3F4: Data Transmission

Handout 6: Coded modulation

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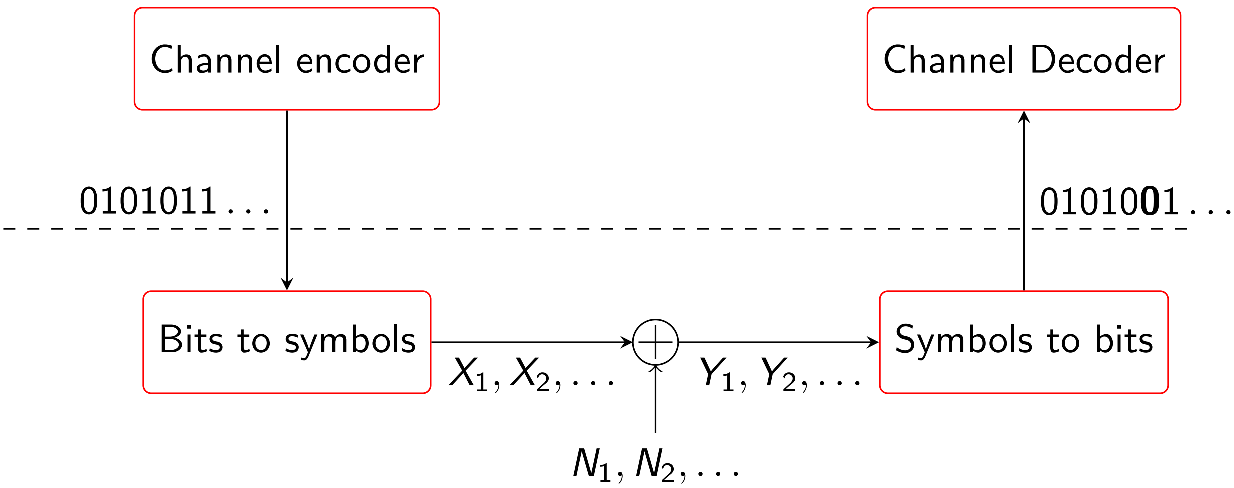
The material in this handout will not be examined, but provides useful context about how the coding and modulation operations are combined in practical communication systems.

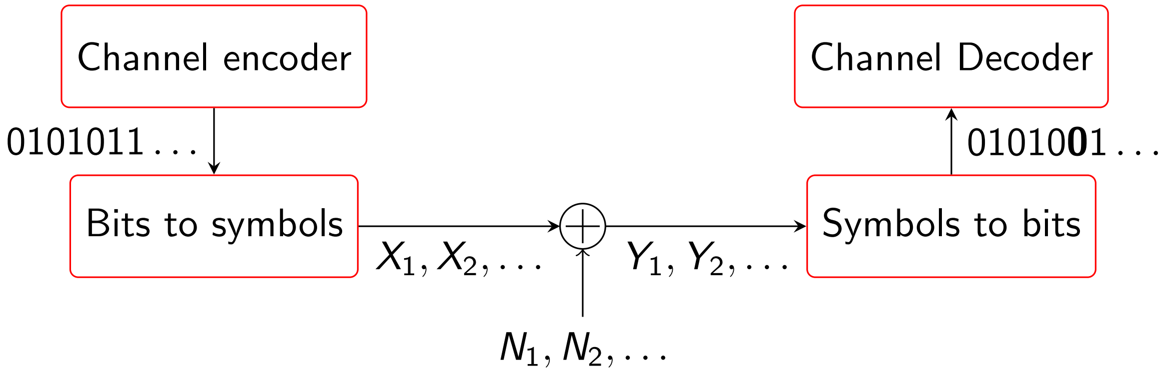


The bottom of the figure shows the *effective* discrete-time channel $Y_j = X_j + N_j$, where:

- X_j are symbols from a PAM/QAM constellation
- $N_1, N_2, \ldots$, are i.i.d Gaussian $\mathcal{N}(0, N_0/2)$

This discrete-time channel models what is obtained at the output of the matched-filter (or signal space) demodulator.

In the set-up above, the 'symbols-to-bits' block first detects the symbols $\hat{X}_1, \hat{X}_2, \ldots$ from $Y_1, Y_2, \ldots$. It then maps $\hat{X}_1, \hat{X}_2, \ldots$ to bits $0101001\ldots$ which go into the channel decoder.



For example, with BPSK, let the bit-to-symbol mapping be $0 \to A$, $1 \to -A$. Then the detector decodes $\{Y_1, Y_2, \ldots\}$ to

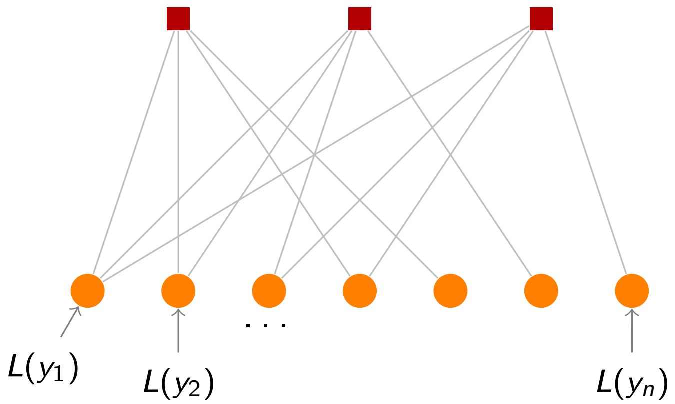
$$\{\hat{X}_1, \hat{X}_2, \ldots\} = A, -A, A, -A, A, A, -A \ldots \longrightarrow 0101001 \ldots$$

- In the above figure, the detector makes **hard decisions** on the symbols, and the corresponding bits are fed into the channel decoder
- For state of the art codes, like LDPC and turbo codes, error performance can often be significantly improved by using *soft inputs* to the channel decoder
- *Intuition*: A highly positive value of Y_j indicates a strong belief (posterior probability) that $X_j = A$, while a small positive value of Y_k indicates a weaker belief that $X_j = A$
- This 'soft information' provided by Y_k can be valuable for the channel decoder, and is lost when detector makes hard decisions

Consider BPSK with $X_j \in \{\pm A\}$ over an AWGN channel:

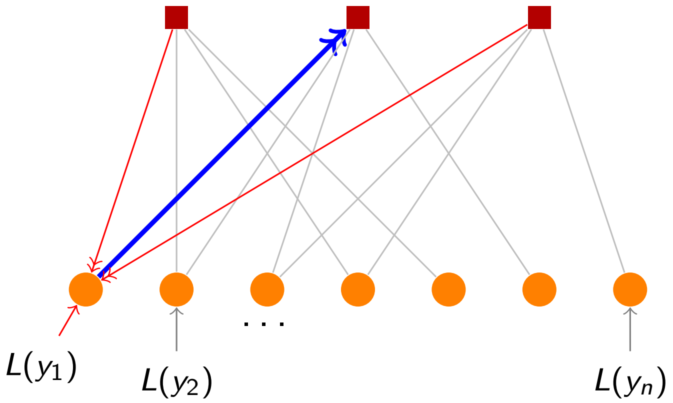
$$Y_j = X_j + N_j, \qquad N_j \text{ i.i.d. } \sim \mathcal{N}(0, N_0/2)$$

An example of using soft information in LDPC codes:



- An LDPC code represents k message bits with n coded bits ($k < n$)
- The code can be represented by a factor graph:

  Circles represent code bits, and squares represent constraints that the code bits connected to them have to satisfy
- Each code bit is mapped to a BPSK symbol: $0 \to A$, $1 \to -A$.
- The BPSK symbols $X_1, \ldots, X_n$ are transmitted, and the receiver needs to recover them from $Y_1, \ldots, Y_n$

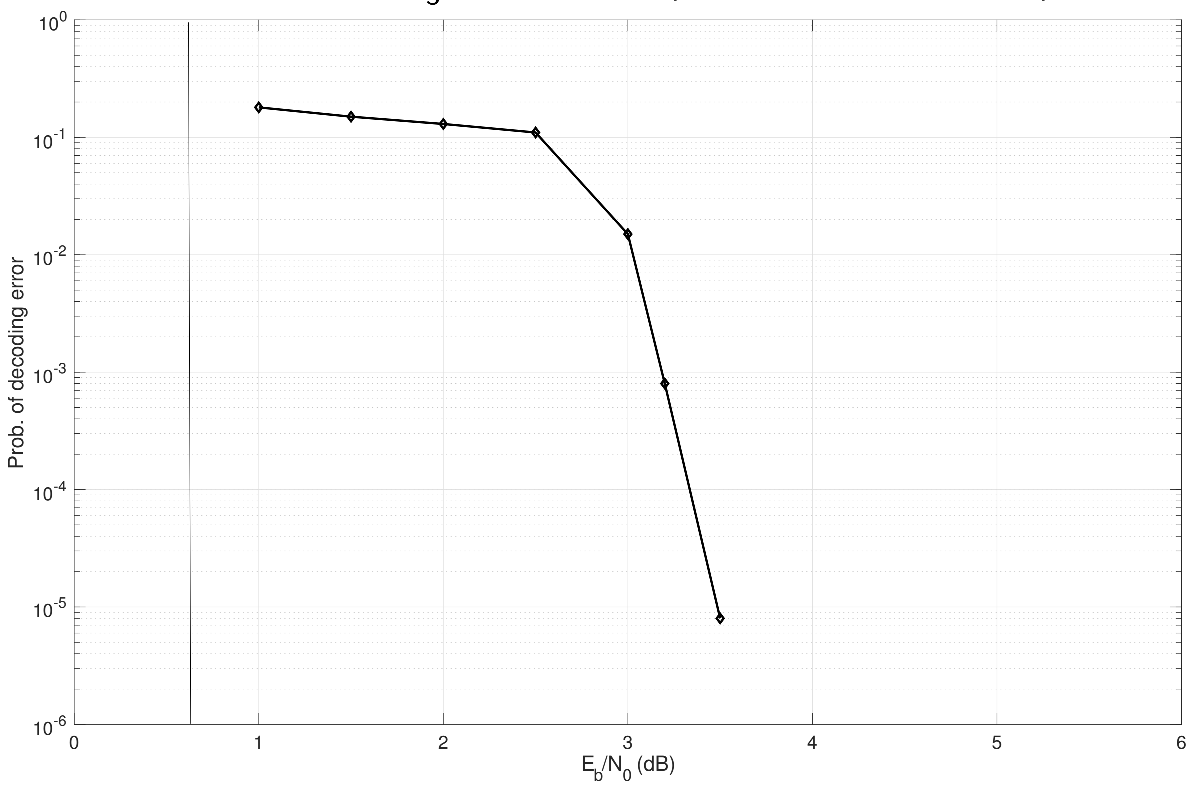


- The belief propagation decoder passes messages back and forth along the edges of the graph to refine *beliefs* about the code bits (equivalently, the BPSK symbols).
- The decoder is initialised with log-likelihood ratios (LLRs) computed using the observed channel outputs $y_1, \ldots, y_n$.
- For $j = 1, \ldots, n$, the initial LLRs are

$$L(y_j) = \ln \frac{f(y_j | X_j = A)}{f(y_j | X_j = -A)} = \ln \frac{e^{-(y_j - A)^2/N_0}}{e^{-(y_j + A)^2/N_0}} = \frac{4A}{N_0} y_j$$

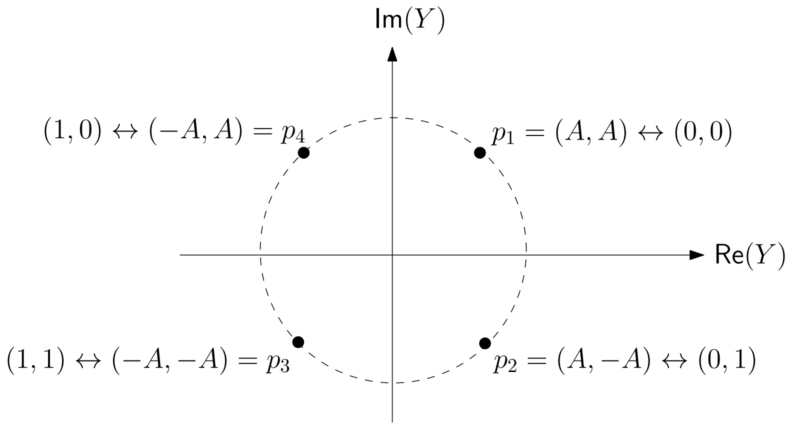
- Notice that the LLRs used to initialise the decoder are just scaled channel output symbols $y_1, \ldots, y_n$ (*soft information*)

BPSK with rate $\frac{4}{5}$ LDPC code (code length $n = 5120$)



- Note that with LDPC code, the user rate is $\frac{4}{5}$ bits/transmission. We get $P_e < 10^{-5}$ at $\frac{E_b}{N_0} \sim 3$ dB higher the Shannon limit for $R = 4/5$.
- In comparison, to get $P_e = 10^{-5}$ with uncoded BPSK, we need $\frac{E_b}{N_0}$ around 7.5 dB higher than the Shannon limit (for $R = 1$).

Coded modulation with QPSK:

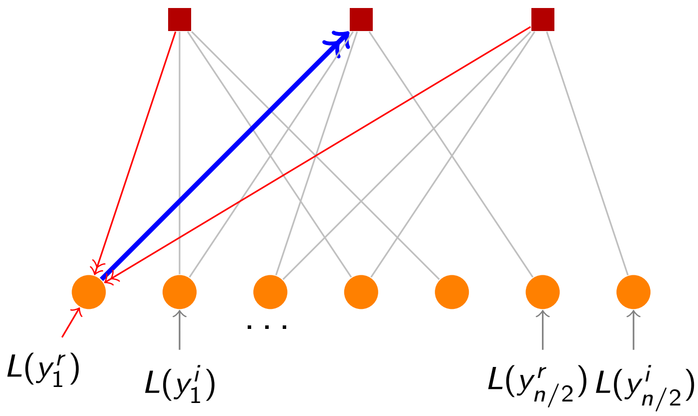


Now we can consider a block of n LDPC coded bits mapped to $n/2$ QPSK symbols.

- The jth QPSK symbol $X_j = (X_j^r, X_j^i)$ for $j = 1, \ldots, n/2$
- We can label n code bits by $c_1^r, c_1^i, \ldots, c_{n/2}^r, c_{n/2}^i$ with:

$$c_1^r \to X_1^r, \quad c_1^i \to X_1^i, \quad \ldots, \quad c_{n/2}^r \to X_{n/2}^r, \; c_{n/2}^i \to X_{n/2}^i$$

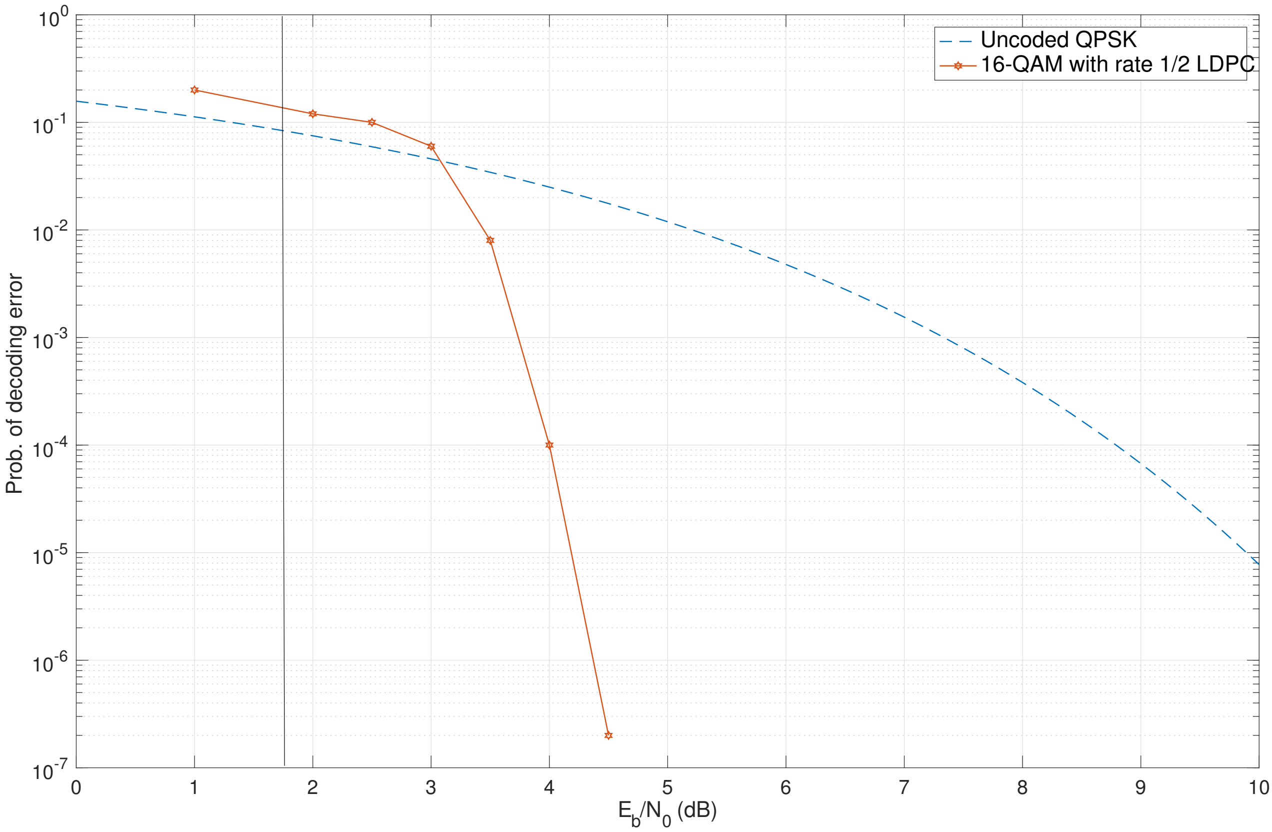
- For each of these, we again use the mapping $0 \to A$, $1 \to -A$



- The decoder is initialised with LLRs computed using the observed channel outputs $y_1^r, y_1^i, \ldots, y_{n/2}^r, y_{n/2}^i$.
- Using an argument similar to that for BPSK, the initial LLRs are

$$L(y_j^r) = \frac{4A}{N_0} y_j^r, \quad L(y_j^i) = \frac{4A}{N_0} y_j^i, \qquad j = 1, \ldots, \frac{n}{2}$$

- The beliefs on $\{X_1^r, X_1^i, \ldots, X_{n/2}^r, X_{n/2}^i\}$ are iteratively refined by the belief propagation decoder



- Uncoded QPSK and 16-QAM with rate $\frac{1}{2}$ LDPC both have user rate of 2 user bits/transmission
- With 16-QAM + rate $\frac{1}{2}$ LDPC (with block length $n = 2304$) we get very low error rates at ~ 2.7 dB from Shannon limit

Summary

- A good outer binary code can give significant improvement in the error performance of PAM/QAM
- For a given P_e (say 10^{-5}), the difference in the required $\frac{E_b}{N_0}$ between coded and uncoded transmission is called the *coding gain*
- A well-designed code can provide a coding gain of several dB
- State of the art decoders typically use soft information directly from the demodulator output
- LDPC codes, convolutional codes etc. can all be used with either soft or hard information at the channel decoder
- Hard decision detection before channel decoding typically reduces the coding gain

- The rate 4/5 LDPC code used in slide 6 is from CCSDS: `https://public.ccsds.org/Pubs/131x0b3e1.pdf`
- The rate 1/2 LDPC code used in slide 9 is from the WiMax standard IEEE 802.16e. Coded modulation implementation using the CML toolkit: `http://www.iterativesolutions.com/Matlab.htm`