Proposal of a New Standard for Wave Realizations in Time-Domain Simulations

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Abstract

With the increasing computer performance, time-domain simulations of ship responses in waves are becoming feasible tools in research as well as in design. When simulating rare and complex hydrodynamic phenomena such as parametric rolling and slamming, efficient and accurate representation of the wave environment is crucial. The most common approach for numerical wave realization is to represent the wave surface as a Fourier series of a finite number of harmonic wave components based on a standardized target spectrum. In time-domain simulations the computational cost is generally proportional to the number of wave components, and it is hence desirable to use the minimum number of wave components that gives a wave process with sufficient statistical quality. This paper evaluates four approaches to discretization of the target spectrum regarding computational cost and statistical quality. The presented results highlight the need for careful consideration when performing numerical wave realization. Based on the findings a new standard for irregular wave realization is proposed where the target spectrum is discretized in the period domain. It is shown to yield excellent wave sequence quality with as few as 100 wave components. Establishment of a unified wave realization standard would have large benefits, for example in simulation code benchmarking and in development of criteria such as the direct stability assessments on level 3 in the IMO second generation intact stability criteria.

1 Introduction

With the increasing computer performance, time-domain simulations of ship responses in waves are becoming feasible tools in research as well as in design. When simulating rare and complex hydrodynamic phenomena such as parametric rolling and slamming, efficient and accurate representation of the wave environment is crucial. Ocean wave models can be divided into three classes: linear models, second-order models and high-order models (e.g. ISSC 2009). Most common in marine engineering applications is the use of linear wave models, which were first developed during the 1950's (e.g. St. Denis & Pierson 1953). The use of linear wave models, where the wave process is modeled as Gaussian, typically based on standardized target wave spectra, is well established. Vast amounts of research regarding the benefits and possible limitations in using a linear wave model have been published, for example regarding wave spectrum formulations (Pierson Moskovitz 1964, Hasselman et al 1973, Michel 1999), wave sequence periodicity (Belenky 2005) and wave grouping (Kimura 1980, Tucker et al 1984, Elgar et al 1985, Rodriguez 2000). Similarly, the number of studies demonstrating the practical use of a linear wave model in analysis of ship responses is countless (e.g. Baarholm & Moan 2000, Wang & Moan 2004, Garme & Rosén 2003).

The wave spectrum $S(\omega)$ describes the statistical character of the wave elevation process and can in principle be seen as a Fourier transform of samples from an irregular wave surface, which are broken down into harmonic components. Based on the spectrum moments defined as

$$m_n = \int_0^\infty \omega^n S(\omega) d\omega \tag{1}$$

where ω is the angular frequency and n is an integer, sea state parameters such as the significant wave height and characteristic period can be calculated. Likewise, a representative spectrum can be determined if the statistical parameters are measured. An irregular sea surface elevation process, which is stationary, ergodic, random and follows a Gaussian distribution with a zero mean, can be formulated directly from the continuous target wave spectrum as a Fourier integral

$$\varsigma(t) = \int_0^\infty \sqrt{2S(\omega)} \cos\left(\omega t + \varepsilon(\omega)\right) d\omega \tag{2}$$

where ε are the random phases. For numerical simulations of for example ship responses, evaluation of Equation (2) using an infinite number of wave components is however practically impossible. Instead, the time-series of wave elevation is usually formulated based on the central-limit theorem (Kendall & Stuart 1994) and following St. Denis and Pierson (1954), as a Fourier series of a finite number of N harmonic wave components,

$$\varsigma(t) = \sum_{i=1}^{N} A_i \cos\left(\omega_i t + \varepsilon_i\right) \tag{3}$$

Here A_i are the component amplitudes integrated from the target spectrum and ε_i the phases for the respective frequencies ω_i. The phases are uniformly distributed $\varepsilon = \left[0, 2\pi\right]$ and mutually independent of each other and the component amplitudes. When the target wave spectrum is defined as a continuous function, between 0 and ∞, there is a clear conflict between the frequency resolution and the spectrum truncation. The details of the spectrum discretization, i.e. the number of wave components, how they are distributed in the frequency domain and how the amplitude carrier frequencies, ω_i, are selected, are therefore important. An interesting alternative for generation of linear or nonlinear waves is the ARM-model presented in e.g. Reed et al (2016), however, this model has not yet been practically implemented for seakeeping analysis. The Fourier formulation is still the most commonly applied model in practical analysis of ship responses.

In theory, Equation (3) is only valid for $N \to \infty$. However, from a computational cost point of view it is generally beneficial to use as few wave components as possible. Hence, the critical question is how to compose a wave with sufficient statistical quality with as few wave components as possible. At present there is no unified recommendation as to how this discretization and truncation shall be performed. There are recommendations in for example DNV (2010) and SNAME (2008) but these recommendations do not cover all aspects of wave spectrum discretization and truncation and are vastly different in their content. In notable works, e.g. Wu and Moan (2006a,b) and Longuet-Higgins (1984), there are also differences in how spectrum discretization and truncation is approached. The lack of unification in recommended practice is problematic from a simulation code development and benchmarking point of view, especially in the context of ship standards development.

The aim of this paper is to examine how the computational effort and the statistical quality of numerically realized wave elevation sequences, using a linear wave model, are affected by the wave spectrum discretization. The paper reviews three common approaches to spectrum discretization and discusses yet another alternative approach proposed in Huss (2010):

- A The continuous wave spectrum is discretized in the frequency domain using frequency equidistant wave components.
- B The continuous wave spectrum is discretized equidistantly in the frequency domain, but each frequency component is assigned to a random carrier frequency within each equidistant interval.
- C The continuous wave spectrum is discretized in the frequency domain such that each wave component carries the same amount of energy.
- D The continuous wave spectrum is discretized using equidistant wave components in the period domain.

The quality of the realized wave process for the different spectrum discretization approaches, A-D, is evaluated using a number of different statistical quality indicators. Some practical implications of the spectrum discretization for analysis of ship responses are exemplified. Finally, a new standard wave spectrum discretization approach is proposed.

2 Statistical quality of wave sequences

To evaluate the effect of the different spectrum discretization approaches the following statistical quality indicators are used and compared with the same quality indicators derived empirically directly from the realized wave elevation process. Particular attention is given to wave sequence periodicity and wave grouping.

- The most probable largest crest height, ζ_{max}, and the exceedance probability as a function of some threshold level.
- The skewness of the wave elevation process, γ_1, which for a Gaussian process is equal to 0.
- The kurtosis of the wave elevation process, γ_2, which for a Gaussian process is equal to 3.
- The mean wave run length, $\bar{\kappa}$, defined as the average number of consecutive waves that exceed a certain wave height.
- The distribution of wave run lengths, $p(\kappa)$.
- The average wave period, within wave groups, as a function of wave run lengths.
- The wave sequence periodicity, evaluated using the autocorrelation of the wave elevation, $R(\tau)$.

2.1 Distribution of crest heights

The maximum crest height is important, as the correlation between the realized wave heights and the maximum ship responses are strong. Assuming that the wave elevation is Gaussian distributed, the probability for a wave crest not exceeding a certain height can be approximately expressed as (e.g. Prevosto et al 2000)

$$p(\varsigma \le x) = \left(1 - e^{\left(-x^2/2m_0\right)}\right)^n \tag{4}$$

where $n = t / T_z$, t is the considered time period and T_z is the average zero-crossing period. This approximation is valid only for relatively small bandwidth spectra or large x values but is still widely used for estimating extreme value distributions. Following e.g. Ochi (1981) the most probable largest crest height given n number of recorded waves can then be expressed as

$$\varsigma_{max,theory} = \sqrt{m_0}\sqrt{2\ln n} \tag{5}$$

The effect of the spectrum discretization on the probability of the wave crest height being larger than some threshold value, here expressed in terms of multiples of the wave elevation standard deviation $\sigma = \sqrt{m_0}$, is also studied and compared to the theoretical exceedance probabilities using Equation (4) based on the zeroth spectrum moment following Equation (1).

2.2 Wave grouping

It is well known that large waves can appear in sequence and this phenomenon has been shown to be important is estimating the extreme responses for marine structures (e.g. Ochi & Sahinoglou 1989). Proper numerical modeling of the waves regarding the presences of wave groups is therefore important. Wave grouping has, as already mentioned in the introduction, been extensively studied analytically, numerically and experimentally (e.g. Kimura 1980, Tucker et al 1984, Elgar et al 1985, Rodriguez 2000). Here, a group of waves is defined by the appearance of one or more consecutive waves that are larger than the significant wave height in the wave elevation record, as illustrated in Figure 1. The number of waves in a group of waves is here denoted run length.

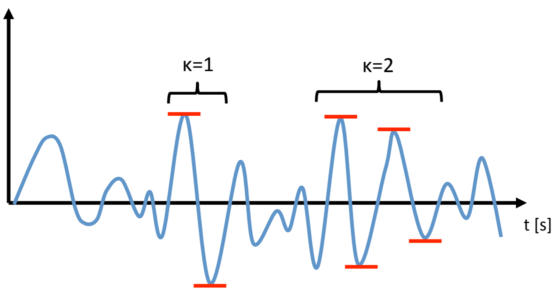

Figure 1. Illustration of the definition of wave groups.

According to Kimura (1980) the time series of zero-upcrossing wave heights have statistical properties close to the Markov chain where two states are considered: state 1 where the wave height is larger than some wave height threshold, $H>H^*$, and state 0 where the wave height is smaller than or equal to the threshold, $H \le H^*$. The initial distribution P_0 can then be expressed as

$$P_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{6}$$

and the transition probability matrix between the two states expressed as

$$p = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \tag{7}$$

The probability of being in either state 0 or 1 after *n* number of wave heights, can be expressed as

$$p_n = P_0 p^n \tag{8}$$

For *n*=1 the probabilities are

$$p_1 = \begin{pmatrix} p_{21}, & p_{22} \end{pmatrix} \tag{9}$$

Since the Markov chain is started in state 1, i.e. the wave height is larger than the threshold, $p_1(1)$ gives the probability than the run length is equal to 1. By induction the probability for a run length of *n* can be expressed as

$$p(n) = p_{22}{}^{n-1}(1 - p_{22}) \tag{10}$$

and the mean run length as

$$\bar{\kappa} = \frac{1}{1 - p_{22}} \tag{11}$$

where

$$p_{22} = \left[H_i > H^* \mid H_{i-1} > H^* \right] \tag{12}$$

The transition probability p_{22} is expressed as

$$p_{22} = \frac{\int_{H^*}^{\infty}\int_{H^*}^{\infty} P_{H_1,H_2}(H_1,H_2)\,dH_1\,dH_2}{\int_{H^*}^{\infty} P_{H_1}(H_1)\,dH_1} \tag{13}$$

where the wave height probability distribution P_{H1} is given by the Rayleigh distribution and the probability distribution for two successive wave height is given by the joint Rayleigh distribution which according to e.g. Rice (1944, 1945) can be expressed as

$$P_{H_1,H_2}(H_1,H_2) = \frac{\pi H_1 H_2}{4(1-\rho)H_m{}^4} \exp\left[-\frac{\pi}{4(1-\rho^2)} \frac{H_1{}^2 + H_2{}^2}{H_m{}^2} \right] I_0 \left[\frac{\pi\rho}{2(1-\rho^2)} \frac{H_1 H_2}{H_m{}^2} \right] \tag{14}$$

where I_0 is the modified Bessel function of the zeroth order and first kind, and H_m is the mean wave height. The correlation coefficient ρ is determined following Battjes and Vledder (1984) based on the target spectrum as

$$\rho = \frac{1}{m_0} \sqrt{\left[\int_0^{\infty} S(\omega)\cos(\omega T_z)\,d\omega \right]^2 + \left[\int_0^{\infty} S(\omega)\sin(\omega T_z)\,d\omega \right]^2} \tag{15}$$

In the run length results presented in this study, the limit wave height has been set equal to the significant wave height, $H^*=H_s$.

2.3 Return periods and periodicity

Wave sequence return periods and what we here refer to as practical periodicity are important statistical quality indicators, particularly when performing long time domain simulations. The return period is here defined as that time period in which the wave elevation realization repeats itself exactly. The condition for such repetitions is that all harmonic wave components of the superimposed wave profile must have run through an integer number of full periods at the same time. For a constant frequency step this can be formulated according to Huss (2010) as

$$k_i = \left(\frac{\omega_i}{2\pi} + (i-1)\frac{\Delta\omega}{2\pi}\right)T_r \tag{16}$$

where ω_1 is the first frequency component, $\Delta\omega$ the frequency step, T_r the return period and k_i an integer number. Since

$$k_1 = \frac{\omega_1}{2\pi}T_r \tag{17}$$

is an integer, Equation (16) breaks down to satisfying the following condition

$$k_2 = k_1\left(\frac{\Delta\omega}{\omega_1} + 1\right),\ [\omega_1 > 0] \text{ or } k_2 = 1,\ T_r = \frac{2\pi}{\Delta\omega},\ [\omega_1 = 0] \tag{18}$$

which will satisfy all other conditions for $i = 3,4..n$ because $k_1 + (i-1)(k_2 - k_1) = k_i$ must be an integer if k_1 and k_2 are integers. In principle Equation (18) implies that the theoretical return period is independent of the number of amplitude components, if $\omega_1 \neq 0$.

Practical periodicity is here defined using the autocorrelation, as a measure of the wave elevation self-similarity. Theoretically the autocorrelation can be expressed based on the wave elevation target spectrum through the Laplace transform as

$$R(\tau) = \int_0^\infty S(\omega)\cos(\omega\tau)d\omega \tag{19}$$

and should tend to zero as the time shift, τ, increases. The autocorrelation can also be calculated based on wave elevation samples. Practical periodicity in a wave elevation sample, which is a statistical anomaly, typically shows up as periodic pulses in the autocorrelation function. In Section 6.2 return periods and practical periodicity is further discussed, and it is clear that the practical periodicity is strongly related to the number of wave components and the spectrum discretization approach.

3 Spectrum discretization

In the following sections the wave spectrum discretization approaches, which were outlined in Section 1 are presented in more detail. Approaches A-C discretize the continuous wave spectrum, as most commonly done, in the frequency domain, while Approach D uses a today unconventional discretization approach in the period domain. For further details regarding standard wave spectra and associated statistical properties refer to the excellent review of Michel (1999).

3.1 Equidistant frequency spectrum discretization with equidistant carrier frequencies - A

One approach is to discretize the continuous spectrum into N components with amplitudes

$$A_i = \sqrt{2 \int_{\omega_i^c - 0.5\Delta\omega}^{\omega_i^c + \Delta\omega} S(\omega) d\omega} \tag{20}$$

as illustrated in Figure 2. The wave components are uniformly distributed in the interval $\omega_i^c - \Delta\omega/2 < \omega_i \leq \omega_i^c + \Delta\omega/2$ where $\Delta\omega = (\omega_{max} - \omega_{min})/N$, ω_{max} and ω_{min} are the upper and lower frequency limits. The carrier frequency for each wave components is

$$\omega_i^c = \frac{\omega_{i+1} - \omega_i}{2} \tag{21}$$

The problem with using equidistant carrier frequencies is that the wave elevation process will contain significant periodic self-similarity as a function of the frequency step $\Delta\omega$. Therefore simulation of large time durations either requires splicing of a number of shorter sequences, each with different sets of random phases ε_i, or a very large number of wave components. This approach is for example recommended in DNV (2010).

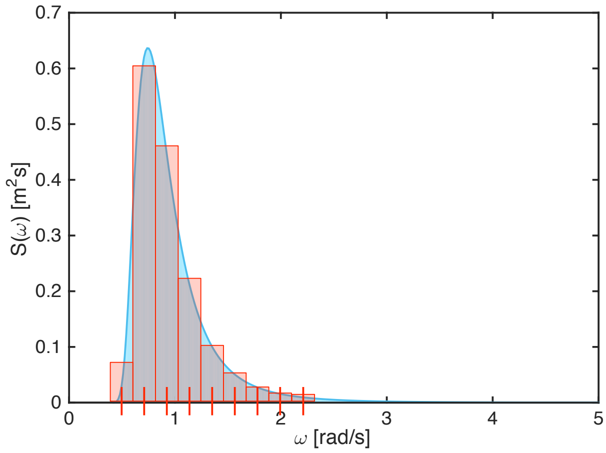

Figure 2. Illustration of a frequency spectrum with equidistant frequency division and uniformly distributed carrier frequencies.

3.2 Equidistant frequency spectrum discretization with random carrier frequencies - B

As noted for the equidistant frequency component approach the practical wave sequence periodicity is directly related to the frequency step, and hence the number of wave components, used in the discretization of the continuous spectrum. One approach to achieve a practically infinite wave sequence periodicity is to choose the components carrier frequencies randomly within each component frequency interval (e.g. Borgman 1967). In principle the wave components are determined using Equation (20), however instead of assigning carrier frequencies according to Equation (21), the carrier frequencies are also stochastic variables uniformly distributed in the interval $\omega_i^c - \Delta\omega/2 < \omega \leq \omega_i^c + \Delta\omega/2$ as illustrated in Figure 3.

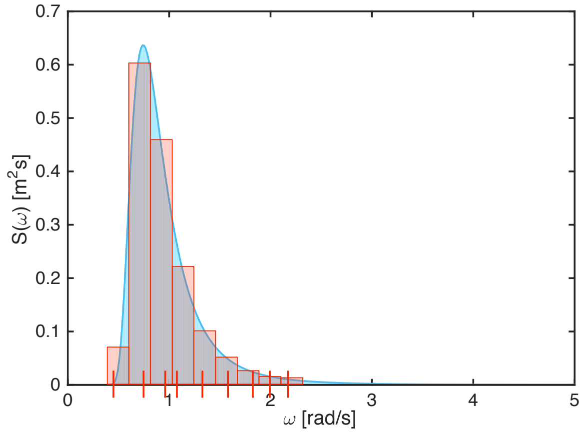

Figure 3. Illustration of frequency spectrum with equidistant frequency division and random carrier frequencies.

3.3 Energy equivalent frequency spectrum discretization - C

Another approach is to discretize the continuous spectrum into N components with amplitudes as

$$A_i = \sqrt{2\left(S_e(\omega_i) - S_e(\omega_{i-1})\right)} \tag{22}$$

where $S_e(\omega_i)$ is the cumulative spectrum defined as

$$S_e(\omega_i) = \int_{\omega_{min}}^{\omega_i} S(\omega)d\omega \tag{23}$$

By selecting ω_i such that

$$S_e(\omega_i) - S_e(\omega_{i-1}) = \frac{S_e(\omega_{max}) - S_e(\omega_{\min})}{N} \tag{24}$$

where N in the number of wave components, the spectrum is discretized into wave components which are energy equivalent as illustrated in Figure 4. Due to the non-uniform distribution of wave components practically infinite wave sequence periodicity is achieved, as for approach B. This approach is for example recommended in SNAME (2008).

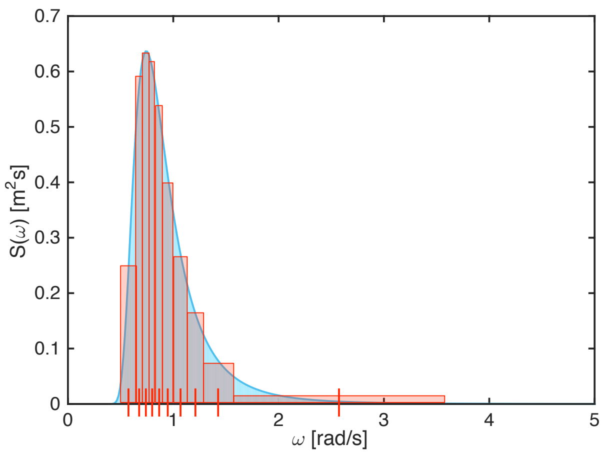

Figure 4. Illustration of frequency spectrum with energy equivalent wave components.

3.4 Spectrum discretization in the period domain - D

Here the spectrum is equidistantly discretized in the period domain following Huss (2010), as illustrated in Figure 5, with component amplitudes calculated as

$$A_i = \sqrt{2 \int_{T_i^c - 0.5\Delta T}^{T_i^c + 0.5\Delta T} S(T) dT} \tag{25}$$

S(*T*) is the continuous wave spectrum formulated in the period domain. The condition for transforming the spectrum from the frequency domain to the period domain is that the total energy is to be maintained as

$$\int_{\omega_{min}}^{\omega_{max}} S(\omega) d\omega = \int_{T_{min}}^{T_{max}} S(T) dT \tag{26}$$

and with $\omega = 2\pi/T$ the transformation condition is obtained as

$$\int_{\omega_{min}}^{\omega_{max}} S_\omega(\omega) d\omega = \int_{T_{max}=\frac{2\pi}{\omega_{min}}}^{T_{min}=\frac{2\pi}{\omega_{max}}} S(\omega) \frac{d\omega}{dT} dT = \int_{T_{min}}^{T_{max}} S_\omega\left(\frac{2\pi}{T}\right) \frac{2\pi}{T^2} dT = \int_{T_{min}}^{T_{max}} S_T(T) dT \tag{27}$$

where

$$S_T(T) = S_\omega\left(\frac{2\pi}{T}\right) \frac{2\pi}{T^2} = S_\omega(\omega) \frac{\omega^2}{2\pi} \tag{28}$$

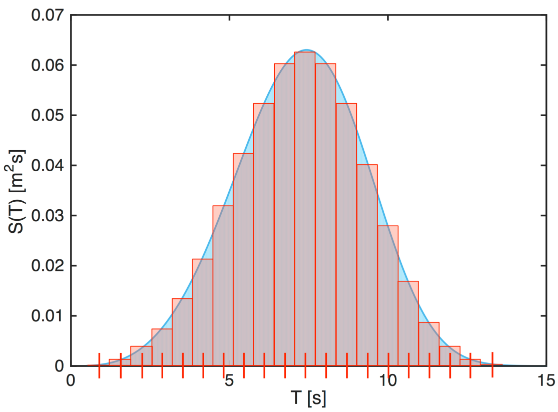

Figure 5. Illustration of spectrum density formulated in the period domain with equidistant wave components.

4 Simulations

To investigate the effects of the spectrum discretization approach on the statistical quality of the realized wave process, the wave elevation is modeled using Equation (3) with *t*=10 800 seconds and a time-step of d*t*=0.1 s. For spectrum discretization approaches B-D the statistical quality indicators are calculated based on 100 realizations each with different sets of component phases ε_i. For approach A however, reaching a total sample size which corresponds to 100 realizations of length *t*=10 800 s, requires splicing of many shorter sequences depending on the periodicity, which is related to the frequency step.

A standard 2-parameter Bretschneider spectrum is here used which is defined as

$$S(\omega \mid H_s, T_z) = \frac{H_s^{\,2} T_z}{8\pi^2} \left(\frac{2\pi}{\omega T_z} \right)^5 e^{-\frac{1}{\pi}\left(\frac{2\pi}{\omega T_z}\right)^4} \tag{29}$$

Three different sea states with different wave slopes

$$\theta_s = \frac{2\pi}{g} \frac{H_s}{T_z^2} \tag{30}$$

have been studied according to Table 1, however the effect of the wave slope on the resulting statistical quality for the different discretization approaches was negligible. Therefore, the results are here only presented for a wave slope of 0.04. The number of wave components that is used in the evaluation is $N_A = [21,\ 51,\ 101,\ 201,\ 401,\ 801,\ 1601,\ 3201]$.

Table 1. Sea states studied.

Case	H_s [m]	T_z [s]	θ_s
#1	3.0	10.0	0.02
#2	2.3	6.0	0.04
#3	2.0	4.0	0.08

5 Spectrum truncation

As stated there is a conflict between the frequency resolution and the spectrum truncation. Mathematic stringency would require inclusion of the complete spectrum over $0 \le \omega \le \infty$ or $0 \le T \le \infty$. However, this is practically impossible and truncation at the high frequency end of the spectrum is therefore necessary. The choice of truncation limits is directly coupled to the frequency resolution; given the same number of wave components a higher truncation limit gives a lower resolution. Further, the spectrum energy content is typically concentrated to a fairly narrow band of frequencies, which also makes truncation practical. Criteria for spectrum truncation could for example be expressed in terms of minimum energy inclusion as

$$\int_{\omega_{min}}^{\omega_{max}} S(\omega) d\omega \ge (1-\alpha) \int_0^{\infty} S(\omega) d\omega \tag{31}$$

where α is some small number. Other criteria such as truncation at the low and high frequency ends of the spectrum based on the value relative the spectrum peak, or based on the spectrum peak frequency are also used (e.g. Wu and Moan 2006a). Alternatively, truncation could be made based on the realized sea state parameters, such as the skewness, kurtosis or max crest elevation (SNAME 2008).

The effect of spectrum truncation on the theoretical significant wave height and zero-crossing period, which can be expressed using the spectrum moments as

$$\begin{aligned} H_s &= 4\sqrt{m_0} \\ T_z &= 2\pi \sqrt{\frac{m_0}{m_2}} \end{aligned} \tag{32}$$

is here studied. In Figure 6 the theoretical significant wave height, the zero-crossing period and the cut-off frequencies are presented as functions of spectrum energy inclusion. The significant wave height and zero-crossing period are normalized with the target values used to define the spectrum (Equation 29). The significant wave height is directly proportional to the square root of the energy. Thus 95% energy truncation yields 97.5% of the target significant wave height. The average zero-crossing period is however more sensitive to the truncation due to the second spectrum moment. It can be seen that a 95% energy truncation results in a theoretical zero-crossing period of approximately 8% larger than the target value and that convergence towards the target value shows a very strong non-linearity.

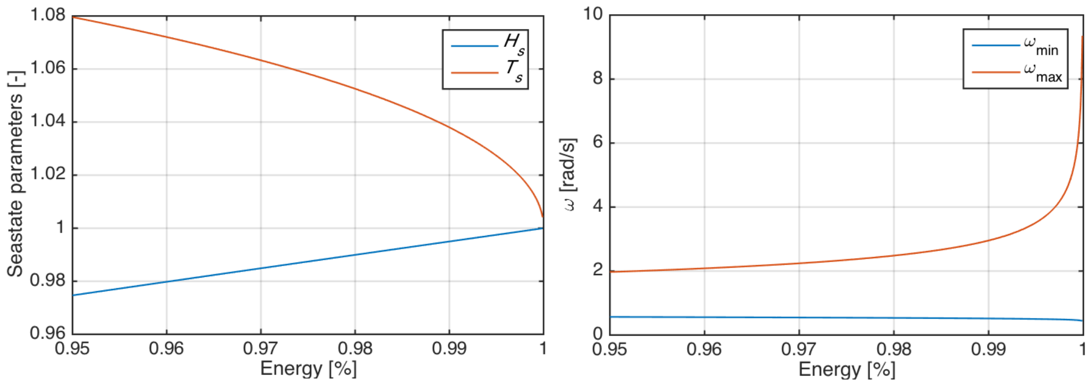

Figure 6. Sea state parameters (*a*) and cut-off frequencies (*b*) as a function of spectrum energy inclusion for a significant wave height of 2.3 m and a zero-crossing period of 6 s.

For approach A and B where the wave components are uniformly distributed in the frequency domain it is clear that the cost for approaching the target values, in terms of the high-frequency cut-off, is very large. Spectrum truncation for these approaches is therefore necessary to reduce the required number of wave components. For approach C and D spectrum truncation is, however, from a computational cost point of view not an issue due to the focused distribution of the wave components; for approach C due to the energy equivalence, which implies that the component density is a direct function of the spectrum energy distribution, and for approach D due to the inverse relation between frequency and period. The spectrum truncation also affects the wave physics. For example, the horizontal particle velocities can be expressed as

$$u_x = -a\omega e^{kz}\sin(kx - \omega t + \varepsilon) \tag{33}$$

where a is the wave amplitude, $k = \omega^2/g$ is the wave number assuming deep water, x and z are spatial coordinates. As seen the largest velocities is related to the high frequencies and the corresponding wave amplitudes. By assuming that the limit for the linear wave model is for a wave slope of $2a_{\max}/\lambda = 0.1$, where $\lambda = 2\pi g/\omega^2$, the maximum wave amplitude can be written as a function of wave frequency as

$$a_{max} = 0.1\frac{\pi g}{\omega^2} \tag{34}$$

Further, in the linear wave model it is also assumed that the waves are small and the free surface boundary condition is defined on the mean surface level. Therefore the wave kinematics are not predicted above the mean water level, i.e. for z>0. To enable calculation of wave particle velocities for the instantaneous surface elevation, z>0, without obtaining unrealistically large particle velocities due to the exponential term, methods such as vertical stretching, Wheeler stretching or extrapolation stretching are used. By combining Equation (33) and (34), assuming that the oscillatory term is equal to 1 and that z=0, the maximum particle velocity as a function of wave frequency can be calculated, as shown in Figure 7. This is a conceptual approach, and the particle velocities will in reality depend on the spectrum discretization and truncation, and the stochastic nature of the process. Nevertheless, it is clear that even as the wave frequencies increase the corresponding wave amplitudes limits the largest particle velocities. As the wave frequency goes to zero the maximum velocities increase exponentially, however these frequencies correspond to very long waves and in reality the amplitudes for the low frequencies will be limited by the wave spectrum, as shown in e.g. Figure 4.

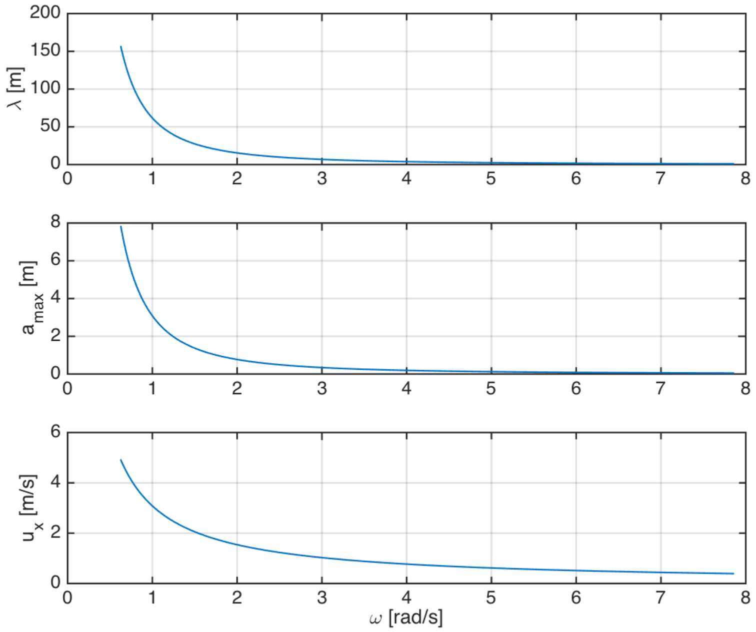

Figure 7. The wave length, the limit wave amplitude and the corresponding horizontal particle velocity for z=0.

Summarizing, there are two important aspects that need to be carefully considered and balanced when truncating the target wave spectrum: the computational cost and the realization of the target seastate parameters. In the following analysis the spectra are truncated at $\frac{2\pi}{2.5T_z} \leq \omega \leq \frac{2\pi}{0.2T_z}$ and $0.2T_z \leq T \leq 2.5T_z$, which principally includes 100% of the spectrum energy and results in seastate parameters that are approximately within 1% of the target values. This truncation also follows the proposed standard outlined in Section 8. Spectrum truncation is further discussed in Section 7.2.

6 Results

In Figure 8 the most probable largest crest height, the skewness, the kurtosis and the mean wave run length are displayed for the four different spectrum discretization methods, with amplitude components determined from equations (20), (22) and (25), for all considered number of wave components. The most probable largest crest height and the average run length are normalized with the theoretical predictions from equations (5) and (11). The wave elevation kurtosis is normalized with the theoretical value of 3.

As seen the effects of spectrum discretization on the statistical convergence is large. It is clear that approaches C and D converge faster regarding for example the most probable largest crest height and the kurtosis, and that 51-101 wave components are typically sufficient to obtain converged statistics. For approaches A and B 201 wave components are typically required to obtain converged statistics. It can also be observed that there is no significant effect of the number of wave components on the resulting skewness.

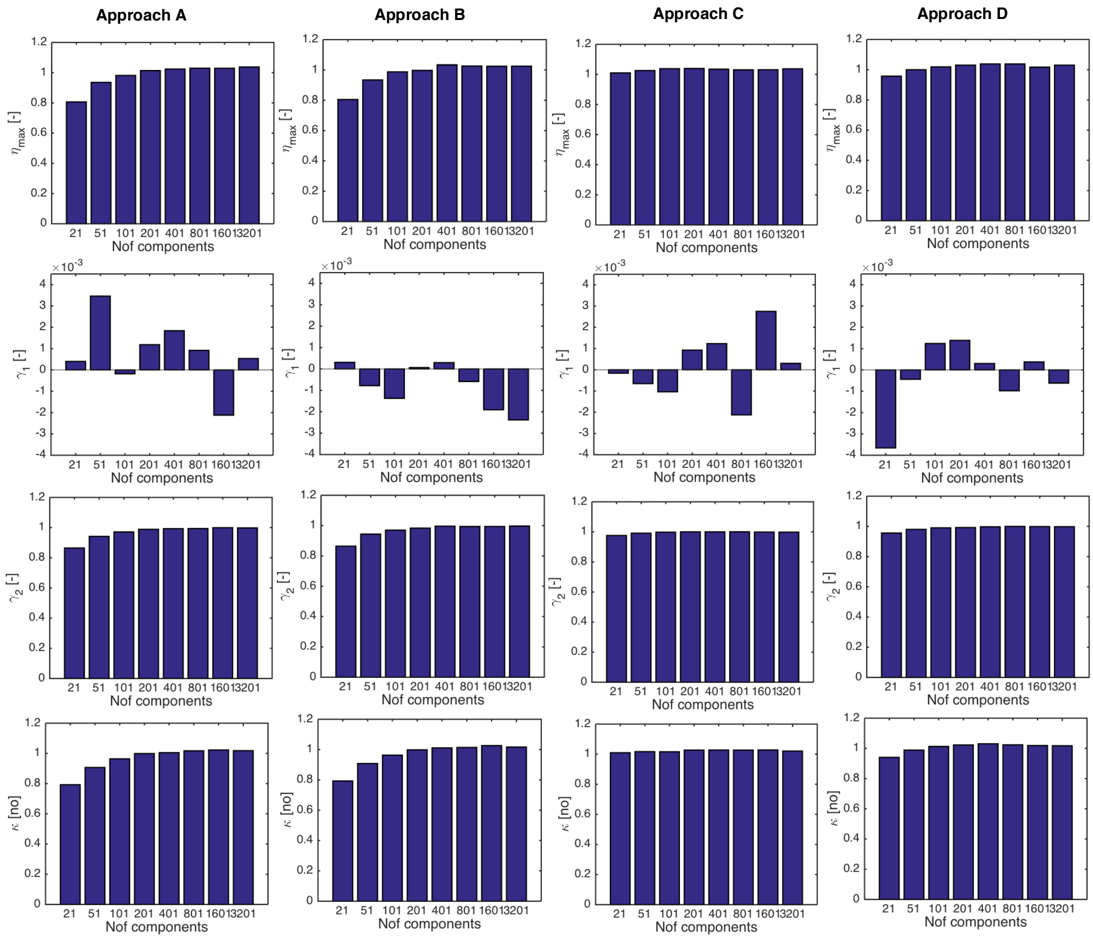

Figure 8. The normalized maximum crest height, the skewness, the kurtosis and the mean run length for a wave slope of 0.04.

In Figure 9 the empirical wave crest exceedance probabilities are presented for two threshold values: 2σ and 3σ. The results are presented for discretization approaches A-D as a function of the number of considered wave components. For reference the theoretical exceedance probabilities based on equation (4) are also presented. The exceedance probabilities for the 2σ level are principally converged for as few components as 51, for all approaches. For approaches C and D 101 wave components are required for the 3σ level statistics to converge within ±5% of the value for 3201 components, while 401 components are required for approaches A and B. Hence, accurate description of the distribution tail probabilities, which is related to the largest waves, requires more wave components.

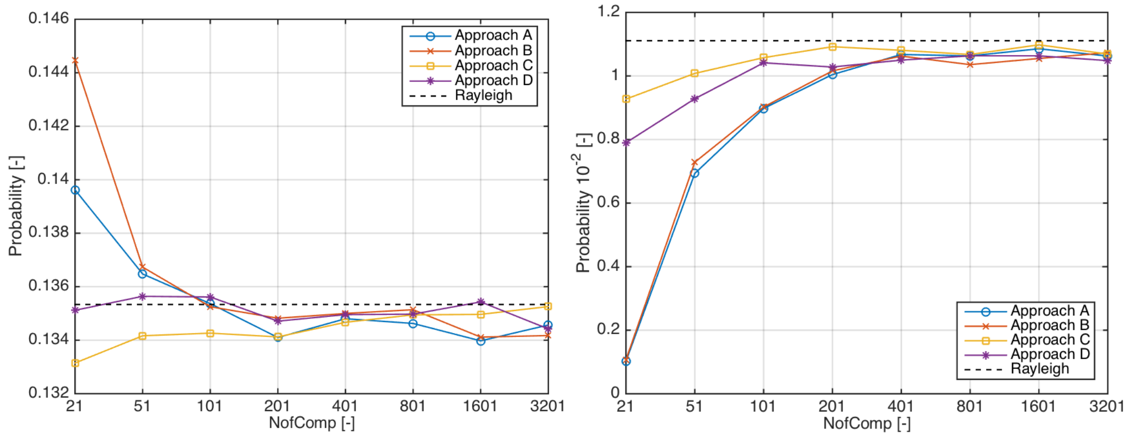

Figure 9. The exceedance probabilities for the 2σ (a) and 3σ (b) levels for approaches A-D as a function of the number of wave components.

6.1 Wave groups

In Figure 10 the mean run length for approaches A-D is displayed as a function of the number of considered wave components in relation to the theoretical mean wave run length according to Kimura (1980). As also observed in Figure 8, the run length for approaches A and B converges to within ±5% for 201 components, approach C requires 21 components and approach D 51 wave components.

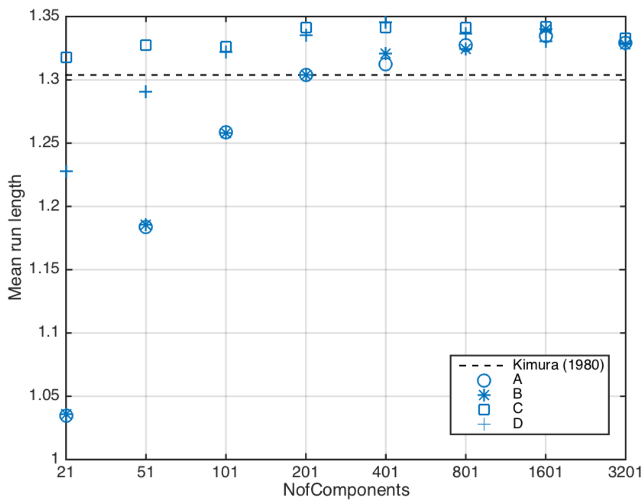

Figure 10. Mean run length for approaches A-D as a function of number of considered wave components for a wave slope of 0.04.

Figure 11 shows the probability distribution of the number of wave groups with a certain wave group length for the different spectrum discretization approaches A-D. The probability is determined by dividing the total number of zero-crossing waves belonging to each wave group size by the theoretical number of zero-crossing waves for the entire simulation length.

It is clear that similar conclusion as based on Figure 10 can be made. Convergence regarding group length probabilities is similar for approach A and B, while for approaches C and D the statistics converge faster. It can also be observed that convergence regarding wave group probability is slower for the more rare events corresponding to longer run lengths.

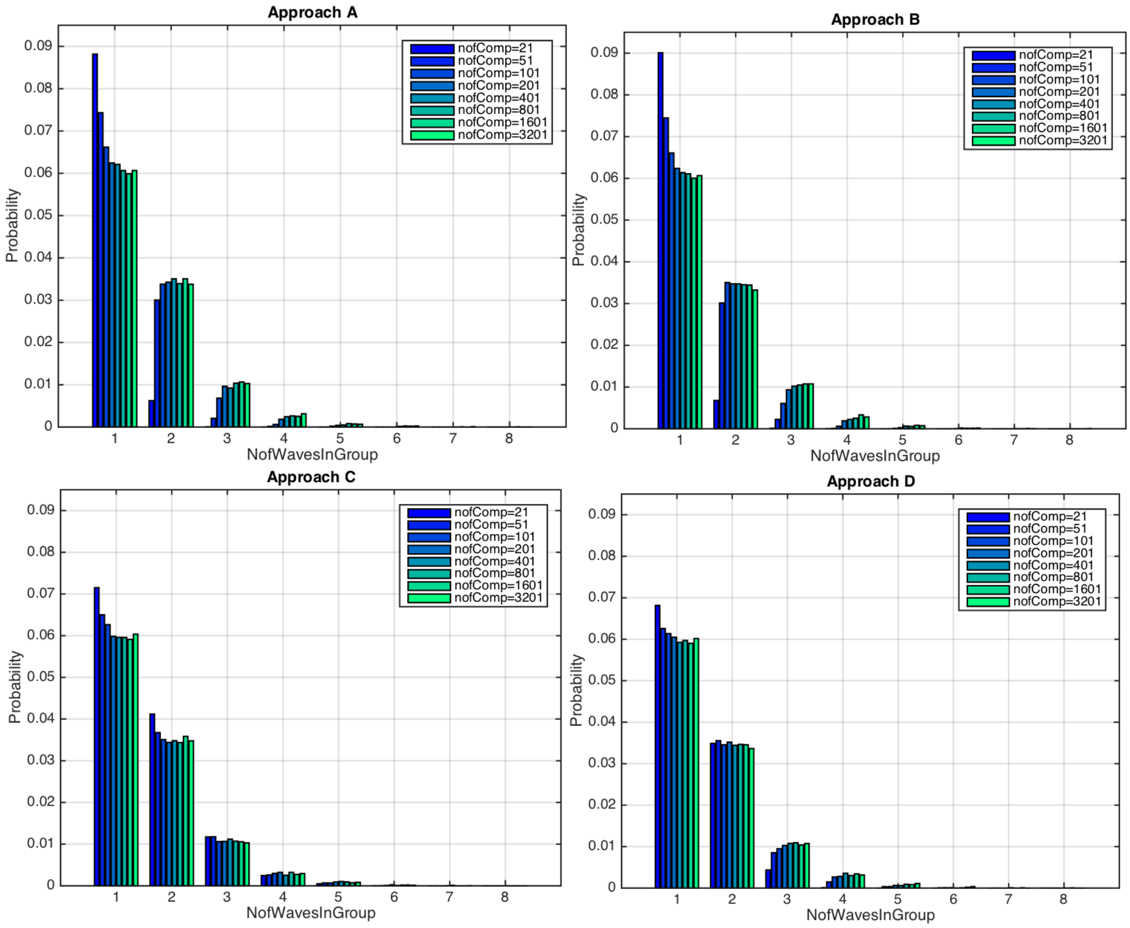

Figure 11. Distribution of number of wave groups with different number of waves for a wave slope of 0.04.

In Figure 12 the average zero crossing periods within each wave group are displayed for each spectrum discretization approach and for the different number of wave components. The convergence regarding the wave period is faster for approaches C and D, and wave periods corresponding to large and rare run lengths require more components to converge. It is also interesting to note that the wave periods increase with increasing group lengths. From a ship responses point of view, for example for parametric excitation phenomena, the appearance or non appearance of such wave groups may well be decisive for the triggering of such phenomena.

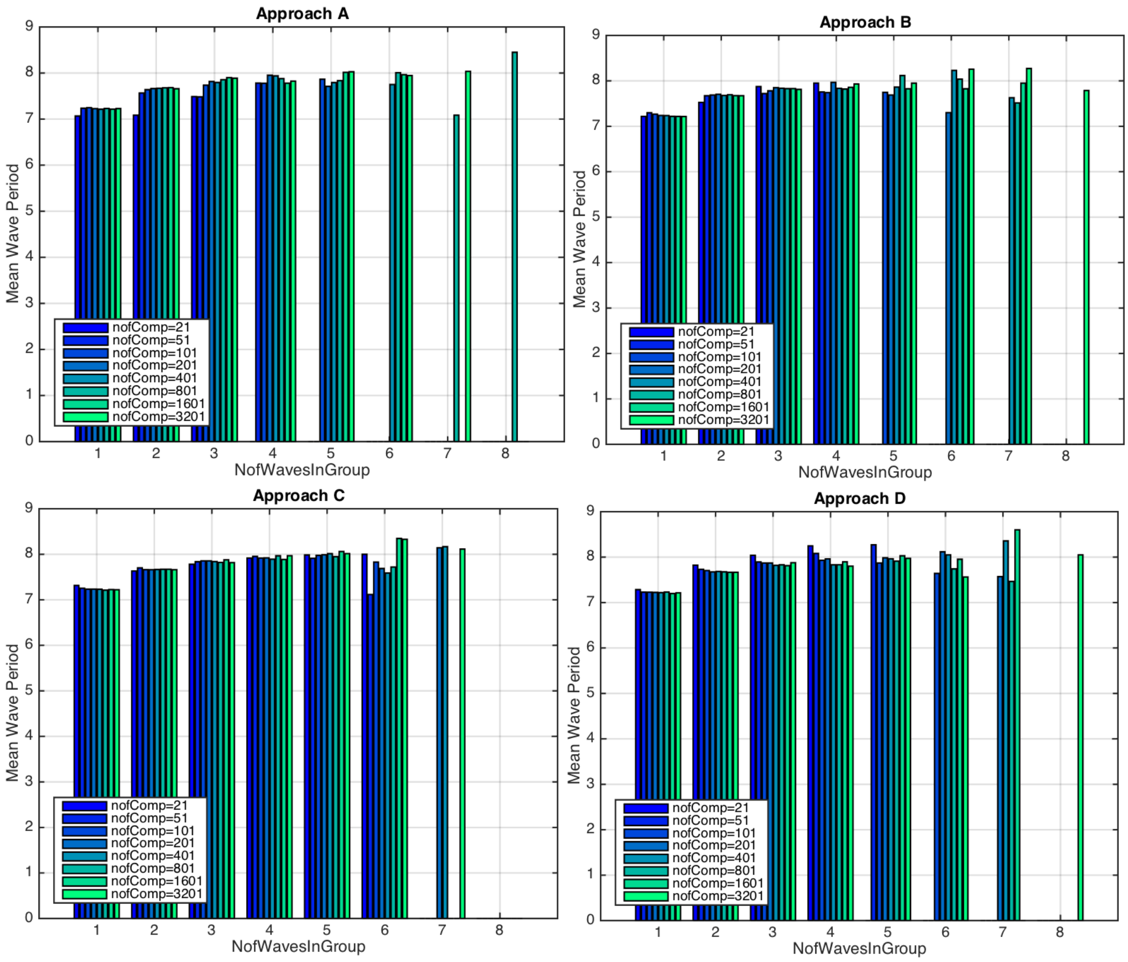

Figure 12. The average zero-crossing period within each wave group size for a wave slope of 0.04.

In Figure 13 the probability for a wave group of a certain run length for the different spectrum discretization approaches are displayed together with the theoretical run lengths in Kimura (1980). It can be seen that all approaches in general shows good agreement to the theoretical probabilities. Generally, it can also be observed that for a given run length approach A and B requires more wave components for the statistics to converge towards the theoretical values. For approach A and B realization of run lengths ≥4 requires at least 401 wave components, while for approaches C and D 51 and 101 components are sufficient.

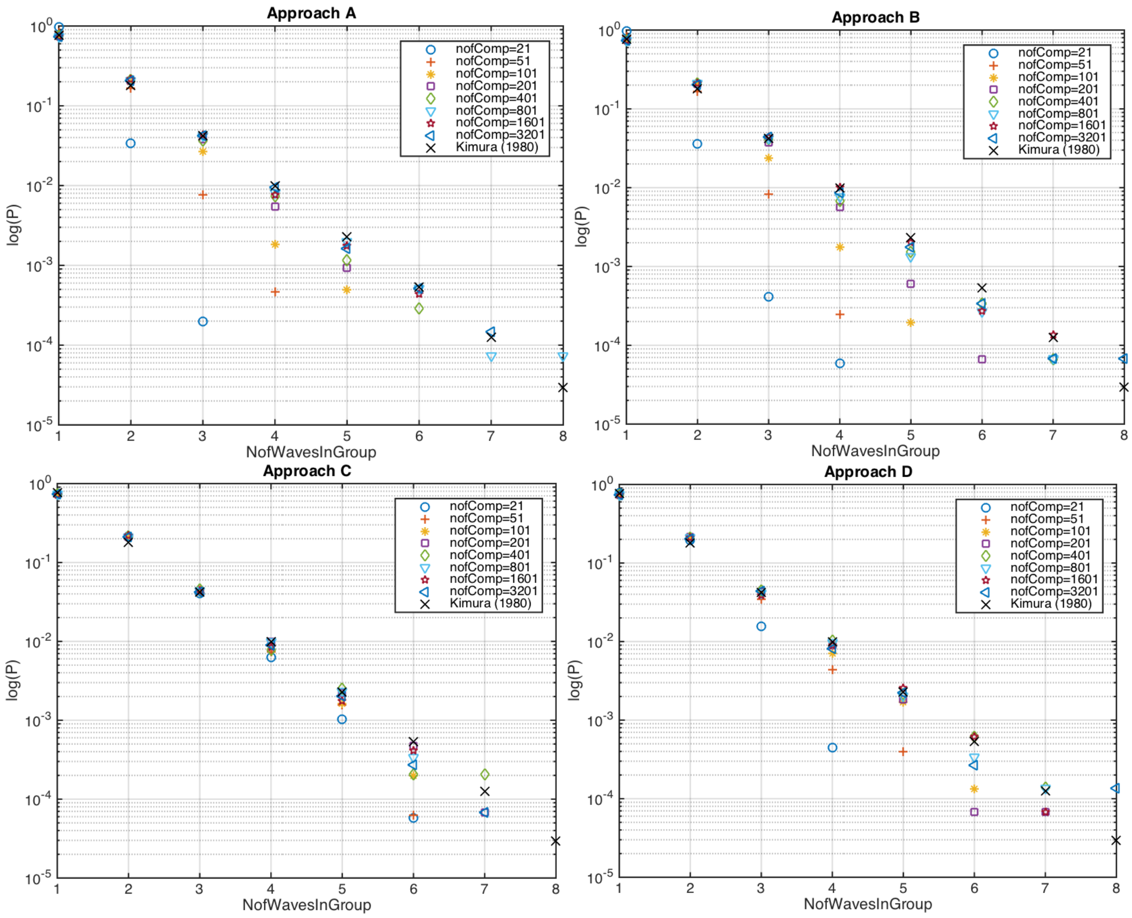

Figure 13. Simulated run length probabilities compared to analytical solution by Kimura (1980).

6.2 Return periods and autocorrelation

In Figure 14 examples of the autocorrelation for the four different spectrum discretization approaches are displayed. The number of wave components is 101 and 401. For these examples the frequency steps are $\Delta\omega = 0.052$ and 0.013 rad/s and $\omega_1 = 0.469$ rad/s for 101 and 401 components respectively, which according to Equation (18) formally gives T_r=6283 and T_r=18850 seconds. However, for approach A the autocorrelation indicates significant practical periodicity in term of wave elevation self-similarity approximately every $2\pi/\Delta\omega = 121$ and 484 seconds for 101 and 401 wave components respectively. Using approach A therefore naturally requires careful consideration of this periodicity and splicing of many shorter wave realizations.

For approaches B-D there is no clear periodicity in the autocorrelation, however the autocorrelation does not tend to zero, instead there is a low level noise which grows to some stationary level related to the number of considered wave components. It can be noted that the magnitude of the low level noise decreases with an increasing number of wave components. The meaning and the possible practical importance of these observations could be further investigated; this is however outside the scope of this study.

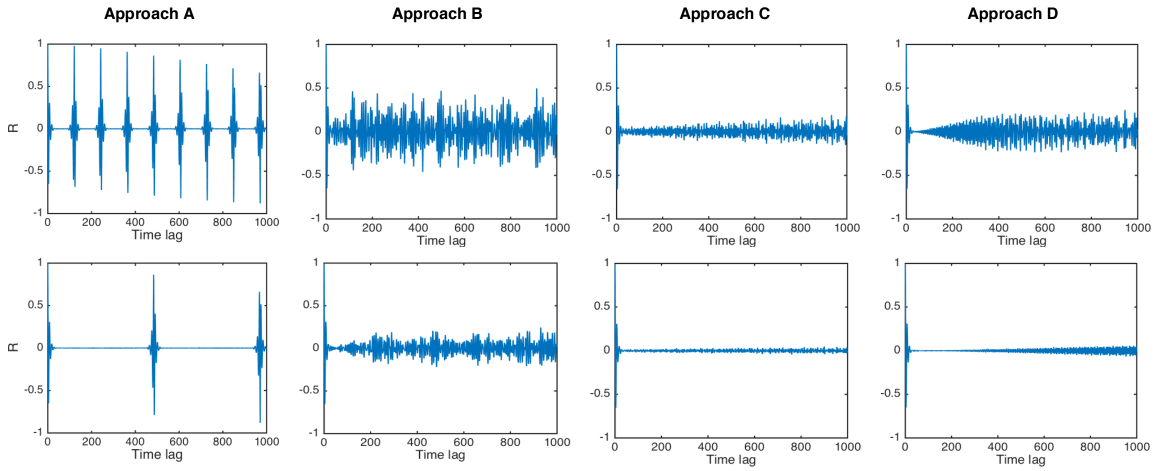

Figure 14. Autocorrelation for time shifts up to 1000 seconds for simulations using 101 and 401 wave components.

7 Practical implications

In the following sections some important considerations in selecting discretization approach and numerically realizing wave sequences are discussed from practical application points of view.

7.1 Statistical quality and computational effort

It is clear based on the results presented in Section 6 that in general approaches C and D result in statistics that converge using fewer wave components compared to approaches A and B. For basic statistical averages, such as significant wave height, and for high probability events, the differences are rather small, and 100 wave components generally is sufficient for all the considered approaches. However, studying statistical quality indicators, which are related to more rarely occurring phenomena, the situation is not quite so simple. For example, the probability for run lengths larger than 2 converge significantly slower for approaches A and B compared to approaches C and D. Similarly, for approach A and B the exceedance probabilities regarding the 3σ level converge considerably slower compared to approaches C and D. It can be concluded that for phenomena related to the tail of the probability distributions, it is beneficial to use approaches C or D. The reason for this is mainly the focused distribution of the wave components in the frequency domain such that the resolution is higher where the main energy content is located. For approach C, the non-uniform carrier frequency distribution is obviously due to the wave component energy equivalence, and for approach D due to the inverse relation between frequency and period.

In Belenky (2004, 2005) it is argued that low-level noise observed in the autocorrelation for Approaches B-D in Figure 14 is related to the periodic pulses seen for Approach A, but that it is spread out over time since the wave components are non-uniformly dispersed in the frequency domain. In Belenky (2004, 2005) it is argued that Approach A therefore is more stringent from a statistical point of view. The statistical quality indicators studied in the previous section does however not indicate any benefits with Approach A. It can further be argued that Approach A is somewhat impractical in that it requires splicing of a large number of realizations for long simulations.

As stated in Section 1 the wave elevation process will approach Gaussian as the number of wave components tends to infinity, given that no wave component dominates the other (Kendall and

Stuart 1994). From a statistical stringency point of view approach C, where all component amplitudes are equal, would therefore be preferable. However, integration of energy equivalent wave components must be performed with significant care and adds complexity, if the spectrum cannot be integrated analytically. In addition to the fast convergence and high statistical quality reached with relatively few components, Approach D has the practical benefit that the period spectrum, in contrast to the frequency ditto, is more intuitive in relation to wave statistics and observed wave conditions.

7.2 Spectrum truncation

As commented in Section 5, spectrum truncation is particular important for approach A and B from a computational cost point of view, therefore an alternative truncation criteria is here investigated which gives better frequency resolution by reducing ω_{max}. The spectrum is here truncated with the criteria that the significant wave height and the average zero-crossing period, based on the spectrum moments, are within less than ±1% and ±5% of the target values. This implies that 99.9% and 98.2% of the spectrum energy is included.

In Figure 15 the effect of the spectrum truncation on the crest height exceedance probabilities are displayed for approach B for a wave slope of 0.04. As seen the truncation criteria has an effect on how the exceedance probabilities converge. For the probabilities related to the threshold value of 2σ, the convergence rate for the two different truncation criteria are very similar. It can also be observed that the statistics converge to slightly different levels due to the energy that is omitted, the difference is however only 3%. For the exceedance level of 3σ, the effect of reducing ω_{max} is that the exceedance probability converges faster. Similarly as for the 2σ level, the probabilities converge to a lower value due to the energy that is omitted, here however the difference is 10%. It can be concluded that the dilemma between resolution in the high-energy part of the spectrum and truncation is not trivial, and the best choice may depend on the purpose. With few components, a significant truncation is necessary in order to obtain reasonable quality. For approach C and D this problem is however non-existing due to the focused distribution of the frequency components.

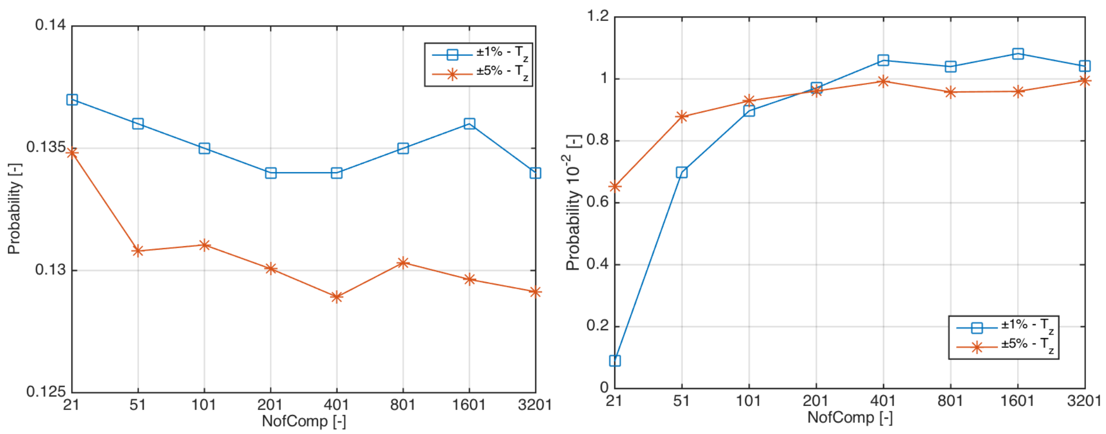

Figure 15. The wave crest probability of exceedance for levels 2σ (a) and 3σ (b) for discretization approach B using two different truncation criteria, ±1% and ±5% regarding the average zero-crossing period.

7.3 High-speed craft accelerations

A practical example is here used to illustrate the importance of wave sequence quality from a ship response statistics point of view. The focus here is on high-speed craft vertical acceleration statistics, which form the basis for evaluation of onboard working environment and structural design of such craft (e.g. Koelbel 1995, DNV 2013 and Razola et al 2016).

High-speed craft vertical accelerations are here simulated using a non-linear strip method (Garme and Rosén 2003, Garme 2005). The simulation method has been extensively validated regarding the global motions and accelerations, for example in Rosén and Garme (2004). The craft has a length of 10.5 meters, a beam of 2.5 meters, a mass of 6500 kg and a deadrise of 22 degrees at the transom. This craft has been extensively studied both experimentally (Garme & Rosén 2003, Rosén and Garme 2004) and numerically (Garme 2005). The irregular waves are realized using the standard 2-paramter Bretschneider spectrum defined according to Equation (29) with a significant wave height of H_s=0.67 m and a zero crossing period of T_z=3.58 s. The craft has a constant forward speed of V=16.3 m/s. The simulation time is 1800 s. The wave spectrum is discretized using discretization approaches B and D using 51 and 201 wave components and numbered according to Table 2. For approach B the two different truncation criteria in Section 7.2 are used, and for approach D truncation is made according to the standard proposed in Section 8.

Table 2. Numbering of simulated conditions 1-6, where discretization approach B and D are considered. For approach B the wave spectrum is truncated so that the sea state parameters are within ±1% and ±5% of the target values.

Approach \ NofComp	51		201	
	T_z±5%	T_z ±1%	T_z±5%	T_z ±1%
B	1	2	3	4
D	5		6	

The resulting acceleration levels are here characterized by the most probable largest value, and the average of the largest 1/100th peak accelerations. In Figure 16 the results from simulations 1-6 are presented where the 95 percent confidence intervals regarding the statistics are included based on a simple percentile Bootstrapping method (Razola et al 2016).

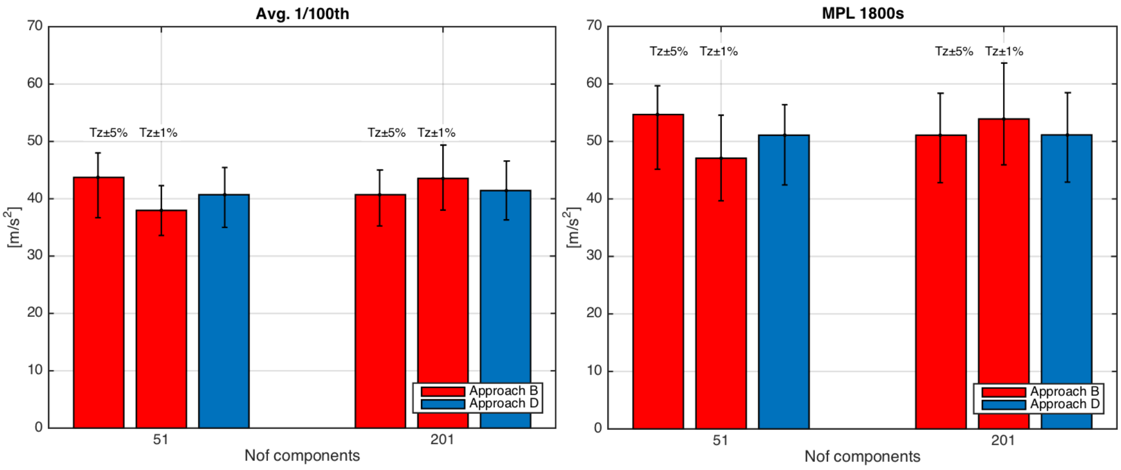

Figure 16. Average of the largest 1/100th peak acceleration values (*a*) and the most probable largest values (*b*) for the simulated cases (Table 3).

As shown in Figure 16 some interesting effects of spectrum discretization and truncation can be observed. First it can be observed that the simulated acceleration levels for approach D are almost identical for 51 and 201 wave components, both regarding the average of the largest 1/100th peak values and the most probable largest values.

For approach B however there are significant differences related to the different truncation criteria and the number of wave components (discretization). For 51 components and a ±5% truncation criteria the acceleration levels predicted are larger compared to approach D. However, including more of the spectrum energy using the ±1% truncation criteria results in significantly lower acceleration level, this is likely related to a poor resolution of the target spectrum. It can also be observed that even with 201 wave components there is a difference in the resulting acceleration statistics due to the different truncation criteria. Here the benefit of approach D is evident, since the dilemma of resolution and truncation can be disregarded.

8 A possible new standard

Based on the results in Section 6 and the discussion in Section 7, it is clear for us that approaches C and D are significantly more efficient than A and B. Both show favorable quality ranking and convergence characteristics. However, in our opinion D has the advantage of being much more simple and intuitive to apply with equidistant period steps. Therefore, as a new standard for high quality wave spectrum truncation and discretization we propose a formulation with the wave components distributed in the period domain between $T_1 \leq T \leq T_N$. The wave elevation is expressed as

$$\varsigma(t) = \sum_{i=1}^{N} \left[\sqrt{2 \int_{T_{c,i}-0.5\Delta T}^{T_{c,i}+0.5\Delta T} S_T(T)\, dT} \cos\left(\frac{2\pi}{T_{c,i}} t + \varepsilon_i \right) \right] \tag{35}$$

where $S_T(T)$ is the period spectrum according to Equation (28). Two alternatives regarding the equation parameters are proposed: one where the spectrum is discretized using a fixed number of N=100 wave components, and one with a fixed period increment of 0.1 seconds. The truncation limits T_1 and T_N, the period increment, ΔT , the wave component carrier frequencies, $T_{c,i}$, and the number of wave components, N, for the two alternatives are outlined in Table 2.

Table 2. Parameters for the standard formulation in equation (35).

	Alternative 1	Alternative 2
T_1	$0.2T_z$	0.1
T_N	$2.5T_z$	$0.1N$
ΔT	$\frac{2.3T_z}{N}$	0.1
$T_{c,i}$	$T_1 + \frac{\Delta T(2i-1)}{2}$	$0.1i$
N	100	$\left\lfloor \frac{2.5T_z}{0.1} \right\rfloor$ *

*Rounded to the nearest lower integer this gives 100-300 wave components for wave periods between $4 \leq T_z \leq 12$ seconds

This formulation will yield excellent wave sequence quality and, as seen in Figure 17, practically negligible energy truncation. The practical periodicity will be infinite for any foreseeable irregular wave condition of interest for ship analysis. Alternative 1 with the lower and upper truncation at $0.2T_z$ and $2.5T_z$ is recommended to keep the number of component low, and to have a similar resolution of the spectrum energy regardless of the wave period. As shown 100 wave components is typically sufficient to obtain high quality realizations. However if very high quality wave realization is required, for example, to properly account for rarely occurring phenomena such as large wave groups, the number of components could be increased. Alternative 2, with fixed period increments, enable further standardization of the wave modeling so that actual simulation sequences can be identically repeated. The random phase lag between components could be locked to any fixed value (e.g. unity) so that specific wave patterns can be identified just by referring to a specific sequence in time. "The simulation was based on a standard period discretization with a 3h sequence starting at t_0=7000…"

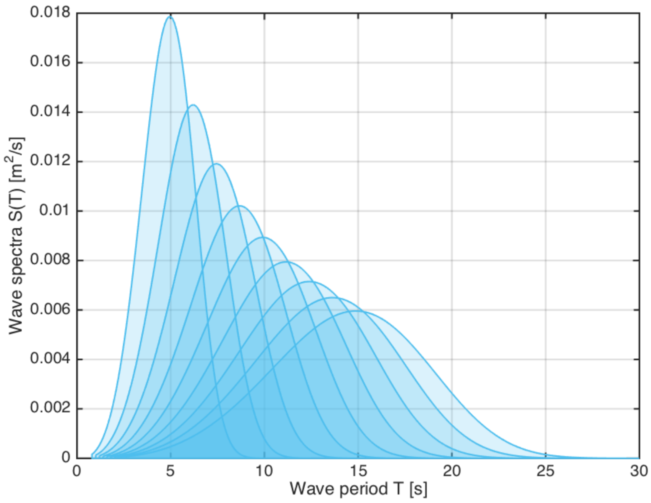

Figure 17. Standard 2-parameter Bretschneider spectra for different zero-crossing periods 4-12 s shown in the period domain (plotted with period increment as in the proposed standard). Significant wave height 1.0 m for all spectra. Note the concentrated symmetric energy distribution and the short tails.

The presented standard, that covers all kinds of wave spectra and all kinds of simulations, would facilitate simulation code development and enable better benchmarking opportunities. The period spectrum formulation has been successfully applied by the authors in various recent studies considering both slamming and parametric roll (e.g. Razola et al (2016) and Huss (2016)), and we consider it from a practical point of view superior to any of the more commonly used approaches.

9 Conclusions

In this paper four different wave spectrum discretization approaches are studied regarding the resulting quality of the realized wave elevation process. Particular focus is on wave realization for long time-domain simulations, where computational cost is crucial. A set of statistical quality indicators, for example wave crest exceedance probabilities, return periods and wave grouping probabilities, are established and used to highlight some important effects of wave spectrum truncation and discretization. The quality of the wave elevation realization is also discussed in relation to the resulting ship responses; here particular focus is on high-speed craft acceleration statistics. The main conclusions are:

- Basic and high probability statistics converge for a very limited number of wave components regardless of the discretization approach. However, for approaches C and D convergence is faster.

- Statistics that relate to more rarely occurring phenomena, i.e. the tail of the distribution, requires a larger number of wave components to converge. For approaches C and D 100 wave components is typically sufficient, while approaches A and B typically require 400 wave components.

- Approaches A and B are sensitive to truncation limits regarding the required number of wave components for the statistics to converge. Approach C is also sensitive to truncation limits if the number of wave components is particularly small. For approach D the truncation is in practice not an issue due to the distribution of wave components period domain.

- If the considered wave spectrum cannot be integrated analytically, approach C requires careful numerical integration and selection of wave components to ensure energy equivalence. For Approach D the period domain formulation per definition distributes the wave components such that a finer discretization in the high-energy region of the spectrum is achieved. It is therefore practical and from a wave environment description point of view more intuitive to formulate the spectrum in the period domain.

The presented results highlight the need for careful consideration when performing numerical wave realization. A new standard for realization of irregular waves in time-domain simulations, where the target spectrum is discretized in the period domain, is proposed. It is shown to yield excellent wave sequence quality and practically unlimited return periods with a low number of wave components, thus limiting the computational effort significantly. Establishment of a unified wave realization standard would have large benefits, for example in simulation code benchmarking and in development of criteria such as the direct stability assessments on level 3 in the IMO second generation intact stability criteria. Further work could concern the significance of the autocorrelation for approaches B-D, and practical implementation of the standard using different types of wave spectra.

10 References

Baarholm G. S., Moan T., "Estimation of Nonlinear Long-Term Extremes of Hull Girder Loads in Ships" Marine Structures 13(6): pp. 495–516, 2000.

Battjes J. A., Van Vledder G. P., "Verification of Kimura's Theory for Wave Group Statistics" Coastal Engineering Proceedings 1(19): pp. 642-648, 1984.

Belenky V. L., "On Risk Evaluation at Extreme Seas", In Proceedings of the 7th International Ship Stability Workshop, Shanghai, China, November, 2004.

Belenky V.L., "On Long Numerical Simulations at Extreme Seas" In Proceedings of the 8th International Ship Stability Workshop, Istanbul, Turkey, October, 2005.

Bitner-Gregersen E. M., Ellermann K., Ewans K. C., Falzarano J. M., Johnson M. C., Nielson U. D., Nilva A., Queffeulou P., Smith T. W. P., Waseda T., "Committee I.1 Environment" In the Proceedings of the 17th International Ship and Offshore Structures Congress, Rostock, Germany, September, 2009.

Borgman L.E., "Ocean Wave Simulation for Engineering Design", Technical report HEL-9-13, University of California, Berkley, 1967.

DNV, "Environmental Conditions and Environmental Loads", DNV Recommend Practice DNV-RP-C205, 2010.

DNV, "Rules for Classification of High Speed, Light Craft and Naval Surface Craft", Det Norske Veritas, 2013.

Elgar S., Guza R. T., Seymour R.J., "Wave Group Statistics From Numerical Simulations of a Random Sea", Applied Ocean Research 7(2): pp. 93–96, 1985.

Garme K., "Improved Time Domain Simulation of Planing Hulls in Waves by Correction of the Near-Transom Lift", International Shipbuilding Progress 52(3): pp. 201–30, 2005.

Garme K., Rosén A., "Time-Domain Simulations and Full-Scale Trials on Planing Craft in Waves", International Shipbuilding Progress 50(3): pp. 177–208, 2003.

Hasselmann K., Barnett T. P., Bouws E., Carlson H., Cartwright D. E., Enke K., Ewing J. A., Gienapp H., Hasselmann D. E., Kruseman P., "Measurements of Wind-Wave Growth and Swell Decay During the Joint North Sea Wave Project (JONSWAP)." *Ergnzungsheft Zur Deutschen Hydrographischen Zeitschrift Reihe*, A8(12), 1973.

Huss M., "Notes on the Modeling of Irregular Seas in Time Simulations", available online since November 2010:http://www.mhuss.se/documents/Downloads/101129mh_Notes_IrrSea_R2.pdf, 2010.

Huss M., "Operational Stability Beyond Rule Compliance", Proceedings of the 15th International Ship Stability Workshop (ISSW2016), 13-15 June, Stockholm, Sweden, 2016.

Kimura A., "Statistical Properties of Random Wave Groups" Coastal Engineering Proceedings 1(17): pp. 2955-2973, 1980.

Longuet-Higgins M. S., "Statistical Properties of Wave Groups in a Random Sea State" Philosophical Transactions of the Royal Society of London a: Mathematical, Physical and Engineering Sciences 312(1521): pp. 219–250, 1984.

Michel W. H., "Sea Spectra Revisited", Marine Technology 36(4): pp. 211–27, 1999.

Ochi M.K., "Principles of Extreme Value Statistics and Their Application" Extreme Loads Response Symposium, SNAME, Arlington, Virginia, October 1981.

Ochi M. K., Sahinoglou I. I., "Stochastic Characteristics of Wave Groups in Random Seas: Part 1 - Time Duration of and Number of Waves in a Wave Group", Applied Ocean Research 11(1): pp. 39–50, 1989.

Pierson W. J., Moskowitz L., "A Proposed Spectral Form for Fully Developed Wind Seas Based on the Similarity Theory of S. A. Kitaigorodskii", Journal of Geophysical Research 69(24): pp. 5181–90, 1964.

Prevosto M., Krogstad H. E., Robin A., "Probability Distributions for Maximum Wave and Crest Heights", Coastal Engineering 40(4): pp. 329–60, 2000.

Razola M., Olausson K., Garme K., Rosén A., "On High-Speed Craft Acceleration Statistics" Ocean Engineering 114(C): pp. 115–33, 2016.

Reed A. M., Degtyarev A.B., Gankevich I., Weems K., "New Models of Irregular Waves", Proceedings of the 15th International Ship Stability Workshop (ISSW2016), 13-15 June, Stockholm, Sweden, 2016.

Rice S. O., "Mathematical Analysis of Random Noise", Bell System Technical Journal 23(3): pp. 282–332, 1944.

Rice S. O., "Mathematical Analysis of Random Noise", Bell System Technical Journal 24(1): pp. 46–15, 1945.

Rodriguez G., Soares C. G., Pacheco M., Pérez-Martell E., "Wave Height Distribution in Mixed Sea States", Journal of Offshore Mechanics and Arctic Engineering 124(1): pp. 34–37, 2002.

Rosén, A., Garme K., "Model Experiments Adressing the Impact Pressure Distribution on Planing Craft in Waves", International Journal of Small Craft Technology 146, 2004.

St Denis M., Pierson W. J. Jr., "On the Motions of Ships in Confused Seas." SNAME Transactions 61: pp. 280–357, 1953.

Stuart A., Keith O., "Kendall's Advanced Theory of Statistics", 6 Ed., Wiley, ISBN 978-0470665305, 1994.

Tucker M. J., Challenor P. G., Carter D. J. T., "Numerical Simulation of a Random Sea: a Common Error and Its Effect Upon Wave Group Statistics", Applied Ocean Research 6(2): pp. 118–22, 1984.

Wang L., and Moan T., “Probabilistic Analysis of Nonlinear Wave Loads on Ships Using Weibull, Generalized Gamma, and Pareto Distributions”, Journal of Ship Research 48(3): pp. 202–17, 2004.

Wu M., Moan T., “Numerical Prediction of Wave-Induced Long-Term Extreme Load Effects in a Flexible High-Speed Pentamaran” Journal of Marine Science and Technology 11(1): pp. 39–51, 2006a.

Wu M., Moan T., “Statistical Analysis of Wave-Induced Extreme Nonlinear Load Effects Using Time-Domain Simulations”, Applied Ocean Research 28(6): pp. 386–97, 2006b.

SNAME, “Guidelines for Site Specific Assessment of Mobile Jack-Up Units”, T&R Bulletin 5-5 and 5-5A, The Society of Naval Architects and Marine Engineers, 2008.