1. Two cars are moving in the same direction in parallel lanes along a highway. At some instant, the velocity of car A exceeds the velocity of car B. Does that mean that the acceleration of car A is greater than that of car B? Explain by analytical analysis in equations with concrete examples.

No, the fact that car A has a greater velocity than car B at some instant does not necessarily mean that the acceleration of car A is greater than that of car B. Velocity and acceleration are related, but they are different concepts: velocity can be defined as the rate at which the position changes over time and acceleration can be described as the rate at which the velocity changes over time. Here we assume 3 cases to prove our answer.

Case 1

Car A is moving with Uniform Velocity, Car B is decelerating

Here, lets assume that:

Car A has a constant velocity of V_A= 25m/s

Car B is decelerating with acceleration of a_B= -2m/s^2

For Car A, since it maintains a constant velocity, its acceleration a_A is 0

V_A = 25m/s

a_A= 0m/s2

For Car B:

V_B= u_B + a_Bt

= 35m/s-2t

Lets say at t=5, the velocity of Car B will be less than that of Car A. Then

VB(5)= 35m/s- 2*5

= 25m/s

Here, we see that Car A has greater velocity than Car B without any acceleration. Also, if we compare the magnitudes of their acceleration , $|a_A|$ is more than , $|a_B|$ indicating that Car A has higher acceleration than of Car B.

Case 2:

Car A has higher acceleration than Car B

Here we assume that Car A accelerates at $5m/s^2$ and Car B at $3m/s^2$

Then , $a_A = 5m/s^2$

$a_B = 3m/s^2$

Here the velocity of car A increases due to its acceleration and will surpass Car B.

Case 3:

Car A is decelerating but Car B has uniform velocity

Lets suppose Car A is decelerating but Car B is maintaining constant velocity

Car A's acceleration $a_A = -1.5m/s^2$

Car B's constant velocity $V_B = 20m/s$

Since, Car B has constant velocity, its acceleration a_B is 0. Then,

$V_B = 20m/s$

$a_B = 0m/s^2$

Calculating when velocity of Car A will exceed that of Car B

$V_A = u_A + a_A t$

$20 = u_A - 1.5t$

Lets assume that initial velocity uA is higher than that of Car B's initially. Let $u_A = 25$. Then,

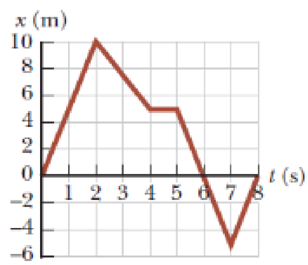
$20 = 25 - 1.5t$

$T = 3.33$ sec

Thus, after 3.33sec, Car A’s velocity will exceed that of Car B. However, since the acceleration is negative but velocity still exceeds that of Car B showing that velocity alone is not sufficient to prove that acceleration is greater or not.

In conclusion we can say that a higher velocity at a given instant does not lead to a higher rate of acceleration at that time instants. The two quantities cannot be combined and considered as a whole; they need to be calculated independently of one another.

2. The position versus time for a certain particle moving along the x axis is shown in the following figure. Find the average velocity in the time intervals (a) 0 to 2s, (b) 0 to 4s, (c) 2s to 4s, (d) 4s to 7s, and (e) 0 to 8s. Notice that the detailed calculation process must be shown.



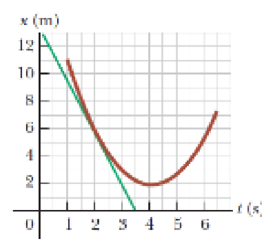
Here, in the graph we are given the time values but we need to find the position values from the graph plotted for the given time values for the given time interval. For instance it is known that the slope of the displacement-time represents the velocity of the body in motion. We know,

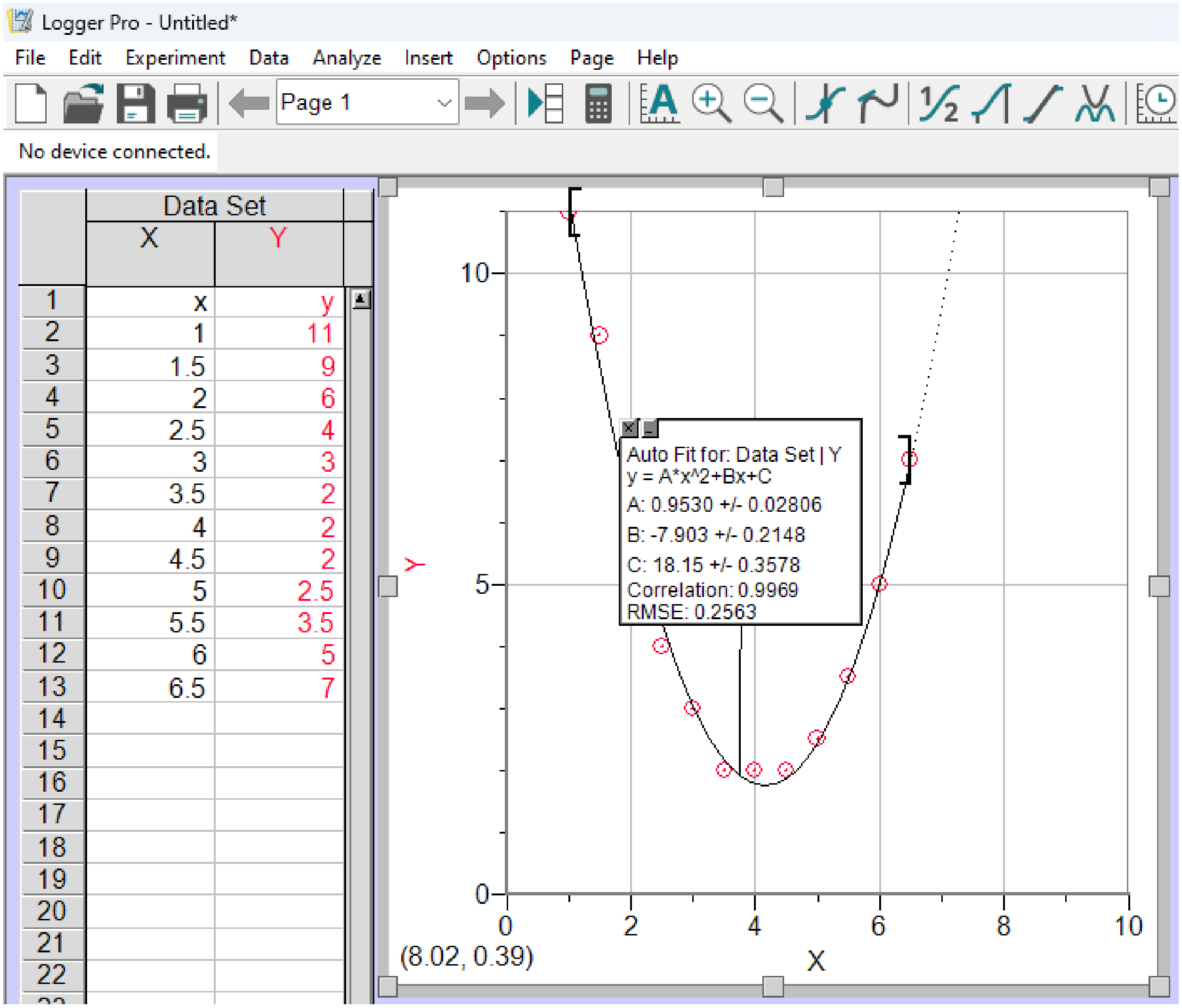
Average velocity(Vav)= x2-x1/ t2-t1. Thus

a. 0 to 2s
   X1= 0m, X2= 10m, t1=0s, t2=2s
   Vav= 10-0/ 2-0
   = 5m/s

b. 0 to 4s
   X1= 0m, X2= 5m, t1= 0s, t2=4s
   Vav= 5-0/4-0
   = 1.25m/s
c. 2 to 4s
   X1= 10m, X2= 5m, t1=2s, t2=4s
   Vav= 5-10/4-2
   = -2.5m/s
d. 4s to 7s
   X1= 5m, X2= -5m, t1=4s, t2=7s
   Vav= -5-5/7-4
   = -3.3m/s
e. 0s to 8s
   X1= 0m, X2= 0m, t1=0s, t2=8s
   Vav= 0-0/8-0
   =0m/s

3. A position - time graph for a particle moving along the x axis is shown in the figure below. (a) Find the average velocity in the time interval t = 1.50s to t = 4.00s. (b) Determine the instantaneous velocity at t = 2.00s by measuring the slope of the tangent line shown in the graph. (c) At what value of t is the velocity zero? Notice that before calculation, the function expression of fitting curve must be found based on the given graph by Logger Pro or Excel through enough observed points on the curve.





With the function expression of fitting curve in logger pro, we come with our curve equation to be:

$Y= 0.953x^2-7.903x+18.15$

Taking derivatives, we get

d/dx (0.953x2-7.903x+18.15)

= 1.906x-7.903

(a) Find the average velocity in the time interval t = 1.50s to t = 4.00s.
We use our equation. Here,
$F(1.50)= 0.953x^2-7.903x+18.15$
$= 0.953(1.50)^2 – 7.903(1.50) +18.15$
$= 8.439$

Again,

$F(4) = 0.953x^2 - 7.903x + 18.15$

$= 0.953(4)^2 - 7.903(4) + 18.15$

$= 1.786$

Average velocity= $F(4) - F(1.5) / t2 - t1$

$= 1.786 - 8.439 / (4 - 1.5)$

$= -2.66$m/s

(b) Determine the instantaneous velocity at t = 2.00s by measuring the slope of the tangent line shown in the graph.

Since, instantaneous velocity is derivative of our derivative function Y. So,

$V(t) = 1.906t - 7.903$

$V(2) = 1.906 * 2 - 7.903$

$= -4.091$m/s

(c) At what value of t is the velocity zero?

The velocity is 0 when slope of the tangent line is 0. Here we can use:

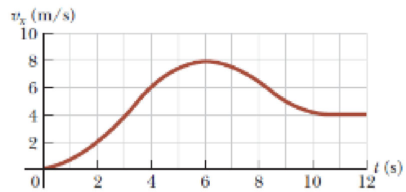
$V(t)=0$. So,

$V(t) = 1.906t - 7.903$

$0 = 1.906t - 7.903$

T= **4.14 seconds**

4. The figure shows a graph of v_x versus t for the motion of a motorcyclist as he starts from rest and moves along the road in a straight line. (a) Find the average acceleration for the time interval t = 0 to t = 6.00s. (b) Estimate the time at which the acceleration has its greatest positive value and the value of the acceleration at that instant. (c) When is the acceleration zero? (d) Estimate the maximum negative value of the acceleration and the time at which it occurs? Notice that the detailed calculation process must be shown.



a) Find the average acceleration for the time interval t = 0 to t = 6.00s.

From the graph, we can see the velocities for these times v1 = v(t1)

= v(6 s) = 8 m/s

vo = v(to)

= v(0 s)

= 0 m/s

velocity change = v1 - vo = 8 m/s - 0 = 8 m/s

change in time = 6.0 s

a = change in velocity/change in time

= [8 m/s] / [6 s] = **1.33 m/s²**

(b) Estimate the time at which the acceleration has its greatest positive value and the value of the acceleration at this instant.

The slope looks highest at about t = 3 s if we estimate the slope by sketching a tangent line which goes through the points, we get

At (t = 1 s, v = 0) and (t = 6 s, v = 10 m/s) change in velocity = v_f - v_i

= 10 m/s - 0

= 10 m/s

change in time = 6.0 s

a = change in velocity /change in time

= [10 m/s] / [6 s]

= 1.67 m/s^2

c. When is the acceleration zero?
It is 0 when the slope of velocity time graph is 0. Here a=0 for **t= 6s . Also for t> 10s**, the acceleration is 0.

d) Estimate the maximum negative value of the acceleration and the time at which it occurs? Notice that the detailed calculation process must be shown.

The slope of line on velocity time graph denotes acceleration. Here from t=0 to t=10, the slope is negative. **It is greatest negative at t=8**. If we estimate the slope by sketching a tangent line which goes through the points. We get,
when t = 6 s, v = 10 m/s and t = 11 s, v = 0 m/s
change in velocity= 0-10
= -10m/s
change in time= 11-6 = 5sec

acceleration= change in velocity/change in time
= -10/5
= -2m/s^2