1 Week 1

1.1 Excersise 1: Point Vortex Stream Function

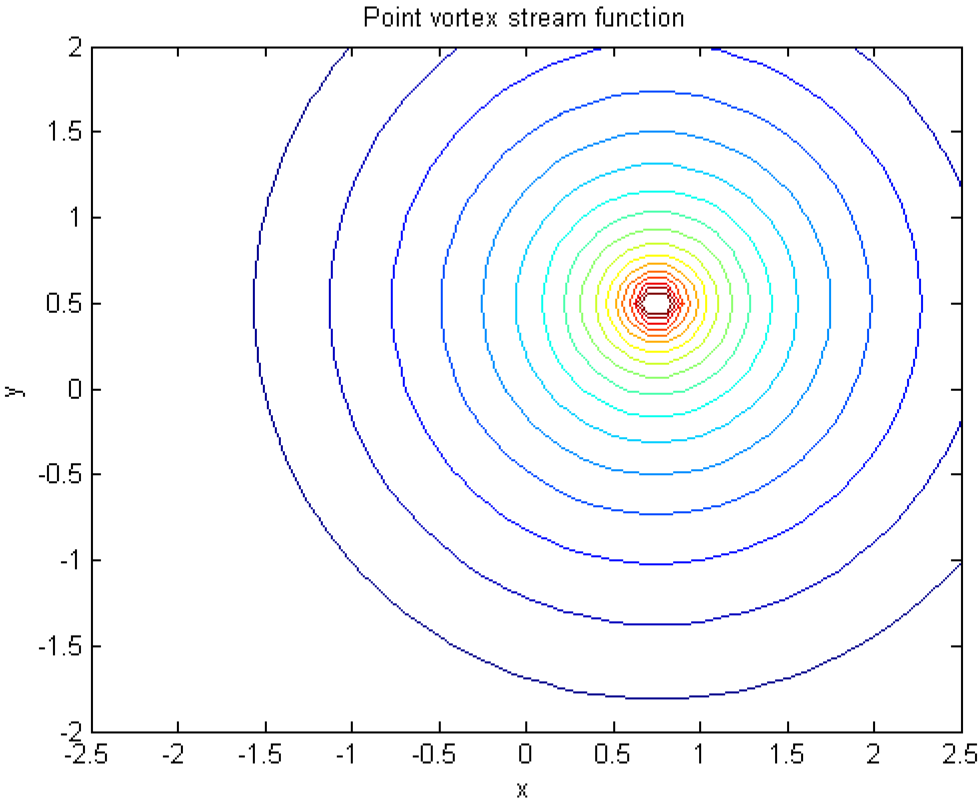
1.1.1 Listing of psipv.m

```
function psixy = psipv1(xc,yc,Gamma,x,y)
r2 = (x-xc).^2 + (y-yc).^2;
psixy = -(Gamma/(4*pi)).*log(r2);
```

1.1.2 Listing of script

```
clear
close all
xmin=-2.5; xmax=2.5; ymin=-2;ymax=2; %domain
xc=0.75; yc=0.50; %vortex location
Gamma=3.0; nx=51; ny=41;

%matrices whose (i,j)th entries give values of x, y and psi at the (i,j)th
%grid point
for i=1:nx
    for j=1:ny
        xm(i,j)=xmin+(i-1)*(xmax-xmin)/(nx-1);
        ym(i,j)=ymin+(j-1)*(ymax-ymin)/(ny-1);
        psi(i,j)=psipv1(xc,yc,Gamma,xm(i,j),ym(i,j));
    end
end

%plotting contours
c=-0.4:0.1:1.2;
axis equal
contour(xm,ym,psi,c)
xlabel('x')
ylabel('y')
title('Point vortex stream function')
```

1.1.3 Contour Plot



1.2 Exercise 2: Reference vortex-sheet-panel stream function

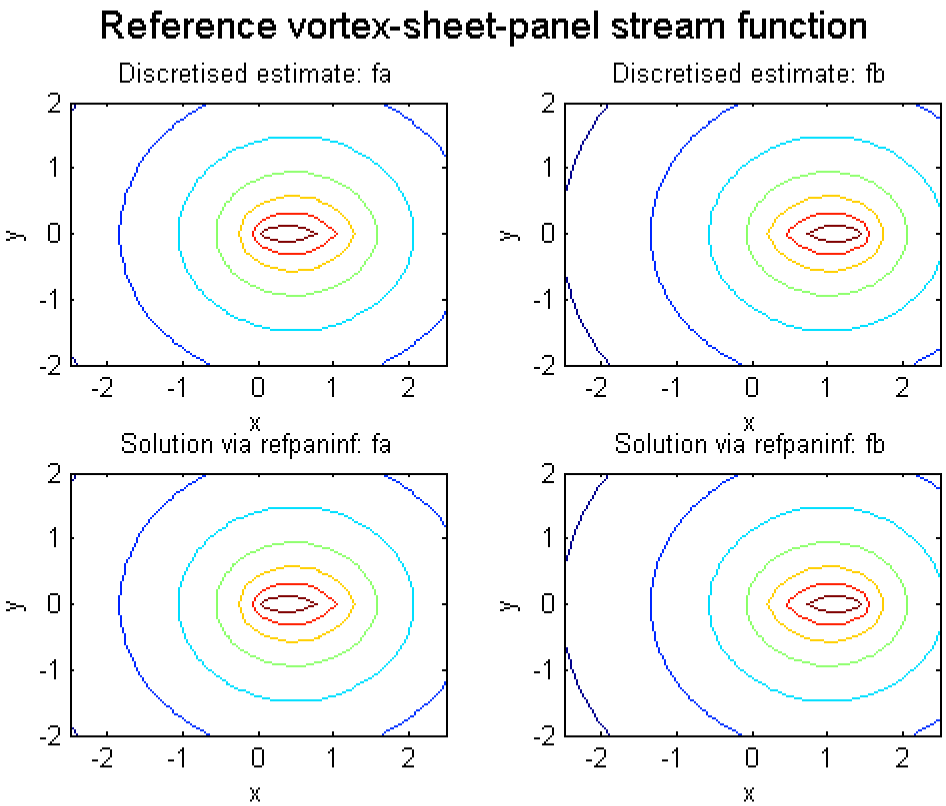
1.2.1 Lisitng of refpaninf.m

```
function [infa, infb] = refpaninf1(del,X,Yin)
for i = 1:size(Yin,1)
    for j = 1:size(Yin,2)
        if abs(Yin(i,j)) < 1e-6
            Y(i,j)=1e-6;
        else
            Y(i,j)=Yin(i,j);
        end
    end
end

I0 = (-1/(4*pi))*(X.*log(X.^2+Y.^2)-((X-del).*log((X-del).^2+Y.^2))-
2*del+2.*Y.*(atan(X./Y)-atan((X-del)./Y)));

I1 = (1/(8*pi)).*((X.^2+Y.^2).*log(X.^2+Y.^2)-((X-del).^2+Y.^2).*log((X-
del).^2+Y.^2)-2.*X.*del+del.^2);

infa= (1-X./del).*I0-I1./del;
infb= (X./del).*I0+I1./del;
```

1.2.2 Listing of script

```
clear
close all
xmin=-2.5; xmax=2.5; ymin=-2;ymax=2; xc=0.75; yc=0.50;
Gamma=3.0; nx=51; ny=41; del=1.5; nv=100; w=del/nv;
%w is the vortex spacing, nv is number of vortices

for i=1:nx %setting (x,y) domain
    for j=1:ny
        xm(i,j)=xmin+(i-1)*(xmax-xmin)/(nx-1);
        ym(i,j)=ymin+(j-1)*(ymax-ymin)/(ny-1);
    end
end

[infa, infb] = refpaninf1(del,xm,ym);
for a=1:nv
    GamA(a) = (1 - (2*a-1).*(w/2)./del).*w; %Capital Gammas and B,
circulations
    GamB(a) = ((2*a-1).*(w/2)./del).*w;      %of each vortex
end

%summing contributions of all vortices
psiA = psipv1((w/2),0,GamA(1),xm,ym);
psiB = psipv1((w/2),0,GamB(1),xm,ym);
for b = 2:nv
     psiA=psiA+psipv1((2*b-1).*(w/2),0,GamA(b),xm,ym);
     psiB=psiB+psipv1((2*b-1).*(w/2),0,GamB(b),xm,ym);
end

c=-0.15:0.05:0.15;
figure
subplot(2,2,1);
contour(xm,ym,psiA,c);
title('Discretised estimate: fa')
[xlabel('x'), ylabel('y')];

subplot(2,2,2);
contour(xm,ym,psiB,c);
title('Discretised estimate: fb')
[xlabel('x'), ylabel('y')];

subplot(2,2,3);
contour(xm,ym,infa,c);
title('Solution via refpaninf: fa')
[xlabel('x'), ylabel('y')];

subplot(2,2,4);
contour(xm,ym,infb,c);
title('Solution via refpaninf: fb')
[xlabel('x'), ylabel('y')];
```

1.2.3 Contour plots showing exact and approximate values of f_a, f_b



1.3 Exercise 3: General vortex-sheet-panel stream function

1.3.1 Listing of panelinf.m

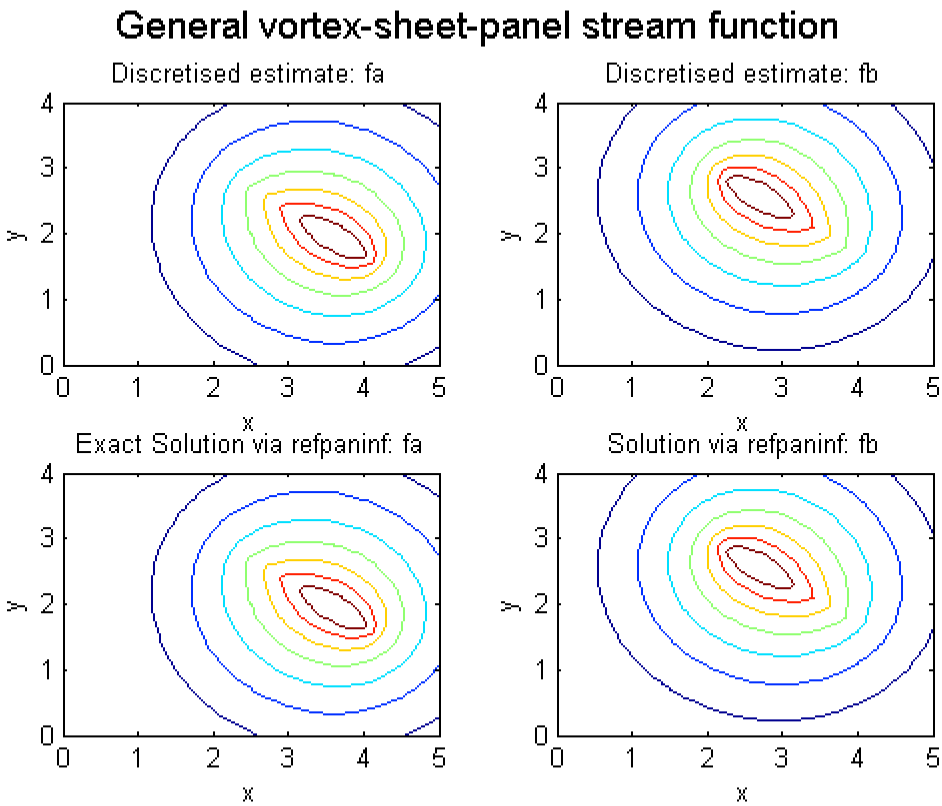
```
function [infa,infb] = panelinf1(xa,ya,xb,yb,x,y)
t0=[xb-xa, yb-ya]; n0=[ya-yb, xb-xa]; %tangential and normal vectors
t=t0./norm(t0); %normalised vectors
n=n0./norm(n0);
qx=x-xa; qy=y-ya; %coordinates referred to xa and ya

X=qx.*t(1,1) + qy.*t(1,2); %r.t
Y=qx.*n(1,1) + qy.*n(1,2); %r.n
[infa, infb] = refpaninf1(norm(t0),X,Y);
```

1.3.2 Listing of script

```
clear
close all
xmin=0; xmax=5; ymin=0;ymax=4; xc=0.75; yc=0.50;
Gamma=3.0; nx=51; ny=41; nv=100; %100 vortices
xa=4.1;ya=1.6; xb=2.2;yb=2.9;

t0=[xb-xa, yb-ya];n0=[ya-yb, xb-xa]; %vectors of panel, tangential and
normal
t=t0./norm(t0); %normalised vectors
n=n0./norm(n0);
del=norm(t0);  w=del/nv; %vortex spacing
```

```
%x,y grid
for i=1:nx
    for j=1:ny
        xm(i,j)=xmin+(i-1)*(xmax-xmin)/(nx-1);
        ym(i,j)=ymin+(j-1)*(ymax-ymin)/(ny-1);
    end
end

[infa, infb] = panelinf1(xa,ya,xb,yb,xm,ym);
for a=1:nv
    GamA(a) = (1 - (2*a-1).*(w/2)./del).*w; %Capital Gammas A and B,
    GamB(a) = ((2*a-1).*(w/2)./del).*w;      %circulations of each vortex
end

%summing contributions of each vortex
psiA = psipv1(xa+(w/2)*t(1,1),ya+(w/2)*t(1,2),GamA(1),xm,ym);
psiB = psipv1(xa+(w/2)*t(1,1),ya+(w/2)*t(1,2),GamB(1),xm,ym);
for b = 2:nv
     psiA=psiA+psipv1(xa+(2*b-1)*(w/2)*t(1,1),ya+(2*b-
1)*(w/2)*t(1,2),GamA(b),xm,ym);
     psiB=psiB+psipv1(xa+(2*b-1)*(w/2)*t(1,1),ya+(2*b-
1)*(w/2)*t(1,2),GamB(b),xm,ym);
end

%plotting contours
c=-0.15:0.05:0.15;
figure

subplot(2,2,1);
contour(xm,ym,psiA,c);
title('Discretised estimate: fa')
[xlabel('x'), ylabel('y')];

subplot(2,2,2);
contour(xm,ym,psiB,c);
title('Discretised estimate: fb')
[xlabel('x'), ylabel('y')];

subplot(2,2,3);
contour(xm,ym,infa,c);
title('Exact Solution via refpaninf: fa')
[xlabel('x'), ylabel('y')];

subplot(2,2,4);
contour(xm,ym,infb,c);
title('Solution via refpaninf: fb')
[xlabel('x'), ylabel('y')];

suptitle('General vortex-sheet-panel stream function');
```

1.3.3 Contour plots showing exact and approximate values of f_a, f_b



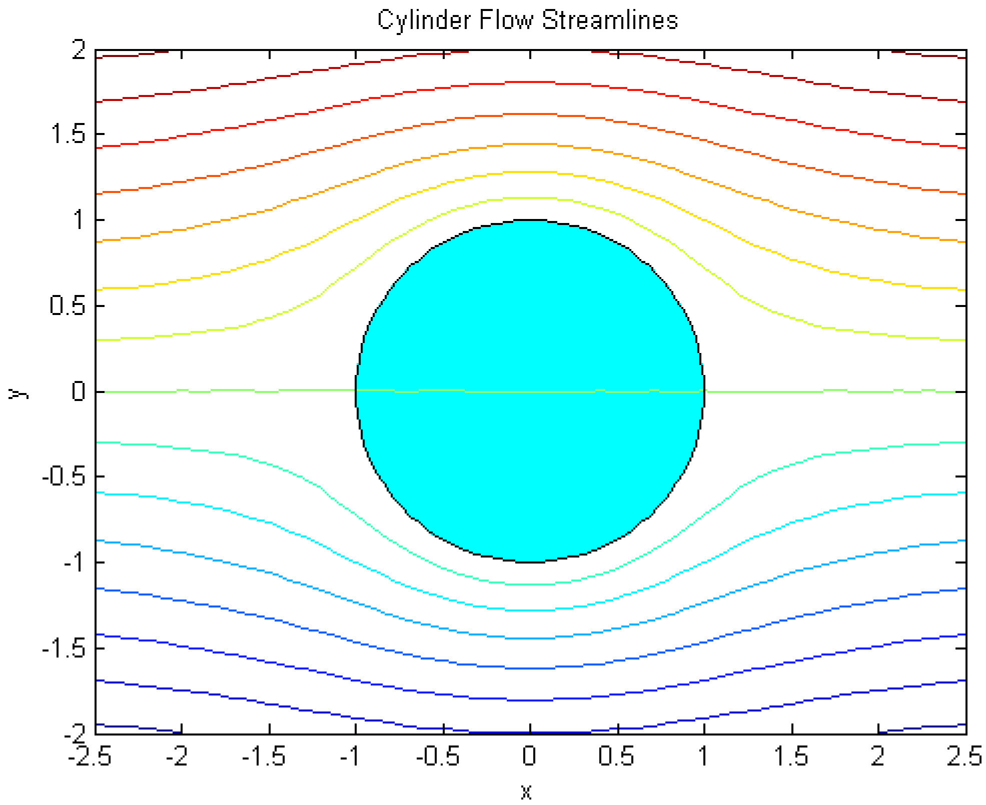
1.4 Exercise 4: Cylinder flow streamlines

1.4.1 Listing of script

```
clear
close all
np=100; xmin=-2.5; xmax=2.5; ymin=-2;ymax=2;nx=51; ny=41;
theta = (0:np)*2*pi/np; %array of panel angles

for i = 1:np %panel edges definition
    xs(i) = cos(theta(i));
    ys(i) = sin(theta(i));
    gamma(i) = -2*sin(theta(i));
end
for i=1:nx %x,y grid
    for j=1:ny
        xm(i,j)=xmin+(i-1)*(xmax-xmin)/(nx-1);
        ym(i,j)=ymin+(j-1)*(ymax-ymin)/(ny-1);
        psi(i,j) = ym(i,j);
    end
end

%streamfunction contributions
psitotal=psi;
for i=1:np-1
    [infa,infb] = panelinf1(xs(i),ys(i),xs(i+1),ys(i+1),xm,ym);
    psitotal=psitotal+ gamma(i)*infa + gamma(i+1)*infb;
```

```
end
[infa,infb] = panelinf1(xs(np),ys(np),xs(1),ys(1),xm,ym);
psitotal=psitotal+ gamma(np)*infa + gamma(1)*infb;

c = -1.75:0.25:1.75;
contour(xm,ym,psitotal,c)
hold on
fill(xs,ys,'c')
hold off
[xlabel('x'), ylabel('y')];
title('Cylinder Flow Streamlines')
```

1.4.2 Streamline plot



1.5 Exercise 5: Panel method solution for the cylinder flow

1.5.1 Listing of build_lhs.m

```
function lhsmat = build_lhs1(xs,ys)
np = length(xs)-1;
psip=zeros(np,np+1);
for i=1:np
    [infa,infb] = panelinf1(xs(1),ys(1),xs(2),ys(2),xs(i),ys(i));
    psip(i,1)=infa;
    [infa,infb] = panelinf1(xs(np),ys(np),xs(1),ys(1),xs(i),ys(i));
    psip(i,np+1)=infb;
```

```
    for j=2:np
        [infa,infb] = panelinf1(xs(j),ys(j),xs(j+1),ys(j+1),xs(i),ys(i));
        [infa0,infb0] = panelinf1(xs(j-1),ys(j-1),xs(j),ys(j),xs(i),ys(i));
        psip(i,j)=infa+infb0;
    end
end
A = zeros(np+1,np+1);
for i=1:np-1
    for j=1:np+1
        A(i,j)=psip(i+1,j)-psip(i,j);
    end
end
A(np,1)=1;
A(np+1,np+1)=1;
lhsmat = A;
size(A);
end
```

1.5.2 Listing of build_rhs.m

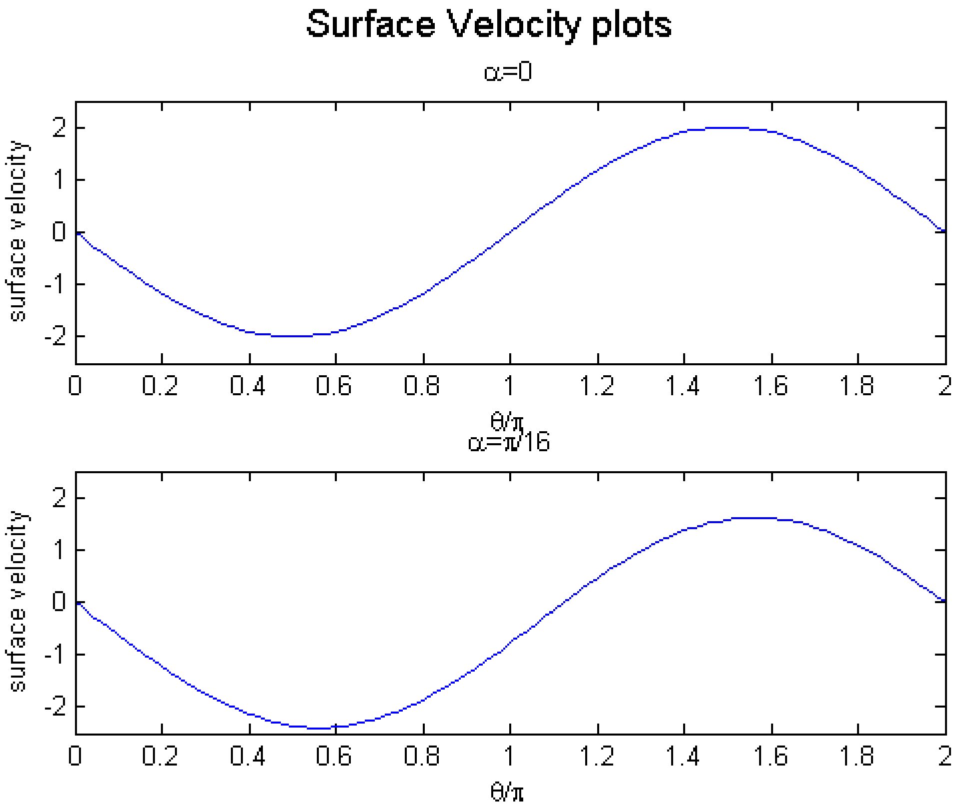
```
function rhsvec = build_rhs1(xs,ys,alpha)
np = length(xs)-1;
b = zeros(np+1,1);
for i = 1:np-1
    b(i) = ys(i)*cos(alpha) - xs(i)*sin(alpha) - ys(i+1)*cos(alpha) +
xs(i+1)*sin(alpha);
end
b(np) = 0;
b(np+1) = 0;
rhsvec = b;
end
```

1.5.3 Listing of script

```
clear
close all
np=100; xmin=-2.5; xmax=2.5; ymin=-2;ymax=2;nx=51; ny=41;
theta = (0:np)*2*pi/np; %array of panel angles
for i = 1:np+1 %panel edges definition
    xs(i) = cos(theta(i));
    ys(i) = sin(theta(i));

end
for i=1:nx %x,y grid
    for j=1:ny
        xm(i,j)=xmin+(i-1)*(xmax-xmin)/(nx-1);
        ym(i,j)=ymin+(j-1)*(ymax-ymin)/(ny-1);
        psitotal0(i,j) = ym(i,j);
        psitotal1(i,j) = ym(i,j)*cos(pi/16) - xm(i,j)*sin(pi/16);
    end
end

A = build_lhs1(xs,ys);
b0 = build_rhs1(xs,ys,0);
bpi_16 = build_rhs1(xs,ys,pi/16);
gam0 = A\b0; %gamma vector for alpha=0
gampi_16 = A\bpi_16; %gamma vector for alpha=pi/16
```

```
theta0 = theta/pi;
circ=0; circ1=0; %total circulation
for i = 1:np
   circ = circ + gam0(i)*((2*pi)/np);
   circ1 = circ1 + gampi_16(i)*((2*pi)/np);
end

for i=1:np-1 %streamfunction contributions for streamlines of incidence
pi/16
     [infa,infb] = panelinf1(xs(i),ys(i),xs(i+1),ys(i+1),xm,ym);
    psitotal0=psitotal0+ gam0(i)*infa + gam0(i+1)*infb;
    psitotal1=psitotal1+ gampi_16(i)*infa + gampi_16(i+1)*infb;
end
[infa,infb] = panelinf1(xs(np),ys(np),xs(1),ys(1),xm,ym);
psitotal0=psitotal0+ gam0(np)*infa + gam0(1)*infb;
psitotal1=psitotal1+ gampi_16(np)*infa + gampi_16(1)*infb;

disp('circulation for alpha=0')
disp(circ)
disp('circulation for alpha=pi/16')
disp(circ1)

subplot(2,1,1)
plot(theta0,gam0')
title('\alpha=0')
axis([0 2 -2.5 2.5]);
xlabel('\theta/\pi')
ylabel('surface velocity')

subplot(2,1,2)
plot(theta0,gampi_16')
title('\alpha=\pi/16')
axis([0 2 -2.5 2.5]);
xlabel('\theta/\pi')
ylabel('surface velocity')
suptitle('Surface Velocity plots');
figure

c = -1.75:0.15:1.75;
contour(xm,ym,psitotal1,c)
axis equal;
hold on
fill(xs,ys,'c')
hold off
[xlabel('x'), ylabel('y')];
title('Streamlines at incidence of \pi/16')
```

1.5.4 Surface velocity plots for two values of α



1.5.5 Values of circulations

```
circulation for alpha=0

   0

circulation for alpha=pi/16

   -2.4528
```

1.5.6 Streamline plot at incidence of $\alpha = \pi/16$