Forecasting

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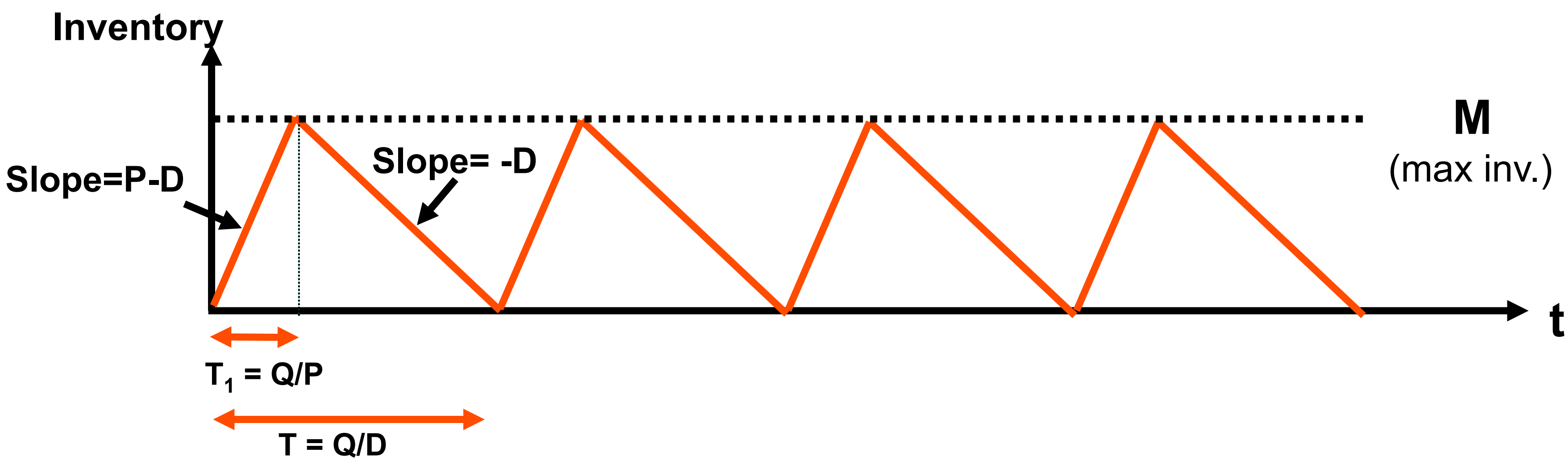
Recap of Last Lecture

- Understand different types of inventory
- Understand basic inventory dynamics
- Understand and calculate Little's Law
- Application of different inventory models to minimise holding and setup costs (EOQ/EPQ/POQ/LUC)
- Appreciate limitations of EOQ/EPQ models in reality

Economic Production Quantity (EPQ)

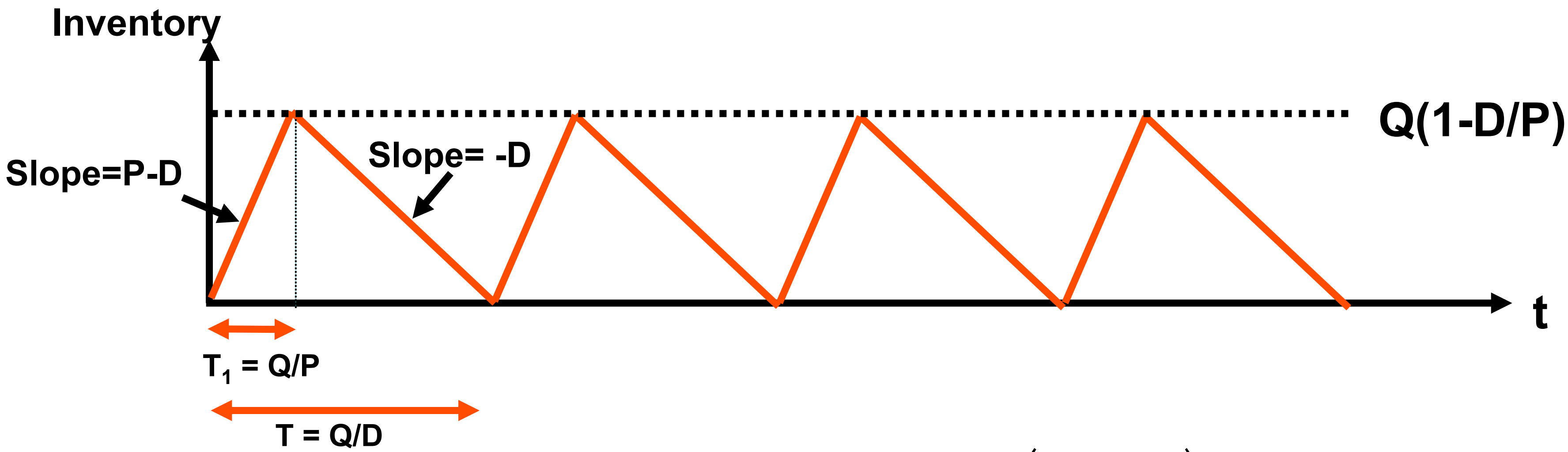
Assume a constant production rate of P>D for each batch



$$\text{Max Inv.} = M = \frac{Q}{P} * (P - D) = Q * \left(1 - \frac{D}{P}\right)$$

Economic Production Quantity (EPQ)

Assume a constant production rate of P>D for each batch

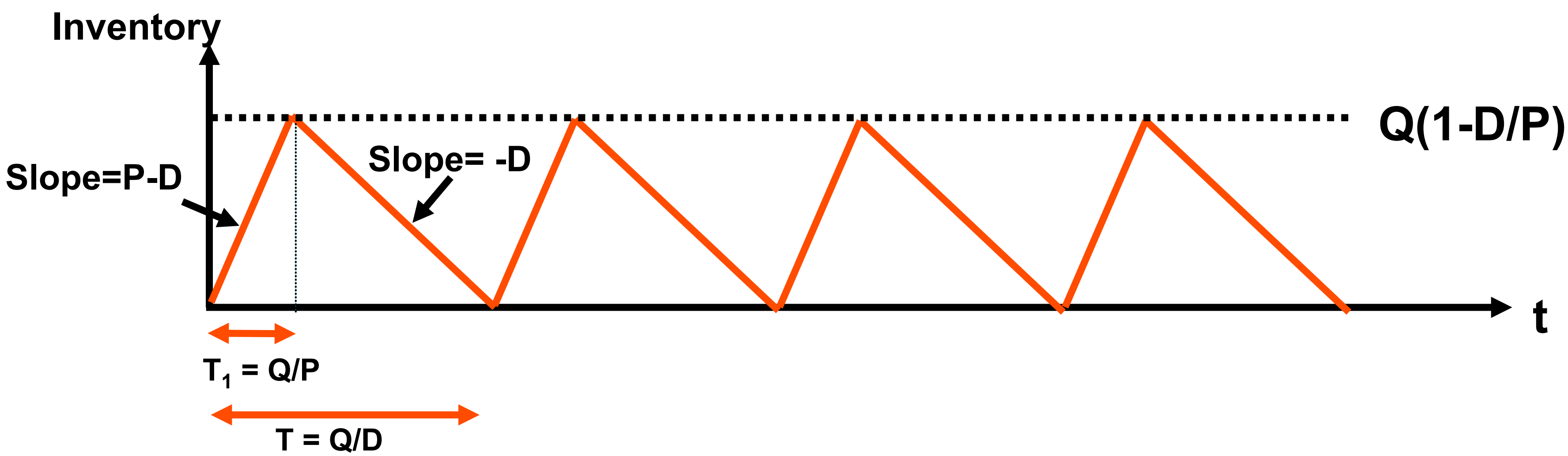


$$\text{Avg. Inv} = \frac{M}{2} = \frac{Q}{2}\left(1-\frac{D}{P}\right)$$

$$TC(Q) = \frac{M}{2}C_H + \frac{D}{Q}C_S = \frac{Q}{2}\left(1-\frac{D}{P}\right)C_H + \frac{D}{Q}C_S$$

Economic Production Quantity (EPQ)

Assume a constant production rate of P>D for each batch

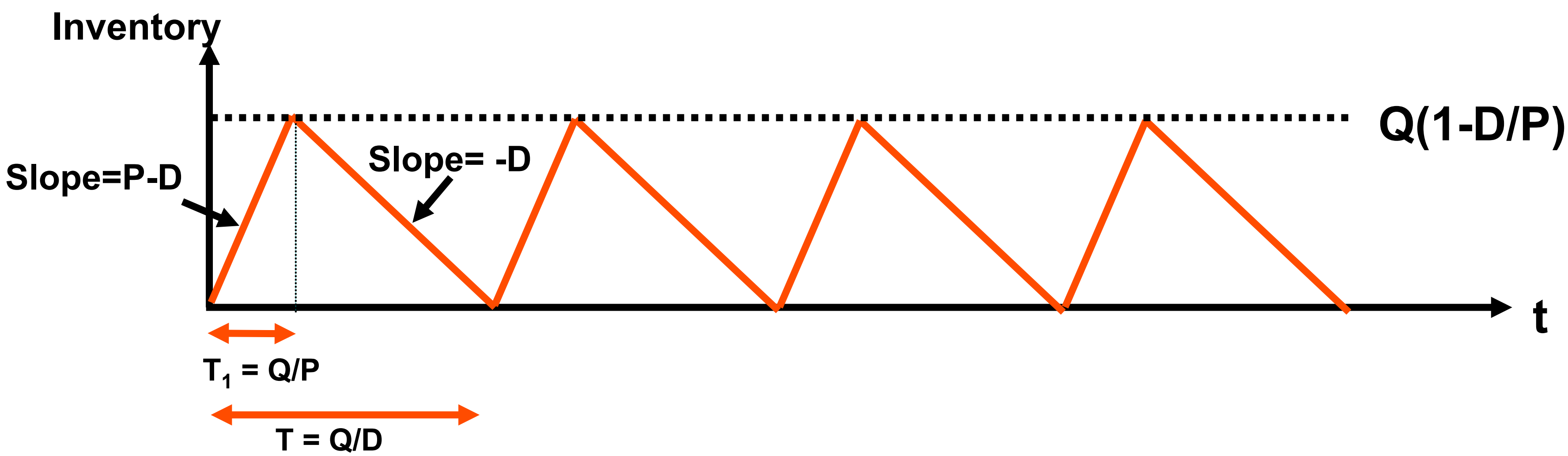


Analysis is as for EOQ, except C_H is replaced by $C_H(1 – D/P)$ reflecting lower average stock held over the period

$$EPQ = \sqrt{\frac{2DC_S}{C_H(1-D/P)}}$$

Economic Production Quantity (EPQ)

Assume a constant production rate of P>D for each batch



$$\frac{T_1}{T} = \frac{Q/P}{Q/D} = \frac{D}{P}$$

Objectives for Today

- Understand different **order fulfilment strategies**
- Understand the differences between **independent and dependent demand** for goods
- Appreciate the need for **forecasting** of independent demand
- Understand **strengths** and **limitations** of different forecasting methods

Order Fulfilment Strategies

Conflicting Objectives

Conflicting objectives:

- 100% utilisation of resources
- Zero stock
- Any volume and product mix
- No lead time

Supply

Demand

This requires coordination within and between firms aiming at:

- Balanced flow of work - all tasks take the same time
- Steady demand for products - and hence for inputs

Ideal demand is smooth and predictable:

- Total demand = maximum output capacity of resources
- Any changes are perfectly forecast in sufficient time to allow capacity change

But …

- Real demand is usually not predictable
- Demand has peaks - lunchtime/Saturday/summer/etc.
- Demand varies through product life cycle and competition

How to Deal with Customer Orders

Order fulfilment process

- From “order to delivery” (OTD)
- Key process, day-to-day challenge!
- Determines customer service and majority of cost

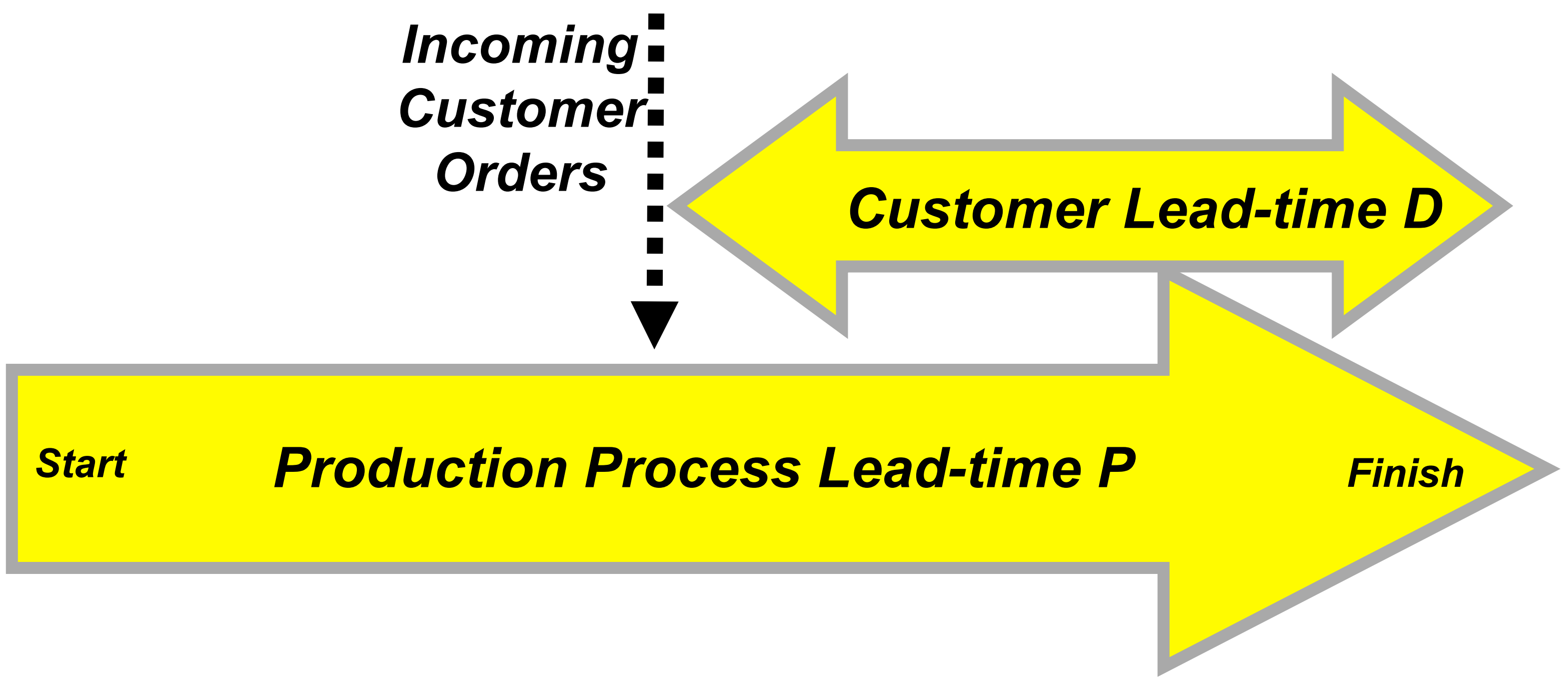
Order fulfilment strategy

- How far back does the order go into the supply chain?
- Which is appropriate under which circumstances?

Two basic factors are important

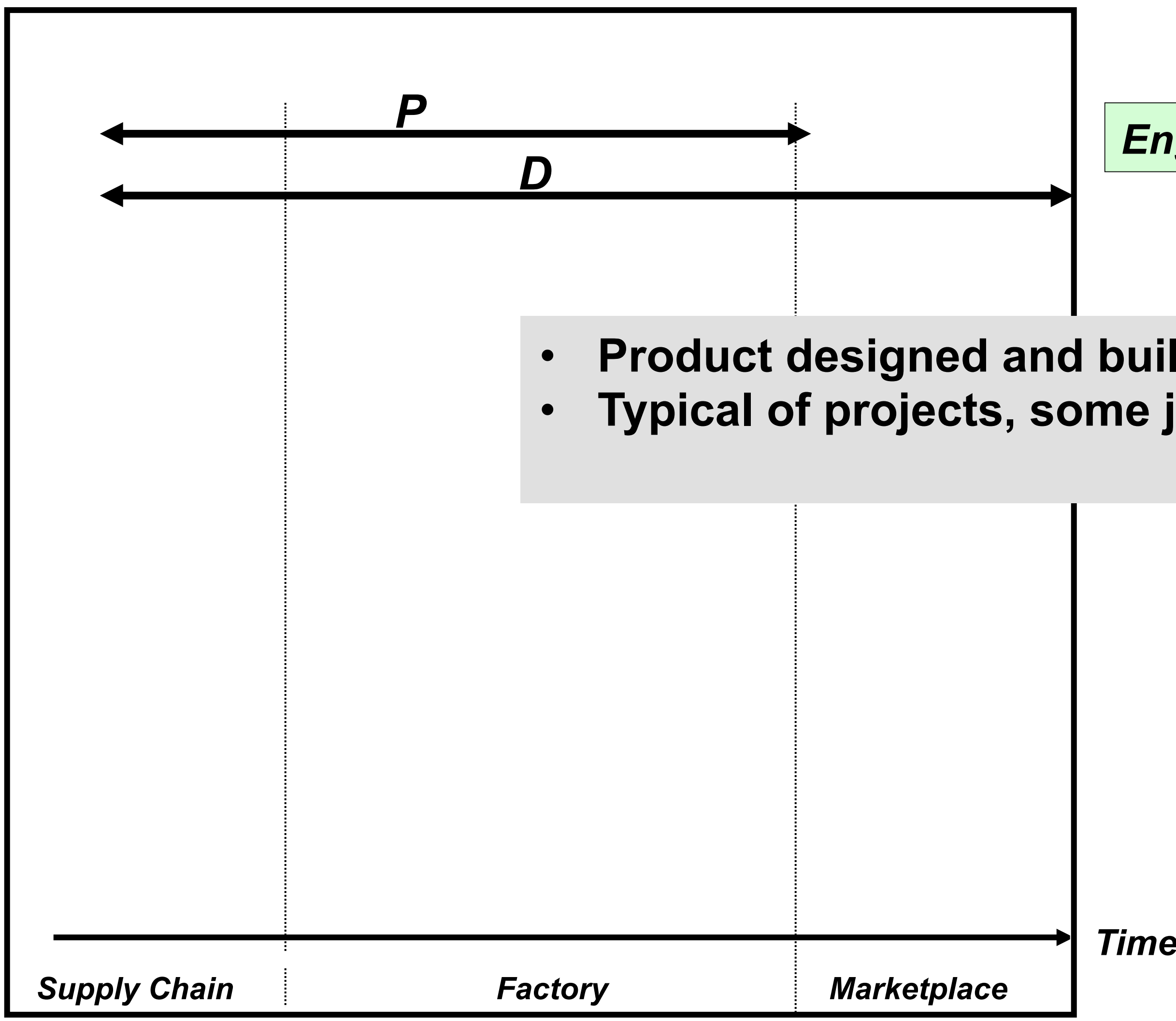
- **Production Lead-time P**: How long does it take to make the product?
- **Demand Lead-time D**: How long is the customer willing to wait for the product?

The P:D Ratio



Source: Hal Mather, *Competitive Manufacturing*, Prentice Hall, 1988

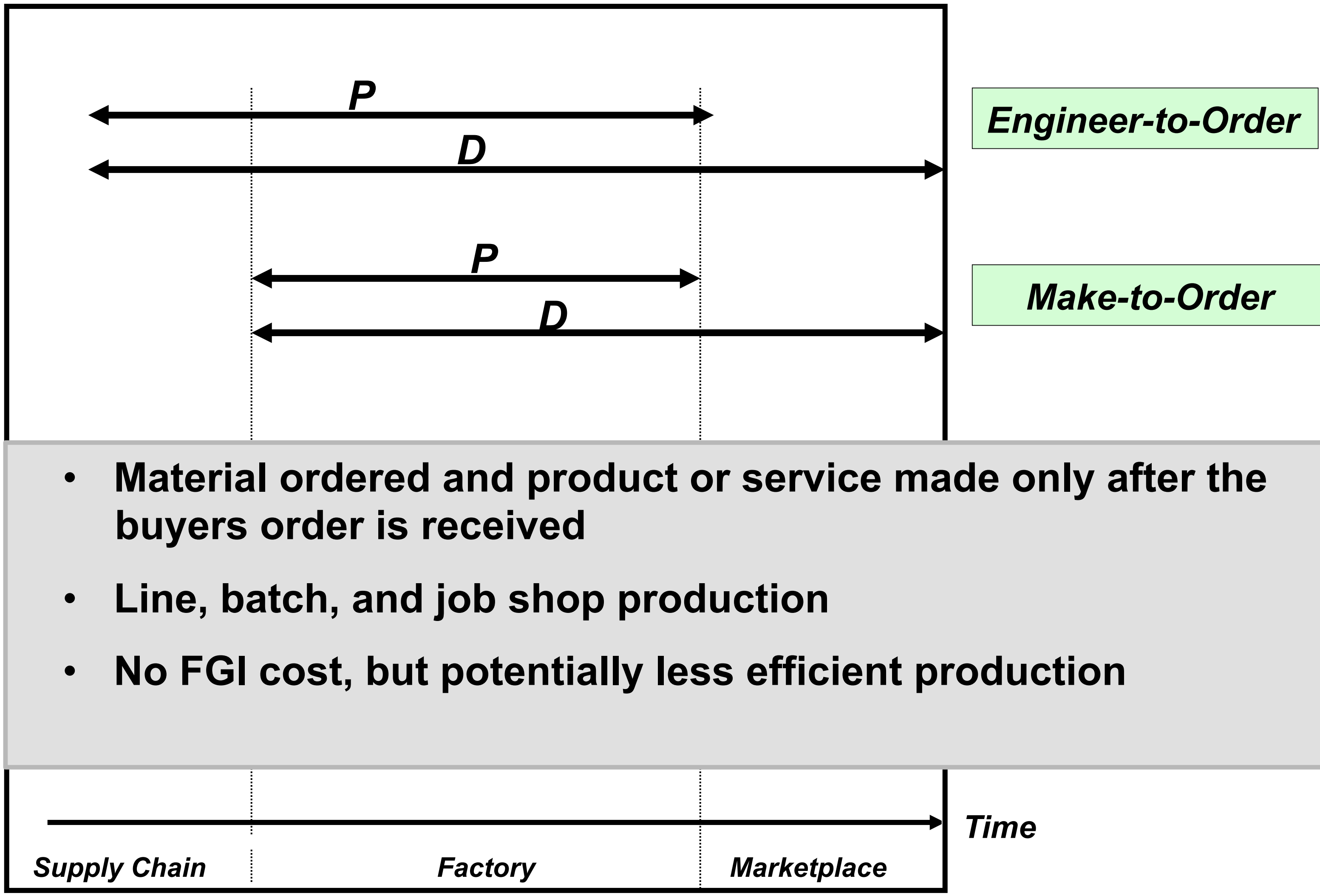
The P:D Ratio



Engineer-to-Order

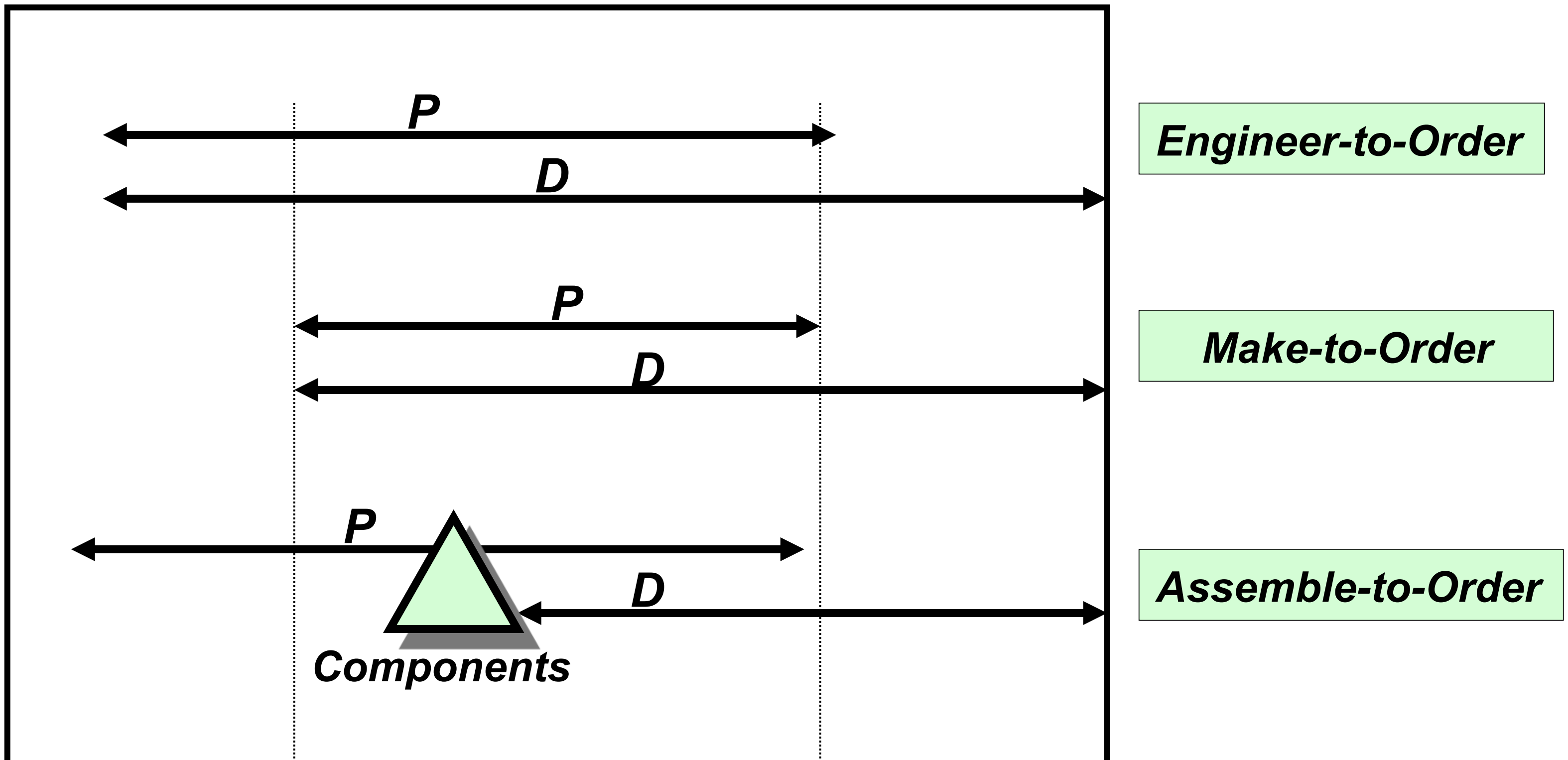
- **Product designed and built to customer order**
- **Typical of projects, some job shop**

The P:D Ratio



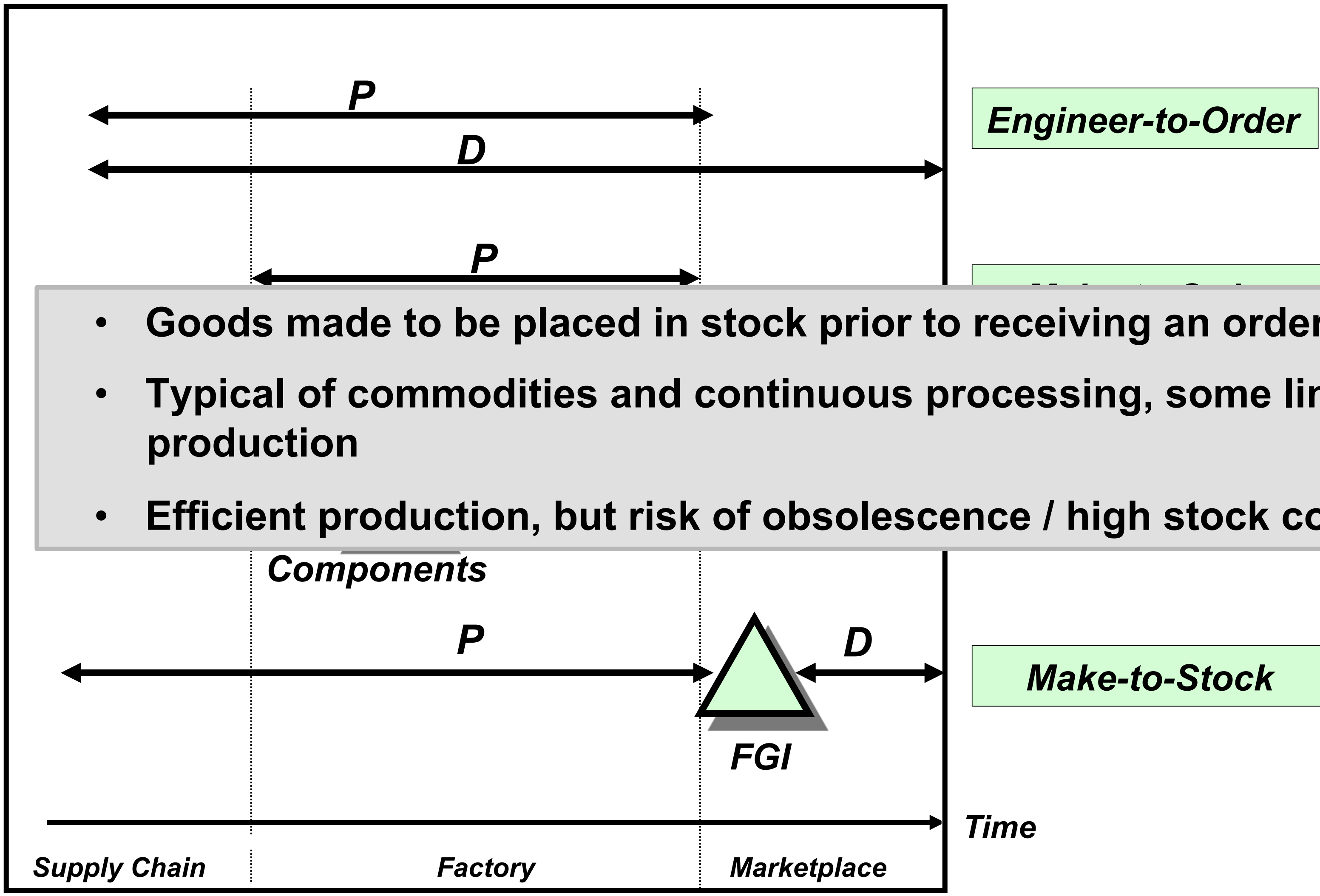
- **Material ordered and product or service made only after the buyers order is received**
- **Line, batch, and job shop production**
- **No FGI cost, but potentially less efficient production**

The P:D Ratio



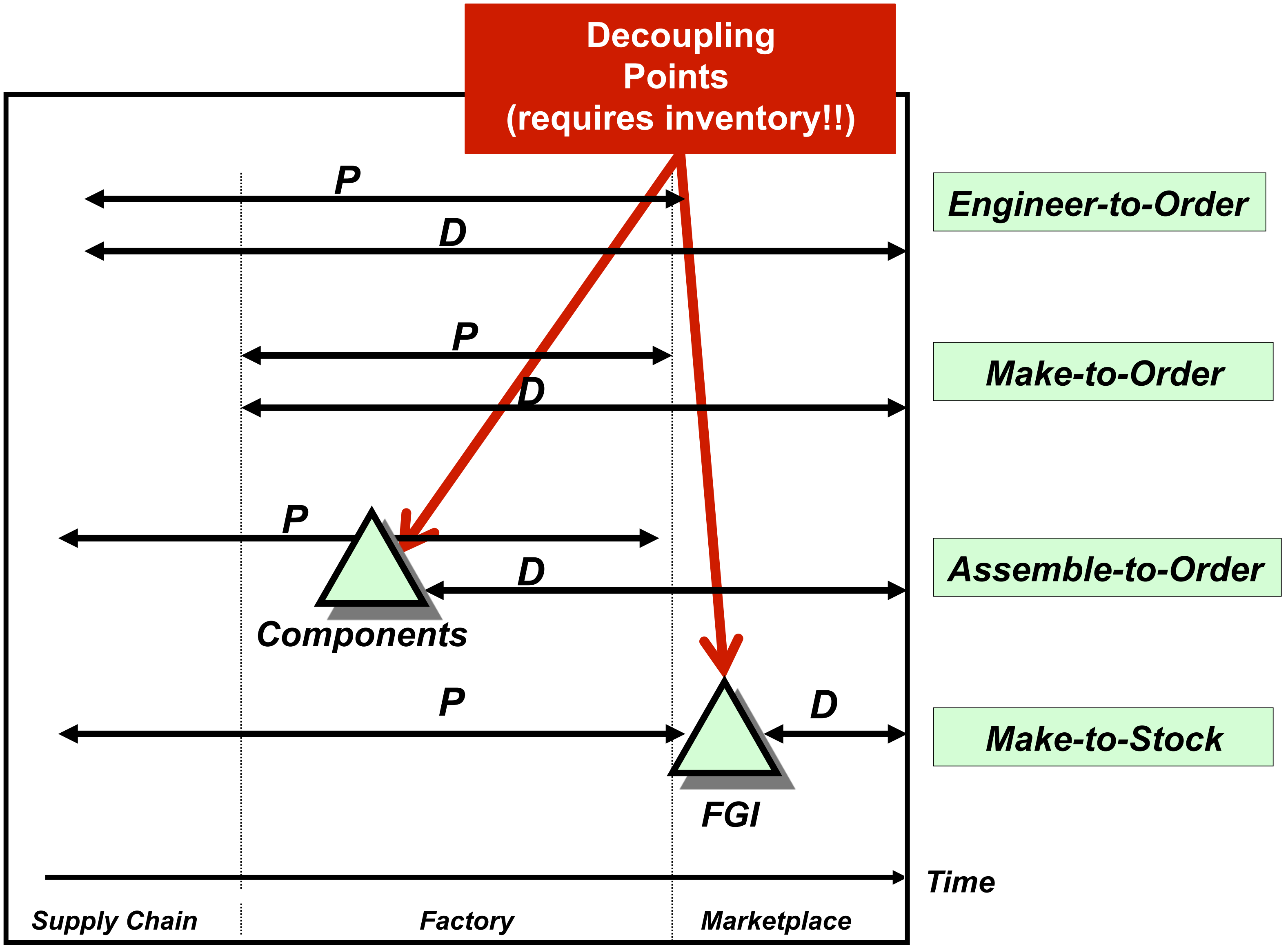
- Producers hold components stock to assemble an order as required by the customer
- Typical of line/ batch production
- Responsive for customised products, but still cost of stock

The P:D Ratio



- Goods made to be placed in stock prior to receiving an order
- Typical of commodities and continuous processing, some line production
- Efficient production, but risk of obsolescence / high stock cost

The P:D Ratio


Decoupling Points (requires inventory!!)
P
D
P
D
P
D
Components
P
D
FGI
Engineer-to-Order
Make-to-Order
Assemble-to-Order
Make-to-Stock
Time
Supply Chain
Factory
Marketplace

MTS, ATO, MTO, ETO

Make-to-Stock (MTS/BTS) / Make-to-Forecast (MTF/BTF)

- Goods made to be placed in stock prior to receiving an order; typical of commodities and continuous processing, some line production
- Efficient production, but risk of obsolescence / high stock cost

Assemble-to-Order (ATO)

- Producers hold components stock to assemble an order as required by the customer; typical of line/ batch production
- Responsive for customised products, but still cost of stock

Make-to-Order / Build-to-Order (MTO/BTO)

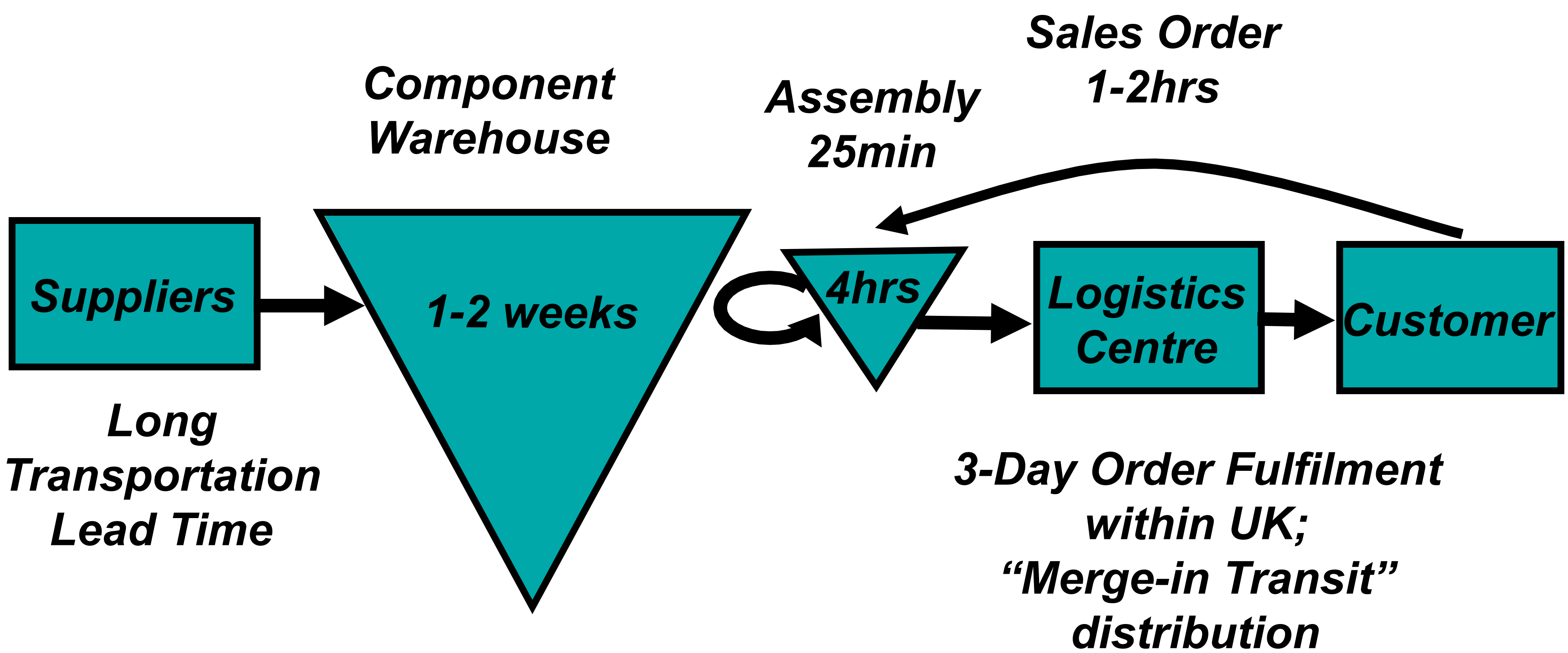
- Material ordered and product or service made only after the buyers order is received
- Line, batch and job shop production
- No FGI cost, but potentially less efficient production

Engineer-to-Order (ETO)

- Product designed and built to customer order; typical of projects, some job shop

MTS, ATO, MTO, ETO?

Dell's "Build-To-Order" Process



Mass Customization

… an oxymoron?

- The need to “customize” mass-produced goods to customer needs
- An umbrella concept
  - Build-to-Order
  - Assemble-to-Order
  - Late configuration/postponement/delayed differentiation
  - Customisation at point of use
  - Customisation through service

Effective Mass Customization

- Modular product design
- Modular process design
- Agile supply networks

Approaches to mass customization

- Collaborative
- Adaptive
- Cosmetic
- Transparent

External Product Variety: Cars

Model	Bodies	Power trains	Paint/ Trim Combinations	No. of Factory-fitted Options	Total No of Variances	European Sales in 2002 [units]
Peugeot 206	3	8	70	5	1,739	596,531
VW Golf	3	16	221	26	1,999,813,504	595,465
Ford Focus	4	11	64	19	366,901,933	523,356
Renault Clio	2	10	57	9	81,588	502,497
Peugeot 307	4	8	70	9	41,590	441,468
GM Astra	4	11	83	14	27,088,176	440,567
GM Corsa	2	9	77	17	36,690,436	420,296
Fiat Punto	2	5	51	8	39,364	416,843
VW Polo	2	9	195	27	52,612,300,800	357,539
BMW 3-Series	3	18	280	45	64,081,043,660,000,000	350,723
Ford Fiesta	2	5	57	13	1,190,784	294,360
Renault Megane	2	6	52	14	3,451,968	261,383
Mercedes C-Class	2	16	312	59	1,131,454,740,000,000,000,000	254,836
Toyota Yaris	2	6	30	8	34,320	194,256
Fiat Stilo	3	7	93	25	10,854,698,500	173,453
Mercedes E-Class	2	15	285	70	3,347,807,348,000,000,000,000,000	157,584
Toyota Corolla	4	5	24	6	162,752	139,837
Nissan Micra	2	6	30	4	676	106,428
Mini (BMW)	1	5	418	44	50,977,207,350,000,000	105,617

Forecasting

Discussion

- Why do we need to forecast?
- Under which circumstances is the information given by customer orders insufficient?

Why Do We Need to Forecast? Example I

- You are the plant manager at Manufacturing Excellence Ltd., a local producer of metal parts for the automotive industry
- You are currently running at over 100% of your capacity, causing a significant increase in unit costs (e.g., through overtime)
- However, you can still not satisfy all of your customers' orders
- Expanding your production capacity would take 6 months and cost about £2 million
- Should you make the investment?

Why Do We Need to Forecast? Example II

- You are the store manager at one of AllFoodYouNeed Plc.'s stores
- Recently, you have noticed that you lose money on selling freshly squeezed orange juice due to a high percentage of unsold juice that cannot be kept overnight
- You have made the following observations over the past two weeks:

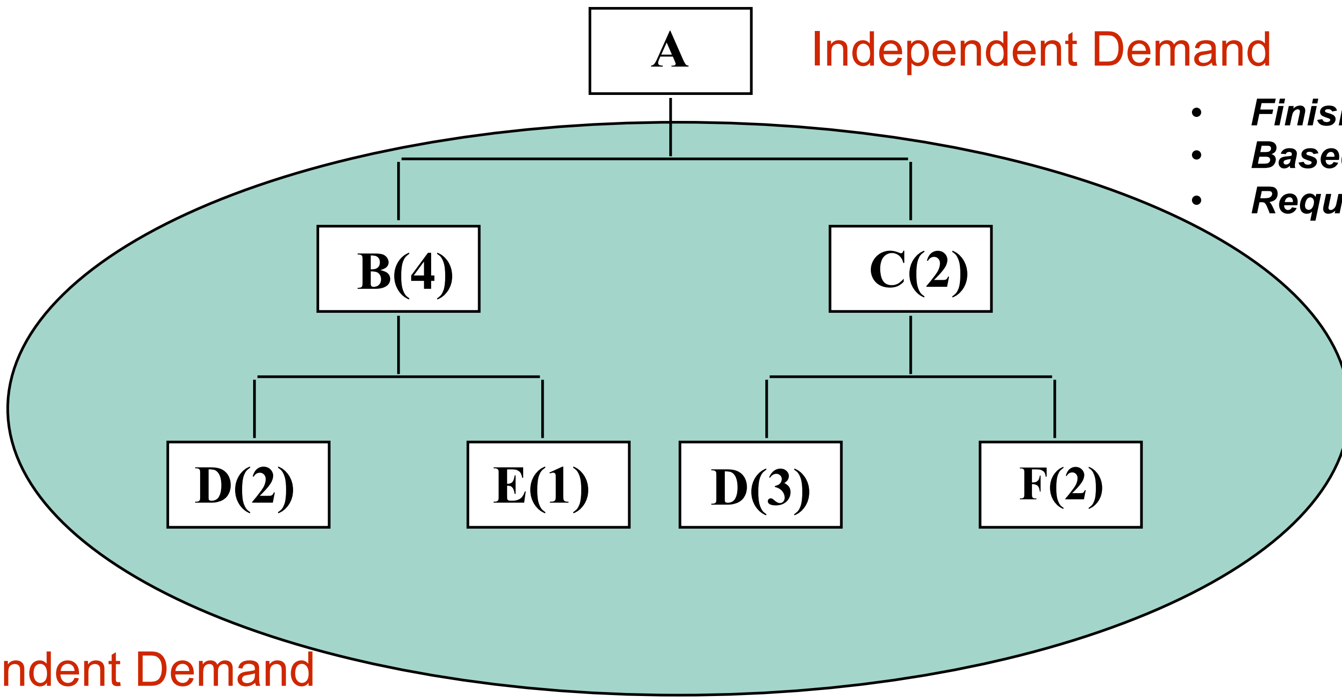
Week	1							2						
Day	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su
Juice produced (litres)	100	100	100	100	100	100	100	100	100	100	100	100	100	100
Juice unsold (litres)	19	27	38	27	18	7	5	17	28	36	29	17	6	4
Demand (litres)	81	73	62	73	82	93	95	83	72	64	71	83	94	96

- What improvements would you suggest?

Why Do We Need to Forecast?

- Forecasts are vitally important to organizations
- They are used to plan facilities, production schedules, staffing allocation, capacity planning, and other things
- The goal of a business forecast is not to have a perfect forecast but to have a reasonable forecast that helps us plan

Two Forms of Demand



Independent Demand

- ***Finished products***
- ***Based on market demand***
- ***Requires forecasting***

Dependent Demand

- ***Parts that go into the finished products***
- ***Dependent demand is a known function of independent demand***
- ***No forecasting is required***

Forecasting Methods: Overview

1. Qualitative (long-term, years)

- Market surveys
- Delphi study: ask the experts... (see, e.g., Rand Corp. 1964: Research report: *"Research Report on a Long-Range Forecasting Study")*
- Problems: bias, ignorance

2. Quantitative (short to medium term, days/months)

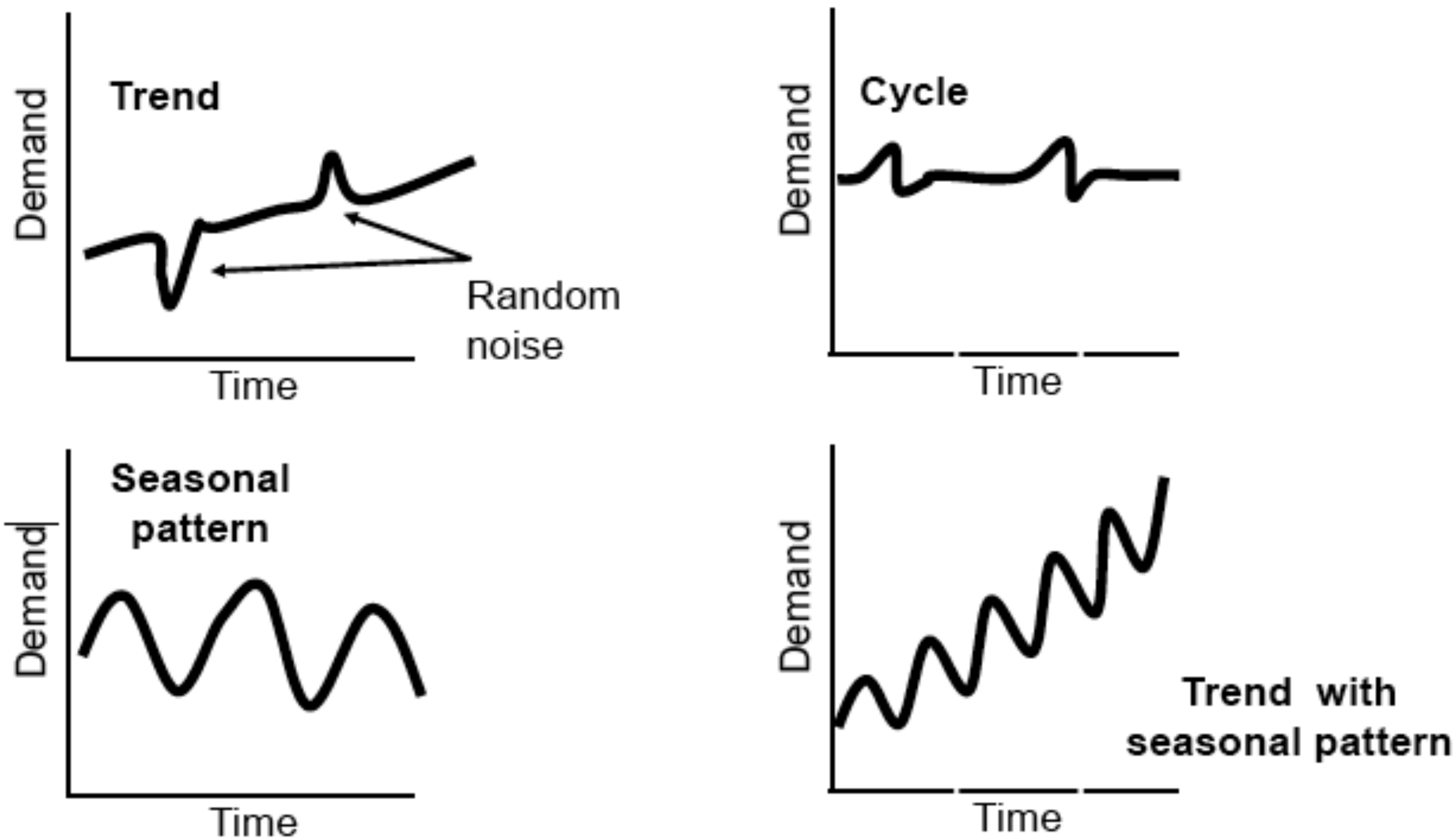
A. Extrinsic (based on external patterns beyond firm level)

- Econometrics models including contextual variables, medium term (1-2 years)
- Problem: will miss unusual events and short term issues

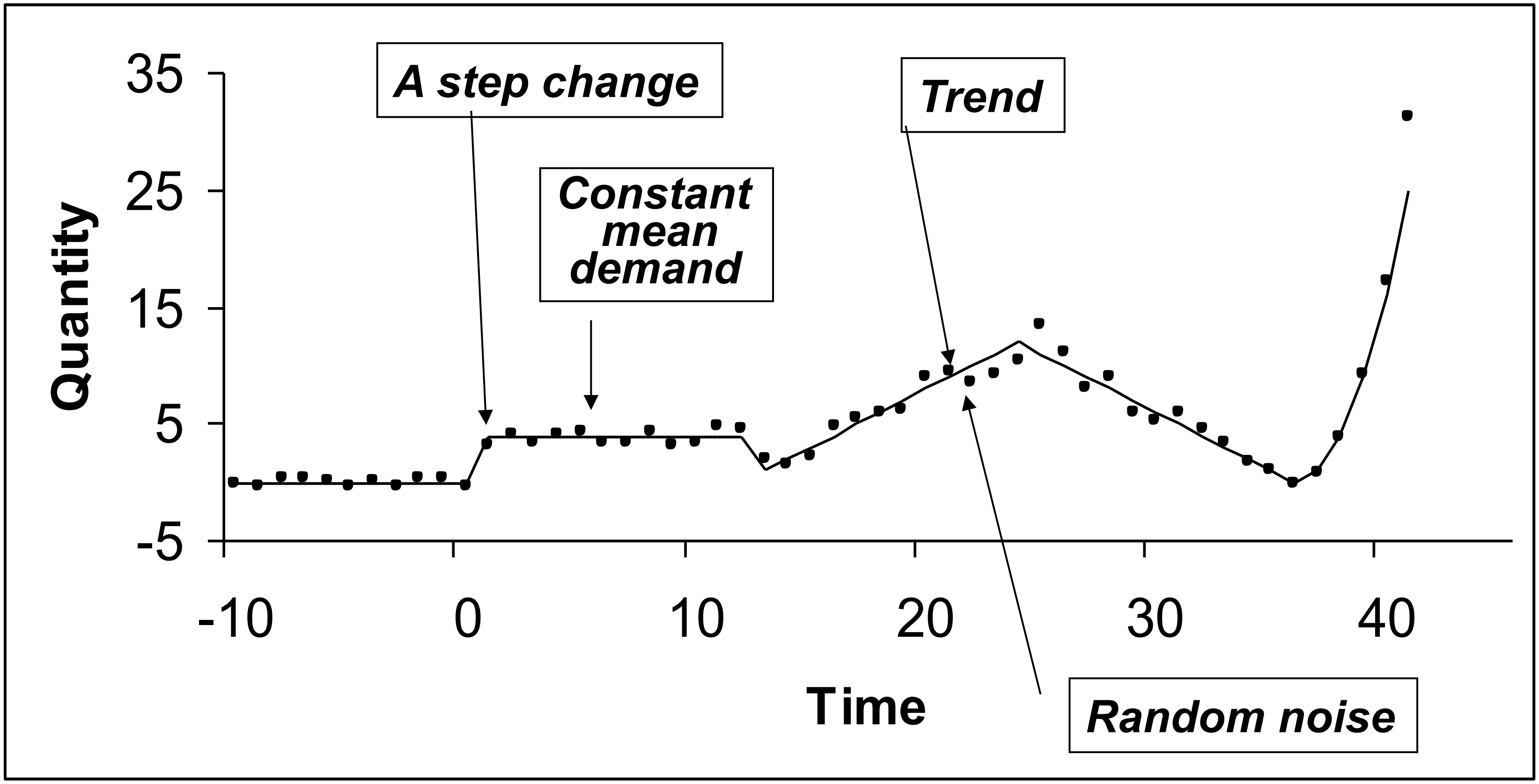
B. Intrinsic (based on patterns of actual data at firm level)

- Short term (up to 12 months)
- Moving average, exponential smoothing (extrapolation methods)
- Regression analysis
- Time series, decomposition analysis
- Problem: almost exclusively based on historical data!

Patterns of Demand Fluctuation



Patterns of Demand Fluctuation



Forecast via Moving Average (MA)

- Most basic approach; we assume mean demand constant
- To forecast demand in next period we average demands in the recent past periods
- A 4-period moving average is the average of the last 4 time periods
- The general formula for moving average is:

$$S_t = (x_t + x_{t-1} + .. + x_{t-n+1}) / n$$

NB: by convention, S_t is an average based on data up to time t, but used as a forecast for time t+1

4-Period MA Forecasting: Exercise

Time	Demand	Forecast
-3	504	
-2	484	
-1	493	
0	423	
1		476
2		
3		
4		
5		
6		

Forecast made in week 0 = our best guess of what we will sell in week 1

$$S_0 = (504 + 484 + 493 + 423) / 4$$

4-Period MA Forecasting: Exercise

Time	Demand	Forecast
-3	504	
-2	484	
-1	493	
0	423	
1	458	476
2		465
3		
4		
5		
6		

Forecast made in week 1 = our best guess of what we will sell in week 2

$$S_1 = (484 + 493 + 423 + 458) / 4$$

4-Period MA Forecasting: Exercise

Time	Demand	Forecast
-3	504	
-2	484	
-1	493	
0	423	
1	458	476
2	440	465
3		454
4		
5		
6		

Forecast made in week 2 = our best guess of what we will sell in week 3

$$S_2 = (493 + 423 + 458 + 440) / 4$$

4-Period MA Forecasting: Exercise

Time	Demand	Forecast
-3	504	
-2	484	
-1	493	
0	423	
1	458	476
2	440	465
3	485	454
4	395	452
5	368	445
6	344	422

Age of Data and Weights

The more data periods are used, the older the data gets:

- 1 period MA: average age of data is 1 period
- 2 period MA: average age is (1+2)/2 = 1.5 periods
- 3 period MA: average age is (1+2+3)/3 = 2 periods
- …
- n period MA: average age is (1+2+ … +n)/n=(n+1)/2 periods

- In a MA forecast, each period has a weight of 1/n; i.e., the oldest data has the same weight as the most recent
- Perhaps the more recent past should have greater weight (as it is arguably more relevant information)?

Exponential Smoothing (ES)

- Like a moving average, but weight given to each period is a fixed proportion of weight given to succeeding period

$$S_t = k \cdot \left\{ x_t + (1-\alpha) \cdot x_{t-1} + (1-\alpha)^2 \cdot x_{t-2} + (1-\alpha)^3 \cdot x_{t-3} + \ldots \right\}$$

- $0<\alpha<1$, so weights get smaller as we recede into the past

$$k \quad k(1-\alpha) \quad k(1-\alpha)^2 \quad k(1-\alpha)^3 \quad \ldots$$

- The constant k is chosen so that the weights sum to 1:

$$k + k(1-\alpha) + k(1-\alpha)^2 + k(1-\alpha)^3 + \ldots = 1$$

- And this (it can be shown) requires $k=\alpha$

ES Forecast

- Exponential smoothing forecasts contain information on all previous demands, each demand is given a weight that is decreasing exponentially back in time.
- Smoothing constant: $0 < \alpha < 1$
- The general formula for exponential smoothing is:

$$S_t = \alpha \cdot x_t + \alpha \cdot (1-\alpha) x_{t-1} + \alpha \cdot (1-\alpha)^2 x_{t-2} + \alpha \cdot (1-\alpha)^3 x_{t-3} + \ldots$$

- S_t is based on all (available) data up to period t to forecast x_{t+1}

ES: Updating Formula

Forecast for period (t+1) can be quickly calculated from the forecast for period t:

$$
\begin{aligned}
S_t &= \alpha \cdot x_t + \alpha \cdot (1-\alpha) x_{t-1} + \alpha \cdot (1-\alpha)^2 x_{t-2} + \alpha \cdot (1-\alpha)^3 x_{t-3} + \ldots \\
&= \alpha \cdot x_t + (1-\alpha)\left[\alpha \cdot x_{t-1} + \alpha \cdot (1-\alpha) x_{t-2} + \alpha \cdot (1-\alpha)^2 x_{t-3} + \ldots\right] \\
&= \alpha \cdot x_t + (1-\alpha) S_{t-1}
\end{aligned}
$$

Alternative notations:

$$S_t = S_{t-1} + \alpha \cdot \varepsilon_t \quad \text{where } \varepsilon_t = x_t - S_{t-1}$$

ES Forecasting: Example

Time	Demand	Forecast α=0.1	Forecast α=0.9
0	504		
1	484		
2	493		
3	423		
4	458		
5	440		
6	485		
7	395		
8	368		
9	344		

ES Forecasting: Example

Time	Demand	Forecast α=0.1	Forecast α=0.9	
0	504			
1	484	504	504	
2	493	502	486	
3	423	501	492	
4	458	493	430	476
5	440	490	455	465
6	485	485	442	454
7	395	485	481	452
8	368	476	404	445
9	344	465	372	422

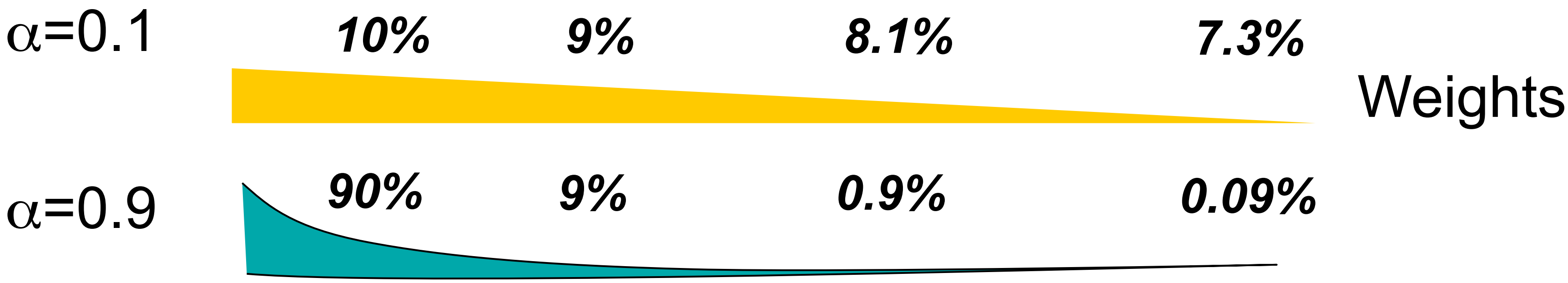
Compare this to the 4-period moving average…

ES: α = ?

Choosing α: depends on how rapidly we want the smoothed value to respond to changes in demand

- Usually α=0.1 to 0.3
- The larger α, the more responsive (nervous) is the forecast
- The smaller α, the less reactive and more stable the forecast

$$S_t = \alpha \cdot x_t + \alpha \cdot (1-\alpha) x_{t-1} + \alpha \cdot (1-\alpha)^2 x_{t-2} + \alpha \cdot (1-\alpha)^3 x_{t-3} + \ldots$$

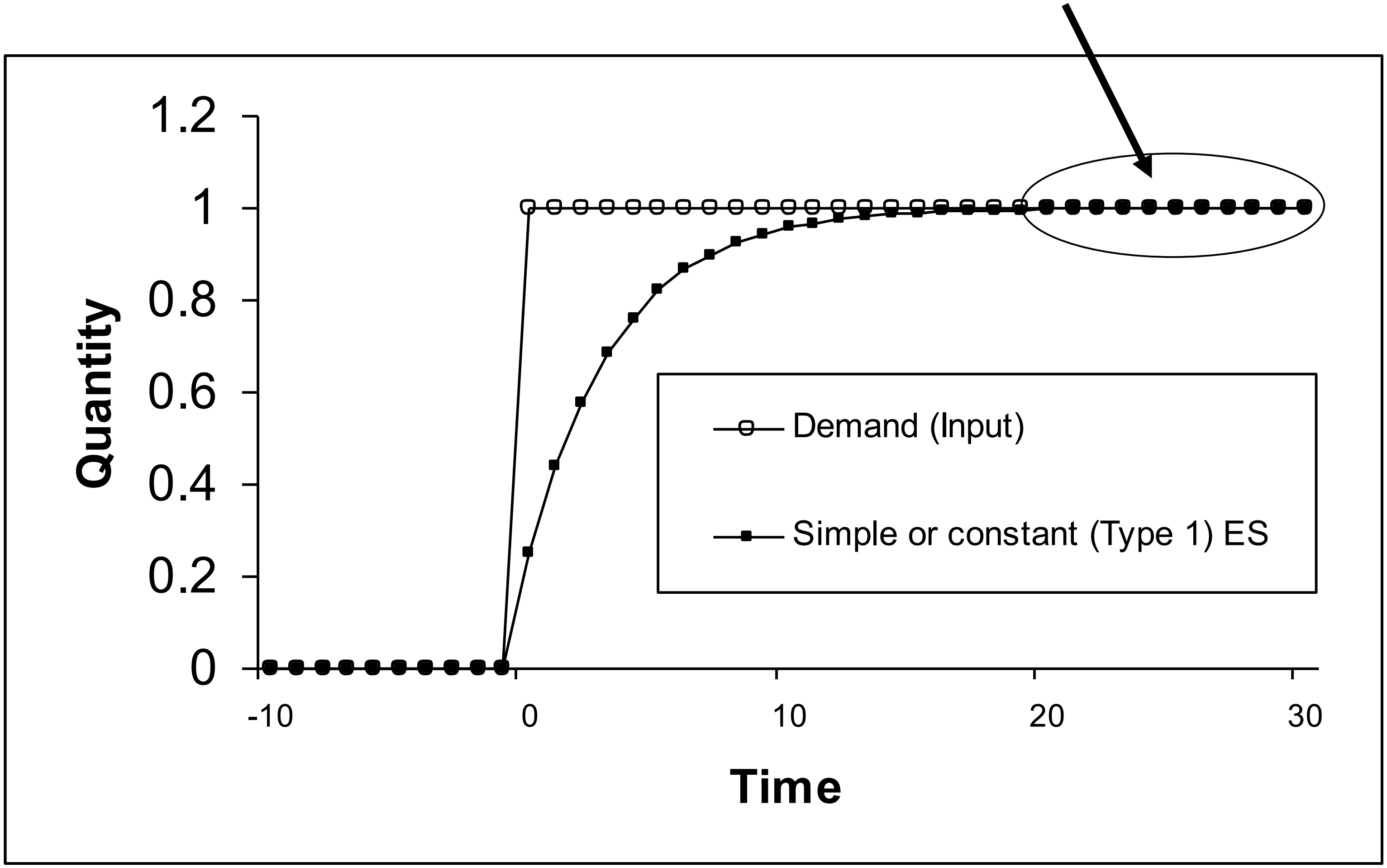


It can be shown that the average age of data = $1/\alpha$

NB For MA average age of data = (n+1)/2

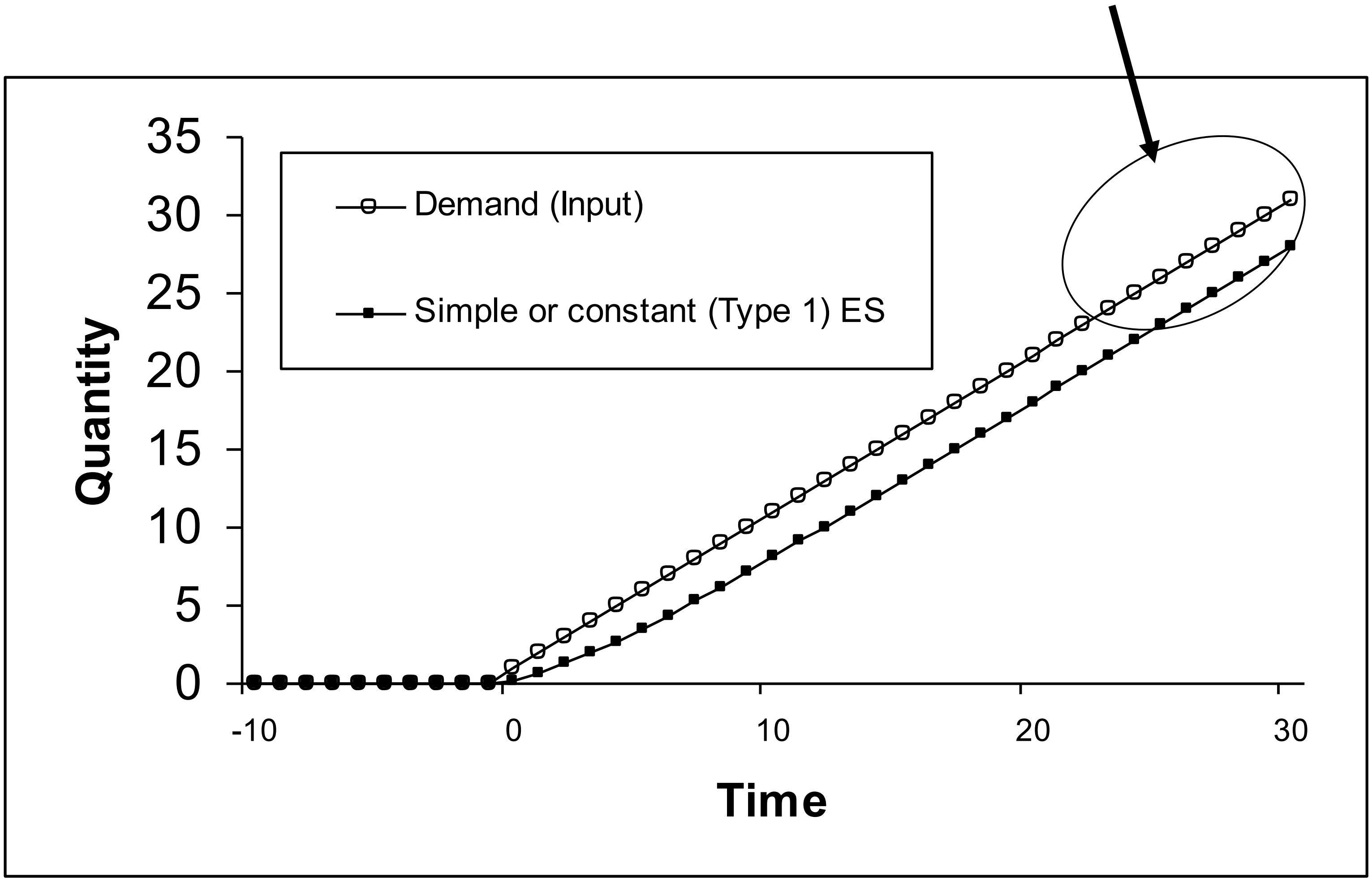
ES: Step Change in Demand

The forecast "locks-on" to the demand

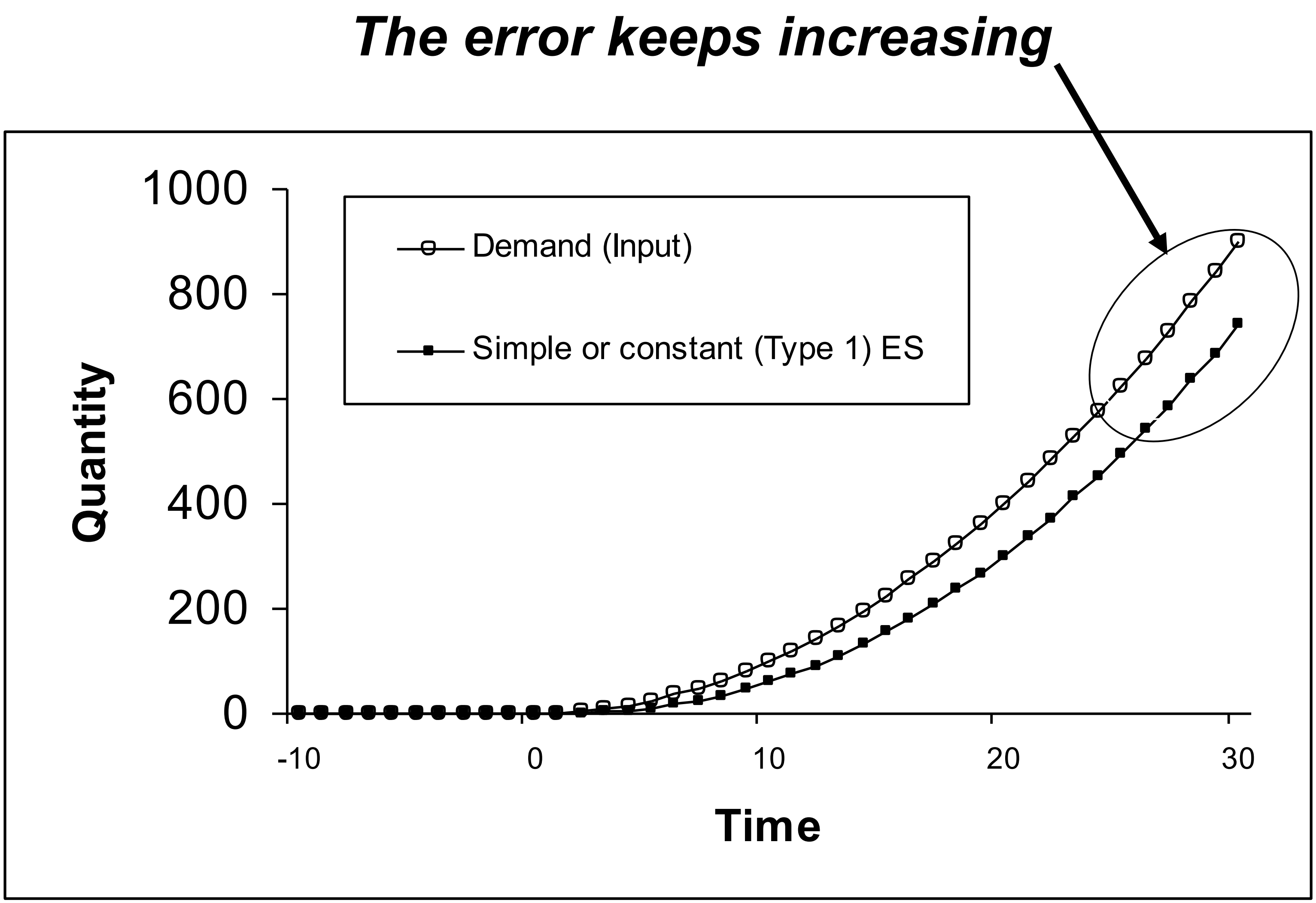


ES: "Ramp" (Linear Trend) Demand

The forecast does not "lock-on"



ES: “Parabolic” Input



Summary of ES

- The method we have seen so far is known as simple ES
- It copes OK with step changes in demand
- It does not cope well with linear trends
- An adaptation of simple ES can cope with linear trends: double ES (sometimes known as Type 2 ES)
- Triple ES is needed for quadratic trends

Double Exponential Smoothing

- Basic idea is to introduce a trend estimate
- Similar to single exponential smoothing but we have two equations
- Need to choose two smoothing rates, α and β
- Also called Holt's Linear Trend Model or Trend Adjusted Exponential Smoothing
- Trend dominates after a few periods in forecasts so forecasts are only good for a short term

Trend adjusted level equation

$$A_t = \alpha x_t + (1-\alpha)(A_{t-1} + T_{t-1})$$

Trend estimate

Trend equation

$$T_t = \beta(A_t - A_{t-1}) + (1-\beta)T_{t-1}$$

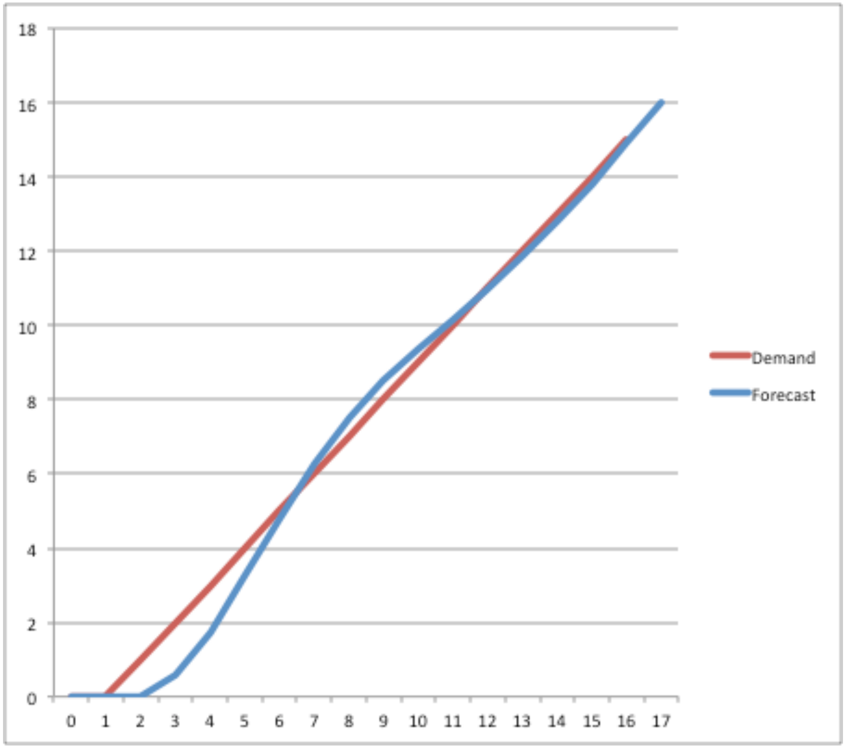
Forecast

$$S_t = A_t + T_t$$

Example of Double ES

ALPHA 0.3 **BETA** 1

t	Demand, x_t	A_t	T_t	S_t
0	0	0	0	0
1	0	0	0	0
2	1	0.3	0.3	0.6
3	2	1.0	0.7	1.7
4	3	2.1	1.1	3.2
5	4	3.5	1.3	4.8
6	5	4.8	1.4	6.2
7	6	6.2	1.3	7.5
8	7	7.3	1.2	8.5
9	8	8.4	1.0	9.4
10	9	9.3	0.9	10.2
11	10	10.1	0.9	11.0
12	11	11.0	0.9	11.8
13	12	11.9	0.9	12.8
14	13	12.9	1.0	13.8
15	14	13.9	1.0	14.9
16	15	14.9	1.1	16.0



Triple Exponential Smoothing

- Basic idea is to introduce a trend estimate and a seasonality estimate
- Similar to double exponential smoothing but we have three equations
- Need to choose three smoothing rates, α, β, and γ
- Also called Winter's Linear and Seasonal ES model
- Forecasts are only good for a short term

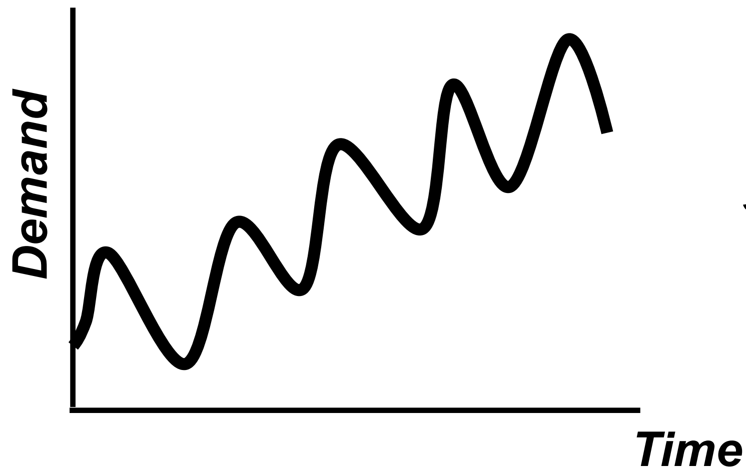
Summary of Extrapolation Forecast Models

- MA and single ES work well for models with constant mean demand (or irregular step changes)
- Double ES works well for models with linear trends
- Triple ES works well for models with quadratic trends (but these are rare in practice)

Further Approaches to Forecasting

- One approach is to extrapolate past data (MA, ES)
  - This works fine for minor variability
  - Problem: response to step changes not immediate, forecast may be late in reacting to trends
- Another approach is to learn about the underlying properties of the demand time series
  - E.g., demand for turkeys goes up every Christmas
- How to analyse the underlying demand pattern:
  - Regression analysis: what factors matter? E.g., weather and sporting events to predict beer consumption
  - Decomposition/Fourier analysis: the idea is that one complex demand time series can be decomposed into a set of simpler series

Decomposition Models



Trend with seasonal pattern

- Additive Model:

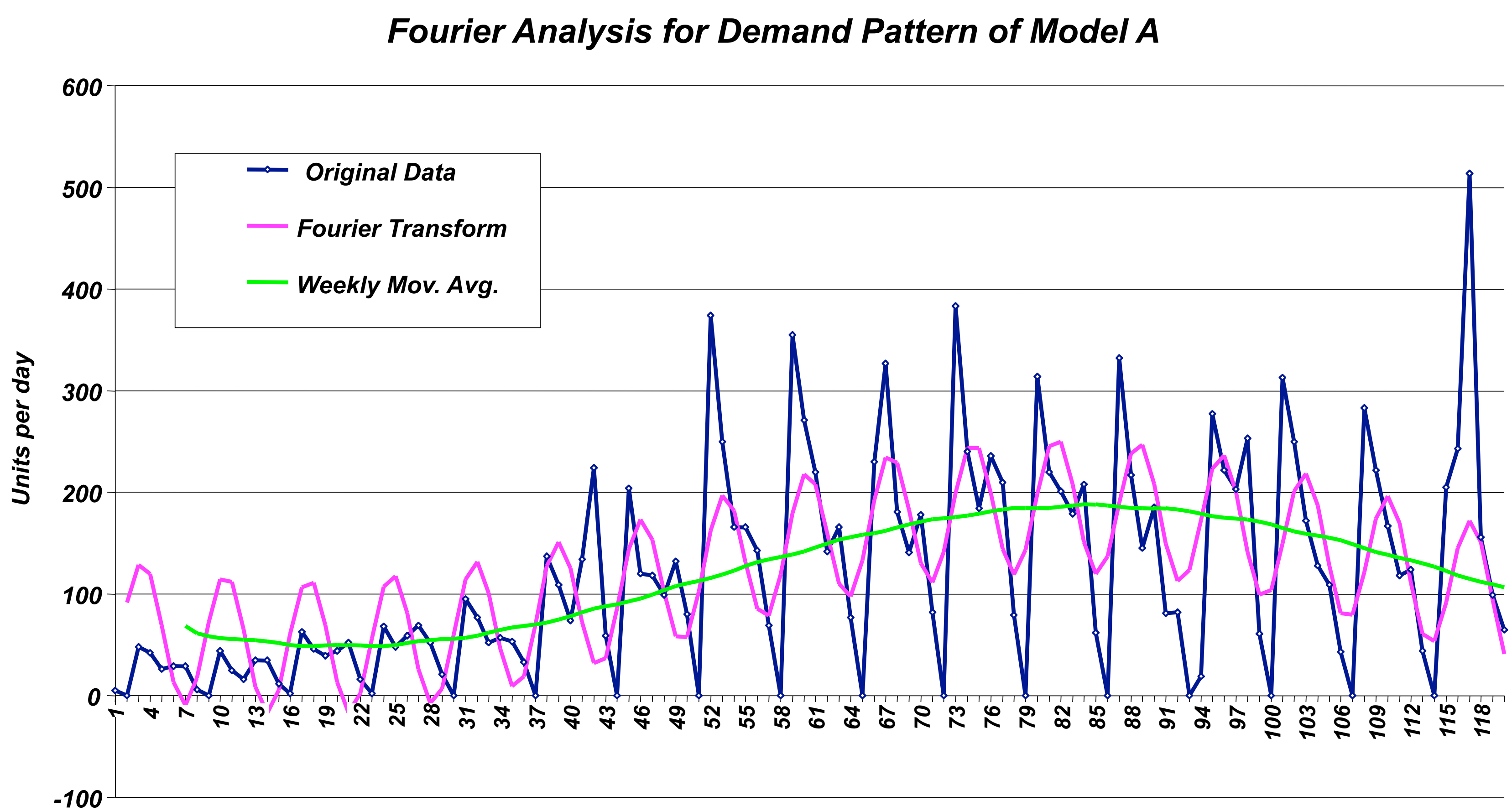
  Demand = (Trend) + (Seasonal) + (Cyclic) + (Randomness)

- Multiplicative Model:

  Demand = (Trend) * (Seasonal) * (Cyclic) * (Randomness)

- Cyclic component is like the seasonal component, but with a longer cycle period: simple models ignore it

Fourier Analysis Example: New Vehicle Orders



Forecast Accuracy

General rules

1. The forecast is always wrong!
2. The longer the forecast horizon, the worse the forecast
3. The less aggregated, the worse the forecast

"The two 'L' s of forecasting: Lucky or Lousy."

Forecast Error

- We define $e_t = x_t - S_{t-1}$ as the forecast error in period t
- Let $e_1, e_2, \ldots, e_n$ be the forecast errors observed over n periods

Forecast Error (cont'd)

- A measure of forecast error over n periods is the **Mean Absolute Deviation (MAD):**

$$MAD = \frac{\sum_{i=1}^{n} |e_i|}{n}$$

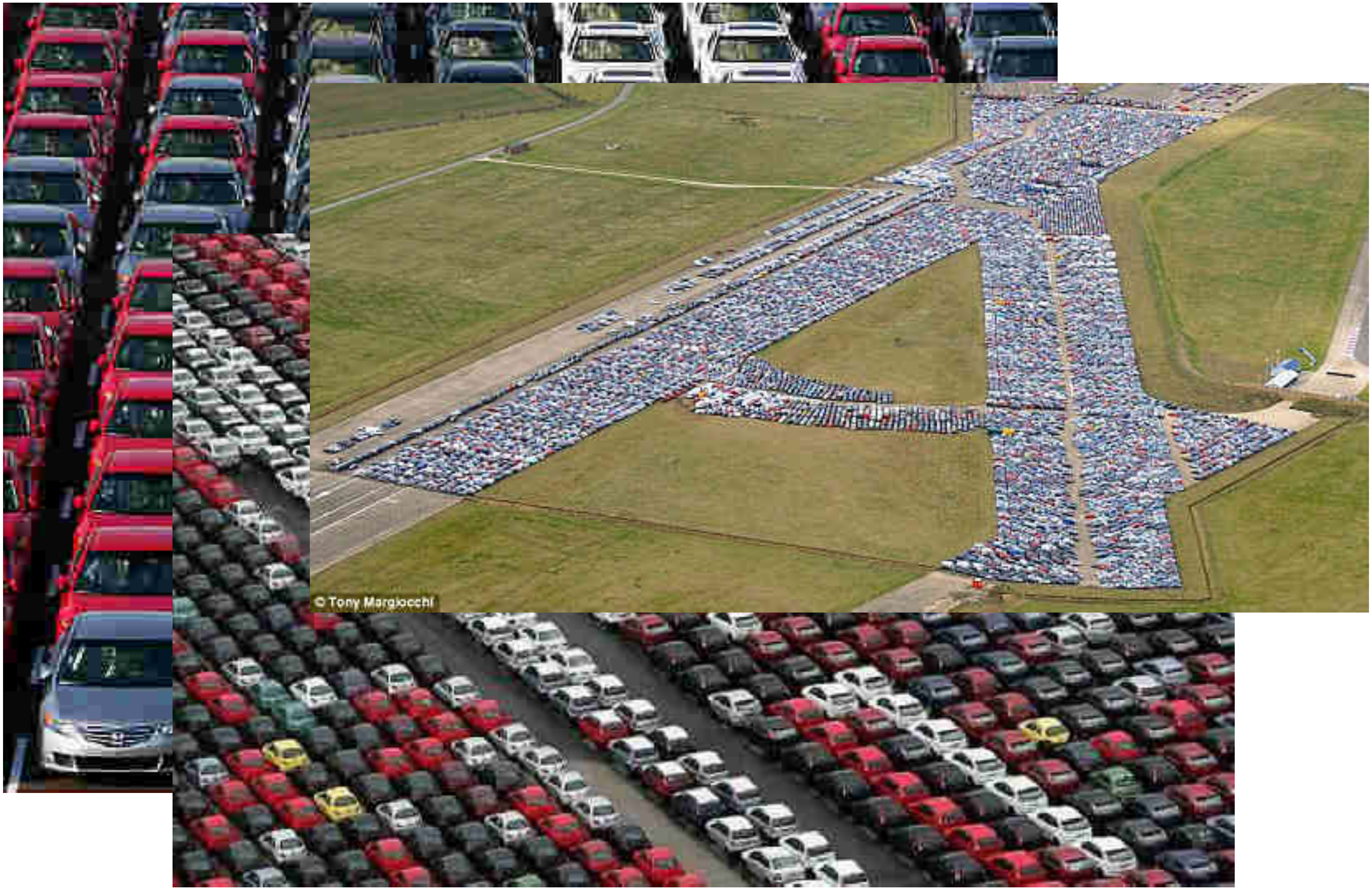
- Another measure of forecast error is the **Mean Squared Error (MSE):**

$$MSE = \frac{\sum_{i=1}^{n} {e_i}^2}{n}$$

- Yet another measure of forecast error is the **Mean Absolute Percentage Error (MAPE):**

$$MAPE = \frac{\sum_{i=1}^{n} {|e_i|}/{x_i}}{n}$$

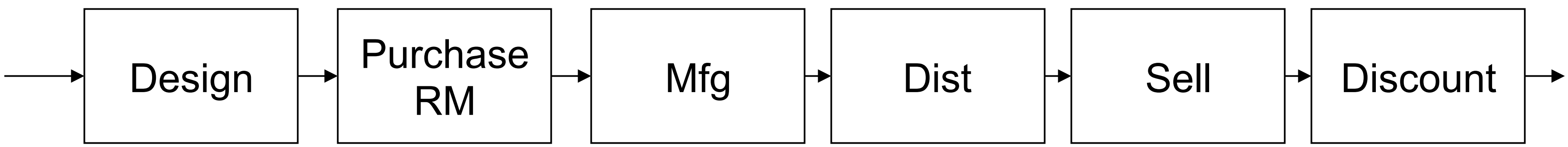
Consequences of Poor Forecasting



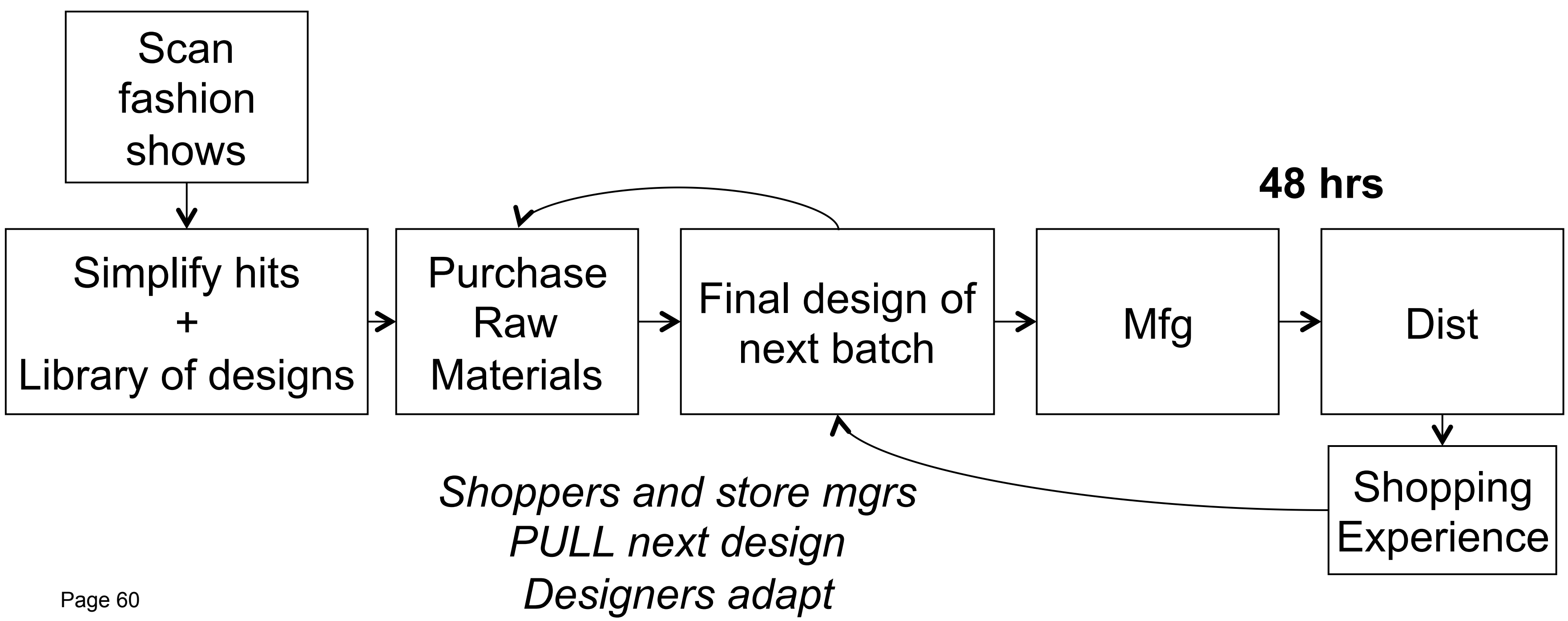
http://www.zerohedge.com/news/2014-05-16/where-worlds-unsold-cars-go-die

How Can We Reduce Reliance on Forecast Quality? Example

Traditional fashion process flow



Zara's model



Group Exercise: Forecasting iPad Sales

You are working for Apple Inc and about to release a new product, the iPad Air 3.

- What information would take into account when forecasting initial sales?
- What techniques might you use to refine your forecast?

Takeaways from Today

- There are different order fulfilment strategies dependent on the P:D ratio.
- Reliable forecasting is critical for many organisations at both strategic and operational levels
- Forecasting methods can be quantitative or qualitative
  - Qualitative methods, where little historical data available
  - Quantitative methods can be: time series methods (ES, MA) or causal methods (regression)
  - Each comes with its own limitations

Preparation for Next Class

Readings:

- Slack et al., "Operations Management": Chp. 10 (The Nature of Planning and Control)

Preparatory Questions:

- What algorithms and heuristics can be used to schedule jobs on a single machine?
- What algorithms and heuristics can be used to schedule jobs involving two machines?

Operations Management

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