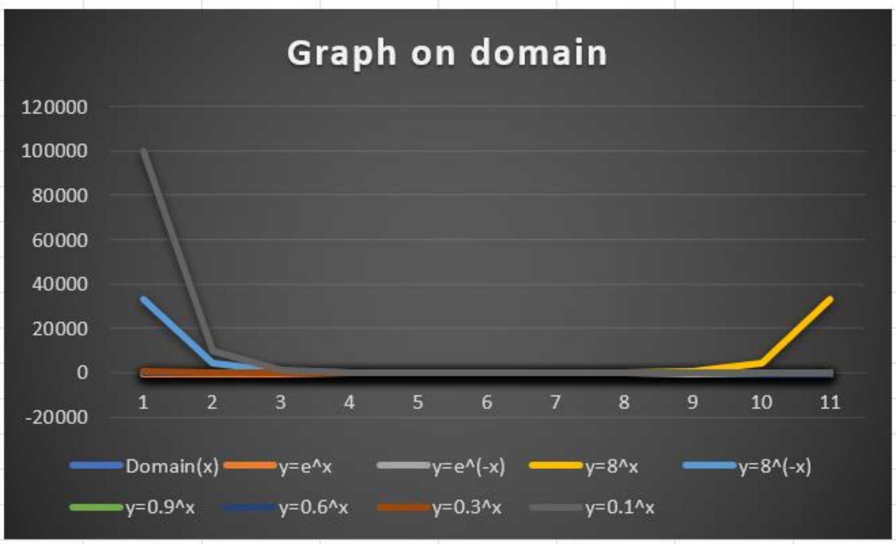
1. Plot each following group of functions in one graph respectively by Excel, covering the appropriate domain of x and y..

a. $y = e^x,\ y = e^{-x}, y = 8^x, y = 8^{-x}$

b. $y = 0.9^x,\ y = 0.6^x, y = 0.3^x, y = 0.1^x$

Answer of no:1

Domain(x)	y=e^x	y=e^(-x)	y=8^x	y=8^(-x)	y=0.9^x	y=0.6^x	y=0.3^x	y=0.1^x
-5	0.0067	148.4132	0.0000	32768.0000	1.6935	12.8601	411.5226	100000.0000
-4	0.0183	54.5982	0.0002	4096.0000	1.5242	7.7160	123.4568	10000.0000
-3	0.0498	20.0855	0.0020	512.0000	1.3717	4.6296	37.0370	1000.0000
-2	0.1353	7.3891	0.0156	64.0000	1.2346	2.7778	11.1111	100.0000
-1	0.3679	2.7183	0.1250	8.0000	1.1111	1.6667	3.3333	10.0000
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	2.7183	0.3679	8.0000	0.1250	0.9000	0.6000	0.3000	0.1000
2	7.3891	0.1353	64.0000	0.0156	0.8100	0.3600	0.0900	0.0100
3	20.0855	0.0498	512.0000	0.0020	0.7290	0.2160	0.0270	0.0010
4	54.5982	0.0183	4096.0000	0.0002	0.6561	0.1296	0.0081	0.0001
5	148.4132	0.0067	32768.0000	0.0000	0.5905	0.0778	0.0024	0.0000



2. Given $f(x) = 10^x$, prove that $\frac{f(x+h)-f(x)}{h} = 10^x(\frac{10^h-1}{h})$ and verify it by the plot in Excel.

Taking LHS:

$= \frac{f(x+h)-f(x)}{h}$

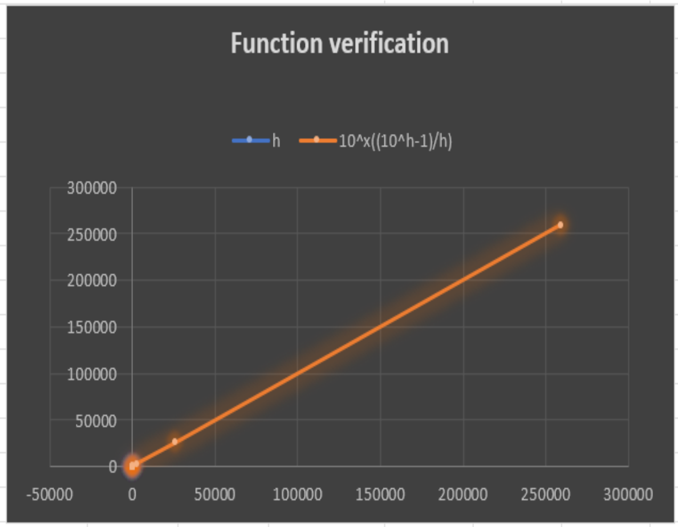
$= \frac{10^{x+h}-10^x}{h}$ [Since, f(x)=10^x]

$= \frac{10^x \cdot 10^h - 10^x}{h}$

$= \frac{10^x \cdot (10^h-1)}{h}$

Hence, LHS=RHS

x	h	(f(x+h)-f(x))/h	10^x((10^h-1)/h)
-5	0.1	2.58925E-05	2.58925E-05
-4	0.1	0.000258925	0.000258925
-3	0.1	0.002589254	0.002589254
-2	0.1	0.025892541	0.025892541
-1	0.1	0.258925412	0.258925412
0	0.1	2.589254118	2.589254118
1	0.1	25.89254118	25.89254118
2	0.1	258.9254118	258.9254118
3	0.1	2589.254118	2589.254118
4	0.1	25892.54118	25892.54118
5	0.1	258925.4118	258925.4118



3. Compare the functions $f(x) = x^5$ and $g(x) = 5^x$ by plotting curve in Excel and which function grows more rapidly when *x* is large? And prove it mathematically.

To compare the functions f(x) = x^5 and g(x) = 5^x and determine which one grows more rapidly when x is large, we can plot their curves in Excel and observe their behavior.

Let's create a table in Excel with the x-values ranging from, for example, 0 to 10. In the adjacent column, we can calculate the corresponding y-values for each function using the formulas f(x) = x^5 and g(x) = 5^x.

After inputting the formulas and generating the values, we can select the data and create a scatter plot in Excel. The x-values will be plotted on the horizontal axis, and the corresponding y-values will be plotted on the vertical axis.

Upon examining the graph, we can observe that both curves start at the point (0, 0) since f(0) = 0^5 = 0 and g(0) = 5^0 = 1. As x increases, the function f(x) = x^5 grows rapidly but still follows a polynomial growth pattern. On the other hand, the function g(x) = 5^x grows even more rapidly and exhibits an exponential growth pattern.

To prove mathematically that g(x) = 5^x grows more rapidly than f(x) = x^5 as x becomes large, we can compare their derivatives.

The derivative of f(x) = x^5 can be calculated as follows:

f'(x) = 5x^4

The derivative of g(x) = 5^x can be determined using the chain rule and logarithmic differentiation:

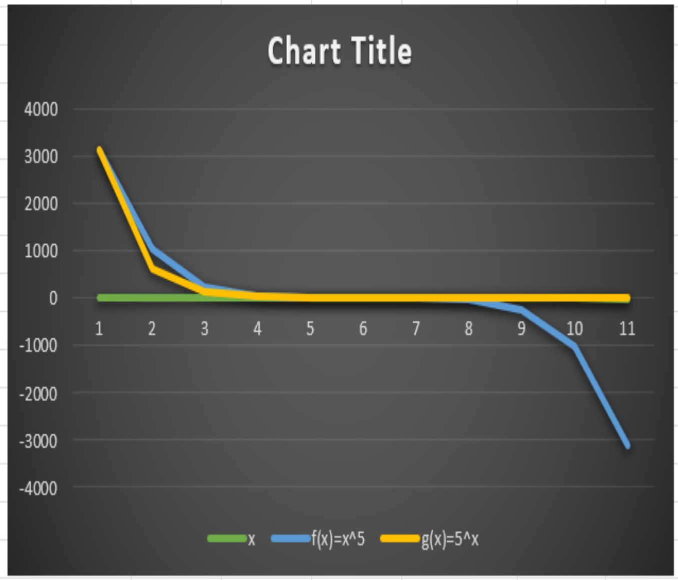
g'(x) = 5^x * ln(5)

When evaluating these derivatives, it becomes evident that the rate of change of f(x) is dependent on the value of x and is limited by the power of x^4. As x approaches infinity, the derivative will tend to infinity but remain constrained by the power term.

However, the derivative of g(x) = 5^x includes the function value 5^x itself, multiplied by the natural logarithm of 5. This implies that the rate of change of g(x) is directly proportional to its function value, resulting in exponential growth. As x becomes large, there is no limiting factor that restricts the growth of g(x).

Therefore, based on the mathematical analysis, we can conclude that g(x) = 5^x grows more rapidly than f(x) = x^5 when x is large. The graph is shown below:

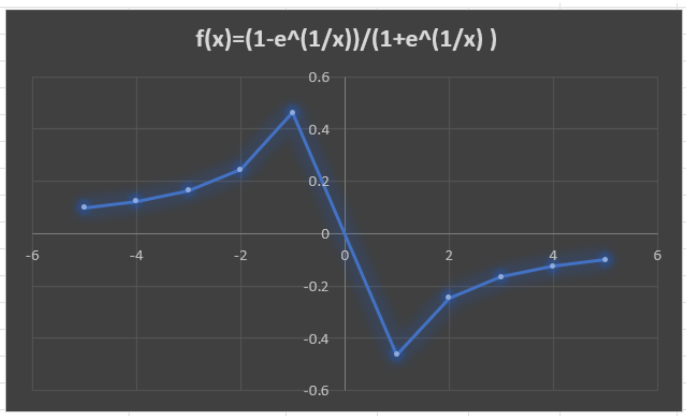
x	f(x)=x^5	g(x)=5^x
5	3125	3125
4	1024	625
3	243	125
2	32	25
1	1	5
0	0	1
-1	-1	0.2
-2	-32	0.04
-3	-243	0.008
-4	-1024	0.0016
-5	-3125	0.00032



4. Plot the function $f(x) = \frac{1-e^{1/x}}{1+e^{1/x}}$ in Excel. And then prove that $f(x)$ is an odd function.

As, we can see from the graph that it is symmetric with respect to origin. Hence, the given function is odd function.

x	f(x)=(1-e^(1/x))/(1+e^(1/x))
5	-0.099667995
4	-0.124353002
3	-0.165140413
2	-0.244918662
1	-0.462117157
-1	0.462117157
-2	0.244918662
-3	0.165140413
-4	0.124353002
-5	0.099667995



5. For the parametrized function $f(x) = \frac{1}{1+ae^{bx}}$

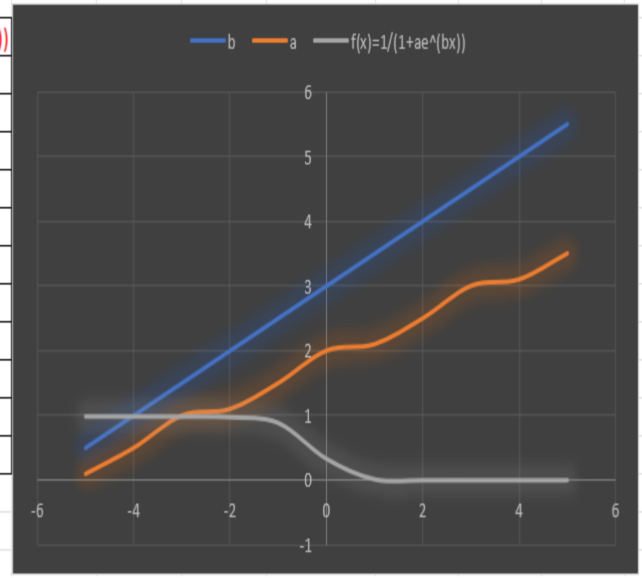
a. **where a > 0. How does the graph change when b changes by showing a group of curves by Excel?**

b. **How does it change when *a* changes in Excel?**

Answer for a: The graph f(x) becomes steeper as b increases when value of a > 0.
Answer for b: As, a increases, the amplitude of graph f(x) increases.

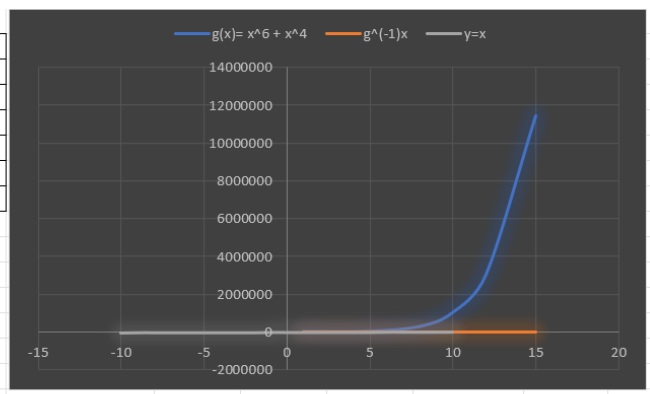
Answer of no: 5

x	b	a	f(x)=1/(1+ae^(bx))
-5	0.5	0.1	0.991858331
-4	1	0.5	0.990925285
-3	1.5	1	0.989013057
-2	2	1.1	0.980250691
-1	2.5	1.5	0.890370863
0	3	2	0.333333333
1	3.5	2.1	0.014175862
2	4	2.5	0.000134167
3	4.5	3	4.56986E-07
4	5	3.1	6.64888E-10
5	5.5	3.5	3.25712E-13



6. If $g(x) = x^6 + x^4, x \geq 0$, find $g^{-1}(x)$ expression. And that, plot $y = g(x), y = x$, and $y = g^{-1}(x)$ in one graph by Excel.

x	g(x)= x^6 + x^4	g^(-1)x
1	2	0.5
5	16250	6.15385E-05
8	266240	3.75601E-06
10	1010000	9.90099E-07
12	3006720	3.32588E-07
15	11441250	8.7403E-08



7. When a camera flash goes off, the batteries immediately begin to recharge the flash's capacitor, which stores electric charge given by

$$Q(t) = Q_0(1 - e^{-t/a})$$

(The maximum charge capacity is Q_0 and t is measured in seconds.)

a. Find the inverse of this function and explain its meaning.

b. How long does it take to recharge the capacitor to 90% of capacity if a = 2 showing in the plot by **Excel**?

Answer for a

Given, $that\ Q(t) = Q_0(1 - e^{-t/a})$
$\frac{Q}{Q_0} = 1 - e^{-t}/a$

Rearranging the equations, we get

$$1 - \frac{Q}{Q_0} = e^{-t}/a$$

Taking log on both sides, we get

$$\ln\left(1 - \frac{Q}{Q_0}\right) = -t/a$$

$$\mathrm{t} = -\mathrm{a} * \ln\left(1 - \frac{Q}{Q_0}\right)$$ **[eqn- 1]**

Therefore, $\mathrm{Q}'(\mathrm{t}) = -\mathrm{a} * \ln\left(1 - \frac{Q}{Q_0}\right)$

Answer for b

Given:

a = 2

Q(t) = 0.9Q0 (90% capacity represented by 0.9 times Q0)

Substituting these values into the equation:

t = -2 * ln(1 - 0.9)

t = -2 * ln(0.1)

t = -2 * (-2.3)

t = 4.6 seconds