Paper 3A3: Fluid mechanics

TWO-DIMENSIONAL COMPRESSIBLE FLOW

Examples paper 1

Compressible potential flow and similarity

1. Measurements of the pressure distribution on a NACA009 aerofoil section at zero incidence in a low-speed wind tunnel show a minimum pressure coefficient of –0.25.

   (a) Assuming that linearised flow theory may be applied, calculate the minimum pressure coefficient when the aerofoil is at zero incidence in a flow of Mach number 0.5.

   (b) Using a suitable isentropic flow equation, find the local Mach number corresponding to this pressure coefficient.

   (c) If this local Mach number is equal to one, evaluate the free-stream Mach number and corresponding minimum pressure coefficient. (A graphical method, or numerical trial and error, is recommended.)

   The free-stream Mach number you have calculated in (c) is known as the critical Mach number for the aerofoil section. It is important because, at higher speeds, the flow over part of the aerofoil surface is supersonic, and terminates in a shock wave. This causes a marked increase in drag (the 'drag-divergence'), mainly due to boundary-layer separation induced by the shock wave.

   (d) Comment briefly on the validity of using linearised flow theory for the calculation in (c).

Method of characteristics

2. Figure 1 shows a Prandtl-Meyer expansion fan, with an upstream Mach number of 1.86 and a flow deflection angle of 20°.

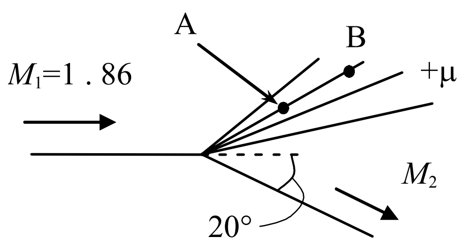

Fig. 1

(a) Calculate the downstream Mach number, M_2.

(b) Calculate the Mach number and flow angle at the point A, which lies on the characteristic at 30° to the local flow direction. At what angle is this characteristic to the original flow direction.

(c) Explain why conditions at B are the same as at A and why the characteristic through A and B is straight.

3. A duct is designed to reduce the Mach number of a flow from 1.8 to 1.2 through a two-dimensional isentropic compression, as shown in Figure 2.

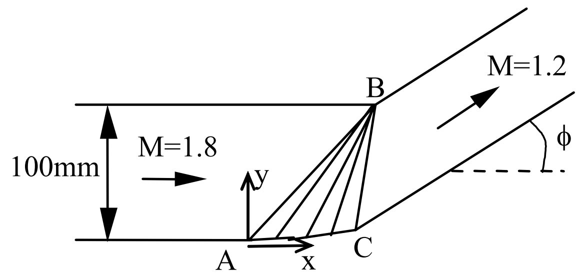

Fig. 2

The shape of the corner AC is such that the waves generated by it all converge on the point B where the upper wall has a sharp corner. Since no subsequent waves are generated from B, the upper wall is said to be 'designed for cancellation'. Find the angle ϕ, the pressure ratio across the wave region, and the co-ordinates of B and C.

4. (a) Figure 3(a) shows a discretised lattice of characteristics for the flow in a supersonic nozzle. On the line AA' the Mach number and flow angle distributions are

$$M = 2.0 + 0.5\cos\left(180\, y/b\right) \qquad \theta = 12\, y/b \qquad \text{(angles in degrees)},$$

and the lattice points 1–5 are equispaced. We wish to calculate the downstream flow.

(i) Explain how symmetry may be invoked to reduce the computational load.

(ii) Using the <u>lattice</u> method, calculate M and θ at points 6 and 7.

(iii) Calculate the position of point 6. (NB The best estimate of the slopes of the 1-6 and 2-6 characteristics would be based on the average of conditions at each end.)

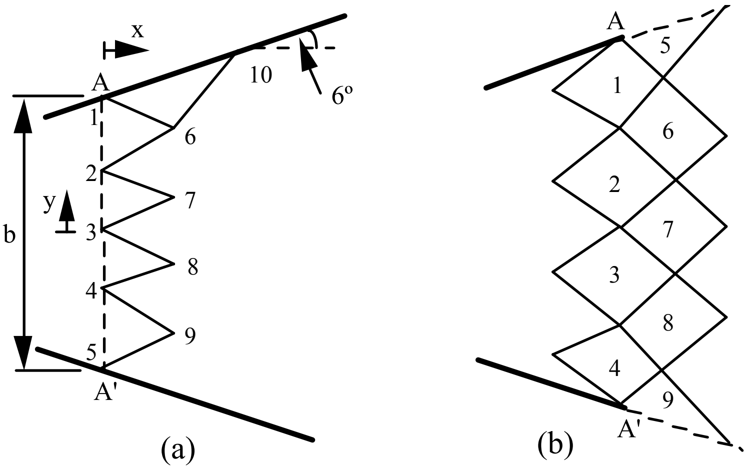

Fig. 3

(b) Figure 3(b) shows a nozzle of the same geometry, ending at A–A', where the flow conditions are the same as in (a). The ambient static pressure is p_a, and the stagnation pressure of the nozzle flow is $11.04p_a$.

  (i) Using the field method, calculate M and θ in regions 5–7.

  (ii) What is the slope of the characteristic between regions 1 and 6?

5. On some aircraft the Coanda effect, whereby a jet blown along a curved surface tends to stick to that surface, is used to augment lift. If the jet is subsonic this is a complicated flow to try and understand; however the supersonic case is a straightforward expansion (Figure 4).

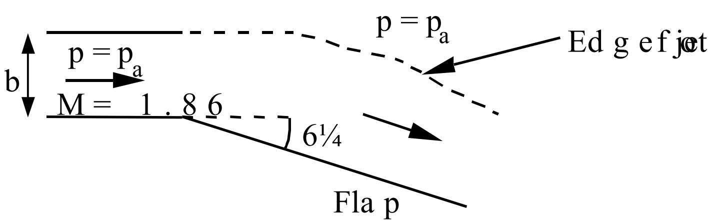

Fig. 4

Using the field method, and a 2° discretisation,

(a) sketch the resulting wave pattern and determine the pressure distribution on the flap up to the point where it first rises above atmospheric,

(b) find the length of the minimum pressure region on the flap.

Compare your results with those obtained using a 6° discretisation.

Answers

1 (a) –0.289 (b) 0.573 (c) 0.805, –0.421

2 (a) 2.65, (b) 2.00, –3.92°, 26.08°

3 17.1°, 2.37, (149.7, 100), (125.4, 17.6)

4 (a) (ii) $M_6 = 2.23$, $M_7 = 2.49$, $\theta_6 = -0.11°$, $\theta_7 = -0.24°$

(iii) $x_6 = 0.252b$, $y_6 = 0.380b$

(iv) $M_{10} = 2.48$

(b) (i) $M_5 = 2.22$, $M_6 = 2.39$, $M_7 = 2.53$, $\theta_5 = 5.25°$, $\theta_6 = -0.36°$, $\theta_7 = 0°$

(ii) 28.05°

5 (a) Three regions: $0.715p_a$, $0.897p_a$, $1.11p_a$ (lengths of regions not required).

(b) $3.55b$

6° discretisation: $0.715p_a$, $1.37p_a$, $3.59b$

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