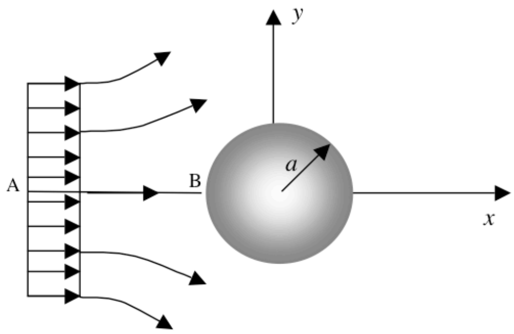
Module 3A1: Fluid Mechanics I

INCOMPRESSIBLE FLOW

Examples paper

To find out which topic a question is associated with, refer to the cross-references in the lecture handout. Questions denoted 'challenge' can (optionally) be omitted during your first attempt at this examples paper.

1. An incompressible, inviscid fluid flows steadily past a sphere of radius a. The fluid velocity along the streamline $A-B$ is given by $\boldsymbol{u} = u(x)\boldsymbol{i} = U(1+a^3/x^3)\boldsymbol{i}$, where U is the velocity far upstream of the sphere.



   (a) Determine the acceleration experienced by fluid particles as they flow from A to B and sketch the variation of this acceleration with x.

   (b) What is the maximum deceleration of the fluid particles as they flow along this line, and where does it occur?

   (c) A cricket ball is typically bowled at around 30m/s. It has diameter 70mm. Evaluate the maximum deceleration found in (b), and compare it with the acceleration due to gravity. Comment.

2. A two-dimensional flow has streamfunction $\psi = Axy$, with A constant.

   (a) Show that the flow is irrotational and find the velocity potential.

   (b) Use Matlab to plot contours of constant streamfunction (i.e. streamlines). Add the equipotentials to your plot (sketching these by hand is fine). Identify two practical situations where such a flow would occur.

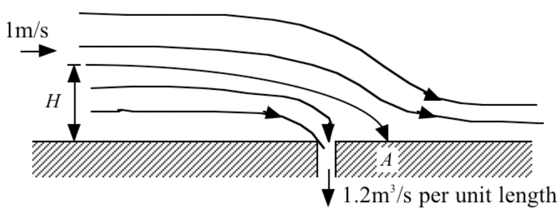
   (c) Show that the isobars (curves of constant pressure) are circles centred at the origin.

3. A flow has velocity potential $\phi = Ar^{6/5}\cos(6\theta/5)$, with A constant.

   (a) Determine the streamfunction $\psi(r,\theta)$. Plot the streamlines for $0 \le \theta \le 5\pi/3$ and show that this can describe flow onto a wedge of half-angle $\pi/6$. (If you didn't use Matlab for question 2, you're *strongly* advised to start now.)

   (b) Find the variation of surface pressure with distance s from the apex of the wedge. Is the pressure gradient adverse or favourable?

4. A river flows over a flat bed at 1m/s, as shown in the figure. A pump draws off water through a narrow slit, at a volumetric rate of $1.2\text{m}^3/\text{s}$ per metre length of the slit. The fluid can be assumed incompressible and inviscid, and the flow can be represented by the combination of a uniform velocity and a sink.

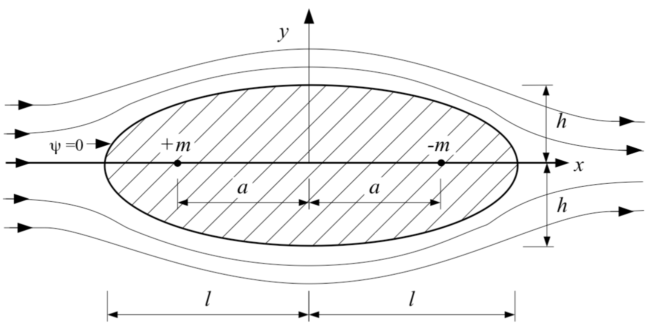


   (a) Locate the stagnation point (A) on the wall. Where might separation occur?

   (b) If the origin of coordinates is at the slit, what is the equation for the stagnation streamline?

   (c) Contaminated water is discharged upstream, at height H above the river bed. Use your result from (b) to find the minimum value of H if this water is not to be sucked into the slit. Compare your answer with the offtake rate; is this a coincidence?

5. The Rankine oval shown in the figure is formed by a source and a sink, each of strength m, placed in a uniform flow and separated by a distance $2a$.



   (a) Show that the half-length l and the half-height h are given by:

$$\frac{l}{a} = \left(\frac{m}{\pi U a} + 1\right)^{\frac{1}{2}} \quad \text{and} \quad \frac{h}{a} = \frac{m}{\pi U a}\cot^{-1}\left(\frac{h}{a}\right).$$

   Hence confirm that $m/Ua = 0.1$ leads to a body of aspect ratio around 20:1.

(b) For this value of m/Ua, find the pressure coefficient on the body surface at the maximum thickness point. Comment on the implications for the flow over the downstream half of the body.

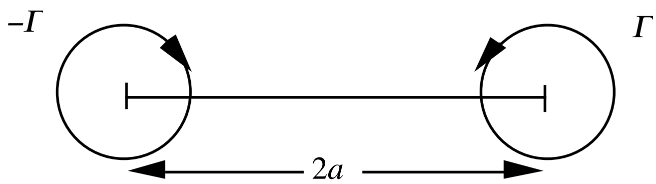
6. What is the complex potential for uniform flow of speed U at an angle α to the x direction?

7. A crude approximation to a stationary hurricane is to model the flow field outside its core by the two-dimensional flow due to a superimposed line vortex and a line sink.

   (a) Sketch the streamlines.

   (b) The wind velocity at a location A, 20 km away from the centre of a hurricane, makes an angle of 65° with the line towards the centre. The pressure at A is 550N/m^2 less than the pressure far away. What is the rate of volume influx of air per metre height (i.e. what is the strength of the apparent sink) and what is the circulation?

8. The trailing vortex pattern left by an aircraft can, as a first approximation, be modelled by a pair of vortices of equal strength but opposite sign, as shown below.



   (a) What is the downwards speed of the vortex pair?

   (b) In a frame of reference in which the vortices are at rest, at $(\mp a, 0)$, there is an oval-shaped streamline enclosing the vortices. Show that the equation of this streamline is

$$x = a \ln\left(\frac{(x+a)^2 + y^2}{(x-a)^2 + y^2}\right).$$

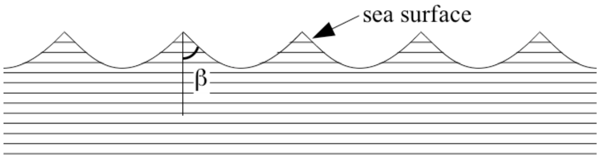
   (c) Show that there are stagnation points on this streamline at $x = 0$, $y = \mp\sqrt{3}a$. Hence find the ratio of the oval's major to minor axes.

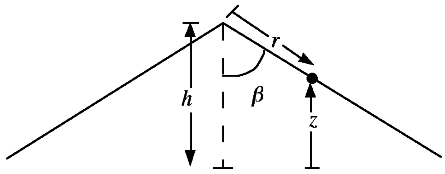
9. A flow has complex potential $F(z) = Az^{\alpha}$, with A constant.

   (a) Plot the flow streamlines, for $\alpha = 3, 2, 3/2, 2/3$ and $1/2$. Identify possible bounding walls (i.e. straight streamlines). For which of these values of α does the potential represent a physically realisable flow?

   (b) What is the variation of pressure along the line $\theta = 0$, neglecting any gravity effect?

(c) *Challenge*: use your results to estimate the 'breaking angle' of a water wave.



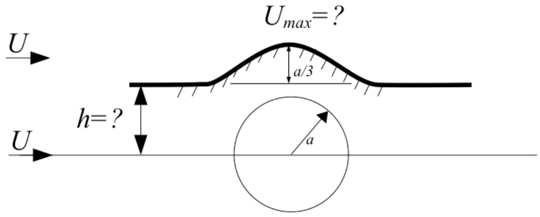


(i) In the frame of reference moving with the crest, the flow is steady and the sea surface is a streamline. Near the crest (right-hand figure), we have one of our corner flows. Express the parameter α in terms of the crest half-angle, β.

(ii) What is Bernoulli's equation for the surface streamline? (N.B. Refer back to (b), but note that gravity must now be included.)

(iii) Your result from (ii) must apply for all values of r. What is the unique value for β that follows from this observation?

10. Flow over a two-dimensional bump is to be analysed by approximating the bump geometry to a streamline in the flow passing over a cylinder, as shown in the figure.



(a) The bump is to be $a/3$ high, where a is the cylinder radius. What is the (upstream) elevation, h, of the streamline?

(b) What is the flow speed at the bump's crest, in terms of the free-stream velocity U?

11. Find an expression for the (idealised) force on a circular cylinder with circulation Γ in a uniform flow of velocity U, by integrating the potential-flow pressure distribution over the surface of the cylinder.

12. Find the position of the stagnation point on a flat plate at an angle of incidence α in a uniform stream, when the plate's circulation is given by the Kutta condition. Express the distance from the leading edge to the stagnation point as a fraction of the chord length. *Challenge*: use your result, and the streamline plot in the handout, to sketch the pressure variation over the plate's lower surface. What about the upper?

13. *Challenge*: analyse the flow whose complex potential $F(z)$ is given implicitly by

$$z = F - e^{-F}.$$

(a) Plot the streamlines in Matlab, as follows. First generate a matrix **F**, such that $F_{ij} = \phi_i + i\psi_j$, where ϕ_i is the i'th element of a vector of potential values (use 51 elements, ranging from –3 to 24), and ψ_j is the j'th element of a vector of streamfunction values (21 elements, ranging from –π to +π). Now form an associated matrix of z values, according to the equation above. The command `plot(z)` will then display the streamlines (why?).

(b) Check the salient features of your plot analytically; write $F = \phi + i\psi$ and show that:

(i) the real and imaginary parts of the equation are $x = \phi - e^{-\phi}\cos\psi$ and $y = \psi + e^{-\phi}\sin\psi$ respectively;

(ii) the streamline $\psi = 0$ lies along $y = 0$ for $-\infty \le x \le \infty$;

(iii) the streamline $\psi = \pi$ lies along $y = \pi$ for $1 \le x \le \infty$ (how does x vary as ϕ increases from $-\infty$?);

(iv) the streamline $\psi = -\pi$ lies along $y = -\pi$ for $1 \le x \le \infty$;

(v) for ϕ large and positive, $F \approx z$ (what is this flow?);

(vi) for ϕ large in magnitude but negative, $F \approx -\ln z + \text{constant}$ (what is this flow?).

14. *Challenge*: analyse the (inviscid and incompressible) flow over a rigid corrugated wall of infinite extent. The shape of the wall is given by $y = a\sin kx$, and the height of the corrugations is very small compared with the distance between successive corrugations, so that $ka << 1$.

(a) Determine the fluid velocity in the vicinity of the wall when, far from the wall, there is a uniform velocity U in the x direction, as follows.

(i) Consider the variable $\zeta = \xi + i\eta$, where $\zeta = z - ae^{ikz}$. Show that, as z moves over the curve $y = a\sin kx$, and x varies from $-\infty$ to $+\infty$, ζ moves (approximately) along the line $\eta = 0$. (Hint: exploit the fact that $ka << 1$.)

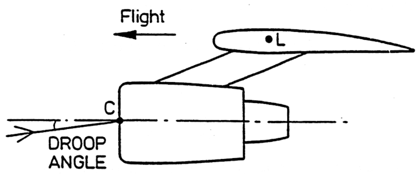
(ii) Write down the complex potential for a uniform flow parallel to the rigid surface $\eta = 0$. Hence deduce the complex potential $F(z)$ for the flow over the corrugated wall.

(iii) Differentiate $F(z)$ to find the fluid velocity.

(b) What is the difference between the maximum and minimum pressure on the wall?

N.B. In this example, the effect of the corrugations decays exponentially with distance away from the surface (the correction to uniform flow decreases with y like e^{-ky}). This exponential decay always happens for an incompressible fluid with x dependence of the form $\sin kx$. In an analogous way, the fluid velocity due to waves on the sea surface decreases exponentially with distance from the surface, leading to calm conditions at only modest depths.

15. A large aeroplane weighs 400 tonnes, has a wing span of 100m and is flying at 250m/s through air of density 0.3kg/m^3. The engines are mounted under and ahead of the wing, as sketched in the figure. The centre of the intake inlet plane C is 4m ahead of, and 2m below, the wing centre of lift L. Assuming the lift to be distributed uniformly across the span of the wing, estimate the 'droop' angle at which the engine inlet must be set such that the flow is parallel to the centre line at point C.



16. A small-diameter porous pipe spans a river whose flow speed is U. The pipe is placed well below the river surface, at a height h above the bed. A pump draws off water at a volume flow rate m per unit length of pipe.

   (a) Write down an expression for the complex potential. (Use coordinates centred on the river bed below the pipe, with the x axis horizontal and the y axis vertical.)

   (b) (i) Determine the position(s) of the stagnation point(s).

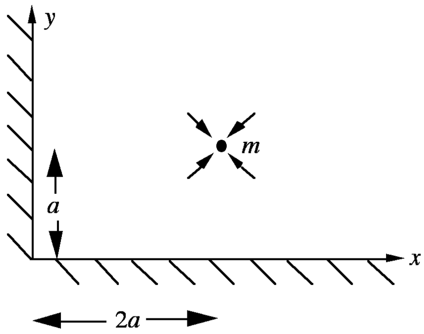
   (ii) Write the dimensionless complex potential $\tilde{F} = F/(Uh)$ as a function of the dimensionless coordinate $\tilde{z} = z/h$.

   (iii) Hence plot the streamline pattern for: $m = 2\pi Uh$; $m = 4\pi Uh$; $m = \pi Uh$. (Plot range: $-5 \le x \le 5$, $0 \le y \le 5$. Use `linspace(-2*pi,4*pi,31)` to set the streamfunction values at your contours. Don't worry about the weird vertical line going upwards from $(0,1)$. For $m = \pi Uh$, you will need to add the stagnation streamline by hand.)

   (iv) *Challenge*: what happens if you use `linspace(-2*pi,4*pi,30)` for the contour values? Can you explain why?

   (c) Water is extracted at a volume flow rate of 5m^3/s from a river flowing at 1m/s. The water is drawn off uniformly through a porous pipe of length 3 m. What is the minimum height of the pipe above the river bed, if the silty water near the bed is not to be extracted?

17. A sink is placed in a right-angle corner, as shown.



(a) Set up an image system for the flow field, and find the variation of velocity along the wall $y = 0,\ x \geq 0$. Where is the stagnation point?

(b) Plot the velocity variation derived in part (a).

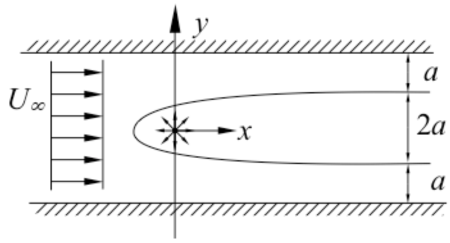
(c) From your plot, determine the region of this wall in which there is a possibility of separation.

18. The trailing vortices left by an aircraft are modelled by two infinitely long line vortices of equal strength, Γ, but opposite sign, a distance $2a$ apart.

(a) Calculate the velocity of the vortex pair when it is far above the ground.

(b) Show that, as the vortices approach the ground ($y = 0$), they follow the trajectories $x = \mp ay/(y^2 - a^2)^{\frac{1}{2}}$. Sketch these trajectories.

19. A source of strength m is placed on the centre-line of a wind tunnel of height $4a$.



(a) Determine the fluid velocity far upstream and far downstream of the source when the tunnel is switched off.

(b) A half-body in a wind tunnel with upstream flow speed U_∞ is to be represented by superposition of this line source and a uniform stream of velocity U_0. For the particular dimensions shown in the figure, find the required values of m and U_0.

(c) Determine the image system for the line source, and calculate the position of the nose of the body. You can assume, without proof, that

$$\sum_{n=-\infty}^{\infty} \frac{1}{s + in4a} = \frac{\pi}{4a}\coth\left(\frac{\pi s}{4a}\right).$$

20. *Challenge*: confirm that the image system required for a line vortex of strength Γ, at a distance d from the centre of a rigid cylinder of radius a $(a < d)$, consists of line vortices of strength $-\Gamma$ at a^2/d and $+\Gamma$ at the origin. In particular:

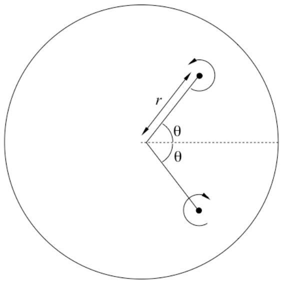
(a) show that

(i) $$\mathrm{Im}\left[F\left(ae^{i\theta}\right)\right] = -\frac{\Gamma}{2\pi}\ln\left[\left|\frac{ae^{i\theta}-d}{ae^{i\theta}-a^2/d}\right|\right] - \frac{\Gamma}{2\pi}\ln(a), \text{ and}$$

(ii) $$\left|\frac{ae^{i\theta}-d}{ae^{i\theta}-a^2/d}\right| = \frac{d}{a}, \text{ independent of } \theta;$$

(b) explain why the vortex at the origin is necessary.

21. *Challenge*: analyse the motion of two line vortices, of strength Γ and $-\Gamma$, inside a tube of internal radius a. At the initial instant the vortices are both at a radial distance r and subtend an angle 2θ at the centre, as shown in the figure.



(a) Determine the velocities of the two vortices in terms of r, θ, Γ and a.

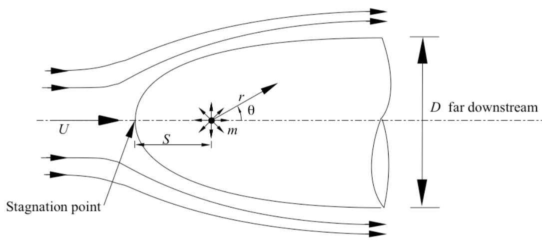
(b) Comment on the velocities when:

(i) θ is small and r is not close to a; and

(ii) r is nearly equal to a and θ is not small.

(c) Sketch the trajectories of the vortices, assuming that the approximation of (b)(i) holds at the initial instant.

22. A three-dimensional Rankine half-body is produced by a point source of strength m and a uniform flow U.

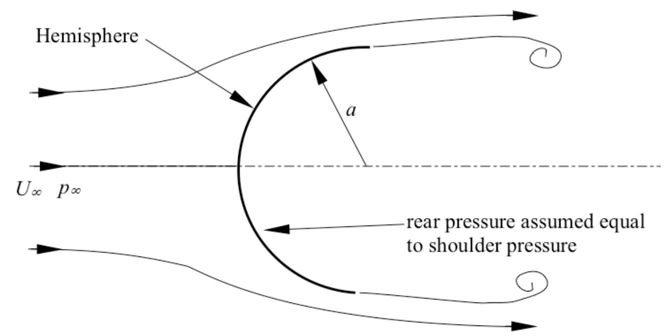


(a) Find the distance from the source to the stagnation point.

(b) What is the body diameter, D, far downstream?

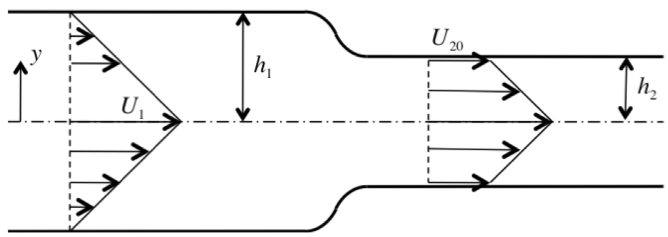
(c) Until what distance from the stagnation point does the centre-line flow velocity remain within 5% of its upstream value? Express your answer in terms of D, and compare it with the corresponding result for a two-dimensional half-body (see lecture notes).

23. Estimate the form drag on a hemisphere in a uniform flow, by assuming that the flow on the upstream side of the hemisphere is given by inviscid sphere theory and that the pressure in the rear equals the shoulder pressure.



Comment on the defects of this analysis.

24. The flow in a two-dimensional channel, of height $2h_1$, has a bi-linear velocity profile. (This can be regarded either as a rough approximation to the fully-developed, quadratic profile expected in viscous laminar flow, or as a possible developing flow.) It then enters a short constriction, and emerges into a channel of height $2h_2$.



(a) Assuming that the flow traverses the constriction too rapidly for viscosity to have any significant influence, derive an expression for the downstream velocity profile in terms of: the upstream centre-line speed, U_1; the downstream 'wall' speed, U_{20}; h_1; h_2; and y. (Only the $y > 0$ form is required.)

(b) Find U_{20} in terms of U_1, h_1 and h_2.

(c) How should your result for U_{20} be interpreted, in the light of the no-slip condition that must be satisfied at the wall in reality? Discuss the downstream evolution of the velocity profile.

(d) A sudden expansion, with $h_2 > h_1$, could be analysed in the same way. Comment on the value taken by U_{20} in this case, and discuss the implications for the real flow.

25. (a) Verify that the velocity field $\boldsymbol{u} = (-\frac{1}{2}\alpha x - \Omega y e^{\alpha t}, -\frac{1}{2}\alpha y + \Omega x e^{\alpha t}, \alpha z)$ satisfies $\nabla \cdot \boldsymbol{u} = 0$.

(b) (i) What is the vorticity?

(ii) A fluid line initially lies in the z direction and has length l. What is the length L of this line at time t?

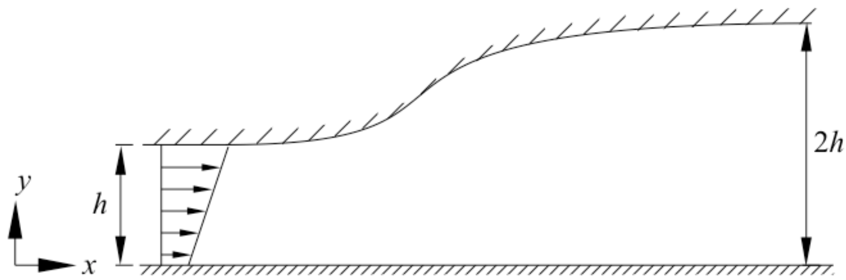
(iii) Verify that vorticity/L is independent of time, for the appropriate component of vorticity.

(c) Sketch the flow. (Hint: start by considering the x–y plane.)

26. The entry flow to the diffuser shown in the figure has a velocity profile

$$u = U\left[1 + \frac{y}{4h}\right].$$

The depth of the diffuser is constant and the flow may be treated as incompressible and inviscid.



(a) Determine:

(i) the velocity profile at the exit;

(ii) the pressure rise across the diffuser;

(iii) the exit height of the streamline that enters the diffuser at $y=h/2$.

(b) A second design of diffuser maintains a constant height h in the y direction, but varies the depth in the z direction. Calculate the area ratio of this second diffuser which is necessary to achieve the same pressure recovery as the original design.

27. Consider a steady, viscous, laminar flow through a straight horizontal tube, whose constant elliptical cross-section is described by the equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

where x and y are the cross-section coordinates. The streamlines are straight and parallel.

(a) Confirm that a velocity field $\mathbf{u} = (0,0,w)$, with

$$w = A\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right),$$

satisfies the mass conservation equation and the wall boundary conditions, and find the equations for the pressure field.

(b) With this velocity distribution, what is the relationship between the pressure gradient dp/dz along the tube and the mass flow rate $\dot{m}$ through the tube?

28. An infinite plate is oscillated tangentially with speed $U_0 \cos\omega t$ in a viscous fluid which is at rest far from the plate.

(a) If the plate is taken to lie in the (x–z) plane, and to oscillate in the x direction:

(i) show that $\mathbf{u} = (u(y,t),0,0)$ is a possible solution, and find the equation that must be satisfied by $u(y,t)$;

(ii) show that this equation is satisfied by $u = \mathrm{Re}\left(U_0 e^{i\omega t + my}\right)$, and find the complex constant m;

(iii) obtain the peak-to-peak amplitude of velocity fluctuation a distance y above the plate.

(b) Determine the unsteady shear force/unit area on the plate.

29. A swirling flow has velocity components

$$u_r = 0, \quad u_\theta = \frac{\Gamma_0}{2\pi r}\left(1 - e^{-r^2/R^2}\right), \quad u_z = 0$$

in cylindrical polar coordinates. (The parameter R is a characteristic radial distance.)

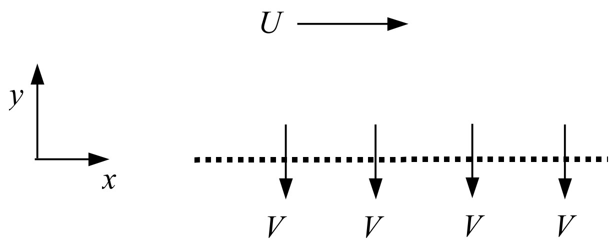
(a) Plot $u_\theta R/\Gamma_0$ against r/R.

(b) Obtain approximate expressions for u_θ in the regions $r^2/R^2 >> 1$ and $r^2/R^2 << 1$. Hence describe the fluid motion in these regions.

(c) Find the vorticity. Is your expression consistent with the flow characterisation you gave for (b)?

(d) The characteristic radius, R, is time-dependent. Write $R^2 = S$ and use the vorticity equation to obtain an expression for dS/dt. Hence find how R changes with time. What physical mechanism is responsible for this behaviour?

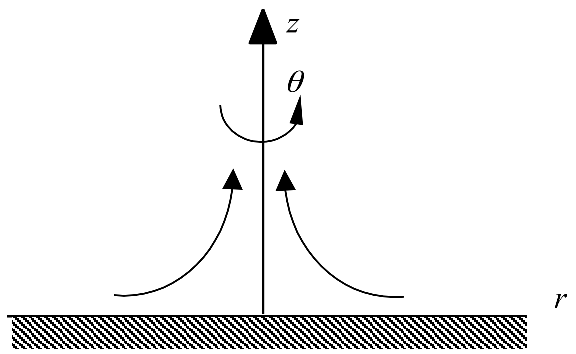
30. A viscous fluid flows over a fixed plane boundary through which there is a constant suction velocity, V. Far away from the surface $\mathbf{u}$ tends to $(U, -V, 0)$ and $\boldsymbol{\omega}$ decays to zero.



Porous surface

Show that there is a steady solution in which $\mathbf{u} = (u(y), -V, 0)$ and $\boldsymbol{\omega} = (0, 0, \omega(y))$, and find expressions for $u(y)$, $\omega(y)$.

31. A tornado is to be modelled as an axisymmetric velocity field (u_r, u_θ, u_z) in the cylindrical polar co-ordinate system (r, θ, z). We seek a solution of the form $u_r = -\frac{1}{2}\alpha r$, $u_z = \alpha z$. In addition, it is required to have unidirectional vorticity: $\boldsymbol{\omega} = (0, 0, \omega_z)$.



(a) Check that the proposed flow is incompressible.

(b) Show that the restrictions placed on the solution imply that u_θ, and hence also ω_z, can depend only on r and t.

(c) Now show that, for this flow, the vorticity transport equation becomes

$$\frac{\partial \omega_z}{\partial t} = \frac{1}{2}\alpha r \frac{\partial \omega_z}{\partial r} + \alpha \omega_z + \nu \left(\frac{\partial^2 \omega_z}{\partial r^2} + \frac{1}{r}\frac{\partial \omega_z}{\partial r} \right).$$

(d) Confirm that a steady solution to the equation is $\omega_z = \omega_0 e^{-\alpha r^2/(4\nu)}$, for some constant ω_0. Hence:

(i) find the swirl component of velocity, u_θ;

(ii) sketch the u_θ and ω_z variations; and

(iii) indicate the length scale of the vortical region.

Answers

1 (b) $0.610\ U^2/a$ at $x = -1.205a$ (c) 15.7×10^3m/s^2

2 (a) $\frac{1}{2}A\left(x^2 - y^2\right)$

3 (a) $Ar^{6/5}\sin\left(6\theta/5\right)$ (b) $p = p_0 - \dfrac{18}{25}\rho A^2 s^{2/5}$, where p_0 is the stagnation pressure.

4 (a) 0.382m downstream of the slit (b) $y - 0.382\theta = 0$ (c) 1.2m

5 (b) –0.065

6 $Uze^{-i\alpha}$

7 (b) 1.6×10^6 m^2/s, 3.45×10^6 m^2/s.

8 (a) $\Gamma/4\pi a$ (c) 1.21

9 (a) $\alpha > 1$ (b) $p = p_0 - \frac{1}{2}\rho A^2\alpha^2 r^{2(\alpha-1)}$
(c) (i) $\pi/(2\beta)$ (ii) $A^2\alpha^2 r^{2(\alpha-1)} + 2g(h - r\cos\beta) = \text{const}$ (iii) 60°

10 (a) $8a/3$ (b) $1.11U$

11 $\rho U\left(-\Gamma\right)$, perpendicular to flow.

12 $\left(1 - \cos 2\alpha\right)/2$

14 (a) $u = U\left(1 + ka\sin(kx)e^{-ky}\right)$, $v = Uka\cos(kx)e^{-ky}$ (b) $2ka\rho U^2$

15 3.9°

16 (a) $F = Uz - \dfrac{m}{2\pi}\left[\ln\left(z - ih\right) + \ln\left(z + ih\right)\right]$
(b) (i) $\left(\alpha \mp (\alpha^2 - h^2)^{\frac{1}{2}}, 0\right)$ for $\alpha \geq h$, $\left(\alpha, (h^2 - \alpha^2)^{\frac{1}{2}}\right)$ for $\alpha \leq h$, where $\alpha = m/2\pi U$.
(b) (ii) $\tilde{F} = \tilde{z} - \dfrac{m}{2\pi U h}\ln\left(\tilde{z}^2 + 1\right)$ (c) 0.27m

17 (a) $-\dfrac{m}{\pi}\left[\dfrac{x - 2a}{\left(x - 2a\right)^2 + a^2} + \dfrac{x + 2a}{\left(x + 2a\right)^2 + a^2}\right]$, $\sqrt{3}a$ (c) Between $1.1a$ and $2.9a$.

18 (a) $\Gamma/4\pi a$

19 (a) $\mp m/8a$ (b) $4aU_\infty$, $\frac{3}{2}U_\infty$ (c) $0.44a$ upstream of the source.

21 (a) $u=\dfrac{\Gamma}{2\pi}\left[\dfrac{\sin\theta}{r-a^2/r}+\dfrac{1}{2r\sin\theta}-\dfrac{\left(r+a^2/r\right)\sin\theta}{\left(r-a^2/r\right)^2\cos^2\theta+\left(r+a^2/r\right)^2\sin^2\theta}\right]$

$$v=\pm\frac{\Gamma}{2\pi}\left[-\frac{\cos\theta}{r-a^2/r}+\frac{\left(r-a^2/r\right)\cos\theta}{\left(r-a^2/r\right)^2\cos^2\theta+\left(r+a^2/r\right)^2\sin^2\theta}\right]$$

22 (a) $(m/4\pi U)^{\frac{1}{2}}$ (b) $(4m/\pi U)^{\frac{1}{2}}$ (c) $0.87D$

23 $\frac{9}{16}\pi\rho U^2a^2$

24 (a) $U_{20}+\dfrac{U_1}{h_1}(h_2-y)$ (b) $\dfrac{U_1}{2}\left(\dfrac{h_1}{h_2}-\dfrac{h_2}{h_1}\right)$

25 (b) (i) $(0,0,2\Omega e^{\alpha t})$ (ii) $le^{\alpha t}$

26 (a) (i) $U\left(\dfrac{5}{16}+\dfrac{y}{4h}\right)$ (ii) $\dfrac{231}{512}\rho U^2$ (iii) $y=1.16h$ (b) 2

27 (b) $\dot{m}=-\dfrac{dp}{dz}\dfrac{\pi}{4\nu}\dfrac{a^3b^3}{a^2+b^2}$

28 (a) (i) $\dfrac{\partial u}{\partial t}=\nu\dfrac{\partial^2 u}{\partial y^2}$ (ii) $-\left(\dfrac{\omega}{2\nu}\right)^{1/2}(1+i)$ (iii) $2U_0\exp\left[-\left(\dfrac{\omega}{2\nu}\right)^{1/2}y\right]$

(b) $\rho U_0\left(\dfrac{\omega\nu}{2}\right)^{1/2}(\sin\omega t-\cos\omega t)$, or (equivalently) $-\rho U_0(\omega\nu)^{1/2}\cos\left(\omega t+\dfrac{\pi}{4}\right)$

29 (b) $\dfrac{\Gamma_0}{2\pi r}$, $\dfrac{1}{2}\dfrac{\Gamma_0}{\pi R^2}r$ (c) $\omega_z\mathbf{e}_z$, with $\omega_z=\dfrac{\Gamma_0}{\pi R^2}e^{-r^2/R^2}$ (d) $\dfrac{dS}{dt}=4\nu$, $R=\sqrt{R_0^2+4\nu t}$

30 $u(y)=U\left(1-e^{-Vy/\nu}\right)$, $\omega(y)=-(UV/\nu)e^{-Vy/\nu}$

31 (d) (i) $(\omega_0 2\nu/\alpha r)\left(1-e^{-\alpha r^2/4\nu}\right)$

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