MCMC Algorithms and Bayesian Inference

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February 1, 2017

Abstract

This second homework covers the main concepts behind Monte Carlo Markov Chain simulation (MCMC).

Contents

Glossary

1. p for ”probability”, the cumulative distribution function (c. d. f.)
2. q for ”quantile”, the inverse c. d. f.
3. d for ”density”, the density function (p. f. or p. d. f.)
4. r for ”random”, a random variable having the specified distribution

Markov Chains Essentials

Markov Process are sthocastics process which are usually defined as a collection of random variables. Markov Chains are useful to model random process which has a short memory dependence.

We can have four types of Markov Chains:

Types of Markov Chains		
Time/State-Space	Countable State Space	General State Space
Discrete-Time	MC on a finite state space	MC on a general state space
Continuos-Time	Markov Process	Stochastic Process w/ Markov Property

In this case we are instrested in defining the probability law of Markov Chains on a generals state space. Let $t \in \{0, 1, 2, ...\}$ be the index of the process and $S \subset R^k$ the general state space of states:

1. $\mu \rightarrow$ initial distribution at time t = 0
2. Transition Kernel $K_t(x, A) = Pr\{X_{t+1} \in A | X_t = x\}$ for each $t = \{1, 2, ...\}$

Where the transition kernel is a function $K(\cdot, \cdot) : S \times \mathcal{B}(\mathcal{S}) \rightarrow [0, 1]$

- $\forall x \in \mathcal{S} : K(x, \cdot)$ is a probability measure
- $\forall A \in \mathcal{B}(S) : K(\cdot, A)$ is measurable

Markov Chain as an approximation tool

Markov Chains have several properties among which **Invariant Measure** and **Stationarity** at steady-state. More formally given a finite Markov Chain $X_t, t \in \mathcal{T}$ which is irreducible and poistive recurrent than after t stpes I get a random value: $\theta_t \sim P_y^t(\cdot) = K^t(y, \cdot)$. This is true thanks to the ergodic theorem, which holds in under the previous properties.

In the end we obtain $P_y^t(\cdot) \rightarrow P_y^\infty = \pi(\cdot)$ or more specifically,

$$\hat{I} = \frac{1}{t} \sum_{i=T_0}^{T_0+t} h(\theta_i) \rightarrow E_\pi[h(\theta)] = I \quad for\ t \rightarrow \infty \tag{1}$$

Monte Carlo Markov Chain Essentials

Puppet Markov Chain

Coal Mining Disaster

The Dugongs Strike back