.2813	1.1061	1.9738	0.99569
	8.000	33.827	1.5832	1.3845	1.1435	1.8987	0.99020

$\Rightarrow$ 2nd shock angle *relative to the flow direction* , β_2 = and M_3 =

i.e. M_3 = and the angle of the shock *relative to the wall* =