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Design loads and long term distribution of mooring line response of a large weathervaning vessel in a tropical cyclone environment

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ABSTRACT

The mooring design of a floating offshore structure requires the estimation of mooring responses corresponding to annual exceedance probabilities of 10^{-2} (extreme event) and sometimes 10^{-4} (survival event). The most straightforward method to determine the extreme design response under a specified design sea state, is to carry out N time domain simulations, so as to capture the inherent randomness of this sea state and use the N maximum values to estimate the most probable maximum response for design. However, this requires typically 30–40 time-domain analyses of the same design sea state, which is computationally extensive. In this paper it is shown that the required number of time domain simulations can be reduced significantly by utilising the peaks of the mooring tension time series, obtained from time domain simulations, to derive a distribution for the maxima. Different variations of using these peaks are explored and a "best practice" for this technique is proposed. In order to establish a robust benchmark for evaluating and validating this "best practice", extensive time domain simulations have been carried out for a large permanently connected, weathervaning vessel, with catenary mooring system, in a tropical cyclone environment. Both extreme and survival conditions are explored, by running 170 3-h simulations for each condition, thereby representing in detail the random nature of each sea state. It is shown that a reliable distribution for the maxima, (within ± 4% from the benchmark) can be obtained in a manner which is simple and computationally efficient, based on just 4–7 time domain analyses. Thus by using more peaks from the time domain analyses, there is a significant gain in terms of accuracy and efficiency. The above "best practice" is used to calculate the most probable maximum mooring line response and the variability of this maximum (short term variability) within a 3-h sea state for environmental conditions with annual exceedance probabilities of 10^{-2} and 10^{-4}. It is shown that the short term variability is not invariant but may be described in terms of a Gumbel distribution whose parameters depend on the magnitude of the response. These expressions provide a means of calculating the long term distribution of mooring line load, accounting for the short term variability, which can be used to address the reliability of a mooring system.

1. Introduction

In order to design the mooring system of a floating platform, the engineer is interested in the maximum load that a mooring line will experience, under each of the specified design sea states. For a permanently manned installation the design sea states are based

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on exceedance probability of 10^{-2}/annum (return period 100yr) for the extreme design event [1] and 10^{-4}/annum (return period 10,000yr) for the abnormal or survival design event. For a weathervaning vessel which is the focus of this paper, the derivation of design sea states with the above exceedance probabilities is an important and complex aspect, since the extreme response depends on several metocean variables, namely significant wave height, H_S, peak wave period, T_P, wind speed, angle between waves and winds, current speed, current direction etc. Thus many joint design conditions need to be developed (see Section 2) and the floating system needs to be analysed for all of these conditions, since it is not known beforehand which ones will lead to maximum response. To cover all these environmental design conditions and different vessel drafts, the number of different load combinations can easily reach more than 500. Furthermore, in the time domain, each case needs to be analysed several times in order to account for the random nature of the environment. The design mooring load is usually described in terms of the most probable maximum (MPM) load in the 3-h simulation. The simplest method of obtaining MPM is to analyse each load case N times and obtain N mooring line response time series. From each time series a maximum peak, which represents the maximum load for that analysis, is selected. The MPM load is estimated from this population of N maxima. Stanisic et al. [2] explored this method and concluded that N should be between 30 and 40 in order to achieve an accurate estimate (within ± 4%) of the true MPM load.

The design process executed in the manner described above becomes exceedingly computationally intensive and time consuming due to the complexity of the mooring response (e.g. sensitivity to low frequency excitation, coupling effects, line stiffness characteristics). To deal with this problem, this paper explores the utilisation of more peaks from the time series, via the Peak Distribution Method (hereafter PDM), as a means of reducing the number of simulations. This method involves fitting a probability distribution on to the peaks of the mooring line load time series. The fitted distribution (typically Weibull) is used as a basis for developing a distribution for the maximum response for a 3-h period, from which the MPM is selected as the design value. PDM utilises all the peaks from a mooring time series, instead of using only the maximum one. As such, more information from a single simulation is utilised and hence it is expected that a smaller number of simulations is sufficient to achieve the desired accuracy (within ± 4% of true MPM response).

Several design codes and regulatory bodies propose or mention PDM. DNVGL [3] states that Weibull distribution should be fitted to the peak responses of a simulation of sufficient duration and the number of peaks should be used to derive a distribution for the maxima from which the MPM is selected. In API [4] and Lloyd'sRegister [5], PDM is recognised as a method that requires fewer realisations but entails substantial knowledge and experience. No guidance is provided on its execution. Some papers have explored certain variations of the PDM, by fitting Weibull distribution to the peaks of the mooring response. The peaks used are obtained from a single simulation of duration of 3-h or shorter. The results of these studies are evaluated against the "true response" which is defined as a particular statistic of 10–30 maximum peaks obtained from 10 to 30 realisations respectively. In some of these studies the Weibull distribution is fitted to the upper 40% of the selected peaks only, in order to improve the estimation of the fit in the upper tail of the distribution ([6,7]). Cheng and Kuang [8] investigate the sensitivity of the PDM to selecting different portion of the peaks (top 5%, 10%, 20%, 30%, 50%). The results from this study indicate that the use of 30%–40% of top peaks has a close fit to the observed data, however this does not improve the consistency in prediction of the extreme value. Chen and Mills [9] fitted the peaks of mooring line response of different offshore facilities on to the 3 and 2 parameter Weibull distribution and concluded that the use of 3 parameter Weibull showed poor results for highly non-linear responses. Sagrilo et al. [10] proposed a duration correction factor which guarantees, with given confidence, that the extreme value estimated from peaks of durations 3-h or shorter, is equal to or higher than the "true extreme value".

Although the PDM has been researched in the literature, it is not commonly used in the industry, for design, due to the lack of accepted systematic approach and lack of evidence that this method may lead consistently to accurate results. This paper develops a 'best practice' procedure for PDM and demonstrates that this approach can predict the design mooring load consistently with similar accuracy (within ± 4%) as when a large number of simulations is carried out. The PDM method is evaluated extensively in this paper for two environmental load cases, one corresponding to a 100yr and another to 10,000yr return period. Instead of using only 10–30 simulations, as in previous studies ([7,9,10,11]), to establish the "true response" (benchmark) against which the method is assessed, results are used from 170 coupled time domain simulations for each environmental load case. By using a large number of simulations (170), the benchmark was established utilising only a single maximum from each simulation [2]. In addition to establishing a reliable benchmark value, use of a large number of simulations allowed for comprehensive investigation of the method which would be challenging with only 10–30 simulations. A key aspect of the "best practice" procedure is the length of the time domain simulation which is needed to establish a reliable probability distribution for the peaks and hence a reliable design value for the mooring load. Clear guidance on this aspect is given as well. Although the focus of this paper is on a mooring line response, it is also demonstrated that the proposed "best practice" of PDM can be used for other responses, namely the total turret excursion.

The PDM is assessed with respect to a large, turret moored floating vessel, with catenary mooring system in water depth of 580 m for a tropical cyclone environment. The coupled model has been validated against experimental data. The design basis of this vessel requires analysis of a large number of joint metocean conditions with return periods of 100yr and 10,000yr. Some background on the method used to derive the metocean conditions is given in Section 2, although the derivation is not in the scope of this study. All of these conditions were checked as part of the vessel design and the two most critical environmental load cases (one for 100yr and one for 10,000yr) are used here. These design cases are demanding for the PDM method due to the system's sensitivity to low frequency excitation (as well as wave frequency) and consideration of load levels where line stiffness increases non-linearly with offset. Hence the "best practice" procedure developed is expected to work well for a wide range of floating system responses.

The establishment of the best practice for PDM enables accurate determination of the variability in maximum mooring line load for a specified metocean condition (short term variability). These results are used for developing the long term distribution of mooring line load, i.e. long term statistics of mooring line response which accounts for short term variability. The long term distribution is developed for application in reliability analysis. In a full long term approach the response of the floating system is evaluated for all seastates in a hindcast [12] or wave scatter diagram [13]. This approach may be too cumbersome if the system under consideration is complex, such as a weathervaning vessel with many mooring lines and risers, which is the case in this study. In the simplification considered in this paper, the long term distribution is developed by analysing only joint metocean conditions (see Section 2) for return period of 100yr, 10,000yr and 1,000yr. A long term response distribution (excluding short term variability) is developed using the most critical MPM value for each of the return periods. The short term variability is accurately quantified (through the PDM "best practice" model) and then convoluted with the long term MPM response to develop the long term distribution. The justification for this method is discussed in Section 6 of this paper.

Design standards for mooring lines (e.g. Refs. [3,4]) at present define the mooring design load as the MPM for the design sea states without direct reference to the effect of short term variability. Thus it could be argued that quantification of short term variability is not needed for design. This is justified provided that the contribution from short term variability is taken into account when deriving (or validating) the factors of safety of the design standard using reliability analysis. Thus, the prime purpose of quantifying the short term variability and incorporating it into the long term response, is for reliability considerations. These considerations become important when a new floating system is developed or used in a new environment. Both aspects are applicable to the floating system considered in this work which is a Floating LNG, designed to be permanently connected and permanently manned in an environment dominated by tropical cyclones. In addition to the above, the Floating LNG application is breaking new ground in terms of size of the vessel and consequences of failure and these aspects provided additional motivation for this work.

2. Environmental design conditions

A set of design metocean conditions corresponding to annual exceedance probabilities of 10^{-2} and 10^{-4} has been developed and specified as an input to this study. The derivation of these conditions is briefly outlined but a detailed description is outside the scope of this paper. The starting point for developing design metocean conditions is a hindcast database for the site which provides "historical" values of the metocean variables (e.g. wind, wind waves, swell and current) for a regular time interval of t hours, over a long period of typically 50 years. For a North Sea environment t is typically 3-h since conditions may be regarded as stationary over this duration, whereas for a tropical cyclone environment conditions change more rapidly and t = 1-h is more appropriate. The location of the floating vessel studied here lies in the most severe sector of cyclone activity off the coast of Western Australia. The severity of extreme cyclonic conditions is summarised in Table 1 in terms of independent conditions for 100yr and 10,000yr return period.

For a weathervaning vessel the derivation of design sea states is complex since the extreme response depends on several metocean variables. A method which accounts rigorously for the joint extremes of these variables is hence required for design. The method which has been used to derive the design conditions, used in this paper, is based on the model for conditional extremes developed by Heffernan and Tawn [14] which was subsequently applied by Jonathan et al. [15] to develop joint metocean extremes of relevance to floating systems. In this paper it is referred to as joint extremes method. A more commonly used method to develop joint metocean conditions is inverse first order reliability method (IFORM) [16,17]. It was demonstrated that the method of joint extremes yields results which compare well with the IFORM method and furthermore, this method is well suited to problems that extend beyond two variables [15]. The joint conditions provide the most likely combination of environmental parameters with one parameter being dominant (e.g. wave). From an engineer's perspective these conditions cover the H_S-T_P contour when the angle between the wind and waves is zero but include additional H_S-T_P contour lines covering cases when the angle between wind and waves is 22.5°, 45°, 67.5°, 90°. This is done for wave-dominated events and wind-dominated events.

The hindcast, used to develop the design conditions, provides a record of metocean conditions at 1-h intervals so as to provide adequate granularity for tropical cyclone events, e.g. wind direction can change rapidly, as well as the wind speed. Joint conditions have been derived initially for 1-h and they were subsequently converted to equivalent 3-h stationary conditions. The conversion is done by reducing H_S with factor of 0.96. This follows from the basis that water surface elevation is a Gaussian process and hence wave height follows Rayleigh distribution. The T_p is obtained by regression relationship between H_S and T_p, namely $\propto T_p/(H_S)^{1/2}$. These two sets of conditions should produce analogous results when the extreme response is dominated by the maximum wave height. Here the response is not entirely dominated by wave height. It is noted that a conversion to 3-h conditions is not necessary but it was carried out to maintain consistency with other design metocean conditions forming part of the design process. Table 2 provides two example design metocean conditions for each return period. Since the majority of design conditions will not be critical from the point of view of mooring response, the mooring analyses are initially carried out in the frequency domain, screening out a large number of design conditions; thus only a small number of cases are analysed in the time domain. The screening analyses do not form part of this

Table 1
Independent environmental conditions with return period of 100 and 10,000 years (wave parameters are for 1-h sea state).

Independent Criteria	100yr Return Period	10,000yr Return period
1-h mean wind speed (m/s)	44.2	65.5
H_S (m)	13.7	20.7
T_P (s)	13.4	16.8
Current mean speed at 3 m below MSL (m/s)	1.8	2.67

Table 2
Joint environmental conditions with return period of 100 and 10,000 years.

Environment		100yr Joint Probability Condition		10,000yr Joint Probability Condition	
		Condition 1	Condition 2	Condition 3	Condition 4
Wave	Spectrum	JONSWAP	JONSWAP	JONSWAP	JONSWAP
	H_S (m)	12.1	9.0	13.4	19.5
	T_P (s)	14	13	13.9	16.6
Angle between Wind and Wave		45°	67.5°	90°	0°
Wind	Spectrum	NPD	NPD	NPD	NPD
	1-h Mean Speed (m/s)	34.8	29.5	37.9	42.2
Current	Mean Speed at 12 m below MSL (m/s)	1.2	1.2	1.2	1.3
Angle between Current and Wind		45°	67.5°	22.5°	0°

paper. Condition 1 and condition 3 have resulted in maximum mooring line responses under 100yr and 10,000yr conditions respectively and have been selected for this study.

Another method of developing design conditions is to adopt a response-based approach as done by Tromans and Vanderschuren [12] and van Zutphen and Christou [18] whereby each sea state in the hindcast database (above a certain threshold H_S) is converted into a mooring response by simulating the motion of the floating system (see Appendix A for more details). The final step is to identify credible combinations of metocean variables which result in these responses. These become the response-based design conditions. Some of the conditions used in the mooring design of this floater have been derived in this manner. The two sets of conditions, (based on joint extremes method and response-based), have been used in parallel, complementing each other.

3. Extreme value analysis

The peaks of the mooring line load can be represented as a random variable, X, with distribution $F_x(x)$. The largest value (Y_n) from a set of observations of X has a probability distribution $F_{Yn}(y)$ which can be established as a product of the probabilities that each sampled observation X is less than some value y, assuming independence between observations. Hence, the distribution of the largest value, in this case the extreme mooring load, has the following distribution:

$$F_{Yn}(y) = [F_x(y)]^n \tag{1}$$

where $F_{Yn}(y)$ is a cumulative probability distribution of the largest value Y_n, and n is the size of the sample of the random variable X. In this application n represents the number of peaks in the mooring line load resulting from a 3-h time domain simulation. By differentiating Eq. (1) the probability distribution of the extreme value can be expressed as:

$$f_{Yn}(y) = n\,[F_x(y)]^{n-1} f_x(y) \tag{2}$$

The limit of Eq. (2) when the size of the sample (n) becomes very large was initially derived by Fisher and Tippett [19]. Later Gumbel [20] identified that there are three main asymptotic forms of Eq. (2): Type I, Type II and Type III. The tail behaviour of the initial distribution determines which limiting form its extreme distribution will tend to. Type I asymptotic form, to which Weibull distribution belongs, is often said to be in Gumbel domain where the maxima are attracted to the Gumbel distribution. In the context of mooring design, the design sea states are based on return periods of 100yr (extreme event) or 10,000yr (survival event) and the extreme response is usually the most probable maximum of the probability distribution of the extreme value, $f_{Yn}(y)$. Fig. 1 shows a typical plot of probability density finction (PDF), $f_x(y)$, of the primary random variable X (where $f_x(y)$ is a Weibull distribution) and PDF of the extreme value $f_{Yn}(y)$.

4. Peak Distribution Method- prediction of extreme mooring line load from single 3-h simulation

Application of the PDM method may lead to variable success depending on how the method is executed. This section investigates different variations of the PDM by breaking the method in to several steps: (i) how peaks are defined, (ii) selecting probability distribution model and (iii) method of fitting the distribution model to the peaks. At each step several options or variations of the method are described and selection of the best option is made based on these investigations. In Section 5 the accuracy of the PDM method is improved by extending the length of the simulation beyond 3-h.

An overall view of the mooring system used for extreme response analysis in this paper is shown in Fig. 2. The vessel natural periods for heave and pitch lie in the range of 10–20s, while for roll it is about 25s. As there is very little wave energy at 25s, the roll response is small. The natural period for low frequency oscillations is about 200s. The coupled model details are presented in Stanisic et al. [2] and will not be repeated here. The response time series, used to investigate the PDM, are also the same as in the previous study. Each load case was analysed 170 times using different random seeds. While the previous study [2] concentrated on estimating the extreme response from the maxima peaks only, this study looks at reducing the number of simulations required, by including more peaks from a single simulation. The PDM is used to evaluate the extreme mooring response for each of the 170 simulations, of 3-h duration, for two load conditions. The extreme response obtained by PDM is compared to the benchmark value (7830 kN and 15290 kN for 100yr and 10,000yr respectively), constructed from data of all 170 simulations. The benchmark was obtained directly

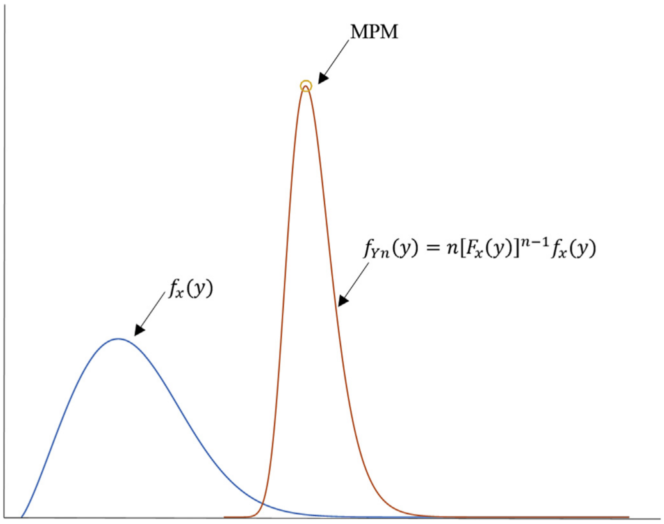


Fig. 1. Parent distribution and extreme value distribution.

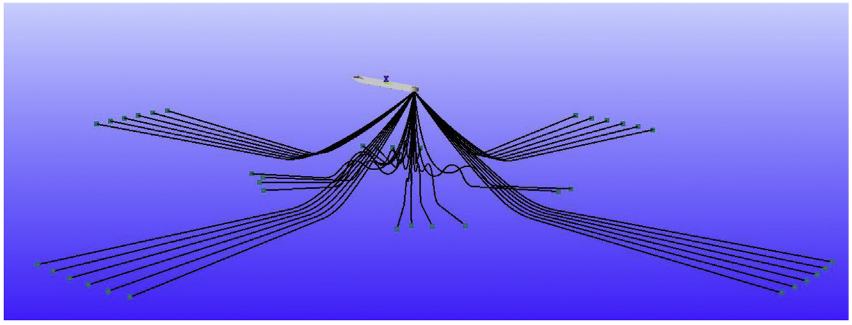


Fig. 2. Schematic view of the coupled model.

and accurately from the distribution of the maxima by sampling only the highest peak from each simulation. A Gumbel distribution was fitted on to the data of N = 170 maxima, using Method of Moments (MoM), and the MPM was obtained. Convergence in results was obtained after N = 30 thereby confirming the robustness of this benchmark (see Stanisic et al. [2] for more details). A prediction error is calculated for each estimate, using PDM as follows:

$$\%Prediction\ Error = \frac{Predicted\ Value - Benchmark}{Benchmark} \times 100 \tag{3}$$

For each environmental case, 170 prediction errors are obtained. The PDM is assessed based on the statistics of these prediction errors. The statistics utilised are the mean, minimum, maximum and variance. The mean (μ) represents the bias, while the minimum and maximum give an indication of the total spread of errors. Variance, which also measures the spread, was calculated with respect to the mean of the errors (σ^2) and with respect to the benchmark value ($\sigma^2(t)$). The former gives an indication of how close the errors are to their mean, while the later measures the spread around the benchmark value (how close are the errors to 0%). If the mean of the prediction errors is 0% the two variance values will be identical. A good fitting method has a bias close to zero and low values of minimum, maximum and variance.

4.1. Step 1- Peak definition of the mooring line top tension response

A turret moored vessel is sensitive to low frequency motions as well as wave frequency, resulting in the overall mooring load time series to consist of both low frequency and wave frequency contributions which can be seen in the typical time series for 100yr and 10,000yr mooring tension response (see Fig. 3). At the 100yr response, the low frequency process is clearly visible and has a period of about 200s. For the 10,000yr response, it is harder to identify the low frequency process, which is nevertheless present. This can be explained by the large angle between the wind and the wave for the 10,000yr case (90°). The vessel changes its heading between time-varying wind and wave, taking the heading of the predominant environment, which in turn affects its motions. The vertical motion at the turret (due to heave and pitch) has a big effect on mooring tension for 10,000yr case and this makes wave-frequency contributions more dominant. Therefore, both processes are important and selecting peaks based on one of the two processes (such as

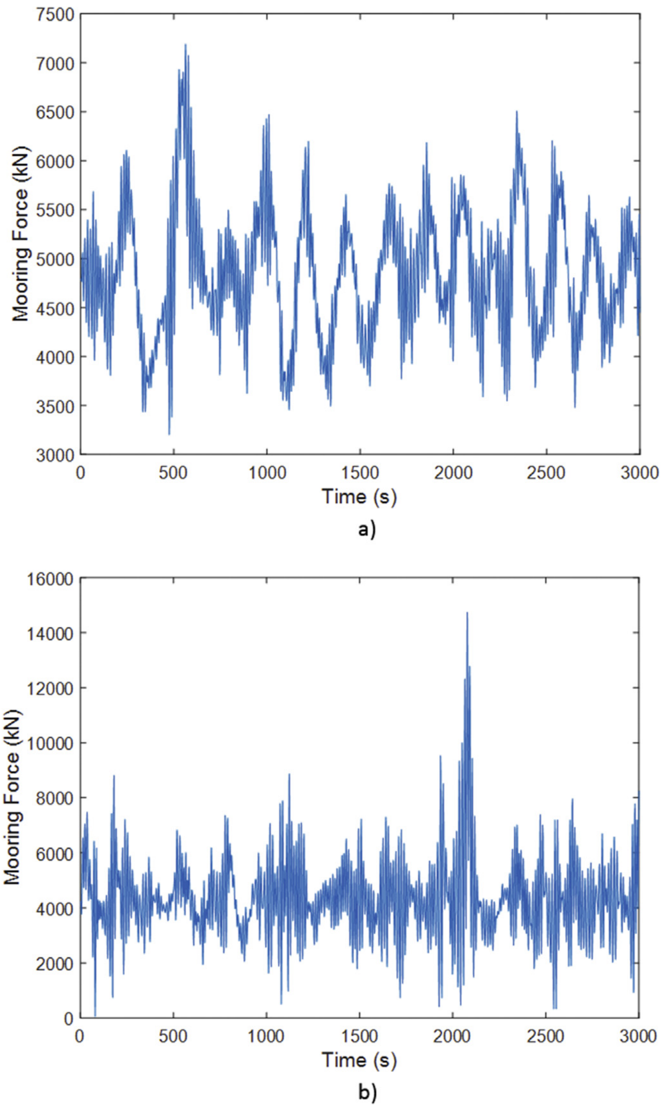


Fig. 3. Typical time series for mooring tension for a) 100yr critical environment and b) 10,000yr critical environment.

selecting only one peak riding on the crest of the low frequency motion) may lead to biased and inconsistent results. The aim of this paper is to present a consistent and unbiased approach, which does not assume that one of the two processes dominates.

The extreme value analysis assumes that the response peaks are random and statistically independent, meaning that the occurrence of one does not affect the probability of occurrence of the other. Peaks can be selected so as to reduce their inter-dependence by applying a minimum time gap between them or utilising Peak over Threshold Method (POT), where only peaks above certain threshold are selected. Some methods take in to account the peak dependence, such as ACER method [21]. The ACER method has been used by Sagrilo et al. [7]. It was shown that it produced unbiased results when compared to using Weibull distribution, however with higher sampling uncertainty, due to large number of open parameters. Rather than considering the dependence, this paper concentrates on looking at reducing the correlation by peak selection and subsequently observing the result of the estimate properties. In DNVGL [3] a peak is defined as the global peak or mean up crossing peak which is a maximum response between two successive mean up crossings on the time series. This application will result in selection of both wave frequency and low frequency processes. However, if the response is mainly governed by one of the processes, the selected peaks will reflect this. The mean tension over the 3-h simulation is estimated and used to define a peak after each mean up crossing. This definition is also used by Chen and Mills [9], Sagrilo et al. [7] and Sagrilo et al. [10] and it follows from the definition of a wave as a surface elevation between two successive downward or upward zero crossings [22].

Different definitions of a peak have an impact on estimation of the extreme value using PDM. Firstly the sensitivity of peak dependence is investigated as PDM assumes that peaks are independent. Autocorrelation, the correlation of the time series with itself at different time intervals, is used to test dependence between successive peaks of the mooring tension. The autocorrelation coefficient can be calculated for a single lag (correlation between successive peaks) as well as for two, three lags etc. The autocorrelation coefficient for lags higher than one was found to be almost negligible, hence only single lag results are considered. The average autocorrelation coefficient between successive mean up crossing peaks (global peaks) for 170 simulations is 0.59 for 100yr condition, while for 10,000yr condition it is 0.69. As both coefficients indicate some dependence between the successive peaks, it was investigated to see if applying a minimum time separation between the peaks or selecting peaks that are above a chosen threshold value (POT) decreases the recorded correlations. The effects of POT are examined in Section 4.3, where only a portion of largest peaks is used. A time separation of 14 s has been examined, as it corresponds approximately to the peak period in the wave spectrum. Other time separations were investigated as well, however no improvement in results was observed.

Besides describing peaks as mean up crossing peaks and mean up crossing peaks with minimum 14s separation, peaks were also represented by local peaks which are defined as a point that is larger than the two neighbouring points in the time series (see Fig. 4 for illustrative representation of peak definitions). The fitting method used to execute PDM for the purpose of studying the peak definition was MoM, due to its robustness (see Section 4.3.2 and Section 4.3.5). The results for different peak definitions are presented in Table 3 in terms of various statistics of all prediction errors for 170 simulations. Number of peaks selected and autocorrelation coefficients are also presented for each peak definition. The evaluating measures indicate that defining peaks other than as mean up crossing peaks increases the spread of prediction errors (large minimum, maximum and variance) especially for 10,000yr condition. When minimum 14s time separation is applied to mean up crossing peaks, the range of errors reduces slightly, however for 10,000yr condition the bias becomes −2.3% which indicates that the method does not show improvement in results. When all local peaks are selected, the number of peaks considerably increases, especially for 10,000yr case where the number exceeds the wave frequency peaks. Furthermore, the spread and bias become worst. It is therefore concluded that defining peaks as maximum between two mean up crossings, without any time constraint, produces more accurate and precise estimates of the extreme mooring response and is the adopted definition of peaks in all subsequent work.

4.2. Step 2- Picking peak distribution model

Most widely used probability function to represent the distribution of peaks of a mooring line is the Weibull probability function. The three parameter Weibull probability distribution function can be expressed as follows:

$$f(x) = \frac{\beta}{\eta}\left(\frac{x-\gamma}{\eta}\right)^{\beta-1}\exp\left(-\left(\frac{x-\gamma}{\eta}\right)^{\beta}\right) \tag{4}$$

where β is referred to as shape parameter, η as scale parameter and γ as the location parameter. Weibull distribution is well known for its flexibility and can be related to several other distributions (exponential and Rayleigh). As a result it is believed to be a good fit for a process such as mooring line tension which is comprised of both wave and low frequency components. The location parameter is a threshold or minimum value that the variate takes. Often the Weibull distribution is represented by a two parameter Weibull distribution, where the location parameter is set to zero. Estimating parameters for a three parameter Weibull distribution has proven, using recognised classical fitting methods, to be a challenge as estimates are inconsistent or are difficult to evaluate [23,24]. This is confirmed in the application of the three parameter Weibull distribution in PDM, where poor estimates of the extreme value were generated in comparison to those produced by using two parameter Weibull distribution, [9]. Hence, for the purpose of this study the two parameter Weibull distribution will be fitted to the fluctuating part of the time series of the response i.e. the mean value of the time series is subtracted before the fitting.

The Kolmogotov-Smirnov (KS) goodness of fit test is used in this paper to justify the utilisation of the two parameter Weibull distribution in PDM to predict extreme mooring response. The KS test is independent on the data size and data selection intervals [25,26], hence it is utilised due to its supremacy over the commonly used Chi-square test. The test is based on accepting or rejecting a hypothesis that the data follows a given distribution function. The fitness of peaks on the Weibull distribution was tested for each of the 170 simulations, of both conditions. None of the 170 fits (both for 100yr and 10,000yr case) were rejected by the KS test. This provides confidence that the two parameter Weibull distribution combined with a peak selection method, based on mean up crossing peaks, is suitable to be used in the PDM for the application to mooring tension.

4.3. Step 3- Fitting peak data on to the probability function

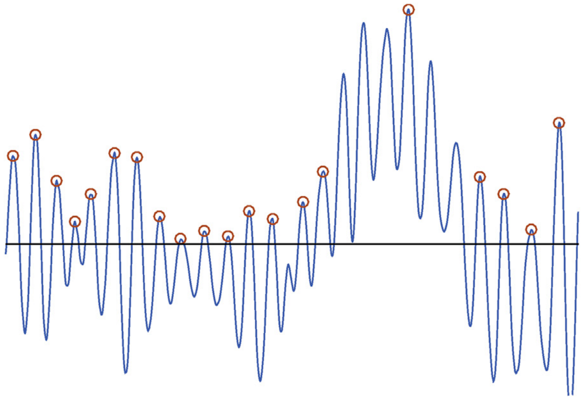
There are several methods available for fitting data on to a probability distribution. Most commonly used fitting methods are reviewed below with reference to the two parameter Weibull distribution for the application of PDM.

4.3.1. Maximum likelihood method (MLM)

This method is the most common statistical method in estimating parameters of a distribution. It is based on maximising the likelihood that the parameters estimated will yield the sampled data. Assuming independent and identically distributed sample ($x_1, x_2, \ldots x_n$), one can write a joint density function in terms of the distribution parameters β and η as follows:

$$L(x_1, x_2, \ldots, x_{n\beta}, \eta) = f(x_{1\beta}, \eta) \times f(x_{2\beta}, \eta) \times \ldots \times f(x_{n\beta}, \eta) \tag{5}$$

Eq. (5) represents the likelihood function that the sample ($x_1, x_2, \ldots x_n$) will be observed. The estimated distribution parameters are then evaluated by differentiating Eq. (5) and equating it to zero. The expressions obtained for the parameters of the Weibull



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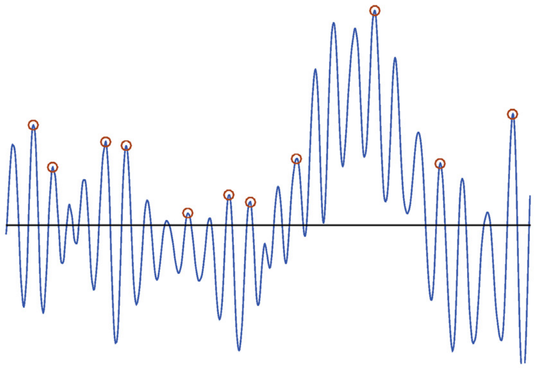
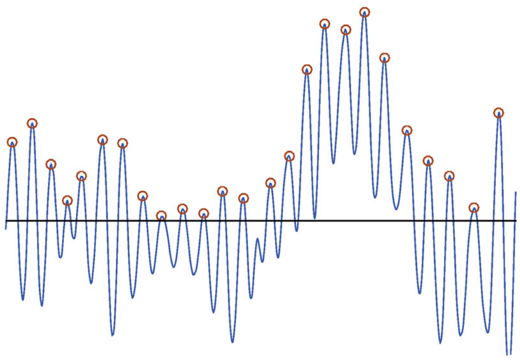


b)



c)

Fig. 4. Peak definitions a) Peak as maximum between two mean up crossings (adopted as preferred), b) Peak as maximum between two mean up crossings with minimum separation of 14s, c) Peak as local peak.

distribution using this method are presented in Eqs. (6) and (7):

$$\eta = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^{\beta}\right)^{1/\beta} \tag{6}$$

Table 3
Results from different peak definitions.

Statistic	Global Peak		Global Peak (min 14s)		All Local Peaks	
	100yr	10,000yr	100yr	10,000yr	100yr	10,000yr
Average Number of Peaks	399	592	285	388	764	1866
Bias (μ)	0.7	1.0	−0.2	−2.3	5.6	17.1
Range	16.2	35.2	16	33	30.2	56
Variance (σ^2)	9.6	42.3	9.4	36	20.8	100.6
Autocorrelation Coefficient	0.59	0.69	0.46	0.53	0.81	0.87

$$\beta = \left[\frac{\sum_{i=1}^{n}(x_i^{\beta}\ln(x_i))}{\sum_{i=1}^{n}x_i^{\beta}} - \frac{1}{n}\sum_{i=1}^{n}x_i\right]^{-1} \tag{7}$$

The parameters attained will maximise the likelihood of obtaining a given sample. The distribution parameters estimated using MLM, for very large samples, have a minimum variance and the estimators approach the actual probability parameters [27].

4.3.2. Method of moments (MoM)

Method of Moments utilises moments of the sampled data to estimate the parameters of the distribution that is to be fitted. Hence, the method suggests that the moments of the sampled data are true representation of the moments of the actual population. For a two parameter Weibull distribution, the parameters can be estimated from the 1st and 2nd moments, represented by Eq. (8) and Eq. (9) respectively.

$$E(X) = \eta \times \Gamma\left(\frac{1}{\beta} + 1\right) \tag{8}$$

$$E(X^2) = \eta^2 \times \Gamma\left(\frac{2}{\beta} + 1\right) \tag{9}$$

Where Γ represents the Gamma function. The equations are solved simultaneously for η and β. The first and second moment statistics are used as they have the potential to produce unbiased estimators, unlike higher moment statistics such as skewness and kurtosis [28].

4.3.3. Weibull distribution tail fitting

The distribution of the extreme values will depend on the behaviour of the tail of the initial distribution. Hence, when one is interested in extreme values, it is common to use methods which emphasise the tail of the parent distribution. One such method is presented in detail in Ref. [29]. Firstly a cumulative distribution of the peaks of the desired response is obtained. A two parameter Weibull distribution is then fitted on to seven different thresholds of the obtained cumulative distribution using standard linear regression analysis (least squares fit). These seven levels correspond to the upper 60%, 65%, 70%, 75%, 80% 85% and 90% of the data. The parameters of the parent distribution are taken to be the average of the seven parameter estimates obtained.

4.3.4. Exponential distribution tail fitting

Here the focus is again on the upper tail of the distribution but instead of Weibull, it is investigated whether a simpler distribution namely an Exponential distribution may lead to an improved fit. A study by Breiman et al. [30] found that when the exponential distribution is fitted to the upper 10% of observations the estimate of the 99.45th percentile demonstrated better results than using Weibull distribution, even when the observations follow Weibull distribution. The wave induced low frequency portion of the mooring load is proportional to the square of the wave height. Given that wave surface elevation peaks follow Rayleigh distribution and that square of a Rayleigh distributed random variable is an exponential random variable, it is expected that the peaks of the low frequency component of the mooring load will be exponentially distributed. Hence, in this paper, a one parameter exponential distribution will be fitted on to the tail using the method presented above by Sagrilo et al. [29]. As discussed previously, the mooring load is comprised of both low frequency and wave frequency components. When only the top portion of the peaks are considered, it is mainly the peaks which occur at the maximum of the low frequency that are selected. Hence it is a reasonable assumption to test the relevance of this fitting method in the application of the mooring line load peaks. The portion of the peaks selected will be varied in order to optimise this method.

4.3.5. Comparison of the different fitting methods

The peaks (defined as maximum between two mean up crossings) for each of the 170 simulations were fitted using the four fitting methods discussed above, thereby leading to four PDM options. The results are presented in Fig. 5, where the histogram of prediction error for all 170 simulations is shown together with evaluating statistics. Since each of these simulations covers a 3-h period with certain number of peaks (see Table 3), there will be natural variability in the simulation results leading to a "prediction error" which is not the result of the fitting method. This type of "prediction error" will reduce systematically when the length of simulation is

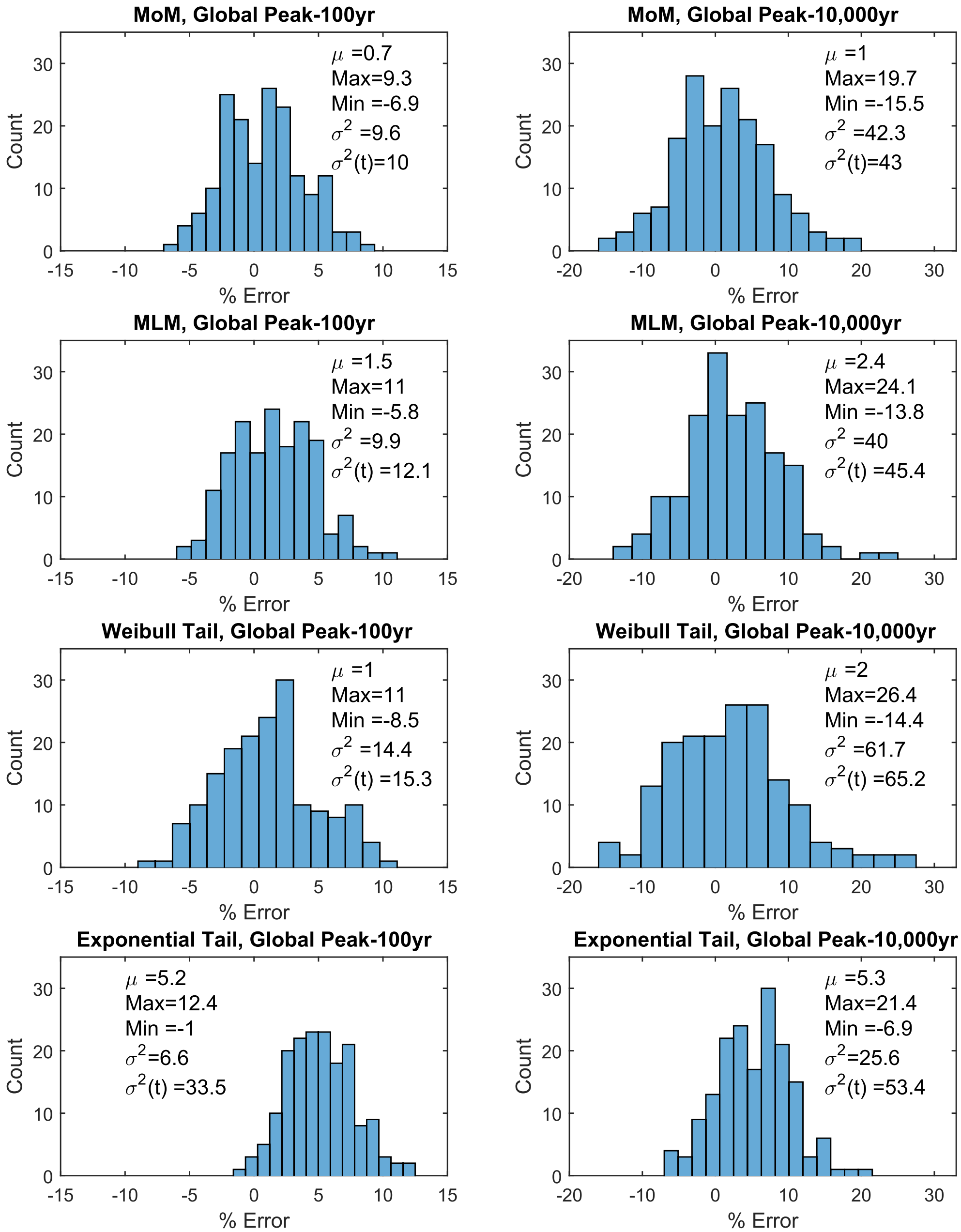


Fig. 5. Prediction errors for mooring top tension of 170 simulations across different fitting methods.

increased from 3-h to say 6-h, 9-h and 12-h, as is done later in Section 4. Hence the success of the PDM method can be evaluated holistically only after Section 5.

From Fig. 5 it is evident that regardless of the fitting method, the 10,000yr condition prediction errors are more divergent when compared to the 100yr results. As the environmental conditions become harsher, the loads that are experienced by the mooring line increase. Due to the stiffness characteristics of the mooring line, the response becomes more complex and harder to predict, as the load increases. Hence, a larger spread of the prediction error is observed for the 10,000yr condition. Different fitting methods yield varying prediction errors across the **same** environmental condition. The Exponential tail fitting method produces results that on average over predict the extreme value but have the lowest spread (lowest variance measured with respect to the mean (σ^2). Due to a large positive bias (over prediction) the variance evaluated with respect to the benchmark (σ^2 (t)) is high. The Weibull tail fitting method produces results with the highest variance out of all methods for both conditions (100yr and 10,000yr) indicating that the method yields inconsistent estimates. A sensitivity of Weibull and Exponential tail method was performed to investigate the effects on prediction error if the top 30%, 20% and 10% of peaks is selected instead of top 40%. For both environmental conditions as the peak threshold increased, the bias had a negligible change, whereas the variance increased.

The above results are consistent with the findings of Cheng and Kuang [8] that taking top 30–40% peaks did not produce consistent estimates of the extreme mooring tension. This can be partially explained by the decrease in peak data as the threshold is increased, which influences the distribution parameter estimation and in turn, produces inconsistent estimates of the extreme response. Furthermore, both tail methods only consider the top portion of the peaks, which mainly captures the peaks of the low frequency process and exclude many of the wave frequency component peaks. Given that both Weibull and exponential distribution tail fit produced poorer results (large variability and large bias respectively) than other methods that use all global peaks, it can also indicate that both wave and low frequency peaks should be selected when applying PDM. The MLM method has the second highest mean after Exponential tail method for both environmental conditions, indicating bias in the results. Overall, the use of MoM to fit peaks produces the smallest bias, the smallest range, variance and variance with respect to the benchmark, indicating that it yields best estimates of the extreme response and it should be the preferred fitting method for execution of PDM.

5. Prediction of the extreme mooring value from multiple simulations

5.1. Effects of increasing number of simulations

As discussed in Section 3.3.5, the "prediction error" estimated from a 3-h simulation is expected to reduce systematically when the length of simulation is increased to say 6- h, 9 -h, 12- h. As the length of simulation is increased, the number of peaks increases and this leads to a more accurate definition of the parent distribution $f_x(y)$ and hence a more accurate distribution for the maxima, $f_{Yn}(y)$. Analogous to increasing the length of a simulation is to increase the number of 3-h simulations. For N number of simulations taken together, the extreme response (Y) can be taken as the mean of N extreme responses, each estimated from the peaks of a one 3-h simulation:

$$Y = \frac{Y_1 + \ldots Y_N}{N} \tag{10}$$

The standard error of the mean of N prediction errors obtained from N number of 3-h simulations can be presented as follows:

$$\varepsilon = \frac{\sigma}{\sqrt{N}} \tag{11}$$

where σ is the standard deviation of the population of prediction errors. In this case, 170 different simulations are considered, hence it is assumed that the population standard deviation can be approximated by the sample standard deviation ($\sigma = \hat{\sigma}$) [31]. The number of simulations required to achieve desired accuracy can be calculated from this expression.

As indicated in Ref. [2], the benchmark value selected was estimated by fitting Gumbel distribution to 170 maxima using MoM and selecting the MPM of the estimated distribution. In Stanisic et al. [2] other methods of estimating the MPM value are also utilised and the results exhibit some variability (within 1% of one another). The PDM executed according to the 'best practice' developed here achieves a bias less than 1% (which is within the margin of error of the benchmark) and hence PDM can be regarded as unbiased (μ = 0). On this basis a confidence interval can be evaluated within which the prediction errors lie. This is based on Central Limit Theory. The interval will depend on the level of confidence chosen and can be captured by confidence factor F as follows:

$$Interval = \mu \pm F \times \varepsilon \tag{12}$$

If the errors are distributed normally, 90% of data will lie within 1.65 standard deviations from the mean (μ ± 1.65σ). Hence F = 1.65 for a confidence level of 90%.

Table 4 indicates the number of 3-h simulations that should be considered, so that the interval of ±4%, will contain the evaluated prediction error with probability of 0.9. Results for mooring tension for both environmental conditions are presented. For the 100yr condition, two 3-h simulations are required, while for the 10,000yr condition peaks from seven 3-h simulations need to be utilised. The reduction of error as the number of simulations is increased is visually presented in Fig. 6. The histograms are obtained using all different combinations of prediction errors from the 170 time domain simulations for 100yr and 10,000yr conditions for N = 1, 2, 3 and 4. For the 100yr conditions at $N = 4$ the prediction error will be within 2.5% with 90% confidence. When choosing 2 simulations to predict the extreme response, 90% of such samples will generate a response which is accurate to within ± 3.6%. For the 10,000yr condition, the error at N = 4 is ± 5.3%, indicates that more simulations are required to achieve the desired accuracy.

5.2. Turret offset

In order to validate the above recommendations, the response of the total turret excursion was studied using PDM. The proposed

Table 4
Number of 3-h simulations required for 90% confidence interval to achieve results within± 4%.

	Mooring Tension		Turret Excursion	
Simulation Number (X)	100yr	10,000yr	100yr	10,000yr
	2	7	4	6

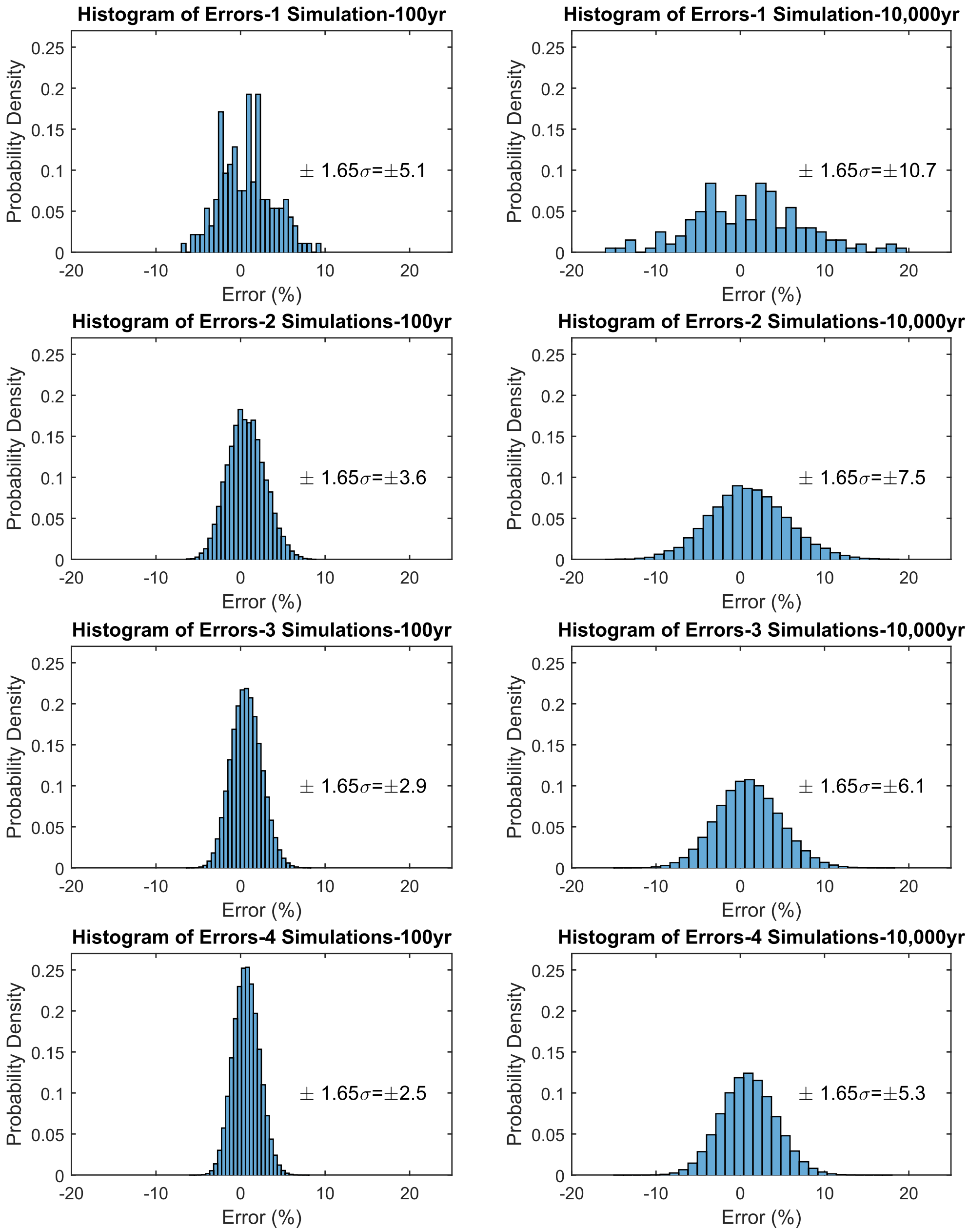


Fig. 6. Prediction errors of mooring line load from peaks of several simulations, N = 1, 2, 3, 4.

'best practice' approach of the method was applied and 170 prediction errors were estimated for each environmental case. Similar to Fig. 6, Fig. 7 shows how the intervals that contain the prediction errors vary as the number of simulations is increased from 1 to 4. After 4 simulations, the prediction errors are accurate to within ± 3.8% for 100yr and ± 5% for 10,000yr. It is calculated that six simulations are required in order to achieve accuracy within ± 4% for 10,000yr. The required number of simulations for turret excursions are summarised in Table 4, together with those for mooring tension. It can be seen that for both mooring load and for turret offset, results accurate to within ± 4%, can be obtained with four 3-h simulations for the 100yr condition and seven 3-h simulations for the 10,000yr condition.

5.3. Discussion of extreme responses

It is noted that for 100yr conditions the spread of prediction errors is somewhat higher for the turret than for the mooring tension. The main reason for this is that the number of peaks in 3-h simulation for the turret offset in this application (typically 45) is much smaller than the number of peaks in mooring line response (typically 400). This can be explained by the fact that the mooring line

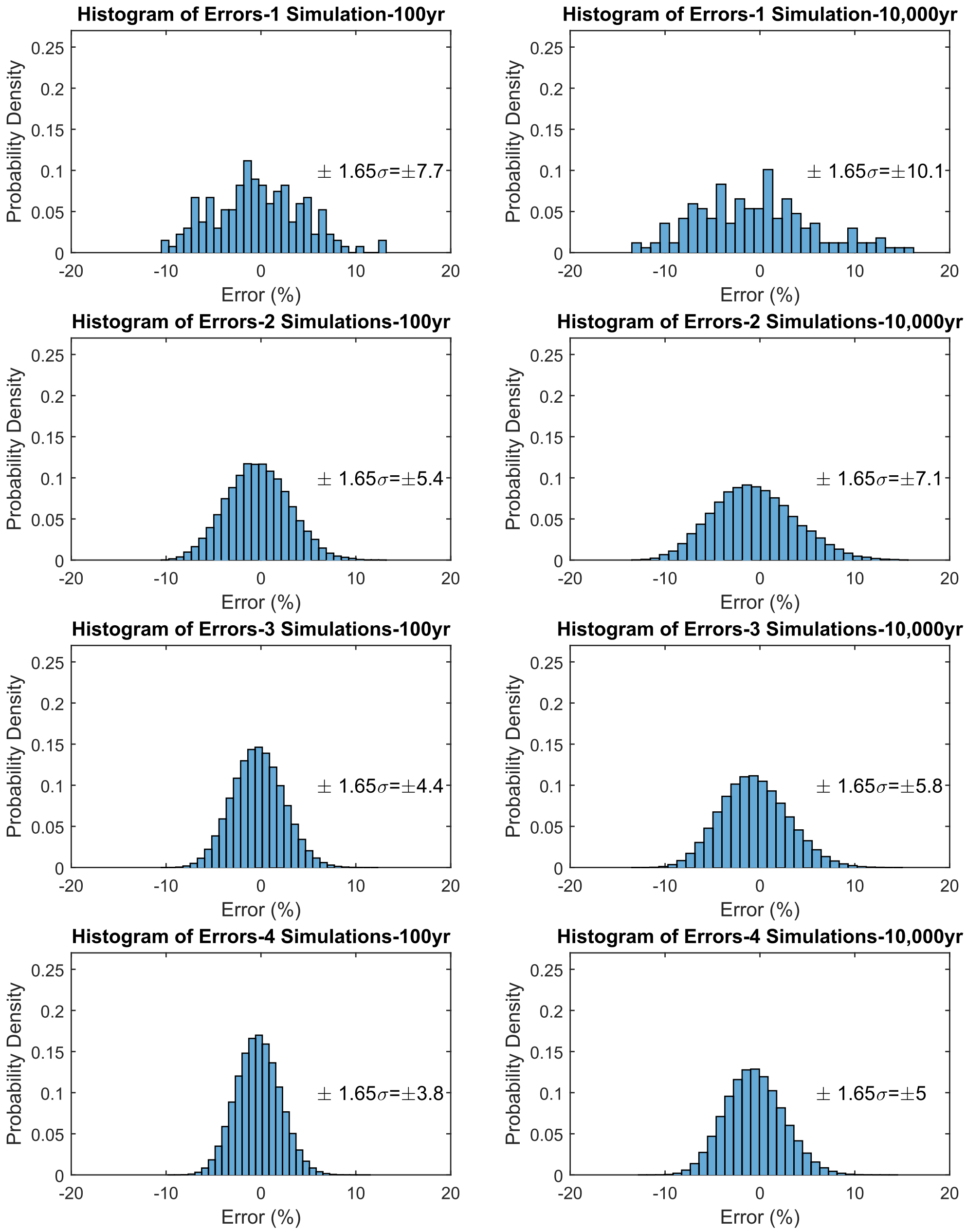


Fig. 7. Prediction errors of turret excursion from peaks of several simulations, N = 1, 2, 3, 4.

load is influenced by two processes (at wave frequency and low frequency) while the turret offset is dominated by low frequency components. The dominance of the low frequency in turret excursion results in higher number of simulations required for a desired accuracy of the extreme response (four simulations vs two 3-h simulations required for mooring tension). It is worth noting that the ratio of the number of simulations required (4/2 = 2) is not the same as the ratio of the number of peaks (400/45 = 8.9).

The 10,000yr turret response is also mainly governed by low frequency process (mooring tension has typically 600 peaks while turret excursion gives 80 peaks in one 3-h interval). However, the number of simulations required is slightly higher for mooring tension (see Table 4). This can be explained by the non-linear characteristic of the mooring line response which is evident in the 10,000yr conditions. The non-linearity increases the spread of the response and the prediction error for the mooring line load compared to the turret. Hence larger number of simulations is required for 10,000yr tension response compared to the turret offset. It follows that the number of response peaks has an influence on the number of simulations required, however other properties of the response (non-linearity of mooring line) can out govern this. Overall, it is recommended to use four and seven simulations respectively for 100yr and 10,000yr condition, both for mooring and turret response. The case studied in this paper is quite demanding for the PDM (weathervaning vessel, low frequency and wave frequency play a role, non-linear stiffness characteristic) and hence the "best practice" procedure developed is expected to work well for many applications. However, the number of simulations needed for good convergence can be case specific.

The relative importance of low frequency and wave frequency mooring tension responses is further illustrated below examining the standard deviation of responses. For 100yr conditions, the standard deviation of wave frequency of 362 kN is smaller than that of low frequency of 524 kN, indicating that low frequency dominates the response. For 10,000yr conditions, the wave frequency standard deviation increases to 1208 kN as opposed to low frequency which increases to 761 kN. One important reason for this is the non-linear mooring stiffness characteristic, which makes the wave frequency more pronounced. This also leads to a higher number of peaks as seen in Table 3.

6. Long term distribution

An efficient and accurate method of estimating the MPM response for a given sea state has been presented in Section 4 and Section 5. This method can be used to develop the design mooring responses based on design sea states. It can also be used to develop the long term distribution of mooring responses for use in a reliability analysis. This latter aspect is addressed in this section.

In a full long term approach the response of the floating system is evaluated for all seastates in a wave scatter diagram [13] or for each sea state (above a certain threshold) in a hindcast [12], [18]. In the latter the metocean time history is transformed into a mooring response time history. This is used to develop the response distribution to a random cyclone and by taking into account the arrival rate of cyclones as a Poisson process, the long term distribution of mooring responses is developed on MPM basis. This is convoluted with the short term distribution of maximum line response (within a stationary sea state) to develop long term statistics of mooring load. A full approach may be too time consuming if the system under consideration is complex. To evaluate vessel response for every 1-h sea state above the threshold H_S, in a 50 year hindcast would require several thousands of response calculations. To overcome this challenge, the approach adopted by van Zutphen and Christou [18] is to use a simplified and automated algorithm for vessel response, recognising that this comes at the expense of accuracy.

In this section the long term response distribution is developed by using the most critical joint metocean conditions (see Section 2) for return period of 100yr, 10,000yr and 1,000yr as a basis for the long term response distribution founded initially on MPM responses (i.e. excluding short term variability). The short term variability is accurately quantified, through the PDM "best practice" method and convoluted with the long term MPM response to develop the long term distribution. Some approximation is introduced here through the assertion that use of the joint metocean conditions, with annual exceedance probability of 10^{-2} in a response analysis, will result in MPM response with the same annual exceedance probability.

The two distinct sources of randomness (variability) which are being quantified and convoluted into the long term distribution of mooring response are:

1 Long Term storm intensity variability: All the joint metocean conditions, as described in Section 2, with annual exceedance probabilities of 10^{-2} and 10^{-4} are used to identify the conditions which result in the most critical mooring responses. The PDM method is then used to estimate MPM responses of the mooring system for these nominal exceedance probabilities. Additional conditions for an exceedance probability of 10^{-3}/annum would be useful. A probability distribution through these results provides the long term response intensity.
2 Short term variability: This is quantified by using a detailed model of the weathervaning vessel together with the PDM method to develop the PDF for the maxima in a 3-h storm. The short term variability is described by a Gumbel PDF whose parameters vary with the relative magnitude of the response.

The long term distribution is developed step-by-step and is summarised below.

Step 1- Carry out mooring analyses for each of the design metocean conditions, corresponding to 100yr, 10,000yr and 1,000yr (if available) return period, to identify the critical conditions leading to maximum response for each return period. Obtain accurate MPM values for each of these critical conditions using the "best practice" PDM method. The maximum responses may be denoted by their MPM values as: Y_{MPM100}, $Y_{MPM10,000}$ and $Y_{MPM1,000}$.

Step 2- Fit appropriate extreme value distribution through available Y_{MPM} thereby developing the long term distribution of mooring line load. This is based on the MPM values and therefore excludes the short term variability. For the mooring response in this study, the fitted long term distribution is:

$$P(y > Y) = 1.26e^{-4.83\frac{Y}{Y_{MPM100}}} \tag{13}$$

The distribution was developed using three values Y_{MPM100}, $Y_{MPM10,000}$ and $Y_{MPM1,000}$. An exponential format was fitted to these points by utilising least squares regression, leading to the constants given in Eq. (13). For the exponential to represent a "true" probability distribution function, the first constant needs to be unity. However, Eq. (13) leads to more accurate predictions over the range of interest from 10^{-1}/annum to 10^{-5}/annum, and not on the entire range. A Weibull distribution can be used in place of Eq. (13) and identical results will be obtained (less than 1% difference). For $Y_{MPM1,000}$, the distribution was estimated by using seven simulations, as recommended for 10,000yr condition in Section 5.

Step 3- Obtain an expression for the short term variability in a 3-h sea state by examining the distributions obtained in Step 1

Table 5
Values of parameter A for short term variability of mooring line loads.

Return Period	Mooring Response (kN)	Value of A
100yr	7830	20
1,000yr	11498	13
10,000yr	15290	9.5

(based on Eq. (1) and fitting a Gumbel distribution to each). The distributions of short term variability developed in this paper for mooring line responses are described by Eq. (14) with the parameter A varying with return period (or mooring line load) as per Table 5. Intermediate values of A are interpolated, with the basis that A varies with the log of the return period.

$$P\left(\frac{Y}{Y_{MPMi}}\right) = \exp\left[-\exp\left(-A\left(\frac{Y}{Y_{MPMi}} - 1\right)\right)\right] \tag{14}$$

Fig. 8 presents Eq. (14) graphically and shows how the distribution becomes broader as the return period increases from 100yr through to 1,000yr and 10,000yr. This is due to the non-linearity of the mooring response as the line stiffness increases non-linearly with offset at higher load levels. The distributions in Fig. 8 are normalised with respect to 10,000yr MPM values for comparison purposes.

Step 4- Combine long term variability from Step 2 with short term variability from Step 3 to obtain long term variability including contribution from short term. This convolution is carried out in two distinct ways resulting in identical results, thereby confirming that the evaluation method is sound.

Step 4 – Alternative A: the convolution is represented as a product of: 1) probability interval δY_{MPM} of getting a sea state which leads to a mooring response Y_{MPM} and 2) probability that the mooring line response exceeds Y, given that the MPM value of mooring response is Y_{MPM} (see Fig. 9). The above needs to be integrated over all values of mooring response (from 0 to infinity) but a more practical response range is (half of 100yr response to twice the 10,000yr response). The integral can be represented by the following expression:

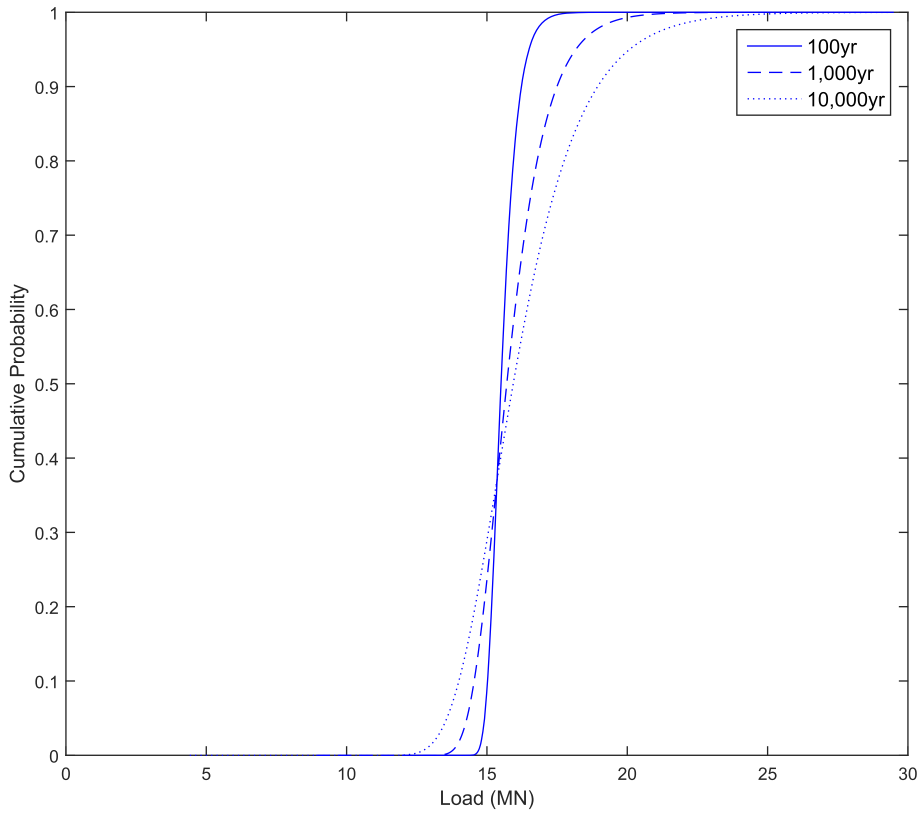


Fig. 8. Short term variability cumulative distribution functions for 100yr, 1,000yr and 10,000yr scaled to 10,000yr MPM load.

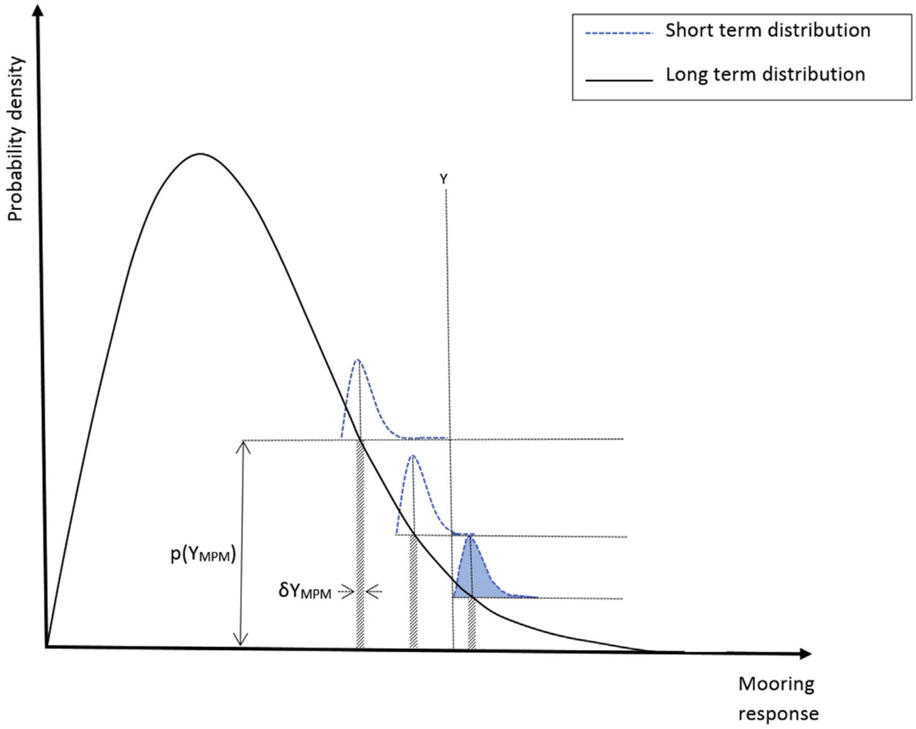


Fig. 9. Short term variability convolution.

$$P(y > Y) = \int_0^{\infty} \left[1 - P\left(\frac{Y}{Y_{MPM}}|Y_{MPM}\right)\right]. \, p(Y_{MPM}). \, dY_{MPM} \tag{15}$$

In Eq. (15) $p(Y_{MPM})$ is obtained by differentiating Eq. (13), namely:

$$p(Y_{MPM}) = 1.26*\frac{4.83}{Y_{MPM100}}*e^{-4.83\frac{Y_{MPM}}{Y_{MPM100}}} \tag{16}$$

Step 4- Alternative B: the convolution is represented as a product of: 1) probability interval δq of getting a sea state of intensity defined by return period r in the range r_i to $r_{(i+1)}$ and 2) probability that the mooring line load exceeds Y, given that the sea state intensity lies between r_i to $r_{(i+1)}$. The above needs to be integrated over all return periods. It can be represented by the following expression:

$$P(y > Y) = \int \left[1 - P\left(\frac{Y}{Y_{MPMi}}\right)\right] \delta q \tag{17}$$

The integral in Eq. (17) has to be solved numerically as $P(Y/Y_{MPMi})$ is a function of the sea state of intensity r_i. The probability interval δq is related to the return periods r_i and $r_{(i+1)}$ through:

$$\delta q = \left(1 - \exp\left(-\frac{1}{r_i}\right)\right) - \left(1 - \exp\left(-\frac{1}{r_{i+1}}\right)\right) = \exp\left(-\frac{1}{r_{i+1}}\right) - \exp\left(-\frac{1}{r_i}\right) \tag{18}$$

Hence Eq. (17) can be written as:

$$P(y > Y) = \sum_{r=5}^{r=100,000} \left(\exp\left(-\frac{1}{r_{i+1}}\right) - \exp\left(-\frac{1}{r_i}\right)\right). \left(1 - P\left(\frac{Y}{Y_{MPMi}}\right)\right) \tag{19}$$

The range $r = 5$–100,000yrs is selected to cover the entire range of interest. Y_{MPMi} is a function of return period r_i.

Approach 4A and 4B produce same results. The long term distribution of mooring line response for the tropical cyclone environment considered here is shown in Fig. 10, (i) excluding short term variability and (ii) including short term variability. As expected the long term distribution which accounts for short term variability leads to a higher response than the one which does not account for short term variability, as seen in Table 6. The contribution from short term variability is about 3% at the 100yr level and 8% at the 10,000yr level. It is seen from Table 6 that at the 100yr level the 3% contribution needed to account for short term variability corresponds to moving from the MPM value (37th percentile) to the 60th percentile, while at the 10,000yr level the 8%

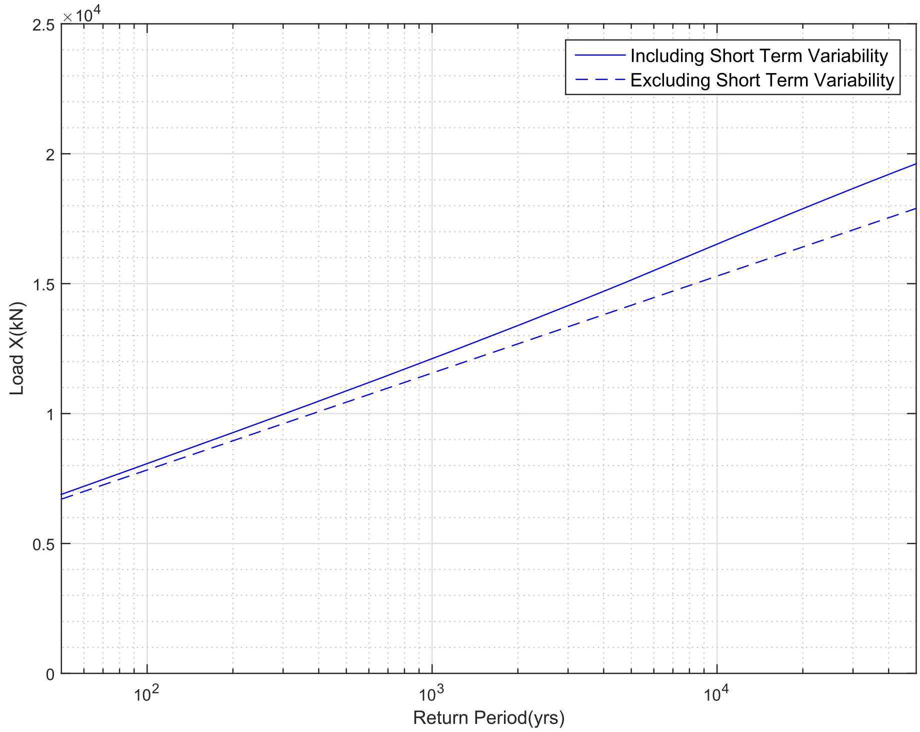


Fig. 10. Long term distribution of mooring line load excluding and including short term variability.

Table 6
Long term distribution extreme loads and equivalent percentiles.

Return Period	Extreme Load-Short Term Variability Excluded (kN)	Extreme Load-Short Term Variability Included (kN)	Adequate Percentile to compensate for Lack of Short Term Variability
100yr	7830	8072	60th
10,000yr	15290	16520	60th

contribution corresponds to moving from the MPM value to again the 60th percentile. The 60th percentile is very close to the mean for Gumbel distribution. These percentiles are much lower than the 90th percentile recommended in Refs. [32] and [33]. The main reason for this is that in a tropical cyclone environment, which is the case in this paper, the long term distribution of wave height is rather steep and this leads to a steep distribution of mooring line response (the 10,000yr response is twice as high as the 100yr response). As a consequence the contribution from long term distribution is dominant and short term variability in a tropical cyclone environment is less pronounced, compared to a winter storm environment such as North Sea.

Discussion of methods for estimating long term response: A method has been used here for estimating the long term mooring response for a weathervaning vessel. The prime reason which drove the authors to this simplification is that the full long term distribution method is too cumbersome requiring the response of the mooring system to many thousands of sea states. In parallel with this, a response-based method as described by van Zutphen and Christou [18] (see Appendix A), based on calculating the response to every relevant sea state in the hindcast, has also been applied to this problem. The prime objective of the response-based method is to develop metocean conditions for design, which are appropriate for the mooring response. Although a detailed comparison between the two approaches is beyond the scope of this paper, it is relevant to note that the two approaches lead to comparable results, when used as intended. Since the representation of vessel and mooring used in response-based method is simplified, the prediction of mooring responses is not as accurate. However, the method does well in identifying the metocean conditions which result in 10^{-2} or 10^{-4} responses. Thus, when these metocean conditions are used with a detailed coupled model, so as to compare directly with the most critical conditions used in this paper, accounting for short term variability effect, the agreement is rather good (typically within 15%) given that extrapolation is performed to long return periods.

This increases the confidence that both methods are credible and reliable. An advantage of the response-based method is that it results in a small number of metocean design cases. An important attribute of the method presented in this paper, is that although the initial list of joint metocean conditions for a specified exceedance probability is large, the identification of the most critical conditions can be done efficiently by simplified methods for screening purposes, since these results do not contribute to the extremes. Furthermore, the development of joint metocean conditions for specified exceedance probabilities, is required as part of the design process, to capture extreme responses for other purposes, e.g. maximum accelerations at various locations on the topsides for the design of topsides structures, maximum vertical motion at the turret, most extreme situations for green water freeboard exceedance, etc. Different metocean conditions from this list govern different aspects of the design. Hence each of these methods has its merits and they may be considered as complementary.

7. Conclusion

A "best practice" procedure has been presented for applying the Peak Distribution Method (PDM) to determine the extreme design response for mooring lines under 100yr and 10,000yr conditions. The study was performed with respect to a large turret moored vessel with catenary mooring system in tropical cyclone environment. The water depth for which the system was analysed is 580 m. The method has been extensively compared against an accurate benchmark obtained by carrying out a large number (170) of 3-h time domain simulations. While conventional methods which use only the maximum value in each 3-h simulation need 30–40 simulations to achieve sufficient accuracy, it is demonstrated that the PDM method can achieve consistently the same accuracy with a very small number of simulations. The main elements of the "best practice" procedure are:

(i) Peaks in mooring line response are defined as maximum values between two mean up crossings
(ii) Peaks are fitted to a two parameter Weibull distribution using the entire population of peaks rather than upper tail
(iii) The method of moments is used for the fit
(iv) The number of peaks needed for accurate fit corresponds to four 3-h simulations under 100yr conditions and seven 3-h simulations for 10,000yr conditions.

These conclusions are based on very demanding application of the PDM and hence are expected to work well for many applications. However, the number of simulations needed for good convergence can be case specific.

The PDM method is used to develop a long term distribution for mooring line response, which accounts for short term variability, for use in reliability analysis. In order to achieve this, in this paper, the short term variability of the maximum line load, in a given 3-h storm, was combined with the long term distribution of mooring line loads, defined by the MPM load as a function of the return period. It was found that the contribution from short term variability increases the long term load by 3% at the 100yr level and by 8% at the 10,000yr level. These surprisingly low contributions arise because the tropical cyclone environment exhibits a steep long term response. It is shown that for this tropical cyclone environment, the contribution from short term variability can be considered to be included by selecting the 60th percentile rather than the MPM (37th percentile of the Gumbel distribution) to represent the mooring design load.

Acknowledgements

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Appendix A. Overview of Response Based Method

Long term response methodology involves the transformation of environmental variables into responses. It can either be done through a wave scatter diagram as described by Naess and Moan [13] or through response based methodology.

Using the response-based methodology, the long term data set of responses can be obtained from long term data set of environmental variables through a response function. Tromans and Vanderschuren (1995) described the response function for base shear and overturning moment of a fixed steel jacket in terms of a function involving the wind, wave and current. A similar methodology was later applied to a weathervaning FPSO by van Zutphen and Christou [18]. This methodology is summarised in Fig. A1 (where P(Y) denotes a cumulative distribution while p(Y) denotes a probability density function). The starting point is a long record of hindcast metocean conditions providing simultaneous values of all the relevant metocean parameters over a period of typically 30–50 years. The main steps of the response-based methodology are summarised below.

a) From the hindcast record identify storms S_1, S_2, S_3 .... S_N as periods when the significant wave height exceeds a threshold H_T (see Fig. A1 (a)).
b) For each storm S_1, S_2,. S_N use a response model to calculate vessel response distribution P(Y| storm S_i) and its most probable maximum value Y_{MPM} (see Fig. A1 (b)). This response model uses a simplified definition of the mooring lines, the risers and the vessel so as to achieve computational efficiency. It is not an analytical function of the metocean variables but a numerical algorithm for calculating vessel heading and thereafter the vessel offset.
c) From the results in Step (b) estimate short term variability assuming that responses are similar (assumed identical) when normalised with respect to their respective most probable maximum value. This is an approximation. The short term variability is denoted by $P(Y|Y_{MPM})$, (see Fig. A1 (c)).
d) Use Y_{MPM} values to establish the long term exceedance probability, $P(\,Y_{MPM})$, based on most probable exceedances (see Fig. A1

(d)).

e) The short term variability of the response $P(Y|Y_{MPM})$ established in Step (c), is convoluted with the long term distribution of Y_{MPM} (from Step (d)) and the extreme response for any random storm is obtained as follows (see Fig. A1 (e)):

$$P(Y|r.\ s.) = \int_0^\infty P(Y|Y_{MPM})p(Y_{MPM})dY_{MPM} \tag{A1}$$

f) Considering the storm arrival as a Poisson process, with arrival rate ν, the extreme response over an interval of T years has the probability distribution:

$$P(Y|\nu T) = [P(Y|r.\ s.)]^{\nu T} \tag{A2}$$

as seen in Fig. A1 (f)). The MPM value from this distribution is taken to be the T-year response, Y_T, corresponding to return period T.

One can then estimate the extremes of response variable which correspond to a certain return period. The final step is to obtain the response based environmental conditions. One way to achieve this is to study the extreme events in the hindcast data and magnify them to reach the extreme response.

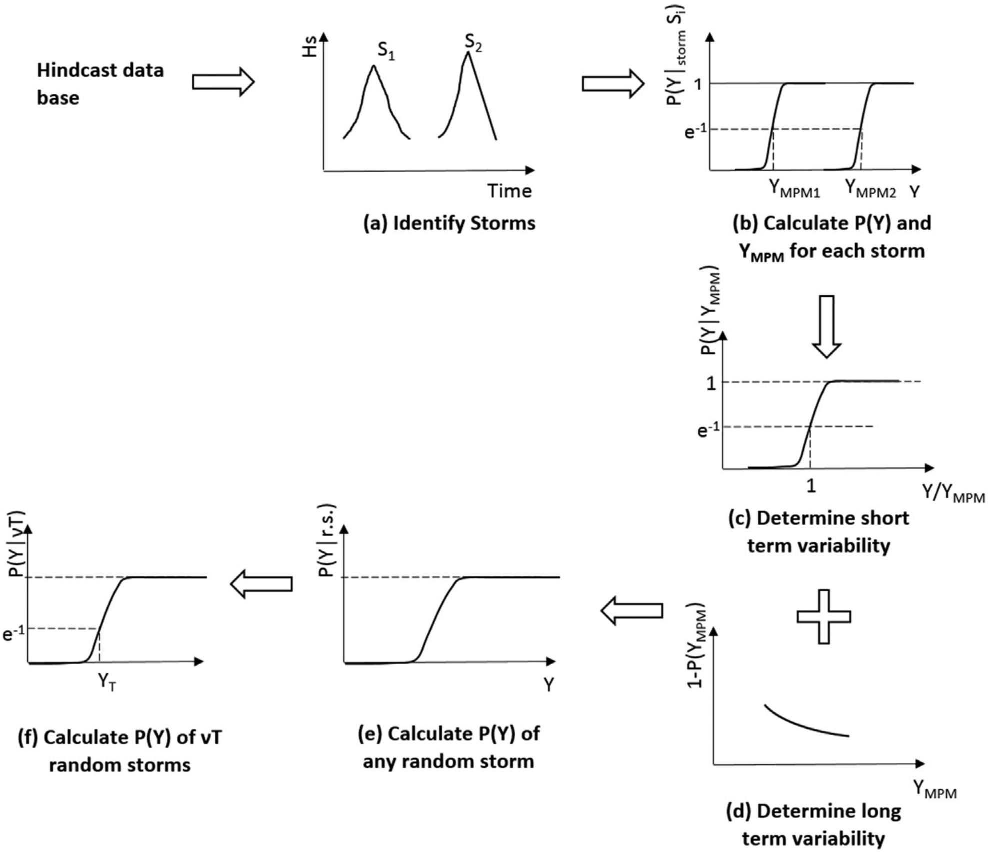

Fig. A1. Evaluation of Extreme Response Statistics (Tromans and Vanderschuren [12]).

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