Module 3A1: Fluid Mechanics I

APPLICATIONS TO EXTERNAL FLOWS

Examples paper 1—The flow around aerofoil sections

2D panel methods

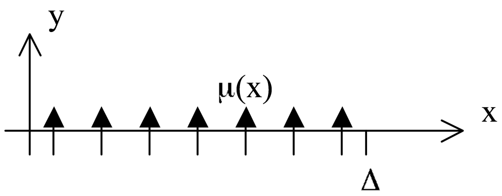

Fig. 1

1. Figure 1 shows a doublet panel. The stream function associated with this panel is:

$$\psi(x,y) = \int_0^\Delta -\frac{\mu(l)}{2\pi}\frac{x-l}{(x-l)^2+y^2}\,dl$$

Use integration by parts to show that this stream function is equal to that for a vortex sheet panel combined with two point vortices, and find:

(a) the point vortex circulations;

(b) the point vortex locations;

(c) the vortex sheet strength.

2. The stream function for the constant-strength vortex panel shown in Figure 2 was derived in the lectures. It is:

$$\psi(x,y) = -\frac{\gamma_0}{4\pi}\left\{x\log\left(x^2+y^2\right)-(x-\Delta)\log\left((x-\Delta)^2+y^2\right)-2\Delta+2y\left[\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-\Delta}{y}\right)\right]\right\}$$

(a) What is the corresponding velocity field, (u,v)?

(b) Find u and v in the region $0 < x < \Delta$, for $y = 0^+$ and $y = 0^-$. Comment on your results.

(c) *Challenge*. Find the largest contributor to the stream function as $r^2 = x^2 + y^2 \to \infty$. Is this what you expect? (Note that $\tan^{-1}(z+\varepsilon) \approx \tan^{-1} z + \varepsilon/\left(1+z^2\right)$ for $\varepsilon/z << 1$.)

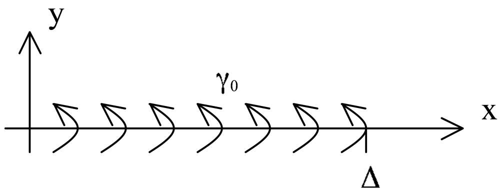

Fig. 2

Thin Aerofoil Theory

3. A thin aerofoil of chord c has the parabolic camber line

$$y_c = 4h\frac{x}{c}\left(1-\frac{x}{c}\right)$$

(a) What is the pitching moment coefficient about the quarter-chord point?

(b) Where is the centre of pressure (ie the point about which there is no pitching moment) when the aerofoil is at zero incidence?

4. Aerofoils with both positive and negative mean-line curvature are often described as having 'reflex camber'. This can be represented by the equation

$$y_c = h\frac{x}{c}\left(1-\frac{x}{c}\right)\left(\kappa-\frac{x}{c}\right)$$

(a) Find the coefficients in the Fourier series for $-2\,dy_c/dx$.

(b) What value of κ gives the aerofoil zero pitching moment about the quarter-chord point? Sketch the corresponding camber line.

(c) Modern civil aircraft have aerofoils with initially negative camber, followed by positive (giving benefits at high subsonic Mach numbers). A mean line of this form can be represented by setting κ = ½, h = –H (H > 0). Sketch the camber line, and find the pitching moment coefficient about the quarter chord.

(d) *Challenge/discussion*. Imagine an aircraft with unswept wings whose aerofoil section is that of (c). For stability reasons, the centre of gravity of the aircraft is at the wing quarter-chord point. What direction force on the tail is required to maintain moment equilibrium? What is the implication for this aircraft's cruising efficiency, relative to one equipped with the section in (b)?

5. The ‘inverse problem’ of thin aerofoil theory is the determination of a camber line, given a required loading distribution at the design incidence angle, α_d.

   (a) What is the boundary condition linking the normal perturbation velocity component, v', the camber-line coordinate, $y_c(x)$, and the incidence, α_d? Ignore the contribution of the aerofoil thickness.

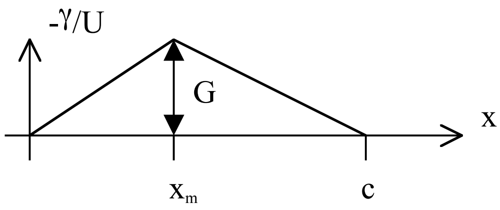

Fig. 3

   (b) The desired loading is shown, in terms of the vortex sheet strength, in Figure 3. Find the associated distribution of v' on the aerofoil surface. You may assume that $\int_a^b (x'-x)^{-1} dx' = \log|(b-x)/(a-x)|$, even if x lies between a and b.

   (c) Use your favourite analysis program to plot the parameter $dy_c/dx - \alpha_d$, for G = 0.2, $x_m = 0.25c$.

   (d) *Challenge*. Integrate your result for (c) numerically. Hence find α_d and plot the camber line. (Matlab’s `cumsum` function will come in handy here.)

6. Figure 4 shows a bi-linear camber line, defined in terms of the angles ψ_1 and ψ_2. The location of maximum camber is $x = x_p$.

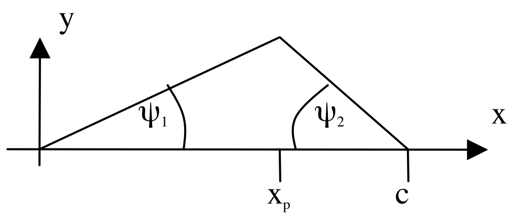

Fig. 4

   (a) Find the first two coefficients in the Fourier series for $-2dy_c/dx$. Express your results in terms of ψ_1, ψ_2 and θ_p, where $\cos\theta_p = 2x_p/c - 1$. You may assume that ψ_1 and ψ_2 are small.

   (b) A symmetrical aerofoil section sits at incidence in a flow. The rearward quarter chord is then deflected downwards through an angle ψ.
      (i) What is the change in incidence of the chord line? Find also the angles ψ_1 and ψ_2 in terms of ψ.
      (ii) What is the change in lift coefficient?

   (c) Starting again from the symmetrical section, find the change in lift coefficient when the forward quarter chord is deflected downwards through ψ.

(d) *Discussion*. With reference to your results, comment on the use of trailing edge flaps and leading edge slats on aircraft wings. Consider not only the change in lift coefficient, but also modifications to the pressure distribution.

7. A symmetrical aerofoil has thickness distribution

$$\frac{t}{c} = T\sqrt{\frac{x}{c}}\left(1-\frac{x}{c}\right),$$

where x is the horizontal coordinate, c is the chord, and T is a constant.

(a) Sketch the thickness distribution, and find its maximum value.

(b) Find the pressure coefficient at zero incidence as a function of the variable θ, where $x/c = (1+\cos\theta)/2$. Note that

$$\int_0^{\pi} \sqrt{\frac{x}{c}}\left(1-\frac{x}{c}\right)\sin n\theta \, \mathrm{d}\theta = \frac{n}{4n^2-1} - \frac{n}{4n^2-9}.$$

(c) Using Matlab/Octave (recommended) or Excel (if you must), write code to evaluate the pressure coefficient expression (five terms in the summation are sufficient). Compare your results with the calculated values for non-linearised potential flow, plotted in Figure 5, and comment.

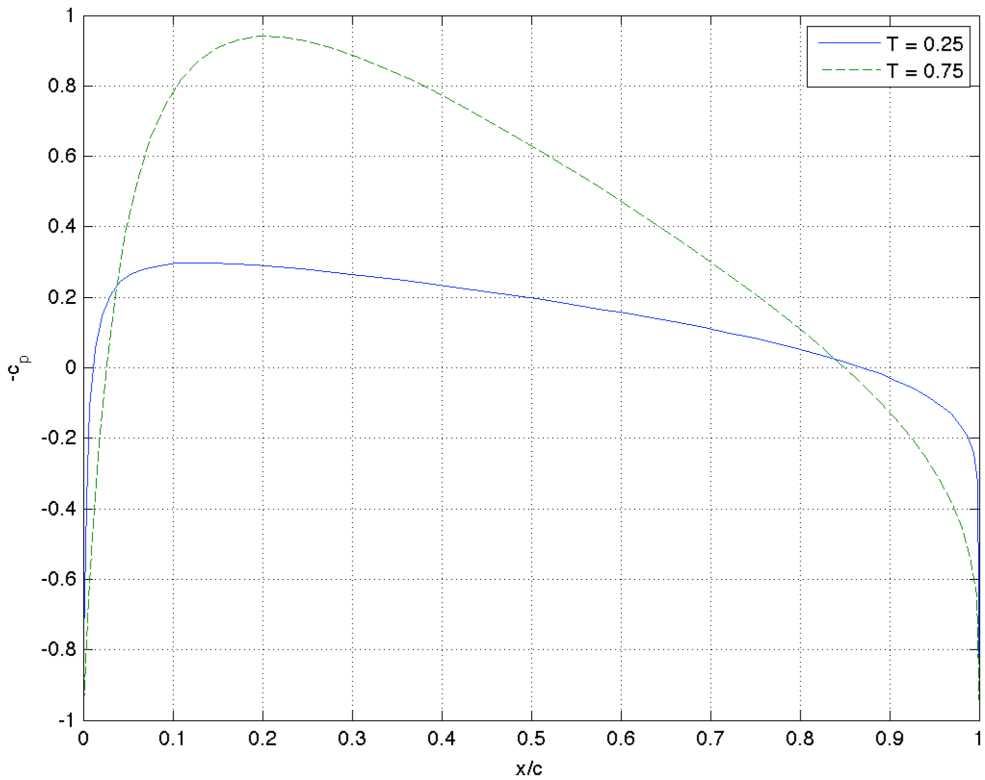

Fig. 5. Pressure coefficients for two values of the thickness parameter, T.

8. *Challenge (optional).* The shape known in aerodynamic parlance as a 'brick' is defined (in the usual coordinate system) by the thickness distribution

$$t = t_0\big[H(x) - H(x-c)\big]$$

where H(x) is the Heaviside step function. It sits at zero incidence in a flow with free-stream velocity U.

(a) Find the equivalent source distribution for this shape, and hence the vertical perturbation velocity component due to the brick along the line y = 2h.

(b) A ground plane is now inserted a distance h below the brick. What image is required to represent the influence of this plane, and what vertical velocity does the image cause at the brick?

(c) By considering the flow boundary condition on the brick, show that the influence of the image velocity is equivalent to a camber line, and sketch its gradient.

(d) Without performing any calculations, characterise the first two coefficients, g_0 and g_1, of the standard Fourier series associated with the equivalent camber line.

(e) What do your results tell you about the thickness contribution to (2D) ground effect? Can you give a physical explanation for your conclusion? Is it good or bad news for the designers of front wings on Formula 1 cars?

Lumped parameter models

9. Figure 6 shows two symmetrical aerofoils in series, at incidence α in a flow of free-stream velocity U.

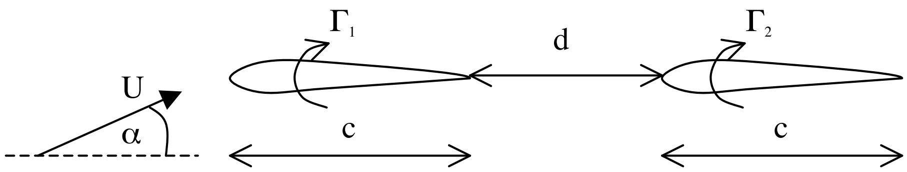

Fig. 6

(a) Estimate the circulations, Γ_1 and Γ_2, of the aerofoils.

(b) *Discussion.* Comment on the implications for the effectiveness of the horizontal tail surfaces on aircraft.

10. Figure 7 shows two identical, symmetrical aerofoils in a biplane arrangement.

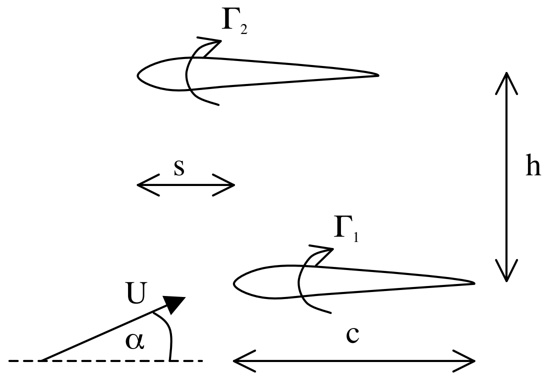

Fig. 7

(a) Estimate the circulations, Γ_1 and Γ_2, of the aerofoils, for:
   (i) s = 0 (no 'stagger');
   (ii) s = c/2.

(b) For the case s = 0, compare the lift of the combination with that obtained from a symmetrical section with chord 2c.

(c) *Discussion*. Briefly comment on the pros and cons of stagger.

11. (a) Find the effective three-quarter-chord slope in the lumped-parameter model of the parabolic camber aerofoil of Qu. 3.

(b) Calculate the actual three-quarter-chord slope for this aerofoil, and comment on the result.

Viscous effects and stall

12. Dig out a copy of Abbott and von Doenhoff's 'Theory of Wing Sections' (CUED library shelfmark TD61; ISBN 486-60586-8), and tabulate or plot the minimum drag coefficients and maximum lift coefficients for the following examples:

(a) NACA 63_1-012, 63_1-212, 63_1-412, against camber, for Reynolds number 6×10^6;

(b) NACA 23012, 23015, 23018, 23021, 23024, against percentage section thickness, for Reynolds number 6×10^6;

(c) NACA 4412, against Reynolds number.

Answers

1 (a) $\mu(0)$, $-\mu(\Delta)$ (b) $x = 0, x = \Delta$ (c) $d\mu/dx$

2 (a) $u = -\frac{\gamma_0}{2\pi}\left[\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-\Delta}{y}\right)\right]$ $v = \frac{\gamma_0}{4\pi}\left\{\log\left(x^2+y^2\right) - \log\left((x-\Delta)^2+y^2\right)\right\}$

(b) $u = \mp\frac{\gamma_0}{2}$ $v = \frac{\gamma_0}{4\pi}\left\{\log\left(x^2\right) - \log\left((x-\Delta)^2\right)\right\}$ (c) $\psi(x,y) \approx -\frac{\gamma_0\Delta}{4\pi}\log\left(x^2+y^2\right)$

3 (a) $\pi h/c$ (b) $x = c/2$

4 (a) $g_0 = -h/4c$, $g_1 = (2\kappa-1)h/c$, $g_2 = -3h/4c$

(b) 7/8 (c) $3\pi H/32c$

5 (a) $v'/U = -\alpha_d + dy_c/dx$ (b) $v' = \frac{GU}{2\pi}\left\{\frac{x}{x_m}\log\left|\frac{x_m - x}{x}\right| + \frac{c-x}{c-x_m}\log\left|\frac{c-x}{x_m-x}\right|\right\}$

6 (a) $g_0 = 2\frac{(\psi_1+\psi_2)}{\pi}\theta_p - 2\psi_1$, $g_1 = 4\frac{(\psi_1+\psi_2)}{\pi}\sin\theta_p$

(b) (i) $\psi/4$, $\psi_1 = \psi/4, \psi_2 = 3\psi/4$ (ii) 3.83ψ (c) -0.36ψ

7 (a) Maximum $\frac{t}{c} = 0.385T$ (b) $c_p = -\frac{4T}{\pi}\sum_{n=1}^{\infty}\left(\frac{n^2}{4n^2-1} - \frac{n^2}{4n^2-9}\right)\frac{\sin n\theta}{\sin\theta}$

8 (a) $\frac{Ut_0}{2\pi}\left[\frac{2h}{x^2+4h^2} - \frac{2h}{(x-c)^2+4h^2}\right]$ (d) $g_0 = 0, g_1 < 0$

9 (a) $\Gamma_1 = \pi U\alpha c\left[1 + \frac{1}{2}\frac{c}{c+d}\right], \Gamma_2 = \pi U\alpha c\left[1 - \frac{1}{2}\frac{c}{c+d}\right]$

10 (a) (i) $\Gamma_1 = \Gamma_2 = \pi U\alpha c \Big/ \left[1 + c^2/\left(c^2+4h^2\right)\right]$ (ii) $\Gamma_1 = \pi U\alpha c\left[1 - \frac{1}{2}\frac{c^2}{\left(c^2+h^2\right)}\right], \Gamma_2 = \pi U\alpha c$

(b) Smaller, by a factor $\left[1 + c^2/\left(c^2+4h^2\right)\right]$

11 (a) -2h/c (b) -2h/c

The material in this paper is examinable from 2007 onwards. For further revision practice, find the old G22 papers in the library and try: ’87 qu7, ’88 qu6, ’89 qu5, ’90 qu6, ’91 qu6, ’92 qu4, ‘93, qu7. Note, however, that these questions are not necessarily representative of current Tripos standard.

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