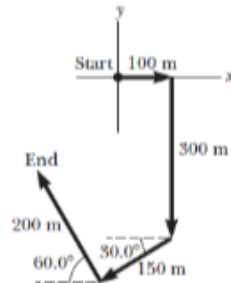


1. A person going for a walk follows the path shown in the figure below. The total trip consists of four straight line paths. At the end of the walk, what is the person's resultant displacement measured from the starting point? Please calculate based on matrix(vector) concept and write the Python program.



Here, writing as vector notation

$$d_1 \vec{=} 100\hat{i} \text{ m}$$

$$d_2 \vec{=} -300\hat{j} \text{ m}$$

$$d_3 \vec{=} (-150\cos 30)\hat{i} \text{ m} + (-150\sin 30)\hat{j} \text{ m} = -130\hat{i} \text{ m} - 75\hat{j} \text{ m} \text{ (since it is angle 30 from horizontal)}$$

$$d_4 \vec{=} (-200\cos 60)\hat{i} \text{ m} + (200\sin 60)\hat{j} \text{ m} = -100\hat{i} \text{ m} + 173\hat{j} \text{ m} \text{ (since it is angle 60 from horizontal)}$$

Adding the components together, we get

$$R_x = d_{1x} + d_{2x} + d_{3x} + d_{4x} = (100 + 0 - 130 - 100) = -130 \text{ m}$$

$$R_y = d_{1y} + d_{2y} + d_{3y} + d_{4y} = (0 - 300 - 75 + 173) \text{ m} = -202 \text{ m}.$$

Hence,

$$R \vec{=} d_1 \vec{+} d_2 \vec{+} d_3 \vec{+} d_4 \vec{=} (-130\hat{i} - 202\hat{j}) \text{ m}$$

The magnitude is

$$|R| = \sqrt{(-130)^2 + (-202)^2} = 240 \text{ m}$$

$$\text{Angle is } \tan^{-1} (R_y/R_x) = \tan^{-1} (-202/-130) = 57.2 \text{ degree}$$

we would consider angles counterclockwise as positive, so we would write this as

$$\text{angle} = -57.2 \text{ degree}$$

```
[1] import numpy as np
import math

# Define vectors with descriptive variable names
vector_r1 = np.array([100, 0])
vector_r2 = np.array([0, -300])

# Calculate components of vectors r3 and r4
magnitude_r3 = 150
angle_r3 = 30
magnitude_r4 = 200
angle_r4 = 60

# Calculate components of vectors r3 and r4
vector_r3 = np.array([-magnitude_r3 * math.cos(math.radians(angle_r3)), -magnitude_r3 * math.sin(math.radians(angle_r3))])
vector_r4 = np.array([-magnitude_r4 * math.cos(math.radians(angle_r4)), magnitude_r4 * math.sin(math.radians(angle_r4))])

# Calculate the resultant vector
vector_resultant = vector_r1 + vector_r2 + vector_r3 + vector_r4

# Calculate the magnitude of the resultant vector
magnitude_resultant = np.linalg.norm(vector_resultant)

# Calculate the direction (angle with the positive x-axis)
angle_resultant = np.degrees(np.arctan2(vector_resultant[1], vector_resultant[0]))

# Adjust the angle to be measured clockwise from the negative x-axis
if angle_resultant < 0:
    angle_resultant += 360

# Direction relative to the negative x-axis
direction_from_negative_x_axis = angle_resultant - 180 if angle_resultant > 180 else angle_resultant

# Print the results
print(f"Resultant Displacement Vector: {vector_resultant}")
print(f"Magnitude: {magnitude_resultant:.2f} meters")
print(f"Direction from negative x-axis: {direction_from_negative_x_axis:.2f} degrees")
```

Resultant Displacement Vector: [-129.90381057 -201.79491924]  
Magnitude: 239.99 meters  
Direction from negative x-axis: 57.23 degrees

((Python code is in .ipynb file))

2. After a ball rolls off the edge of a horizontal table at time  $t = 0$ , its velocity as a function of time is given by

$$\vec{v} = 1.2\hat{i} - 9.8t\hat{j}$$

where  $\vec{v}$  is in meters per second and  $t$  is in seconds. The ball's displacement away from the edge of the table, during the time interval of  $0.380\text{ s}$  for which the ball is in flight, is given by

$$\Delta\vec{r} = \int_0^{0.380\text{s}} \vec{v} dt$$

To perform the integral, you can use the calculus theorem

$$\int [A + Bf(x)]dx = \int A dx + B \int f(x) dx$$

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You can think of the units and unit vectors as constants, represented by A and B. Perform the integration to calculate the displacement of the ball from the edge of the table at 0.380s.

Here, we integrate the velocity vector  $\vec{v}$  over the given time interval.

Given,

$$\vec{v} = 1.2\hat{i} - 9.8t\hat{j}$$

displacement  $\Delta\vec{r}$  is given by the integral of velocity  $\vec{v}$  with respect to time

$$\begin{aligned}\Delta\vec{r} &= \int_0^{0.380} \vec{v} \, dt \\ &= \int_0^{0.380} 1.2\hat{i} - 9.8t\hat{j} \, dt\end{aligned}$$

Integrating each components separately

For the  $\hat{i}$  component, we get

$$\begin{aligned}&\int_0^{0.380} 1.2 \, dt \\ &= 1.2 * t \Big|_0^{0.380} \\ &= 1.2 * 0.380 \\ &= 0.456\text{m}\end{aligned}$$

For the  $\hat{j}$  component, we get

$$\begin{aligned}&\int_0^{0.380} -9.8t \, dt \\ &= -9.8 * t^2/2 \Big|_0^{0.380} \\ &= -9.8 * (0.380)^2/2 \\ &= -0.710\text{m}\end{aligned}$$

Hence, the displacement vector  $\Delta\vec{r}$  is:

$$\Delta\vec{r} = 0.456\hat{i} - 0.710\hat{j} \, \text{m}$$

This is the displacement of the ball from the edge of the table at 0.380 seconds

3. Ecotourists use their global positioning system indicator to determine their location inside a botanical garden as latitude  $0.002, 43$  degrees south of the equator, longitude  $75.642, 38$  degrees west. They wish to visit a tree at latitude  $0.001, 62$  degrees north, longitude  $75.644, 26$  degrees west. (a) Determine the straight-line distance and the direction in which they can walk to reach the tree as follows. First model the Earth as a sphere of radius  $6.37 \times 10^6$  m to determine the westward and northward displacement components required, in meters. Then model the Earth as a flat surface to complete the calculation. (b) Explain why it is possible to use these two geometrical models together to solve the problem. Please calculate by Python programs.

Given,

The latitude of the Ecotourist inside botanical garden =  $\theta_1 = -0.00243^\circ$

The longitude of the Ecotourist inside botanical garden =  $\phi_1 = 75.64238^\circ$

The latitude of the tree inside botanical garden =  $\theta_2 = 0.00162^\circ$

The longitude of the tree inside botanical garden =  $\phi_2 = 75.64426^\circ$

The radius of Earth is  $R = 6.37 \times 10^6$  m

Since, The distance of Ecotourist is found by calculating the distance covered due to the change in latitude and distance covered due to change in longitude, then taking the vector sum of both distances.

The westward displacement of the Ecotourist is given as:

$$d_x = R(\phi_2 - \phi_1)$$

$$= (6.37 \times 10^6) \text{ m} * (75.64426^\circ - 75.64238^\circ)$$

$$= (1.1975 \times 10^5 \text{ m}) * (\pi \text{ rad} / 180^\circ)$$

$$= 209 \text{ m}$$

The northward displacement of the Ecotourist is given as:

$$dy = R(\theta_2 - \theta_1)$$

$$= (6.37 \times 10^6 \text{m}) * (0.00162^\circ - (-0.00243^\circ))$$

$$= (0.00257 * 10^6 \text{m}) (\pi \text{rad} / 180^\circ)$$

$$= 450 \text{m}$$

The straight-line distance of the ecotourist is given as:

$$d = \text{root}((dx)^2 + (dy)^2)$$

For the given values, the above equation becomes:

$$d = \text{root}(209\text{m}^2 + (450\text{m})^2)$$

$$d = 496.42 \text{m}$$

The direction of the ecotourist is given as:

$$\tan \alpha = dy/dx$$

For the given values, the above equation becomes-

$$\tan \alpha = 450 \text{m} / 209 \text{m}$$

$$\alpha = 65.1^\circ$$

Therefore, the northward component of displacement is 450 m, the westward component of displacement is 290 m, the straight line distance of the ecotourist assuming earth as flat surface is 496.17m and the direction of the ecotourist is 65.1° north of west.

((Python code is in .ipynb file))

```
import math

# Constants
earth_radius = 6.37 * 10**6 # Radius of Earth in meters
ecotourist_latitude = 0.00243 # Latitude of ecotourists' location in degrees south
ecotourist_longitude = 75.64238 # Longitude of ecotourists' location in degrees west
tree_latitude = 0.00162 # Latitude of tree in degrees north
tree_longitude = 75.64426 # Longitude of tree in degrees west

# Convert degrees to radians
ecotourist_lat_rad = math.radians(ecotourist_latitude)
ecotourist_lon_rad = math.radians(ecotourist_longitude)
tree_lat_rad = math.radians(tree_latitude)
tree_lon_rad = math.radians(tree_longitude)

# Calculate displacement components
westward_distance = (tree_longitude - ecotourist_longitude) * math.pi * earth_radius * math.cos(ecotourist_lat_rad) / 180
northward_distance = (tree_latitude + ecotourist_latitude) * math.pi * earth_radius / 180

# Calculate total distance
total_distance = math.sqrt(westward_distance**2 + northward_distance**2)

# Calculate direction
direction = math.degrees(math.atan2(northward_distance, westward_distance))

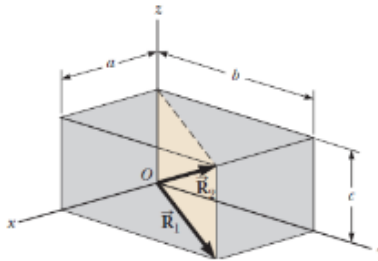
# Print results
print("Straight-line distance to tree: {:.2f} meters".format(total_distance))
print("Direction to walk: {:.2f} degrees north of west".format(direction))
```

Straight-line distance to tree: 496.42 meters  
Direction to walk: 65.10 degrees north of west

(b) Explain why it is possible to use these two geometrical models together

It is valid to use both spherical and flat Earth approximations together because: Spherical model gives us an accurate calculation of the great-circle distance between two points on the Earth's surface. Flat Earth approximation simplifies the calculation of the straight-line distance and direction over short distances, assuming the curvature of the Earth is negligible over the small distance involved.

4. A rectangular parallelepiped has dimensions  $a$ ,  $b$ , and  $c$  as shown in the following figure (a) Obtain a vector expression for the face diagonal vector  $\vec{R}_1$  in matrix (vector). (b) What is the magnitude of this vector? (c) Notice that  $\vec{R}_1$ ,  $c\hat{k}$  and  $\vec{R}_2$  make a right triangle. Obtain a vector expression for the body diagonal vector  $\vec{R}_2$  in matrix (vector).



- (a)  $R_1$  is the hypotenuse of a right triangle in the  $xy$  plane or the diagonal of the rectangle in the  $xy$  plane. The sides are  $a$  (along  $x$ ) and  $b$  (along  $y$ ). Therefore,

$$\vec{R}_1 = a\hat{i} + b\hat{j}$$

- (b) The magnitude of this vector is:

$$R_1 = \sqrt{a^2 + b^2}$$

**For part c**

$$\vec{R}_2 = \vec{R}_1 + c\hat{k}$$

$$= a\hat{i} + b\hat{j} + c\hat{k}$$

$$\text{Magnitude of } R_2 = \sqrt{a^2 + b^2 + c^2}$$