3A1 Data Sheet for Applications to External Flows

Aerodynamic Coefficients

For a flow with free-stream density, ρ, velocity U and pressure p_∞:

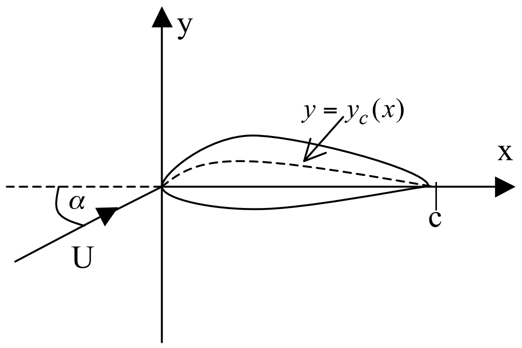
Pressure coefficient: $$c_p = \frac{p - p_\infty}{\frac{1}{2}\rho U^2}$$

Section lift and drag coefficients: $$c_l = \frac{\text{lift}\,(N/m)}{\frac{1}{2}\rho U^2 c}, \quad c_d = \frac{\text{drag}\,(N/m)}{\frac{1}{2}\rho U^2 c} \qquad \text{(section chord c)}$$

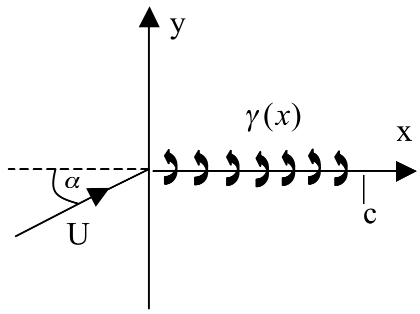
Wing lift and drag coefficients: $$C_L = \frac{\text{lift}\,(N)}{\frac{1}{2}\rho U^2 S}, \quad C_D = \frac{\text{drag}\,(N)}{\frac{1}{2}\rho U^2 S} \qquad \text{(wing area S)}$$

Thin Aerofoil Theory

Geometry



Approximate representation



Pressure coefficient: $$c_p = \pm\gamma / U$$

Pitching moment coefficient: $$c_m = (\text{moment about } x = 0) / \tfrac{1}{2}\rho U^2 c^2$$

Coordinate transformation: $$x = c(1 + \cos\theta)/2 = c\cos^2(\theta/2)$$

Incidence solution: $$\gamma = -2U\alpha\frac{1-\cos\theta}{\sin\theta}, \quad c_l = 2\pi\alpha, \quad c_m = c_l/4$$

Camber solution: $$\gamma = -U\left[g_0\frac{1-\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} g_n \sin n\theta\right], \text{ where}$$

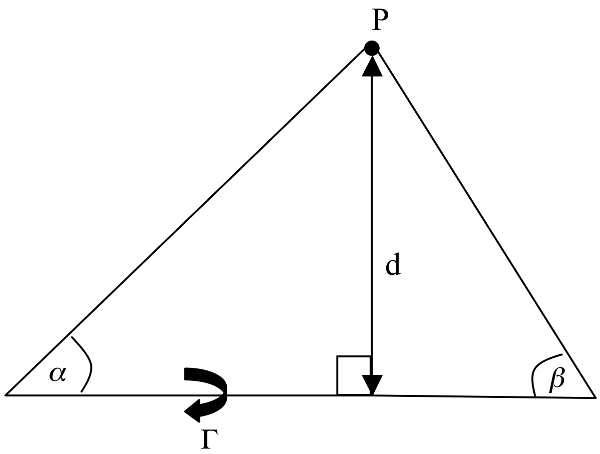
$$g_0 = \frac{1}{\pi}\int_0^\pi\left(-2\frac{dy_c}{dx}\right)d\theta, \quad g_n = \frac{2}{\pi}\int_0^\pi\left(-2\frac{dy_c}{dx}\right)\cos n\theta\, d\theta$$

$$c_l = \pi\left(g_0 + \frac{g_1}{2}\right), \quad c_m = \frac{\pi}{4}\left(g_0 + g_1 + \frac{g_2}{2}\right) = \frac{c_l}{4} + \frac{\pi}{8}(g_1 + g_2)$$

Glauert Integral

$$\int_{o}^{\pi} \frac{\cos n\phi}{\cos\phi - \cos\theta} d\phi = \pi \frac{\sin n\theta}{\sin\theta}$$

Line Vortices



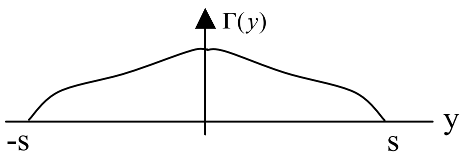
A straight element of circulation Γ induces a velocity at P of

$$\frac{\Gamma}{4\pi d}(\cos\alpha + \cos\beta)$$

perpendicular to the plane containing P and the element.

Lifting-Line Theory

Spanwise circulation distribution:



Aspect ratio: $A_R = 4s^2 / S$

Wing lift: $L = \rho U \int_{-s}^{s} \Gamma(y) dy$

Downwash angle: $\alpha_d(y) = \dfrac{1}{4\pi U} \displaystyle\int_{-s}^{s} \dfrac{d\Gamma(\eta)/d\eta}{y - \eta}\, d\eta$

Induced drag: $D_i = \rho U \int_{-s}^{s} \Gamma(y) \alpha_d(y) dy$

Fourier series for circulation: $\Gamma(y) = Us \sum_{\text{n odd}} G_n \sin n\theta$, with $y = -s\cos\theta$

Relation between lift and induced drag:

$$C_{Di} = (1+\delta)\frac{C_L^2}{\pi A_R}, \text{ where } \delta = 3\left(\frac{G_3}{G_1}\right)^2 + 5\left(\frac{G_5}{G_1}\right)^2 + \ldots$$