

Review

A review of wave-energy extraction

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Abstract

Comparing ocean-wave energy with its origin, wind energy, the former is more persistent and spatially concentrated. In this paper wave spectrum parameters related to transport, distribution and variability of wave energy in the sea are educed. Many different types of wave-energy converters, of various categories, have been proposed. It is useful to think of primary conversion of wave energy by an oscillating system as a wave-interference phenomenon. Corresponding to optimum wave interference, there is an upper bound to the amount of energy that can be extracted from a wave by means of a particular oscillating system. Taking physical limitations into account, another upper bound, for the ratio of extracted energy to the volume of the immersed oscillating system, has been derived. Finally, the significance of the two different upper bounds is discussed.

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Keywords: Spectral energy parameters; Wave absorption as wave interference; Upper bounds to converted energy

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1. Introduction

Impressed by the force of ocean waves, inventors have, for more than two centuries, proposed many different devices for utilising wave power for human purposes [1–5]. As petroleum became the most important modern source of energy, the interest for wave-energy utilisation faded after the First World War. In the late 1940s, the Japanese wave-power pioneer Yoshio Masuda [6] started to test and develop wave-energy devices. Two inventive European pioneers, Stephen Salter and Kjell Budal, initiated in 1973 wave-power research at universities in Scotland and Norway, respectively. In the US, Michael E. McCormick was an early academic wave-power researcher. In years following the oil crisis in 1973, many researchers at universities and other institutions took up the subject of wave energy. Larger government-funded R&D programmes were started, during the late 1970s, in some European countries, the UK, Sweden and Norway—subsequently also in other countries. During the early 1980s, when the petroleum price declined, wave-energy funding was drastically reduced [7]. A few first-generation prototypes were, nevertheless, tested in the sea. More recently, following the Kyoto protocol on reduction of CO₂ emission to the atmosphere, there is again a growing interest for wave-energy R&D in many countries. As many new young researchers are now entering into this research field, the present paper is intended to convey an overview on knowledge accrued until now, but in particular during years around 1980.

The global power potential represented by waves that hit all coasts worldwide, has been estimated to be in the order of 1 TW (1 terawatt = 10^{12} W) [8]. If wave energy is harvested on open oceans, energy that is otherwise lost in friction and wave breaking, may be utilised. Then the global wave-power input is estimated to be one order of magnitude larger ($\sim 10^{13}$ W), a quantity that is comparable with the world's present power consumption. Although this is only a small proportion of the world's wind power potential, which, in turn, is only a small portion of global solar power, ocean waves represent an enormous source of renewable energy. As solar energy is converted to wind energy, the time-averaged power flow is spatially concentrated, from an intensity of typically 0.1–0.3 kW/m² horizontal surface of the earth to 0.5 kW/m² envisaged area perpendicular to wind direction. As wind energy is converted to wave energy, even more spatial concentration takes place. Just below the ocean surface, average power flow intensity is typically 2–3 kW/m² of envisaged area perpendicular to direction of wave propagation. This increase in power intensity, and also the fact that wave energy is more persistent than wind energy, stimulate motivation and hope for developing the, still rather undeveloped, wave-power technology to a prosperous mature level in the future. If the technology can be successfully developed, the market potential is enormous.

In the present paper, the main subject of the next section is the energy associated with ocean waves. Then follow, first, a section on fundamental principles for absorption of wave energy and on various ways of classifying wave-energy converters into different categories, and secondly, a section on mathematical description of wave-energy extraction. Before the final section with concluding remarks, there is a section, where upper bounds to the extracted wave energy are discussed.

2. Ocean waves and their energy resource

The term wind sea is used for waves that are actively growing due to forcing from local wind. These waves travel in or close to the local wind direction. Swell is the term used to

describe long-period waves that have moved out from the storm area where they were generated. Swells spread out over the ocean with little energy loss. They are somehow analogous to waves spreading out from the splash of a stone thrown into a pond. Swells in deep water will, typically, have wavelengths of 100–500 m whilst wind seas may range from a few metres to 500 m depending on the wind speed. In this context, deep water is understood to mean that the water depth exceeds about one third of the wavelength. Then the seabed has a negligible influence on the wave. An instantaneous picture of the ocean offshore will generally reveal several wave trains with different wavelengths and directions. Swells may coexist with wind sea. In contrast to a single-frequency sinusoidal wave propagating in a particular direction, a real sea wave may be considered as composed of many elementary waves of different frequencies and directions.

Per unit area of sea surface a stored energy amounting to an average of

$$E = \rho g H_{m0}^2 / 16 = \rho g \int_0^\infty S(f) df, \quad (1)$$

is associated with the wave, where $\rho = 1030 \text{ kg/m}^3$ is the mass density of sea water, and $g = 9.81 \text{ m/s}^2$ is the acceleration of gravity, whereas H_{m0} is the significant wave height for the actual sea state. This stored energy is equally partitioned between kinetic energy, due to the motion of the water, and potential energy. The latter half-portion is due to mechanical work performed when the flat water surface is being deformed to a wavy. This work corresponds to water lifted against the gravity force from wave troughs to wave crests. For wavelengths exceeding a few centimetres, the capillary force (surface tension) has a negligible contribution to the potential energy. In Eq. (1), the integrand $S(f)$ is the wave spectrum [9, Section 2.2]. Its unit is m^2/Hz , and it describes quantitatively how the different wave frequencies f contribute to the wave energy. In practice, the integral in Eq. (1) is, as an approximation, replaced by a summation over a finite number of wave frequencies. For a sinusoidal wave with amplitude $H/2$, where H is the wave height (the vertical distance between crest and trough of the wave), Eq. (1) for E is applicable provided H_{m0} is replaced by $H\sqrt{2}$. Taking as a typical value, $H = 2 \text{ m}$ or $H_{m0} = 2.83 \text{ m}$, we get $E = 5.05 \text{ kJ/m}^2$.

By means of wave measurement during a certain time, e.g. 2048 s, the approximate actual wave spectrum $S(f)$ is determined through Fourier analysis. To obtain long-term statistics, such wave measurements and analyses are repeated every 3 h. In cases of “fully developed wind sea”, that is when a constant wind has blown for a sufficiently long time along a sufficiently long fetch of the ocean, then the semi-empirical Pierson–Moskowitz (PM) spectrum

$$S(f) = (A/f^5) \exp(-B/f^4). \quad (2)$$

matches fairly well with experimentally obtained wave spectra. Here $A = BH_{m0}^2/4 = 0.00050 \text{ m}^2 \text{ Hz}^4$ and $B = (5/4)f_p^4 = 0.74 g^4/(2\pi U)^4$, where $f_p = 1/T_p$ is the peak frequency (at which S has its maximum), where T_p is the peak period, and where U is the mean wind speed at a level of 19.5 m [9, Section 5.5]. For situations where the fetch is limited, the JONSWAP spectrum is more commonly applied. It is more narrow-banded than the PM spectrum.

The spectral moment of order j is defined as

$$m_j = \int_0^\infty f^j S(f) df. \quad (3)$$

Thus, the significant wave height may be defined, in terms of the zero order moment, as $H_{m0} = 4\sqrt{m_0}$. In the following discussion, where we, for simplicity, shall assume that the wave is propagating in a certain direction, say the x -direction, we shall encounter also spectral moments of other orders, such as m_{-1} and m_1 , and also spectrally defined characteristic wave periods, such as $T_{-1,0} \equiv m_{-1}/m_0$ and $T_{0,1} \equiv m_0/m_1$.

For a sinusoidal wave of period $T = 1/f$, the wave energy is transported with an energy velocity equal to the group velocity c_g . The wave-power level—defined as the transport of energy per unit width of the progressing wave front—is $J = c_g E = c_g \rho g H^2/8$. For a real sea wave, the wave-power level may be expressed in terms of the wave spectrum as

$$J = \rho g \int_0^\infty c_g(f) S(f) df = \rho g^2 m_{-1}/4\pi = \rho g^2 T_J H_{m0}^2/64\pi, \quad (4)$$

where we have assumed deep water, for which the group velocity is $c_g = gT/4\pi = g/4\pi f$. The energy period is defined as $T_J = T_{-1,0} \equiv m_{-1}/m_0$ [9, p. 53]. Taking, as typical values $H_{m0} = 2.83$ m and $T_J = 9$ s, we get $J = 35$ kW/m.

The wave-power level J may be considered to result from integration of the power flow intensity $I(z)$ over all vertical coordinates z for which there is wave motion in the sea water, $J = \int I(z) dz$. The wave power flow intensity has its maximum

$$I(0) = 2\pi \rho g m_1 = (\pi/8) \rho g H_{m0}^2/T_{0,1} \quad (5)$$

just below the sea surface $z = 0$, and it diminishes downwards in the water. For a fully developed wind sea, we have

$$I(0) = 0.0325 \rho g^{3/2} H_{m0}^{3/2} = 5 I_{\text{wind}}, \quad (6)$$

as obtained from the PM spectrum. Then the maximum wave power flow $I(0)$ just below the sea surface is five times larger than the wind power flow $I_{\text{wind}} = (\rho_{\text{air}}/2) U^3$ at level 19.5 m above the sea surface. For a mean wind speed of $U = 10$ m/s, we get $I_{\text{wind}} = 0.6$ kW/m², $I(0) = 3.2$ kW/m², $J = 14$ kW/m, $H_{m0} = 2.13$ m, $T_p = 7.3$ s, $T_J = 6.3$ s and $T_{0,1} = 5.6$ s. For a deep-water sinusoidal wave of amplitude $H/2$ and period T , Eqs. (4) and (5) are applicable, provided H_{m0} is replaced by $H\sqrt{2}$ and both of T_J and $T_{0,1}$ by T . Taking, as representative values, $H = 2$ m and $T = 9$ s, we get $J = 35$ kW/m and $I(0) = 3.5$ kW/m². The wave power flow intensity varies with the vertical coordinate z as $I(z) = I(0) \exp(2kz)$, where $k = 2\pi/\lambda$ is the angular repetency (wave number), and $\lambda = gT^2/2\pi = (1.56 \text{ m/s}^2)T^2$ is the wavelength. Integration of $I(z)$ over the interval $-\lambda/4 < z < 0$ accounts for 96% of J , while integration over the interval $-\infty < z < -\lambda/4$ accounts for the remaining 4%. In the upper z interval, the wave power flow intensity has an average value of $I_{\text{average}} = 0.30 I(0)$. Integration of $I(z)$ over the interval $z_5 < z < 0$ accounts for 80% of J , where $z_5 = -0.13 \lambda = -(0.20 \text{ m/s}^2)T^2$, is the vertical coordinate at which the wave power flow intensity is reduced by a factor of five, that is, $I(0)/I(z_5) = 5$. In the latter interval, the wave power flow intensity has an average value of $I_{\text{average}} = 0.50 I(0)$. For $T = 9$ s, $z_5 = -16$ m.

Assume that the water depth h decreases slowly as the wave approaches the coast. Then also the wavelength λ decreases monotonically, but the group velocity c_g increases to a value 20% above the deep-water value $gT/4\pi$ when h decreases to $gT^2/4\pi^2$. With further decrease of h also c_g decreases monotonically. In the shallow-water approximation, $\lambda \rightarrow T\sqrt{gh} \rightarrow 0$ and $c_g \rightarrow \sqrt{gh} \rightarrow 0$ as $h \rightarrow 0$. If the wave-power level $J = c_g E = c_g \rho g H^2/8$ remained invariant, as if the wave propagated towards the beach without energy loss, then the wave height H should increase to infinity as the group velocity is approaching zero. In

reality this is not the case, as the wave loses energy, in particular in shallow water, mainly by wave breaking and by friction against the seabed. If the shore is rocky and steep sufficiently down into the water, then wave reflection may be more important than wave dissipation.

The variability of wave conditions in coastal waters is, generally, very large compared to offshore waters. Near-shore variation in the wave climate is compounded by shallow-water physical processes such as wave refraction, which may cause local “hot spots” of high energy due to wave focusing particularly at headlands and areas of low energy in bays due to defocusing. In addition, other coastal wave processes such as wave reflection, diffraction, bottom friction and depth-induced breaking effects may have some influence.

As averaged over years, offshore wave-power levels in the range of 30–100 kW/m are found at latitudes 40–50°, and less power levels further south and north. In most tropical waters, the average wave-power level is below 20 kW/m. Offshore wave-power levels may vary from a few kW/m during calm weather to several MW/m during storms. Wave-power levels will vary over time, on many different time scales: hours ($\sim 10^4$ s), days ($\sim 10^5$ s), weeks ($\sim 10^6$ s), months, seasons ($\sim 10^7$ s) and years ($\sim 10^8$ s). There are also important wave variations on shorter time scales: wave periods ($\sim 10^1$ s) and duration of and intervals between wave groups ($\sim 10^2$ s). In spite of their importance, information on wave groups are not always taken care of by wave spectra obtained from wave records (during $\sim 10^3$ s). Availability of time series, in addition to wave spectra, from wave records, is also very desirable, concerning practical wave-energy conversion. The variation in offshore wave-power levels is quite large. According to Torsethaugen [10], there is “a factor of two between the highest and lowest yearly mean for wave energy at one particular location. The average wave energy for a winter month can be 5–10 times the mean value for a summer month. The wave energy can vary 10 times from one week to the next. The wave energy during one storm can be five times higher than the mean value for the week the storm occurs. Wave energy in wave groups can be up to 50 times the wave energy between wave groups”. Extreme storm seas contain very much wave energy and contribute significantly to yearly mean values of wave-power level. The power-capacity limitation of a wave-power plant reduces, however, the usefulness of this extreme-state contribution. It may be said that for a wave-power plant, the income has to be provided by the prevalent moderate waves, while extreme waves may be as catastrophic as for other ocean structures.

Real sea waves are composed of elementary waves propagating in different directions. A generalisation of Eq. (1) is

$$E = \rho g H_{m0}^2 / 16 = \rho g \int_0^\infty \int_{\beta_1}^{\beta_2} s(f, \beta) df d\beta, \quad (7)$$

where $s(f, \beta)$ is the direction-resolved energy spectrum. Moreover, β is the angle of incidence (with respect to some chosen x -axis), and angles in the interval $\beta_1 < \beta < \beta_2$ contribute to the wave spectrum. If waves are incident from all directions, then $\beta_1 = -\pi$ and $\beta_2 = \pi$. The power that is passing an envisaged vertical strip of unit width (1 m) with its normal pointing horizontally in direction θ , is [11]

$$J_\theta = (\rho g^2 / 4\pi) \int_0^\infty \int_{\beta_1}^{\beta_2} f^{-1} s(f, \beta) \cos(\beta - \theta) df d\beta. \quad (8)$$

If we choose $\beta_1 = \theta - \pi/2$ and $\beta_2 = \theta - \pi/2$, we have selected only wave components for which the group velocity has a positive component in the direction that makes an angle θ with the x -axis. In contrast to J_θ , the wave power level J , given by Eq. (4), is the power that, irrespectively of wave direction, passes an envisaged vertical cylinder of unit diameter (1 m). Thus, J_θ is necessarily smaller than J , unless waves always have the same direction of incidence. As an example, off the coast of the Hebrides, Mollison [11] reported values of $J = 67 \text{ kW/m}$ and of $J_\theta = 49 \text{ kW/m}$ provided an optimum value for θ was chosen, corresponding to most of the wave energy propagating from west to east. In locations off the western coasts of Europe, best values of J_θ/J may be found typically in the range of 0.6–0.75. Moreover, predominating wave directions are from south to west in the north and from north to west in the south of Europe.

The phenomenon of wave grouping, which is important in relation to wave-energy conversion, has been addressed, recently, by Saulnier and Pontes [12]. Because of wave groups, the available wave energy may vary significantly from one min to the next minute.

3. Principles for extraction of wave energy

The physical law of conservation of energy requires that the energy-extracting device must interact with the waves such as to reduce the amount of wave energy that is otherwise present in the sea. The device must generate a wave, which interferes destructively with the sea waves [13]. “In order for an oscillating system to be a good wave absorber it should be a good wave generator” [14]. It should be considered as an advantage that practically all the volume, of e.g. a heaving-float system (cf. Fig. 1), could be “used to displace fluid and thus to generate outgoing waves” [13]. Several proposed wave-energy converters have, however, relatively large proportions of “dead” volume not participating in such wave generation.

If an incident sinusoidal plane wave of power level $J = c_g E$, is interfering with a ring-shaped outgoing wave radiated from e.g. a heaving axisymmetric body, this wave-generating body can at most absorb a power of $P_{\max} = J\lambda/2\pi$, which corresponds to optimum destructive interference in this wave-geometrical case [13,15,16]. Here λ is the

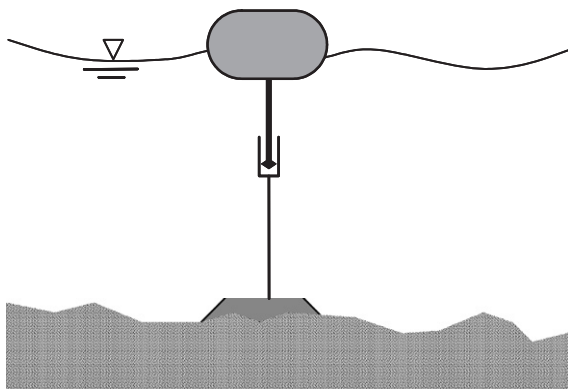


Fig. 1. Heaving body reacting against a fixed anchor. The indicated pump represents a hydraulic power take-off machinery.

wavelength. On deep water, where $\lambda = (g/2\pi)T^2$ and $J = \rho g^2 T H^2 / (32\pi)$, we then have the following upper limit for the absorbed wave power P ,

$$P < P_A \equiv c_\infty T^\delta H^2, \quad (9)$$

where $c_\infty = \rho(g/\pi)^3/128 = 245 \text{ W m}^{-2} \text{ s}^{-3}$ and $\delta = 3$. According to inequality (9) the values of absorbed power, as well as of converted useful power are bound to the region below the, fully drawn, increasing curve in Fig. 2.

On the other hand, if the incident wave interferes with equally large plane waves radiating in opposite directions from a symmetrical two-dimensional body, then this body can absorb at most half of the incident wave power [15–17]. However, if wave radiation from a two-dimensional body is sufficiently non-symmetrical, such as with a wave-maker in a wave channel or with a horizontal cylinder, which is submerged on open sea and moves in a circular orbit about a sufficiently eccentric axis (cf. Fig. 3), then all incident wave energy is potentially absorbable [18]. The famous Salter Duck [19] is an early proposal of a non-symmetrical wave-power device (cf. Fig. 4) attempting to approach complete wave-energy absorption. For complete absorption in the two-dimensional case for a width d perpendicular to the direction of wave incidence, the values $c_\infty = \rho g^2 d / (32\pi) = (986 \text{ W m}^{-3} \text{ s}^{-1})d$ and $\delta = 1$ apply in inequality (9). In this ideal case $P \rightarrow Jd$.

It depends on the phase of the generated wave, relative to that of the incident wave, whether the wave interference is constructive or destructive. Hence, optimum destructive

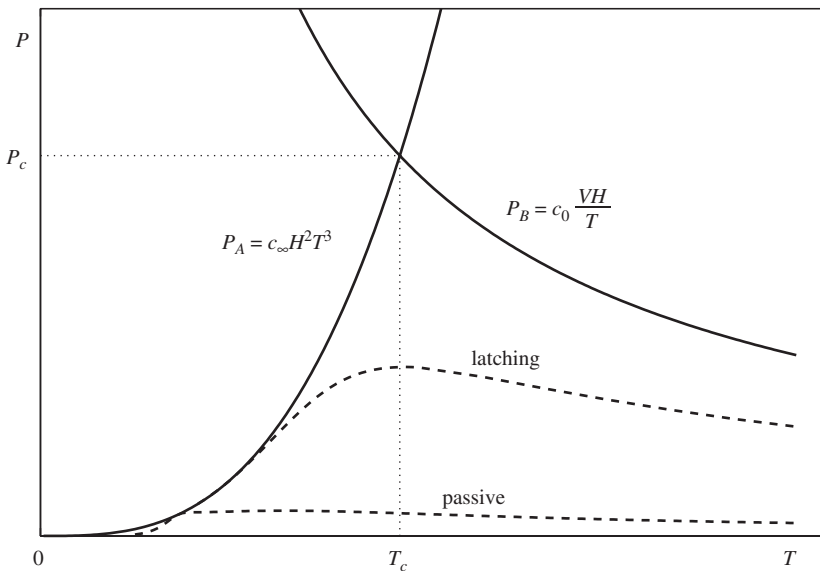


Fig. 2. Two upper bounds, P_A and P_B , for the power P that can be absorbed from a sinusoidal wave of height H and period T by means of an immersed body of volume V . The actual absorbed power P approaches the monotonically increasing curve P_A if the volume is sufficiently large ($V \rightarrow \infty$). P can approach the monotonically declining curve P_B only if the volume is sufficiently small ($V \rightarrow 0$). The two dashed curves represent the power absorbed by a semi-submerged finite-volume sphere heaving with optimum amplitude (optimum load). For the lowest curve there is no phase control, whereas for the second lowest curve phase control by the latching method is assumed.

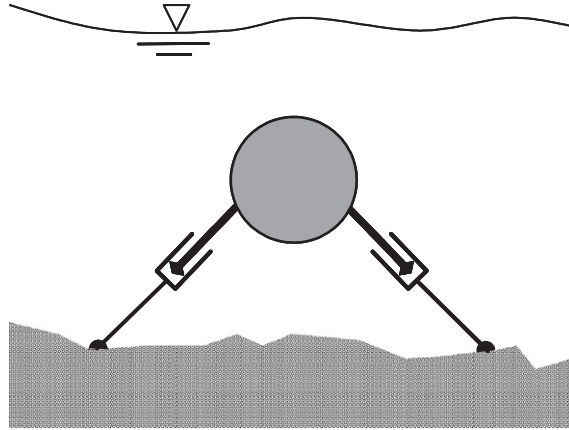


Fig. 3. The “Bristol cylinder”. A submerged horizontal cylinder moving in a circular orbit. The motion corresponds to oscillation with equal amplitudes in surge and heave, but with phases differing by 90° . Hydraulic power take-off is envisaged.

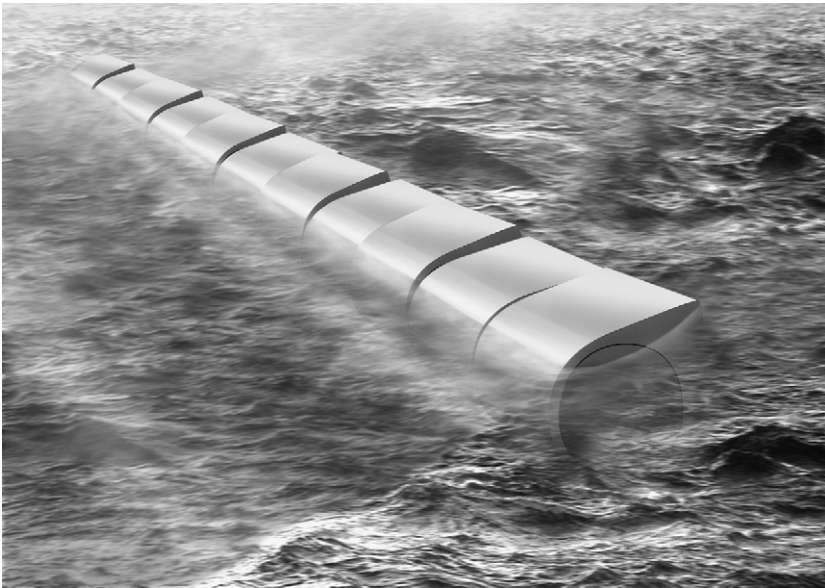


Fig. 4. A set of nodding-duck shaped bodies, performing pitch oscillation, with different phases. Wave-induced pitch moments react against a common cylindrical spine. The relative motion is utilised to run a hydraulic power take-off machinery (not shown).

interference, corresponding to maximum absorbed wave energy, is directly related to an optimum phase of the absorber’s oscillation. It has long been known that for a single-oscillator system that interacts with a sinusoidal incident wave, optimum phase is obtained at resonance [20]. If the oscillating body is of sufficiently large geometrical size, it has

a large enough bandwidth for obtaining approximate optimum phase for all frequencies within the wave spectrum. Otherwise, for reasonable body sizes, the resonance bandwidth is narrow, and then phase control methods may be adopted for approaching optimum phase for wave frequencies outside the bandwidth.

Furthermore, optimum destructive interference requires an optimum amplitude of the generated wave, and hence of the oscillator system. Optimum amplitude may be obtained by selecting an optimum load on the oscillator system [13,15].

Except in the simplest case, the oscillatory motion has more than one degree of freedom. For a multi-oscillator system, optimum phases are not necessarily obtained at resonance [21]. In order to obtain optimum oscillation, Newton's law demands an optimum total force for each degree of freedom. In addition to wave forces and other hydrodynamic forces, as well as friction forces, the total force may include contributions from control devices and energy-conversion machinery. For various wave conditions, these latter force contributions should be selected by human or computer intelligence in order to achieve an oscillation that is as close to optimum as possible. In order to apply these force contributions, some kind of force reaction is needed. A force applied to an immersed body could, for instance, react against an anchor on the seabed (cf. Figs. 1 and 3), or alternatively, against another immersed body, which oscillates with an amplitude and/or a phase different from the former body (cf. Figs. 4 and 5).

Primary conversion of wave energy may be described as follows: as a result of the destructive wave interference, energy is transferred from the sea to the oscillating system, where it may be found as kinetic and/or potential energy. The oscillating system could be one or several oscillating floating bodies or oscillating solid or flexible members. It could alternatively be oscillating water within one or several structures, floating or based on seabed or on shore.

In a second conversion step, the mechanical energy, which has been captured by the oscillating system, may be made more useful by means of conversion machinery that may deliver useful energy, e.g. through a rotating shaft. In 19th century proposals, the primary mechanical energy may be transmitted to pumps or other suitable energy converting

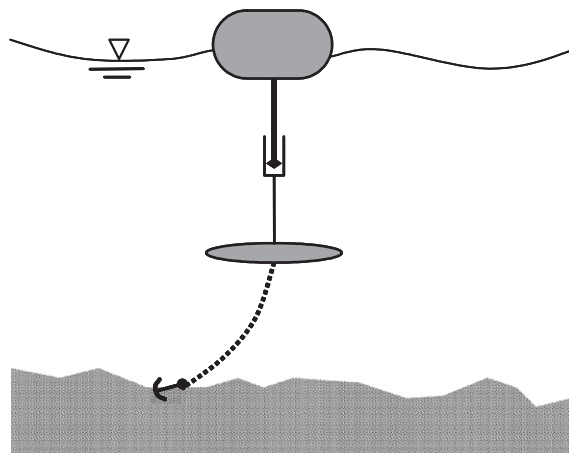


Fig. 5. Heaving body reacting against a submerged body. Hydraulic power take-off is envisaged. Slack mooring is indicated to keep the system in location.

machinery by mechanical means (such as racks and pinions, ratchet wheels, ropes and levers). In contrast, devices for control and power take-off in modern proposals also may comprise controllable valves, hydraulic rams and various hydraulic and pneumatic components (including turbines), as well as electronic hardware and software. If the end use is electric energy, an electric generator may serve for a tertiary conversion step in the power train. In order to even the effect of wave groups, which represent a very large variation of wave-energy input, it is strongly recommended that a short-time ($\sim 10^2$ s) energy storage is incorporated, as early as possible, in the power train [22].

The many different proposals and principles for wave energy conversion may be classified in several ways. These are useful for seeing the differences and similarities between various wave energy converters (WECs). They may be classified, e.g. according to location (off-shore, near-shore or onshore; floating, submerged or bottom-standing), according to type of energy conversion machinery (mechanical, hydraulic, pneumatic or directly electrical), and according to type of energy for end use (electricity, water pumping, desalination of seawater, refrigeration, water heating, propulsion).

WECs may also be classified according to their horizontal extension and orientation. If the extension is very small compared to a typical wavelength, the WEC is called a *point absorber*. On the contrary, if the extension is comparable to or larger than a typical wavelength, the WEC is called a line absorber, but the terms attenuator and terminator are more frequently used. A line absorber is called an *attenuator* or a *terminator* if it is aligned parallel or normal to the prevailing direction of wave propagation, respectively. An early example of a terminator was proposed by Salter, several ducks pitching with respect to a common horizontal cylindrical cylinder, the so-called spine see Fig. 4. It was found necessary to divide the long cylinder into shorter cylindrical sections, hinged together. The spine still resists twisting, but complies to bending moments. This spine development has now evolved to the Pelamis [23,24], which is a device of the attenuator type. A typical device of the point-absorber type is a heaving axisymmetric body [13], a pulsating submerged volume, such as the AWS device [25], or an open-sea located oscillating water column (OWC) device [20,26]. Most of the proposed OWC devices have pneumatic power take-off.

4. Mathematical description of wave-energy extraction

As a mathematical illustration of wave-energy extraction, we shall, for simplicity, consider a body oscillating in one mode only, e.g. the heave mode. We shall, in the following, assume that amplitudes of waves and oscillations are sufficiently small to make linear theory applicable. In cases where latching control [14,27–29] is applied, the system is not time invariant. Then instead of studying system dynamics in the frequency domain, it is better to apply time-domain analysis, as follows. The excursion $s(t)$ and the velocity $u(t)$ $\dot{s}(t)$ of the oscillating body are determined by the dynamic equation [27,28]

$$(m + A_{r\infty})\ddot{s}(t) + B_f\dot{s}(t) + k_r(t)\dot{s}(t) + Cs(t) = F_e(t) + F_u(t) \equiv F_{\text{ext}}(t), \quad (10)$$

where $F_e(t)$ is the excitation force resulting from the incident wave, and $F_u(t)$ is a force applied intentionally for control and power take-off. Further, m is the body's mass, C is the stiffness (restoring-force coefficient). The hydrodynamic parameters $A_{r\infty}$ and $k_r(t)$ are explained below. Further, B_f is a mechanical loss resistance, due to e.g. friction and viscosity. In a numerical calculation, the simplified loss force $F_f(t) = -B_f u(t)$ as used here,

could preferably be replaced by a more realistic, non-linear loss force $F_f(t,s,u)$. In Eq. (10), the star (*) denotes the operation of convolution. Moreover, $k_r(t)$ is the radiation-force impulse-response function, which is causal (that is, $k_r(t) = 0$ for all negative times, $t < 0$), and which is the inverse Fourier transform of

$$K_r(\omega) = Z_r(\omega) - i\omega A_{r\infty} = B_r(\omega) + i\omega(A_r(\omega) - A_{r\infty}) \equiv B_r(\omega) + iD_r(\omega), \quad (11)$$

where $Z_r(\omega)$ is the radiation impedance, $B_r(\omega)$ is the radiation resistance (damping coefficient), $A_r(\omega)$ the “added” mass and $A_{r\infty} = A_r(\infty)$. In correspondence with the last expression in Eq. (11), we write $k_r(t)$ as

$$k_r(t) = b_r(t) + d_r(t). \quad (12)$$

Note that, even if $k_r(t)$ is causal, either of $b_r(t)$ and $d_r(t)$ are non-causal; $b_r(t) = b_r(-t)$ is an even function of t and its Fourier transform $B_r(\omega)$ is an even function of ω , while $d_r(t) = -d_r(-t)$ is an odd function of t and $D_r(\omega)$ is an odd function of ω . Thus, as $k_r(t) = 0$ for $t < 0$, $k_r(t) = 2b_r(t) = 2d_r(t)$ for $t > 0$.

In a more compact form, the dynamic Eq. (10) may be written as [28,29]

$$g_i(t)*s(t) = z_i(t)*\dot{s}(t) = F_e(t) + F_u(t) = F_{\text{ext}}(t), \quad (13)$$

where

$$g_i(t) = \dot{z}_i(t) = B_f\dot{\delta}(t) + \dot{b}_r(t) + \dot{d}_r(t) + (m + A_{r\infty})\ddot{\delta}(t) + C\delta(t), \quad (14)$$

where $\delta(t)$ is the delta distribution (Dirac delta function). Also the two impulse-response functions $g_i(t)$ and $z_i(t)$ are causal, that is, they are zero for $t < 0$. On the right-hand side of Eq. (14) the two first terms are odd, and the three last terms are even, functions of t . Each term in Eqs. (10) and (13) represents a force. Multiplying each term by the velocity $u(t) = \dot{s}(t)$ and rearranging terms, we find the instantaneous power $P_u(t)$ delivered to the control-and-power-take-off machinery

$$P_u(t) \equiv -F_u(t)u(t) = P_b(t) + P_d(t), \quad (15)$$

where

$$P_b(t) = F_e(t)u(t) - [b_r(t)*u(t)]u(t) - B_f[u(t)]^2, \quad (16)$$

is the instantaneous active power, and where

$$P_d(t) = -(m + A_{r\infty})\dot{u}(t)u(t) - [d_r(t)*u(t)]u(t) - Cs(t)\dot{s}(t), \quad (17)$$

is the instantaneous reactive power that contributes nothing to the average delivered power, but only represents back-and-forth exchange of stored energy between the machinery and the oscillating system [29]. Thus the average power delivered to the machinery is

$$P_u \equiv \overline{P_u(t)} = \overline{P_b(t)} = \overline{F_e(t)u(t)} - \overline{[b_r(t)*u(t)]u(t)} - \overline{B_f[u(t)]^2}, \quad (18)$$

where the overbar denotes averaging over a chosen time interval, which is sufficiently long for the contribution from $P_d(t)$ to be negligible. With a periodic wave, and consequently periodic oscillation, one period is a sufficiently long time interval.

For convenience, we may write the three terms on the right-hand side of Eq. (18) as $P_e - P_r - P_f$, where the two first terms represent the absorbed wave power $P_a = P_e - P_r$, which is the power removed from the interfering-waves system, while the third term is power lost by dissipative processes, e.g. friction. The second term, which is the radiated

power, should not be considered as a loss, but as a necessity. To absorb a wave means to generate a wave that interferes destructively with the incident wave!. If an optimum velocity $[u(t)]_{\text{opt}}$ that satisfies the condition [28]

$$B_f[u(t)]_{\text{opt}} + b_r(t)*[u(t)]_{\text{opt}} = (1/2)F_e(t), \quad (19)$$

can be realised that maximises the delivered power, we have [30]

$$P_u = P_{u,\text{max}} = (1/2)\overline{F_e(t)[u(t)]_{\text{opt}}} = (1/2)P_{e,\text{opt}} = [P_r + P_f]_{\text{opt}}. \quad (20)$$

From this we see that in the case of an ideal (lossless) two-dimensional wave-power converter that absorbs 100% of the incident wave energy, then $P_r = P_u = P_a$ and, moreover, $P_e = 2P_r$. The optimum applied machine force $F_u(t)$ that corresponds to the optimum condition (19) is $[F_u(t)]_{\text{opt}} = z_f(-t)*u(t) = z_f(-t)*[u(t)]_{\text{opt}}$. Observe that—since $z_f(t)$ is causal—the convolution operand $z_f(-t)$ is anti-causal, that is, it is vanishing for $t > 0$, but not for $t < 0$. Consequently, the optimum machine (control and load) force $[F_u(t)]_{\text{opt}}$ is not influenced by the past, but only by present and future values of the velocity $u(t)$. Because of the mirroring symmetry about $t = 0$ we may say that known values of $u(t)$ are needed so long time into the future as the system “remembers” into the past [31,32]. By considering the Fourier transform of Eq. (19), it can be shown that $[u(t)]_{\text{opt}}$ depends on past, present and future values of $F_e(t)$. It follows that, unless the time-varying values $u(t)$ and/or $F_e(t)$ are known sufficiently far into the future, as e.g. for the case of a sinusoidal wave, it is not possible to determine accurate online values of the optimum quantities $[F_u(t)]_{\text{opt}}$ and $[u(t)]_{\text{opt}}$, respectively. It is, however, possible to find approximate optimum values by application of a reasonably good prediction of the wave force some seconds into the future. Another possibility for approximate optimum control is to replace the non-causal impulse-response functions by approximate causal ones, which may have to be chosen differently for different wave-state situations [33].

Because of this causality problem, and also because of practical constraints, such as amplitude bounds and power-capacity limitations, the converted power may be slightly or substantially less than given by Eq. (20). Observe that, since $[u(t)]_{\text{opt}}$ is linearly related to the excitation force $F_e(t)$, which is again linearly related to the incident-wave-elevation A , the maximum delivered power $P_{u,\text{max}}$ according to Eq. (20) is quadratically related to the incident-wave elevation A . Disregarding Eqs. (19) and (20), however, let us now, for a while, consider $u(t)$ and $F_e(t)$ to be two independent variables. Thus the machine (control-and-load) force $F_u(t)$ has to be at our disposal in Eqs. (10) and (13). Then, on the right-hand side of Eq. (18), the first term, the “excitation power” P_e is linearly related to the excitation force $F_e(t)$ and also to the velocity $u(t)$. The second and the third terms, the radiated power P_r and the lost power P_f , are both quadratically related to $u(t)$, but not related to $F_e(t)$. For economic and practical reasons, it may be necessary to avoid too large excursion, velocity and acceleration of the oscillating body, in particular in rough wave-state situations. The optimum condition described by Eqs. (19) and (20) may be a goal only for cases with small or moderate waves. Only then there will be a maximum destructive interference between the radiated wave and the incident wave. Otherwise, in wave cases where $F_e(t)$ is larger, there will be a smaller ratio between the radiated power P_r and the “excitation” power P_e . Thus a larger fraction of the incident wave energy will remain in the sea than for the case of maximum destructive interference.

Let us now, for simplicity, consider a sinusoidal wave, for which the excitation force is $F_e(t) = F_{e,0}\cos(\omega t)$, and the heave velocity is $u(t) = u_0 \cos(\omega t - \varphi)$. Then Eq. (18)

specialises to [34]

$$P_u = (1/2)F_{e,0}u_0 \cos(\varphi) - (1/2)B_r(\omega)u_0^2 - (1/2)B_f u_0^2 \equiv P_e - P_r - P_f. \quad (21)$$

From this, it is obvious that if the phase angle φ between the velocity and the excitation force could be made equal to zero, this would give the largest useful power P_u . In the case of sinusoidal wave and oscillation, the solution of Eq. (19) yields an optimum velocity in phase with the excitation force, thus $\varphi_{\text{opt}} = 0$, which evidently maximises the first term in Eq. (21), and an optimum velocity amplitude $u_{0,\text{opt}} = (1/2) F_{e,0}/[B_r(\omega) + B_f]$. This amplitude can be realised only for waves for which the excitation force amplitude is below a certain critical value $F_{e,0} < 2[B_r(\omega) + B_f] \omega s_{\text{max}}$, where s_{max} is the specified maximum heave amplitude for the designed heaving body. To increase the allowable s_{max} requires probably additional investment expenditure, while wave energy in the sea is free. For an economically designed system s_{max} is so small that the situation will be $F_{e,0} > 2[B_r(\omega) + B_f] \omega s_{\text{max}}$ during a substantial fraction of the year. When $F_{e,0} \gg 2[B_r(\omega) + B_f] \omega s_{\text{max}}$, the two last terms in Eq. (21) are negligible in comparison with the first term, $P_e = (1/2) F_{e,0}u_0 \cos(\varphi) \approx P_u$. Observe that, now the useful power per unit oscillation amplitude, P_u/u_0 , is twice as large as in the case of Eq. (20), where the ratio between the useful power and the square of the wave amplitude was maximised. In the situation where $P_u \approx P_e$, a large fraction of the free wave energy remains in the sea. Only a little fraction is absorbed by the oscillating system. But more power is absorbed per unit of oscillation amplitude. At present, it is not easy to conclude generically on where the economic optimum is.

5. Budal's upper bound

By extending the above arguments, Budal presented [35] an upper bound to the wave power that can be absorbed by a given immersed oscillating volume. As a more detailed derivation is published previously [36], we shall here just indicate the derivation. Based on Eq. (21) we have the inequality

$$P_u < (1/2)F_{e,0}u_0 \cos(\varphi) \leq (1/2)F_{e,0}u_0 < \omega \rho g A_0 V, \quad (22)$$

where V is the volume of the heaving body, and A_0 is the elevation amplitude of the incident wave. In the last step we took the maximum heave amplitude s_{max} into consideration, and we simply applied Archimedes' law to find an upper bound for the heave excitation force amplitude $F_{e,0}$. This corresponds to the case of negligible diffraction effects, which is valid only if the body volume V approaches zero ($V^{1/3} \ll \lambda$). Expressed in terms of the wave period T and the wave height $H = 2A_0$, Budal's upper bound is

$$P_u < P_B \equiv c_0 V H / T, \quad (23)$$

where $c_0 = \alpha \pi \rho g / 4 = \alpha 7.9 \text{ kW/m}^{-4} \text{ s}$, with $\alpha = 1$. If, however, the heaving body is placed at a totally reflecting vertical wall, instead of in the open sea, then the excitation force is twice as large, in which case $\alpha = 2$. The right-hand side of inequality (23) is represented by the declining P_B curve in the diagram in Fig. 2. Observe that the same factor α is associated with the coefficient $c_\infty = \alpha \rho (g/\pi)^3 / 128$ that enters into inequality (9), defining P_A , represented by the increasing curve in Fig. 2.

Observe that it is not easy to satisfy all the necessary conditions for approaching the upper limit for P/V as given by inequality (23). Apart for the condition of small volume V ,

it is necessary that the oscillating body is a source-type, and not a dipole-type, wave radiator. Moreover, it is required that the full volume V of the body is a swept volume participating in wave generation. Finally, it is necessary to keep $\cos(\varphi)$ close to 1 [see inequality (22)], and the full design-specified heave stroke should be utilised for the wave in question. For several proposed wave-energy devices, P/V has a value that is at least an order of magnitude below Budal's upper bound [37]. Control strategies, like reactive control and latching control [28], may be applied to keep $\cos(\varphi)$ close to 1. A more sub-optimum, but simpler strategy, is to prevent $\cos(\varphi)$ from becoming too small by increasing the load resistance [37].

For a practical device, the converted power is necessarily below the upper bounds given by the two curves P_A and P_B in Fig. 2. If, for low wave periods T , the converted power P_u approaches the increasing curve, P_A say, then the fraction of removed energy from the wave is large, but the volume of the device is far from being fully utilised. Contrary, if for large wave periods the converted power P_u approaches the declining curve, P_B say, in Fig. 2, the immersed oscillating volume is well utilised, but only a tiny fraction of the free ocean energy is utilised. While wave energy in the ocean is free, the oscillating immersed device volume requires economic expenditure. This indicates that wave-energy converters should preferably be designed with technical specifications such that their full capacity should be utilised during a significant proportion of their lifetime. Consequently, much wave energy will remain in the sea except during time spans of low, or very moderate, wave activity. Then the limited capacity does not prevent to convert a larger fraction of the incident wave energy.

The regions above each of the two curves P_A and P_B are forbidden for absorbed power. This does not mean, however, that the whole region below both curves is allowed. The point of intersection between the two curves, may be defined mathematically as $(T, P) = (T_c, P_c)$, say, where

$$T_c = \{(c_0 V)/(c_\infty H)\}^{1/(\delta+1)} \quad \text{and} \quad P_c = c_\infty T_c^\delta H^2 = c_0 V H / T_c. \quad (24)$$

This curve-intersection point has not, however, a direct physical significance, since all conditions for approaching the two upper bounds cannot be satisfied simultaneously. For the case of a point absorber—or the more general case of any oscillating body generating axisymmetrical (circular) waves—Eqs. (24) specialise to

$$T_c = (32\pi^4 g^{-2} V / H)^{1/4} \quad \text{and} \quad P_c = \pi \rho g V H / (4 T_c) = (\rho g^{3/2} / 8) (V^3 H^5 / 2)^{1/4}. \quad (25)$$

The maximum power P_{\max} that can be absorbed with optimum load control, but no phase control, appears (with wave periods $T \approx T_c$) to be an order of magnitude lower than the curve-intersection value P_c . This is illustrated by the relatively flat maximum of the lowest dashed curve in Fig. 2. However, if there is also phase control, then it appears that values of maximum absorbed power P_{\max} up to about half of the curve-intersection value P_c can be attained. See the second lowest dashed curve in Fig. 2. While the shape of the P_A and the P_B upper-bound curves are universal for an axisymmetric system, the actual shape of the dashed curves depends on variables H and V , and also on the geometrical shape of the immersed oscillating body.

The two dashed curves in Fig. 2 represent the maximum power that can be absorbed, from a sinusoidal wave of height $H = 1$ m and period T , by means of a heaving semi-submerged sphere of diameter 10 m, when there is no phase control (passive case), and when there is latching control. For either case, and for each wave period T , different optimum values for the load resistance have had to be chosen. Details are given by Hals

et al. [29]. Since the volume of the sphere is $V = 524 \text{ m}^3$, we find from Eq. (25) that $T_c = 11.4 \text{ s}$ and $P_c = 361 \text{ kW}$. The lowest curve (passive case) has its largest value 24.0 kW for $T = 7.7 \text{ s}$, while the second lowest curve (latching-control case) has its largest value 160.2 kW for $T = 11.5 \text{ s}$. The passive-case curve touches the upper-bound P_A curve at heave-resonance period 4.3 s . The latching-case curve is slightly below the P_A curve for wave periods above resonance, but for periods exceeding about 9 s , the deviation becomes appreciable because the heave amplitude is limited to 3 m .

6. Concluding remarks

In the first part of this paper, we have discussed some wave spectrum parameters that are related to transport, distribution and variability of wave energy in the sea. For a fully developed wind sea, Eq. (6) shows that the power flow intensity is up to five times larger for ocean waves than for the wind that generates the waves. Moreover, wave energy is more persistent than wind energy. These facts give good hope for developing wave-energy technology, which is, however, still less mature than wind-energy technology.

Very many different wave-energy converters have been proposed. Hagerman [38] has classified many of the proposals into twelve different groups, according to which oscillation modes (heave, surge, pitch) are utilised, according to applied method of force reaction (against seabed or against another, differently oscillating, body), and according to type of wave-oscillator interface (solid or flexible structure, or water-air interface of oscillating-water column). Unfortunately, it is not yet possible to say which of the many different proposals will emerge as feasible future wave-power plants. In a recent paper, French [39] addresses this problem and advocates that an economic wave-energy converter should have a large working area (wave-oscillator interface) relative to its size, and that this area should have a relatively large oscillating speed. Secondly, he indicates that the working area should preferably be resonant or “quasi-resonant”. This corresponds to keeping $\cos(\varphi)$ close to its maximum value 1; see Eq. (21).

We have found that it is useful to think of primary conversion of wave energy by an oscillating system as a wave-interference phenomenon. Corresponding to optimum wave interference, there is an upper bound (9) to the amount of energy that can be extracted from a wave by means of a particular oscillating system. Taking physical limitations into account, another limit, Budal’s upper bound (23), for the ratio of extracted energy to the volume of the immersed oscillating system, has been derived. Finally, the significance of the two different upper bounds is discussed; see Fig. 2. Unless the recommendations of French [39] are followed, the performance of a WEC may typically correspond to figures that are one to two orders of magnitude below Budal’s upper bound.

To develop a commercial WEC is not a straightforward task. Many inventions still have to be made, and many challenging problems need to be solved. We need a broad basis of knowledge, so that we know what is necessary to invent. We should avoid re-inventing old inventions and repeating old mistakes. One mistake could be to believe: “My invention is the best one.” Instead we need to co-operate and work together.

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