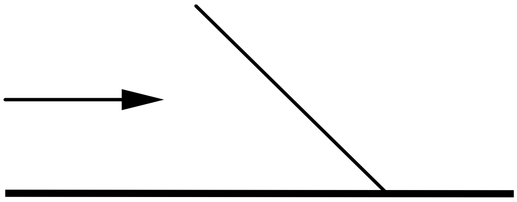
Mach Reflection

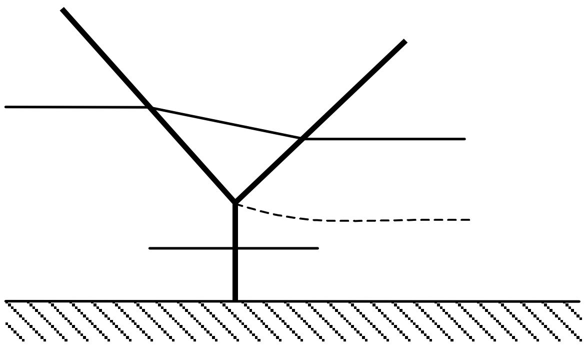
What would have happened in the example above if the turning required by the second shock, the wedge half-angle = θ , had turned out to be greater than the maximum possible turning, θ_{max}, for this value of M_2 ?



E.g. $M_1 =$ and $\theta =$

Tables/Chart $\Rightarrow$ $\beta_1 =$ and $M_2 =$

and θ_{max} for $M =$ is about



Get *Mach Reflection* .

In the downstream region there is a 'slip line', which in inviscid flow is a discontinuity of velocity, but in practice is a thin shear layer.

The two regions either side of the slip line have the same static pressure, but different levels of velocity.

Note

These types of reflections assume inviscid flow near the wall. In practice, there is a boundary layer there. The incident shock is a sudden pressure rise and may or may not separate the boundary layer depending on the shape and state of the boundary layer flow. This is a highly complex situation and an area of active research.

Supersonic Air Intakes

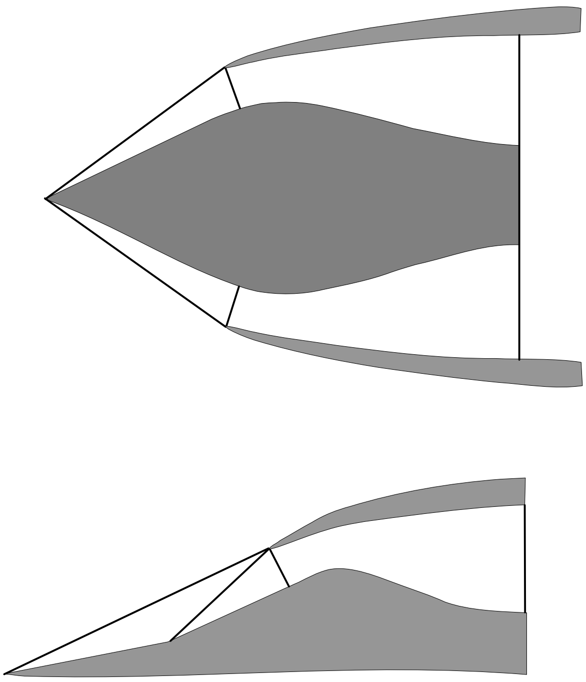
The flow through an aero-engine is always subsonic (typically M =) and this poses quite a design problem for intakes for supersonic aircraft. The air intakes of the *TSR2* and *Tornado* aircraft show two common approaches to this problem. Note the sharp angles, which are intended to induce only oblique shocks, wherever possible.

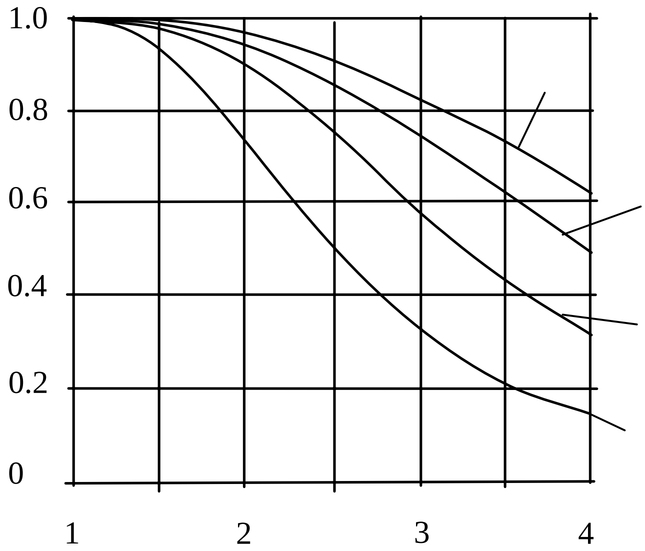
The flow through typical conical centre-body type intakes is shown for design speed, along with the improvement in intake performance obtained by using oblique shock compression for the design of a particular intake. Oblique shocks form a very efficient compression of the air immediately prior to the intake. This slows the flow down considerably such that the final normal shock, after which the flow is subsonic, is at a modest Mach number and hence has relatively small entropy rise.

TSR2 semi-circular conical intake

Tornado two-dimensional ramp intake

Note that both of these particular designs are variable geometry.

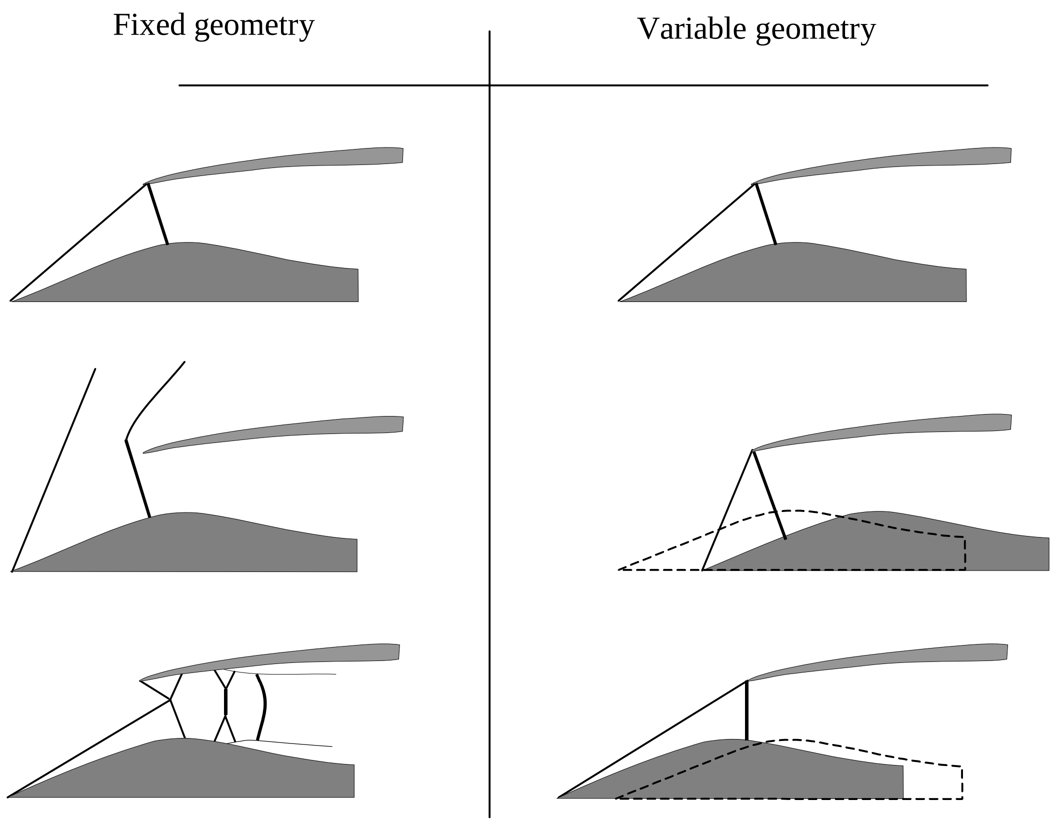




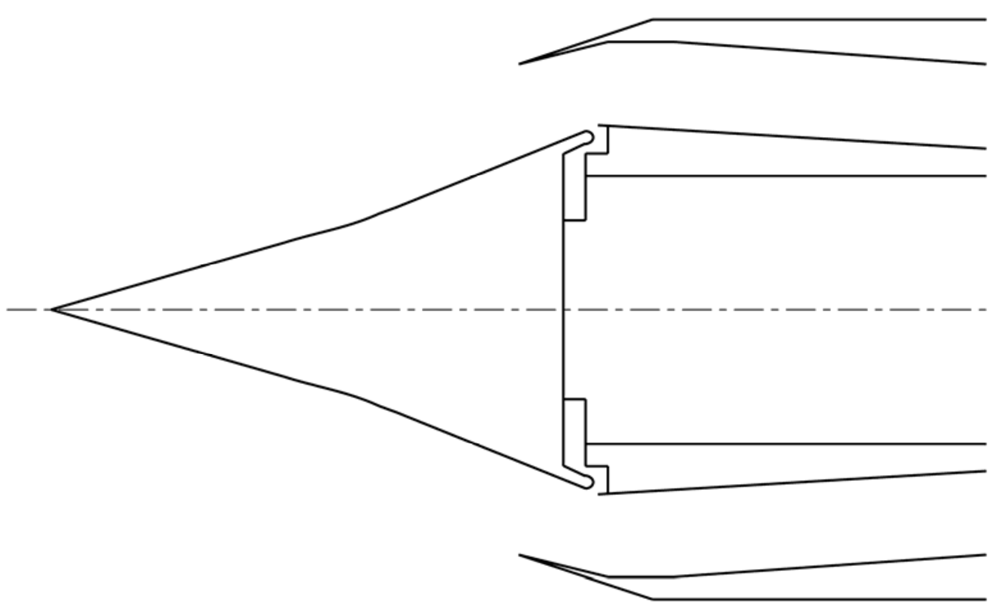
For the oblique (i.e. weak) shocks, the loss of stagnation pressure across them is relatively small and the main loss is across the normal shock. As the number of shocks N increases, the more the flow can be slowed down by the oblique shocks and the lower the Mach Number ahead of the normal shock, and hence the smaller the loss across it.

More shocks = more complexity = greater cost + lower reliability.

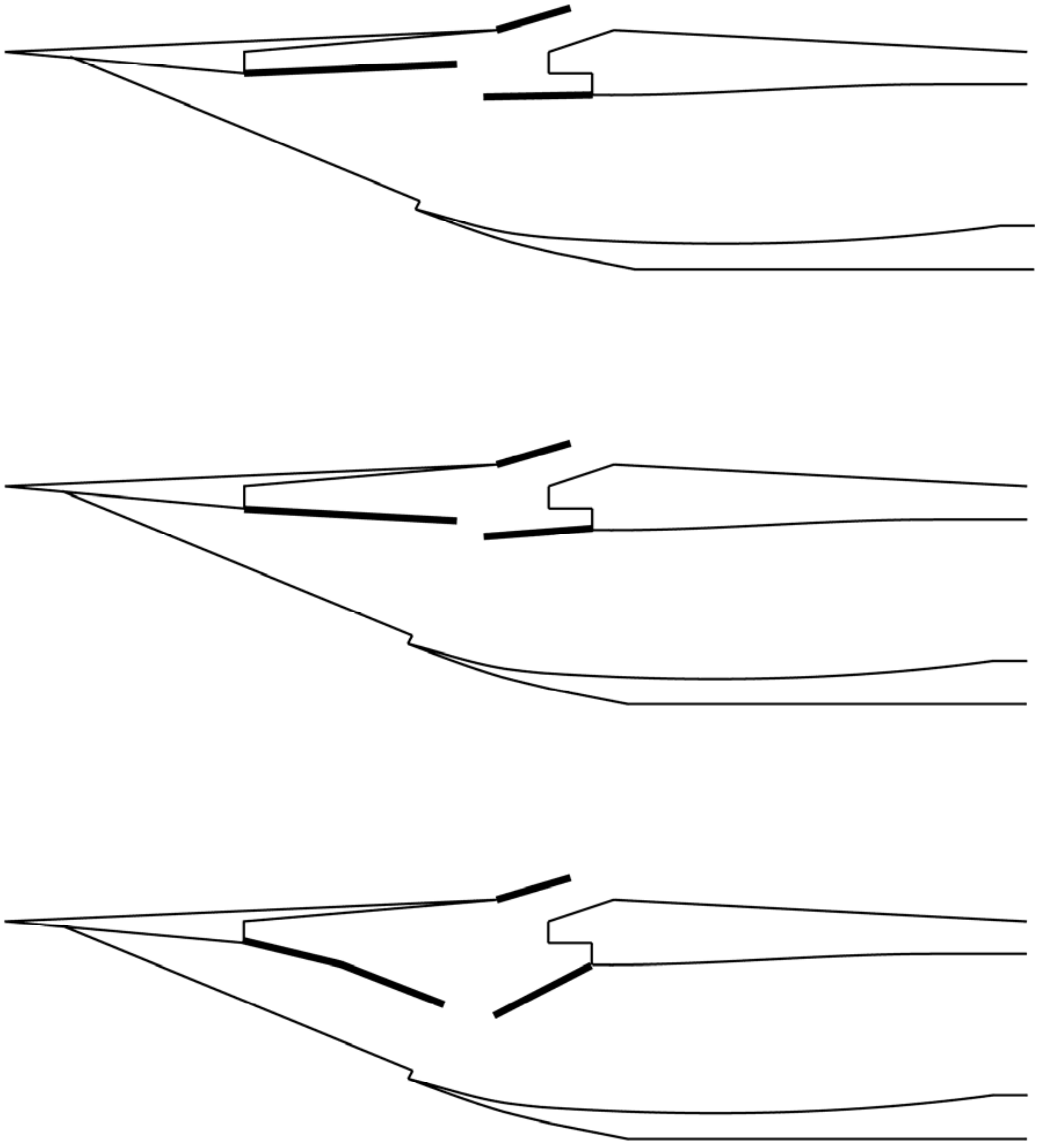
As the mach number of the aircraft increases or as the proportion of flight time spent in supersonic flight increases, the inlets become more complex. There are two things to balance: the aircraft speed and the engine flow demand. Good intake recovery (low total pressure loss) is obtained by using variable geometry intakes with spill doors for excess air.



The SR-71 intake operating at design Mach number has multiple external oblique shocks all designed to intersect just inside the intake on the outer casing, with reflection as a single oblique shock..



A schematic of the SR-71 intake – an axisymmetric *mixed compression* design



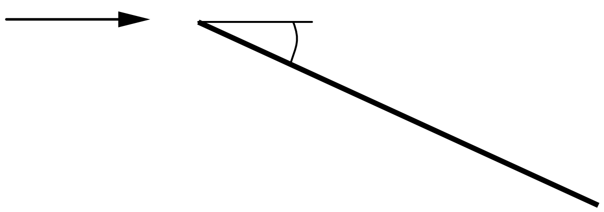
A schematic of the F-14 intake – an *unfocussed* design

This illustrates the design ingenuity needed to balance the competing demands of good intake recovery (low total pressure loss) over a wide range of flow speeds, coupled with varying engine flow demands and geometric simplicity.

FURTHER EXAMPLES (SHOCK-EXPANSION THEORY)

The supersonic flow around a variety of shapes can be calculated by patching together various combinations of shocks and isentropic expansions, *provided that the individual elements do not interact significantly in regions of interest.*

Flat Plate at Incidence



Transition $\infty \rightarrow A$

This is an *expansion* across a $+\mu$ ch'ic. Thus

$\nu_\infty =$ $\theta_\infty =$ $\theta_A =$

$\Rightarrow$ $\Rightarrow$

We will see later that ,to complete the solution, we need to calculate the static pressure here. This is done by remembering that the expansion is isentropic, so that there is no loss of stagnation pressure. This gives

Transition $\infty \rightarrow B$

This is a shock, with upstream Mach number and a deflection of H & B gives

M_1	$\frac{p_2}{p_1} = \frac{p_B}{p_\infty}$	$M_2 = M_B$	
2.40	4.9648	1.0705	
2.45	4.8671	1.1325	
2.41	4.8653	1.0829	$\nu_B = 1.01$

<u>Transitions A $\rightarrow$ C and B $\rightarrow$ D</u>

To match the flow at the trailing edge, the flow in region A must be compressed (by a shock) and that in region B expanded. The shocks at the leading and trailing edge will, however, have different strengths in general and the drop of total pressure across each of them will thus be different. Downstream of the trailing edge there will thus be two streams. These must have the same static pressure and flow angle and but different velocities and Mach numbers. There is no simple way of determining what value of flow angle is appropriate and it is necessary to iterate.

θ	δ_{AC}	$\frac{p_2}{p_1}$	$\frac{p_C}{p_\infty}$	$\Delta\theta_{BD}$	ν_D	M_D	$\frac{p_D}{p_\infty}$
0	28	8.1982	.8625	28	29.01	2.099	1.1148
2	30	9.1018	.9575	30	31.01	2.175	.9900
4	32	10.0742	1.0598	32	33.01	2.214	.9315

Interpolating between the last two values
$\theta = x\,4 + (1-x)\,2$ and equal static pressures in the
two streams means that

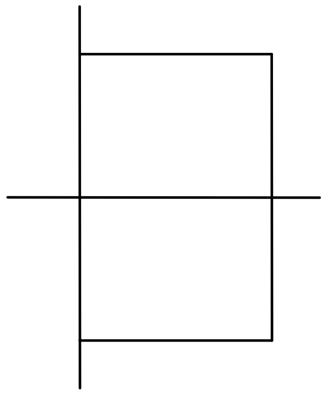
Notes

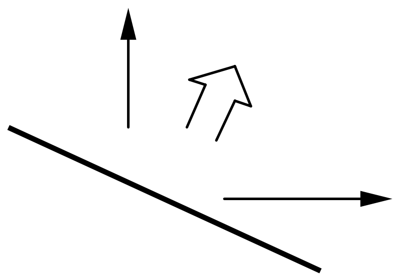
(i) There is *no singularity* at the leading edge. Hence no suction force parallel to plate.

i.e. force on plate $\perp$ plate.

The drag on the plate is referred to as 'wave drag' since it is caused by the momentum carried away by the wave system.

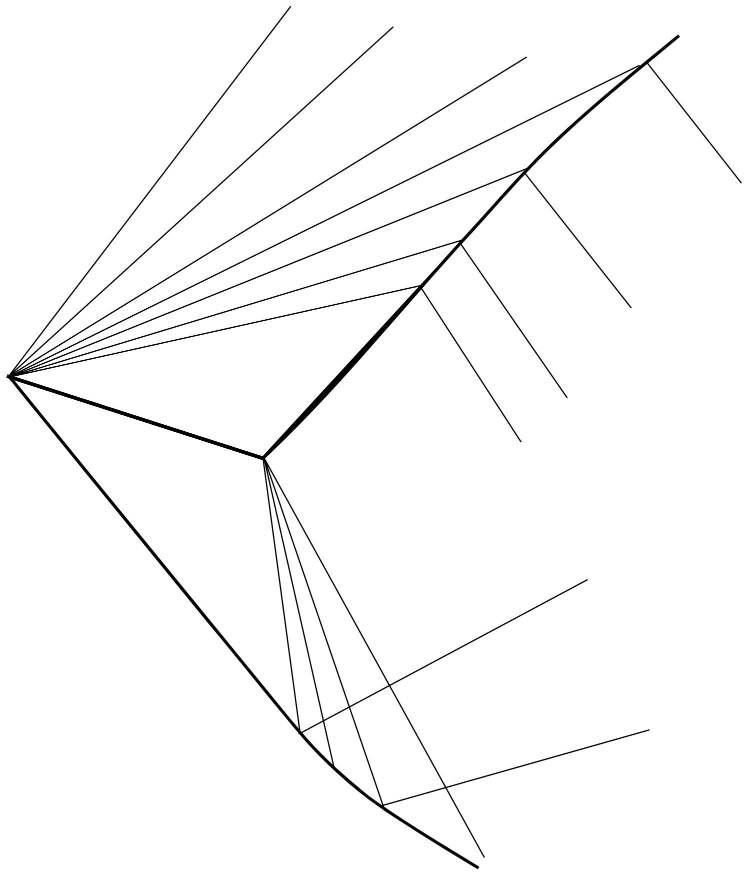
Contrast this behaviour with incompressible (or for that matter subsonic) flow. For that case, there is a leading edge singularity and lift but no drag.



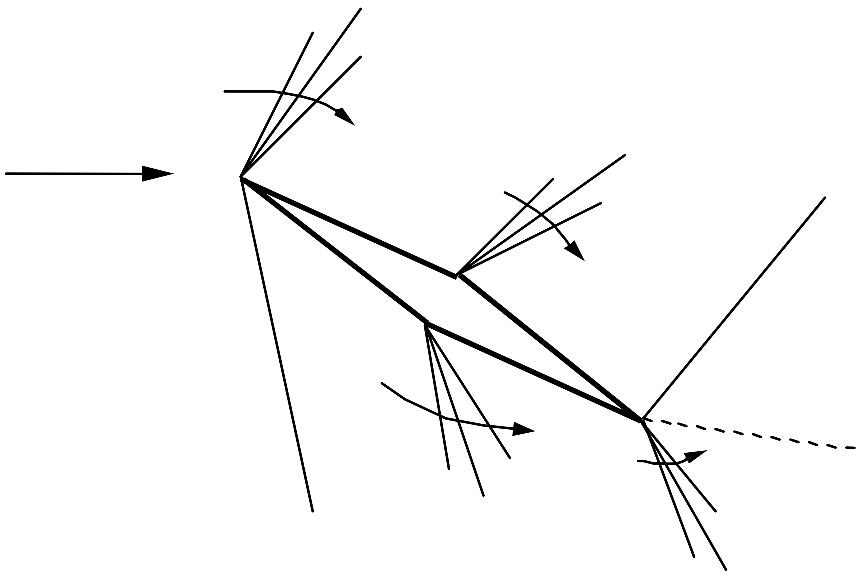


(ii) The pressures on the upper and lower surfaces do not equalise at the trailing edge (i.e. no Kutta condition as for the subsonic case). There is turning of the flow downstream of the trailing edge (called 'supersonic deviation').

(iii) The solution we have obtained assumes that the various shocks, expansions and slip line do not interact (i.e. intersect). At some distance from the plate they will do so



Double Wedge



In this case, there is not only lift and drag, there is also a pitching moment.