1. If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height in

meters t seconds later is given by $y = 10t — 1.86t2$

(a) Find the average velocity over the given time intervals

To find the average velocity over the given time intervals, we need to calculate the displacement of the rock during each interval and divide it by the duration of the interval.

(a) Average velocity over the given time intervals:

(i) [1, 2]:

To find the average velocity from time t = 1 to t = 2, we subtract the initial position (t = 1) from the final position (t = 2) and divide it by the time interval (2 - 1 = 1).

Initial position (t = 1):

y(1) = 10(1) - 1.86(1^2) = 10 - 1.86 = 8.14 m

Final position (t = 2):

y(2) = 10(2) - 1.86(2^2) = 20 - 7.44 = 12.56 m

Displacement: y(2) - y(1) = 12.56 - 8.14 = 4.42 m

Average velocity: Displacement / Time interval = 4.42 m / 1 s = 4.42 m/s

(ii) [1, 1.5]:

Similarly, for the time interval from t = 1 to t = 1.5:

Initial position (t = 1):

y(1) = 8.14 m (from the previous calculation)

Final position (t = 1.5):

y(1.5) = 10(1.5) - 1.86(1.5^2) = 15 - 4.185 = 10.815 m

Displacement: y(1.5) - y(1) = 10.815 - 8.14 = 2.675 m

Average velocity: Displacement / Time interval = 2.675 m / 0.5 s = 5.35 m/s

(iii) [1, 1.1]:

For the time interval from t = 1 to t = 1.1:

Initial position (t = 1):

y(1) = 8.14 m (from the previous calculation)

Final position (t = 1.1):

y(1.1) = 10(1.1) - 1.86(1.1^2) = 11 - 2.0466 = 8.7494 m

Displacement: y(1.1) - y(1) = 8.7494 - 8.14 = 0.6094 m

Average velocity: Displacement / Time interval = 0.6094 m / 0.1 s = 6.094 m/s

(iv) [1, 1.01]:

For the time interval from t = 1 to t = 1.01:

Initial position (t = 1):

y(1) = 8.14 m (from the previous calculation)

Final position (t = 1.01):

y(1.01) = 10(1.01) - 1.86(1.01^2) = 10.1 - 1.8666 = 8.2027 m

Displacement: y(1.01) - y(1) = 8.2027 - 8.14 = 0.0627 m

Average velocity: Displacement / Time interval = 0.0627 m / 0.01 s = 6.27 m/s

(v) [1, 1.001]:

For the time interval from t = 1 to t = 1.001:

Initial position (t = 1):

y(1) = 8.14 m (from the previous calculation)

Final position (t = 1.001):

y(1.001) = 10(1.001) - 1.86(1.001^2) = 10.01 - 1.86603 = 8.1463 m

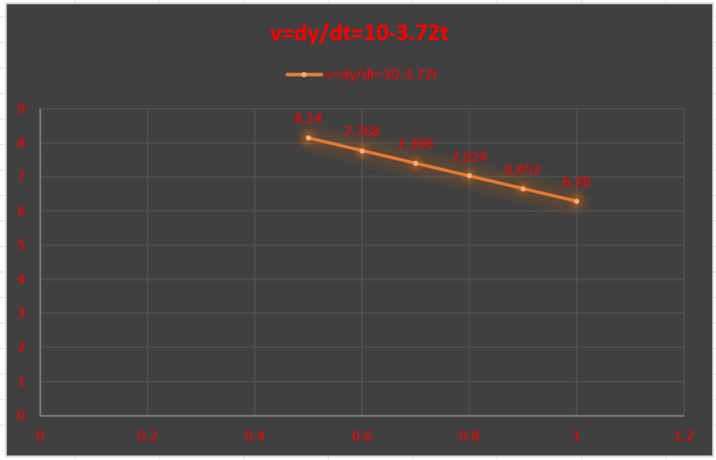
Displacement: y(1.001) - y(1) = 8.1463 - 8.14 = 0.0063 m

Average velocity: Displacement / Time interval = 0.0063 m / 0.001 s = 6.3 m/s

(b) Estimate the instantaneous velocity in Excel when t = 1

Answer of 1b

t	y=10t-1.86t^2	v=dy/dt=10-3.72t
0.5	4.535	8.14
0.6	5.3304	7.768
0.7	6.0886	7.396
0.8	6.8096	7.024
0.9	7.4934	6.652
1	8.14	6.28



2. The displacement (in centimeters) of a particle moving back and forth along a straight line is given by the equation of motion s = 2sin (πt) +3cos (πt), where t is measured in seconds.

(a) Find the average velocity during each time interval:

To find the average velocity for each time interval, we need to calculate the

displacement during that interval and divide it by the duration of the interval.

Here, Given:

s = 2sin (πt) + 3cos (πt)

(i) [1, 2]

At t=1,

s (1) = 2 sin (π*1) + 3 cos (π*1)

s (1) = 2 sin (π) + 3 cos (π)

s (1) = 2 (0) + 3 (-1)

s (1) = -3 cm

At t=2,

s (2) = 2 sin (π*2) + 3 cos (π*2)

s (2) = 2 sin (2π) + 3 cos (2π)

s (2) = 2 (0) + 3 (1)

s (2) = 3 cm

Displacement during the interval is,

$\Delta s = s(2) - s(1)$

$\Delta s = 3-(-3)$

$\Delta s = 6cm$

The time duration of the interval (Δt) is 2-1=1 sec.

So, average velocity= $\Delta s/\Delta t$

$= 6/1$

$=6$ cm/s

(ii) [1, 1.1]

At t=1,

s (1) = -3 cm

At t=1.1,

$s(1.1) = 2\sin(\pi*1.1) + 3\cos(\pi*1.1)$

$s(1.1) = 2\sin(1.1\pi) + 3\cos(1.1\pi)$

s (1.1) = -3.4712 cm

Displacement during the interval is,

$\Delta s = s(1.1) - s(1)$

$\Delta s = -3.4712-(-3)$

$\Delta s = -0.4712$ cm

The time duration of the interval (Δt) is (1.1)-1=0.1 sec.

So, average velocity= $\Delta s/\Delta t$

$= -0.4712/0.1$

$=-4.712$ cm/s

(iii) [1, 1.01]

At t=1,

s (1) = -3 cm

At t=1.01,

s (1.01) = 2 sin (π*1.01) + 3 cos (π*1.01)

s (1.01) = 2 sin (1.01π) + 3 cos (1.01π)

s (1.01) = -3.06134 cm

Displacement during the interval is,

Δs = s (1.01) -s (1)

Δs = -3.06134- (-3)

Δs = -0.06134 cm

The time duration of the interval (Δt) is (1.01)-1=0.01 sec.

So, average velocity= Δs/Δt

= -0.06134/0.01

=-6.134 cm/s

(iv) [1, 1.001]

At t=1,

s (1) = -3 cm

At t=1.001,

s (1.001) = 2 sin (π*1.001) + 3 cos (π*1.001)

s (1.001) = 2 sin (1.001π) + 3 cos (1.001π)

s (1.001) = -3.006268 cm

Displacement during the interval is,

Δs = s (1.001) -s (1)

Δs = -3.006268- (-3)

Δs = -0.006268 cm

The time duration of the interval (Δt) is (1.001)-1=0.001 sec.

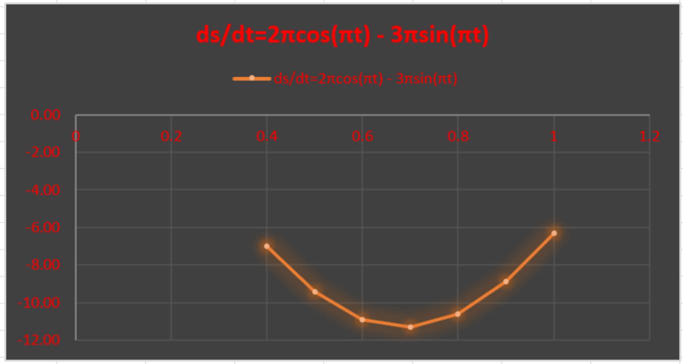
So, average velocity= Δs/Δt

= -0.006268/0.001

=-6.268 m/s

(b) Estimate the instantaneous velocity of the particle in Excel when $t = 1$

Answer of 2b		
x	s=2 sin(πt)+3cos(πt)	ds/dt=2πcos(πt) - 3πsin(πt)
0.4	2.83	-7.02
0.5	2.00	-9.42
0.6	0.98	-10.91
0.7	-0.15	-11.32
0.8	-1.25	-10.62
0.9	-2.24	-8.89
1	-3.00	-6.28



3a. Estimate the value of

$$\lim_{x \to 0} \frac{\sin(x)}{\sin(\pi x)}$$

by graphing the function $f(x)$ = (sin x) / (sin πx) in Excel. State your answer correct to two decimal places.

Here we can see that (0/0) the indeterminate from now we can apply the L'hospital rule.

$$L = \lim_{x \to 0} \frac{\sin(x)}{\sin\,(\pi x)}$$

$$= \lim_{x \to 0} \frac{\frac{d}{dx} sinx}{\frac{d}{dx}\sin(\pi x).\pi}$$

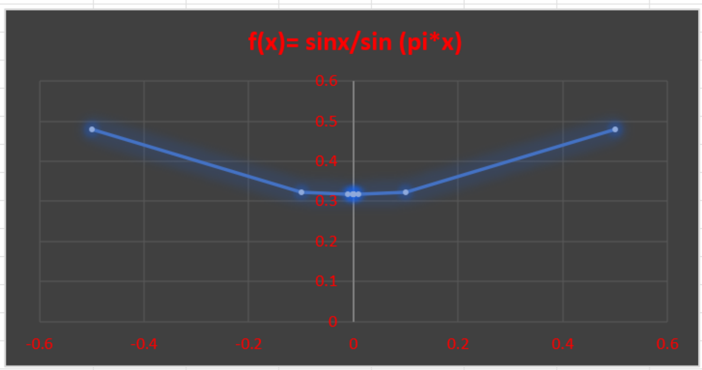
$$= \frac{\cos(x)}{\cos(\pi x).\pi}$$

$$= \frac{1}{\pi}\left[\frac{\cos(0)}{\cos\pi(0)}\right] \quad = \frac{1}{\pi} \times 1 \quad = \frac{1}{\pi} = 0.32$$

b) Check your answer in part (a) by evaluating $f(x)$ for values of x that approaches 0 in Excel

Answer of no 3b

x	f(x)= sinx/sin (pi*x)
-0.5	0.479425539
-0.1	0.323067723
-0.01	0.318356946
-0.001	0.318310357
-0.0001	0.318309891
0.0001	0.318309891
0.001	0.318310357
0.01	0.318356946
0.1	0.323067723
0.5	0.479425539



4a. Estimate the value of the limit $\lim_{x \to 0} (1+x)^{\frac{1}{x}}$ to five decimal places. Does this number look familiar?

$$\lim_{x \to 0} (1+x)^{1/x}$$

$$= \lim_{x \to 0} \left[1 + 1 + \frac{1(1-x)}{2!}x^2 + \frac{1-(1-x)(1-2x)}{3!}x^3 + \cdots\right]$$

$$= \left[1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots\right]$$

$$= e = 2.71$$

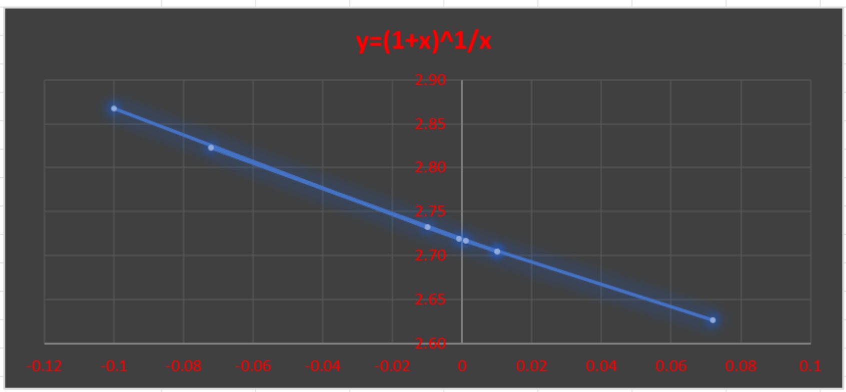
Using binomial expansion function, we get:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots\ldots\ldots\ldots$$

b. Illustrate part (a) by graphing the function $y = (1+x)1/x$ in Excel

Answer of No:4(b)

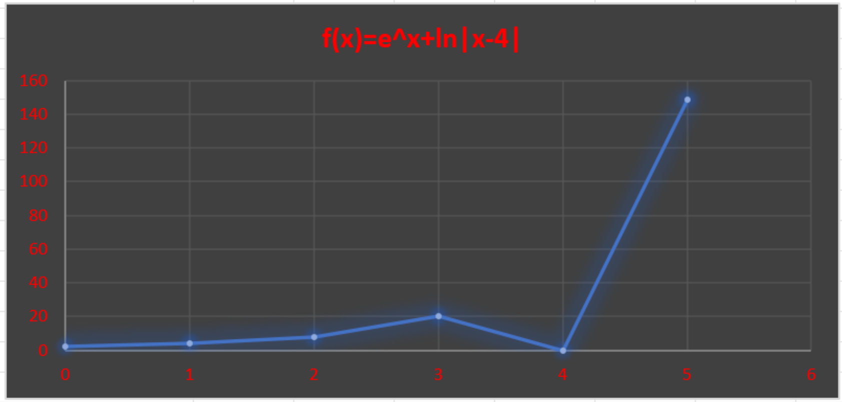
x	y=(1+x)^1/x
-0.1	2.87
-0.01	2.73
-0.001	2.72
-0.072	2.82
0.01	2.70
0.01	2.70
0.001	2.72
0.072	2.63



5. Graph the function $f(x) = ex + \ln|x - 4|$ for $0 \le x \le 5$ in Excel. Do you think the graph is an accurate representation of f?

Answer of No:5(a)

x	f(x)=e^x+ln|x-4|
0	2.386294361
1	3.816894117
2	8.082203279
3	20.08553692
4	#NUM!
5	148.4131591



We can see that the graph is undefined at x=4. The range is taken from 1 to 5. I think if more smaller values are taken, the graph would be more accurate.

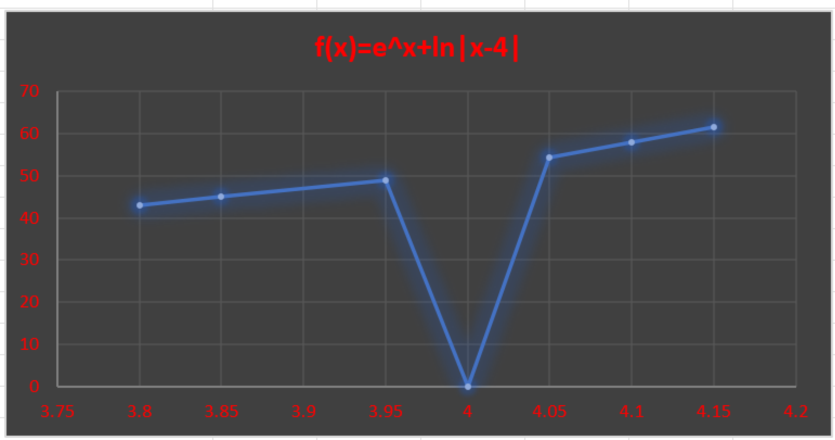
(b) How would you get a graph that represents f better?

To get a graph that represents the function better, we can specify the domain over

the range of $3.8 \le x \le 4.15$ as shown below.

Answer of no 5b

x	f(x)=e^x+ln|x-4|
3.8	43.09174658
3.85	45.09594325
3.95	48.93963456
4	#NUM!
4.05	54.40172477
4.1	58.0377025
4.15	61.53688031



6. Use numerical to find the value of the limit and verify it in Excel.

$$\lim_{x \to 1} \frac{x^3 - 1}{\sqrt{x} - 1}$$

$\Rightarrow \lim_{x \to 1} \frac{x^3-1}{\sqrt{x}-1}$

$\Rightarrow \lim_{x \to 1} \frac{(x-1)(x^2+x_1)\,(\sqrt{x}+1)}{(\sqrt{x}+1)(\sqrt{x}+1)}$

$\Rightarrow \lim_{x \to 1} \frac{(x-1)(x^2+x+1)(\sqrt{x}+1)}{(\sqrt{x})^2+1}$

$\Rightarrow \lim_{x \to 1} \frac{(x-1)(x^2+x+1)(\sqrt{x}+1)}{x-1}$

$\Rightarrow \lim_{x \to 1} (x^2 + x + 1)\,(\sqrt{x} + 1)$

$\Rightarrow (1^2 + 1 + 1)\,(1+1)$

$\Rightarrow 3 * 2$

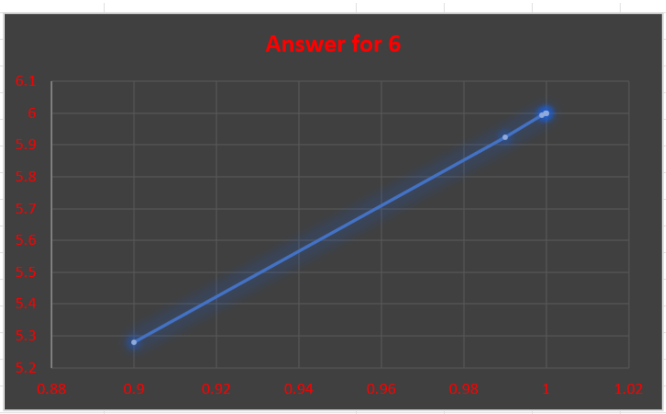
$\Rightarrow 6$

$\Rightarrow \lim_{x \to 1} \frac{x^3-1}{\sqrt{x}-1}$

$\Rightarrow 6$

b. How close to 1 does x have to be to ensure that the function in part (a) is within a distance 0.5 of its limit?

	Answer for 6
x	y
0.9	5.2809317
0.99	5.9253122
0.999	5.9925031
0.9999	5.99925
0.99999	5.999925
0.999999	5.9999925
0.932	5.5043505



From the graph we can see that x has to be approx. 0.932 to be within distance of 0.5 from the limit.