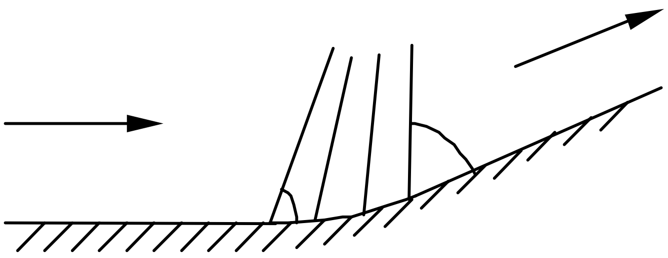
**Corner Compression**



The first characteristic from A, at the start of the curve is at the Mach angle of the $M_1$ flow, while the last, emanating from B, is at the Mach angle to the $M_2$ flow.

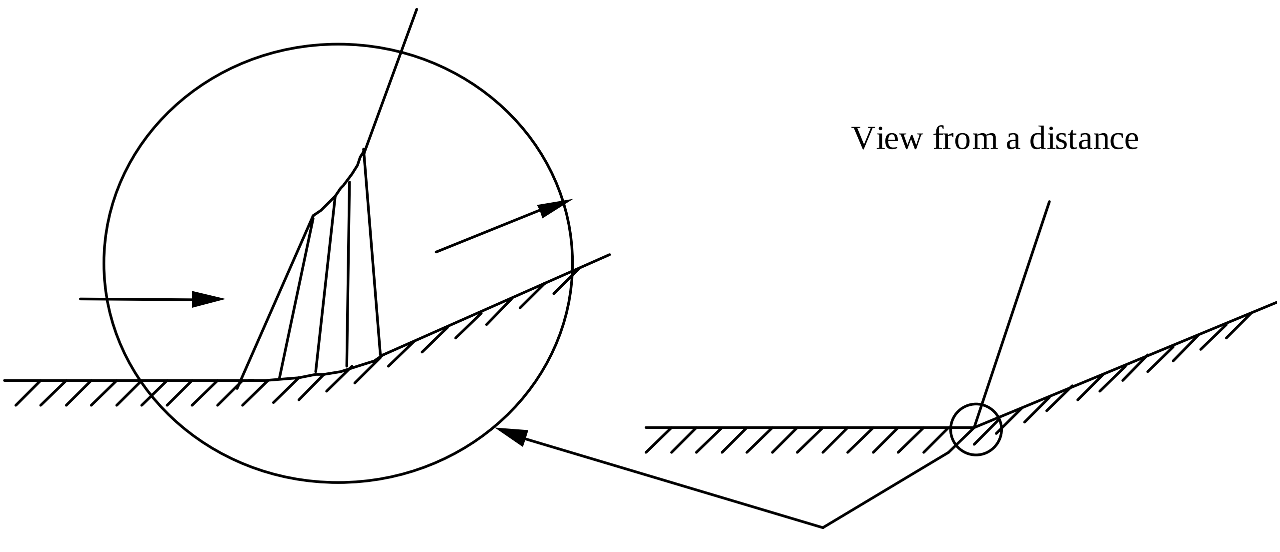
In this case $\quad \nu_1 = \nu_1(M_1)$ , $\theta_1 = 0$, $\quad \theta_2 > \theta_1 \quad \Rightarrow \; \nu_2 < \nu_1$ i.e. $M_2 < M_1$

Flow slows down $\Rightarrow$ (isentropic) compression.

*Notes*

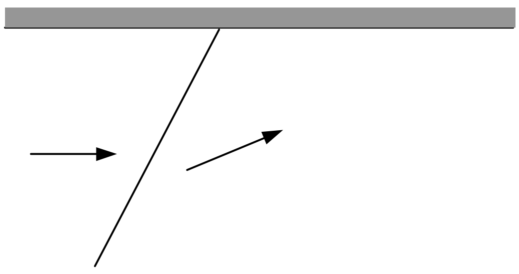
(i) 'Compression' waves run together.

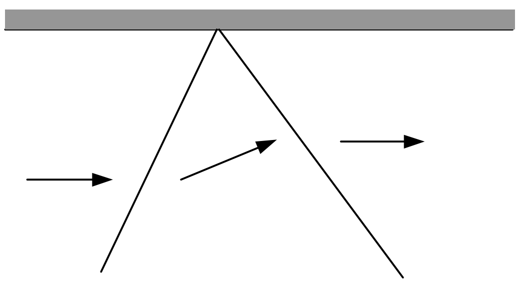
(ii) when two ch'ics of the same family meet $\Rightarrow$ discontinuity i.e. shock




Shock starts at point where first ch'ics cross. Gets stronger as more ch'ics run into it (at P, $M_s = M_C$ , and at Q, $M_s = M_B$ )

**Reflection of characteristics**



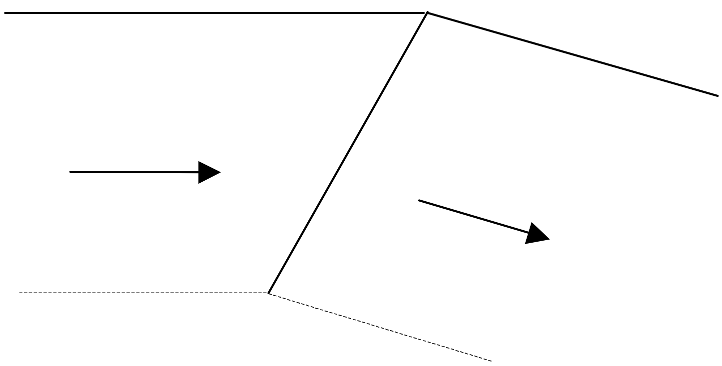


Consider an incoming (compression) characteristic.  This turns the flow and in order to satisfy the boundary condition at the wall, a second compression is needed.

At <u>solid walls</u>
- expansion reflects as an expansion
- compression reflects as a compression

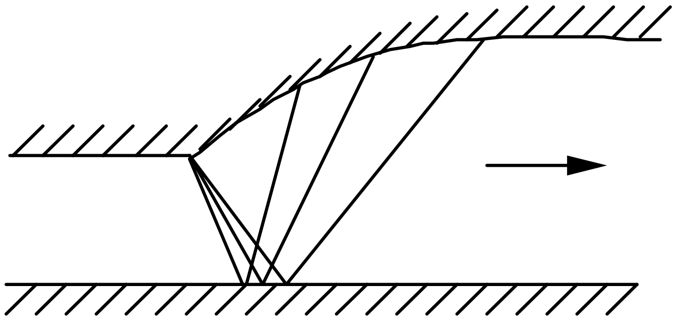
At <u>constant pressure</u> boundary
- expansion reflects as a compression
- compression reflects as an expansion

We can only have no reflection if the angle of the wall downstream of where the incoming characteristic meets it happens (or is designed) to have the same angle as the flow downstream of the characteristic.  The characteristic is then said to have been cancelled.



**Example Wind Tunnel Expansion**

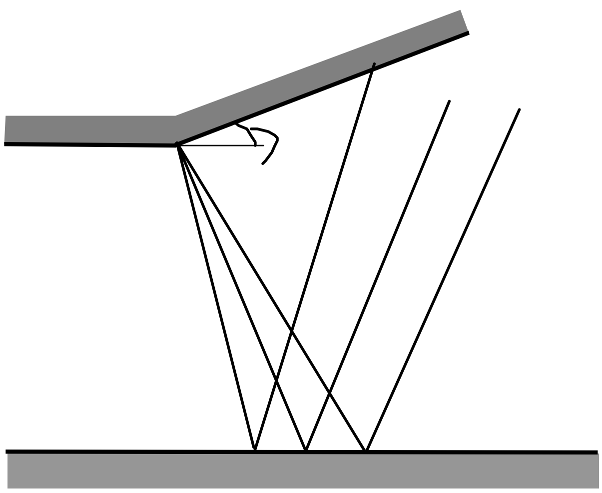
For a supersonic wind tunnel try to profile end wall to avoid further reflection $\Rightarrow$ flow uniform in working section. This is a) difficult to do and b) each different $M_2$ needs a different shape.



The equation

(2.1)

determines the area ratio, but gives no indication of the geometry of the transitional piece AC.



The first section AB can be straight and the turning at A ($\theta_E$ say) will cause an expansion fan of characteristics to emanate from A. The last point on the straight portion of the upper wall, B, will be where the first of the reflected characteristics meets this upper wall. From B to C the end wall must be contoured to the local flow direction implied by these incoming characteristics.

Across the fan emanating from A,

(2.2)

Across the reflected fan

(2.3)

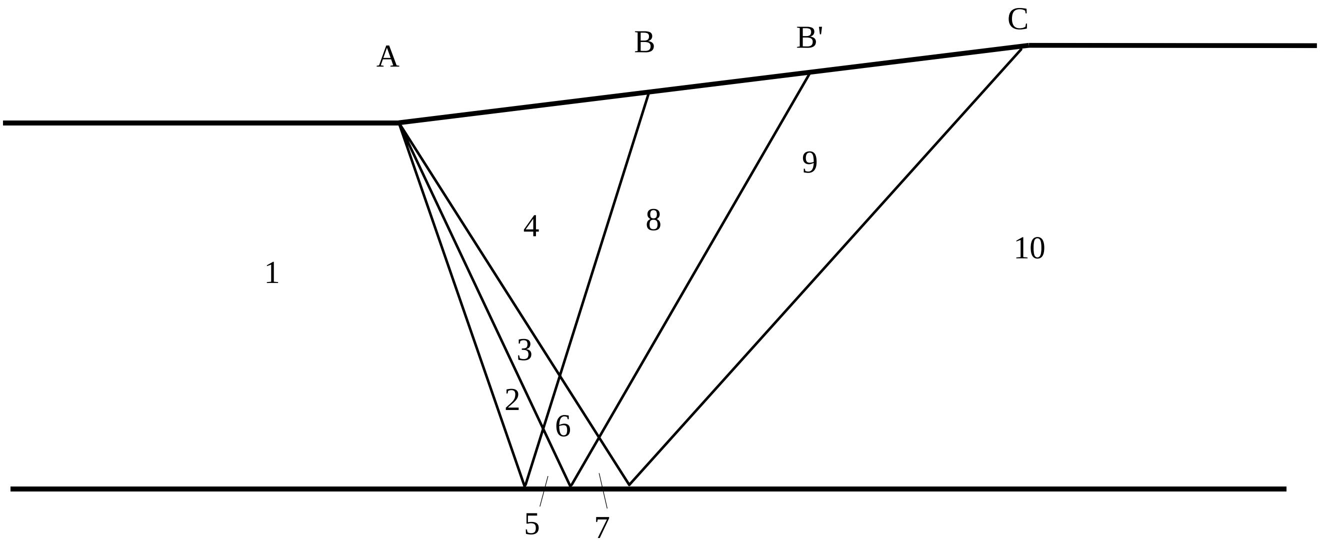
Equations (2.2) and (2.3) determine $\theta_E$ .

As a concrete example (chosen to make numbers easy !)

$M_1 =$ and $M_2 =$ $\nu_1 =$ $\nu_2 =$

Thus

$\nu_E - \theta_E =$ and $\nu_E + \theta_E =$ $\Rightarrow$ $\theta_E =$




To find the shape of the wall downstream of B, Use the field method with a discretisation

Prepare data from tables or calculator

| $\nu°$ | $M$ | $\mu°$ |
|---|---|---|
| 12 | 1·503 | 41·71 |
| 14 | 1·571 | 39·53 |
| 16 | 1·638 | 37·63 |
| 18 | 1·707 | 35·86 |
| 20 | 1·775 | 34·29 |
| 22 | 1·844 | 32·84 |
| 24 | 1·916 | 31·46 |

A) Expansion fan region 1 $\rightarrow$ 4

| Region | $\theta°$ | $\nu°$ |
|---|---|---|
| 1 | 0 | 12 |
| 2 | 2 | 14 |
| 3 | 4 | 16 |
| 4 | 6 | 18 |

(by choice of angle increment)

B) Region 5 (reflection from the lower wall).

$\nu_2 + \theta_2 = \nu_5 + \theta_5 =$ and $\theta_5 =$ $\Rightarrow$ $\nu_5 =$

C) Region 6

$$\left.\begin{aligned} \nu_3 + \theta_3 &= \nu_6 + \theta_6 = \\ \nu_5 - \theta_5 &= \nu_6 - \theta_6 = \end{aligned}\right\} \Rightarrow \quad \theta_6 = \qquad \Rightarrow \; \nu_6 =$$

D) Region 8

$$\left.\begin{aligned} \nu_4 + \theta_4 &= \nu_8 + \theta_8 = \\ \nu_6 - \theta_6 &= \nu_8 - \theta_8 = \end{aligned}\right\} \Rightarrow \quad \theta_8 = \qquad \Rightarrow \; \nu_8 =$$

E) Region 7

$$\nu_7 + \theta_7 = \nu_6 + \theta_6 = \qquad \text{and} \quad \theta_7 = \qquad \Rightarrow \; \nu_7 =$$

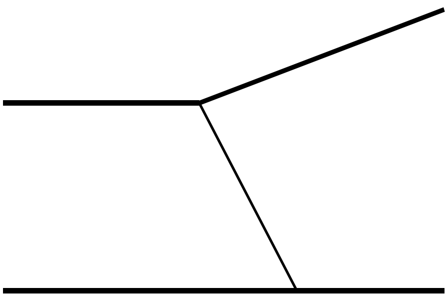
F) Region 9

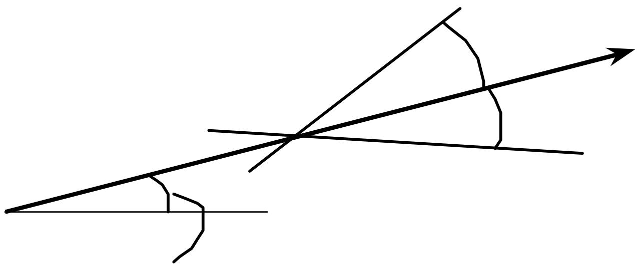
$$\left.\begin{aligned} \nu_8 + \theta_8 &= \nu_9 + \theta_9 = \\ \nu_7 - \theta_7 &= \nu_9 - \theta_9 = \end{aligned}\right\} \Rightarrow \quad \theta_9 = \qquad \Rightarrow \; \nu_9 =$$

G) Region 10

$$\nu_9 + \theta_9 = \nu_{10} + \theta_{10} = \qquad \text{and} \quad \theta_{10} = \qquad \Rightarrow \; \nu_{10} =$$

Finally, it remains only to find the co-ordinates of B, B' and C between which the local wall angle will be 4° and 2° respectively. These follow from the Mach angle of the various characteristics





e.g. for the ch'ic separating regions 1 and 2, could take ch'ic at Mach angle to the upstream flow, but probably more accurate to use the average of conditions in regions 1 and 2. Remembering that $\mu$ is the angle of the ch'ic to the local <u>flow</u> direction,